Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.4-e-x-^m-a+b-x^2-^p-c+d-x^2-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1156]. This is test number [21].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1156)	% 0. (0)
Mathematica	% 100. (1156)	% 0. (0)
Maple	% 86.68 (1002)	% 13.32 (154)
Maxima	% 24.48 (283)	% 75.52 (873)
Fricas	% 73.79 (853)	% 26.21 (303)
Sympy	% 50.52 (584)	% 49.48 (572)
Giac	% 69.2 (800)	% 30.8 (356)

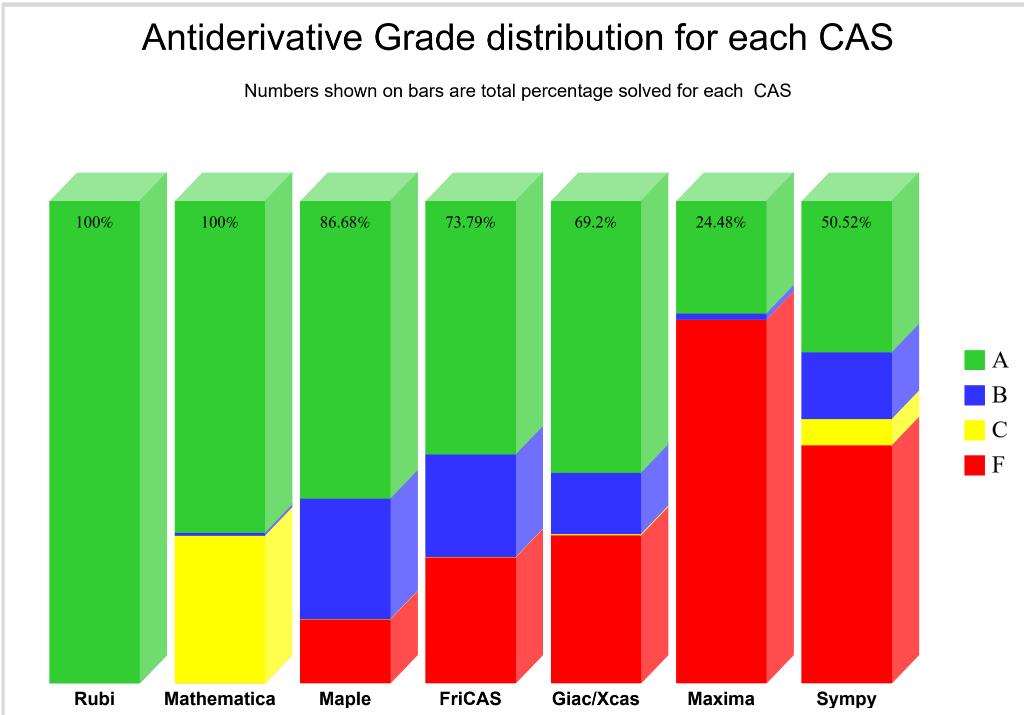
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

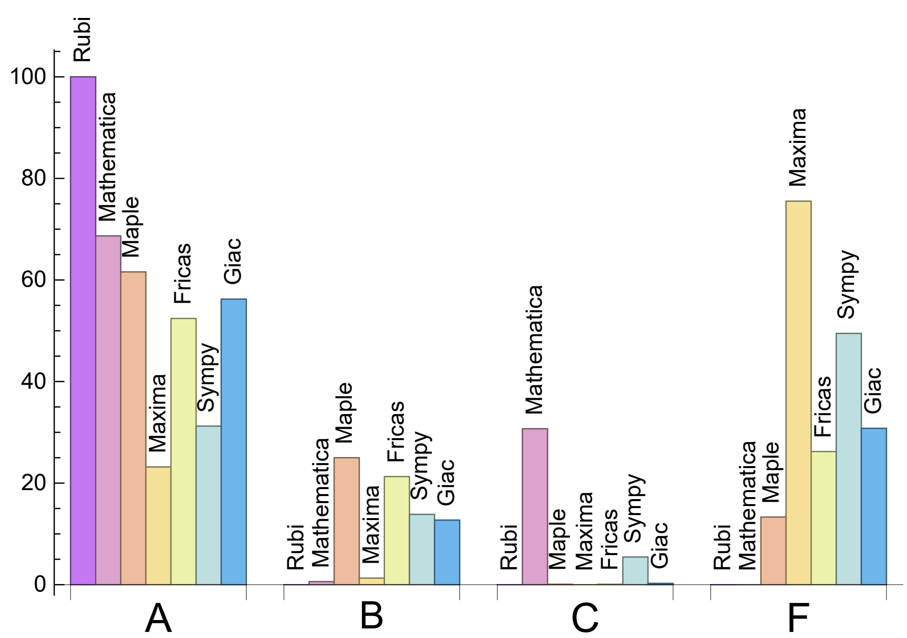
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	68.69	0.61	30.71	0.
Maple	61.59	25.	0.09	13.32
Maxima	23.18	1.3	0.	75.52
Fricas	52.42	21.28	0.09	26.21
Sympy	31.23	13.84	5.45	49.48
Giac	56.23	12.72	0.26	30.8

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	179.5	1.	124.	1.
Mathematica	0.25	134.09	0.85	100.	0.87
Maple	0.01	601.46	3.15	196.	1.31
Maxima	1.08	126.19	1.52	115.	1.38
Fricas	3.9	1038.33	6.31	512.	4.84
Sympy	17.27	277.88	2.54	133.	1.38
Giac	1.37	295.48	2.	182.	1.58

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {336, 337, 338, 341, 342, 452, 453, 454, 455, 456, 457, 458, 459, 460, 711, 717, 724, 726, 762, 769, 771, 776, 778, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1091, 1092, 1142, 1148, 1149, 1150}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

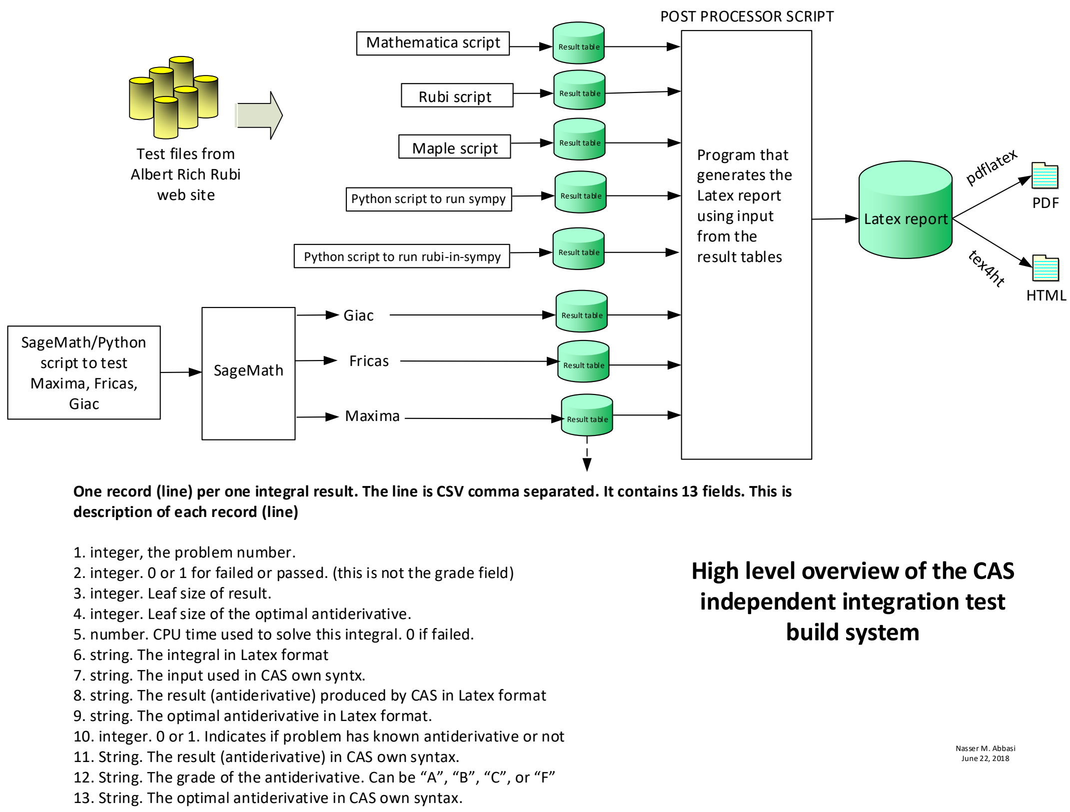
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 339, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 377, 379, 381, 383, 385, 387, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 536, 538, 539, 540, 541, 542, 543, 544, 545, 546, 552, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 581, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 595, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 654, 655, 656, 658, 660, 661, 662, 663, 664, 666, 668, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 712, 713, 714, 715, 719, 721, 722, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 744, 745, 747, 748, 749, 750, 751, 753, 754, 755, 756, 758, 759, 760, 761, 763, 764, 765, 766, 773, 775, 780, 782, 784, 934, 935, 936, 937, 938, 939, 942, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 964, 965, 966, 967, 969, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 993, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1023, 1024, 1025, 1042, 1043, 1044, 1045, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086, 1093, 1094, 1095, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131, 1139, 1140, 1141, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

B grade: { 31, 47, 108, 990, 991, 1007, 1142 }

C grade: { 331, 332, 336, 337, 338, 341, 342, 374, 376, 378, 380, 382, 384, 386, 388, 390, 424, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 516, 518, 520, 530, 531, 532, 535, 537, 547, 548, 549, 550, 551, 554, 567, 578, 580, 582, 592, 594, 596, 624, 635, 653, 657, 659, 665, 667, 669, 682, 693, 711, 716, 717, 718, 720, 723, 724, 725, 726, 727, 729, 743, 746, 752, 757, 762, 767, 768, 769, 770, 771, 772, 774, 776, 777, 778, 779, 781, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 943, 950, 951, 952, 953, 960, 961, 962, 963, 968, 975, 976, 977, 978, 979, 992, 994, 1004, 1005, 1006, 1014, 1015, 1016, 1017, 1018, 1022, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 316, 317, 318, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 536, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 551, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 820, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 881, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 965, 966, 967, 968, 975, 976, 977, 978, 979, 982, 984, 985, 986, 987, 988, 989, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1007, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade: { 29, 31, 47, 108, 118, 218, 219, 221, 227, 251, 252, 254, 256, 257, 280, 281, 283, 284, 310, 311, 312, 315, 319, 320, 321, 325, 326, 327, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 428, 429, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 533, 535, 537, 550, 552, 554, 604, 624, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 815, 816, 817, 818, 819, 821, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 969, 970, 971, 972, 973, 974, 980, 981, 983, 990, 991 }

C grade: { 1006 }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 107, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 131, 133, 134, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 241, 243, 245, 247, 249, 257, 261, 263, 265, 267, 269, 272, 274, 276, 278, 281, 283, 285, 287, 290, 292, 294, 296, 298, 300, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 590, 591, 965, 967, 1008, 1009, 1010, 1019, 1020, 1021, 1022, 1031, 1032, 1033, 1034, 1042, 1043, 1044, 1045, 1061, 1062, 1063, 1064, 1081, 1082, 1083, 1084

B grade: { 31, 47, 108, 251, 253, 255, 259, 302, 304, 306, 308, 311, 313, 315, 317 }

C grade: { }

F grade: { 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 98, 99, 100, 101, 102, 103, 104, 105, 106, 116, 128, 130, 132, 135, 137, 139, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 264, 266, 268, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 289, 291, 293, 295, 297, 299, 301, 303, 305, 307, 309, 310, 312, 314, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 188, 189, 190, 191, 192, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 282, 286, 288, 289, 290, 291, 292, 293, 295, 297, 299, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 706, 707, 708, 709, 715, 716, 731, 732, 738, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 763, 765, 766, 934, 935, 936, 938, 939, 944, 945, 946, 949, 954, 955, 956, 957, 958, 959, 965, 969, 970, 973, 974, 982, 984, 986, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1042, 1043, 1044, 1045, 1046, 1047, 1057, 1058, 1061, 1062, 1063, 1081, 1082, 1083, 1084, 1085, 1086, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade: { 31, 47, 95, 96, 108, 162, 184, 187, 193, 194, 196, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 278, 281, 283, 284, 285, 287, 294, 296, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 542, 553, 616, 625, 627, 703, 704, 705, 710, 711, 712, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 733, 734, 735, 736, 737, 739, 759, 760, 761, 762, 764, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 937, 947, 948, 967, 971, 972, 980, 981, 983, 985, 987, 988, 989, 990, 991, 1016, 1031, 1032, 1033, 1034, 1035, 1036, 1039, 1050, 1053, 1054, 1055, 1056, 1059, 1060, 1064, 1065, 1066, 1069, 1073, 1075, 1076, 1077, 1078, 1079, 1080, 1089, 1105, 1106, 1126, 1127 }

C grade: { 1074 }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 56, 57, 59, 60, 61, 63, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 194, 195, 196, 197, 198, 200, 201, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 261, 262, 263, 264, 265, 267, 268, 269, 272, 274, 276, 278, 281, 283, 285, 287, 319, 320, 321, 325, 326, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 417, 419, 420, 421, 444, 445, 446, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 518, 520, 521, 523, 525, 527, 529, 530, 538, 539, 540, 542, 544, 546, 548, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 569, 570, 571, 572, 573, 574, 575, 576, 577, 580, 582, 584, 586, 588, 590, 592, 598, 599, 600, 601, 602, 603, 607, 608, 614, 616, 618, 620, 622, 625, 627, 629, 631, 633, 635, 637, 639, 640, 641, 642, 643, 644, 645, 649, 651, 653, 661, 663, 665, 677, 679, 681, 686, 688, 690, 695, 697, 699, 705, 706, 716, 718, 725, 727, 967, 1042, 1043, 1044, 1045, 1084 }

B grade: { 29, 31, 47, 58, 62, 64, 66, 68, 70, 80, 84, 86, 101, 104, 105, 108, 116, 130, 132, 135, 137, 162, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 193, 199, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 228, 229, 230, 231, 232, 233, 235, 237, 239, 241, 242, 243, 244, 245, 246, 248, 251, 252, 253, 254, 255, 266, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 289, 290, 291, 292, 293, 295, 301, 302, 303, 304, 305, 310, 311, 313, 507, 515, 516, 517, 519, 522, 524, 526, 528, 531, 532, 533, 534, 535, 536, 537, 541, 543, 545, 547, 549, 550, 551, 552, 553, 556, 566, 567, 568, 578, 579, 581, 583, 585, 587, 589, 591, 593, 594, 595, 596, 597, 604, 605, 606, 609, 610, 611, 612, 613, 615, 617, 619, 621, 623, 624, 626, 628, 630, 632, 634, 636, 638, 646, 647 }

C grade: { 107, 322, 323, 324, 328, 331, 332, 333, 334, 339, 785, 786, 787, 788, 789, 790, 791, 794, 795, 796, 797, 798, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 818, 822, 823, 824, 825, 826, 827, 828, 834, 835, 836, 837, 840, 841, 842, 843, 844, 1093, 1094, 1095, 1100, 1101, 1102, 1105, 1106, 1112, 1113, 1117, 1122, 1123 }

F grade: { 53, 54, 55, 234, 236, 238, 240, 247, 249, 250, 256, 257, 258, 259, 260, 294, 296, 297, 298, 299, 300, 306, 307, 308, 309, 312, 314, 315, 316, 317, 318, 329, 330, 335, 336, 337, 338, 340, 341, 342, 367, 368, 370, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 418, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 554, 648, 650, 652, 654, 655, 656, 657, 658, 659, 660, 662, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 678, 680, 682, 683, 684, 685, 687, 689, 691, 692, 693, 694, 696, 698, 700, 701, 702, 703, 704, 707, 708, 709, 710, 711, 712, 713, 714, 715, 717, 719, 720, 721, 722, 723, 724, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 792, 793, 799, 807, 808, 813, 814, 815, 816, 817, 819, 820, 821, 829, 830, 831, 832, 833, 838, 839, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1096, 1097, 1098, 1099, 1103, 1104, 1107, 1108, 1109, 1110, 1111, 1114, 1115, 1116, 1118, 1119, 1120, 1121, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 302, 304, 310, 312, 313, 314, 316, 318, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 452, 453, 454, 455, 456, 457, 458, 459, 460, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 518, 520, 522, 524, 526, 527, 528, 529, 531, 533, 535, 537, 539, 541, 543, 544, 545, 546, 547, 548, 550, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 613, 615, 617, 618, 619, 620, 621, 622, 624, 626, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 683, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 709, 710, 711, 713, 714, 715, 716, 717, 718, 719, 720, 721, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 768, 770, 772, 774, 775, 777, 779, 781, 783, 934, 935, 936, 944, 945, 946, 954, 955, 956, 957, 965, 967, 969, 970, 971, 980, 981, 990, 991, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086 }

B grade: { 31, 47, 108, 129, 162, 177, 179, 229, 231, 233, 235, 237, 239, 249, 251, 257, 259, 281, 283, 296, 301, 303, 305, 307, 309, 311, 315, 317, 319, 320, 321, 325, 326, 327, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 481, 496, 497, 498, 499, 500, 501, 513, 515, 517, 519, 521, 523, 525, 530, 532, 534, 536, 538, 540, 542, 549, 551, 553, 564, 566, 568, 579, 581, 583, 595, 597, 608, 609, 610, 611, 612, 614, 616, 623, 625, 627, 634, 644, 646, 656, 658, 668, 670, 682, 684, 693, 702, 712, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 755, 757, 758, 760, 762, 764, 766, 767, 769, 771, 773, 776, 778, 780, 782, 784, 937, 947, 948, 958, 972, 973, 982, 983, 984, 985, 986, 987, 988, 989 }

C grade: { 467, 468, 469 }

F grade: { 228, 230, 232, 234, 236, 238, 240, 306, 308, 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 461, 462, 463, 464, 465, 466, 488, 489, 490, 491, 492, 493, 494, 495, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 938, 939, 940, 941, 942, 943, 949, 950, 951, 952, 953, 959, 960, 961, 962, 963, 964, 966, 968, 974, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.02	0.006	0.001	0.991	1.247	0.058	1.122

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.032	0.007	0.	0.975	1.271	0.057	1.134

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.013	0.005	0.	0.981	1.239	0.056	1.141

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	38	65	27	41
normalized size	1	1.	1.	0.97	1.31	2.24	0.93	1.41
time (sec)	N/A	0.021	0.009	0.002	1.029	1.399	0.251	1.17

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	32	61	20	31
normalized size	1	1.	1.	0.92	1.23	2.35	0.77	1.19
time (sec)	N/A	0.016	0.009	0.003	1.002	1.431	0.245	1.184

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	38	70	26	57
normalized size	1	1.	1.	0.9	1.31	2.41	0.9	1.97
time (sec)	N/A	0.02	0.011	0.005	1.012	1.477	0.321	1.142

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	35	63	26	38
normalized size	1	1.	1.04	0.96	1.35	2.42	1.	1.46
time (sec)	N/A	0.015	0.012	0.004	0.986	1.436	0.332	1.092

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	41	73	27	53
normalized size	1	1.	1.07	0.97	1.41	2.52	0.93	1.83
time (sec)	N/A	0.02	0.017	0.004	1.014	1.468	0.475	1.133

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	39	70	32	42
normalized size	1	1.	1.06	0.9	1.26	2.26	1.03	1.35
time (sec)	N/A	0.015	0.011	0.004	1.021	1.456	0.483	1.105

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	35	28	39	69	32	42
normalized size	1	1.	1.06	0.85	1.18	2.09	0.97	1.27
time (sec)	N/A	0.022	0.009	0.005	1.479	1.411	0.603	1.1

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	128	56	72
normalized size	1	1.	1.	0.95	1.25	2.33	1.02	1.31
time (sec)	N/A	0.036	0.008	0.001	1.022	1.251	0.065	1.118

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	128	53	72
normalized size	1	1.	1.21	1.24	1.64	3.05	1.26	1.71
time (sec)	N/A	0.064	0.011	0.001	1.023	1.246	0.067	1.114

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	120	53	68
normalized size	1	1.	1.	0.98	1.3	2.4	1.06	1.36
time (sec)	N/A	0.022	0.007	0.001	1.	1.28	0.065	1.116

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	70	116	49	72
normalized size	1	1.	1.19	1.19	1.63	2.7	1.14	1.67
time (sec)	N/A	0.032	0.015	0.002	0.97	1.428	0.279	1.123

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	65	116	48	65
normalized size	1	1.	1.	1.02	1.35	2.42	1.	1.35
time (sec)	N/A	0.027	0.015	0.003	0.976	1.425	0.276	1.133

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	70	122	48	95
normalized size	1	1.	0.96	0.98	1.37	2.39	0.94	1.86
time (sec)	N/A	0.046	0.023	0.006	0.987	1.47	0.362	1.621

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	68	109	49	68
normalized size	1	1.	1.04	0.96	1.42	2.27	1.02	1.42
time (sec)	N/A	0.03	0.018	0.004	0.966	1.427	0.38	1.126

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	51	73	122	49	97
normalized size	1	1.	0.98	1.	1.43	2.39	0.96	1.9
time (sec)	N/A	0.038	0.024	0.006	0.964	1.456	0.644	1.105

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	69	119	51	72
normalized size	1	1.	1.	0.94	1.44	2.48	1.06	1.5
time (sec)	N/A	0.028	0.019	0.005	0.985	1.398	0.703	1.137

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	52	74	127	53	89
normalized size	1	1.	1.06	1.02	1.45	2.49	1.04	1.75
time (sec)	N/A	0.037	0.025	0.006	0.976	1.445	1.114	1.176

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	48	72	126	56	74
normalized size	1	1.	1.06	0.91	1.36	2.38	1.06	1.4
time (sec)	N/A	0.027	0.016	0.004	1.01	1.381	1.134	1.164

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	48	72	119	56	74
normalized size	1	1.	1.15	1.	1.5	2.48	1.17	1.54
time (sec)	N/A	0.031	0.016	0.005	0.991	1.404	1.676	1.261

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	316	136	169
normalized size	1	1.	1.	1.06	1.38	2.7	1.16	1.44
time (sec)	N/A	0.153	0.017	0.001	1.073	1.232	0.084	1.337

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	320	138	169
normalized size	1	1.	1.	1.06	1.38	2.74	1.18	1.44
time (sec)	N/A	0.092	0.017	0.001	0.977	1.36	0.082	1.191

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	117	124	161	313	136	169
normalized size	1	1.	0.96	1.02	1.32	2.57	1.11	1.39
time (sec)	N/A	0.28	0.015	0.001	0.994	1.291	0.083	1.265

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	316	136	169
normalized size	1	1.	1.	1.06	1.38	2.7	1.16	1.44
time (sec)	N/A	0.07	0.014	0.	0.968	1.254	0.083	1.254

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	161	302	133	167
normalized size	1	1.	1.13	1.31	1.69	3.18	1.4	1.76
time (sec)	N/A	0.213	0.024	0.	0.993	1.28	0.084	1.124

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	313	136	169
normalized size	1	1.	1.	1.06	1.38	2.68	1.16	1.44
time (sec)	N/A	0.07	0.015	0.001	1.013	1.28	0.083	1.135

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	114	124	159	294	131	166
normalized size	1	1.	1.7	1.85	2.37	4.39	1.96	2.48
time (sec)	N/A	0.143	0.016	0.001	0.982	1.307	0.082	1.155

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	304	134	167
normalized size	1	1.	1.	1.06	1.38	2.6	1.15	1.43
time (sec)	N/A	0.066	0.015	0.001	0.985	1.313	0.082	1.154

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	161	296	133	167
normalized size	1	1.	2.55	2.95	3.83	7.05	3.17	3.98
time (sec)	N/A	0.066	0.023	0.002	0.98	1.269	0.081	1.194

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	155	286	129	163
normalized size	1	1.	1.	1.11	1.42	2.62	1.18	1.5
time (sec)	N/A	0.06	0.017	0.001	0.976	1.289	0.08	1.159

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	162	265	134	170
normalized size	1	1.	1.28	1.41	1.84	3.01	1.52	1.93
time (sec)	N/A	0.065	0.029	0.003	1.027	1.451	0.368	1.332

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	121	157	271	126	162
normalized size	1	1.	1.	1.12	1.45	2.51	1.17	1.5
time (sec)	N/A	0.062	0.029	0.003	0.981	1.394	0.368	1.155

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	162	279	131	196
normalized size	1	1.	1.02	1.09	1.43	2.47	1.16	1.73
time (sec)	N/A	0.113	0.042	0.005	1.006	1.43	0.463	1.626

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	110	118	159	263	126	165
normalized size	1	1.	1.02	1.09	1.47	2.44	1.17	1.53
time (sec)	N/A	0.061	0.034	0.007	1.002	1.414	0.481	1.135

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	124	165	271	126	201
normalized size	1	1.	1.	1.11	1.47	2.42	1.12	1.79
time (sec)	N/A	0.104	0.038	0.007	1.005	1.468	0.795	1.149

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	113	162	270	126	166
normalized size	1	1.	1.	1.02	1.46	2.43	1.14	1.5
time (sec)	N/A	0.062	0.035	0.006	1.	1.392	0.882	1.137

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	116	124	166	269	124	204
normalized size	1	1.	1.02	1.09	1.46	2.36	1.09	1.79
time (sec)	N/A	0.101	0.038	0.007	1.	1.42	1.528	1.443

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	108	162	270	126	167
normalized size	1	1.	1.	0.97	1.46	2.43	1.14	1.5
time (sec)	N/A	0.062	0.037	0.006	1.053	1.373	1.691	1.162

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	116	124	166	271	124	203
normalized size	1	1.	1.04	1.11	1.48	2.42	1.11	1.81
time (sec)	N/A	0.097	0.051	0.006	1.002	1.424	3.037	1.204

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	115	102	161	263	122	166
normalized size	1	1.	1.06	0.94	1.49	2.44	1.13	1.54
time (sec)	N/A	0.066	0.032	0.007	1.016	1.443	3.392	1.152

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	116	123	166	281	122	198
normalized size	1	1.	1.03	1.09	1.47	2.49	1.08	1.75
time (sec)	N/A	0.089	0.057	0.007	1.035	1.491	6.398	1.106

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	101	161	275	122	169
normalized size	1	1.	1.13	0.94	1.49	2.55	1.13	1.56
time (sec)	N/A	0.062	0.041	0.007	0.985	1.37	7.756	1.127

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	118	124	166	279	124	186
normalized size	1	1.	1.3	1.36	1.82	3.07	1.36	2.04
time (sec)	N/A	0.056	0.057	0.006	1.018	1.466	14.284	1.139

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	163	286	128	171
normalized size	1	1.	1.05	0.92	1.44	2.53	1.13	1.51
time (sec)	N/A	0.062	0.033	0.006	1.004	1.439	16.901	1.163

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	163	265	128	171
normalized size	1	1.	2.46	2.17	3.4	5.52	2.67	3.56
time (sec)	N/A	0.03	0.032	0.006	1.03	1.403	25.928	1.157

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	294	128	171
normalized size	1	1.	1.03	0.89	1.39	2.51	1.09	1.46
time (sec)	N/A	0.059	0.034	0.005	1.039	1.492	37.398	1.133

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	121	104	163	271	128	171
normalized size	1	1.	1.59	1.37	2.14	3.57	1.68	2.25
time (sec)	N/A	0.051	0.03	0.005	0.997	1.43	59.32	1.145

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	306	128	171
normalized size	1	1.	1.	0.89	1.39	2.62	1.09	1.46
time (sec)	N/A	0.059	0.047	0.005	1.025	1.436	86.174	1.115

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	275	128	171
normalized size	1	1.	1.03	0.89	1.39	2.35	1.09	1.46
time (sec)	N/A	0.084	0.029	0.006	0.994	1.454	111.735	1.124

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	313	128	171
normalized size	1	1.	1.	0.89	1.39	2.68	1.09	1.46
time (sec)	N/A	0.059	0.043	0.005	1.003	1.381	176.26	1.184

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	282	0	171
normalized size	1	1.	1.03	0.89	1.39	2.41	0.	1.46
time (sec)	N/A	0.083	0.031	0.007	1.028	1.398	0.	1.115

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	313	0	171
normalized size	1	1.	1.	0.89	1.39	2.68	0.	1.46
time (sec)	N/A	0.062	0.042	0.007	1.021	1.369	0.	1.119

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	290	0	171
normalized size	1	1.	1.03	0.89	1.39	2.48	0.	1.46
time (sec)	N/A	0.085	0.031	0.007	0.984	1.173	0.	1.145

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	116	0	487	173	146
normalized size	1	1.	1.	1.18	0.	4.97	1.77	1.49
time (sec)	N/A	0.061	0.071	0.003	0.	1.269	0.501	1.126

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	100	155	65	104
normalized size	1	1.	0.95	1.15	1.33	2.07	0.87	1.39
time (sec)	N/A	0.087	0.032	0.003	0.994	1.194	0.438	1.131

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	0	381	150	115
normalized size	1	1.	1.	1.19	0.	4.95	1.95	1.49
time (sec)	N/A	0.05	0.048	0.002	0.	1.261	0.479	1.138

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	108	44	70
normalized size	1	1.	0.87	1.15	1.26	2.	0.81	1.3
time (sec)	N/A	0.056	0.019	0.001	1.017	1.212	0.412	1.133

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	0	277	90	77
normalized size	1	1.	0.98	1.17	0.	4.78	1.55	1.33
time (sec)	N/A	0.036	0.046	0.003	0.	1.236	0.446	1.106

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	65	27	43
normalized size	1	1.	0.89	1.14	1.2	1.86	0.77	1.23
time (sec)	N/A	0.03	0.011	0.003	0.992	1.329	0.375	1.164

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	223	82	46
normalized size	1	1.	1.03	1.15	0.	5.72	2.1	1.18
time (sec)	N/A	0.017	0.025	0.002	0.	1.21	0.4	1.193

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	74	26	49
normalized size	1	1.	1.	1.09	1.38	2.18	0.76	1.44
time (sec)	N/A	0.034	0.012	0.004	1.005	1.217	0.655	1.147

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	48	0	228	82	49
normalized size	1	1.	0.98	1.12	0.	5.3	1.91	1.14
time (sec)	N/A	0.021	0.025	0.003	0.	1.225	0.462	1.129

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	65	111	41	96
normalized size	1	1.	0.98	1.12	1.3	2.22	0.82	1.92
time (sec)	N/A	0.048	0.02	0.006	1.009	1.188	0.791	1.141

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	72	0	296	129	77
normalized size	1	1.	1.02	1.22	0.	5.02	2.19	1.31
time (sec)	N/A	0.039	0.052	0.005	0.	1.306	0.563	1.144

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	70	81	95	158	61	135
normalized size	1	1.	1.01	1.17	1.38	2.29	0.88	1.96
time (sec)	N/A	0.06	0.028	0.006	1.022	1.18	0.986	1.148

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	96	0	398	163	109
normalized size	1	1.	0.98	1.2	0.	4.97	2.04	1.36
time (sec)	N/A	0.05	0.052	0.006	0.	1.188	0.689	1.164

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	96	107	130	211	88	170
normalized size	1	1.	1.03	1.15	1.4	2.27	0.95	1.83
time (sec)	N/A	0.082	0.036	0.007	1.024	1.228	1.164	1.356

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	120	0	506	187	143
normalized size	1	1.	1.02	1.21	0.	5.11	1.89	1.44
time (sec)	N/A	0.068	0.07	0.005	0.	1.28	0.84	1.238

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	113	146	177	365	126	215
normalized size	1	1.	0.9	1.16	1.4	2.9	1.	1.71
time (sec)	N/A	0.169	0.077	0.009	1.325	1.234	0.905	1.179

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	134	155	0	755	233	188
normalized size	1	1.	1.02	1.18	0.	5.76	1.78	1.44
time (sec)	N/A	0.15	0.103	0.009	0.	1.294	0.906	1.12

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	122	144	309	102	182
normalized size	1	1.	0.89	1.17	1.38	2.97	0.98	1.75
time (sec)	N/A	0.126	0.064	0.01	0.987	1.206	0.849	1.127

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	132	0	637	206	155
normalized size	1	1.	1.01	1.2	0.	5.79	1.87	1.41
time (sec)	N/A	0.114	0.085	0.008	0.	1.282	0.854	1.151

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	72	98	111	251	78	143
normalized size	1	1.	0.88	1.2	1.35	3.06	0.95	1.74
time (sec)	N/A	0.087	0.065	0.01	1.017	1.178	0.798	1.167

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	105	0	513	128	119
normalized size	1	1.	1.02	1.21	0.	5.9	1.47	1.37
time (sec)	N/A	0.07	0.072	0.008	0.	1.223	0.78	1.116

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	81	165	56	123
normalized size	1	1.	0.83	1.23	1.35	2.75	0.93	2.05
time (sec)	N/A	0.058	0.035	0.009	0.994	1.239	0.702	1.128

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	82	0	433	114	80
normalized size	1	1.	1.01	1.22	0.	6.46	1.7	1.19
time (sec)	N/A	0.05	0.069	0.007	0.	1.266	0.666	1.146

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	92	36	88
normalized size	1	1.	1.	1.15	1.32	2.24	0.88	2.15
time (sec)	N/A	0.036	0.012	0.008	1.035	1.225	0.508	1.235

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	381	112	77
normalized size	1	1.	1.	1.08	0.	6.05	1.78	1.22
time (sec)	N/A	0.022	0.044	0.007	0.	1.323	0.543	1.164

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	151	46	85
normalized size	1	1.	0.9	1.04	1.35	2.96	0.9	1.67
time (sec)	N/A	0.045	0.028	0.011	1.028	1.212	0.558	1.123

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	85	0	447	114	84
normalized size	1	1.	0.99	1.2	0.	6.3	1.61	1.18
time (sec)	N/A	0.051	0.032	0.008	0.	1.377	0.632	1.137

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	86	103	248	70	111
normalized size	1	1.	0.84	1.13	1.36	3.26	0.92	1.46
time (sec)	N/A	0.075	0.046	0.012	0.992	1.198	1.039	1.137

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	0	532	184	115
normalized size	1	1.	1.	1.22	0.	5.91	2.04	1.28
time (sec)	N/A	0.106	0.073	0.01	0.	1.321	0.804	1.148

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	114	143	327	100	203
normalized size	1	1.	0.88	1.18	1.47	3.37	1.03	2.09
time (sec)	N/A	0.095	0.097	0.013	1.035	1.342	1.307	1.113

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	112	136	0	653	218	151
normalized size	1	1.	0.99	1.2	0.	5.78	1.93	1.34
time (sec)	N/A	0.186	0.075	0.011	0.	1.31	1.003	1.144

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	110	143	184	385	129	240
normalized size	1	1.	0.89	1.15	1.48	3.1	1.04	1.94
time (sec)	N/A	0.13	0.098	0.014	0.981	1.416	1.585	1.621

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	136	182	223	489	163	247
normalized size	1	1.	0.91	1.21	1.49	3.26	1.09	1.65
time (sec)	N/A	0.23	0.087	0.012	1.029	1.484	1.953	1.123

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	116	158	190	431	138	215
normalized size	1	1.	0.91	1.23	1.48	3.37	1.08	1.68
time (sec)	N/A	0.166	0.074	0.011	1.027	1.246	1.798	1.108

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	134	157	360	116	178
normalized size	1	1.	0.86	1.23	1.44	3.3	1.06	1.63
time (sec)	N/A	0.121	0.065	0.011	0.994	1.249	1.657	1.12

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	109	127	289	94	126
normalized size	1	1.	1.05	1.24	1.44	3.28	1.07	1.43
time (sec)	N/A	0.09	0.037	0.009	1.021	1.164	1.445	1.15

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	80	97	184	70	82
normalized size	1	1.	0.97	1.21	1.47	2.79	1.06	1.24
time (sec)	N/A	0.066	0.024	0.009	0.996	1.298	1.1	1.136

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	57	86	42	38
normalized size	1	1.	0.94	1.22	1.78	2.69	1.31	1.19
time (sec)	N/A	0.02	0.013	0.008	0.998	1.19	0.659	1.112

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	104	250	75	103
normalized size	1	1.	0.87	1.	1.53	3.68	1.1	1.51
time (sec)	N/A	0.06	0.047	0.011	1.013	1.231	0.743	1.159

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	87	118	147	412	107	186
normalized size	1	1.	0.86	1.17	1.46	4.08	1.06	1.84
time (sec)	N/A	0.105	0.057	0.014	1.041	1.173	1.34	1.127

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	108	150	185	474	136	180
normalized size	1	1.	0.87	1.21	1.49	3.82	1.1	1.45
time (sec)	N/A	0.13	0.079	0.017	1.015	1.301	1.825	1.15

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	135	180	230	560	165	271
normalized size	1	1.	0.91	1.21	1.54	3.76	1.11	1.82
time (sec)	N/A	0.166	0.119	0.014	1.	1.282	2.423	1.141

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	158	198	0	1025	274	219
normalized size	1	1.	1.	1.25	0.	6.49	1.73	1.39
time (sec)	N/A	0.283	0.092	0.011	0.	1.348	1.715	1.189

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	133	174	0	892	250	186
normalized size	1	1.	0.96	1.26	0.	6.46	1.81	1.35
time (sec)	N/A	0.218	0.102	0.01	0.	1.32	1.609	1.672

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	147	0	763	212	150
normalized size	1	1.	0.97	1.27	0.	6.58	1.83	1.29
time (sec)	N/A	0.152	0.085	0.009	0.	1.319	1.478	1.489

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	122	0	672	194	108
normalized size	1	1.	0.97	1.3	0.	7.15	2.06	1.15
time (sec)	N/A	0.086	0.075	0.009	0.	1.263	1.216	1.161

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	89	0	624	153	105
normalized size	1	1.	0.93	1.	0.	7.01	1.72	1.18
time (sec)	N/A	0.062	0.082	0.009	0.	1.334	0.882	1.43

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	90	0	621	150	105
normalized size	1	1.	0.91	0.98	0.	6.75	1.63	1.14
time (sec)	N/A	0.032	0.064	0.008	0.	1.289	0.726	1.194

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	125	0	686	194	111
normalized size	1	1.	0.99	1.29	0.	7.07	2.	1.14
time (sec)	N/A	0.1	0.058	0.012	0.	1.297	0.879	1.186

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	116	152	0	782	226	146
normalized size	1	1.	0.99	1.3	0.	6.68	1.93	1.25
time (sec)	N/A	0.165	0.089	0.013	0.	1.326	1.185	1.486

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	139	177	0	909	260	182
normalized size	1	1.	0.98	1.25	0.	6.4	1.83	1.28
time (sec)	N/A	0.33	0.1	0.015	0.	1.349	1.648	1.159

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	16	34	26	16
normalized size	1	1.	1.	1.17	1.33	2.83	2.17	1.33
time (sec)	N/A	0.006	0.006	0.003	1.513	1.207	0.289	1.153

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	28	34	31	76	22	34
normalized size	1	1.	2.55	3.09	2.82	6.91	2.	3.09
time (sec)	N/A	0.007	0.009	0.003	0.997	1.271	0.3	1.144

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	14	19	7	15
normalized size	1	1.	0.91	1.45	1.27	1.73	0.64	1.36
time (sec)	N/A	0.003	0.004	0.005	0.972	1.232	0.08	1.141

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	18	5	9
normalized size	1	1.	1.	1.11	1.33	2.	0.56	1.
time (sec)	N/A	0.003	0.004	0.004	1.016	1.174	0.083	1.153

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	58	14	20
normalized size	1	1.	1.	0.84	1.05	3.05	0.74	1.05
time (sec)	N/A	0.004	0.007	0.005	1.487	1.282	0.099	1.128

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	59	15	20
normalized size	1	1.	1.	0.84	1.05	3.11	0.79	1.05
time (sec)	N/A	0.004	0.007	0.005	1.526	1.245	0.099	1.114

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	53	10	19
normalized size	1	1.	1.	1.07	1.36	3.79	0.71	1.36
time (sec)	N/A	0.004	0.006	0.006	1.498	1.304	0.093	1.644

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	15	19	22	8	19
normalized size	1	1.	1.17	1.25	1.58	1.83	0.67	1.58
time (sec)	N/A	0.004	0.007	0.006	0.966	1.308	0.317	1.17

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	15	19	22	8	19
normalized size	1	1.	1.17	1.25	1.58	1.83	0.67	1.58
time (sec)	N/A	0.004	0.005	0.005	1.009	1.188	0.315	1.127

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	0	225	75	49
normalized size	1	1.	1.	0.95	0.	5.77	1.92	1.26
time (sec)	N/A	0.018	0.021	0.003	0.	1.202	0.408	1.17

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	25	41	116	27	34
normalized size	1	1.	1.	0.71	1.17	3.31	0.77	0.97
time (sec)	N/A	0.007	0.009	0.007	1.505	1.164	0.124	1.15

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	30	22	35	17	15
normalized size	1	1.	1.	2.14	1.57	2.5	1.21	1.07
time (sec)	N/A	0.008	0.006	0.011	0.982	1.221	0.128	1.144

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.	0.	0.002	0.993	1.094	0.054	1.172

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	15	5	8
normalized size	1	1.	1.	0.88	1.	1.88	0.62	1.
time (sec)	N/A	0.002	0.	0.	1.018	1.26	0.067	1.177

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	15	5	8
normalized size	1	1.	1.	0.88	1.	1.88	0.62	1.
time (sec)	N/A	0.001	0.	0.	0.989	1.198	0.066	1.129

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	15	5	8
normalized size	1	1.	1.	0.88	1.	1.88	0.62	1.
time (sec)	N/A	0.002	0.	0.	0.994	1.187	0.065	1.144

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	7	2	4
normalized size	1	1.	1.	1.33	1.33	2.33	0.67	1.33
time (sec)	N/A	0.001	0.	0.	0.988	1.248	0.062	1.123

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	9	14	3	7
normalized size	1	1.	1.	1.25	2.25	3.5	0.75	1.75
time (sec)	N/A	0.001	0.	0.002	0.998	1.224	0.068	1.144

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	3	8
normalized size	1	1.	1.	1.17	1.33	1.33	0.5	1.33
time (sec)	N/A	0.001	0.	0.	0.995	1.191	0.068	1.115

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	16	7	8
normalized size	1	1.	1.	0.88	1.	2.	0.88	1.
time (sec)	N/A	0.001	0.	0.001	0.997	1.197	0.07	1.163

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	54	22	63
normalized size	1	1.	1.	0.9	1.17	1.86	0.76	2.17
time (sec)	N/A	0.021	0.004	0.003	1.006	1.196	0.291	1.655

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	176	58	38
normalized size	1	1.	1.	0.88	0.	5.33	1.76	1.15
time (sec)	N/A	0.015	0.008	0.003	0.	1.304	0.298	1.143

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	32	12	85
normalized size	1	1.	1.	0.94	1.19	2.	0.75	5.31
time (sec)	N/A	0.005	0.002	0.001	0.997	1.242	0.111	1.129

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	0	157	54	22
normalized size	1	1.	1.	0.68	0.	6.28	2.16	0.88
time (sec)	N/A	0.008	0.004	0.003	0.	1.25	0.132	1.188

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	34	54	17	35
normalized size	1	1.	1.	0.96	1.42	2.25	0.71	1.46
time (sec)	N/A	0.013	0.005	0.003	1.	1.223	0.19	1.157

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	0	181	66	42
normalized size	1	1.	1.	0.89	0.	5.03	1.83	1.17
time (sec)	N/A	0.015	0.013	0.003	0.	1.22	0.325	1.169

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	49	88	32	63
normalized size	1	1.	0.97	0.92	1.29	2.32	0.84	1.66
time (sec)	N/A	0.024	0.007	0.007	1.038	1.253	0.409	1.181

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	32	46	84	31	43
normalized size	1	1.	0.8	0.91	1.31	2.4	0.89	1.23
time (sec)	N/A	0.027	0.009	0.007	0.998	1.214	0.34	1.23

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	0	279	80	50
normalized size	1	1.	1.	0.81	0.	5.94	1.7	1.06
time (sec)	N/A	0.015	0.02	0.006	0.	1.214	0.354	1.167

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	22	32	15	20
normalized size	1	1.	1.	0.94	1.29	1.88	0.88	1.18
time (sec)	N/A	0.004	0.002	0.001	0.983	1.225	0.306	1.68

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	0	277	80	50
normalized size	1	1.	1.	0.81	0.	5.89	1.7	1.06
time (sec)	N/A	0.013	0.023	0.003	0.	1.22	0.361	1.212

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	38	54	120	36	69
normalized size	1	1.	0.83	0.93	1.32	2.93	0.88	1.68
time (sec)	N/A	0.03	0.014	0.01	0.979	1.27	0.433	1.127

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	49	0	309	92	68
normalized size	1	1.	0.93	0.82	0.	5.15	1.53	1.13
time (sec)	N/A	0.022	0.036	0.007	0.	1.27	0.446	1.171

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	50	77	173	51	76
normalized size	1	1.	0.79	0.94	1.45	3.26	0.96	1.43
time (sec)	N/A	0.041	0.035	0.011	1.02	1.247	0.529	1.159

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	131	56	72
normalized size	1	1.	1.	0.95	1.25	2.38	1.02	1.31
time (sec)	N/A	0.035	0.009	0.001	1.011	1.157	0.068	1.146

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	131	53	72
normalized size	1	1.	1.	0.95	1.25	2.38	0.96	1.31
time (sec)	N/A	0.061	0.008	0.	1.015	1.	0.067	1.135

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	128	56	72
normalized size	1	1.	1.	0.95	1.25	2.33	1.02	1.31
time (sec)	N/A	0.03	0.007	0.001	1.002	1.132	0.067	1.188

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	128	53	72
normalized size	1	1.	1.21	1.24	1.64	3.05	1.26	1.71
time (sec)	N/A	0.059	0.012	0.001	1.	1.039	0.067	1.129

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	120	53	68
normalized size	1	1.	1.	0.98	1.3	2.4	1.06	1.36
time (sec)	N/A	0.025	0.007	0.	1.008	1.073	0.067	1.147

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	70	116	49	72
normalized size	1	1.	1.19	1.19	1.63	2.7	1.14	1.67
time (sec)	N/A	0.031	0.014	0.002	1.006	1.249	0.282	1.199

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	65	116	48	65
normalized size	1	1.	1.	1.02	1.35	2.42	1.	1.35
time (sec)	N/A	0.026	0.016	0.003	1.004	1.263	0.281	1.161

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	70	122	48	95
normalized size	1	1.	0.96	0.98	1.37	2.39	0.94	1.86
time (sec)	N/A	0.042	0.022	0.005	0.981	1.238	0.369	1.167

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	68	109	49	68
normalized size	1	1.	1.04	0.96	1.42	2.27	1.02	1.42
time (sec)	N/A	0.028	0.019	0.004	1.071	1.117	0.381	1.153

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	115	225	100	127
normalized size	1	1.	1.	1.03	1.32	2.59	1.15	1.46
time (sec)	N/A	0.062	0.015	0.001	0.998	1.088	0.079	1.135

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	115	223	92	127
normalized size	1	1.	0.93	1.03	1.32	2.56	1.06	1.46
time (sec)	N/A	0.106	0.025	0.	0.99	1.149	0.076	1.181

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	115	220	100	127
normalized size	1	1.	1.	1.03	1.32	2.53	1.15	1.46
time (sec)	N/A	0.053	0.018	0.	1.009	1.027	0.077	1.147

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	81	90	115	220	94	127
normalized size	1	1.	1.14	1.27	1.62	3.1	1.32	1.79
time (sec)	N/A	0.105	0.022	0.001	1.014	1.014	0.076	1.143

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	209	97	123
normalized size	1	1.	1.	1.06	1.35	2.55	1.18	1.5
time (sec)	N/A	0.039	0.016	0.	1.215	1.147	0.075	1.132

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	90	115	178	85	124
normalized size	1	1.	1.	1.12	1.44	2.22	1.06	1.55
time (sec)	N/A	0.075	0.024	0.002	0.976	1.282	0.327	1.179

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	91	112	189	92	122
normalized size	1	1.	1.	1.12	1.38	2.33	1.14	1.51
time (sec)	N/A	0.043	0.038	0.003	1.005	1.28	0.315	1.329

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	93	115	188	87	154
normalized size	1	1.	0.99	1.11	1.37	2.24	1.04	1.83
time (sec)	N/A	0.078	0.041	0.007	0.996	1.269	0.412	1.194

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	81	113	185	90	119
normalized size	1	1.	1.	1.01	1.41	2.31	1.12	1.49
time (sec)	N/A	0.053	0.041	0.005	1.	1.176	0.427	1.153

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	323	143	182
normalized size	1	1.	1.	1.01	1.35	2.54	1.13	1.43
time (sec)	N/A	0.088	0.027	0.001	1.033	1.125	0.086	1.148

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	119	128	171	319	138	182
normalized size	1	1.	1.12	1.21	1.61	3.01	1.3	1.72
time (sec)	N/A	0.225	0.032	0.	1.007	1.138	0.084	1.138

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	315	143	182
normalized size	1	1.	1.	1.01	1.35	2.48	1.13	1.43
time (sec)	N/A	0.072	0.021	0.	0.991	1.042	0.084	1.169

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	119	128	171	308	136	181
normalized size	1	1.	1.68	1.8	2.41	4.34	1.92	2.55
time (sec)	N/A	0.118	0.028	0.001	1.006	1.054	0.083	1.165

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	296	136	177
normalized size	1	1.	1.	1.02	1.37	2.43	1.11	1.45
time (sec)	N/A	0.054	0.021	0.	0.99	1.084	0.081	1.121

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	123	132	173	275	133	181
normalized size	1	1.	1.	1.07	1.41	2.24	1.08	1.47
time (sec)	N/A	0.104	0.03	0.002	0.978	1.201	0.363	1.133

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	131	167	279	131	176
normalized size	1	1.	1.	1.09	1.39	2.32	1.09	1.47
time (sec)	N/A	0.056	0.038	0.003	0.991	1.218	0.361	1.123

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	120	134	173	282	133	216
normalized size	1	1.	0.98	1.09	1.41	2.29	1.08	1.76
time (sec)	N/A	0.101	0.048	0.007	0.989	1.256	0.453	1.157

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	124	170	281	129	174
normalized size	1	1.	1.	1.03	1.42	2.34	1.08	1.45
time (sec)	N/A	0.06	0.042	0.005	0.984	1.283	0.472	1.149

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	176	0	649	240	207
normalized size	1	1.	1.	1.69	0.	6.24	2.31	1.99
time (sec)	N/A	0.074	0.093	0.003	0.	1.305	0.658	1.201

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	82	124	135	212	83	144
normalized size	1	1.	1.04	1.57	1.71	2.68	1.05	1.82
time (sec)	N/A	0.084	0.041	0.003	0.98	1.164	0.554	1.169

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	135	0	489	192	153
normalized size	1	1.	1.	1.63	0.	5.89	2.31	1.84
time (sec)	N/A	0.064	0.069	0.003	0.	1.281	0.615	1.155

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	85	88	140	51	90
normalized size	1	1.	0.8	1.39	1.44	2.3	0.84	1.48
time (sec)	N/A	0.049	0.023	0.003	0.993	1.266	0.481	1.172

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	390	172	97
normalized size	1	1.	0.94	1.51	0.	6.19	2.73	1.54
time (sec)	N/A	0.039	0.048	0.	0.	1.229	0.541	1.148

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	69	82	128	41	84
normalized size	1	1.	0.98	1.35	1.61	2.51	0.8	1.65
time (sec)	N/A	0.046	0.022	0.005	0.996	1.336	1.298	1.154

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	85	0	344	165	85
normalized size	1	1.	1.	1.55	0.	6.25	3.	1.55
time (sec)	N/A	0.049	0.045	0.006	0.	1.321	0.67	1.179

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	81	95	159	49	123
normalized size	1	1.	1.03	1.4	1.64	2.74	0.84	2.12
time (sec)	N/A	0.058	0.028	0.005	0.986	1.338	1.426	1.14

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	98	0	408	172	96
normalized size	1	1.	0.97	1.48	0.	6.18	2.61	1.45
time (sec)	N/A	0.055	0.052	0.006	0.	1.314	0.783	1.156

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	116	130	213	66	188
normalized size	1	1.	0.96	1.55	1.73	2.84	0.88	2.51
time (sec)	N/A	0.067	0.044	0.006	1.009	1.264	1.352	1.154

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	143	0	506	207	151
normalized size	1	1.	0.99	1.64	0.	5.82	2.38	1.74
time (sec)	N/A	0.065	0.07	0.006	0.	1.262	0.95	1.184

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	108	160	181	290	105	248
normalized size	1	1.	1.1	1.63	1.85	2.96	1.07	2.53
time (sec)	N/A	0.083	0.061	0.006	0.983	1.281	1.623	1.15

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	138	196	0	851	280	211
normalized size	1	1.	0.95	1.35	0.	5.87	1.93	1.46
time (sec)	N/A	0.135	0.092	0.01	0.	1.307	1.166	1.166

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	87	142	144	327	97	220
normalized size	1	1.	0.97	1.58	1.6	3.63	1.08	2.44
time (sec)	N/A	0.098	0.063	0.01	0.994	1.378	1.11	1.153

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	105	156	0	713	245	154
normalized size	1	1.	0.89	1.32	0.	6.04	2.08	1.31
time (sec)	N/A	0.11	0.071	0.008	0.	1.501	1.008	1.179

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	97	100	200	68	149
normalized size	1	1.	0.9	1.56	1.61	3.23	1.1	2.4
time (sec)	N/A	0.058	0.045	0.008	0.988	1.41	0.912	1.169

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	129	0	612	236	128
normalized size	1	1.	1.09	1.57	0.	7.46	2.88	1.56
time (sec)	N/A	0.107	0.058	0.	0.	1.517	0.85	1.181

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	70	94	116	228	80	134
normalized size	1	1.	1.04	1.4	1.73	3.4	1.19	2.
time (sec)	N/A	0.064	0.044	0.012	1.01	1.46	1.334	1.121

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	131	0	625	238	138
normalized size	1	1.	0.86	1.24	0.	5.9	2.25	1.3
time (sec)	N/A	0.076	0.06	0.009	0.	1.554	1.014	1.16

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	114	135	316	92	147
normalized size	1	1.	0.89	1.41	1.67	3.9	1.14	1.81
time (sec)	N/A	0.079	0.093	0.014	0.982	1.467	1.529	1.176

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	161	0	737	248	150
normalized size	1	1.	0.85	1.28	0.	5.85	1.97	1.19
time (sec)	N/A	0.135	0.064	0.011	0.	1.484	1.179	1.127

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	148	223	0	1107	238	208
normalized size	1	1.	0.91	1.37	0.	6.79	1.46	1.28
time (sec)	N/A	0.158	0.089	0.011	0.	1.508	2.252	1.134

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	114	155	162	365	122	144
normalized size	1	1.	1.15	1.57	1.64	3.69	1.23	1.45
time (sec)	N/A	0.1	0.05	0.011	0.982	1.473	2.655	1.202

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	130	196	0	963	223	180
normalized size	1	1.	1.02	1.54	0.	7.58	1.76	1.42
time (sec)	N/A	0.125	0.1	0.008	0.	1.529	1.768	1.197

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	75	105	117	216	87	103
normalized size	1	1.	1.12	1.57	1.75	3.22	1.3	1.54
time (sec)	N/A	0.068	0.024	0.009	1.019	1.456	1.661	1.185

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	121	147	0	921	223	170
normalized size	1	1.	1.04	1.27	0.	7.94	1.92	1.47
time (sec)	N/A	0.077	0.095	0.	0.	1.543	1.191	1.176

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	103	112	147	324	107	149
normalized size	1	1.	1.2	1.3	1.71	3.77	1.24	1.73
time (sec)	N/A	0.083	0.047	0.011	1.002	1.46	1.485	1.15

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	151	133	199	0	976	224	182
normalized size	1	0.99	0.88	1.31	0.	6.42	1.47	1.2
time (sec)	N/A	0.109	0.088	0.012	0.	1.538	1.492	1.167

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	99	149	192	524	139	239
normalized size	1	1.	0.93	1.41	1.81	4.94	1.31	2.25
time (sec)	N/A	0.114	0.089	0.014	1.006	1.481	2.194	1.188

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	148	227	0	1131	240	204
normalized size	1	1.	0.92	1.41	0.	7.02	1.49	1.27
time (sec)	N/A	0.191	0.072	0.014	0.	1.558	1.793	1.186

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	100	155	65	104
normalized size	1	1.	0.95	1.15	1.33	2.07	0.87	1.39
time (sec)	N/A	0.085	0.03	0.004	0.972	1.449	0.442	1.163

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	0	381	150	113
normalized size	1	1.	1.	1.19	0.	4.95	1.95	1.47
time (sec)	N/A	0.052	0.051	0.003	0.	1.484	0.473	1.178

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	108	44	70
normalized size	1	1.	0.87	1.15	1.26	2.	0.81	1.3
time (sec)	N/A	0.056	0.018	0.002	0.99	1.442	0.412	1.167

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	0	277	90	78
normalized size	1	1.	0.98	1.17	0.	4.78	1.55	1.34
time (sec)	N/A	0.034	0.041	0.003	0.	1.539	0.445	1.142

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	65	27	43
normalized size	1	1.	0.89	1.14	1.2	1.86	0.77	1.23
time (sec)	N/A	0.031	0.011	0.003	0.987	1.428	0.378	1.165

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	223	82	45
normalized size	1	1.	1.03	1.15	0.	5.72	2.1	1.15
time (sec)	N/A	0.015	0.026	0.003	0.	1.502	0.408	1.136

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	74	26	49
normalized size	1	1.	1.	1.09	1.38	2.18	0.76	1.44
time (sec)	N/A	0.031	0.012	0.003	0.998	1.465	0.651	1.153

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	48	0	230	82	50
normalized size	1	1.	0.98	1.12	0.	5.35	1.91	1.16
time (sec)	N/A	0.02	0.025	0.005	0.	1.502	0.459	1.152

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	65	109	41	97
normalized size	1	1.	0.98	1.12	1.3	2.18	0.82	1.94
time (sec)	N/A	0.045	0.02	0.005	0.976	1.472	0.782	1.145

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	72	0	294	129	77
normalized size	1	1.	1.02	1.22	0.	4.98	2.19	1.31
time (sec)	N/A	0.036	0.051	0.005	0.	1.516	0.561	1.205

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	116	165	185	285	119	200
normalized size	1	1.	1.13	1.6	1.8	2.77	1.16	1.94
time (sec)	N/A	0.122	0.052	0.003	0.999	1.387	0.57	1.206

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	176	0	649	240	207
normalized size	1	1.	1.	1.68	0.	6.18	2.29	1.97
time (sec)	N/A	0.069	0.095	0.003	0.	1.69	0.645	1.158

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	124	136	212	83	144
normalized size	1	1.	1.02	1.55	1.7	2.65	1.04	1.8
time (sec)	N/A	0.086	0.039	0.004	0.982	1.65	0.536	1.147

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	135	0	489	192	153
normalized size	1	1.	1.	1.61	0.	5.82	2.29	1.82
time (sec)	N/A	0.064	0.075	0.003	0.	1.724	0.599	1.149

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	85	89	140	51	90
normalized size	1	1.	0.8	1.39	1.46	2.3	0.84	1.48
time (sec)	N/A	0.049	0.023	0.002	0.988	1.427	0.504	1.126

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	390	172	97
normalized size	1	1.	0.94	1.51	0.	6.19	2.73	1.54
time (sec)	N/A	0.042	0.052	0.	0.	1.502	0.545	1.173

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	69	82	128	41	84
normalized size	1	1.	0.98	1.35	1.61	2.51	0.8	1.65
time (sec)	N/A	0.048	0.022	0.005	1.031	1.499	1.345	1.135

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	85	0	344	165	85
normalized size	1	1.	1.	1.55	0.	6.25	3.	1.55
time (sec)	N/A	0.05	0.051	0.005	0.	1.462	0.691	1.157

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	81	93	159	49	122
normalized size	1	1.	1.03	1.4	1.6	2.74	0.84	2.1
time (sec)	N/A	0.058	0.028	0.006	1.016	1.516	1.516	1.176

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	98	0	408	172	97
normalized size	1	1.	1.03	1.53	0.	6.38	2.69	1.52
time (sec)	N/A	0.055	0.062	0.006	0.	1.504	0.787	1.185

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	128	263	296	447	187	321
normalized size	1	1.	0.93	1.91	2.14	3.24	1.36	2.33
time (sec)	N/A	0.178	0.077	0.005	1.042	1.451	0.729	1.158

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	276	0	964	338	325
normalized size	1	1.	1.	1.97	0.	6.89	2.41	2.32
time (sec)	N/A	0.097	0.048	0.003	0.	1.496	0.815	1.205

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	125	205	227	342	136	243
normalized size	1	1.	1.09	1.78	1.97	2.97	1.18	2.11
time (sec)	N/A	0.125	0.056	0.003	1.024	1.4	0.673	1.149

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	218	0	757	275	248
normalized size	1	1.	0.99	1.83	0.	6.36	2.31	2.08
time (sec)	N/A	0.085	0.039	0.003	0.	1.559	0.746	1.228

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	149	161	244	88	167
normalized size	1	1.	0.94	1.71	1.85	2.8	1.01	1.92
time (sec)	N/A	0.081	0.03	0.003	0.985	1.444	0.625	1.166

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	161	0	613	240	174
normalized size	1	1.	0.94	1.64	0.	6.26	2.45	1.78
time (sec)	N/A	0.056	0.066	0.	0.	1.506	0.72	1.177

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	116	132	209	63	134
normalized size	1	1.	0.89	1.59	1.81	2.86	0.86	1.84
time (sec)	N/A	0.077	0.03	0.004	1.02	1.526	1.912	1.157

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	135	0	520	221	140
normalized size	1	1.	0.99	1.75	0.	6.75	2.87	1.82
time (sec)	N/A	0.064	0.032	0.005	0.	1.513	0.931	1.133

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	114	131	217	63	162
normalized size	1	1.	1.03	1.56	1.79	2.97	0.86	2.22
time (sec)	N/A	0.076	0.037	0.007	0.993	1.516	2.381	1.146

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	135	0	531	221	135
normalized size	1	1.	1.	1.82	0.	7.18	2.99	1.82
time (sec)	N/A	0.066	0.039	0.007	0.	1.452	1.238	1.15

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	92	142	201	0
normalized size	1	1.	0.94	0.93	1.31	2.03	2.87	0.
time (sec)	N/A	0.067	0.03	0.007	0.985	1.572	3.121	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	73	0	810	921	576
normalized size	1	1.	0.95	0.94	0.	10.38	11.81	7.38
time (sec)	N/A	0.083	0.09	0.007	0.	1.645	4.579	1.301

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	66	92	144	0
normalized size	1	1.	0.81	0.94	1.25	1.74	2.72	0.
time (sec)	N/A	0.049	0.02	0.006	1.023	1.543	1.744	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	630	570	176
normalized size	1	1.	0.87	0.79	0.	9.	8.14	2.51
time (sec)	N/A	0.034	0.042	0.006	0.	1.59	2.113	1.178

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	55	69	138	0
normalized size	1	1.	0.69	0.93	1.22	1.53	3.07	0.
time (sec)	N/A	0.026	0.016	0.006	1.067	1.55	0.879	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	608	712	257
normalized size	1	1.	0.87	0.79	0.	8.69	10.17	3.67
time (sec)	N/A	0.027	0.042	0.001	0.	1.64	2.234	1.205

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	82	123	0	0
normalized size	1	1.	0.87	0.95	1.32	1.98	0.	0.
time (sec)	N/A	0.061	0.027	0.008	1.073	1.861	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	76	0	819	1093	520
normalized size	1	1.	0.94	0.94	0.	10.11	13.49	6.42
time (sec)	N/A	0.086	0.083	0.008	0.	1.654	4.769	1.65

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	117	200	0	0
normalized size	1	1.	1.01	1.	1.34	2.3	0.	0.
time (sec)	N/A	0.091	0.039	0.01	1.02	2.853	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	98	0	1131	1353	728
normalized size	1	1.	1.01	0.98	0.	11.31	13.53	7.28
time (sec)	N/A	0.177	0.122	0.011	0.	1.75	9.505	1.286

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	124	158	255	0	0
normalized size	1	1.	1.	1.04	1.33	2.14	0.	0.
time (sec)	N/A	0.127	0.054	0.013	1.1	7.426	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	135	141	0	1358	1504	890
normalized size	1	1.	1.01	1.05	0.	10.13	11.22	6.64
time (sec)	N/A	0.227	0.123	0.012	0.	1.999	21.341	1.305

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	147	184	223	311	0	0
normalized size	1	1.	0.95	1.19	1.44	2.01	0.	0.
time (sec)	N/A	0.169	0.064	0.015	1.04	10.595	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	91	136	176	321	348	205
normalized size	1	1.	0.98	1.46	1.89	3.45	3.74	2.2
time (sec)	N/A	0.087	0.046	0.012	1.082	1.734	5.721	1.166

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	144	0	1472	1850	163
normalized size	1	1.	1.	1.33	0.	13.63	17.13	1.51
time (sec)	N/A	0.085	0.129	0.009	0.	2.043	13.895	1.213

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	95	142	243	253	123
normalized size	1	1.	1.	1.28	1.92	3.28	3.42	1.66
time (sec)	N/A	0.064	0.034	0.01	1.058	1.549	2.378	1.178

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	90	134	0	1485	1530	149
normalized size	1	1.	0.87	1.29	0.	14.28	14.71	1.43
time (sec)	N/A	0.063	0.131	0.009	0.	2.125	7.072	1.161

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	90	134	217	248	115
normalized size	1	1.	0.94	1.29	1.91	3.1	3.54	1.64
time (sec)	N/A	0.054	0.028	0.01	1.053	1.585	2.182	1.155

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1451	2033	165
normalized size	1	1.	0.87	1.32	0.	13.31	18.65	1.51
time (sec)	N/A	0.079	0.163	0.009	0.	2.233	15.121	1.165

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	139	186	444	0	250
normalized size	1	1.	0.98	1.39	1.86	4.44	0.	2.5
time (sec)	N/A	0.101	0.095	0.014	1.192	6.303	0.	1.211

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	123	169	0	2049	2526	221
normalized size	1	1.	0.85	1.17	0.	14.23	17.54	1.53
time (sec)	N/A	0.198	0.249	0.013	0.	3.69	71.003	1.135

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	117	170	254	601	0	347
normalized size	1	1.	0.93	1.35	2.02	4.77	0.	2.75
time (sec)	N/A	0.144	0.237	0.018	1.175	14.249	0.	1.158

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	142	191	0	2558	0	223
normalized size	1	1.	0.75	1.01	0.	13.53	0.	1.18
time (sec)	N/A	0.272	0.372	0.016	0.	7.904	0.	1.171

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	99	218	319	571	418	313
normalized size	1	1.	0.85	1.88	2.75	4.92	3.6	2.7
time (sec)	N/A	0.113	0.1	0.011	1.244	1.603	4.606	1.191

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	154	299	0	3168	3390	275
normalized size	1	1.	0.98	1.9	0.	20.18	21.59	1.75
time (sec)	N/A	0.172	0.248	0.009	0.	3.636	65.07	1.145

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	77	177	293	513	410	235
normalized size	1	1.	0.77	1.77	2.93	5.13	4.1	2.35
time (sec)	N/A	0.092	0.114	0.01	1.091	1.651	4.313	1.179

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	151	298	0	3168	3386	278
normalized size	1	1.	0.97	1.92	0.	20.44	21.85	1.79
time (sec)	N/A	0.14	0.228	0.012	0.	3.832	58.704	1.188

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	176	285	506	391	235
normalized size	1	1.	1.	1.8	2.91	5.16	3.99	2.4
time (sec)	N/A	0.075	0.047	0.01	1.038	1.567	4.262	1.157

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	158	310	0	3217	0	293
normalized size	1	1.	0.99	1.94	0.	20.11	0.	1.83
time (sec)	N/A	0.189	0.231	0.	0.	6.451	0.	1.147

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	141	286	375	1027	0	425
normalized size	1	1.	0.95	1.92	2.52	6.89	0.	2.85
time (sec)	N/A	0.152	0.28	0.016	1.193	23.227	0.	1.133

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	172	335	0	4054	0	319
normalized size	1	1.	0.82	1.59	0.	19.21	0.	1.51
time (sec)	N/A	0.309	0.389	0.017	0.	12.578	0.	1.172

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	171	322	491	1257	0	482
normalized size	1	1.	0.96	1.81	2.76	7.06	0.	2.71
time (sec)	N/A	0.213	0.443	0.018	1.137	52.7	0.	1.184

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	196	362	0	4841	0	346
normalized size	1	1.	0.73	1.34	0.	17.93	0.	1.28
time (sec)	N/A	0.434	0.443	0.017	0.	32.683	0.	1.156

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	51	15	23
normalized size	1	1.	1.	0.86	1.1	2.43	0.71	1.1
time (sec)	N/A	0.013	0.005	0.005	1.296	1.465	0.111	1.148

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	105	0	513	128	119
normalized size	1	1.	1.02	1.21	0.	5.9	1.47	1.37
time (sec)	N/A	0.071	0.075	0.008	0.	1.558	0.767	1.113

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	80	165	56	122
normalized size	1	1.	0.83	1.23	1.33	2.75	0.93	2.03
time (sec)	N/A	0.06	0.035	0.008	1.464	1.54	0.69	1.133

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	82	0	433	114	78
normalized size	1	1.	1.01	1.22	0.	6.46	1.7	1.16
time (sec)	N/A	0.051	0.068	0.009	0.	1.518	0.647	1.174

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	93	36	88
normalized size	1	1.	1.	1.15	1.32	2.27	0.88	2.15
time (sec)	N/A	0.036	0.012	0.008	1.038	1.517	0.496	1.117

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	381	112	77
normalized size	1	1.	1.	1.08	0.	6.05	1.78	1.22
time (sec)	N/A	0.022	0.044	0.	0.	1.544	0.53	1.195

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	150	46	85
normalized size	1	1.	0.9	1.04	1.35	2.94	0.9	1.67
time (sec)	N/A	0.044	0.028	0.012	1.112	1.499	0.55	1.124

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	85	0	447	114	86
normalized size	1	1.	0.99	1.2	0.	6.3	1.61	1.21
time (sec)	N/A	0.053	0.033	0.01	0.	1.565	0.627	1.158

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	86	105	248	70	113
normalized size	1	1.	0.84	1.13	1.38	3.26	0.92	1.49
time (sec)	N/A	0.074	0.046	0.013	1.092	1.555	1.001	1.164

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	0	529	184	116
normalized size	1	1.	1.	1.22	0.	5.88	2.04	1.29
time (sec)	N/A	0.106	0.07	0.011	0.	1.52	0.795	1.162

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	138	196	0	851	280	211
normalized size	1	1.	0.95	1.35	0.	5.87	1.93	1.46
time (sec)	N/A	0.132	0.089	0.009	0.	1.643	1.126	1.146

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	87	142	144	327	97	220
normalized size	1	1.	0.99	1.61	1.64	3.72	1.1	2.5
time (sec)	N/A	0.106	0.06	0.01	1.163	1.483	1.097	1.143

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	105	156	0	713	245	154
normalized size	1	1.	0.91	1.34	0.	6.15	2.11	1.33
time (sec)	N/A	0.111	0.074	0.008	0.	1.518	1.021	1.122

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	97	99	200	68	150
normalized size	1	1.	0.92	1.59	1.62	3.28	1.11	2.46
time (sec)	N/A	0.058	0.047	0.01	1.065	1.44	0.906	1.137

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	129	0	612	236	127
normalized size	1	1.	1.07	1.57	0.	7.46	2.88	1.55
time (sec)	N/A	0.099	0.062	0.009	0.	1.478	0.889	1.124

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	70	94	116	228	80	134
normalized size	1	1.	1.04	1.4	1.73	3.4	1.19	2.
time (sec)	N/A	0.065	0.043	0.011	1.049	1.554	1.379	1.159

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	91	131	0	625	238	139
normalized size	1	1.	0.88	1.27	0.	6.07	2.31	1.35
time (sec)	N/A	0.078	0.069	0.01	0.	1.593	1.017	1.156

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	114	135	316	92	147
normalized size	1	1.	0.9	1.42	1.69	3.95	1.15	1.84
time (sec)	N/A	0.081	0.097	0.014	1.179	1.528	1.611	1.122

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	125	107	161	0	737	248	151
normalized size	1	0.98	0.84	1.27	0.	5.8	1.95	1.19
time (sec)	N/A	0.143	0.07	0.011	0.	1.564	1.187	1.163

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	151	302	0	1196	382	325
normalized size	1	1.	0.89	1.79	0.	7.08	2.26	1.92
time (sec)	N/A	0.156	0.084	0.01	0.	1.58	1.6	1.166

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	106	229	235	518	158	336
normalized size	1	1.	0.91	1.96	2.01	4.43	1.35	2.87
time (sec)	N/A	0.148	0.092	0.011	0.982	1.428	1.693	1.14

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	247	0	1061	337	248
normalized size	1	1.	0.85	1.68	0.	7.22	2.29	1.69
time (sec)	N/A	0.19	0.067	0.01	0.	1.591	1.444	1.127

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	127	168	167	365	112	247
normalized size	1	1.	1.44	1.91	1.9	4.15	1.27	2.81
time (sec)	N/A	0.093	0.044	0.01	1.002	1.479	1.371	1.155

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	205	0	896	313	205
normalized size	1	1.	1.	1.93	0.	8.45	2.95	1.93
time (sec)	N/A	0.091	0.06	0.007	0.	1.541	1.249	1.156

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	111	146	165	352	110	203
normalized size	1	1.	1.26	1.66	1.88	4.	1.25	2.31
time (sec)	N/A	0.084	0.099	0.017	0.994	1.571	2.808	1.138

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	94	189	0	798	309	193
normalized size	1	1.	0.72	1.44	0.	6.09	2.36	1.47
time (sec)	N/A	0.133	0.06	0.013	0.	1.674	1.848	1.124

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	156	190	413	128	212
normalized size	1	1.	0.89	1.59	1.94	4.21	1.31	2.16
time (sec)	N/A	0.106	0.094	0.014	1.013	1.574	3.808	1.139

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	109	209	0	921	321	203
normalized size	1	1.	0.74	1.42	0.	6.27	2.18	1.38
time (sec)	N/A	0.141	0.062	0.011	0.	1.681	2.26	1.151

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1476	1850	165
normalized size	1	1.	0.87	1.32	0.	13.54	16.97	1.51
time (sec)	N/A	0.085	0.148	0.009	0.	2.41	14.407	1.179

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	95	142	242	253	124
normalized size	1	1.	1.	1.28	1.92	3.27	3.42	1.68
time (sec)	N/A	0.065	0.033	0.01	0.981	1.801	2.245	1.192

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	134	0	1485	1530	149
normalized size	1	1.	1.	1.29	0.	14.28	14.71	1.43
time (sec)	N/A	0.065	0.135	0.009	0.	1.993	7.143	1.145

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	90	134	219	248	115
normalized size	1	1.	0.94	1.29	1.91	3.13	3.54	1.64
time (sec)	N/A	0.052	0.028	0.01	0.971	1.6	2.181	1.149

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	109	144	0	1451	2033	163
normalized size	1	1.	1.01	1.33	0.	13.44	18.82	1.51
time (sec)	N/A	0.08	0.142	0.002	0.	2.221	15.639	1.143

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	97	139	185	443	0	247
normalized size	1	1.	0.98	1.4	1.87	4.47	0.	2.49
time (sec)	N/A	0.105	0.108	0.016	1.015	6.411	0.	1.195

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	123	169	0	2049	2526	221
normalized size	1	1.	0.85	1.17	0.	14.23	17.54	1.53
time (sec)	N/A	0.206	0.189	0.015	0.	3.68	127.24	1.136

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	119	170	255	601	0	347
normalized size	1	1.	0.94	1.35	2.02	4.77	0.	2.75
time (sec)	N/A	0.146	0.153	0.018	1.015	14.497	0.	1.154

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	142	191	0	2558	0	223
normalized size	1	1.	0.75	1.01	0.	13.53	0.	1.18
time (sec)	N/A	0.277	0.271	0.015	0.	7.853	0.	1.161

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	155	209	348	706	0	379
normalized size	1	1.	0.97	1.31	2.17	4.41	0.	2.37
time (sec)	N/A	0.194	0.193	0.018	1.022	35.303	0.	1.182

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	179	234	0	3002	0	279
normalized size	1	1.	0.72	0.94	0.	12.01	0.	1.12
time (sec)	N/A	0.409	0.304	0.019	0.	21.376	0.	1.151

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	202	268	458	817	0	478
normalized size	1	1.	0.96	1.28	2.18	3.89	0.	2.28
time (sec)	N/A	0.249	0.266	0.021	1.043	54.161	0.	1.125

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	133	222	0	2822	2378	1670
normalized size	1	1.	0.82	1.37	0.	17.42	14.68	10.31
time (sec)	N/A	0.164	0.19	0.013	0.	2.635	28.342	1.46

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	86	188	308	595	507	240
normalized size	1	1.	0.8	1.76	2.88	5.56	4.74	2.24
time (sec)	N/A	0.108	0.067	0.017	0.987	1.602	4.393	1.224

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	137	222	0	2805	2399	1481
normalized size	1	1.	0.93	1.51	0.	19.08	16.32	10.07
time (sec)	N/A	0.134	0.178	0.017	0.	3.321	34.427	1.404

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	77	143	290	508	408	220
normalized size	1	1.	0.84	1.55	3.15	5.52	4.43	2.39
time (sec)	N/A	0.079	0.068	0.016	0.988	1.632	3.844	1.157

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	136	238	0	3294	3662	1928
normalized size	1	1.	0.81	1.43	0.	19.72	21.93	11.54
time (sec)	N/A	0.198	0.317	0.	0.	5.88	143.969	1.478

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	133	225	398	1042	0	0
normalized size	1	1.	0.94	1.6	2.82	7.39	0.	0.
time (sec)	N/A	0.169	0.233	0.021	1.052	25.75	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	158	261	0	4182	0	2430
normalized size	1	1.	0.72	1.2	0.	19.18	0.	11.15
time (sec)	N/A	0.311	0.294	0.017	0.	12.563	0.	1.53

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	157	254	514	1287	0	0
normalized size	1	1.	1.01	1.63	3.29	8.25	0.	0.
time (sec)	N/A	0.209	0.2	0.024	1.062	53.814	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	178	285	0	4880	0	2608
normalized size	1	1.	0.66	1.05	0.	18.01	0.	9.62
time (sec)	N/A	0.448	0.372	0.02	0.	27.704	0.	1.645

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	166	388	0	5736	4041	406
normalized size	1	1.	0.8	1.87	0.	27.71	19.52	1.96
time (sec)	N/A	0.277	0.353	0.013	0.	5.27	146.596	1.16

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	121	283	560	1200	780	360
normalized size	1	1.	0.85	1.99	3.94	8.45	5.49	2.54
time (sec)	N/A	0.153	0.097	0.017	1.067	1.472	15.802	1.188

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	171	391	0	5824	0	428
normalized size	1	1.	0.86	1.96	0.	29.12	0.	2.14
time (sec)	N/A	0.252	0.402	0.015	0.	9.219	0.	1.2

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	234	532	1007	643	309
normalized size	1	1.	0.85	1.86	4.22	7.99	5.1	2.45
time (sec)	N/A	0.109	0.134	0.016	1.086	1.378	15.078	1.17

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	197	403	0	6472	0	448
normalized size	1	1.	0.86	1.75	0.	28.14	0.	1.95
time (sec)	N/A	0.301	0.407	0.013	0.	19.284	0.	1.179

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	187	374	711	2079	0	635
normalized size	1	1.	0.97	1.95	3.7	10.83	0.	3.31
time (sec)	N/A	0.241	0.302	0.023	1.088	60.738	0.	1.187

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	210	428	0	7524	0	581
normalized size	1	1.	0.71	1.44	0.	25.33	0.	1.96
time (sec)	N/A	0.497	0.44	0.02	0.	23.778	0.	1.208

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	208	405	879	2446	0	861
normalized size	1	1.	0.97	1.88	4.09	11.38	0.	4.
time (sec)	N/A	0.294	0.321	0.028	1.189	86.576	0.	1.173

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	230	455	0	8631	0	495
normalized size	1	1.	0.61	1.21	0.	22.89	0.	1.31
time (sec)	N/A	0.686	0.451	0.025	0.	66.872	0.	1.172

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	0	865	2069	801
normalized size	1	1.	0.93	4.94	0.	9.01	21.55	8.34
time (sec)	N/A	0.065	0.102	0.007	0.	1.072	2.951	1.196

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	0	490	1044	448
normalized size	1	1.	0.93	3.69	0.	6.9	14.7	6.31
time (sec)	N/A	0.042	0.05	0.004	0.	1.078	1.712	1.234

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	110	0	211	410	193
normalized size	1	1.	0.93	2.44	0.	4.69	9.11	4.29
time (sec)	N/A	0.021	0.03	0.005	0.	1.02	0.876	1.136

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0
normalized size	1	1.	0.83	0.	0.	0.	2.88	0.
time (sec)	N/A	0.033	0.059	0.031	0.	0.	4.792	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0
normalized size	1	1.	0.86	0.	0.	0.	9.74	0.
time (sec)	N/A	0.044	0.056	0.038	0.	0.	35.635	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	3053	0
normalized size	1	1.	0.86	0.	0.	0.	32.83	0.
time (sec)	N/A	0.042	0.057	0.053	0.	0.	132.097	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	141	976	0	1740	4345	1609
normalized size	1	1.	0.93	6.46	0.	11.52	28.77	10.66
time (sec)	N/A	0.089	0.104	0.01	0.	1.02	5.016	1.204

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	101	569	0	984	2363	949
normalized size	1	1.	0.93	5.22	0.	9.03	21.68	8.71
time (sec)	N/A	0.062	0.072	0.007	0.	0.858	3.036	1.191

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	0	490	1044	448
normalized size	1	1.	0.93	3.69	0.	6.9	14.7	6.31
time (sec)	N/A	0.037	0.063	0.006	0.	0.882	1.684	1.155

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	118	0	0	0	299	0
normalized size	1	1.	1.26	0.	0.	0.	3.18	0.
time (sec)	N/A	0.06	0.097	0.046	0.	0.	9.363	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	118	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.108	0.048	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	166	118	0	0	0	0	0
normalized size	1	0.97	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.108	0.064	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	0	0	0	411	0
normalized size	1	1.	0.86	0.	0.	0.	3.09	0.
time (sec)	N/A	0.086	1.36	0.046	0.	0.	16.986	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	299	0
normalized size	1	1.	0.9	0.	0.	0.	3.18	0.
time (sec)	N/A	0.058	0.414	0.043	0.	0.	9.324	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0
normalized size	1	1.	0.83	0.	0.	0.	2.88	0.
time (sec)	N/A	0.031	0.055	0.031	0.	0.	4.815	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	85	0	0	0	354	0
normalized size	1	1.	0.83	0.	0.	0.	3.47	0.
time (sec)	N/A	0.044	0.054	0.053	0.	0.	12.089	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.083	0.068	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	54	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	0.051	0.056	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	2524	0	0	0	0	0
normalized size	1	1.	12.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	4.783	0.051	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	895	0	0	0	0	0
normalized size	1	1.	7.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	1.986	0.051	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0
normalized size	1	1.	0.86	0.	0.	0.	9.74	0.
time (sec)	N/A	0.042	0.064	0.038	0.	0.	37.355	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.072	0.	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	54	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	0.085	0.054	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	54	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.614	0.16	0.072	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	89	46	39
normalized size	1	1.	0.85	0.82	0.92	2.28	1.18	1.
time (sec)	N/A	0.016	0.016	0.002	1.028	0.845	11.125	1.138

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	88	46	39
normalized size	1	1.	0.85	0.82	0.92	2.26	1.18	1.
time (sec)	N/A	0.016	0.015	0.003	1.041	0.852	5.669	1.135

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	85	46	39
normalized size	1	1.	0.85	0.82	0.92	2.18	1.18	1.
time (sec)	N/A	0.016	0.013	0.003	1.059	0.882	2.569	1.212

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	81	37	39
normalized size	1	1.	0.85	0.82	0.92	2.08	0.95	1.
time (sec)	N/A	0.015	0.015	0.002	1.026	0.89	1.68	1.118

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	74	44	39
normalized size	1	1.	0.89	0.86	0.97	2.	1.19	1.05
time (sec)	N/A	0.015	0.014	0.003	1.003	0.828	0.764	1.123

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	36	74	44	39
normalized size	1	1.	0.95	0.86	0.97	2.	1.19	1.05
time (sec)	N/A	0.015	0.01	0.002	1.003	0.853	0.983	1.179

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	32	36	74	42	39
normalized size	1	1.	0.97	0.86	0.97	2.	1.14	1.05
time (sec)	N/A	0.016	0.012	0.003	1.032	0.664	1.251	1.161

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	32	39	74	42	42
normalized size	1	1.	0.97	0.86	1.05	2.	1.14	1.14
time (sec)	N/A	0.016	0.013	0.003	1.039	0.859	1.866	1.139

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	146	80	72
normalized size	1	1.	0.84	0.89	1.1	2.32	1.27	1.14
time (sec)	N/A	0.03	0.031	0.005	1.052	0.798	20.99	1.127

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	147	80	72
normalized size	1	1.	1.	0.89	1.1	2.33	1.27	1.14
time (sec)	N/A	0.03	0.029	0.005	1.053	0.816	11.472	1.153

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	143	80	72
normalized size	1	1.	0.84	0.89	1.1	2.27	1.27	1.14
time (sec)	N/A	0.03	0.028	0.005	1.036	0.741	5.924	1.144

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	136	66	72
normalized size	1	1.	0.84	0.89	1.1	2.16	1.05	1.14
time (sec)	N/A	0.029	0.029	0.006	1.044	0.912	2.468	1.152

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	131	78	72
normalized size	1	1.	0.87	0.92	1.13	2.15	1.28	1.18
time (sec)	N/A	0.032	0.028	0.005	1.081	0.904	2.122	1.164

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	60	56	69	130	78	72
normalized size	1	1.	0.98	0.92	1.13	2.13	1.28	1.18
time (sec)	N/A	0.031	0.02	0.006	1.046	0.887	2.51	1.161

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	69	124	76	72
normalized size	1	1.	0.93	0.92	1.13	2.03	1.25	1.18
time (sec)	N/A	0.029	0.018	0.005	1.071	0.724	3.012	1.131

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	72	130	76	74
normalized size	1	1.	0.93	0.92	1.18	2.13	1.25	1.21
time (sec)	N/A	0.029	0.016	0.005	1.016	0.797	4.143	1.128

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	205	114	104
normalized size	1	1.	1.	0.94	1.16	2.41	1.34	1.22
time (sec)	N/A	0.042	0.042	0.004	1.027	0.766	34.03	1.149

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	201	114	104
normalized size	1	1.	1.	0.94	1.16	2.36	1.34	1.22
time (sec)	N/A	0.041	0.038	0.005	1.06	0.87	20.641	1.151

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	198	114	104
normalized size	1	1.	1.	0.94	1.16	2.33	1.34	1.22
time (sec)	N/A	0.04	0.036	0.005	1.116	0.83	11.391	1.136

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	99	192	95	104
normalized size	1	1.	0.84	0.94	1.16	2.26	1.12	1.22
time (sec)	N/A	0.041	0.038	0.004	1.086	0.855	3.659	1.138

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	80	99	184	112	104
normalized size	1	1.	0.86	0.96	1.19	2.22	1.35	1.25
time (sec)	N/A	0.04	0.041	0.005	1.473	0.7	4.953	1.118

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	81	80	99	181	110	104
normalized size	1	1.	0.98	0.96	1.19	2.18	1.33	1.25
time (sec)	N/A	0.044	0.022	0.005	1.355	0.78	5.608	1.332

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	80	99	177	110	104
normalized size	1	1.	0.94	0.96	1.19	2.13	1.33	1.25
time (sec)	N/A	0.043	0.026	0.006	1.032	0.865	6.684	1.134

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	80	101	176	107	107
normalized size	1	1.	0.96	0.99	1.25	2.17	1.32	1.32
time (sec)	N/A	0.043	0.022	0.006	1.042	0.769	8.912	1.127

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	227	330	0	1477	0	402
normalized size	1	1.	0.82	1.2	0.	5.35	0.	1.46
time (sec)	N/A	0.259	0.277	0.009	0.	0.933	0.	1.145

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	110	308	0	1823	0	356
normalized size	1	1.	0.43	1.2	0.	7.09	0.	1.39
time (sec)	N/A	0.205	0.136	0.009	0.	0.989	0.	1.162

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	208	299	0	1365	382	355
normalized size	1	1.	0.82	1.17	0.	5.35	1.5	1.39
time (sec)	N/A	0.202	0.205	0.008	0.	1.001	34.86	1.178

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	95	280	0	1698	0	339
normalized size	1	1.	0.4	1.18	0.	7.16	0.	1.43
time (sec)	N/A	0.178	0.072	0.008	0.	0.948	0.	1.174

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	166	277	0	1347	371	339
normalized size	1	1.	0.71	1.18	0.	5.73	1.58	1.44
time (sec)	N/A	0.176	0.129	0.007	0.	0.925	10.189	1.179

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	74	277	0	1735	374	339
normalized size	1	1.	0.31	1.18	0.	7.38	1.59	1.44
time (sec)	N/A	0.181	0.086	0.009	0.	0.914	18.256	1.163

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	168	280	0	1347	364	339
normalized size	1	1.	0.71	1.18	0.	5.68	1.54	1.43
time (sec)	N/A	0.177	0.135	0.009	0.	0.989	54.609	1.16

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	46	299	0	1793	0	362
normalized size	1	1.	0.18	1.17	0.	7.03	0.	1.42
time (sec)	N/A	0.206	0.014	0.011	0.	1.022	0.	1.171

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	385	339	0	1775	0	402
normalized size	1	1.	1.24	1.09	0.	5.73	0.	1.3
time (sec)	N/A	0.243	0.429	0.014	0.	1.009	0.	1.17

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	136	317	0	2141	0	382
normalized size	1	1.	0.47	1.1	0.	7.41	0.	1.32
time (sec)	N/A	0.219	0.192	0.013	0.	0.999	0.	1.164

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	353	323	0	1594	0	382
normalized size	1	1.	1.24	1.14	0.	5.61	0.	1.35
time (sec)	N/A	0.209	0.389	0.012	0.	0.888	0.	1.151

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	95	305	0	1968	0	369
normalized size	1	1.	0.36	1.17	0.	7.54	0.	1.41
time (sec)	N/A	0.182	0.119	0.012	0.	0.997	0.	1.21

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	203	305	0	1559	0	369
normalized size	1	1.	0.78	1.17	0.	5.97	0.	1.41
time (sec)	N/A	0.181	0.245	0.012	0.	1.015	0.	1.212

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	117	323	0	2026	0	375
normalized size	1	1.	0.4	1.12	0.	7.01	0.	1.3
time (sec)	N/A	0.212	0.213	0.013	0.	1.073	0.	1.216

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	355	317	0	1700	0	382
normalized size	1	1.	1.23	1.1	0.	5.88	0.	1.32
time (sec)	N/A	0.214	0.401	0.014	0.	0.9	0.	1.184

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	151	339	0	2310	0	409
normalized size	1	1.	0.49	1.09	0.	7.45	0.	1.32
time (sec)	N/A	0.237	0.418	0.016	0.	1.031	0.	1.173

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	402	363	0	1787	0	410
normalized size	1	1.	1.27	1.15	0.	5.66	0.	1.3
time (sec)	N/A	0.24	0.466	0.015	0.	0.914	0.	1.191

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	137	325	0	2226	0	396
normalized size	1	1.	0.47	1.11	0.	7.6	0.	1.35
time (sec)	N/A	0.216	0.213	0.015	0.	0.986	0.	1.194

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	389	334	0	1817	0	402
normalized size	1	1.	1.31	1.12	0.	6.1	0.	1.35
time (sec)	N/A	0.212	0.442	0.014	0.	1.076	0.	1.163

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	62	335	0	2276	0	402
normalized size	1	1.	0.21	1.12	0.	7.64	0.	1.35
time (sec)	N/A	0.216	0.053	0.014	0.	1.007	0.	1.179

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	230	325	0	1782	0	396
normalized size	1	1.	0.78	1.11	0.	6.08	0.	1.35
time (sec)	N/A	0.212	0.28	0.015	0.	0.951	0.	1.161

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	147	363	0	2244	0	405
normalized size	1	1.	0.46	1.13	0.	6.97	0.	1.26
time (sec)	N/A	0.234	0.194	0.019	0.	1.069	0.	1.222

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	400	357	0	1901	0	410
normalized size	1	1.	1.24	1.11	0.	5.9	0.	1.27
time (sec)	N/A	0.237	0.431	0.017	0.	1.112	0.	1.181

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	189	381	0	2507	0	440
normalized size	1	1.	0.55	1.11	0.	7.31	0.	1.28
time (sec)	N/A	0.262	0.473	0.02	0.	0.884	0.	1.219

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	146	80	72
normalized size	1	1.	0.84	0.89	1.1	2.32	1.27	1.14
time (sec)	N/A	0.033	0.032	0.005	1.049	0.756	20.147	1.157

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	147	80	72
normalized size	1	1.	1.	0.89	1.1	2.33	1.27	1.14
time (sec)	N/A	0.029	0.028	0.005	1.074	0.819	11.211	1.148

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	143	80	72
normalized size	1	1.	0.84	0.89	1.1	2.27	1.27	1.14
time (sec)	N/A	0.03	0.028	0.004	1.045	0.744	5.931	1.155

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	136	66	72
normalized size	1	1.	0.84	0.89	1.1	2.16	1.05	1.14
time (sec)	N/A	0.029	0.03	0.005	1.064	0.708	2.492	1.172

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	131	78	72
normalized size	1	1.	0.87	0.92	1.13	2.15	1.28	1.18
time (sec)	N/A	0.029	0.031	0.005	1.115	0.908	2.14	1.167

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	60	56	69	130	78	72
normalized size	1	1.	0.98	0.92	1.13	2.13	1.28	1.18
time (sec)	N/A	0.031	0.021	0.003	1.078	0.628	2.465	1.173

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	69	124	76	72
normalized size	1	1.	0.93	0.92	1.13	2.03	1.25	1.18
time (sec)	N/A	0.029	0.02	0.005	1.018	0.87	2.97	1.173

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	72	130	76	74
normalized size	1	1.	0.93	0.92	1.18	2.13	1.25	1.21
time (sec)	N/A	0.029	0.016	0.005	1.113	0.861	4.188	1.139

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	227	136	127
normalized size	1	1.	1.	1.	1.19	2.34	1.4	1.31
time (sec)	N/A	0.049	0.035	0.004	1.086	0.759	34.539	1.145

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	225	136	127
normalized size	1	1.	1.	1.	1.19	2.32	1.4	1.31
time (sec)	N/A	0.046	0.033	0.006	1.048	0.748	20.577	1.144

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	220	136	127
normalized size	1	1.	1.	1.	1.19	2.27	1.4	1.31
time (sec)	N/A	0.05	0.032	0.004	1.105	0.884	11.379	1.6

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	97	115	213	110	127
normalized size	1	1.	0.86	1.	1.19	2.2	1.13	1.31
time (sec)	N/A	0.047	0.034	0.006	1.078	0.737	3.738	1.218

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	97	115	208	134	127
normalized size	1	1.	1.	1.02	1.21	2.19	1.41	1.34
time (sec)	N/A	0.046	0.031	0.005	1.021	0.702	4.985	1.178

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	115	203	134	127
normalized size	1	1.	0.87	1.02	1.21	2.14	1.41	1.34
time (sec)	N/A	0.048	0.032	0.004	1.031	0.721	5.545	1.622

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	115	201	133	127
normalized size	1	1.	0.87	1.02	1.21	2.12	1.4	1.34
time (sec)	N/A	0.047	0.032	0.006	1.029	0.803	6.633	1.152

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	117	204	133	130
normalized size	1	1.	0.87	1.02	1.23	2.15	1.4	1.37
time (sec)	N/A	0.047	0.037	0.006	1.062	0.754	8.948	1.166

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	332	192	182
normalized size	1	1.	1.	0.99	1.23	2.39	1.38	1.31
time (sec)	N/A	0.065	0.043	0.006	1.037	0.841	55.167	1.171

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	328	192	182
normalized size	1	1.	1.	0.99	1.23	2.36	1.38	1.31
time (sec)	N/A	0.065	0.04	0.005	1.037	0.77	35.058	1.188

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	319	192	182
normalized size	1	1.	1.	0.99	1.23	2.29	1.38	1.31
time (sec)	N/A	0.063	0.037	0.006	1.06	0.762	21.254	1.174

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	316	155	182
normalized size	1	1.	1.	0.99	1.23	2.27	1.12	1.31
time (sec)	N/A	0.064	0.038	0.007	1.041	0.808	5.207	1.178

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	305	190	182
normalized size	1	1.	1.	1.01	1.25	2.23	1.39	1.33
time (sec)	N/A	0.065	0.039	0.007	1.089	0.822	10.474	1.691

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	304	189	182
normalized size	1	1.	1.	1.01	1.25	2.22	1.38	1.33
time (sec)	N/A	0.064	0.051	0.007	1.067	0.861	11.564	1.155

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	298	189	182
normalized size	1	1.	1.	1.01	1.25	2.18	1.38	1.33
time (sec)	N/A	0.064	0.052	0.007	1.081	0.906	12.552	1.189

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	138	174	293	185	185
normalized size	1	1.	0.88	1.01	1.27	2.14	1.35	1.35
time (sec)	N/A	0.063	0.048	0.006	1.065	0.806	16.219	1.161

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	299	545	0	2813	0	589
normalized size	1	1.	0.96	1.75	0.	9.05	0.	1.89
time (sec)	N/A	0.313	0.134	0.015	0.	1.117	0.	1.405

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	276	504	0	3549	0	520
normalized size	1	1.	0.95	1.74	0.	12.24	0.	1.79
time (sec)	N/A	0.254	0.104	0.01	0.	1.186	0.	1.203

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	276	495	0	2633	661	520
normalized size	1	1.	0.96	1.72	0.	9.14	2.3	1.81
time (sec)	N/A	0.24	0.102	0.01	0.	1.124	126.589	1.162

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	249	461	0	3380	0	487
normalized size	1	1.	0.93	1.72	0.	12.61	0.	1.82
time (sec)	N/A	0.233	0.113	0.009	0.	1.163	0.	1.168

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	452	0	2581	612	486
normalized size	1	1.	0.94	1.7	0.	9.7	2.3	1.83
time (sec)	N/A	0.212	0.11	0.009	0.	1.153	30.749	1.154

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	261	439	0	3394	597	464
normalized size	1	1.	1.	1.69	0.	13.05	2.3	1.78
time (sec)	N/A	0.266	0.122	0.011	0.	1.239	28.832	1.212

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	261	439	0	2591	581	464
normalized size	1	1.	1.	1.69	0.	9.97	2.23	1.78
time (sec)	N/A	0.265	0.117	0.011	0.	1.096	52.912	1.163

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	254	452	0	3421	0	477
normalized size	1	1.	0.95	1.69	0.	12.81	0.	1.79
time (sec)	N/A	0.28	0.114	0.012	0.	1.2	0.	1.182

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	254	461	0	2587	0	478
normalized size	1	1.	0.94	1.71	0.	9.62	0.	1.78
time (sec)	N/A	0.276	0.115	0.013	0.	1.071	0.	1.174

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	101	495	0	3498	0	527
normalized size	1	1.	0.35	1.72	0.	12.15	0.	1.83
time (sec)	N/A	0.316	0.196	0.013	0.	1.119	0.	1.686

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	372	563	0	3289	0	594
normalized size	1	1.	0.99	1.5	0.	8.77	0.	1.58
time (sec)	N/A	0.437	0.346	0.017	0.	1.202	0.	1.18

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	337	523	0	4166	0	558
normalized size	1	1.	0.97	1.51	0.	12.04	0.	1.61
time (sec)	N/A	0.317	0.183	0.017	0.	1.209	0.	1.211

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	333	523	0	3036	0	551
normalized size	1	1.	0.96	1.51	0.	8.77	0.	1.59
time (sec)	N/A	0.329	0.178	0.014	0.	1.089	0.	1.212

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	319	499	0	3846	0	524
normalized size	1	1.	1.03	1.61	0.	12.41	0.	1.69
time (sec)	N/A	0.282	0.179	0.014	0.	1.148	0.	1.201

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	318	496	0	2944	0	524
normalized size	1	1.	1.02	1.59	0.	9.44	0.	1.68
time (sec)	N/A	0.342	0.171	0.016	0.	1.054	0.	1.188

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	317	495	0	3861	0	525
normalized size	1	1.	0.95	1.49	0.	11.59	0.	1.58
time (sec)	N/A	0.328	0.188	0.016	0.	1.216	0.	1.232

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	315	498	0	2965	0	518
normalized size	1	1.	0.95	1.5	0.	8.93	0.	1.56
time (sec)	N/A	0.343	0.184	0.017	0.	0.968	0.	1.208

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	333	524	0	3997	0	541
normalized size	1	1.	0.92	1.44	0.	11.01	0.	1.49
time (sec)	N/A	0.377	0.199	0.02	0.	1.2	0.	1.211

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	383	590	0	3676	0	609
normalized size	1	1.	0.87	1.34	0.	8.35	0.	1.38
time (sec)	N/A	0.381	0.367	0.019	0.	0.966	0.	1.197

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	363	562	0	4574	0	576
normalized size	1	1.	0.91	1.4	0.	11.41	0.	1.44
time (sec)	N/A	0.343	0.236	0.018	0.	1.181	0.	1.223

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	361	568	0	3345	0	575
normalized size	1	1.	0.9	1.41	0.	8.32	0.	1.43
time (sec)	N/A	0.33	0.372	0.017	0.	0.965	0.	1.209

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	339	514	0	4366	0	562
normalized size	1	1.	0.93	1.41	0.	11.99	0.	1.54
time (sec)	N/A	0.293	0.203	0.017	0.	1.296	0.	1.294

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	339	514	0	3345	0	562
normalized size	1	1.	0.93	1.41	0.	9.19	0.	1.54
time (sec)	N/A	0.297	0.196	0.018	0.	1.33	0.	1.201

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	398	364	568	0	4365	0	576
normalized size	1	1.	0.91	1.42	0.	10.94	0.	1.44
time (sec)	N/A	0.395	0.408	0.022	0.	1.514	0.	1.204

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	401	365	562	0	3515	0	575
normalized size	1	1.	0.91	1.4	0.	8.74	0.	1.43
time (sec)	N/A	0.406	0.26	0.022	0.	1.369	0.	1.224

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	438	382	590	0	4849	0	599
normalized size	1	1.	0.87	1.34	0.	11.05	0.	1.36
time (sec)	N/A	0.45	0.381	0.023	0.	1.631	0.	1.245

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	132	721	0	5611	0	717
normalized size	1	1.	0.4	2.2	0.	17.11	0.	2.19
time (sec)	N/A	0.293	0.385	0.01	0.	2.072	0.	1.193

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	133	712	0	4181	0	717
normalized size	1	1.	0.41	2.18	0.	12.83	0.	2.2
time (sec)	N/A	0.277	0.377	0.01	0.	1.775	0.	1.201

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	97	659	0	5393	0	662
normalized size	1	1.	0.32	2.15	0.	17.62	0.	2.16
time (sec)	N/A	0.245	0.36	0.01	0.	1.662	0.	1.197

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	96	650	0	4100	889	662
normalized size	1	1.	0.32	2.14	0.	13.49	2.92	2.18
time (sec)	N/A	0.249	0.344	0.01	0.	1.386	106.066	1.217

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	89	622	0	5403	847	624
normalized size	1	1.	0.31	2.19	0.	19.02	2.98	2.2
time (sec)	N/A	0.29	0.335	0.012	0.	1.566	103.445	1.243

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	89	616	0	4078	842	622
normalized size	1	1.	0.31	2.17	0.	14.36	2.96	2.19
time (sec)	N/A	0.267	0.357	0.013	0.	1.38	157.665	1.447

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	88	616	0	5407	0	614
normalized size	1	1.	0.31	2.18	0.	19.11	0.	2.17
time (sec)	N/A	0.275	0.353	0.015	0.	2.053	0.	1.247

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	88	622	0	4086	0	614
normalized size	1	1.	0.31	2.2	0.	14.44	0.	2.17
time (sec)	N/A	0.258	0.357	0.013	0.	1.743	0.	1.18

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	101	650	0	5445	0	652
normalized size	1	1.	0.33	2.15	0.	17.97	0.	2.15
time (sec)	N/A	0.291	0.365	0.015	0.	2.018	0.	1.224

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	102	659	0	4089	0	652
normalized size	1	1.	0.33	2.16	0.	13.41	0.	2.14
time (sec)	N/A	0.268	0.358	0.015	0.	1.361	0.	1.167

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	148	712	0	5557	0	724
normalized size	1	1.	0.46	2.19	0.	17.1	0.	2.23
time (sec)	N/A	0.306	0.394	0.015	0.	1.579	0.	1.231

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	409	409	419	804	0	5115	0	810
normalized size	1	1.	1.02	1.97	0.	12.51	0.	1.98
time (sec)	N/A	0.467	2.574	0.018	0.	1.429	0.	1.222

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	377	748	0	6029	0	745
normalized size	1	1.	1.01	2.	0.	16.12	0.	1.99
time (sec)	N/A	0.415	2.032	0.017	0.	1.659	0.	1.27

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	386	386	377	748	0	4809	0	745
normalized size	1	1.	0.98	1.94	0.	12.46	0.	1.93
time (sec)	N/A	0.529	2.112	0.016	0.	1.399	0.	1.198

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	355	706	0	6134	0	697
normalized size	1	1.	0.94	1.88	0.	16.31	0.	1.85
time (sec)	N/A	0.426	2.109	0.019	0.	1.649	0.	1.226

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	358	697	0	4316	0	690
normalized size	1	1.	1.05	2.05	0.	12.69	0.	2.03
time (sec)	N/A	0.388	2.02	0.016	0.	1.375	0.	1.223

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	368	368	355	682	0	6132	0	680
normalized size	1	1.	0.96	1.85	0.	16.66	0.	1.85
time (sec)	N/A	0.431	1.995	0.02	0.	1.642	0.	1.269

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	350	682	0	4639	0	676
normalized size	1	1.	0.95	1.86	0.	12.64	0.	1.84
time (sec)	N/A	0.411	2.134	0.019	0.	1.373	0.	1.184

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	353	697	0	5704	0	682
normalized size	1	1.	0.94	1.85	0.	15.17	0.	1.81
time (sec)	N/A	0.429	1.897	0.02	0.	1.656	0.	1.253

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	353	706	0	4648	0	687
normalized size	1	1.	0.94	1.88	0.	12.36	0.	1.83
time (sec)	N/A	0.416	1.799	0.021	0.	1.407	0.	1.2

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	411	351	0	2858	0	0
normalized size	1	1.	0.86	0.73	0.	5.98	0.	0.
time (sec)	N/A	0.557	0.245	0.012	0.	9.177	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	409	339	0	2757	0	0
normalized size	1	1.	0.86	0.71	0.	5.79	0.	0.
time (sec)	N/A	0.489	0.188	0.011	0.	2.322	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	142	328	0	2741	0	0
normalized size	1	1.	0.31	0.71	0.	5.92	0.	0.
time (sec)	N/A	0.359	0.113	0.012	0.	1.668	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	364	304	0	2542	0	0
normalized size	1	1.	0.79	0.66	0.	5.49	0.	0.
time (sec)	N/A	0.363	0.119	0.011	0.	1.358	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	364	304	0	2596	0	0
normalized size	1	1.	0.79	0.66	0.	5.61	0.	0.
time (sec)	N/A	0.358	0.133	0.011	0.	1.388	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	364	328	0	2714	0	0
normalized size	1	1.	0.79	0.71	0.	5.86	0.	0.
time (sec)	N/A	0.347	0.123	0.012	0.	1.928	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	409	339	0	2808	0	2300
normalized size	1	1.	0.86	0.71	0.	5.9	0.	4.83
time (sec)	N/A	0.535	0.233	0.015	0.	2.798	0.	3.838

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	411	351	0	2877	0	2246
normalized size	1	1.	0.86	0.73	0.	6.02	0.	4.7
time (sec)	N/A	0.481	0.236	0.013	0.	19.891	0.	5.292

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	437	375	0	3024	0	1754
normalized size	1	1.	0.88	0.75	0.	6.07	0.	3.52
time (sec)	N/A	0.687	0.274	0.015	0.	25.655	0.	2.854

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	563	582	0	7129	0	969
normalized size	1	1.	0.99	1.02	0.	12.51	0.	1.7
time (sec)	N/A	0.846	0.376	0.016	0.	161.814	0.	1.557

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	527	566	0	7397	0	919
normalized size	1	1.	0.98	1.06	0.	13.8	0.	1.71
time (sec)	N/A	0.592	0.328	0.017	0.	142.469	0.	1.553

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	523	533	0	6624	0	903
normalized size	1	1.	0.98	1.	0.	12.45	0.	1.7
time (sec)	N/A	0.529	0.319	0.015	0.	31.931	0.	1.509

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	522	528	0	6947	0	922
normalized size	1	1.	0.99	1.	0.	13.16	0.	1.75
time (sec)	N/A	0.591	0.304	0.015	0.	51.417	0.	1.58

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	522	528	0	6483	0	884
normalized size	1	1.	0.99	1.	0.	12.28	0.	1.67
time (sec)	N/A	0.475	0.308	0.014	0.	18.256	0.	1.472

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	523	533	0	7121	0	946
normalized size	1	1.	0.98	0.99	0.	13.29	0.	1.76
time (sec)	N/A	0.598	0.317	0.016	0.	69.956	0.	1.553

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	526	566	0	6876	0	909
normalized size	1	1.	0.98	1.06	0.	12.83	0.	1.7
time (sec)	N/A	0.533	0.328	0.013	0.	100.731	0.	1.474

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	540	582	0	7727	0	979
normalized size	1	1.	0.95	1.02	0.	13.56	0.	1.72
time (sec)	N/A	0.75	0.673	0.018	0.	141.711	0.	1.561

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	542	588	0	0	0	969
normalized size	1	1.	0.95	1.03	0.	0.	0.	1.7
time (sec)	N/A	0.823	0.66	0.018	0.	0.	0.	1.554

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	563	612	0	0	0	965
normalized size	1	1.	0.91	0.99	0.	0.	0.	1.56
time (sec)	N/A	0.962	0.805	0.023	0.	0.	0.	1.6

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	640	839	0	0	0	1274
normalized size	1	1.	1.01	1.33	0.	0.	0.	2.02
time (sec)	N/A	0.814	0.569	0.018	0.	0.	0.	1.727

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	544	839	0	0	0	1300
normalized size	1	1.	0.87	1.34	0.	0.	0.	2.07
time (sec)	N/A	0.761	0.787	0.019	0.	0.	0.	1.861

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	627	627	543	848	0	0	0	1277
normalized size	1	1.	0.87	1.35	0.	0.	0.	2.04
time (sec)	N/A	0.71	0.804	0.018	0.	0.	0.	1.845

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	633	620	855	0	0	0	1307
normalized size	1	1.	0.98	1.35	0.	0.	0.	2.06
time (sec)	N/A	0.817	0.98	0.019	0.	0.	0.	1.882

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	633	620	882	0	0	0	1296
normalized size	1	1.	0.98	1.39	0.	0.	0.	2.05
time (sec)	N/A	0.832	0.909	0.018	0.	0.	0.	1.762

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	681	637	900	0	0	0	1332
normalized size	1	1.	0.94	1.32	0.	0.	0.	1.96
time (sec)	N/A	1.006	1.093	0.024	0.	0.	0.	1.999

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	681	639	906	0	0	0	1343
normalized size	1	1.	0.94	1.33	0.	0.	0.	1.97
time (sec)	N/A	0.928	1.012	0.023	0.	0.	0.	2.059

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	743	660	933	0	0	0	1350
normalized size	1	1.	0.89	1.26	0.	0.	0.	1.82
time (sec)	N/A	1.233	1.285	0.026	0.	0.	0.	1.879

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	585	740	0	0	0	0
normalized size	1	1.	0.94	1.19	0.	0.	0.	0.
time (sec)	N/A	0.757	1.147	0.019	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	583	740	0	0	0	0
normalized size	1	1.	0.96	1.22	0.	0.	0.	0.
time (sec)	N/A	0.796	1.103	0.02	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	601	601	575	770	0	0	0	0
normalized size	1	1.	0.96	1.28	0.	0.	0.	0.
time (sec)	N/A	0.691	0.997	0.019	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	589	778	0	0	0	0
normalized size	1	1.	0.94	1.25	0.	0.	0.	0.
time (sec)	N/A	0.858	1.143	0.02	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	593	808	0	0	0	0
normalized size	1	1.	0.94	1.29	0.	0.	0.	0.
time (sec)	N/A	0.862	1.202	0.019	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	606	825	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.
time (sec)	N/A	1.056	1.286	0.025	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	610	825	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.
time (sec)	N/A	0.967	1.253	0.025	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	731	731	630	849	0	0	0	0
normalized size	1	1.	0.86	1.16	0.	0.	0.	0.
time (sec)	N/A	1.276	1.422	0.027	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	604	1066	0	0	0	1611
normalized size	1	1.	0.84	1.48	0.	0.	0.	2.24
time (sec)	N/A	1.042	1.332	0.023	0.	0.	0.	2.481

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	703	604	1067	0	0	0	1671
normalized size	1	1.	0.86	1.52	0.	0.	0.	2.38
time (sec)	N/A	1.02	1.828	0.023	0.	0.	0.	2.668

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	703	603	1094	0	0	0	1643
normalized size	1	1.	0.86	1.56	0.	0.	0.	2.34
time (sec)	N/A	1.005	1.77	0.023	0.	0.	0.	2.479

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	691	1100	0	0	0	1665
normalized size	1	1.	0.94	1.49	0.	0.	0.	2.25
time (sec)	N/A	1.159	2.058	0.023	0.	0.	0.	2.66

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	692	1124	0	0	0	1692
normalized size	1	1.	0.94	1.52	0.	0.	0.	2.29
time (sec)	N/A	0.978	2.01	0.022	0.	0.	0.	2.204

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	805	706	1143	0	0	0	1800
normalized size	1	1.	0.88	1.42	0.	0.	0.	2.24
time (sec)	N/A	1.4	2.247	0.03	0.	0.	0.	2.438

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	805	707	1143	0	0	0	1725
normalized size	1	1.	0.88	1.42	0.	0.	0.	2.14
time (sec)	N/A	1.322	2.124	0.027	0.	0.	0.	2.25

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	881	881	729	1170	0	0	0	1740
normalized size	1	1.	0.83	1.33	0.	0.	0.	1.98
time (sec)	N/A	1.691	2.202	0.034	0.	0.	0.	2.719

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	75	77	0	219	212	144
normalized size	1	1.	0.73	0.75	0.	2.13	2.06	1.4
time (sec)	N/A	0.087	0.056	0.006	0.	2.29	1.446	1.113

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	130	181	0	587	286	178
normalized size	1	1.	0.84	1.17	0.	3.79	1.85	1.15
time (sec)	N/A	0.072	0.268	0.01	0.	2.662	13.51	1.147

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	166	162	107
normalized size	1	1.	0.78	0.73	0.	2.27	2.22	1.47
time (sec)	N/A	0.062	0.04	0.006	0.	1.833	0.705	1.141

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	139	0	475	226	135
normalized size	1	1.	0.89	1.14	0.	3.89	1.85	1.11
time (sec)	N/A	0.059	0.21	0.007	0.	1.888	8.624	1.133

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	113	110	63
normalized size	1	1.	0.74	0.67	0.	2.46	2.39	1.37
time (sec)	N/A	0.036	0.023	0.003	0.	1.812	0.329	1.111

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	96	0	370	144	93
normalized size	1	1.	0.98	1.1	0.	4.25	1.66	1.07
time (sec)	N/A	0.028	0.147	0.006	0.	1.789	5.168	1.129

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	300	76	81
normalized size	1	1.	1.	0.97	0.	5.08	1.29	1.37
time (sec)	N/A	0.044	0.033	0.008	0.	1.581	15.253	1.127

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	93	0	320	107	113
normalized size	1	1.	0.85	1.11	0.	3.81	1.27	1.35
time (sec)	N/A	0.033	0.148	0.008	0.	1.576	3.461	1.135

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	63	106	0	332	107	92
normalized size	1	1.	0.75	1.26	0.	3.95	1.27	1.1
time (sec)	N/A	0.063	0.042	0.009	0.	1.661	19.891	1.113

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	81	75	0	331	107	204
normalized size	1	1.	1.23	1.14	0.	5.02	1.62	3.09
time (sec)	N/A	0.025	0.166	0.009	0.	1.591	2.557	1.207

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	93	153	0	394	144	162
normalized size	1	1.	1.06	1.74	0.	4.48	1.64	1.84
time (sec)	N/A	0.069	0.07	0.01	0.	1.604	46.176	1.11

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	126	119	313
normalized size	1	1.	0.75	0.7	0.	2.38	2.25	5.91
time (sec)	N/A	0.021	0.013	0.004	0.	1.547	2.326	1.134

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	61	197	0	500	226	189
normalized size	1	1.	0.51	1.64	0.	4.17	1.88	1.58
time (sec)	N/A	0.094	0.021	0.013	0.	1.766	59.657	1.111

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	63	59	0	178	442	389
normalized size	1	1.	0.75	0.7	0.	2.12	5.26	4.63
time (sec)	N/A	0.034	0.03	0.005	0.	1.732	3.051	1.142

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	62	239	0	612	286	262
normalized size	1	1.	0.4	1.53	0.	3.92	1.83	1.68
time (sec)	N/A	0.12	0.022	0.016	0.	2.14	98.791	1.613

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	81	83	0	231	957	464
normalized size	1	1.	0.69	0.71	0.	1.97	8.18	3.97
time (sec)	N/A	0.055	0.038	0.006	0.	2.178	4.134	1.167

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	62	281	0	745	347	311
normalized size	1	1.	0.33	1.49	0.	3.94	1.84	1.65
time (sec)	N/A	0.147	0.022	0.024	0.	2.339	114.89	1.153

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	0	284	260	323
normalized size	1	1.	0.76	0.75	0.	2.76	2.52	3.14
time (sec)	N/A	0.077	0.058	0.006	0.	1.615	3.997	1.114

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	150	219	0	693	345	215
normalized size	1	1.	0.8	1.16	0.	3.69	1.84	1.14
time (sec)	N/A	0.096	0.306	0.01	0.	2.014	34.035	1.127

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	217	209	247
normalized size	1	1.	0.78	0.73	0.	2.97	2.86	3.38
time (sec)	N/A	0.059	0.039	0.005	0.	1.626	2.322	1.128

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	130	177	0	597	287	180
normalized size	1	1.	0.84	1.14	0.	3.85	1.85	1.16
time (sec)	N/A	0.067	0.249	0.008	0.	1.789	22.243	1.134

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	161	158	162
normalized size	1	1.	0.74	0.67	0.	3.5	3.43	3.52
time (sec)	N/A	0.032	0.023	0.003	0.	1.629	1.264	1.097

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	109	131	0	482	253	138
normalized size	1	1.	0.92	1.11	0.	4.08	2.14	1.17
time (sec)	N/A	0.041	0.21	0.003	0.	1.729	13.376	1.115

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	70	0	408	71	107
normalized size	1	1.	1.	0.92	0.	5.37	0.93	1.41
time (sec)	N/A	0.053	0.065	0.007	0.	1.593	35.731	1.344

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	87	125	0	424	216	154
normalized size	1	1.	0.8	1.15	0.	3.89	1.98	1.41
time (sec)	N/A	0.041	0.191	0.008	0.	1.633	8.71	1.172

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	80	132	0	404	184	139
normalized size	1	1.	0.73	1.2	0.	3.67	1.67	1.26
time (sec)	N/A	0.08	0.05	0.008	0.	1.687	25.617	1.122

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	168	0	402	202	279
normalized size	1	1.	0.7	1.41	0.	3.38	1.7	2.34
time (sec)	N/A	0.047	0.077	0.007	0.	1.596	5.485	1.148

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	59	184	0	436	216	177
normalized size	1	1.	0.51	1.6	0.	3.79	1.88	1.54
time (sec)	N/A	0.084	0.028	0.01	0.	1.649	56.681	1.144

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	115	0	439	184	319
normalized size	1	1.	0.88	1.34	0.	5.1	2.14	3.71
time (sec)	N/A	0.035	0.036	0.01	0.	1.63	4.285	1.149

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	119	233	0	506	253	215
normalized size	1	1.	0.99	1.94	0.	4.22	2.11	1.79
time (sec)	N/A	0.096	0.08	0.011	0.	1.706	82.447	1.124

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	173	518	464
normalized size	1	1.	0.75	0.7	0.	3.26	9.77	8.75
time (sec)	N/A	0.022	0.017	0.004	0.	1.632	4.679	1.133

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	62	275	0	621	287	262
normalized size	1	1.	0.4	1.76	0.	3.98	1.84	1.68
time (sec)	N/A	0.123	0.024	0.017	0.	1.755	136.73	1.147

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	63	59	0	230	1408	540
normalized size	1	1.	0.75	0.7	0.	2.74	16.76	6.43
time (sec)	N/A	0.036	0.033	0.005	0.	1.807	6.338	1.183

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	61	317	0	722	345	286
normalized size	1	1.	0.33	1.72	0.	3.92	1.88	1.55
time (sec)	N/A	0.146	0.027	0.027	0.	2.066	162.342	1.127

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	0	340	313	545
normalized size	1	1.	0.76	0.75	0.	3.3	3.04	5.29
time (sec)	N/A	0.076	0.062	0.005	0.	1.637	9.159	1.146

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	172	257	0	844	405	263
normalized size	1	1.	0.78	1.16	0.	3.82	1.83	1.19
time (sec)	N/A	0.103	0.346	0.012	0.	2.452	61.418	1.627

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	273	260	431
normalized size	1	1.	0.78	0.73	0.	3.74	3.56	5.9
time (sec)	N/A	0.056	0.042	0.005	0.	1.612	5.966	1.131

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	151	215	0	732	348	223
normalized size	1	1.	0.8	1.14	0.	3.89	1.85	1.19
time (sec)	N/A	0.088	0.295	0.007	0.	1.999	40.913	1.154

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	211	209	304
normalized size	1	1.	0.74	0.67	0.	4.59	4.54	6.61
time (sec)	N/A	0.035	0.026	0.005	0.	1.628	3.58	1.097

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	130	166	0	603	316	181
normalized size	1	1.	0.87	1.11	0.	4.05	2.12	1.21
time (sec)	N/A	0.054	0.22	0.005	0.	1.819	25.858	1.209

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	85	0	529	88	131
normalized size	1	1.	0.93	0.89	0.	5.57	0.93	1.38
time (sec)	N/A	0.064	0.093	0.007	0.	1.679	48.334	1.138

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	158	0	547	306	197
normalized size	1	1.	0.92	1.16	0.	4.02	2.25	1.45
time (sec)	N/A	0.053	0.334	0.008	0.	1.665	16.838	1.127

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	105	161	0	527	296	188
normalized size	1	1.	0.78	1.19	0.	3.9	2.19	1.39
time (sec)	N/A	0.1	0.07	0.008	0.	1.652	30.728	1.144

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	84	204	0	514	299	321
normalized size	1	1.	0.58	1.4	0.	3.52	2.05	2.2
time (sec)	N/A	0.06	0.037	0.01	0.	1.67	10.45	1.145

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	60	213	0	516	279	231
normalized size	1	1.	0.42	1.49	0.	3.61	1.95	1.62
time (sec)	N/A	0.101	0.031	0.009	0.	1.725	61.969	1.157

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	84	251	0	525	292	433
normalized size	1	1.	0.55	1.65	0.	3.45	1.92	2.85
time (sec)	N/A	0.062	0.036	0.011	0.	1.805	7.31	1.17

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	61	266	0	559	306	225
normalized size	1	1.	0.41	1.79	0.	3.75	2.05	1.51
time (sec)	N/A	0.11	0.028	0.012	0.	1.727	90.288	1.148

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	78	155	0	560	592	432
normalized size	1	1.	0.72	1.44	0.	5.19	5.48	4.
time (sec)	N/A	0.046	0.067	0.013	0.	1.801	6.802	1.177

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	140	311	0	628	316	263
normalized size	1	1.	0.92	2.05	0.	4.13	2.08	1.73
time (sec)	N/A	0.118	0.092	0.02	0.	1.822	159.292	1.164

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	223	1489	616
normalized size	1	1.	0.75	0.7	0.	4.21	28.09	11.62
time (sec)	N/A	0.022	0.017	0.004	0.	1.863	8.26	1.174

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	62	353	0	761	0	311
normalized size	1	1.	0.33	1.87	0.	4.03	0.	1.65
time (sec)	N/A	0.148	0.026	0.036	0.	2.094	0.	1.172

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	78	77	0	173	172	140
normalized size	1	1.	0.78	0.77	0.	1.73	1.72	1.4
time (sec)	N/A	0.076	0.055	0.005	0.	1.593	1.378	1.104

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	143	0	490	235	144
normalized size	1	1.	0.82	1.17	0.	4.02	1.93	1.18
time (sec)	N/A	0.053	0.095	0.009	0.	1.715	9.834	1.137

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	53	0	117	121	99
normalized size	1	1.	0.79	0.75	0.	1.65	1.7	1.39
time (sec)	N/A	0.055	0.036	0.004	0.	1.637	0.844	1.15

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	101	0	382	150	101
normalized size	1	1.	0.83	1.13	0.	4.29	1.69	1.13
time (sec)	N/A	0.036	0.056	0.008	0.	1.628	6.037	1.133

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	30	0	69	70	58
normalized size	1	1.	0.77	0.7	0.	1.6	1.63	1.35
time (sec)	N/A	0.034	0.022	0.003	0.	1.565	0.506	1.119

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	62	0	275	126	65
normalized size	1	1.	0.98	1.07	0.	4.74	2.17	1.12
time (sec)	N/A	0.017	0.017	0.005	0.	1.557	2.49	1.116

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	45	0	246	61	51
normalized size	1	1.	1.	1.05	0.	5.72	1.42	1.19
time (sec)	N/A	0.033	0.02	0.008	0.	1.587	6.524	1.13

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	41	0	255	99	78
normalized size	1	1.	1.	0.87	0.	5.43	2.11	1.66
time (sec)	N/A	0.018	0.017	0.008	0.	1.581	1.22	1.122

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	79	0	301	66	84
normalized size	1	1.	1.03	1.36	0.	5.19	1.14	1.45
time (sec)	N/A	0.046	0.032	0.007	0.	1.648	13.178	1.12

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	81	70	162
normalized size	1	1.	0.74	0.68	0.	1.53	1.32	3.06
time (sec)	N/A	0.02	0.013	0.005	0.	1.612	1.683	1.138

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	83	119	0	406	150	163
normalized size	1	1.	0.92	1.32	0.	4.51	1.67	1.81
time (sec)	N/A	0.068	0.156	0.007	0.	1.702	28.918	1.114

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	62	59	0	130	355	238
normalized size	1	1.	0.74	0.7	0.	1.55	4.23	2.83
time (sec)	N/A	0.035	0.019	0.006	0.	1.712	2.345	1.142

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	61	161	0	514	235	213
normalized size	1	1.	0.5	1.31	0.	4.18	1.91	1.73
time (sec)	N/A	0.094	0.02	0.01	0.	1.764	48.634	1.127

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	84	83	0	185	819	313
normalized size	1	1.	0.72	0.71	0.	1.58	7.	2.68
time (sec)	N/A	0.047	0.029	0.005	0.	1.69	3.315	1.118

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	131	185	0	717	233	184
normalized size	1	1.	0.86	1.22	0.	4.72	1.53	1.21
time (sec)	N/A	0.068	0.146	0.015	0.	1.787	17.798	1.126

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	77	77	0	186	172	131
normalized size	1	1.	0.78	0.78	0.	1.88	1.74	1.32
time (sec)	N/A	0.076	0.051	0.006	0.	1.612	1.524	1.143

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	108	141	0	603	177	140
normalized size	1	1.	0.91	1.18	0.	5.07	1.49	1.18
time (sec)	N/A	0.051	0.119	0.008	0.	1.62	10.778	1.117

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	55	52	0	131	117	88
normalized size	1	1.	0.82	0.78	0.	1.96	1.75	1.31
time (sec)	N/A	0.056	0.032	0.003	0.	1.539	0.928	1.127

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	97	0	482	114	95
normalized size	1	1.	1.04	1.17	0.	5.81	1.37	1.14
time (sec)	N/A	0.057	0.098	0.006	0.	1.628	6.297	1.13

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	30	0	80	66	46
normalized size	1	1.	0.73	0.73	0.	1.95	1.61	1.12
time (sec)	N/A	0.032	0.021	0.003	0.	1.541	0.589	1.108

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	70	54	0	370	60	69
normalized size	1	1.	1.3	1.	0.	6.85	1.11	1.28
time (sec)	N/A	0.018	0.053	0.005	0.	1.531	3.629	1.135

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	60	0	365	48	70
normalized size	1	1.	1.	1.13	0.	6.89	0.91	1.32
time (sec)	N/A	0.039	0.033	0.006	0.	1.611	8.915	1.119

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	85	68	77
normalized size	1	1.	0.77	0.77	0.	1.81	1.45	1.64
time (sec)	N/A	0.019	0.012	0.005	0.	1.495	4.804	1.121

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	57	109	0	506	262	134
normalized size	1	1.	0.66	1.27	0.	5.88	3.05	1.56
time (sec)	N/A	0.067	0.018	0.009	0.	1.715	20.967	1.136

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	61	58	0	143	284	244
normalized size	1	1.	0.74	0.71	0.	1.74	3.46	2.98
time (sec)	N/A	0.031	0.018	0.005	0.	1.554	8.175	1.134

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	60	153	0	628	180	185
normalized size	1	1.02	0.51	1.3	0.	5.32	1.53	1.57
time (sec)	N/A	0.088	0.019	0.01	0.	1.643	41.586	1.134

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	60	83	0	194	593	397
normalized size	1	1.	0.52	0.72	0.	1.69	5.16	3.45
time (sec)	N/A	0.045	0.023	0.005	0.	1.579	14.523	1.155

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	62	197	0	741	236	243
normalized size	1	1.	0.41	1.29	0.	4.84	1.54	1.59
time (sec)	N/A	0.117	0.021	0.01	0.	1.659	72.196	1.167

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	71	107	0	246	1030	549
normalized size	1	1.	0.48	0.72	0.	1.66	6.96	3.71
time (sec)	N/A	0.061	0.027	0.005	0.	1.78	25.465	1.174

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	98	101	0	259	437	167
normalized size	1	1.	0.77	0.79	0.	2.02	3.41	1.3
time (sec)	N/A	0.102	0.066	0.006	0.	1.623	2.932	1.131

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	139	181	0	856	804	200
normalized size	1	1.	0.93	1.21	0.	5.74	5.4	1.34
time (sec)	N/A	0.062	0.231	0.017	0.	1.759	29.448	1.151

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	76	0	201	337	124
normalized size	1	1.	0.75	0.78	0.	2.07	3.47	1.28
time (sec)	N/A	0.076	0.05	0.004	0.	1.563	1.827	1.129

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	116	134	0	732	675	151
normalized size	1	1.	1.02	1.18	0.	6.42	5.92	1.32
time (sec)	N/A	0.09	0.249	0.007	0.	1.975	17.879	1.132

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	54	53	0	155	240	82
normalized size	1	1.	0.79	0.78	0.	2.28	3.53	1.21
time (sec)	N/A	0.054	0.035	0.006	0.	1.761	1.159	1.113

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	96	92	0	531	352	93
normalized size	1	1.	1.25	1.19	0.	6.9	4.57	1.21
time (sec)	N/A	0.03	0.156	0.008	0.	1.639	12.432	1.123

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	30	68	111	143	43
normalized size	1	1.	0.77	0.68	1.55	2.52	3.25	0.98
time (sec)	N/A	0.033	0.023	0.003	1.414	1.536	1.065	1.098

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	92	115	144	54
normalized size	1	1.	0.79	0.72	1.96	2.45	3.06	1.15
time (sec)	N/A	0.01	0.013	0.003	1.244	1.548	10.201	1.142

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	75	0	521	66	89
normalized size	1	1.	0.85	1.04	0.	7.24	0.92	1.24
time (sec)	N/A	0.049	0.02	0.007	0.	1.986	17.791	1.125

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	59	0	161	265	136
normalized size	1	1.	0.78	0.77	0.	2.09	3.44	1.77
time (sec)	N/A	0.027	0.02	0.004	0.	1.84	23.141	1.146

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	57	140	0	756	1608	136
normalized size	1	1.	0.5	1.24	0.	6.69	14.23	1.2
time (sec)	N/A	0.087	0.021	0.01	0.	1.707	49.816	1.154

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	79	82	0	209	524	302
normalized size	1	1.	0.73	0.76	0.	1.94	4.85	2.8
time (sec)	N/A	0.049	0.025	0.006	0.	1.634	43.603	1.153

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	150	60	187	0	880	1323	223
normalized size	1	1.03	0.41	1.28	0.	6.03	9.06	1.53
time (sec)	N/A	0.11	0.021	0.011	0.	1.738	106.168	1.197

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	72	107	0	267	944	454
normalized size	1	1.	0.49	0.73	0.	1.83	6.47	3.11
time (sec)	N/A	0.057	0.038	0.004	0.	1.731	108.123	1.16

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	132	149	0	398	389	248
normalized size	1	1.	0.84	0.95	0.	2.54	2.48	1.58
time (sec)	N/A	0.129	0.092	0.008	0.	1.614	2.74	1.132

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	99	108	0	304	308	192
normalized size	1	1.	0.87	0.95	0.	2.67	2.7	1.68
time (sec)	N/A	0.096	0.076	0.007	0.	1.636	1.598	1.136

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	225	226	130
normalized size	1	1.	0.87	0.9	0.	2.92	2.94	1.69
time (sec)	N/A	0.06	0.042	0.008	0.	1.593	0.79	1.544

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	93	100	0	481	90	136
normalized size	1	1.	1.01	1.09	0.	5.23	0.98	1.48
time (sec)	N/A	0.084	0.098	0.01	0.	1.655	25.106	1.126

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	87	132	0	474	148	120
normalized size	1	1.	0.8	1.21	0.	4.35	1.36	1.1
time (sec)	N/A	0.087	0.077	0.01	0.	1.696	30.815	1.101

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	140	104	207	0	497	219	207
normalized size	1	0.98	0.73	1.45	0.	3.48	1.53	1.45
time (sec)	N/A	0.16	0.075	0.01	0.	1.683	73.117	1.112

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	142	281	0	608	291	300
normalized size	1	1.	0.95	1.89	0.	4.08	1.95	2.01
time (sec)	N/A	0.149	0.118	0.013	0.	1.746	88.973	1.151

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	188	157	259	0	767	411	235
normalized size	1	0.98	0.82	1.36	0.	4.02	2.15	1.23
time (sec)	N/A	0.186	0.102	0.011	0.	1.934	16.701	1.127

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	122	190	0	585	291	173
normalized size	1	1.	0.82	1.28	0.	3.93	1.95	1.16
time (sec)	N/A	0.095	0.066	0.007	0.	1.716	10.148	1.112

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	130	99	163	0	483	219	170
normalized size	1	0.98	0.74	1.23	0.	3.63	1.65	1.28
time (sec)	N/A	0.086	0.091	0.011	0.	1.648	6.552	1.126

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	91	122	0	473	170	254
normalized size	1	1.	0.82	1.1	0.	4.26	1.53	2.29
time (sec)	N/A	0.065	0.096	0.009	0.	1.602	4.096	1.124

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	123	0	512	199	544
normalized size	1	1.	1.01	1.19	0.	4.97	1.93	5.28
time (sec)	N/A	0.058	0.084	0.012	0.	1.66	3.446	1.163

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	100	76	78	0	238	510	662
normalized size	1	1.01	0.77	0.79	0.	2.4	5.15	6.69
time (sec)	N/A	0.075	0.026	0.004	0.	1.832	3.629	1.158

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	144	108	117	0	316	1061	782
normalized size	1	1.01	0.76	0.82	0.	2.21	7.42	5.47
time (sec)	N/A	0.132	0.066	0.007	0.	2.58	4.862	1.152

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	190	141	158	0	410	1856	902
normalized size	1	1.01	0.75	0.84	0.	2.17	9.82	4.77
time (sec)	N/A	0.171	0.078	0.008	0.	3.333	6.921	1.192

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	278	225	389	0	1116	598	355
normalized size	1	0.99	0.8	1.38	0.	3.97	2.13	1.26
time (sec)	N/A	0.266	0.143	0.02	0.	3.38	62.218	1.151

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	108	0	398	384	437
normalized size	1	1.	0.88	0.95	0.	3.49	3.37	3.83
time (sec)	N/A	0.09	0.089	0.006	0.	1.51	4.19	1.126

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	232	193	321	0	953	505	296
normalized size	1	0.99	0.82	1.37	0.	4.06	2.15	1.26
time (sec)	N/A	0.218	0.117	0.007	0.	1.963	41.306	1.114

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	305	303	315
normalized size	1	1.	0.87	0.9	0.	3.96	3.94	4.09
time (sec)	N/A	0.06	0.046	0.005	0.	1.392	2.383	1.125

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	159	249	0	776	440	236
normalized size	1	1.	0.81	1.27	0.	3.96	2.24	1.2
time (sec)	N/A	0.124	0.092	0.008	0.	1.829	26.212	1.12

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	110	115	0	648	109	163
normalized size	1	1.	0.99	1.04	0.	5.84	0.98	1.47
time (sec)	N/A	0.094	0.076	0.01	0.	1.455	59.476	1.122

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	172	135	221	0	660	367	234
normalized size	1	0.98	0.77	1.26	0.	3.77	2.1	1.34
time (sec)	N/A	0.119	0.146	0.012	0.	1.388	16.626	1.138

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	108	161	0	610	303	170
normalized size	1	1.	0.79	1.18	0.	4.49	2.23	1.25
time (sec)	N/A	0.109	0.088	0.011	0.	1.412	40.15	1.139

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	181	118	241	0	603	352	354
normalized size	1	0.98	0.64	1.31	0.	3.28	1.91	1.92
time (sec)	N/A	0.13	0.092	0.011	0.	1.384	10.932	1.153

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	178	116	256	0	605	332	246
normalized size	1	0.98	0.64	1.41	0.	3.34	1.83	1.36
time (sec)	N/A	0.207	0.097	0.011	0.	1.511	89.426	1.162

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	113	203	0	609	304	549
normalized size	1	1.	0.77	1.38	0.	4.14	2.07	3.73
time (sec)	N/A	0.09	0.105	0.012	0.	1.505	7.248	1.212

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	184	92	335	0	674	367	350
normalized size	1	0.98	0.49	1.79	0.	3.6	1.96	1.87
time (sec)	N/A	0.222	0.047	0.013	0.	1.494	118.042	1.169

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	99	108	0	490	468	749
normalized size	1	1.	0.87	0.95	0.	4.3	4.11	6.57
time (sec)	N/A	0.09	0.098	0.007	0.	1.432	9.425	1.144

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	278	226	383	0	1131	602	358
normalized size	1	0.99	0.8	1.36	0.	4.02	2.14	1.27
time (sec)	N/A	0.258	0.147	0.014	0.	2.73	77.37	1.13

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	389	384	564
normalized size	1	1.	0.87	0.9	0.	5.05	4.99	7.32
time (sec)	N/A	0.059	0.05	0.003	0.	1.401	6.138	1.147

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	192	308	0	963	537	298
normalized size	1	1.	0.8	1.28	0.	4.01	2.24	1.24
time (sec)	N/A	0.152	0.123	0.008	0.	1.915	50.072	1.164

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	132	0	824	128	190
normalized size	1	1.	0.93	1.	0.	6.24	0.97	1.44
time (sec)	N/A	0.111	0.108	0.009	0.	1.458	85.568	1.141

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	214	174	278	0	855	496	296
normalized size	1	0.99	0.8	1.28	0.	3.94	2.29	1.36
time (sec)	N/A	0.141	0.116	0.009	0.	1.714	33.549	1.129

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	122	193	0	807	518	223
normalized size	1	1.	0.75	1.19	0.	4.98	3.2	1.38
time (sec)	N/A	0.132	0.187	0.01	0.	1.465	50.403	1.136

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	219	155	298	0	776	490	414
normalized size	1	0.98	0.7	1.34	0.	3.48	2.2	1.86
time (sec)	N/A	0.175	0.119	0.012	0.	1.527	20.904	1.188

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	219	153	305	0	740	473	327
normalized size	1	0.99	0.69	1.37	0.	3.33	2.13	1.47
time (sec)	N/A	0.253	0.104	0.01	0.	1.444	98.836	1.144

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	225	158	369	0	738	474	689
normalized size	1	0.99	0.69	1.62	0.	3.24	2.08	3.02
time (sec)	N/A	0.16	0.155	0.014	0.	1.522	13.67	1.156

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	219	92	387	0	778	468	386
normalized size	1	0.99	0.41	1.74	0.	3.5	2.11	1.74
time (sec)	N/A	0.252	0.06	0.014	0.	1.49	131.077	1.188

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	265	0	782	422	240
normalized size	1	1.	0.82	1.37	0.	4.03	2.18	1.24
time (sec)	N/A	0.155	0.114	0.016	0.	1.625	18.357	1.131

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	108	0	231	240	203
normalized size	1	1.	0.88	0.96	0.	2.06	2.14	1.81
time (sec)	N/A	0.089	0.071	0.007	0.	1.333	1.484	1.131

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	125	197	0	605	301	182
normalized size	1	1.	0.86	1.35	0.	4.14	2.06	1.25
time (sec)	N/A	0.14	0.086	0.009	0.	1.553	11.646	1.158

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	69	0	151	158	132
normalized size	1	1.	0.89	0.93	0.	2.04	2.14	1.78
time (sec)	N/A	0.054	0.041	0.006	0.	1.35	0.895	1.123

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	91	131	0	440	238	123
normalized size	1	1.	0.85	1.22	0.	4.11	2.22	1.15
time (sec)	N/A	0.059	0.052	0.007	0.	1.384	6.202	1.128

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	63	87	0	365	76	111
normalized size	1	1.	0.84	1.16	0.	4.87	1.01	1.48
time (sec)	N/A	0.071	0.062	0.009	0.	1.337	25.22	1.139

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	88	0	374	155	126
normalized size	1	1.	0.93	1.07	0.	4.56	1.89	1.54
time (sec)	N/A	0.045	0.064	0.009	0.	1.331	3.349	1.137

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	100	0	387	99	109
normalized size	1	1.	0.96	1.25	0.	4.84	1.24	1.36
time (sec)	N/A	0.068	0.053	0.01	0.	1.469	43.084	1.121

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	72	85	0	390	158	211
normalized size	1	1.	0.86	1.01	0.	4.64	1.88	2.51
time (sec)	N/A	0.05	0.071	0.009	0.	1.319	2.209	1.168

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	92	157	0	463	178	189
normalized size	1	1.	0.87	1.48	0.	4.37	1.68	1.78
time (sec)	N/A	0.106	0.086	0.012	0.	1.42	77.579	1.132

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	100	74	78	0	163	391	421
normalized size	1	1.01	0.75	0.79	0.	1.65	3.95	4.25
time (sec)	N/A	0.072	0.023	0.004	0.	1.368	3.167	1.152

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	135	224	0	630	301	325
normalized size	1	1.	0.89	1.48	0.	4.17	1.99	2.15
time (sec)	N/A	0.161	0.232	0.012	0.	1.547	140.095	1.174

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	158	263	0	950	0	236
normalized size	1	1.	0.8	1.34	0.	4.82	0.	1.2
time (sec)	N/A	0.149	0.146	0.015	0.	1.692	0.	1.133

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	108	0	246	236	180
normalized size	1	1.	0.9	1.	0.	2.28	2.19	1.67
time (sec)	N/A	0.086	0.061	0.007	0.	1.319	1.741	1.134

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	124	192	0	759	0	177
normalized size	1	1.	0.82	1.26	0.	4.99	0.	1.16
time (sec)	N/A	0.117	0.111	0.008	0.	1.501	0.	1.128

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	69	0	165	155	108
normalized size	1	1.	0.89	0.95	0.	2.26	2.12	1.48
time (sec)	N/A	0.056	0.038	0.007	0.	1.358	1.011	1.137

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	123	0	595	0	124
normalized size	1	1.	0.88	1.16	0.	5.61	0.	1.17
time (sec)	N/A	0.054	0.1	0.005	0.	1.368	0.	1.131

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	102	0	489	70	111
normalized size	1	1.	0.83	1.36	0.	6.52	0.93	1.48
time (sec)	N/A	0.075	0.04	0.009	0.	1.344	17.924	1.137

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	87	81	99	0	504	0	140
normalized size	1	0.96	0.89	1.09	0.	5.54	0.	1.54
time (sec)	N/A	0.063	0.085	0.009	0.	1.38	0.	1.15

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	96	135	0	620	0	189
normalized size	1	1.	0.93	1.31	0.	6.02	0.	1.83
time (sec)	N/A	0.098	0.083	0.009	0.	1.409	0.	1.16

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	74	77	0	173	0	269
normalized size	1	1.01	0.76	0.79	0.	1.78	0.	2.77
time (sec)	N/A	0.072	0.024	0.004	0.	1.37	0.	1.138

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	89	211	0	782	0	220
normalized size	1	1.	0.61	1.46	0.	5.39	0.	1.52
time (sec)	N/A	0.167	0.034	0.011	0.	1.416	0.	1.172

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	105	117	0	258	0	610
normalized size	1	1.	0.74	0.83	0.	1.83	0.	4.33
time (sec)	N/A	0.114	0.081	0.007	0.	1.505	0.	1.172

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	193	92	281	0	976	0	360
normalized size	1	1.02	0.48	1.48	0.	5.14	0.	1.89
time (sec)	N/A	0.218	0.037	0.012	0.	1.551	0.	1.153

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	156	255	0	1127	0	257
normalized size	1	1.	0.77	1.26	0.	5.58	0.	1.27
time (sec)	N/A	0.155	0.13	0.019	0.	1.584	0.	1.168

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	98	108	0	252	454	173
normalized size	1	1.	0.89	0.98	0.	2.29	4.13	1.57
time (sec)	N/A	0.09	0.059	0.005	0.	1.375	1.977	1.151

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	118	185	0	869	0	176
normalized size	1	1.	0.98	1.53	0.	7.18	0.	1.45
time (sec)	N/A	0.108	0.112	0.008	0.	1.52	0.	1.132

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	68	0	182	303	105
normalized size	1	1.	0.93	0.94	0.	2.53	4.21	1.46
time (sec)	N/A	0.058	0.038	0.005	0.	1.356	1.277	1.163

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	101	136	0	655	0	142
normalized size	1	1.	0.96	1.3	0.	6.24	0.	1.35
time (sec)	N/A	0.05	0.123	0.005	0.	1.38	0.	1.155

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	120	0	651	87	138
normalized size	1	1.	0.76	1.36	0.	7.4	0.99	1.57
time (sec)	N/A	0.092	0.045	0.011	0.	1.464	23.779	1.162

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	78	0	188	0	158
normalized size	1	1.	0.84	0.87	0.	2.09	0.	1.76
time (sec)	N/A	0.046	0.025	0.005	0.	1.303	0.	1.173

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	105	169	0	894	0	173
normalized size	1	1.	0.8	1.29	0.	6.82	0.	1.32
time (sec)	N/A	0.117	0.045	0.012	0.	1.444	0.	1.144

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	130	107	116	0	258	0	348
normalized size	1	0.99	0.82	0.89	0.	1.97	0.	2.66
time (sec)	N/A	0.125	0.072	0.005	0.	1.556	0.	1.158

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	90	265	0	1150	0	284
normalized size	1	1.	0.49	1.43	0.	6.22	0.	1.54
time (sec)	N/A	0.216	0.036	0.013	0.	1.537	0.	1.175

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	142	158	0	359	0	687
normalized size	1	1.	0.78	0.86	0.	1.96	0.	3.75
time (sec)	N/A	0.166	0.099	0.006	0.	1.773	0.	1.193

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	0	315	0	95
normalized size	1	1.	0.78	0.74	0.	4.38	0.	1.32
time (sec)	N/A	0.026	0.024	0.007	0.	1.335	0.	1.122

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	38	0	257	0	62
normalized size	1	1.	0.85	0.73	0.	4.94	0.	1.19
time (sec)	N/A	0.016	0.016	0.004	0.	1.25	0.	1.117

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	24	0	205	0	31
normalized size	1	1.	1.	0.71	0.	6.03	0.	0.91
time (sec)	N/A	0.008	0.007	0.004	0.	1.351	0.	1.121

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	36	0	273	0	65
normalized size	1	1.	0.92	0.72	0.	5.46	0.	1.3
time (sec)	N/A	0.017	0.016	0.006	0.	1.283	0.	1.096

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	58	0	331	0	92
normalized size	1	1.	0.85	0.85	0.	4.87	0.	1.35
time (sec)	N/A	0.024	0.022	0.01	0.	1.363	0.	1.118

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	148	1088	0	1885	0	254
normalized size	1	1.	0.94	6.93	0.	12.01	0.	1.62
time (sec)	N/A	0.231	0.212	0.019	0.	2.391	0.	1.153

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	963	0	636	87	130
normalized size	1	1.	0.97	10.94	0.	7.23	0.99	1.48
time (sec)	N/A	0.081	0.064	0.011	0.	1.534	5.633	1.136

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	108	1010	0	1544	0	185
normalized size	1	1.	0.96	9.02	0.	13.79	0.	1.65
time (sec)	N/A	0.096	0.124	0.01	0.	1.986	0.	1.153

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	936	0	539	61	86
normalized size	1	1.	1.	14.4	0.	8.29	0.94	1.32
time (sec)	N/A	0.054	0.026	0.007	0.	1.432	3.426	1.112

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	948	0	1288	0	151
normalized size	1	1.	1.04	11.7	0.	15.9	0.	1.86
time (sec)	N/A	0.044	0.032	0.008	0.	1.729	0.	1.146

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	984	0	1261	78	117
normalized size	1	1.	0.98	12.3	0.	15.76	0.98	1.46
time (sec)	N/A	0.073	0.034	0.01	0.	1.824	6.257	1.139

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	1017	0	570	0	158
normalized size	1	1.	0.73	14.53	0.	8.14	0.	2.26
time (sec)	N/A	0.051	0.019	0.017	0.	1.397	0.	1.906

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	1054	0	1578	0	163
normalized size	1	1.	0.95	9.33	0.	13.96	0.	1.44
time (sec)	N/A	0.12	0.122	0.013	0.	1.953	0.	1.128

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	1059	0	678	0	290
normalized size	1	1.	0.89	10.09	0.	6.46	0.	2.76
time (sec)	N/A	0.12	5.121	0.013	0.	1.681	0.	2.183

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	196	2081	0	2427	0	358
normalized size	1	1.	0.93	9.91	0.	11.56	0.	1.7
time (sec)	N/A	0.403	0.209	0.019	0.	13.03	0.	1.157

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	1897	0	846	104	204
normalized size	1	1.	0.94	16.5	0.	7.36	0.9	1.77
time (sec)	N/A	0.106	0.091	0.013	0.	1.83	30.734	1.62

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	1973	0	1959	0	267
normalized size	1	1.	0.88	12.49	0.	12.4	0.	1.69
time (sec)	N/A	0.252	0.163	0.012	0.	4.886	0.	1.131

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	83	1856	0	655	80	151
normalized size	1	1.	0.91	20.4	0.	7.2	0.88	1.66
time (sec)	N/A	0.076	0.067	0.011	0.	1.77	17.617	1.113

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	110	1875	0	1582	0	205
normalized size	1	1.	0.97	16.59	0.	14.	0.	1.81
time (sec)	N/A	0.096	0.162	0.009	0.	2.763	0.	1.139

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	102	1919	0	1493	92	158
normalized size	1	1.	1.06	19.99	0.	15.55	0.96	1.65
time (sec)	N/A	0.107	0.062	0.011	0.	3.852	18.17	1.16

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	105	1956	0	1553	0	220
normalized size	1	1.	1.03	19.18	0.	15.23	0.	2.16
time (sec)	N/A	0.091	0.133	0.011	0.	2.492	0.	1.188

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	2003	0	1628	0	182
normalized size	1	1.	0.95	17.57	0.	14.28	0.	1.6
time (sec)	N/A	0.141	0.111	0.012	0.	3.755	0.	1.149

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	53	2089	0	701	0	346
normalized size	1	1.	0.52	20.48	0.	6.87	0.	3.39
time (sec)	N/A	0.126	0.017	0.013	0.	1.825	0.	2.862

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	247	3373	0	3162	0	483
normalized size	1	1.	0.85	11.59	0.	10.87	0.	1.66
time (sec)	N/A	0.575	0.244	0.019	0.	48.843	0.	1.68

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	3127	0	1137	144	308
normalized size	1	1.	0.94	21.72	0.	7.9	1.	2.14
time (sec)	N/A	0.15	0.279	0.011	0.	1.881	62.094	1.14

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	187	3235	0	2535	0	373
normalized size	1	1.	0.86	14.91	0.	11.68	0.	1.72
time (sec)	N/A	0.392	0.145	0.012	0.	16.797	0.	1.145

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	3078	0	873	117	248
normalized size	1	1.	0.95	25.87	0.	7.34	0.98	2.08
time (sec)	N/A	0.107	0.14	0.009	0.	1.889	37.289	1.14

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	140	3101	0	2022	0	290
normalized size	1	1.	0.9	19.88	0.	12.96	0.	1.86
time (sec)	N/A	0.18	0.103	0.011	0.	8.132	0.	1.144

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	114	3148	0	1829	119	227
normalized size	1	1.	0.92	25.39	0.	14.75	0.96	1.83
time (sec)	N/A	0.191	0.11	0.012	0.	9.04	39.244	1.145

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	132	3191	0	1901	0	279
normalized size	1	1.	0.91	22.01	0.	13.11	0.	1.92
time (sec)	N/A	0.197	0.089	0.013	0.	5.546	0.	1.141

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	125	3247	0	1908	0	231
normalized size	1	1.	0.87	22.55	0.	13.25	0.	1.6
time (sec)	N/A	0.244	0.13	0.012	0.	8.535	0.	1.138

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	125	3346	0	1932	0	410
normalized size	1	1.	0.96	25.74	0.	14.86	0.	3.15
time (sec)	N/A	0.178	0.096	0.013	0.	3.506	0.	1.2

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	362	0	807	0	142
normalized size	1	1.	0.89	3.62	0.	8.07	0.	1.42
time (sec)	N/A	0.106	0.119	0.015	0.	1.433	0.	1.641

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	318	0	648	0	86
normalized size	1	1.	1.	4.68	0.	9.53	0.	1.26
time (sec)	N/A	0.064	0.054	0.011	0.	1.533	0.	1.125

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	300	0	498	36	53
normalized size	1	1.	1.	6.12	0.	10.16	0.73	1.08
time (sec)	N/A	0.043	0.015	0.008	0.	1.383	3.767	1.136

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	331	0	1315	63	107
normalized size	1	1.	0.98	4.14	0.	16.44	0.79	1.34
time (sec)	N/A	0.074	0.075	0.01	0.	1.807	6.802	1.117

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	385	0	1620	0	159
normalized size	1	1.	0.95	3.35	0.	14.09	0.	1.38
time (sec)	N/A	0.115	0.274	0.011	0.	2.225	0.	1.128

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	112	386	0	1593	0	184
normalized size	1	1.	0.98	3.39	0.	13.97	0.	1.61
time (sec)	N/A	0.094	0.116	0.012	0.	2.061	0.	1.152

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	85	337	0	1337	0	138
normalized size	1	1.	1.04	4.11	0.	16.3	0.	1.68
time (sec)	N/A	0.05	0.048	0.01	0.	1.878	0.	1.129

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	513	0	95
normalized size	1	1.	1.	6.24	0.	10.47	0.	1.94
time (sec)	N/A	0.019	0.013	0.01	0.	1.723	0.	1.14

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	177	334	0	682	0	150
normalized size	1	1.	2.39	4.51	0.	9.22	0.	2.03
time (sec)	N/A	0.053	3.157	0.011	0.	1.762	0.	1.141

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	96	379	0	849	0	263
normalized size	1	1.	0.87	3.45	0.	7.72	0.	2.39
time (sec)	N/A	0.123	5.107	0.012	0.	1.996	0.	1.893

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	111	720	0	2066	0	201
normalized size	1	1.	1.02	6.61	0.	18.95	0.	1.84
time (sec)	N/A	0.101	0.166	0.016	0.	3.005	0.	1.142

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	653	0	892	0	105
normalized size	1	1.	1.12	8.48	0.	11.58	0.	1.36
time (sec)	N/A	0.072	0.061	0.012	0.	1.557	0.	1.137

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	111	653	0	698	0	139
normalized size	1	1.	1.5	8.82	0.	9.43	0.	1.88
time (sec)	N/A	0.049	0.32	0.011	0.	1.885	0.	1.189

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	618	0	678	61	96
normalized size	1	1.	0.69	8.58	0.	9.42	0.85	1.33
time (sec)	N/A	0.056	0.015	0.01	0.	1.4	8.342	1.135

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	236	628	0	914	0	144
normalized size	1	1.	2.99	7.95	0.	11.57	0.	1.82
time (sec)	N/A	0.037	1.569	0.009	0.	2.011	0.	1.146

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	87	681	0	2030	94	158
normalized size	1	1.	0.81	6.36	0.	18.97	0.88	1.48
time (sec)	N/A	0.112	0.029	0.012	0.	3.328	10.612	1.129

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	102	695	0	1139	0	205
normalized size	1	1.	0.82	5.6	0.	9.19	0.	1.65
time (sec)	N/A	0.117	5.188	0.012	0.	2.617	0.	2.463

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	763	0	2649	0	248
normalized size	1	1.	0.75	4.89	0.	16.98	0.	1.59
time (sec)	N/A	0.216	0.047	0.012	0.	5.522	0.	1.151

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	124	762	0	1412	0	371
normalized size	1	1.	0.7	4.33	0.	8.02	0.	2.11
time (sec)	N/A	0.218	5.206	0.014	0.	3.302	0.	3.697

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	160	1207	0	1088	0	410
normalized size	1	1.	1.37	10.32	0.	9.3	0.	3.5
time (sec)	N/A	0.113	0.303	0.017	0.	4.192	0.	1.12

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	1123	0	1106	0	171
normalized size	1	1.	0.75	10.9	0.	10.74	0.	1.66
time (sec)	N/A	0.092	0.029	0.013	0.	1.86	0.	1.144

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	257	1134	0	1130	0	393
normalized size	1	1.	2.23	9.86	0.	9.83	0.	3.42
time (sec)	N/A	0.088	2.832	0.011	0.	3.686	0.	1.15

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	52	1086	0	1061	85	159
normalized size	1	1.	0.53	11.08	0.	10.83	0.87	1.62
time (sec)	N/A	0.085	0.022	0.009	0.	1.868	13.899	1.108

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	1385	1086	0	1530	0	433
normalized size	1	1.	11.35	8.9	0.	12.54	0.	3.55
time (sec)	N/A	0.101	5.461	0.01	0.	4.223	0.	1.162

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	90	1186	0	3514	133	242
normalized size	1	1.	0.62	8.18	0.	24.23	0.92	1.67
time (sec)	N/A	0.185	0.035	0.011	0.	9.386	17.075	1.172

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	143	1192	0	1868	0	494
normalized size	1	1.	0.8	6.7	0.	10.49	0.	2.78
time (sec)	N/A	0.236	5.252	0.014	0.	5.053	0.	3.228

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	118	1289	0	4504	0	294
normalized size	1	1.	0.56	6.11	0.	21.35	0.	1.39
time (sec)	N/A	0.322	0.053	0.013	0.	19.038	0.	1.157

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	160	1285	0	2218	0	662
normalized size	1	1.	0.65	5.24	0.	9.05	0.	2.7
time (sec)	N/A	0.361	5.344	0.017	0.	7.204	0.	3.806

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	134	2615	0	2172	0	389
normalized size	1	1.	0.89	17.43	0.	14.48	0.	2.59
time (sec)	N/A	0.169	0.175	0.022	0.	2.777	0.	1.21

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	117	2543	0	921	0	150
normalized size	1	1.	0.86	18.7	0.	6.77	0.	1.1
time (sec)	N/A	0.111	0.152	0.011	0.	1.837	0.	1.155

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	118	2547	0	2260	0	339
normalized size	1	1.	0.98	21.22	0.	18.83	0.	2.82
time (sec)	N/A	0.083	0.08	0.011	0.	2.503	0.	1.165

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	1617	0	747	0	107
normalized size	1	1.	1.	20.21	0.	9.34	0.	1.34
time (sec)	N/A	0.062	0.082	0.01	0.	1.621	0.	1.161

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	112	2559	0	765	0	294
normalized size	1	1.	1.37	31.21	0.	9.33	0.	3.59
time (sec)	N/A	0.035	0.291	0.009	0.	2.13	0.	3.04

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	112	2585	0	2221	0	170
normalized size	1	1.	0.94	21.72	0.	18.66	0.	1.43
time (sec)	N/A	0.112	0.221	0.015	0.	2.627	0.	1.141

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	2618	0	956	0	444
normalized size	1	1.	0.89	23.17	0.	8.46	0.	3.93
time (sec)	N/A	0.11	5.071	0.013	0.	2.348	0.	3.687

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	190	2669	0	2222	0	258
normalized size	1	1.	1.19	16.79	0.	13.97	0.	1.62
time (sec)	N/A	0.209	0.23	0.014	0.	3.142	0.	1.174

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	120	2667	0	1238	0	487
normalized size	1	1.	0.82	18.14	0.	8.42	0.	3.31
time (sec)	N/A	0.203	5.14	0.017	0.	2.61	0.	6.57

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	192	4795	0	2678	0	532
normalized size	1	1.	0.97	24.34	0.	13.59	0.	2.7
time (sec)	N/A	0.338	0.169	0.021	0.	3.487	0.	1.193

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	4673	0	888	0	234
normalized size	1	1.	0.77	28.67	0.	5.45	0.	1.44
time (sec)	N/A	0.136	0.105	0.016	0.	1.876	0.	1.138

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	155	4685	0	2142	0	454
normalized size	1	1.	1.04	31.44	0.	14.38	0.	3.05
time (sec)	N/A	0.164	0.165	0.012	0.	2.73	0.	1.193

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	54	2821	0	701	0	161
normalized size	1	1.	0.55	28.49	0.	7.08	0.	1.63
time (sec)	N/A	0.077	0.021	0.011	0.	1.788	0.	1.138

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	141	4689	0	1936	0	425
normalized size	1	1.	1.08	35.79	0.	14.78	0.	3.24
time (sec)	N/A	0.089	0.152	0.013	0.	2.583	0.	1.191

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	122	4718	0	1901	0	223
normalized size	1	1.	0.95	36.57	0.	14.74	0.	1.73
time (sec)	N/A	0.142	0.163	0.013	0.	3.719	0.	1.142

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	52	4764	0	732	0	556
normalized size	1	1.	0.41	37.22	0.	5.72	0.	4.34
time (sec)	N/A	0.124	0.015	0.013	0.	1.81	0.	3.789

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	142	4820	0	2195	0	300
normalized size	1	1.	0.84	28.35	0.	12.91	0.	1.76
time (sec)	N/A	0.255	0.222	0.013	0.	3.685	0.	1.149

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	131	4908	0	934	0	597
normalized size	1	1.	0.79	29.57	0.	5.63	0.	3.6
time (sec)	N/A	0.24	5.128	0.014	0.	2.344	0.	6.473

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	219	7611	0	3663	0	703
normalized size	1	1.	0.85	29.5	0.	14.2	0.	2.72
time (sec)	N/A	0.448	0.279	0.022	0.	17.524	0.	1.231

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	164	7443	0	1223	0	356
normalized size	1	1.	0.83	37.59	0.	6.18	0.	1.8
time (sec)	N/A	0.183	0.4	0.014	0.	2.141	0.	1.162

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	173	7459	0	2938	0	602
normalized size	1	1.	0.89	38.25	0.	15.07	0.	3.09
time (sec)	N/A	0.244	0.221	0.013	0.	8.829	0.	1.199

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	54	4363	0	950	0	257
normalized size	1	1.	0.43	34.63	0.	7.54	0.	2.04
time (sec)	N/A	0.103	0.02	0.011	0.	2.069	0.	1.149

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	143	7451	0	2585	0	549
normalized size	1	1.	0.82	42.82	0.	14.86	0.	3.16
time (sec)	N/A	0.219	0.187	0.012	0.	6.09	0.	1.182

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	158	7477	0	2377	0	290
normalized size	1	1.	0.99	46.73	0.	14.86	0.	1.81
time (sec)	N/A	0.218	0.196	0.016	0.	10.994	0.	1.155

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	150	7529	0	2458	0	740
normalized size	1	1.	0.89	44.82	0.	14.63	0.	4.4
time (sec)	N/A	0.189	0.146	0.014	0.	4.517	0.	1.217

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	175	7590	0	2638	0	390
normalized size	1	1.	0.97	42.17	0.	14.66	0.	2.17
time (sec)	N/A	0.27	0.433	0.017	0.	12.609	0.	1.155

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	54	7705	0	996	0	670
normalized size	1	1.	0.31	43.78	0.	5.66	0.	3.81
time (sec)	N/A	0.247	0.017	0.018	0.	2.646	0.	6.279

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	129	846	0	2236	0	383
normalized size	1	1.	0.98	6.41	0.	16.94	0.	2.9
time (sec)	N/A	0.112	0.19	0.019	0.	4.889	0.	1.187

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	807	0	936	0	157
normalized size	1	1.	0.99	8.15	0.	9.45	0.	1.59
time (sec)	N/A	0.085	0.09	0.011	0.	2.156	0.	1.126

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	124	817	0	871	0	312
normalized size	1	1.	1.39	9.18	0.	9.79	0.	3.51
time (sec)	N/A	0.053	0.424	0.01	0.	2.928	0.	3.189

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	513	0	849	0	124
normalized size	1	1.	0.98	5.9	0.	9.76	0.	1.43
time (sec)	N/A	0.068	0.088	0.009	0.	2.041	0.	1.127

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	405	823	0	954	0	304
normalized size	1	1.	4.05	8.23	0.	9.54	0.	3.04
time (sec)	N/A	0.053	0.766	0.01	0.	3.067	0.	1.14

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	123	838	0	2201	0	207
normalized size	1	1.	0.95	6.45	0.	16.93	0.	1.59
time (sec)	N/A	0.136	0.235	0.011	0.	6.067	0.	1.134

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	116	841	0	1238	0	535
normalized size	1	1.	0.79	5.72	0.	8.42	0.	3.64
time (sec)	N/A	0.136	5.154	0.013	0.	3.367	0.	3.663

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	899	0	2869	0	362
normalized size	1	1.	0.88	4.86	0.	15.51	0.	1.96
time (sec)	N/A	0.243	0.552	0.013	0.	10.805	0.	1.13

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	136	893	0	1536	0	506
normalized size	1	1.	0.66	4.33	0.	7.46	0.	2.46
time (sec)	N/A	0.252	5.234	0.015	0.	5.083	0.	6.436

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	54	1498	0	1134	0	402
normalized size	1	1.	0.42	11.52	0.	8.72	0.	3.09
time (sec)	N/A	0.102	0.021	0.019	0.	3.824	0.	5.247

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	91	1456	0	1492	0	244
normalized size	1	1.	0.68	10.87	0.	11.13	0.	1.82
time (sec)	N/A	0.119	0.03	0.012	0.	2.21	0.	1.185

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	133	1453	0	1512	0	404
normalized size	1	1.	1.08	11.81	0.	12.29	0.	3.28
time (sec)	N/A	0.091	1.059	0.01	0.	4.562	0.	5.324

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	52	989	0	1110	0	203
normalized size	1	1.	0.46	8.75	0.	9.82	0.	1.8
time (sec)	N/A	0.085	0.016	0.011	0.	2.046	0.	1.128

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	210	1461	0	1705	0	429
normalized size	1	1.	1.48	10.29	0.	12.01	0.	3.02
time (sec)	N/A	0.102	4.41	0.01	0.	5.182	0.	5.261

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	123	1672	0	4030	0	319
normalized size	1	1.	0.72	9.84	0.	23.71	0.	1.88
time (sec)	N/A	0.244	0.111	0.013	0.	21.153	0.	1.132

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	145	1524	0	2020	0	748
normalized size	1	1.	0.71	7.43	0.	9.85	0.	3.65
time (sec)	N/A	0.268	5.351	0.015	0.	6.083	0.	6.969

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	189	1778	0	5106	0	510
normalized size	1	1.	0.78	7.38	0.	21.19	0.	2.12
time (sec)	N/A	0.336	0.116	0.016	0.	41.51	0.	1.184

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	167	1608	0	2504	0	656
normalized size	1	1.	0.6	5.81	0.	9.04	0.	2.37
time (sec)	N/A	0.396	5.434	0.016	0.	8.487	0.	11.88

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	133	2463	0	2048	0	802
normalized size	1	1.	0.76	14.16	0.	11.77	0.	4.61
time (sec)	N/A	0.211	0.963	0.023	0.	7.753	0.	6.438

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	93	2400	0	2022	0	351
normalized size	1	1.	0.55	14.12	0.	11.89	0.	2.06
time (sec)	N/A	0.165	0.038	0.015	0.	2.755	0.	1.182

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	211	2369	0	2588	0	803
normalized size	1	1.	1.29	14.53	0.	15.88	0.	4.93
time (sec)	N/A	0.16	2.032	0.014	0.	8.948	0.	6.338

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	54	1639	0	1816	0	301
normalized size	1	1.	0.39	11.71	0.	12.97	0.	2.15
time (sec)	N/A	0.103	0.024	0.012	0.	2.371	0.	1.14

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	170	2405	0	2849	0	836
normalized size	1	1.	0.85	11.97	0.	14.17	0.	4.16
time (sec)	N/A	0.222	5.378	0.013	0.	11.208	0.	6.515

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	114	2837	0	6770	0	410
normalized size	1	1.	0.51	12.61	0.	30.09	0.	1.82
time (sec)	N/A	0.328	0.094	0.015	0.	47.739	0.	1.173

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	188	2513	0	3333	0	1266
normalized size	1	1.	0.67	9.01	0.	11.95	0.	4.54
time (sec)	N/A	0.443	5.534	0.018	0.	10.421	0.	9.192

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	190	2980	0	8246	0	684
normalized size	1	1.	0.62	9.8	0.	27.12	0.	2.25
time (sec)	N/A	0.484	0.12	0.016	0.	69.879	0.	1.213

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	210	2623	0	3780	0	1065
normalized size	1	1.	0.58	7.25	0.	10.44	0.	2.94
time (sec)	N/A	0.596	5.697	0.018	0.	9.494	0.	20.508

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	110	276	0	0	97	0
normalized size	1	1.	0.52	1.3	0.	0.	0.46	0.
time (sec)	N/A	0.16	0.142	0.038	0.	0.	25.69	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	93	414	0	0	95	0
normalized size	1	1.	0.28	1.23	0.	0.	0.28	0.
time (sec)	N/A	0.27	0.087	0.029	0.	0.	3.385	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	93	246	0	0	97	0
normalized size	1	1.	0.53	1.4	0.	0.	0.55	0.
time (sec)	N/A	0.111	0.06	0.023	0.	0.	3.428	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	96	391	0	0	100	0
normalized size	1	1.	0.29	1.17	0.	0.	0.3	0.
time (sec)	N/A	0.261	0.051	0.034	0.	0.	3.765	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	82	234	0	0	100	0
normalized size	1	1.	0.48	1.36	0.	0.	0.58	0.
time (sec)	N/A	0.11	0.074	0.026	0.	0.	13.646	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	95	417	0	0	107	0
normalized size	1	1.	0.28	1.23	0.	0.	0.32	0.
time (sec)	N/A	0.264	0.059	0.032	0.	0.	109.453	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	79	242	0	0	97	0
normalized size	1	1.	0.52	1.59	0.	0.	0.64	0.
time (sec)	N/A	0.093	0.094	0.038	0.	0.	139.503	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	80	439	0	0	0	0
normalized size	1	1.	0.24	1.33	0.	0.	0.	0.
time (sec)	N/A	0.233	0.094	0.041	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	80	270	0	0	0	0
normalized size	1	1.	0.43	1.44	0.	0.	0.	0.
time (sec)	N/A	0.112	0.101	0.028	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	114	300	0	0	199	0
normalized size	1	1.	0.45	1.19	0.	0.	0.79	0.
time (sec)	N/A	0.167	0.159	0.023	0.	0.	84.816	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	97	438	0	0	197	0
normalized size	1	1.	0.26	1.16	0.	0.	0.52	0.
time (sec)	N/A	0.29	0.108	0.023	0.	0.	12.593	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	96	272	0	0	199	0
normalized size	1	1.	0.45	1.27	0.	0.	0.93	0.
time (sec)	N/A	0.136	0.075	0.013	0.	0.	12.205	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	84	421	0	0	202	0
normalized size	1	1.	0.23	1.15	0.	0.	0.55	0.
time (sec)	N/A	0.295	0.072	0.017	0.	0.	14.864	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	85	255	0	0	202	0
normalized size	1	1.	0.4	1.21	0.	0.	0.96	0.
time (sec)	N/A	0.137	0.076	0.015	0.	0.	25.18	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	84	422	0	0	0	0
normalized size	1	1.	0.23	1.16	0.	0.	0.	0.
time (sec)	N/A	0.289	0.068	0.017	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	96	417	0	0	94	0
normalized size	1	1.	0.28	1.23	0.	0.	0.28	0.
time (sec)	N/A	0.258	0.112	0.025	0.	0.	98.259	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	96	250	0	0	94	0
normalized size	1	1.	0.55	1.44	0.	0.	0.54	0.
time (sec)	N/A	0.11	0.105	0.024	0.	0.	14.697	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	80	379	0	0	92	0
normalized size	1	1.	0.27	1.27	0.	0.	0.31	0.
time (sec)	N/A	0.214	0.079	0.013	0.	0.	3.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	79	214	0	0	94	0
normalized size	1	1.	0.57	1.54	0.	0.	0.68	0.
time (sec)	N/A	0.081	0.055	0.016	0.	0.	2.377	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	82	378	0	0	97	0
normalized size	1	1.	0.28	1.3	0.	0.	0.33	0.
time (sec)	N/A	0.223	0.042	0.017	0.	0.	4.858	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	81	223	0	0	97	0
normalized size	1	1.	0.59	1.62	0.	0.	0.7	0.
time (sec)	N/A	0.086	0.046	0.014	0.	0.	25.278	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	82	417	0	0	104	0
normalized size	1	1.	0.24	1.22	0.	0.	0.3	0.
time (sec)	N/A	0.251	0.046	0.018	0.	0.	136.821	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	111	252	0	0	0	0
normalized size	1	1.	0.53	1.19	0.	0.	0.	0.
time (sec)	N/A	0.137	0.143	0.035	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	84	391	0	0	0	0
normalized size	1	1.	0.25	1.16	0.	0.	0.	0.
time (sec)	N/A	0.252	0.112	0.032	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	85	225	0	0	94	0
normalized size	1	1.	0.49	1.29	0.	0.	0.54	0.
time (sec)	N/A	0.112	0.105	0.019	0.	0.	61.248	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	76	382	0	0	94	0
normalized size	1	1.	0.25	1.27	0.	0.	0.31	0.
time (sec)	N/A	0.225	0.103	0.017	0.	0.	12.405	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	75	213	0	0	94	0
normalized size	1	1.	0.52	1.48	0.	0.	0.65	0.
time (sec)	N/A	0.09	0.057	0.021	0.	0.	18.224	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	77	386	0	0	97	0
normalized size	1	1.	0.23	1.16	0.	0.	0.29	0.
time (sec)	N/A	0.257	0.042	0.022	0.	0.	49.587	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	91	232	0	0	0	0
normalized size	1	1.	0.52	1.32	0.	0.	0.	0.
time (sec)	N/A	0.113	0.055	0.02	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	78	417	0	0	0	0
normalized size	1	1.	0.21	1.1	0.	0.	0.	0.
time (sec)	N/A	0.298	0.044	0.021	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	116	439	0	0	0	0
normalized size	1	1.	0.56	2.11	0.	0.	0.	0.
time (sec)	N/A	0.133	0.173	0.034	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	97	767	0	0	0	0
normalized size	1	1.	0.28	2.2	0.	0.	0.	0.
time (sec)	N/A	0.26	0.128	0.037	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	105	429	0	0	0	0
normalized size	1	1.	0.57	2.32	0.	0.	0.	0.
time (sec)	N/A	0.114	0.148	0.018	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	84	764	0	0	94	0
normalized size	1	1.	0.24	2.22	0.	0.	0.27	0.
time (sec)	N/A	0.259	0.105	0.021	0.	0.	162.735	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	108	425	0	0	0	0
normalized size	1	1.	0.58	2.27	0.	0.	0.	0.
time (sec)	N/A	0.112	0.082	0.023	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	86	771	0	0	0	0
normalized size	1	1.	0.23	2.05	0.	0.	0.	0.
time (sec)	N/A	0.292	0.062	0.025	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	120	446	0	0	0	0
normalized size	1	1.	0.56	2.09	0.	0.	0.	0.
time (sec)	N/A	0.142	0.077	0.025	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	225	448	0	0	150	0
normalized size	1	1.	0.78	1.56	0.	0.	0.52	0.
time (sec)	N/A	0.291	0.279	0.063	0.	0.	73.795	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	145	658	0	0	148	0
normalized size	1	1.	0.34	1.55	0.	0.	0.35	0.
time (sec)	N/A	0.415	0.132	0.04	0.	0.	6.681	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	189	401	0	0	150	0
normalized size	1	1.	0.77	1.64	0.	0.	0.61	0.
time (sec)	N/A	0.203	0.197	0.023	0.	0.	7.349	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	129	624	0	0	153	0
normalized size	1	1.	0.31	1.48	0.	0.	0.36	0.
time (sec)	N/A	0.396	0.108	0.034	0.	0.	8.013	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	171	383	0	0	153	0
normalized size	1	1.	0.73	1.64	0.	0.	0.65	0.
time (sec)	N/A	0.191	0.184	0.028	0.	0.	17.452	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	125	648	0	0	160	0
normalized size	1	1.	0.3	1.54	0.	0.	0.38	0.
time (sec)	N/A	0.396	0.127	0.033	0.	0.	147.089	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	210	160	385	0	0	144	0
normalized size	1	0.99	0.75	1.81	0.	0.	0.68	0.
time (sec)	N/A	0.171	0.205	0.046	0.	0.	140.602	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	383	148	659	0	0	0	0
normalized size	1	0.99	0.38	1.71	0.	0.	0.	0.
time (sec)	N/A	0.324	0.142	0.05	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	213	187	403	0	0	0	0
normalized size	1	0.98	0.86	1.86	0.	0.	0.	0.
time (sec)	N/A	0.19	0.157	0.03	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	437	182	706	0	0	0	0
normalized size	1	0.99	0.41	1.6	0.	0.	0.	0.
time (sec)	N/A	0.393	0.173	0.034	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	210	743	0	0	0	0
normalized size	1	1.	0.4	1.4	0.	0.	0.	0.
time (sec)	N/A	0.562	0.168	0.034	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	259	489	0	0	0	0
normalized size	1	1.	0.76	1.44	0.	0.	0.	0.
time (sec)	N/A	0.323	0.263	0.026	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	179	699	0	0	304	0
normalized size	1	1.	0.37	1.45	0.	0.	0.63	0.
time (sec)	N/A	0.469	0.151	0.017	0.	0.	26.803	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	223	444	0	0	306	0
normalized size	1	1.	0.78	1.55	0.	0.	1.07	0.
time (sec)	N/A	0.267	0.23	0.016	0.	0.	30.079	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	161	669	0	0	309	0
normalized size	1	1.	0.34	1.41	0.	0.	0.65	0.
time (sec)	N/A	0.454	0.146	0.02	0.	0.	35.206	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	202	415	0	0	309	0
normalized size	1	1.	0.7	1.44	0.	0.	1.07	0.
time (sec)	N/A	0.242	0.229	0.017	0.	0.	55.449	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	141	668	0	0	0	0
normalized size	1	1.	0.3	1.43	0.	0.	0.	0.
time (sec)	N/A	0.445	0.163	0.022	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	143	661	0	0	0	0
normalized size	1	1.	0.33	1.54	0.	0.	0.	0.
time (sec)	N/A	0.406	0.138	0.028	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	190	405	0	0	144	0
normalized size	1	1.	0.79	1.69	0.	0.	0.6	0.
time (sec)	N/A	0.219	0.212	0.026	0.	0.	46.334	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	111	604	0	0	143	0
normalized size	1	1.	0.3	1.61	0.	0.	0.38	0.
time (sec)	N/A	0.356	0.111	0.017	0.	0.	5.297	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	148	350	0	0	144	0
normalized size	1	1.	0.77	1.81	0.	0.	0.75	0.
time (sec)	N/A	0.156	0.229	0.017	0.	0.	5.214	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	115	595	0	0	148	0
normalized size	1	1.	0.31	1.6	0.	0.	0.4	0.
time (sec)	N/A	0.34	0.111	0.02	0.	0.	6.533	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	165	352	0	0	148	0
normalized size	1	1.	0.9	1.91	0.	0.	0.8	0.
time (sec)	N/A	0.139	0.168	0.017	0.	0.	34.423	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	116	626	0	0	0	0
normalized size	1	1.	0.3	1.62	0.	0.	0.	0.
time (sec)	N/A	0.346	0.124	0.021	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	159	370	0	0	0	0
normalized size	1	1.	0.82	1.92	0.	0.	0.	0.
time (sec)	N/A	0.168	0.19	0.03	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	155	667	0	0	0	0
normalized size	1	1.	0.35	1.52	0.	0.	0.	0.
time (sec)	N/A	0.423	0.182	0.034	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	196	411	0	0	0	0
normalized size	1	1.	0.81	1.7	0.	0.	0.	0.
time (sec)	N/A	0.222	0.236	0.031	0.	0.	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	226	407	0	0	0	0
normalized size	1	1.	0.76	1.38	0.	0.	0.	0.
time (sec)	N/A	0.249	0.243	0.037	0.	0.	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	133	618	0	0	0	0
normalized size	1	1.	0.31	1.42	0.	0.	0.	0.
time (sec)	N/A	0.378	0.138	0.036	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	191	363	0	0	0	0
normalized size	1	1.	0.78	1.48	0.	0.	0.	0.
time (sec)	N/A	0.198	0.196	0.023	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	119	597	0	0	0	0
normalized size	1	1.	0.31	1.55	0.	0.	0.	0.
time (sec)	N/A	0.327	0.121	0.021	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	174	341	0	0	0	0
normalized size	1	1.	0.9	1.77	0.	0.	0.	0.
time (sec)	N/A	0.156	0.151	0.022	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	126	594	0	0	0	0
normalized size	1	1.	0.32	1.51	0.	0.	0.	0.
time (sec)	N/A	0.361	0.107	0.025	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	181	353	0	0	0	0
normalized size	1	1.	0.87	1.71	0.	0.	0.	0.
time (sec)	N/A	0.181	0.18	0.024	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	141	638	0	0	0	0
normalized size	1	1.	0.32	1.47	0.	0.	0.	0.
time (sec)	N/A	0.422	0.125	0.025	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	222	696	0	0	0	0
normalized size	1	1.	0.74	2.3	0.	0.	0.	0.
time (sec)	N/A	0.234	0.28	0.039	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	153	1191	0	0	0	0
normalized size	1	1.	0.35	2.69	0.	0.	0.	0.
time (sec)	N/A	0.384	0.185	0.041	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	204	674	0	0	0	0
normalized size	1	1.	0.82	2.72	0.	0.	0.	0.
time (sec)	N/A	0.194	0.253	0.022	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	147	1176	0	0	0	0
normalized size	1	1.	0.36	2.92	0.	0.	0.	0.
time (sec)	N/A	0.342	0.163	0.023	0.	0.	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	169	660	0	0	0	0
normalized size	1	1.	0.79	3.1	0.	0.	0.	0.
time (sec)	N/A	0.162	0.267	0.025	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	161	1187	0	0	0	0
normalized size	1	1.	0.36	2.69	0.	0.	0.	0.
time (sec)	N/A	0.421	0.147	0.026	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	211	686	0	0	0	0
normalized size	1	1.	0.82	2.66	0.	0.	0.	0.
time (sec)	N/A	0.234	0.267	0.026	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	489	489	181	1231	0	0	0	0
normalized size	1	1.	0.37	2.52	0.	0.	0.	0.
time (sec)	N/A	0.47	0.175	0.03	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	187	1479	0	0	0	0
normalized size	1	1.	0.5	3.98	0.	0.	0.	0.
time (sec)	N/A	0.802	0.215	0.071	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	414	414	143	1491	0	0	0	0
normalized size	1	1.	0.35	3.6	0.	0.	0.	0.
time (sec)	N/A	0.817	0.134	0.043	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	143	1286	0	0	0	0
normalized size	1	1.	0.45	4.08	0.	0.	0.	0.
time (sec)	N/A	0.509	0.117	0.017	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	69	701	0	0	0	0
normalized size	1	1.	0.19	1.92	0.	0.	0.	0.
time (sec)	N/A	0.535	0.042	0.017	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	67	651	0	0	0	0
normalized size	1	1.	0.24	2.3	0.	0.	0.	0.
time (sec)	N/A	0.371	0.038	0.031	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	392	392	143	1274	0	0	0	0
normalized size	1	1.	0.36	3.25	0.	0.	0.	0.
time (sec)	N/A	0.718	0.12	0.038	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	146	1167	0	0	0	0
normalized size	1	1.	0.47	3.79	0.	0.	0.	0.
time (sec)	N/A	0.495	0.131	0.034	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	190	1553	0	0	0	0
normalized size	1	1.	0.42	3.4	0.	0.	0.	0.
time (sec)	N/A	0.984	0.219	0.041	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	183	2183	0	0	0	0
normalized size	1	1.	0.38	4.5	0.	0.	0.	0.
time (sec)	N/A	1.106	0.202	0.033	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	182	1920	0	0	0	0
normalized size	1	1.	0.49	5.16	0.	0.	0.	0.
time (sec)	N/A	0.778	0.189	0.024	0.	0.	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	155	1927	0	0	0	0
normalized size	1	1.	0.37	4.58	0.	0.	0.	0.
time (sec)	N/A	0.823	0.205	0.021	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	153	1721	0	0	0	0
normalized size	1	1.	0.47	5.25	0.	0.	0.	0.
time (sec)	N/A	0.562	0.161	0.023	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	151	1754	0	0	0	0
normalized size	1	1.	0.36	4.21	0.	0.	0.	0.
time (sec)	N/A	0.789	0.143	0.027	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	153	1740	0	0	0	0
normalized size	1	1.	0.46	5.27	0.	0.	0.	0.
time (sec)	N/A	0.592	0.153	0.025	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	459	459	187	2028	0	0	0	0
normalized size	1	1.	0.41	4.42	0.	0.	0.	0.
time (sec)	N/A	1.059	0.222	0.027	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	147	853	0	0	0	0
normalized size	1	1.	0.48	2.8	0.	0.	0.	0.
time (sec)	N/A	0.49	0.128	0.035	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	70	470	0	0	0	0
normalized size	1	1.	0.2	1.35	0.	0.	0.	0.
time (sec)	N/A	0.492	0.05	0.032	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	70	415	0	0	0	0
normalized size	1	1.	0.27	1.59	0.	0.	0.	0.
time (sec)	N/A	0.356	0.042	0.023	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	70	337	0	0	0	0
normalized size	1	1.	0.34	1.66	0.	0.	0.	0.
time (sec)	N/A	0.26	0.041	0.019	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	68	344	0	0	0	0
normalized size	1	1.	0.36	1.83	0.	0.	0.	0.
time (sec)	N/A	0.252	0.044	0.025	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	146	835	0	0	0	0
normalized size	1	1.	0.39	2.2	0.	0.	0.	0.
time (sec)	N/A	0.683	0.117	0.03	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	148	740	0	0	0	0
normalized size	1	1.	0.5	2.49	0.	0.	0.	0.
time (sec)	N/A	0.465	0.132	0.026	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	188	1109	0	0	0	0
normalized size	1	1.	0.42	2.5	0.	0.	0.	0.
time (sec)	N/A	0.921	0.217	0.029	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	148	1039	0	0	0	0
normalized size	1	1.	0.33	2.34	0.	0.	0.	0.
time (sec)	N/A	0.791	0.145	0.044	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	148	825	0	0	0	0
normalized size	1	1.	0.44	2.44	0.	0.	0.	0.
time (sec)	N/A	0.52	0.139	0.039	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	414	414	133	839	0	0	0	0
normalized size	1	1.	0.32	2.03	0.	0.	0.	0.
time (sec)	N/A	0.691	0.098	0.025	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	133	702	0	0	0	0
normalized size	1	1.	0.42	2.24	0.	0.	0.	0.
time (sec)	N/A	0.427	0.097	0.026	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	420	420	148	830	0	0	0	0
normalized size	1	1.	0.35	1.98	0.	0.	0.	0.
time (sec)	N/A	0.717	0.187	0.026	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	147	706	0	0	0	0
normalized size	1	1.	0.45	2.15	0.	0.	0.	0.
time (sec)	N/A	0.488	0.126	0.03	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	493	493	198	1058	0	0	0	0
normalized size	1	1.	0.4	2.15	0.	0.	0.	0.
time (sec)	N/A	0.985	0.229	0.034	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	197	896	0	0	0	0
normalized size	1	1.	0.5	2.26	0.	0.	0.	0.
time (sec)	N/A	0.789	0.248	0.033	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	184	2561	0	0	0	0
normalized size	1	1.	0.51	7.07	0.	0.	0.	0.
time (sec)	N/A	0.697	0.179	0.048	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	163	2542	0	0	0	0
normalized size	1	1.	0.39	6.15	0.	0.	0.	0.
time (sec)	N/A	0.76	0.128	0.046	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	163	2255	0	0	0	0
normalized size	1	1.	0.5	6.88	0.	0.	0.	0.
time (sec)	N/A	0.485	0.126	0.023	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	163	2534	0	0	0	0
normalized size	1	1.	0.39	6.08	0.	0.	0.	0.
time (sec)	N/A	0.72	0.12	0.018	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	161	2251	0	0	0	0
normalized size	1	1.	0.48	6.72	0.	0.	0.	0.
time (sec)	N/A	0.451	0.114	0.029	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	182	2568	0	0	0	0
normalized size	1	1.	0.41	5.78	0.	0.	0.	0.
time (sec)	N/A	0.912	0.178	0.033	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	181	2316	0	0	0	0
normalized size	1	1.	0.51	6.52	0.	0.	0.	0.
time (sec)	N/A	0.652	0.179	0.03	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	233	3790	0	0	0	0
normalized size	1	1.	0.54	8.83	0.	0.	0.	0.
time (sec)	N/A	0.975	0.277	0.036	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	196	3886	0	0	0	0
normalized size	1	1.	0.4	8.01	0.	0.	0.	0.
time (sec)	N/A	1.012	0.232	0.031	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	195	3466	0	0	0	0
normalized size	1	1.	0.51	9.1	0.	0.	0.	0.
time (sec)	N/A	0.729	0.215	0.03	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	189	3858	0	0	0	0
normalized size	1	1.	0.4	8.14	0.	0.	0.	0.
time (sec)	N/A	0.84	0.218	0.029	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	187	2531	0	0	0	0
normalized size	1	1.	0.51	6.92	0.	0.	0.	0.
time (sec)	N/A	0.597	0.178	0.031	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	197	3879	0	0	0	0
normalized size	1	1.	0.38	7.47	0.	0.	0.	0.
time (sec)	N/A	1.12	0.227	0.037	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	412	412	199	3484	0	0	0	0
normalized size	1	1.	0.48	8.46	0.	0.	0.	0.
time (sec)	N/A	0.851	0.222	0.033	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	484	484	184	2956	0	0	0	0
normalized size	1	1.	0.38	6.11	0.	0.	0.	0.
time (sec)	N/A	0.874	0.197	0.031	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	184	2520	0	0	0	0
normalized size	1	1.	0.49	6.7	0.	0.	0.	0.
time (sec)	N/A	0.545	0.19	0.03	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	460	460	168	2548	0	0	0	0
normalized size	1	1.	0.37	5.54	0.	0.	0.	0.
time (sec)	N/A	0.78	0.151	0.028	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	169	2258	0	0	0	0
normalized size	1	1.	0.47	6.22	0.	0.	0.	0.
time (sec)	N/A	0.511	0.136	0.028	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	464	464	181	2545	0	0	0	0
normalized size	1	1.	0.39	5.48	0.	0.	0.	0.
time (sec)	N/A	0.787	0.233	0.027	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	180	2266	0	0	0	0
normalized size	1	1.	0.49	6.17	0.	0.	0.	0.
time (sec)	N/A	0.519	0.172	0.031	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	235	2982	0	0	0	0
normalized size	1	1.	0.44	5.57	0.	0.	0.	0.
time (sec)	N/A	1.077	0.276	0.038	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	234	2622	0	0	0	0
normalized size	1	1.	0.55	6.11	0.	0.	0.	0.
time (sec)	N/A	0.814	0.29	0.035	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	529	529	189	2964	0	0	0	0
normalized size	1	1.	0.36	5.6	0.	0.	0.	0.
time (sec)	N/A	1.08	0.222	0.036	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	420	420	191	2530	0	0	0	0
normalized size	1	1.	0.45	6.02	0.	0.	0.	0.
time (sec)	N/A	0.692	0.203	0.035	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	185	2561	0	0	0	0
normalized size	1	1.	0.38	5.28	0.	0.	0.	0.
time (sec)	N/A	0.958	0.212	0.032	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	186	2277	0	0	0	0
normalized size	1	1.	0.48	5.82	0.	0.	0.	0.
time (sec)	N/A	0.645	0.2	0.033	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	531	531	230	2950	0	0	0	0
normalized size	1	1.	0.43	5.56	0.	0.	0.	0.
time (sec)	N/A	1.09	0.274	0.034	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	426	426	229	2554	0	0	0	0
normalized size	1	1.	0.54	6.	0.	0.	0.	0.
time (sec)	N/A	0.775	0.261	0.038	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	628	628	319	3385	0	0	0	0
normalized size	1	1.	0.51	5.39	0.	0.	0.	0.
time (sec)	N/A	1.403	0.468	0.04	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	318	2871	0	0	0	0
normalized size	1	1.	0.62	5.61	0.	0.	0.	0.
time (sec)	N/A	1.078	0.464	0.045	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	256	5126	0	0	0	0
normalized size	1	1.	0.45	9.02	0.	0.	0.	0.
time (sec)	N/A	1.362	0.404	0.063	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	454	454	252	4403	0	0	0	0
normalized size	1	1.	0.56	9.7	0.	0.	0.	0.
time (sec)	N/A	0.867	0.387	0.061	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	551	551	278	5078	0	0	0	0
normalized size	1	1.	0.5	9.22	0.	0.	0.	0.
time (sec)	N/A	1.296	0.456	0.046	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	275	4403	0	0	0	0
normalized size	1	1.	0.62	9.85	0.	0.	0.	0.
time (sec)	N/A	0.917	0.439	0.043	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	625	625	327	5689	0	0	0	0
normalized size	1	1.	0.52	9.1	0.	0.	0.	0.
time (sec)	N/A	1.431	0.546	0.05	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	328	4776	0	0	0	0
normalized size	1	1.	0.64	9.29	0.	0.	0.	0.
time (sec)	N/A	0.975	0.51	0.05	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	735	735	407	6334	0	0	0	0
normalized size	1	1.	0.55	8.62	0.	0.	0.	0.
time (sec)	N/A	1.843	1.285	0.059	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	606	606	427	5248	0	0	0	0
normalized size	1	1.	0.7	8.66	0.	0.	0.	0.
time (sec)	N/A	1.461	1.369	0.063	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	187	532	0	960	0	304
normalized size	1	1.	0.89	2.55	0.	4.59	0.	1.45
time (sec)	N/A	0.249	0.416	0.051	0.	2.232	0.	1.214

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	138	339	0	752	0	207
normalized size	1	1.	1.01	2.47	0.	5.49	0.	1.51
time (sec)	N/A	0.127	0.3	0.013	0.	1.939	0.	1.217

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	116	198	0	590	0	143
normalized size	1	1.	1.35	2.3	0.	6.86	0.	1.66
time (sec)	N/A	0.075	0.266	0.01	0.	1.915	0.	1.592

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	129	177	0	1708	0	208
normalized size	1	1.	1.4	1.92	0.	18.57	0.	2.26
time (sec)	N/A	0.095	0.135	0.026	0.	2.976	0.	1.202

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	207	0	629	0	0
normalized size	1	1.	1.	2.33	0.	7.07	0.	0.
time (sec)	N/A	0.072	0.061	0.023	0.	2.435	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	125	355	0	795	0	0
normalized size	1	1.	0.87	2.48	0.	5.56	0.	0.
time (sec)	N/A	0.116	0.074	0.026	0.	3.423	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	246	526	0	0	0	0
normalized size	1	1.	0.72	1.53	0.	0.	0.	0.
time (sec)	N/A	0.306	0.425	0.025	0.	0.	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	199	335	0	0	0	0
normalized size	1	1.	0.77	1.29	0.	0.	0.	0.
time (sec)	N/A	0.164	0.254	0.016	0.	0.	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	111	168	0	0	0	0
normalized size	1	1.	0.48	0.72	0.	0.	0.	0.
time (sec)	N/A	0.149	0.266	0.023	0.	0.	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	228	417	0	0	0	0
normalized size	1	1.	0.74	1.36	0.	0.	0.	0.
time (sec)	N/A	0.263	0.757	0.026	0.	0.	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	231	770	0	1265	0	410
normalized size	1	1.	0.84	2.79	0.	4.58	0.	1.49
time (sec)	N/A	0.325	0.575	0.03	0.	2.272	0.	1.231

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	173	532	0	972	0	293
normalized size	1	1.	0.93	2.84	0.	5.2	0.	1.57
time (sec)	N/A	0.172	0.422	0.017	0.	2.062	0.	1.259

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	131	337	0	751	0	201
normalized size	1	1.	1.05	2.7	0.	6.01	0.	1.61
time (sec)	N/A	0.104	0.362	0.013	0.	2.047	0.	1.236

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	195	287	0	2017	0	284
normalized size	1	1.	1.47	2.16	0.	15.17	0.	2.14
time (sec)	N/A	0.14	0.749	0.014	0.	6.189	0.	1.308

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	172	298	0	2095	0	663
normalized size	1	1.	1.26	2.19	0.	15.4	0.	4.88
time (sec)	N/A	0.14	0.941	0.016	0.	5.527	0.	1.967

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	110	352	0	792	0	0
normalized size	1	1.	0.84	2.69	0.	6.05	0.	0.
time (sec)	N/A	0.106	0.071	0.014	0.	3.647	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	305	782	0	0	0	0
normalized size	1	1.	0.71	1.82	0.	0.	0.	0.
time (sec)	N/A	0.518	0.607	0.023	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	245	544	0	0	0	0
normalized size	1	1.	0.73	1.62	0.	0.	0.	0.
time (sec)	N/A	0.34	0.425	0.018	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	206	352	0	0	0	0
normalized size	1	1.	0.84	1.44	0.	0.	0.	0.
time (sec)	N/A	0.16	0.271	0.018	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	227	433	0	0	0	0
normalized size	1	1.	0.73	1.39	0.	0.	0.	0.
time (sec)	N/A	0.292	0.354	0.018	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	271	1054	0	1638	0	536
normalized size	1	1.	0.8	3.1	0.	4.82	0.	1.58
time (sec)	N/A	0.418	1.155	0.031	0.	2.736	0.	1.323

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	214	770	0	1272	0	397
normalized size	1	1.	0.9	3.25	0.	5.37	0.	1.68
time (sec)	N/A	0.23	0.574	0.019	0.	2.271	0.	1.237

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	164	529	0	971	0	284
normalized size	1	1.	1.	3.23	0.	5.92	0.	1.73
time (sec)	N/A	0.141	0.514	0.014	0.	1.987	0.	1.235

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	213	446	0	2361	0	354
normalized size	1	1.	1.14	2.39	0.	12.63	0.	1.89
time (sec)	N/A	0.226	0.637	0.017	0.	18.037	0.	1.235

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	196	423	0	2340	0	752
normalized size	1	1.	1.05	2.26	0.	12.51	0.	4.02
time (sec)	N/A	0.23	0.965	0.017	0.	14.456	0.	2.027

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	206	464	0	2450	0	0
normalized size	1	1.	1.07	2.42	0.	12.76	0.	0.
time (sec)	N/A	0.201	1.099	0.015	0.	13.459	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	379	1047	0	0	0	0
normalized size	1	1.	0.69	1.89	0.	0.	0.	0.
time (sec)	N/A	0.718	1.669	0.028	0.	0.	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	306	782	0	0	0	0
normalized size	1	1.	0.7	1.79	0.	0.	0.	0.
time (sec)	N/A	0.489	0.594	0.019	0.	0.	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	254	568	0	0	0	0
normalized size	1	1.	0.77	1.72	0.	0.	0.	0.
time (sec)	N/A	0.291	0.439	0.018	0.	0.	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	261	583	0	0	0	0
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.
time (sec)	N/A	0.291	0.455	0.019	0.	0.	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	135	0	0	0	0
normalized size	1	1.	0.93	1.36	0.	0.	0.	0.
time (sec)	N/A	0.093	0.092	0.02	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	81	62	185	0	54
normalized size	1	1.	0.68	1.25	0.95	2.85	0.	0.83
time (sec)	N/A	0.049	0.027	0.023	1.458	1.789	0.	1.152

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	86	129	0	0	0	0
normalized size	1	1.	1.23	1.84	0.	0.	0.	0.
time (sec)	N/A	0.046	0.071	0.012	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	60	36	166	66	45
normalized size	1	1.	0.95	1.54	0.92	4.26	1.69	1.15
time (sec)	N/A	0.031	0.012	0.009	1.478	1.803	6.147	1.164

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	127	306	0	0	0	0
normalized size	1	1.	0.53	1.27	0.	0.	0.	0.
time (sec)	N/A	0.15	0.112	0.022	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	154	340	0	761	0	212
normalized size	1	1.	1.09	2.41	0.	5.4	0.	1.5
time (sec)	N/A	0.159	0.253	0.029	0.	2.054	0.	1.232

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	123	200	0	595	0	140
normalized size	1	1.	1.4	2.27	0.	6.76	0.	1.59
time (sec)	N/A	0.093	0.167	0.016	0.	2.176	0.	1.229

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	82	103	0	440	0	73
normalized size	1	1.	1.82	2.29	0.	9.78	0.	1.62
time (sec)	N/A	0.055	0.066	0.014	0.	2.132	0.	1.221

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	103	0	455	0	120
normalized size	1	1.	1.	2.24	0.	9.89	0.	2.61
time (sec)	N/A	0.047	0.012	0.019	0.	2.257	0.	1.21

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	209	0	635	0	558
normalized size	1	1.	1.	2.3	0.	6.98	0.	6.13
time (sec)	N/A	0.079	0.055	0.021	0.	2.497	0.	1.353

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	126	355	0	798	0	0
normalized size	1	1.	0.85	2.38	0.	5.36	0.	0.
time (sec)	N/A	0.137	0.08	0.022	0.	3.582	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	249	546	0	0	0	0
normalized size	1	1.	0.73	1.6	0.	0.	0.	0.
time (sec)	N/A	0.311	0.459	0.026	0.	0.	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	201	333	0	0	0	0
normalized size	1	1.	0.77	1.28	0.	0.	0.	0.
time (sec)	N/A	0.162	0.277	0.02	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	122	129	0	0	0	0
normalized size	1	1.	1.05	1.11	0.	0.	0.	0.
time (sec)	N/A	0.049	0.072	0.017	0.	0.	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	146	224	0	0	0	0
normalized size	1	1.	0.95	1.46	0.	0.	0.	0.
time (sec)	N/A	0.095	0.325	0.019	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	229	435	0	0	0	0
normalized size	1	1.	0.75	1.42	0.	0.	0.	0.
time (sec)	N/A	0.25	0.345	0.022	0.	0.	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	185	553	0	1049	0	254
normalized size	1	1.	1.43	4.29	0.	8.13	0.	1.97
time (sec)	N/A	0.164	0.345	0.038	0.	2.695	0.	1.25

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	118	320	0	790	0	177
normalized size	1	1.	1.42	3.86	0.	9.52	0.	2.13
time (sec)	N/A	0.089	0.495	0.02	0.	2.467	0.	1.226

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	30	0	97	0	95
normalized size	1	1.	0.97	0.88	0.	2.85	0.	2.79
time (sec)	N/A	0.027	0.01	0.004	0.	1.889	0.	1.188

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	214	651	0	1473	0	443
normalized size	1	1.	1.56	4.75	0.	10.75	0.	3.23
time (sec)	N/A	0.141	0.483	0.035	0.	3.484	0.	1.356

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	63	0	265	0	289
normalized size	1	1.	0.61	0.71	0.	2.98	0.	3.25
time (sec)	N/A	0.066	0.026	0.006	0.	2.45	0.	1.253

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	60	0	261	0	174
normalized size	1	1.	0.7	0.81	0.	3.53	0.	2.35
time (sec)	N/A	0.044	0.016	0.006	0.	2.437	0.	1.237

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	91	119	0	522	0	806
normalized size	1	1.	0.59	0.77	0.	3.39	0.	5.23
time (sec)	N/A	0.18	0.054	0.007	0.	4.065	0.	1.399

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	91	125	0	537	0	637
normalized size	1	1.	0.66	0.91	0.	3.89	0.	4.62
time (sec)	N/A	0.101	0.035	0.007	0.	4.324	0.	1.313

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	83	113	0	520	0	328
normalized size	1	1.	0.73	1.	0.	4.6	0.	2.9
time (sec)	N/A	0.064	0.028	0.007	0.	4.142	0.	1.242

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	151	213	0	914	0	1407
normalized size	1	1.	0.7	0.98	0.	4.21	0.	6.48
time (sec)	N/A	0.269	0.076	0.01	0.	8.545	0.	1.489

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	108	108	0	444	0	77
normalized size	1	1.	2.3	2.3	0.	9.45	0.	1.64
time (sec)	N/A	0.057	0.089	0.037	0.	1.917	0.	1.212

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	109	111	0	446	0	77
normalized size	1	1.	2.27	2.31	0.	9.29	0.	1.6
time (sec)	N/A	0.058	0.09	0.04	0.	1.894	0.	1.204

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	72	70	0	0	0	0
normalized size	1	1.	0.65	0.64	0.	0.	0.	0.
time (sec)	N/A	0.045	0.048	0.021	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	59	59	0	0	0	0
normalized size	1	1.	0.68	0.68	0.	0.	0.	0.
time (sec)	N/A	0.07	0.052	0.02	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	76	0	0	0	0
normalized size	1	1.	0.8	0.86	0.	0.	0.	0.
time (sec)	N/A	0.036	0.049	0.016	0.	0.	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	25	0	0	0	0
normalized size	1	1.	0.77	0.81	0.	0.	0.	0.
time (sec)	N/A	0.03	0.031	0.018	0.	0.	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	37	23	0	0	0	0
normalized size	1	1.	1.19	0.74	0.	0.	0.	0.
time (sec)	N/A	0.03	0.036	0.017	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	29	0	0	0	0
normalized size	1	1.	0.8	0.83	0.	0.	0.	0.
time (sec)	N/A	0.031	0.03	0.02	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	33	0	0	0	0
normalized size	1	1.	1.09	0.94	0.	0.	0.	0.
time (sec)	N/A	0.032	0.033	0.018	0.	0.	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	29	0	0	0	0
normalized size	1	1.	0.8	0.83	0.	0.	0.	0.
time (sec)	N/A	0.03	0.032	0.023	0.	0.	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	27	0	0	0	0
normalized size	1	1.	0.8	0.77	0.	0.	0.	0.
time (sec)	N/A	0.032	0.031	0.021	0.	0.	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	0	0	0
normalized size	1	1.	0.88	0.83	0.	0.	0.	0.
time (sec)	N/A	0.028	0.029	0.017	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	35	0	0	0	0
normalized size	1	1.	0.88	0.81	0.	0.	0.	0.
time (sec)	N/A	0.029	0.031	0.017	0.	0.	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	35	0	0	0	0
normalized size	1	1.	0.85	0.74	0.	0.	0.	0.
time (sec)	N/A	0.031	0.033	0.017	0.	0.	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	34	30	0	0	0	0
normalized size	1	1.	0.42	0.38	0.	0.	0.	0.
time (sec)	N/A	0.028	0.028	0.014	0.	0.	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	38	26	0	0	0	0
normalized size	1	1.	0.46	0.32	0.	0.	0.	0.
time (sec)	N/A	0.029	0.028	0.015	0.	0.	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	50	36	0	0	0	0
normalized size	1	1.	0.57	0.41	0.	0.	0.	0.
time (sec)	N/A	0.031	0.033	0.017	0.	0.	0.	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	34	0	0	0	0
normalized size	1	1.	2.18	2.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.035	0.018	0.	0.	0.	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	106	0	146	319	0	0
normalized size	1	1.	0.97	0.	1.34	2.93	0.	0.
time (sec)	N/A	0.088	0.075	0.043	1.482	1.518	0.	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	90	0	131	408	0	0
normalized size	1	1.	0.96	0.	1.39	4.34	0.	0.
time (sec)	N/A	0.065	0.029	0.039	1.477	1.53	0.	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	67	0	116	271	0	0
normalized size	1	1.	0.85	0.	1.47	3.43	0.	0.
time (sec)	N/A	0.052	0.017	0.029	1.479	1.514	0.	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	127	0	0	591	0	0
normalized size	1	1.	0.93	0.	0.	4.35	0.	0.
time (sec)	N/A	0.099	0.036	0.039	0.	1.57	0.	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	0	0	332	0	0
normalized size	1	1.	0.96	0.	0.	3.42	0.	0.
time (sec)	N/A	0.074	0.045	0.044	0.	1.53	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	178	0	0	664	0	0
normalized size	1	1.	1.03	0.	0.	3.86	0.	0.
time (sec)	N/A	0.13	0.07	0.052	0.	1.545	0.	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	536	536	156	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.113	0.046	0.	0.	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	28	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.018	0.035	0.	0.	0.	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	5544	0	0
normalized size	1	1.	1.04	0.	0.	49.06	0.	0.
time (sec)	N/A	0.013	0.031	0.	0.	7.796	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	538	538	161	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.236	0.094	0.046	0.	0.	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	556	556	166	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.116	0.047	0.	0.	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	120	0	170	394	0	0
normalized size	1	1.	0.9	0.	1.28	2.96	0.	0.
time (sec)	N/A	0.086	0.169	0.048	1.493	1.568	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	110	0	155	478	0	0
normalized size	1	1.	0.95	0.	1.34	4.12	0.	0.
time (sec)	N/A	0.077	0.148	0.043	1.469	1.549	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	140	351	0	0
normalized size	1	1.	0.96	0.	1.39	3.48	0.	0.
time (sec)	N/A	0.066	0.091	0.042	1.503	1.534	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	34	0	140	365	0	0
normalized size	1	1.	0.34	0.	1.39	3.61	0.	0.
time (sec)	N/A	0.063	0.008	0.039	1.483	1.515	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	148	0	0	711	0	0
normalized size	1	1.	0.94	0.	0.	4.5	0.	0.
time (sec)	N/A	0.111	0.13	0.053	0.	1.597	0.	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	171	0	0	663	0	0
normalized size	1	1.	0.93	0.	0.	3.62	0.	0.
time (sec)	N/A	0.13	0.149	0.056	0.	1.596	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	194	0	0	778	0	0
normalized size	1	1.	0.93	0.	0.	3.74	0.	0.
time (sec)	N/A	0.147	0.167	0.059	0.	1.638	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	157	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.114	0.05	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	156	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	0.094	0.051	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	157	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.102	0.044	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	168	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.309	0.109	0.052	0.	0.	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	581	581	173	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.37	0.143	0.053	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	46	0	204	814	0	216
normalized size	1	1.	0.34	0.	1.5	5.99	0.	1.59
time (sec)	N/A	0.088	0.03	0.078	1.519	1.343	0.	1.282

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	42	0	189	784	0	189
normalized size	1	1.	0.35	0.	1.56	6.48	0.	1.56
time (sec)	N/A	0.065	0.022	0.061	1.524	1.346	0.	1.721

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	36	0	174	768	0	174
normalized size	1	1.	0.34	0.	1.64	7.25	0.	1.64
time (sec)	N/A	0.046	0.011	0.059	1.57	1.387	0.	1.187

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	34	0	159	574	0	159
normalized size	1	1.	0.37	0.	1.75	6.31	0.	1.75
time (sec)	N/A	0.013	0.007	0.039	1.551	1.376	0.	1.15

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	73	0	0	844	0	292
normalized size	1	1.	0.5	0.	0.	5.82	0.	2.01
time (sec)	N/A	0.083	0.041	0.05	0.	1.542	0.	1.268

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	102	0	0	909	0	259
normalized size	1	1.	0.63	0.	0.	5.58	0.	1.59
time (sec)	N/A	0.133	0.051	0.077	0.	1.455	0.	1.336

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.128	0.066	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	37	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.018	0.045	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	135	0	0	1553	0	0
normalized size	1	1.	1.12	0.	0.	12.94	0.	0.
time (sec)	N/A	0.015	0.027	0.	0.	20.66	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	56	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.036	0.065	0.	0.	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	156	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.155	0.076	0.	0.	0.	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	57	0	100	208	88	101
normalized size	1	1.	0.73	0.	1.28	2.67	1.13	1.29
time (sec)	N/A	0.053	0.041	0.083	1.444	1.287	18.065	1.222

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	0	85	182	75	86
normalized size	1	1.	0.81	0.	1.35	2.89	1.19	1.37
time (sec)	N/A	0.048	0.038	0.07	1.448	1.336	13.977	1.228

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	0	70	165	58	72
normalized size	1	1.	0.92	0.	1.46	3.44	1.21	1.5
time (sec)	N/A	0.033	0.01	0.065	1.513	1.329	10.275	1.24

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	55	131	42	57
normalized size	1	1.	1.	0.	1.67	3.97	1.27	1.73
time (sec)	N/A	0.022	0.006	0.053	1.469	1.341	5.485	1.17

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	63	0	0	649	0	209
normalized size	1	1.	0.36	0.	0.	3.75	0.	1.21
time (sec)	N/A	0.132	0.016	0.066	0.	1.397	0.	1.225

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	77	0	0	720	0	228
normalized size	1	1.	0.4	0.	0.	3.77	0.	1.19
time (sec)	N/A	0.147	0.031	0.087	0.	1.367	0.	1.233

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	177	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.093	0.076	0.	0.	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	52	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.02	0.059	0.	0.	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	281	0	0
normalized size	1	1.	2.08	0.	0.	4.61	0.	0.
time (sec)	N/A	0.009	0.026	0.	0.	24.022	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	64	0	0	0	0	0
normalized size	1	1.	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.028	0.071	0.	0.	0.	0.

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	148	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.144	0.081	0.	0.	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	37	0	0	857	0	0
normalized size	1	1.	0.29	0.	0.	6.64	0.	0.
time (sec)	N/A	0.023	0.038	0.032	0.	1.865	0.	0.

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	37	0	0	871	0	0
normalized size	1	1.	0.31	0.	0.	7.26	0.	0.
time (sec)	N/A	0.023	0.037	0.054	0.	2.002	0.	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	39	0	0	1249	0	0
normalized size	1	1.	0.31	0.	0.	10.07	0.	0.
time (sec)	N/A	0.031	0.046	0.05	0.	1.936	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	39	0	0	1262	0	0
normalized size	1	1.	0.33	0.	0.	10.61	0.	0.
time (sec)	N/A	0.032	0.049	0.051	0.	1.723	0.	0.

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	65	0	0	495	0	0
normalized size	1	1.	0.54	0.	0.	4.12	0.	0.
time (sec)	N/A	0.028	0.05	0.055	0.	1.639	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	65	0	0	501	0	0
normalized size	1	1.	0.54	0.	0.	4.18	0.	0.
time (sec)	N/A	0.026	0.052	0.056	0.	1.61	0.	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	67	0	0	566	0	0
normalized size	1	1.	0.58	0.	0.	4.92	0.	0.
time (sec)	N/A	0.035	0.057	0.056	0.	1.653	0.	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	68	0	0	571	0	0
normalized size	1	1.	0.57	0.	0.	4.8	0.	0.
time (sec)	N/A	0.037	0.06	0.056	0.	1.635	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	179	0	204	622	0	216
normalized size	1	1.	0.95	0.	1.09	3.31	0.	1.15
time (sec)	N/A	0.211	0.097	0.051	1.489	1.708	0.	1.207

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	163	0	189	601	0	189
normalized size	1	1.	0.94	0.	1.09	3.47	0.	1.09
time (sec)	N/A	0.18	0.064	0.048	1.529	1.738	0.	1.208

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	146	0	174	578	0	174
normalized size	1	1.	0.92	0.	1.1	3.66	0.	1.1
time (sec)	N/A	0.152	0.038	0.044	1.495	1.702	0.	1.189

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	117	0	159	741	0	159
normalized size	1	1.	0.82	0.	1.11	5.18	0.	1.11
time (sec)	N/A	0.118	0.026	0.042	1.555	1.68	0.	1.212

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	156	0	0	1019	0	284
normalized size	1	1.	0.79	0.	0.	5.17	0.	1.44
time (sec)	N/A	0.189	0.057	0.048	0.	1.738	0.	1.252

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	210	0	0	1081	0	259
normalized size	1	1.	0.98	0.	0.	5.03	0.	1.2
time (sec)	N/A	0.241	0.078	0.057	0.	1.814	0.	1.266

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	190	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.23	0.055	0.	0.	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.121	0.068	0.	0.	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	37	0	0	871	0	0
normalized size	1	1.	0.31	0.	0.	7.26	0.	0.
time (sec)	N/A	0.023	0.03	0.	0.	1.712	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	67	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.019	0.045	0.	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	37	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.05	0.052	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	37	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.051	0.056	0.	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	0	282	0	0
normalized size	1	1.	0.85	0.	0.	4.62	0.	0.
time (sec)	N/A	0.016	0.039	0.071	0.	1.537	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	0	343	0	0
normalized size	1	1.	0.85	0.	0.	5.62	0.	0.
time (sec)	N/A	0.017	0.042	0.04	0.	1.661	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	54	0	0	707	0	0
normalized size	1	1.	0.75	0.	0.	9.82	0.	0.
time (sec)	N/A	0.024	0.05	0.051	0.	1.647	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	55	0	0	682	0	0
normalized size	1	1.	0.74	0.	0.	9.22	0.	0.
time (sec)	N/A	0.026	0.052	0.049	0.	1.703	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	66	0	0	437	0	0
normalized size	1	1.	0.78	0.	0.	5.14	0.	0.
time (sec)	N/A	0.026	0.053	0.05	0.	1.679	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	67	0	0	443	0	0
normalized size	1	1.	0.79	0.	0.	5.21	0.	0.
time (sec)	N/A	0.024	0.056	0.049	0.	1.647	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	68	0	0	554	0	0
normalized size	1	1.	0.71	0.	0.	5.77	0.	0.
time (sec)	N/A	0.033	0.061	0.051	0.	1.673	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	70	0	0	559	0	0
normalized size	1	1.	0.71	0.	0.	5.7	0.	0.
time (sec)	N/A	0.033	0.059	0.05	0.	1.673	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	100	204	0	101
normalized size	1	1.	0.74	0.	1.28	2.62	0.	1.29
time (sec)	N/A	0.051	0.035	0.075	1.539	1.537	0.	1.23

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	0	85	188	0	86
normalized size	1	1.	0.84	0.	1.35	2.98	0.	1.37
time (sec)	N/A	0.046	0.026	0.065	1.465	1.553	0.	1.214

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	0	70	163	0	72
normalized size	1	1.	0.92	0.	1.46	3.4	0.	1.5
time (sec)	N/A	0.032	0.01	0.063	1.561	1.501	0.	1.264

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	55	132	42	57
normalized size	1	1.	1.	0.	1.67	4.	1.27	1.73
time (sec)	N/A	0.021	0.006	0.057	1.573	1.577	4.238	1.185

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	150	0	0	649	0	209
normalized size	1	1.	0.87	0.	0.	3.75	0.	1.21
time (sec)	N/A	0.126	0.049	0.066	0.	1.688	0.	1.196

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	181	0	0	722	0	228
normalized size	1	1.	0.95	0.	0.	3.78	0.	1.19
time (sec)	N/A	0.146	0.052	0.078	0.	1.602	0.	1.224

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	184	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.204	0.075	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	179	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.082	0.072	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	0	282	0	0
normalized size	1	1.	0.85	0.	0.	4.62	0.	0.
time (sec)	N/A	0.016	0.03	0.	0.	1.534	0.	0.

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	68	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.015	0.057	0.	0.	0.	0.

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	52	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.048	0.073	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	52	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.045	0.074	0.	0.	0.	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	0	0	0	94	0
normalized size	1	1.	0.76	0.	0.	0.	0.54	0.
time (sec)	N/A	0.126	0.166	0.031	0.	0.	142.568	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	112	0	0	0	92	0
normalized size	1	1.	0.82	0.	0.	0.	0.68	0.
time (sec)	N/A	0.094	0.097	0.021	0.	0.	3.841	0.

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	0	0	85	0
normalized size	1	1.	0.88	0.	0.	0.	0.75	0.
time (sec)	N/A	0.08	0.03	0.024	0.	0.	13.998	0.

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	0	101	0	0
normalized size	1	1.	0.66	0.58	0.	1.51	0.	0.
time (sec)	N/A	0.037	0.02	0.004	0.	2.075	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	72	62	0	153	0	0
normalized size	1	1.	0.69	0.6	0.	1.47	0.	0.
time (sec)	N/A	0.051	0.04	0.005	0.	2.636	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	94	86	0	215	0	0
normalized size	1	1.	0.67	0.61	0.	1.52	0.	0.
time (sec)	N/A	0.07	0.048	0.005	0.	1.915	0.	0.

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	123	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.132	0.028	0.	0.	0.	0.

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	97	0	0	0	94	0
normalized size	1	1.	0.7	0.	0.	0.	0.68	0.
time (sec)	N/A	0.109	0.104	0.019	0.	0.	17.616	0.

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	77	0	0	0	78	0
normalized size	1	1.	0.75	0.	0.	0.	0.76	0.
time (sec)	N/A	0.088	0.062	0.023	0.	0.	5.361	0.

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	84	0	0	0	82	0
normalized size	1	1.	0.79	0.	0.	0.	0.77	0.
time (sec)	N/A	0.097	0.044	0.025	0.	0.	111.809	0.

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	88	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.071	0.029	0.	0.	0.	0.

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	88	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.068	0.031	0.	0.	0.	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	77	0	0	2084	94	0
normalized size	1	1.	0.45	0.	0.	12.19	0.55	0.
time (sec)	N/A	0.108	0.114	0.07	0.	2.066	60.24	0.

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	68	0	0	853	83	0
normalized size	1	1.	0.56	0.	0.	6.99	0.68	0.
time (sec)	N/A	0.072	0.062	0.038	0.	1.829	26.967	0.

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	0	124	0	0
normalized size	1	1.	0.67	0.58	0.	1.85	0.	0.
time (sec)	N/A	0.031	0.021	0.004	0.	1.876	0.	0.

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	71	62	0	173	0	0
normalized size	1	1.	0.68	0.6	0.	1.66	0.	0.
time (sec)	N/A	0.047	0.03	0.004	0.	1.605	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	68	86	0	238	0	0
normalized size	1	1.	0.48	0.61	0.	1.69	0.	0.
time (sec)	N/A	0.065	0.04	0.006	0.	1.604	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	112	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.139	0.074	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	85	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.111	0.059	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	77	0	0	0	94	0
normalized size	1	1.	0.78	0.	0.	0.	0.95	0.
time (sec)	N/A	0.048	0.101	0.034	0.	0.	16.91	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	0	0	0	82	0
normalized size	1	1.	0.75	0.	0.	0.	0.8	0.
time (sec)	N/A	0.059	0.041	0.061	0.	0.	104.662	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	78	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.044	0.065	0.	0.	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	82	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.069	0.07	0.	0.	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	77	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.127	0.049	0.	0.	0.	0.

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	69	0	0	0	87	0
normalized size	1	1.	0.55	0.	0.	0.	0.7	0.
time (sec)	N/A	0.076	0.059	0.03	0.	0.	75.21	0.

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	40	0	122	0	0
normalized size	1	1.	0.68	0.62	0.	1.88	0.	0.
time (sec)	N/A	0.031	0.019	0.005	0.	2.162	0.	0.

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	66	62	0	176	0	0
normalized size	1	1.	0.63	0.6	0.	1.69	0.	0.
time (sec)	N/A	0.052	0.026	0.004	0.	2.054	0.	0.

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	88	86	0	228	0	0
normalized size	1	1.	0.62	0.61	0.	1.62	0.	0.
time (sec)	N/A	0.066	0.04	0.005	0.	1.608	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	110	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.138	0.049	0.	0.	0.	0.

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	85	0	0	0	94	0
normalized size	1	1.	0.56	0.	0.	0.	0.62	0.
time (sec)	N/A	0.112	0.113	0.041	0.	0.	160.785	0.

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	0	78	0
normalized size	1	1.	0.68	0.	0.	0.	0.67	0.
time (sec)	N/A	0.093	0.056	0.035	0.	0.	141.547	0.

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	91	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.049	0.047	0.	0.	0.	0.

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	82	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.068	0.053	0.	0.	0.	0.

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	91	0	0	2306	0	0
normalized size	1	1.	0.41	0.	0.	10.43	0.	0.
time (sec)	N/A	0.135	0.116	0.059	0.	2.468	0.	0.

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	77	0	0	949	0	0
normalized size	1	1.	0.52	0.	0.	6.37	0.	0.
time (sec)	N/A	0.087	0.081	0.032	0.	2.207	0.	0.

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	136	0	0
normalized size	1	1.	0.56	0.49	0.	1.72	0.	0.
time (sec)	N/A	0.033	0.046	0.005	0.	2.001	0.	0.

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	65	62	0	204	0	0
normalized size	1	1.	0.62	0.6	0.	1.96	0.	0.
time (sec)	N/A	0.048	0.035	0.004	0.	1.823	0.	0.

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	94	86	0	261	0	0
normalized size	1	1.	0.67	0.61	0.	1.85	0.	0.
time (sec)	N/A	0.069	0.048	0.007	0.	1.587	0.	0.

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	79	110	0	323	0	0
normalized size	1	1.	0.44	0.62	0.	1.81	0.	0.
time (sec)	N/A	0.09	0.054	0.006	0.	1.685	0.	0.

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	140	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.174	0.063	0.	0.	0.	0.

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	116	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.149	0.05	0.	0.	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	98	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.128	0.031	0.	0.	0.	0.

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.101	0.031	0.	0.	0.	0.

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	85	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.042	0.052	0.	0.	0.	0.

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	86	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.043	0.059	0.	0.	0.	0.

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	87	0	0	0	0	0
normalized size	1	1.	0.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.043	0.062	0.	0.	0.	0.

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.113	0.128	0.	0.	0.	0.

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.058	0.063	0.	0.	0.	0.

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.048	0.059	0.	0.	0.	0.

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.07	0.	0.	0.	0.	0.

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.054	0.056	0.	0.	0.	0.

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.087	0.062	0.	0.	0.	0.

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	195	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.27	0.067	0.	0.	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	118	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.092	0.06	0.	0.	0.	0.

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.022	0.057	0.	0.	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.071	0.055	0.	0.	0.	0.

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.07	0.058	0.	0.	0.	0.

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.11	0.061	0.	0.	0.	0.

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.071	0.045	0.	0.	0.	0.

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.055	0.044	0.	0.	0.	0.

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.052	0.042	0.	0.	0.	0.

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.05	0.045	0.	0.	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.057	0.042	0.	0.	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.058	0.044	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1066] had the largest ratio of [0.5833]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	18	0.056
2	A	3	2	1.	16	0.125
3	A	2	1	1.	15	0.067
4	A	3	2	1.	18	0.111
5	A	2	1	1.	18	0.056
6	A	3	2	1.	18	0.111
7	A	2	1	1.	18	0.056
8	A	3	2	1.	18	0.111
9	A	2	1	1.	18	0.056
10	A	3	2	1.	18	0.111
11	A	2	1	1.	20	0.05
12	A	3	2	1.	18	0.111
13	A	2	1	1.	17	0.059
14	A	4	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	2	1	1.	20	0.05
16	A	3	2	1.	20	0.1
17	A	2	1	1.	20	0.05
18	A	3	2	1.	20	0.1
19	A	2	1	1.	20	0.05
20	A	3	2	1.	20	0.1
21	A	2	1	1.	20	0.05
22	A	3	3	1.	20	0.15
23	A	3	2	1.	20	0.1
24	A	2	1	1.	20	0.05
25	A	3	2	1.	20	0.1
26	A	2	1	1.	20	0.05
27	A	3	2	1.	20	0.1
28	A	2	1	1.	20	0.05
29	A	3	2	1.	20	0.1
30	A	2	1	1.	20	0.05
31	A	3	2	1.	18	0.111
32	A	2	1	1.	17	0.059
33	A	4	3	1.	20	0.15
34	A	2	1	1.	20	0.05
35	A	3	2	1.	20	0.1
36	A	2	1	1.	20	0.05
37	A	3	2	1.	20	0.1
38	A	2	1	1.	20	0.05
39	A	3	2	1.	20	0.1
40	A	2	1	1.	20	0.05
41	A	3	2	1.	20	0.1
42	A	2	1	1.	20	0.05
43	A	3	2	1.	20	0.1
44	A	2	1	1.	20	0.05
45	A	4	3	1.	20	0.15
46	A	2	1	1.	20	0.05
47	A	3	3	1.	20	0.15
48	A	2	1	1.	20	0.05
49	A	4	4	1.	20	0.2
50	A	2	1	1.	20	0.05
51	A	3	2	1.	20	0.1
52	A	2	1	1.	20	0.05
53	A	3	2	1.	20	0.1
54	A	2	1	1.	20	0.05
55	A	3	2	1.	20	0.1
56	A	4	3	1.	20	0.15
57	A	3	2	1.	20	0.1
58	A	4	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	3	2	1.	20	0.1
60	A	3	3	1.	20	0.15
61	A	3	2	1.	18	0.111
62	A	2	2	1.	17	0.118
63	A	3	2	1.	20	0.1
64	A	2	2	1.	20	0.1
65	A	3	2	1.	20	0.1
66	A	3	3	1.	20	0.15
67	A	3	2	1.	20	0.1
68	A	4	3	1.	20	0.15
69	A	3	2	1.	20	0.1
70	A	5	3	1.	20	0.15
71	A	3	2	1.	20	0.1
72	A	4	3	1.	20	0.15
73	A	3	2	1.	20	0.1
74	A	4	3	1.	20	0.15
75	A	3	2	1.	20	0.1
76	A	4	3	1.	20	0.15
77	A	3	2	1.	20	0.1
78	A	3	3	1.	20	0.15
79	A	3	2	1.	18	0.111
80	A	2	2	1.	17	0.118
81	A	3	2	1.	20	0.1
82	A	3	3	1.	20	0.15
83	A	3	2	1.	20	0.1
84	A	4	3	1.	20	0.15
85	A	3	2	1.	20	0.1
86	A	4	3	1.	20	0.15
87	A	3	2	1.	20	0.1
88	A	3	2	1.	20	0.1
89	A	3	2	1.	20	0.1
90	A	3	2	1.	20	0.1
91	A	3	2	1.	20	0.1
92	A	3	2	1.	20	0.1
93	A	2	2	1.	18	0.111
94	A	3	2	1.	20	0.1
95	A	3	2	1.	20	0.1
96	A	3	2	1.	20	0.1
97	A	3	2	1.	20	0.1
98	A	5	4	1.	20	0.2
99	A	5	4	1.	20	0.2
100	A	5	4	1.	20	0.2
101	A	4	4	1.	20	0.2
102	A	3	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	3	3	1.	17	0.176
104	A	4	3	1.	20	0.15
105	A	5	4	1.	20	0.2
106	A	5	4	1.	20	0.2
107	A	2	2	1.	15	0.133
108	A	2	2	1.	17	0.118
109	A	1	1	1.	13	0.077
110	A	1	1	1.	15	0.067
111	A	2	2	1.	15	0.133
112	A	2	2	1.	13	0.154
113	A	2	2	1.	13	0.154
114	A	1	1	1.	19	0.053
115	A	1	1	1.	18	0.056
116	A	2	2	1.	18	0.111
117	A	3	3	1.	13	0.231
118	A	2	2	1.	18	0.111
119	A	2	2	1.	17	0.118
120	A	2	2	1.	23	0.087
121	A	2	2	1.	23	0.087
122	A	2	2	1.	21	0.095
123	A	2	2	1.	20	0.1
124	A	2	2	1.	23	0.087
125	A	2	2	1.	23	0.087
126	A	2	2	1.	23	0.087
127	A	4	3	1.	23	0.13
128	A	3	3	1.	23	0.13
129	A	2	2	1.	21	0.095
130	A	2	2	1.	20	0.1
131	A	5	5	1.	23	0.217
132	A	3	3	1.	23	0.13
133	A	4	3	1.	23	0.13
134	A	4	3	1.	23	0.13
135	A	3	3	1.	23	0.13
136	A	2	2	1.	21	0.095
137	A	3	3	1.	20	0.15
138	A	4	3	1.	23	0.13
139	A	4	4	1.	23	0.174
140	A	4	3	1.	23	0.13
141	A	2	1	1.	20	0.05
142	A	3	2	1.	20	0.1
143	A	2	1	1.	20	0.05
144	A	3	2	1.	18	0.111
145	A	2	1	1.	17	0.059
146	A	4	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	2	1	1.	20	0.05
148	A	3	2	1.	20	0.1
149	A	2	1	1.	20	0.05
150	A	2	1	1.	22	0.045
151	A	3	2	1.	22	0.091
152	A	2	1	1.	22	0.045
153	A	3	2	1.	20	0.1
154	A	2	1	1.	19	0.053
155	A	3	2	1.	22	0.091
156	A	2	1	1.	22	0.045
157	A	3	2	1.	22	0.091
158	A	2	1	1.	22	0.045
159	A	2	1	1.	22	0.045
160	A	3	2	1.	22	0.091
161	A	2	1	1.	22	0.045
162	A	3	2	1.	20	0.1
163	A	2	1	1.	19	0.053
164	A	3	2	1.	22	0.091
165	A	2	1	1.	22	0.045
166	A	3	2	1.	22	0.091
167	A	2	1	1.	22	0.045
168	A	3	2	1.	22	0.091
169	A	3	2	1.	22	0.091
170	A	3	2	1.	22	0.091
171	A	3	2	1.	20	0.1
172	A	3	2	1.	19	0.105
173	A	3	2	1.	22	0.091
174	A	3	2	1.	22	0.091
175	A	3	2	1.	22	0.091
176	A	3	2	1.	22	0.091
177	A	3	2	1.	22	0.091
178	A	3	2	1.	22	0.091
179	A	3	2	1.	22	0.091
180	A	5	4	1.	22	0.182
181	A	3	2	1.	22	0.091
182	A	4	4	1.	22	0.182
183	A	3	2	1.	20	0.1
184	A	4	3	1.	19	0.158
185	A	3	2	1.	22	0.091
186	A	3	3	1.	22	0.136
187	A	3	2	1.	22	0.091
188	A	4	4	1.	22	0.182
189	A	5	4	1.	22	0.182
190	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
191	A	4	4	1.	22	0.182
192	A	3	2	1.	20	0.1
193	A	3	3	1.	19	0.158
194	A	3	2	1.	22	0.091
195	A	4	4	0.99	22	0.182
196	A	3	2	1.	22	0.091
197	A	5	4	1.	22	0.182
198	A	3	2	1.	20	0.1
199	A	4	3	1.	20	0.15
200	A	3	2	1.	20	0.1
201	A	3	3	1.	20	0.15
202	A	3	2	1.	18	0.111
203	A	2	2	1.	17	0.118
204	A	3	2	1.	20	0.1
205	A	2	2	1.	20	0.1
206	A	3	2	1.	20	0.1
207	A	3	3	1.	20	0.15
208	A	3	2	1.	22	0.091
209	A	3	2	1.	22	0.091
210	A	3	2	1.	22	0.091
211	A	3	2	1.	22	0.091
212	A	3	2	1.	20	0.1
213	A	3	2	1.	19	0.105
214	A	3	2	1.	22	0.091
215	A	3	2	1.	22	0.091
216	A	3	2	1.	22	0.091
217	A	3	2	1.	22	0.091
218	A	3	2	1.	22	0.091
219	A	3	2	1.	22	0.091
220	A	3	2	1.	22	0.091
221	A	3	2	1.	22	0.091
222	A	3	2	1.	20	0.1
223	A	3	2	1.	19	0.105
224	A	3	2	1.	22	0.091
225	A	3	2	1.	22	0.091
226	A	3	2	1.	22	0.091
227	A	3	2	1.	22	0.091
228	A	3	2	1.	22	0.091
229	A	4	3	1.	22	0.136
230	A	3	2	1.	22	0.091
231	A	3	2	1.	22	0.091
232	A	4	3	1.	20	0.15
233	A	3	2	1.	19	0.105
234	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	4	3	1.	22	0.136
236	A	3	2	1.	22	0.091
237	A	5	4	1.	22	0.182
238	A	3	2	1.	22	0.091
239	A	6	4	1.	22	0.182
240	A	3	2	1.	22	0.091
241	A	3	2	1.	22	0.091
242	A	4	3	1.	22	0.136
243	A	3	2	1.	22	0.091
244	A	4	3	1.	22	0.136
245	A	3	2	1.	20	0.1
246	A	4	3	1.	19	0.158
247	A	3	2	1.	22	0.091
248	A	5	4	1.	22	0.182
249	A	3	2	1.	22	0.091
250	A	6	4	1.	22	0.182
251	A	3	2	1.	22	0.091
252	A	5	4	1.	22	0.182
253	A	3	2	1.	22	0.091
254	A	5	4	1.	22	0.182
255	A	3	2	1.	20	0.1
256	A	5	4	1.	19	0.21
257	A	3	2	1.	22	0.091
258	A	6	5	1.	22	0.227
259	A	3	2	1.	22	0.091
260	A	7	5	1.	22	0.227
261	A	4	3	1.	16	0.188
262	A	4	3	1.	20	0.15
263	A	3	2	1.	20	0.1
264	A	3	3	1.	20	0.15
265	A	3	2	1.	18	0.111
266	A	2	2	1.	17	0.118
267	A	3	2	1.	20	0.1
268	A	3	3	1.	20	0.15
269	A	3	2	1.	20	0.1
270	A	4	3	1.	20	0.15
271	A	5	4	1.	22	0.182
272	A	3	2	1.	22	0.091
273	A	4	4	1.	22	0.182
274	A	3	2	1.	20	0.1
275	A	4	3	1.	19	0.158
276	A	3	2	1.	22	0.091
277	A	3	3	1.	22	0.136
278	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
279	A	4	4	0.98	22	0.182
280	A	4	3	1.	22	0.136
281	A	3	2	1.	22	0.091
282	A	5	4	1.	22	0.182
283	A	3	2	1.	20	0.1
284	A	4	3	1.	19	0.158
285	A	3	2	1.	22	0.091
286	A	4	3	1.	22	0.136
287	A	3	2	1.	22	0.091
288	A	4	3	1.	22	0.136
289	A	4	3	1.	22	0.136
290	A	3	2	1.	22	0.091
291	A	4	3	1.	22	0.136
292	A	3	2	1.	20	0.1
293	A	4	3	1.	19	0.158
294	A	3	2	1.	22	0.091
295	A	5	4	1.	22	0.182
296	A	3	2	1.	22	0.091
297	A	6	4	1.	22	0.182
298	A	3	2	1.	22	0.091
299	A	7	4	1.	22	0.182
300	A	3	2	1.	22	0.091
301	A	5	4	1.	22	0.182
302	A	3	2	1.	22	0.091
303	A	5	4	1.	22	0.182
304	A	3	2	1.	20	0.1
305	A	5	4	1.	19	0.21
306	A	3	2	1.	22	0.091
307	A	6	5	1.	22	0.227
308	A	3	2	1.	22	0.091
309	A	7	5	1.	22	0.227
310	A	6	4	1.	22	0.182
311	A	3	2	1.	22	0.091
312	A	6	4	1.	22	0.182
313	A	3	2	1.	20	0.1
314	A	6	4	1.	19	0.21
315	A	3	2	1.	22	0.091
316	A	7	5	1.	22	0.227
317	A	3	2	1.	22	0.091
318	A	8	5	1.	22	0.227
319	A	2	1	1.	20	0.05
320	A	2	1	1.	20	0.05
321	A	2	1	1.	18	0.056
322	A	2	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	2	2	1.	20	0.1
324	A	2	2	1.	20	0.1
325	A	2	1	1.	22	0.045
326	A	2	1	1.	22	0.045
327	A	2	1	1.	20	0.05
328	A	3	2	1.	22	0.091
329	A	3	3	1.	22	0.136
330	A	3	3	0.97	22	0.136
331	A	3	2	1.	22	0.091
332	A	3	2	1.	22	0.091
333	A	2	2	1.	20	0.1
334	A	3	2	1.	22	0.091
335	A	5	3	1.	22	0.136
336	A	6	4	1.	22	0.182
337	A	4	3	1.	22	0.136
338	A	3	3	1.	22	0.136
339	A	2	2	1.	20	0.1
340	A	5	3	1.	22	0.136
341	A	6	4	1.	22	0.182
342	A	7	4	1.	22	0.182
343	A	2	1	1.	20	0.05
344	A	2	1	1.	20	0.05
345	A	2	1	1.	20	0.05
346	A	2	1	1.	20	0.05
347	A	2	1	1.	20	0.05
348	A	2	1	1.	20	0.05
349	A	2	1	1.	20	0.05
350	A	2	1	1.	20	0.05
351	A	2	1	1.	22	0.045
352	A	2	1	1.	22	0.045
353	A	2	1	1.	22	0.045
354	A	2	1	1.	22	0.045
355	A	2	1	1.	22	0.045
356	A	2	1	1.	22	0.045
357	A	2	1	1.	22	0.045
358	A	2	1	1.	22	0.045
359	A	2	1	1.	22	0.045
360	A	2	1	1.	22	0.045
361	A	2	1	1.	22	0.045
362	A	2	1	1.	22	0.045
363	A	2	1	1.	22	0.045
364	A	2	1	1.	22	0.045
365	A	2	1	1.	22	0.045
366	A	2	1	1.	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	13	9	1.	22	0.409
368	A	12	9	1.	22	0.409
369	A	12	9	1.	22	0.409
370	A	11	8	1.	22	0.364
371	A	11	8	1.	22	0.364
372	A	11	8	1.	22	0.364
373	A	11	8	1.	22	0.364
374	A	12	9	1.	22	0.409
375	A	13	9	1.	22	0.409
376	A	12	9	1.	22	0.409
377	A	12	9	1.	22	0.409
378	A	11	8	1.	22	0.364
379	A	11	8	1.	22	0.364
380	A	12	9	1.	22	0.409
381	A	12	9	1.	22	0.409
382	A	13	9	1.	22	0.409
383	A	13	10	1.	22	0.454
384	A	12	9	1.	22	0.409
385	A	12	9	1.	22	0.409
386	A	12	9	1.	22	0.409
387	A	12	9	1.	22	0.409
388	A	13	10	1.	22	0.454
389	A	13	10	1.	22	0.454
390	A	14	10	1.	22	0.454
391	A	2	1	1.	22	0.045
392	A	2	1	1.	22	0.045
393	A	2	1	1.	22	0.045
394	A	2	1	1.	22	0.045
395	A	2	1	1.	22	0.045
396	A	2	1	1.	22	0.045
397	A	2	1	1.	22	0.045
398	A	2	1	1.	22	0.045
399	A	2	1	1.	24	0.042
400	A	2	1	1.	24	0.042
401	A	2	1	1.	24	0.042
402	A	2	1	1.	24	0.042
403	A	2	1	1.	24	0.042
404	A	2	1	1.	24	0.042
405	A	2	1	1.	24	0.042
406	A	2	1	1.	24	0.042
407	A	2	1	1.	24	0.042
408	A	2	1	1.	24	0.042
409	A	2	1	1.	24	0.042
410	A	2	1	1.	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	2	1	1.	24	0.042
412	A	2	1	1.	24	0.042
413	A	2	1	1.	24	0.042
414	A	2	1	1.	24	0.042
415	A	14	9	1.	24	0.375
416	A	13	9	1.	24	0.375
417	A	13	9	1.	24	0.375
418	A	12	8	1.	24	0.333
419	A	12	8	1.	24	0.333
420	A	12	9	1.	24	0.375
421	A	12	9	1.	24	0.375
422	A	12	9	1.	24	0.375
423	A	12	9	1.	24	0.375
424	A	13	10	1.	24	0.417
425	A	14	10	1.	24	0.417
426	A	13	10	1.	24	0.417
427	A	13	10	1.	24	0.417
428	A	12	9	1.	24	0.375
429	A	12	9	1.	24	0.375
430	A	12	9	1.	24	0.375
431	A	12	9	1.	24	0.375
432	A	13	10	1.	24	0.417
433	A	14	10	1.	24	0.417
434	A	13	10	1.	24	0.417
435	A	13	10	1.	24	0.417
436	A	12	9	1.	24	0.375
437	A	12	9	1.	24	0.375
438	A	13	10	1.	24	0.417
439	A	13	10	1.	24	0.417
440	A	14	11	1.	24	0.458
441	A	13	9	1.	24	0.375
442	A	13	9	1.	24	0.375
443	A	12	8	1.	24	0.333
444	A	12	8	1.	24	0.333
445	A	12	8	1.	24	0.333
446	A	12	8	1.	24	0.333
447	A	12	8	1.	24	0.333
448	A	12	8	1.	24	0.333
449	A	12	8	1.	24	0.333
450	A	12	8	1.	24	0.333
451	A	12	8	1.	24	0.333
452	A	13	9	1.	24	0.375
453	A	13	9	1.	24	0.375
454	A	14	10	1.	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
455	A	13	9	1.	24	0.375
456	A	13	9	1.	24	0.375
457	A	13	9	1.	24	0.375
458	A	13	9	1.	24	0.375
459	A	13	9	1.	24	0.375
460	A	13	9	1.	24	0.375
461	A	22	9	1.	24	0.375
462	A	21	9	1.	24	0.375
463	A	20	8	1.	24	0.333
464	A	20	8	1.	24	0.333
465	A	20	8	1.	24	0.333
466	A	20	8	1.	24	0.333
467	A	22	9	1.	24	0.375
468	A	21	9	1.	24	0.375
469	A	23	10	1.	24	0.417
470	A	22	10	1.	24	0.417
471	A	22	9	1.	24	0.375
472	A	21	9	1.	24	0.375
473	A	22	9	1.	24	0.375
474	A	21	9	1.	24	0.375
475	A	22	9	1.	24	0.375
476	A	21	9	1.	24	0.375
477	A	23	10	1.	24	0.417
478	A	22	10	1.	24	0.417
479	A	24	10	1.	24	0.417
480	A	22	10	1.	24	0.417
481	A	23	10	1.	24	0.417
482	A	22	10	1.	24	0.417
483	A	23	10	1.	24	0.417
484	A	22	10	1.	24	0.417
485	A	24	11	1.	24	0.458
486	A	23	11	1.	24	0.458
487	A	25	11	1.	24	0.458
488	A	22	10	1.	24	0.417
489	A	23	10	1.	24	0.417
490	A	22	10	1.	24	0.417
491	A	23	10	1.	24	0.417
492	A	22	10	1.	24	0.417
493	A	24	11	1.	24	0.458
494	A	23	11	1.	24	0.458
495	A	25	11	1.	24	0.458
496	A	23	10	1.	24	0.417
497	A	24	10	1.	24	0.417
498	A	23	10	1.	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
499	A	24	10	1.	24	0.417
500	A	23	10	1.	24	0.417
501	A	25	11	1.	24	0.458
502	A	24	11	1.	24	0.458
503	A	26	11	1.	24	0.458
504	A	3	2	1.	22	0.091
505	A	6	5	1.	22	0.227
506	A	3	2	1.	22	0.091
507	A	5	5	1.	22	0.227
508	A	3	2	1.	20	0.1
509	A	4	4	1.	19	0.21
510	A	5	5	1.	22	0.227
511	A	4	4	1.	22	0.182
512	A	5	5	1.	22	0.227
513	A	4	4	1.	22	0.182
514	A	5	5	1.	22	0.227
515	A	2	2	1.	22	0.091
516	A	6	6	1.	22	0.273
517	A	3	3	1.	22	0.136
518	A	7	6	1.	22	0.273
519	A	4	3	1.	22	0.136
520	A	8	6	1.	22	0.273
521	A	3	2	1.	22	0.091
522	A	7	5	1.	22	0.227
523	A	3	2	1.	22	0.091
524	A	6	5	1.	22	0.227
525	A	3	2	1.	20	0.1
526	A	5	4	1.	19	0.21
527	A	6	5	1.	22	0.227
528	A	5	4	1.	22	0.182
529	A	6	5	1.	22	0.227
530	A	5	5	1.	22	0.227
531	A	6	6	1.	22	0.273
532	A	5	4	1.	22	0.182
533	A	6	5	1.	22	0.227
534	A	2	2	1.	22	0.091
535	A	7	6	1.	22	0.273
536	A	3	3	1.	22	0.136
537	A	8	6	1.	22	0.273
538	A	3	2	1.	22	0.091
539	A	8	5	1.	22	0.227
540	A	3	2	1.	22	0.091
541	A	7	5	1.	22	0.227
542	A	3	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
543	A	6	4	1.	19	0.21
544	A	7	5	1.	22	0.227
545	A	6	4	1.	22	0.182
546	A	7	5	1.	22	0.227
547	A	6	5	1.	22	0.227
548	A	7	6	1.	22	0.273
549	A	6	5	1.	22	0.227
550	A	7	6	1.	22	0.273
551	A	6	4	1.	22	0.182
552	A	7	5	1.	22	0.227
553	A	2	2	1.	22	0.091
554	A	8	6	1.	22	0.273
555	A	3	2	1.	22	0.091
556	A	5	4	1.	22	0.182
557	A	3	2	1.	22	0.091
558	A	4	4	1.	22	0.182
559	A	3	2	1.	20	0.1
560	A	3	3	1.	19	0.158
561	A	4	4	1.	22	0.182
562	A	3	3	1.	22	0.136
563	A	4	4	1.	22	0.182
564	A	2	2	1.	22	0.091
565	A	5	5	1.	22	0.227
566	A	3	3	1.	22	0.136
567	A	6	5	1.	22	0.227
568	A	4	3	1.	22	0.136
569	A	6	5	1.	22	0.227
570	A	3	2	1.	22	0.091
571	A	5	5	1.	22	0.227
572	A	3	2	1.	22	0.091
573	A	4	4	1.	22	0.182
574	A	3	2	1.	20	0.1
575	A	3	3	1.	19	0.158
576	A	4	4	1.	22	0.182
577	A	2	2	1.	22	0.091
578	A	5	5	1.	22	0.227
579	A	3	3	1.	22	0.136
580	A	6	5	1.02	22	0.227
581	A	4	3	1.	22	0.136
582	A	7	5	1.	22	0.227
583	A	5	3	1.	22	0.136
584	A	3	2	1.	22	0.091
585	A	6	5	1.	22	0.227
586	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
587	A	5	5	1.	22	0.227
588	A	3	2	1.	22	0.091
589	A	4	4	1.	22	0.182
590	A	3	2	1.	20	0.1
591	A	2	2	1.	19	0.105
592	A	5	5	1.	22	0.227
593	A	3	3	1.	22	0.136
594	A	6	5	1.	22	0.227
595	A	4	4	1.	22	0.182
596	A	7	5	1.03	22	0.227
597	A	5	4	1.	22	0.182
598	A	3	2	1.	24	0.083
599	A	3	2	1.	24	0.083
600	A	3	2	1.	22	0.091
601	A	6	5	1.	24	0.208
602	A	6	6	1.	24	0.25
603	A	6	6	0.98	24	0.25
604	A	6	6	1.	24	0.25
605	A	6	6	0.98	24	0.25
606	A	5	5	1.	21	0.238
607	A	5	5	0.98	24	0.208
608	A	5	5	1.	24	0.208
609	A	5	5	1.	24	0.208
610	A	3	3	1.01	24	0.125
611	A	4	4	1.01	24	0.167
612	A	5	4	1.01	24	0.167
613	A	8	6	0.99	24	0.25
614	A	3	2	1.	24	0.083
615	A	7	6	0.99	24	0.25
616	A	3	2	1.	22	0.091
617	A	6	5	1.	21	0.238
618	A	7	5	1.	24	0.208
619	A	6	5	0.98	24	0.208
620	A	7	6	1.	24	0.25
621	A	6	5	0.98	24	0.208
622	A	7	6	0.98	24	0.25
623	A	6	6	1.	24	0.25
624	A	7	7	0.98	24	0.292
625	A	3	2	1.	24	0.083
626	A	8	6	0.99	24	0.25
627	A	3	2	1.	22	0.091
628	A	7	5	1.	21	0.238
629	A	8	5	1.	24	0.208
630	A	7	5	0.99	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
631	A	8	6	1.	24	0.25
632	A	7	5	0.98	24	0.208
633	A	8	6	0.99	24	0.25
634	A	7	6	0.99	24	0.25
635	A	8	7	0.99	24	0.292
636	A	6	5	1.	24	0.208
637	A	3	2	1.	24	0.083
638	A	5	5	1.	24	0.208
639	A	3	2	1.	22	0.091
640	A	4	4	1.	21	0.19
641	A	5	4	1.	24	0.167
642	A	4	4	1.	24	0.167
643	A	5	5	1.	24	0.208
644	A	4	4	1.	24	0.167
645	A	5	5	1.	24	0.208
646	A	3	3	1.01	24	0.125
647	A	6	6	1.	24	0.25
648	A	6	5	1.	24	0.208
649	A	3	2	1.	24	0.083
650	A	5	5	1.	24	0.208
651	A	3	2	1.	22	0.091
652	A	4	4	1.	21	0.19
653	A	5	4	1.	24	0.167
654	A	4	4	0.96	24	0.167
655	A	5	5	1.	24	0.208
656	A	3	3	1.01	24	0.125
657	A	6	6	1.	24	0.25
658	A	4	4	1.	24	0.167
659	A	7	6	1.02	24	0.25
660	A	6	6	1.	24	0.25
661	A	3	2	1.	24	0.083
662	A	5	5	1.	24	0.208
663	A	3	2	1.	22	0.091
664	A	4	4	1.	21	0.19
665	A	5	4	1.	24	0.167
666	A	3	3	1.	24	0.125
667	A	6	6	1.	24	0.25
668	A	4	4	0.99	24	0.167
669	A	7	6	1.	24	0.25
670	A	5	5	1.	24	0.208
671	A	4	3	1.	22	0.136
672	A	3	3	1.	22	0.136
673	A	2	2	1.	20	0.1
674	A	3	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
675	A	4	3	1.	22	0.136
676	A	7	7	1.	24	0.292
677	A	5	5	1.	24	0.208
678	A	6	6	1.	24	0.25
679	A	4	4	1.	22	0.182
680	A	5	5	1.	21	0.238
681	A	6	4	1.	24	0.167
682	A	4	4	1.	24	0.167
683	A	7	5	1.	24	0.208
684	A	5	5	1.	24	0.208
685	A	8	7	1.	24	0.292
686	A	6	5	1.	24	0.208
687	A	7	7	1.	24	0.292
688	A	5	4	1.	22	0.182
689	A	6	6	1.	21	0.286
690	A	7	5	1.	24	0.208
691	A	6	6	1.	24	0.25
692	A	7	5	1.	24	0.208
693	A	5	5	1.	24	0.208
694	A	9	8	1.	24	0.333
695	A	7	5	1.	24	0.208
696	A	8	8	1.	24	0.333
697	A	6	4	1.	22	0.182
698	A	7	7	1.	21	0.333
699	A	8	6	1.	24	0.25
700	A	7	7	1.	24	0.292
701	A	8	6	1.	24	0.25
702	A	7	7	1.	24	0.292
703	A	5	4	1.	24	0.167
704	A	4	4	1.	24	0.167
705	A	3	3	1.	22	0.136
706	A	6	4	1.	24	0.167
707	A	7	5	1.	24	0.208
708	A	6	6	1.	24	0.25
709	A	5	5	1.	24	0.208
710	A	2	2	1.	21	0.095
711	A	4	4	1.	24	0.167
712	A	5	5	1.	24	0.208
713	A	6	6	1.	24	0.25
714	A	4	4	1.	24	0.167
715	A	4	4	1.	24	0.167
716	A	4	4	1.	22	0.182
717	A	3	3	1.	21	0.143
718	A	7	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
719	A	5	5	1.	24	0.208
720	A	8	6	1.	24	0.25
721	A	6	5	1.	24	0.208
722	A	5	5	1.	24	0.208
723	A	5	5	1.	24	0.208
724	A	5	5	1.	24	0.208
725	A	5	4	1.	22	0.182
726	A	5	5	1.	21	0.238
727	A	8	6	1.	24	0.25
728	A	6	6	1.	24	0.25
729	A	9	6	1.	24	0.25
730	A	7	6	1.	24	0.25
731	A	7	7	1.	24	0.292
732	A	5	5	1.	24	0.208
733	A	6	6	1.	24	0.25
734	A	4	4	1.	22	0.182
735	A	3	3	1.	21	0.143
736	A	7	5	1.	24	0.208
737	A	5	5	1.	24	0.208
738	A	8	6	1.	24	0.25
739	A	6	5	1.	24	0.208
740	A	8	8	1.	24	0.333
741	A	6	5	1.	24	0.208
742	A	7	7	1.	24	0.292
743	A	5	5	1.	22	0.227
744	A	6	6	1.	21	0.286
745	A	7	5	1.	24	0.208
746	A	5	5	1.	24	0.208
747	A	8	6	1.	24	0.25
748	A	6	5	1.	24	0.208
749	A	9	8	1.	24	0.333
750	A	7	5	1.	24	0.208
751	A	8	7	1.	24	0.292
752	A	6	5	1.	22	0.227
753	A	7	7	1.	21	0.333
754	A	8	6	1.	24	0.25
755	A	7	7	1.	24	0.292
756	A	8	6	1.	24	0.25
757	A	6	6	1.	24	0.25
758	A	6	6	1.	24	0.25
759	A	4	4	1.	24	0.167
760	A	4	4	1.	24	0.167
761	A	4	4	1.	22	0.182
762	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
763	A	7	5	1.	24	0.208
764	A	5	5	1.	24	0.208
765	A	8	6	1.	24	0.25
766	A	6	5	1.	24	0.208
767	A	5	5	1.	24	0.208
768	A	5	5	1.	24	0.208
769	A	5	5	1.	24	0.208
770	A	5	4	1.	22	0.182
771	A	5	5	1.	21	0.238
772	A	8	6	1.	24	0.25
773	A	6	6	1.	24	0.25
774	A	9	7	1.	24	0.292
775	A	7	6	1.	24	0.25
776	A	6	5	1.	24	0.208
777	A	6	5	1.	24	0.208
778	A	6	5	1.	24	0.208
779	A	6	4	1.	22	0.182
780	A	6	5	1.	21	0.238
781	A	9	6	1.	24	0.25
782	A	7	6	1.	24	0.25
783	A	10	7	1.	24	0.292
784	A	8	6	1.	24	0.25
785	A	5	5	1.	26	0.192
786	A	6	6	1.	26	0.231
787	A	4	4	1.	26	0.154
788	A	6	6	1.	26	0.231
789	A	4	4	1.	26	0.154
790	A	6	6	1.	26	0.231
791	A	4	4	1.	24	0.167
792	A	7	7	1.	24	0.292
793	A	5	5	1.	24	0.208
794	A	6	5	1.	26	0.192
795	A	7	6	1.	26	0.231
796	A	5	4	1.	26	0.154
797	A	7	6	1.	26	0.231
798	A	5	4	1.	26	0.154
799	A	7	7	1.	26	0.269
800	A	6	6	1.	26	0.231
801	A	4	4	1.	26	0.154
802	A	5	5	1.	26	0.192
803	A	3	3	1.	26	0.115
804	A	5	5	1.	26	0.192
805	A	3	3	1.	26	0.115
806	A	6	6	1.	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
807	A	5	5	1.	26	0.192
808	A	6	6	1.	26	0.231
809	A	4	4	1.	26	0.154
810	A	5	5	1.	26	0.192
811	A	3	3	1.	26	0.115
812	A	6	6	1.	26	0.231
813	A	4	4	1.	26	0.154
814	A	7	7	1.	26	0.269
815	A	5	4	1.	26	0.154
816	A	6	6	1.	26	0.231
817	A	4	4	1.	26	0.154
818	A	6	6	1.	26	0.231
819	A	4	4	1.	26	0.154
820	A	7	6	1.	26	0.231
821	A	5	4	1.	26	0.154
822	A	6	6	1.	28	0.214
823	A	7	7	1.	28	0.25
824	A	5	5	1.	28	0.179
825	A	7	7	1.	28	0.25
826	A	5	5	1.	28	0.179
827	A	7	7	1.	28	0.25
828	A	5	5	0.99	26	0.192
829	A	7	7	0.99	26	0.269
830	A	5	5	0.98	26	0.192
831	A	8	8	0.99	26	0.308
832	A	9	8	1.	28	0.286
833	A	7	6	1.	28	0.214
834	A	8	7	1.	28	0.25
835	A	6	5	1.	28	0.179
836	A	8	7	1.	28	0.25
837	A	6	5	1.	28	0.179
838	A	8	7	1.	28	0.25
839	A	7	7	1.	28	0.25
840	A	5	5	1.	28	0.179
841	A	6	6	1.	28	0.214
842	A	4	4	1.	28	0.143
843	A	6	6	1.	28	0.214
844	A	4	4	1.	28	0.143
845	A	6	6	1.	28	0.214
846	A	4	4	1.	28	0.143
847	A	7	7	1.	28	0.25
848	A	5	5	1.	28	0.179
849	A	6	5	1.	28	0.179
850	A	7	7	1.	28	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
851	A	5	5	1.	28	0.179
852	A	6	6	1.	28	0.214
853	A	4	4	1.	28	0.143
854	A	6	6	1.	28	0.214
855	A	4	4	1.	28	0.143
856	A	7	7	1.	28	0.25
857	A	6	6	1.	28	0.214
858	A	7	7	1.	28	0.25
859	A	5	5	1.	28	0.179
860	A	6	6	1.	28	0.214
861	A	4	4	1.	28	0.143
862	A	7	7	1.	28	0.25
863	A	5	5	1.	28	0.179
864	A	8	7	1.	28	0.25
865	A	11	9	1.	30	0.3
866	A	15	12	1.	30	0.4
867	A	10	8	1.	30	0.267
868	A	13	11	1.	30	0.367
869	A	9	7	1.	30	0.233
870	A	15	12	1.	30	0.4
871	A	10	8	1.	30	0.267
872	A	16	13	1.	30	0.433
873	A	16	13	1.	30	0.433
874	A	11	9	1.	30	0.3
875	A	15	12	1.	30	0.4
876	A	10	8	1.	30	0.267
877	A	15	12	1.	30	0.4
878	A	10	8	1.	30	0.267
879	A	16	13	1.	30	0.433
880	A	10	8	1.	30	0.267
881	A	13	11	1.	30	0.367
882	A	9	7	1.	30	0.233
883	A	6	4	1.	30	0.133
884	A	6	4	1.	30	0.133
885	A	15	12	1.	30	0.4
886	A	10	8	1.	30	0.267
887	A	16	13	1.	30	0.433
888	A	15	12	1.	30	0.4
889	A	10	8	1.	30	0.267
890	A	15	12	1.	30	0.4
891	A	10	8	1.	30	0.267
892	A	15	12	1.	30	0.4
893	A	10	8	1.	30	0.267
894	A	16	13	1.	30	0.433

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
895	A	11	9	1.	30	0.3
896	A	11	9	1.	30	0.3
897	A	15	12	1.	30	0.4
898	A	10	8	1.	30	0.267
899	A	15	12	1.	30	0.4
900	A	10	8	1.	30	0.267
901	A	16	13	1.	30	0.433
902	A	11	9	1.	30	0.3
903	A	12	10	1.	30	0.333
904	A	16	13	1.	30	0.433
905	A	11	9	1.	30	0.3
906	A	15	12	1.	30	0.4
907	A	10	8	1.	30	0.267
908	A	16	13	1.	30	0.433
909	A	11	9	1.	30	0.3
910	A	15	12	1.	30	0.4
911	A	10	8	1.	30	0.267
912	A	15	12	1.	30	0.4
913	A	10	8	1.	30	0.267
914	A	15	12	1.	30	0.4
915	A	10	8	1.	30	0.267
916	A	16	13	1.	30	0.433
917	A	11	9	1.	30	0.3
918	A	16	13	1.	30	0.433
919	A	11	9	1.	30	0.3
920	A	16	13	1.	30	0.433
921	A	11	9	1.	30	0.3
922	A	16	13	1.	30	0.433
923	A	11	9	1.	30	0.3
924	A	17	14	1.	30	0.467
925	A	12	10	1.	30	0.333
926	A	17	13	1.	30	0.433
927	A	12	9	1.	30	0.3
928	A	17	13	1.	30	0.433
929	A	12	9	1.	30	0.3
930	A	17	13	1.	30	0.433
931	A	12	9	1.	30	0.3
932	A	18	14	1.	30	0.467
933	A	13	10	1.	30	0.333
934	A	7	7	1.	26	0.269
935	A	6	6	1.	26	0.231
936	A	5	5	1.	24	0.208
937	A	7	7	1.	26	0.269
938	A	4	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
939	A	5	5	1.	26	0.192
940	A	6	6	1.	26	0.231
941	A	5	5	1.	26	0.192
942	A	6	6	1.	26	0.231
943	A	6	6	1.	26	0.231
944	A	8	7	1.	26	0.269
945	A	7	6	1.	26	0.231
946	A	6	5	1.	24	0.208
947	A	8	8	1.	26	0.308
948	A	8	8	1.	26	0.308
949	A	5	4	1.	26	0.154
950	A	7	6	1.	26	0.231
951	A	6	6	1.	26	0.231
952	A	5	5	1.	26	0.192
953	A	6	6	1.	26	0.231
954	A	9	7	1.	26	0.269
955	A	8	6	1.	26	0.231
956	A	7	5	1.	24	0.208
957	A	9	9	1.	26	0.346
958	A	9	9	1.	26	0.346
959	A	9	9	1.	26	0.346
960	A	8	7	1.	26	0.269
961	A	7	7	1.	26	0.269
962	A	6	6	1.	26	0.231
963	A	6	6	1.	26	0.231
964	A	5	5	1.	26	0.192
965	A	6	6	1.	26	0.231
966	A	4	4	1.	26	0.154
967	A	5	5	1.	24	0.208
968	A	5	5	1.	26	0.192
969	A	6	6	1.	26	0.231
970	A	5	5	1.	26	0.192
971	A	4	4	1.	24	0.167
972	A	3	3	1.	26	0.115
973	A	4	4	1.	26	0.154
974	A	6	6	1.	26	0.231
975	A	6	6	1.	26	0.231
976	A	5	5	1.	26	0.192
977	A	2	2	1.	26	0.077
978	A	4	4	1.	26	0.154
979	A	6	6	1.	26	0.231
980	A	6	6	1.	26	0.231
981	A	5	5	1.	26	0.192
982	A	2	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
983	A	6	6	1.	26	0.231
984	A	3	3	1.	26	0.115
985	A	3	3	1.	24	0.125
986	A	4	4	1.	26	0.154
987	A	4	4	1.	26	0.154
988	A	4	3	1.	24	0.125
989	A	5	5	1.	26	0.192
990	A	4	4	1.	25	0.16
991	A	4	4	1.	26	0.154
992	A	2	2	1.	26	0.077
993	A	5	5	1.	26	0.192
994	A	2	2	1.	24	0.083
995	A	3	3	1.	26	0.115
996	A	3	3	1.	26	0.115
997	A	3	3	1.	26	0.115
998	A	3	3	1.	26	0.115
999	A	3	3	1.	26	0.115
1000	A	3	3	1.	26	0.115
1001	A	3	3	1.	24	0.125
1002	A	3	3	1.	24	0.125
1003	A	3	3	1.	26	0.115
1004	A	2	2	1.	24	0.083
1005	A	2	2	1.	24	0.083
1006	A	2	2	1.	26	0.077
1007	A	3	3	1.	26	0.115
1008	A	7	6	1.	22	0.273
1009	A	6	6	1.	22	0.273
1010	A	5	5	1.	20	0.25
1011	A	10	7	1.	22	0.318
1012	A	7	7	1.	22	0.318
1013	A	12	9	1.	22	0.409
1014	A	7	7	1.	22	0.318
1015	A	6	6	1.	22	0.273
1016	A	1	1	1.	19	0.053
1017	A	7	7	1.	22	0.318
1018	A	8	8	1.	22	0.364
1019	A	7	7	1.	22	0.318
1020	A	7	7	1.	22	0.318
1021	A	6	6	1.	22	0.273
1022	A	6	6	1.	20	0.3
1023	A	11	8	1.	22	0.364
1024	A	12	9	1.	22	0.409
1025	A	13	9	1.	22	0.409
1026	A	7	7	1.	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1027	A	7	7	1.	22	0.318
1028	A	7	7	1.	19	0.368
1029	A	8	8	1.	22	0.364
1030	A	9	8	1.	22	0.364
1031	A	10	5	1.	24	0.208
1032	A	7	5	1.	24	0.208
1033	A	4	3	1.	24	0.125
1034	A	1	1	1.	22	0.045
1035	A	8	7	1.	24	0.292
1036	A	14	8	1.	24	0.333
1037	A	6	4	1.	24	0.167
1038	A	4	3	1.	24	0.125
1039	A	1	1	1.	21	0.048
1040	A	5	4	1.	24	0.167
1041	A	8	4	1.	24	0.167
1042	A	7	6	1.	24	0.25
1043	A	7	6	1.	24	0.25
1044	A	6	6	1.	24	0.25
1045	A	5	5	1.	22	0.227
1046	A	16	12	1.	24	0.5
1047	A	17	13	1.	24	0.542
1048	A	12	7	1.	24	0.292
1049	A	7	6	1.	24	0.25
1050	A	1	1	1.	21	0.048
1051	A	8	7	1.	24	0.292
1052	A	14	7	1.	24	0.292
1053	A	1	1	1.	24	0.042
1054	A	1	1	1.	24	0.042
1055	A	1	1	1.	24	0.042
1056	A	1	1	1.	26	0.038
1057	A	1	1	1.	26	0.038
1058	A	1	1	1.	26	0.038
1059	A	1	1	1.	26	0.038
1060	A	1	1	1.	28	0.036
1061	A	20	12	1.	24	0.5
1062	A	17	12	1.	24	0.5
1063	A	14	10	1.	24	0.417
1064	A	11	8	1.	22	0.364
1065	A	18	13	1.	24	0.542
1066	A	24	14	1.	24	0.583
1067	A	11	5	1.	24	0.208
1068	A	8	5	1.	24	0.208
1069	A	1	1	1.	24	0.042
1070	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1071	A	7	5	1.	24	0.208
1072	A	10	5	1.	24	0.208
1073	A	1	1	1.	24	0.042
1074	A	1	1	1.	24	0.042
1075	A	1	1	1.	24	0.042
1076	A	1	1	1.	26	0.038
1077	A	1	1	1.	28	0.036
1078	A	1	1	1.	28	0.036
1079	A	1	1	1.	28	0.036
1080	A	1	1	1.	30	0.033
1081	A	7	6	1.	24	0.25
1082	A	7	6	1.	24	0.25
1083	A	6	6	1.	24	0.25
1084	A	5	5	1.	22	0.227
1085	A	16	12	1.	24	0.5
1086	A	17	13	1.	24	0.542
1087	A	15	6	1.	24	0.25
1088	A	11	6	1.	24	0.25
1089	A	1	1	1.	24	0.042
1090	A	4	4	1.	21	0.19
1091	A	9	6	1.	24	0.25
1092	A	13	6	1.	24	0.25
1093	A	7	7	1.	26	0.269
1094	A	6	6	1.	26	0.231
1095	A	6	6	1.	26	0.231
1096	A	2	2	1.	26	0.077
1097	A	3	3	1.	26	0.115
1098	A	4	3	1.	26	0.115
1099	A	8	7	1.	26	0.269
1100	A	7	7	1.	26	0.269
1101	A	6	6	1.	26	0.231
1102	A	6	6	1.	26	0.231
1103	A	7	7	1.	26	0.269
1104	A	8	7	1.	26	0.269
1105	A	7	7	1.	26	0.269
1106	A	6	6	1.	26	0.231
1107	A	2	2	1.	26	0.077
1108	A	3	3	1.	26	0.115
1109	A	4	3	1.	26	0.115
1110	A	6	5	1.	26	0.192
1111	A	5	5	1.	26	0.192
1112	A	4	4	1.	26	0.154
1113	A	4	4	1.	26	0.154
1114	A	5	5	1.	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1115	A	6	5	1.	26	0.192
1116	A	7	7	1.	26	0.269
1117	A	6	6	1.	26	0.231
1118	A	2	2	1.	26	0.077
1119	A	3	3	1.	26	0.115
1120	A	4	3	1.	26	0.115
1121	A	8	7	1.	26	0.269
1122	A	7	7	1.	26	0.269
1123	A	6	6	1.	26	0.231
1124	A	7	7	1.	26	0.269
1125	A	8	8	1.	26	0.308
1126	A	8	8	1.	26	0.308
1127	A	7	7	1.	26	0.269
1128	A	2	2	1.	26	0.077
1129	A	3	3	1.	26	0.115
1130	A	4	3	1.	26	0.115
1131	A	5	3	1.	26	0.115
1132	A	7	5	1.	26	0.192
1133	A	6	5	1.	26	0.192
1134	A	5	5	1.	26	0.192
1135	A	4	4	1.	26	0.154
1136	A	5	5	1.	26	0.192
1137	A	6	6	1.	26	0.231
1138	A	7	6	1.	26	0.231
1139	A	3	2	1.	24	0.083
1140	A	3	2	1.	22	0.091
1141	A	3	2	1.	22	0.091
1142	A	3	2	1.	19	0.105
1143	A	3	2	1.	22	0.091
1144	A	3	2	1.	22	0.091
1145	A	5	5	1.	22	0.227
1146	A	4	4	1.	22	0.182
1147	A	3	3	1.	20	0.15
1148	A	3	3	1.	22	0.136
1149	A	3	3	1.	22	0.136
1150	A	3	3	1.	22	0.136
1151	A	3	2	1.	26	0.077
1152	A	3	2	1.	26	0.077
1153	A	3	2	1.	26	0.077
1154	A	3	2	1.	26	0.077
1155	A	3	2	1.	26	0.077
1156	A	3	2	1.	26	0.077

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rubi [A] time = 0.0195923, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\int\left(aAx^2+(Ab+aB)x^4+bBx^6\right)\,dx\\&=\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A] time = 0.0055308, size = 33, normalized size = 1.

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Maple [A] time = 0.001, size = 28, normalized size = 0.9

$$\frac{aAx^3}{3} + \frac{(Ab + Ba)\,x^5}{5} + \frac{bBx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

Maxima [A] time = 0.991062, size = 36, normalized size = 1.09

$$\frac{1}{7}\,Bbx^7 + \frac{1}{5}\,(Ba + Ab)x^5 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 1.24705, size = 74, normalized size = 2.24

$$\frac{1}{7}x^7bB + \frac{1}{5}x^5aB + \frac{1}{5}x^5bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/3*x^3*a*A
```

Sympy [A] time = 0.057648, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^7}{7} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

Giac [A] time = 1.12168, size = 39, normalized size = 1.18

$$\frac{1}{7}\,Bbx^7 + \frac{1}{5}\,Bax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

3.2 $\int x(a+bx^2)(A+Bx^2)\,dx$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rubi [A] time = 0.0316592, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 43}

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(aA+(Ab+aB)x+bBx^2\right)dx,x,x^2\right)\\ &= \frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6\end{aligned}$$

Mathematica [A] time = 0.0071658, size = 33, normalized size = 1.

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Maple [A] time = 0., size = 28, normalized size = 0.9

$$\frac{aAx^2}{2} + \frac{(Ab + Ba)\,x^4}{4} + \frac{bBx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

Maxima [A] time = 0.975434, size = 36, normalized size = 1.09

$$\frac{1}{6}\,Bbx^6 + \frac{1}{4}\,(Ba + Ab)x^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 1.27144, size = 74, normalized size = 2.24

$$\frac{1}{6}x^6bB + \frac{1}{4}x^4aB + \frac{1}{4}x^4bA + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + 1/2*x^2*a*A
```

Sympy [A] time = 0.056516, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^6}{6} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**6/6 + x**4*(A*b/4 + B*a/4)
```

Giac [A] time = 1.13427, size = 39, normalized size = 1.18

$$\frac{1}{6}\,Bbx^6 + \frac{1}{4}\,Bax^4 + \frac{1}{4}\,Abx^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/2*A*a*x^2
```

3.3 $\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rubi [A] time = 0.0125876, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aA+(Ab+aB)x^2+bBx^4\right)\,dx\\ &= aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A] time = 0.0053295, size = 28, normalized size = 1.

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Maple [A] time = 0., size = 25, normalized size = 0.9

$$aAx+\frac{(Ab+Ba)\,x^3}{3}+\frac{bBx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A),x)
```

```
[Out] a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

Maxima [A] time = 0.98134, size = 32, normalized size = 1.14

$$\frac{1}{5}Bbx^5 + \frac{1}{3}(Ba + Ab)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Fricas [A] time = 1.23939, size = 66, normalized size = 2.36

$$\frac{1}{5}x^5bB + \frac{1}{3}x^3aB + \frac{1}{3}x^3bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b*B + 1/3*x^3*a*B + 1/3*x^3*b*A + x*a*A
```

Sympy [A] time = 0.055706, size = 26, normalized size = 0.93

$$Aax + \frac{Bbx^5}{5} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

Giac [A] time = 1.141, size = 35, normalized size = 1.25

$$\frac{1}{5}Bbx^5 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

3.4 $\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rubi [A] time = 0.0208088, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(Ab + aB + \frac{aA}{x} + bBx\right) dx, x, x^2\right)\\
&= \frac{1}{2}(Ab + aB)x^2 + \frac{1}{4}bBx^4 + aA\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0086718, size = 29, normalized size = 1.

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Maple [A] time = 0.002, size = 28, normalized size = 1.

$$\frac{bBx^4}{4} + \frac{Ax^2b}{2} + \frac{Bx^2a}{2} + aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x,x)
```

```
[Out] 1/4*b*B*x^4+1/2*A*x^2*b+1/2*B*x^2*a+a*A*ln(x)
```

Maxima [A] time = 1.02901, size = 38, normalized size = 1.31

$$\frac{1}{4}Bbx^4 + \frac{1}{2}(Ba + Ab)x^2 + \frac{1}{2}Aa\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + 1/2*A*a*log(x^2)
```

Fricas [A] time = 1.39864, size = 65, normalized size = 2.24

$$\frac{1}{4}Bbx^4 + \frac{1}{2}(Ba + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] time = 0.250676, size = 27, normalized size = 0.93

$$Aa\log(x) + \frac{Bbx^4}{4} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**4/4 + x**2*(A*b/2 + B*a/2)
```

Giac [A] time = 1.17047, size = 41, normalized size = 1.41

$$\frac{1}{4}\,Bbx^4 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 + \frac{1}{2}\,Aa\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + 1/2*A*a*log(x^2)
```

3.5 $\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=26

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rubi [A] time = 0.0164823, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^2}+bBx^2\right)dx\\ &= -\frac{aA}{x}+(Ab+aB)x+\frac{1}{3}bBx^3\end{aligned}$$

Mathematica [A] time = 0.0088594, size = 26, normalized size = 1.

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\frac{bBx^3}{3}+Abx+Bax-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/3*b*B*x^3+A*b*x+B*a*x-a*A/x
```

Maxima [A] time = 1.00229, size = 32, normalized size = 1.23

$$\frac{1}{3}Bbx^3 + (Ba + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + (B*a + A*b)*x - A*a/x
```

Fricas [A] time = 1.43145, size = 61, normalized size = 2.35

$$\frac{Bbx^4 + 3(Ba + Ab)x^2 - 3Aa}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^4 + 3*(B*a + A*b)*x^2 - 3*A*a)/x
```

Sympy [A] time = 0.245351, size = 20, normalized size = 0.77

$$-\frac{Aa}{x} + \frac{Bbx^3}{3} + x(Ab + Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**3/3 + x*(A*b + B*a)
```

Giac [A] time = 1.18391, size = 31, normalized size = 1.19

$$\frac{1}{3}Bbx^3 + Bax + Abx - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*B*b*x^3 + B*a*x + A*b*x - A*a/x
```

3.6 $\int \frac{(a+bx^2)(A+Bx^2)}{x^3} \, dx$

Optimal. Leaf size=29

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

```
[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rubi [A] time = 0.0202266, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)(A+Bx^2)}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right) dx, x, x^2\right)\\
&= -\frac{aA}{2x^2}+\frac{1}{2}bBx^2+(Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0112234, size = 29, normalized size = 1.

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Maple [A] time = 0.005, size = 26, normalized size = 0.9

$$\frac{bBx^2}{2} + A\ln(x)\,b + B\ln(x)\,a - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^3,x)
```

```
[Out] 1/2*b*B*x^2+A*ln(x)*b+B*ln(x)*a-1/2*a*A/x^2
```

Maxima [A] time = 1.01205, size = 38, normalized size = 1.31

$$\frac{1}{2}\,Bbx^2 + \frac{1}{2}\,(Ba + Ab)\log\left(x^2\right) - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*A*a/x^2
```

Fricas [A] time = 1.47717, size = 70, normalized size = 2.41

$$\frac{Bbx^4 + 2\,(Ba + Ab)x^2\log(x) - Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b*x^4 + 2*(B*a + A*b)*x^2*log(x) - A*a)/x^2
```

Sympy [A] time = 0.320552, size = 26, normalized size = 0.9

$$-\frac{Aa}{2x^2} + \frac{Bbx^2}{2} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**2/2 + (A*b + B*a)*log(x)
```

Giac [A] time = 1.14233, size = 57, normalized size = 1.97

$$\frac{1}{2}Bbx^2 + \frac{1}{2}(Ba + Ab)\log\left(x^2\right) - \frac{Bax^2 + Abx^2 + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*(B*a*x^2 + A*b*x^2 + A*a)/x^2
```

3.7 $\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=26

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

[Out] -(a*A)/(3*x^3) - (A*b + a*B)/x + b*B*x

Rubi [A] time = 0.015107, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*(A + B*x^2))/x^4,x]

[Out] -(a*A)/(3*x^3) - (A*b + a*B)/x + b*B*x

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx &= \int \left(bB+\frac{aA}{x^4}+\frac{Ab+aB}{x^2}\right)dx\\ &= -\frac{aA}{3x^3}-\frac{Ab+aB}{x}+bBx\end{aligned}$$

Mathematica [A] time = 0.0124445, size = 27, normalized size = 1.04

$$\frac{-aB-Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^4,x]

[Out] -(a*A)/(3*x^3) + (-(A*b) - a*B)/x + b*B*x

Maple [A] time = 0.004, size = 25, normalized size = 1.

$$bBx-\frac{Aa}{3\,x^3}-\frac{Ab+Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^4,x)
```

```
[Out] b*B*x-1/3*a*A/x^3-(A*b+B*a)/x
```

Maxima [A] time = 0.986408, size = 35, normalized size = 1.35

$$Bbx - \frac{3\,(Ba + Ab)x^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] B*b*x - 1/3*(3*(B*a + A*b)*x^2 + A*a)/x^3
```

Fricas [A] time = 1.43557, size = 63, normalized size = 2.42

$$\frac{3\,Bbx^4 - 3\,(Ba + Ab)x^2 - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b*x^4 - 3*(B*a + A*b)*x^2 - A*a)/x^3
```

Sympy [A] time = 0.332068, size = 26, normalized size = 1.

$$Bbx - \frac{Aa + x^2\,(3Ab + 3Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**4,x)
```

```
[Out] B*b*x - (A*a + x**2*(3*A*b + 3*B*a))/(3*x**3)
```

Giac [A] time = 1.09227, size = 38, normalized size = 1.46

$$Bbx - \frac{3\,Bax^2 + 3\,Abx^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] B*b*x - 1/3*(3*B*a*x^2 + 3*A*b*x^2 + A*a)/x^3
```

3.8 $\int \frac{(a+bx^2)(A+Bx^2)}{x^5} dx$

Optimal. Leaf size=29

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rubi [A] time = 0.0203407, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)dx,x,x^2\right)\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{2x^2}+bB\log(x)\end{aligned}$$

Mathematica [A] time = 0.0171773, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) + (-(A*b) - a*B)/(2*x^2) + b*B*Log[x]
```

Maple [A] time = 0.004, size = 28, normalized size = 1.

$$bB\ln(x) - \frac{Aa}{4x^4} - \frac{Ab}{2x^2} - \frac{Ba}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^5,x)
```

```
[Out] b*B*ln(x)-1/4*a*A/x^4-1/2/x^2*A*b-1/2/x^2*B*a
```

Maxima [A] time = 1.01377, size = 41, normalized size = 1.41

$$\frac{1}{2}Bb\log\left(x^2\right) - \frac{2(Ba + Ab)x^2 + Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*log(x^2) - 1/4*(2*(B*a + A*b)*x^2 + A*a)/x^4
```

Fricas [A] time = 1.46829, size = 73, normalized size = 2.52

$$\frac{4Bbx^4\log(x) - 2(Ba + Ab)x^2 - Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*B*b*x^4*log(x) - 2*(B*a + A*b)*x^2 - A*a)/x^4
```

Sympy [A] time = 0.47491, size = 27, normalized size = 0.93

$$Bb\log(x) - \frac{Aa + x^2(2Ab + 2Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**5,x)
```

```
[Out] B*b*log(x) - (A*a + x**2*(2*A*b + 2*B*a))/(4*x**4)
```

Giac [A] time = 1.13303, size = 53, normalized size = 1.83

$$\frac{1}{2} Bb \log\left(x^2\right) - \frac{3\, Bbx^4 + 2\, Bax^2 + 2\, Abx^2 + Aa}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b*log(x^2) - 1/4*(3*B*b*x^4 + 2*B*a*x^2 + 2*A*b*x^2 + A*a)/x^4
```

3.9 $\int \frac{(a+bx^2)(A+Bx^2)}{x^6} dx$

Optimal. Leaf size=31

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rubi [A] time = 0.015195, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^6} dx &= \int \left(\frac{aA}{x^6}+\frac{Ab+aB}{x^4}+\frac{bB}{x^2}\right) dx\\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{3x^3}-\frac{bB}{x}\end{aligned}$$

Mathematica [A] time = 0.0109261, size = 33, normalized size = 1.06

$$\frac{-aB-Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) + (-(A*b) - a*B)/(3*x^3) - (b*B)/x
```

Maple [A] time = 0.004, size = 28, normalized size = 0.9

$$-\frac{Ab+Ba}{3x^3}-\frac{Aa}{5x^5}-\frac{Bb}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/3*(A*b+B*a)/x^3-1/5*a*A/x^5-b*B/x
```

Maxima [A] time = 1.02129, size = 39, normalized size = 1.26

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Fricas [A] time = 1.45562, size = 70, normalized size = 2.26

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Sympy [A] time = 0.482671, size = 32, normalized size = 1.03

$$-\frac{3Aa+15Bbx^4+x^2\,(5Ab+5Ba)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

```
[Out] -(3*A*a + 15*B*b*x**4 + x**2*(5*A*b + 5*B*a))/(15*x**5)
```

Giac [A] time = 1.10545, size = 42, normalized size = 1.35

$$-\frac{15\,Bbx^4+5\,Bax^2+5\,Abx^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*B*a*x^2 + 5*A*b*x^2 + 3*A*a)/x^5
```

3.10 $\int \frac{(a+bx^2)(A+Bx^2)}{x^7} \, dx$

Optimal. Leaf size=33

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rubi [A] time = 0.021635, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^7} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^4} \, dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{aA}{x^4}+\frac{Ab+aB}{x^3}+\frac{bB}{x^2}\right) dx, x, x^2\right)\\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{4x^4}-\frac{bB}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0090123, size = 35, normalized size = 1.06

$$\frac{-aB-Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^7,x]

[Out] -(a*A)/(6*x^6) + (-(A*b) - a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$-\frac{Ab+Ba}{4\,x^4}-\frac{Bb}{2\,x^2}-\frac{Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^7,x)

[Out] -1/4*(A*b+B*a)/x^4-1/2*b*B/x^2-1/6*a*A/x^6
```

Maxima [A] time = 1.47851, size = 39, normalized size = 1.18

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="maxima")

[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Fricas [A] time = 1.41149, size = 69, normalized size = 2.09

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="fricas")

[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Sympy [A] time = 0.602801, size = 32, normalized size = 0.97

$$-\frac{2Aa+6Bbx^4+x^2\,(3Ab+3Ba)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**7,x)

[Out] -(2*A*a + 6*B*b*x**4 + x**2*(3*A*b + 3*B*a))/(12*x**6)
```

Giac [A] time = 1.10017, size = 42, normalized size = 1.27

$$-\frac{6\,Bbx^{4} + 3\,Bax^{2} + 3\,Abx^{2} + 2\,Aa}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*B*b*x^4 + 3*B*a*x^2 + 3*A*b*x^2 + 2*A*a)/x^6
```

3.11 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Rubi [A] time = 0.0361749, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx^2\right)dx &= \int \left(a^2Ax^2+a(2Ab+aB)x^4+b(Ab+2aB)x^6+b^2Bx^8\right)dx\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{9}b^2Bx^9\end{aligned}$$

Mathematica [A] time = 0.0077699, size = 55, normalized size = 1.

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{b^2Bx^9}{9}+\frac{\left(b^2A+2\,abB\right)x^7}{7}+\frac{\left(2\,abA+a^2B\right)x^5}{5}+\frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/9*b^2*B*x^9+1/7*(A*b^2+2*B*a*b)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3*a^2*A*x^3
```

Maxima [A] time = 1.02242, size = 69, normalized size = 1.25

$$\frac{1}{9}\,Bb^2x^9 + \frac{1}{7}\left(2\,Bab + Ab^2\right)x^7 + \frac{1}{3}\,Aa^2x^3 + \frac{1}{5}\left(Ba^2 + 2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2*
A*a*b)*x^5
```

Fricas [A] time = 1.25053, size = 128, normalized size = 2.33

$$\frac{1}{9}x^9b^2B + \frac{2}{7}x^7baB + \frac{1}{7}x^7b^2A + \frac{1}{5}x^5a^2B + \frac{2}{5}x^5baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b*a
*A + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.065141, size = 56, normalized size = 1.02

$$\frac{Aa^2x^3}{3} + \frac{Bb^2x^9}{9} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**9/9 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

Giac [A] time = 1.11797, size = 72, normalized size = 1.31

$$\frac{1}{9}\,Bb^2x^9 + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{5}\,Ba^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/5*B*a^2*x^5 + 2/5*A*a*b*x
^5 + 1/3*A*a^2*x^3
```

3.12 $\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rubi [A] time = 0.0643204, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^2(A+Bx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)dx,x,x^2\right)\\
&= \frac{(Ab-aB)\left(a+bx^2\right)^3}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}
\end{aligned}$$

Mathematica [A] time = 0.0111166, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2A+4bx^4(2aB+Ab)+6ax^2(aB+2Ab)+3b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (x^2*(12*a^2*A + 6*a*(2*A*b + a*B)*x^2 + 4*b*(A*b + 2*a*B)*x^4 + 3*b^2*B*x^
6))/24
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{b^2Bx^8}{8}+\frac{\left(b^2A+2\,abB\right)x^6}{6}+\frac{\left(2\,abA+a^2B\right)x^4}{4}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/8*b^2*B*x^8+1/6*(A*b^2+2*B*a*b)*x^6+1/4*(2*A*a*b+B*a^2)*x^4+1/2*a^2*A*x^2
```

Maxima [A] time = 1.02285, size = 69, normalized size = 1.64

$$\frac{1}{8}\,Bb^2x^8+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2*
A*a*b)*x^4
```

Fricas [A] time = 1.24567, size = 128, normalized size = 3.05

$$\frac{1}{8}x^8b^2B+\frac{1}{3}x^6baB+\frac{1}{6}x^6b^2A+\frac{1}{4}x^4a^2B+\frac{1}{2}x^4baA+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2*B + 1/3*x^6*b*a*B + 1/6*x^6*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b*a
*A + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.06673, size = 53, normalized size = 1.26

$$\frac{Aa^2x^2}{2}+\frac{Bb^2x^8}{8}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**8/8 + x**6*(A*b**2/6 + B*a*b/3) + x**4*(A*a*b/2 +
 B*a**2/4)
```

Giac [A] time = 1.11408, size = 72, normalized size = 1.71

$$\frac{1}{8}Bb^2x^8 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{4}Ba^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/4*B*a^2*x^4 + 1/2*A*a*b*x
^4 + 1/2*A*a^2*x^2
```

3.13 $\int \left(a+bx^2\right)^2 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Rubi [A] time = 0.0223412, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(A+Bx^2\right)\,dx &= \int \left(a^2A+a(2Ab+aB)x^2+b(Ab+2aB)x^4+b^2Bx^6\right)dx\\ &= a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A] time = 0.0073365, size = 50, normalized size = 1.

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{b^2Bx^7}{7}+\frac{\left(b^2A+2\,abB\right)x^5}{5}+\frac{\left(2\,abA+a^2B\right)x^3}{3}+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

Maxima [A] time = 1.00006, size = 65, normalized size = 1.3

$$\frac{1}{7}Bb^2x^7 + \frac{1}{5}\left(2\,Bab + Ab^2\right)x^5 + Aa^2x + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)
*x^3
```

Fricas [A] time = 1.28013, size = 120, normalized size = 2.4

$$\frac{1}{7}x^7b^2B + \frac{2}{5}x^5baB + \frac{1}{5}x^5b^2A + \frac{1}{3}x^3a^2B + \frac{2}{3}x^3baA + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2*B + 2/5*x^5*b*a*B + 1/5*x^5*b^2*A + 1/3*x^3*a^2*B + 2/3*x^3*b*a
*A + x*a^2*A
```

Sympy [A] time = 0.065136, size = 53, normalized size = 1.06

$$Aa^2x + \frac{Bb^2x^7}{7} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*A*a*b/3 +
B*a**2/3)
```

Giac [A] time = 1.11558, size = 68, normalized size = 1.36

$$\frac{1}{7}Bb^2x^7 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*x
```

3.14 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} \, dx$

Optimal. Leaf size=43

$$a^2 A \log(x) + aAbx^2 + \frac{B(a+bx^2)^3}{6b} + \frac{1}{4}Ab^2x^4$$

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rubi [A] time = 0.0315544, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 80, 43}

$$a^2 A \log(x) + aAbx^2 + \frac{B(a+bx^2)^3}{6b} + \frac{1}{4}Ab^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx,x,x^2\right)$$
$$= \frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{2}A\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^2\right)$$
$$= \frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{2}A\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx,x,x^2\right)$$
$$= aAbx^2+\frac{1}{4}Ab^2x^4+\frac{B\left(a+bx^2\right)^3}{6b}+a^2A\log(x)$$

Mathematica [A] time = 0.0151837, size = 51, normalized size = 1.19

$$a^2A\log(x)+\frac{1}{4}bx^4(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^6)/6 + a^2*A*L
og[x]
```

Maple [A] time = 0.002, size = 51, normalized size = 1.2

$$\frac{Bb^2x^6}{6}+\frac{Ab^2x^4}{4}+\frac{Bx^4ab}{2}+aAbx^2+\frac{Bx^2a^2}{2}+a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x,x)
```

```
[Out] 1/6*B*b^2*x^6+1/4*A*b^2*x^4+1/2*B*x^4*a*b+a*A*b*x^2+1/2*B*x^2*a^2+a^2*A*ln(
x)
```

Maxima [A] time = 0.969851, size = 70, normalized size = 1.63

$$\frac{1}{6}\,Bb^2x^6+\frac{1}{4}\left(2\,Bab+Ab^2\right)x^4+\frac{1}{2}\,Aa^2\log\left(x^2\right)+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/2*A*a^2*log(x^2) + 1/2*(B*a^2
 + 2*A*a*b)*x^2
```

Fricas [A] time = 1.42842, size = 116, normalized size = 2.7

$$\frac{1}{6}\,Bb^2x^6+\frac{1}{4}\left(2\,Bab+Ab^2\right)x^4+Aa^2\log(x)+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + A*a^2*log(x) + 1/2*(B*a^2 + 2*A
*a*b)*x^2
```

Sympy [A] time = 0.27906, size = 49, normalized size = 1.14

$$Aa^2\log(x) + \frac{Bb^2x^6}{6} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**6/6 + x**4*(A*b**2/4 + B*a*b/2) + x**2*(A*a*b + B
*a**2/2)
```

Giac [A] time = 1.12293, size = 72, normalized size = 1.67

$$\frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Ba^2x^2 + Aabx^2 + \frac{1}{2}Aa^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*B*a^2*x^2 + A*a*b*x^2 +
 1/2*A*a^2*log(x^2)
```

3.15 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2 A}{x}+\frac{1}{3} b x^3 (2 a B+A b)+a x (a B+2 A b)+\frac{1}{5} b^2 B x^5$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Rubi [A] time = 0.026731, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2 A}{x}+\frac{1}{3} b x^3 (2 a B+A b)+a x (a B+2 A b)+\frac{1}{5} b^2 B x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b x^2\right)^2 \left(A+B x^2\right)}{x^2} \, dx &= \int \left(a (2 A b+a B)+\frac{a^2 A}{x^2}+b (A b+2 a B) x^2+b^2 B x^4\right) \, dx\\ &= -\frac{a^2 A}{x}+a (2 A b+a B) x+\frac{1}{3} b (A b+2 a B) x^3+\frac{1}{5} b^2 B x^5\end{aligned}$$

Mathematica [A] time = 0.0154579, size = 48, normalized size = 1.

$$-\frac{a^2 A}{x}+\frac{1}{3} b x^3 (2 a B+A b)+a x (a B+2 A b)+\frac{1}{5} b^2 B x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Maple [A] time = 0.003, size = 49, normalized size = 1.

$$\frac{b^2 B x^5}{5}+\frac{A x^3 b^2}{3}+\frac{2\, B x^3 a b}{3}+2\, abAx+a^2 Bx-\frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^2,x)
```

```
[Out] 1/5*b^2*B*x^5+1/3*A*x^3*b^2+2/3*B*x^3*a*b+2*a*b*A*x+a^2*B*x-a^2*A/x
```

Maxima [A] time = 0.975647, size = 65, normalized size = 1.35

$$\frac{1}{5}\,Bb^{2}x^{5} + \frac{1}{3}\,\left(2\,Bab + Ab^{2}\right)x^{3} - \frac{Aa^{2}}{x} + \left(Ba^{2} + 2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/3*(2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*A*a*b)*x
```

Fricas [A] time = 1.42518, size = 116, normalized size = 2.42

$$\frac{3\,Bb^{2}x^{6} + 5\,\left(2\,Bab + Ab^{2}\right)x^{4} - 15\,Aa^{2} + 15\,\left(Ba^{2} + 2\,Aab\right)x^{2}}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^6 + 5*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 15*(B*a^2 + 2*A*a*
b)*x^2)/x
```

Sympy [A] time = 0.275702, size = 48, normalized size = 1.

$$-\frac{Aa^{2}}{x} + \frac{Bb^{2}x^{5}}{5} + x^{3}\left(\frac{Ab^{2}}{3} + \frac{2Bab}{3}\right) + x\left(2Aab + Ba^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**5/5 + x**3*(A*b**2/3 + 2*B*a*b/3) + x*(2*A*a*b + B*a*
*2)
```

Giac [A] time = 1.13251, size = 65, normalized size = 1.35

$$\frac{1}{5}\,Bb^{2}x^{5} + \frac{2}{3}\,Babx^{3} + \frac{1}{3}\,Ab^{2}x^{3} + Ba^{2}x + 2\,Aabx - \frac{Aa^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^2*x + 2*A*a*b*x - A*a^2
/x
```

3.16 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3} \, dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

```
[Out] -(a^2*A)/(2*x^2) + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2*A*b + a*B
)*Log[x]
```

Rubi [A] time = 0.0456132, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2*A*b + a*B
)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right) dx, x, x^2\right) \\
&= -\frac{a^2A}{2x^2}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{4}b^2Bx^4+a(2Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0234773, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2A}{x^2}+2bx^2(2aB+Ab)+4a\log(x)(aB+2Ab)+b^2Bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*A)/x^2 + 2*b*(A*b + 2*a*B)*x^2 + b^2*B*x^4 + 4*a*(2*A*b + a*B)*Log
[x])/4
```

Maple [A] time = 0.006, size = 50, normalized size = 1.

$$\frac{b^2Bx^4}{4}+\frac{Ax^2b^2}{2}+Bx^2ab+2A\ln(x)ab+B\ln(x)a^2-\frac{Aa^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^3,x)
```

```
[Out] 1/4*b^2*B*x^4+1/2*A*x^2*b^2+B*x^2*a*b+2*A*ln(x)*a*b+B*ln(x)*a^2-1/2*a^2*A/x
^2
```

Maxima [A] time = 0.986909, size = 70, normalized size = 1.37

$$\frac{1}{4}Bb^2x^4+\frac{1}{2}\left(2Bab+Ab^2\right)x^2+\frac{1}{2}\left(Ba^2+2Aab\right)\log\left(x^2\right)-\frac{Aa^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + 1/2*(2*B*a*b + A*b^2)*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*A*a^2/x^2
```

Fricas [A] time = 1.46967, size = 122, normalized size = 2.39

$$\frac{Bb^2x^6+2\left(2Bab+Ab^2\right)x^4+4\left(Ba^2+2Aab\right)x^2\log(x)-2Aa^2}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^6 + 2*(2*B*a*b + A*b^2)*x^4 + 4*(B*a^2 + 2*A*a*b)*x^2*log(x) -
 2*A*a^2)/x^2
```

Sympy [A] time = 0.361602, size = 48, normalized size = 0.94

$$-\frac{Aa^2}{2x^2}+\frac{Bb^2x^4}{4}+a(2Ab+Ba)\log(x)+x^2\left(\frac{Ab^2}{2}+Bab\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**2*(A*b**2/2
+ B*a*b)
```

Giac [A] time = 1.62103, size = 95, normalized size = 1.86

$$\frac{1}{4}Bb^2x^4 + Babx^2 + \frac{1}{2}Ab^2x^2 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)\log\left(x^2\right) - \frac{Ba^2x^2 + 2\,Aabx^2 + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + B*a*b*x^2 + 1/2*A*b^2*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*(B*a^2*x^2 + 2*A*a*b*x^2 + A*a^2)/x^2
```

3.17 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

```
[Out] -(a^2*A)/(3*x^3) - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Rubi [A] time = 0.029933, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^4}+\frac{a(2Ab+aB)}{x^2}+b^2Bx^2\right)dx\\ &= -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+b(Ab+2aB)x+\frac{1}{3}b^2Bx^3\end{aligned}$$

Mathematica [A] time = 0.0177868, size = 50, normalized size = 1.04

$$\frac{a^2(-B)-2aAb}{x}-\frac{a^2A}{3x^3}+bx(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) + (-2*a*A*b - a^2*B)/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Maple [A] time = 0.004, size = 46, normalized size = 1.

$$\frac{b^2Bx^3}{3}+b^2Ax+2\,abBx-\frac{Aa^2}{3\,x^3}-\frac{a\,(2\,Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^4,x)
```

```
[Out] 1/3*b^2*B*x^3+b^2*A*x+2*a*b*B*x-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x
```

Maxima [A] time = 0.96608, size = 68, normalized size = 1.42

$$\frac{1}{3}\,Bb^2x^3 + \left(2\,Bab + Ab^2\right)x - \frac{Aa^2 + 3\left(Ba^2 + 2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

Fricas [A] time = 1.42695, size = 109, normalized size = 2.27

$$\frac{Bb^2x^6 + 3\left(2\,Bab + Ab^2\right)x^4 - Aa^2 - 3\left(Ba^2 + 2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + 3*(2*B*a*b + A*b^2)*x^4 - A*a^2 - 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

Sympy [A] time = 0.379725, size = 49, normalized size = 1.02

$$\frac{Bb^2x^3}{3} + x\left(Ab^2 + 2Bab\right) - \frac{Aa^2 + x^2\left(6Aab + 3Ba^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**2*x**3/3 + x*(A*b**2 + 2*B*a*b) - (A*a**2 + x**2*(6*A*a*b + 3*B*a**2))
/(3*x**3)
```

Giac [A] time = 1.12623, size = 68, normalized size = 1.42

$$\frac{1}{3}\,Bb^2x^3 + 2\,Babx + Ab^2x - \frac{3\,Ba^2x^2 + 6\,Aabx^2 + Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6*A*a*b*x^2 + A*a^
2)/x^3
```

3.18 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*(A*b + 2*a
*B)*Log[x]
```

Rubi [A] time = 0.0383888, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```


```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*(A*b + 2*a
*B)*Log[x]
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^2B+\frac{a^2A}{x^3}+\frac{a(2Ab+aB)}{x^2}+\frac{b(Ab+2aB)}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{2x^2}+\frac{1}{2}b^2Bx^2+b(Ab+2aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0244401, size = 50, normalized size = 0.98

$$b\log(x)(2aB+Ab)-\frac{a^2(A+2Bx^2)+4aAbx^2-2b^2Bx^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -(4*a*A*b*x^2 - 2*b^2*B*x^6 + a^2*(A + 2*B*x^2))/(4*x^4) + b*(A*b + 2*a*B)*
Log[x]
```

Maple [A] time = 0.006, size = 51, normalized size = 1.

$$\frac{b^2Bx^2}{2}+A\ln(x)\,b^2+2\,B\ln(x)\,ab-\frac{Aa^2}{4\,x^4}-\frac{abA}{x^2}-\frac{a^2B}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^5,x)
```

```
[Out] 1/2*b^2*B*x^2+A*ln(x)*b^2+2*B*ln(x)*a*b-1/4*a^2*A/x^4-a/x^2*A*b-1/2*a^2/x^2
*B
```

Maxima [A] time = 0.964059, size = 73, normalized size = 1.43

$$\frac{1}{2}\,Bb^2x^2+\frac{1}{2}\left(2\,Bab+Ab^2\right)\log\left(x^2\right)-\frac{Aa^2+2\left(Ba^2+2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(A*a^2 + 2*(B*a^2 + 2*
A*a*b)*x^2)/x^4
```

Fricas [A] time = 1.45611, size = 122, normalized size = 2.39

$$\frac{2\,Bb^2x^6+4\left(2\,Bab+Ab^2\right)x^4\log(x)-Aa^2-2\left(Ba^2+2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^2*x^6 + 4*(2*B*a*b + A*b^2)*x^4*log(x) - A*a^2 - 2*(B*a^2 + 2*A*
a*b)*x^2)/x^4
```

Sympy [A] time = 0.644465, size = 49, normalized size = 0.96

$$\frac{Bb^2x^2}{2}+b\left(Ab+2Ba\right)\log(x)-\frac{Aa^2+x^2\left(4Aab+2Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**2*x**2/2 + b*(A*b + 2*B*a)*log(x) - (A*a**2 + x**2*(4*A*a*b + 2*B*a**2
))/(4*x**4)
```

Giac [A] time = 1.10515, size = 97, normalized size = 1.9

$$\frac{1}{2}\,Bb^2x^2+\frac{1}{2}\left(2\,Bab+Ab^2\right)\log\left(x^2\right)-\frac{6\,Babx^4+3\,Ab^2x^4+2\,Ba^2x^2+4\,Aabx^2+Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(6*B*a*b*x^4 + 3*A*b^2
*x^4 + 2*B*a^2*x^2 + 4*A*a*b*x^2 + A*a^2)/x^4
```

3.19 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^6} dx$

Optimal. Leaf size=48

$$-\frac{a^2 A}{5x^5} - \frac{a(aB + 2Ab)}{3x^3} - \frac{b(2aB + Ab)}{x} + b^2 Bx$$

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Rubi [A] time = 0.0280537, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2 A}{5x^5} - \frac{a(aB + 2Ab)}{3x^3} - \frac{b(2aB + Ab)}{x} + b^2 Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2)^2 (A + Bx^2)}{x^6} dx &= \int \left(b^2 B + \frac{a^2 A}{x^6} + \frac{a(2Ab + aB)}{x^4} + \frac{b(Ab + 2aB)}{x^2}\right) dx \\ &= -\frac{a^2 A}{5x^5} - \frac{a(2Ab + aB)}{3x^3} - \frac{b(Ab + 2aB)}{x} + b^2 Bx \end{aligned}$$

Mathematica [A] time = 0.0192372, size = 48, normalized size = 1.

$$-\frac{a^2 A}{5x^5} - \frac{a(aB + 2Ab)}{3x^3} - \frac{b(2aB + Ab)}{x} + b^2 Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Maple [A] time = 0.005, size = 45, normalized size = 0.9

$$-\frac{Aa^2}{5x^5} - \frac{a(2Ab + Ba)}{3x^3} - \frac{b(Ab + 2Ba)}{x} + b^2Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

Maxima [A] time = 0.985484, size = 69, normalized size = 1.44

$$Bb^2x - \frac{15\left(2Bab + Ab^2\right)x^4 + 3Aa^2 + 5\left(Ba^2 + 2Aab\right)x^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/15*(15*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2*A*a*b)*x^
2)/x^5
```

Fricas [A] time = 1.39845, size = 119, normalized size = 2.48

$$\frac{15Bb^2x^6 - 15\left(2Bab + Ab^2\right)x^4 - 3Aa^2 - 5\left(Ba^2 + 2Aab\right)x^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*b^2*x^6 - 15*(2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 + 2*A*a*
b)*x^2)/x^5
```

Sympy [A] time = 0.70299, size = 51, normalized size = 1.06

$$Bb^2x - \frac{3Aa^2 + x^4\left(15Ab^2 + 30Bab\right) + x^2\left(10Aab + 5Ba^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**2*x - (3*A*a**2 + x**4*(15*A*b**2 + 30*B*a*b) + x**2*(10*A*a*b + 5*B*a
**2))/(15*x**5)
```

Giac [A] time = 1.13707, size = 72, normalized size = 1.5

$$Bb^2x - \frac{30Babx^4 + 15Ab^2x^4 + 5Ba^2x^2 + 10Aabx^2 + 3Aa^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/15*(30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*A*a*b*x^2 +
3*A*a^2)/x^5
```

3.20 $\int \frac{\left(a+bx^2\right)^2 \left(A+Bx^2\right)}{x^7} \, dx$

Optimal. Leaf size=51

$$-\frac{a^2 A}{6x^6} - \frac{a(aB + 2Ab)}{4x^4} - \frac{b(2aB + Ab)}{2x^2} + b^2 B \log(x)$$

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*Log[x]
```

Rubi [A] time = 0.0367519, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2 A}{6x^6} - \frac{a(aB + 2Ab)}{4x^4} - \frac{b(2aB + Ab)}{2x^2} + b^2 B \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + bx^2\right)^2 \left(A + Bx^2\right)}{x^7} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a + bx)^2 (A + Bx)}{x^4} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2 A}{x^4} + \frac{a(2Ab + aB)}{x^3} + \frac{b(Ab + 2aB)}{x^2} + \frac{b^2 B}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^2 A}{6x^6} - \frac{a(2Ab + aB)}{4x^4} - \frac{b(Ab + 2aB)}{2x^2} + b^2 B \log(x)
\end{aligned}$$

Mathematica [A] time = 0.0251778, size = 54, normalized size = 1.06

$$b^2 B \log(x) - \frac{a^2 \left(2A + 3Bx^2\right) + 6abx^2 \left(A + 2Bx^2\right) + 6Ab^2 x^4}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -(6*A*b^2*x^4 + 6*a*b*x^2*(A + 2*B*x^2) + a^2*(2*A + 3*B*x^2))/(12*x^6) + b
^2*B*Log[x]
```

Maple [A] time = 0.006, size = 52, normalized size = 1.

$$b^2B\ln(x) - \frac{abA}{2x^4} - \frac{a^2B}{4x^4} - \frac{b^2A}{2x^2} - \frac{abB}{x^2} - \frac{Aa^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^7,x)
```

```
[Out] b^2*B*ln(x)-1/2*a/x^4*A*b-1/4*a^2/x^4*B-1/2*b^2/x^2*A-b/x^2*B*a-1/6*a^2*A/x
^6
```

Maxima [A] time = 0.975843, size = 74, normalized size = 1.45

$$\frac{1}{2}Bb^2\log\left(x^2\right) - \frac{6\left(2Bab + Ab^2\right)x^4 + 2Aa^2 + 3\left(Ba^2 + 2Aab\right)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(6*(2*B*a*b + A*b^2)*x^4 + 2*A*a^2 + 3*(B*a^2 + 2
*A*a*b)*x^2)/x^6
```

Fricas [A] time = 1.44511, size = 127, normalized size = 2.49

$$\frac{12Bb^2x^6\log(x) - 6\left(2Bab + Ab^2\right)x^4 - 2Aa^2 - 3\left(Ba^2 + 2Aab\right)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*b^2*x^6*log(x) - 6*(2*B*a*b + A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 +
2*A*a*b)*x^2)/x^6
```

Sympy [A] time = 1.11371, size = 53, normalized size = 1.04

$$Bb^2\log(x) - \frac{2Aa^2 + x^4\left(6Ab^2 + 12Bab\right) + x^2\left(6Aab + 3Ba^2\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**2*log(x) - (2*A*a**2 + x**4*(6*A*b**2 + 12*B*a*b) + x**2*(6*A*a*b + 3*
B*a**2))/(12*x**6)
```

Giac [A] time = 1.17604, size = 89, normalized size = 1.75

$$\frac{1}{2}\,Bb^2\log\left(x^2\right)-\frac{11\,Bb^2x^6+12\,Babx^4+6\,Ab^2x^4+3\,Ba^2x^2+6\,Aabx^2+2\,Aa^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(11*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 3*B*
a^2*x^2 + 6*A*a*b*x^2 + 2*A*a^2)/x^6
```

3.21 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(3*x^3) -
(b^2*B)/x
```

Rubi [A] time = 0.0274398, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(3*x^3) -
(b^2*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^4}+\frac{b^2B}{x^2}\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{3x^3}-\frac{b^2B}{x}\end{aligned}$$

Mathematica [A] time = 0.0161532, size = 56, normalized size = 1.06

$$-\frac{3a^2\left(5A+7Bx^2\right)+14abx^2\left(3A+5Bx^2\right)+35b^2x^4\left(A+3Bx^2\right)}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -(35*b^2*x^4*(A + 3*B*x^2) + 14*a*b*x^2*(3*A + 5*B*x^2) + 3*a^2*(5*A + 7*B*
x^2))/(105*x^7)
```

Maple [A] time = 0.004, size = 48, normalized size = 0.9

$$-\frac{Aa^2}{7x^7}-\frac{a(2Ab+Ba)}{5x^5}-\frac{b(Ab+2Ba)}{3x^3}-\frac{Bb^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

Maxima [A] time = 1.00985, size = 72, normalized size = 1.36

$$-\frac{105\,Bb^2x^6+35\left(2\,Bab+Ab^2\right)x^4+15\,Aa^2+21\left(Ba^2+2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

Fricas [A] time = 1.38145, size = 126, normalized size = 2.38

$$-\frac{105\,Bb^2x^6+35\left(2\,Bab+Ab^2\right)x^4+15\,Aa^2+21\left(Ba^2+2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

Sympy [A] time = 1.13365, size = 56, normalized size = 1.06

$$-\frac{15Aa^2+105Bb^2x^6+x^4\left(35Ab^2+70Bab\right)+x^2\left(42Aab+21Ba^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

```
[Out] -(15*A*a**2 + 105*B*b**2*x**6 + x**4*(35*A*b**2 + 70*B*a*b) + x**2*(42*A*a*
b + 21*B*a**2))/(105*x**7)
```

Giac [A] time = 1.16392, size = 74, normalized size = 1.4

$$-\frac{105\,Bb^2x^6+70\,Babx^4+35\,Ab^2x^4+21\,Ba^2x^2+42\,Aabx^2+15\,Aa^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 70*B*a*b*x^4 + 35*A*b^2*x^4 + 21*B*a^2*x^2 + 42*A*a
*b*x^2 + 15*A*a^2)/x^7
```

3.22 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx^2)^3(Ab-4aB)}{24a^2x^6}-\frac{A(a+bx^2)^3}{8ax^8}$$

```
[Out] -(A*(a + b*x^2)^3)/(8*a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*a^2*x^6)
```

Rubi [A] time = 0.0311508, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 78, 37}

$$\frac{(a+bx^2)^3(Ab-4aB)}{24a^2x^6}-\frac{A(a+bx^2)^3}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -(A*(a + b*x^2)^3)/(8*a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*a^2*x^6)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^3}{8ax^8} + \frac{(-Ab+4aB)\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^4}\,dx, x, x^2\right)}{8a}$$
$$= -\frac{A\left(a+bx^2\right)^3}{8ax^8} + \frac{(Ab-4aB)\left(a+bx^2\right)^3}{24a^2x^6}$$

Mathematica [A] time = 0.0161696, size = 55, normalized size = 1.15

$$-\frac{a^2\left(3A+4Bx^2\right)+4abx^2\left(2A+3Bx^2\right)+6b^2x^4\left(A+2Bx^2\right)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -(6*b^2*x^4*(A + 2*B*x^2) + 4*a*b*x^2*(2*A + 3*B*x^2) + a^2*(3*A + 4*B*x^2)
)/(24*x^8)
```

Maple [A] time = 0.005, size = 48, normalized size = 1.

$$-\frac{b\,(Ab+2\,Ba)}{4\,x^4}-\frac{Aa^2}{8\,x^8}-\frac{Bb^2}{2\,x^2}-\frac{a\,(2\,Ab+Ba)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^9,x)
```

```
[Out] -1/4*b*(A*b+2*B*a)/x^4-1/8*A*a^2/x^8-1/2*B*b^2/x^2-1/6*a*(2*A*b+B*a)/x^6
```

Maxima [A] time = 0.99105, size = 72, normalized size = 1.5

$$-\frac{12\,Bb^2x^6+6\left(2\,Bab+Ab^2\right)x^4+3\,Aa^2+4\left(Ba^2+2\,Aab\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

Fricas [A] time = 1.40432, size = 119, normalized size = 2.48

$$-\frac{12\,Bb^2x^6+6\left(2\,Bab+Ab^2\right)x^4+3\,Aa^2+4\left(Ba^2+2\,Aab\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

Sympy [A] time = 1.67552, size = 56, normalized size = 1.17

$$-\frac{3Aa^2 + 12Bb^2x^6 + x^4\left(6Ab^2 + 12Bab\right) + x^2\left(8Aab + 4Ba^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

```
[Out] -(3*A*a**2 + 12*B*b**2*x**6 + x**4*(6*A*b**2 + 12*B*a*b) + x**2*(8*A*a*b +
4*B*a**2))/(24*x**8)
```

Giac [A] time = 1.26124, size = 74, normalized size = 1.54

$$-\frac{12\,Bb^2x^6 + 12\,Babx^4 + 6\,Ab^2x^4 + 4\,Ba^2x^2 + 8\,Aabx^2 + 3\,Aa^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 4*B*a^2*x^2 + 8*A*a*b*x^
2 + 3*A*a^2)/x^8
```

3.23 $\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB$$

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rubi [A] time = 0.152547, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^5(A+Bx)\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5Ax^4+a^4(5Ab+aB)x^5+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^7+\right.\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4(5Ab+aB)x^{12}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{18}\end{aligned}$$

Mathematica [A] time = 0.0171334, size = 117, normalized size = 1.

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{22}}{22}+\frac{\left(b^5A+5\,ab^4B\right)x^{20}}{20}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{18}}{18}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{16}}{16}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{14}}{14}+\frac{(5\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/20*(A*b^5+5*B*a*b^4)*x^20+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/12*(5*A*a^4*b+B*a^5)*x^12+1/10*a^5*A*x^10
```

Maxima [A] time = 1.07332, size = 161, normalized size = 1.38

$$\frac{1}{22}\,Bb^5x^{22}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{18}\left(2\,Ba^2b^3+Aab^4\right)x^{18}+\frac{5}{8}\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{5}{14}\left(Ba^4b+2\,Aa^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a^4
*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Fricas [A] time = 1.23201, size = 316, normalized size = 2.7

$$\frac{1}{22}x^{22}b^5B+\frac{1}{4}x^{20}b^4aB+\frac{1}{20}x^{20}b^5A+\frac{5}{9}x^{18}b^3a^2B+\frac{5}{18}x^{18}b^4aA+\frac{5}{8}x^{16}b^2a^3B+\frac{5}{8}x^{16}b^3a^2A+\frac{5}{14}x^{14}ba^4B+\frac{5}{7}x^{14}b^2a^3A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 5/9*x^18*b^3*a^2*B +
 5/18*x^18*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*a^2*A + 5/14*x^14*b*
a^4*B + 5/7*x^14*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/10*x^1
0*a^5*A
```

Sympy [A] time = 0.083946, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{22}}{22}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{18}\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{16}\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{14}\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**22/22 + x**20*(A*b**5/20 + B*a*b**4/4) + x**18*
(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b**2/
8) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

Giac [A] time = 1.33724, size = 169, normalized size = 1.44

$$\frac{1}{22}Bb^5x^{22} + \frac{1}{4}Bab^4x^{20} + \frac{1}{20}Ab^5x^{20} + \frac{5}{9}Ba^2b^3x^{18} + \frac{5}{18}Aab^4x^{18} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{14}Ba^4bx^{14} + \frac{5}{7}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 5/9*B*a^2*b^3*x^18 +
 5/18*A*a*b^4*x^18 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/14*B*a^4*b
*x^14 + 5/7*A*a^3*b^2*x^14 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/10*A*a
^5*x^10
```

3.24 $\int x^8 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{9}a^5Ax^9+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Al$$

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rubi [A] time = 0.092316, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{9}a^5Ax^9+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Al$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\int\left(a^5Ax^8+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{14}+5ab^3(Ab+\right.$$
$$=\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{2}{3}a^2b^2(Ab+aB)x^{15}+\frac{5}{17}ab^3(A$$

Mathematica [A] time = 0.0169401, size = 117, normalized size = 1.

$$\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{9}a^5Ax^9+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Al$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{21}}{21}+\frac{(b^5A+5\,ab^4B)\,x^{19}}{19}+\frac{(5\,ab^4A+10\,a^2b^3B)\,x^{17}}{17}+\frac{(10\,a^2b^3A+10\,a^3b^2B)\,x^{15}}{15}+\frac{(10\,a^3b^2A+5\,a^4bB)\,x^{13}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/19*(A*b^5+5*B*a*b^4)*x^19+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/11*(5*A*a^4*b+B*a^5)*x^11+1/9*a^5*A*x^9
```

Maxima [A] time = 0.977337, size = 161, normalized size = 1.38

$$\frac{1}{21}\,Bb^5x^{21}+\frac{1}{19}\,(5\,Bab^4+Ab^5)x^{19}+\frac{5}{17}\,(2\,Ba^2b^3+Aab^4)x^{17}+\frac{2}{3}\,(Ba^3b^2+Aa^2b^3)x^{15}+\frac{1}{9}\,Aa^5x^9+\frac{5}{13}\,(Ba^4b+2\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4*b
 + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Fricas [A] time = 1.36036, size = 320, normalized size = 2.74

$$\frac{1}{21}x^{21}b^5B+\frac{5}{19}x^{19}b^4aB+\frac{1}{19}x^{19}b^5A+\frac{10}{17}x^{17}b^3a^2B+\frac{5}{17}x^{17}b^4aA+\frac{2}{3}x^{15}b^2a^3B+\frac{2}{3}x^{15}b^3a^2A+\frac{5}{13}x^{13}ba^4B+\frac{10}{13}x^{13}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 10/17*x^17*b^3*a^2*
B + 5/17*x^17*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/13*x^13
*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/11*x^11*a^5*B + 5/11*x^11*b*a^4*A + 1/9
*x^9*a^5*A
```

Sympy [A] time = 0.082476, size = 138, normalized size = 1.18

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{21}}{21}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{17}\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{15}\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{13}\left(\frac{10Aa^3b^2}{13}+\frac{5Ba}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**21/21 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**17
*(5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b*
*2/3) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**11*(5*A*a**4*b/11 +
```

```
B*a**5/11)
```

Giac [A] time = 1.19136, size = 169, normalized size = 1.44

$$\frac{1}{21}Bb^5x^{21} + \frac{5}{19}Bab^4x^{19} + \frac{1}{19}Ab^5x^{19} + \frac{10}{17}Ba^2b^3x^{17} + \frac{5}{17}Aab^4x^{17} + \frac{2}{3}Ba^3b^2x^{15} + \frac{2}{3}Aa^2b^3x^{15} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 10/17*B*a^2*b^3*x^1
7 + 5/17*A*a*b^4*x^17 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/9
*A*a^5*x^9
```

3.25 $\int x^7 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=122

$$\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+b\right.}{20b}$$

```
[Out] -(a^3*(A*b - a*B)*(a + b*x^2)^6)/(12*b^5) + (a^2*(3*A*b - 4*a*B)*(a + b*x^2
)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b^5) + ((A*b - 4*a*B)
*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)/(20*b^5)
```

Rubi [A] time = 0.280026, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+b\right.}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a^3*(A*b - a*B)*(a + b*x^2)^6)/(12*b^5) + (a^2*(3*A*b - 4*a*B)*(a + b*x^2
)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b^5) + ((A*b - 4*a*B)
*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)/(20*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^3(-Ab+aB)(a+bx)^5}{b^4}-\frac{a^2(-3Ab+4aB)(a+bx)^6}{b^4}+\frac{3a(-Ab+2aB)}{b^4}\right.\right.\\&=-\frac{a^3(Ab-aB)\left(a+bx^2\right)^6}{12b^5}+\frac{a^2(3Ab-4aB)\left(a+bx^2\right)^7}{14b^5}-\frac{3a(Ab-2aB)\left(a+bx^2\right)^8}{16b^5}+\end{aligned}$$

Mathematica [A] time = 0.014543, size = 117, normalized size = 0.96

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{12}a^3bx^{12}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{18}b^4x^{18}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^12)/
12 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^
4*(A*b + 5*a*B)*x^18)/18 + (b^5*B*x^20)/20
```

Maple [A] time = 0.001, size = 124, normalized size = 1.

$$\frac{b^5Bx^{20}}{20}+\frac{\left(b^5A+5\,ab^4B\right)x^{18}}{18}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{16}}{16}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{14}}{14}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{12}}{12}+\frac{\left(5\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/18*(A*b^5+5*B*a*b^4)*x^18+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/10*(5*A*a^4*b+B*a^5)*x^10+1/8*a^5*A*x^8
```

Maxima [A] time = 0.993744, size = 161, normalized size = 1.32

$$\frac{1}{20}\,Bb^5x^{20}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{1}{8}\,Aa^5x^8+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4*b
 + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Fricas [A] time = 1.29078, size = 313, normalized size = 2.57

$$\frac{1}{20}x^{20}b^5B+\frac{5}{18}x^{18}b^4aB+\frac{1}{18}x^{18}b^5A+\frac{5}{8}x^{16}b^3a^2B+\frac{5}{16}x^{16}b^4aA+\frac{5}{7}x^{14}b^2a^3B+\frac{5}{7}x^{14}b^3a^2A+\frac{5}{12}x^{12}ba^4B+\frac{5}{6}x^{12}b^2a^3A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 5/8*x^16*b^3*a^2*B
+ 5/16*x^16*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3*a^2*A + 5/12*x^12*b
*a^4*B + 5/6*x^12*b^2*a^3*A + 1/10*x^10*a^5*B + 1/2*x^10*b*a^4*A + 1/8*x^8*
a^5*A
```

Sympy [A] time = 0.083165, size = 136, normalized size = 1.11

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{20}}{20}+x^{18}\left(\frac{Ab^5}{18}+\frac{5Bab^4}{18}\right)+x^{16}\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{14}\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{12}\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**20/20 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2
/7) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**10*(A*a**4*b/2 + B*a**5/
10)
```

Giac [A] time = 1.26542, size = 169, normalized size = 1.39

$$\frac{1}{20}Bb^5x^{20} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{5}{8}Ba^2b^3x^{16} + \frac{5}{16}Aab^4x^{16} + \frac{5}{7}Ba^3b^2x^{14} + \frac{5}{7}Aa^2b^3x^{14} + \frac{5}{12}Ba^4bx^{12} + \frac{5}{6}Aa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/8*A*a^
5*x^8
```

3.26 $\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)$$

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rubi [A] time = 0.0700851, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx = \int \left(a^5Ax^6+a^4(5Ab+aB)x^8+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{12}+5ab^3(Ab+\right.$$
$$=\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}ab^3(Ab$$

Mathematica [A] time = 0.0142737, size = 117, normalized size = 1.

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Maple [A] time = 0., size = 124, normalized size = 1.1

$$\frac{b^5Bx^{19}}{19}+\frac{\left(b^5A+5\,ab^4B\right)x^{17}}{17}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{15}}{15}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{13}}{13}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/17*(A*b^5+5*B*a*b^4)*x^17+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/9*(5*A*a^4*b+B*a^5)*x^9+1/7*a^5*A*x^7
```

Maxima [A] time = 0.967849, size = 161, normalized size = 1.38

$$\frac{1}{19}\,Bb^5x^{19}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{1}{3}\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{7}\,Aa^5x^7+\frac{5}{11}\left(Ba^4b+2\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^4*
b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Fricas [A] time = 1.2537, size = 316, normalized size = 2.7

$$\frac{1}{19}x^{19}b^5B+\frac{5}{17}x^{17}b^4aB+\frac{1}{17}x^{17}b^5A+\frac{2}{3}x^{15}b^3a^2B+\frac{1}{3}x^{15}b^4aA+\frac{10}{13}x^{13}b^2a^3B+\frac{10}{13}x^{13}b^3a^2A+\frac{5}{11}x^{11}ba^4B+\frac{10}{11}x^{11}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 2/3*x^15*b^3*a^2*B
+ 1/3*x^15*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*b^3*a^2*A + 5/11*x^1
1*b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/7*x^
7*a^5*A
```

Sympy [A] time = 0.083476, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7}+\frac{Bb^5x^{19}}{19}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{15}\left(\frac{Aab^4}{3}+\frac{2Ba^2b^3}{3}\right)+x^{13}\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{11}\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**19/19 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b**2
/13) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**9*(5*A*a**4*b/9 + B*a
```

```
**5/9)
```

Giac [A] time = 1.25413, size = 169, normalized size = 1.44

$$\frac{1}{19}Bb^5x^{19}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{2}{3}Ba^2b^3x^{15}+\frac{1}{3}Aab^4x^{15}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 5/11*B*a
^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/7*A*
a^5*x^7
```

3.27 $\int x^5 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=95

$$\frac{a^2\left(a+bx^2\right)^6(Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8(Ab-3aB)}{16b^4}-\frac{a\left(a+bx^2\right)^7(2Ab-3aB)}{14b^4}+\frac{B\left(a+bx^2\right)^9}{18b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2)^7
)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (B*(a + b*x^2)^9)/(18
*b^4)
```

Rubi [A] time = 0.213331, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{a^2\left(a+bx^2\right)^6(Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8(Ab-3aB)}{16b^4}-\frac{a\left(a+bx^2\right)^7(2Ab-3aB)}{14b^4}+\frac{B\left(a+bx^2\right)^9}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2)^7
)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (B*(a + b*x^2)^9)/(18
*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^5\left(A+Bx^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^6}{b^3}+\frac{(Ab-3aB)(a\right.\right.}{b^3}\\ &= \frac{a^2(Ab-aB)\left(a+bx^2\right)^6}{12b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^7}{14b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^8}{16b^4}+\frac{B\left(a\right.}{1}\end{aligned}$$

Mathematica [A] time = 0.0236781, size = 107, normalized size = 1.13

$$\frac{x^6\left(840a^2b^2x^6(aB+Ab)+504a^3bx^4(aB+2Ab)+126a^4x^2(aB+5Ab)+168a^5A+63b^4x^{10}(5aB+Ab)+360ab^3x^8(2a\right.}{1008}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^6*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^2 + 504*a^3*b*(2*A*b + a*B)*x^4 +
 840*a^2*b^2*(A*b + a*B)*x^6 + 360*a*b^3*(A*b + 2*a*B)*x^8 + 63*b^4*(A*b +
5*a*B)*x^10 + 56*b^5*B*x^12))/1008
```

Maple [A] time = 0., size = 124, normalized size = 1.3

$$\frac{b^5Bx^{18}}{18}+\frac{\left(b^5A+5\,ab^4B\right)x^{16}}{16}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{14}}{14}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{12}}{12}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{10}}{10}+\frac{(5\,a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/18*b^5*B*x^18+1/16*(A*b^5+5*B*a*b^4)*x^16+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/8*(5*A*a^4*b+B*a^5)*x^8+1/6*a^5*A*x^6
```

Maxima [A] time = 0.99256, size = 161, normalized size = 1.69

$$\frac{1}{18}\,Bb^5x^{18}+\frac{1}{16}\left(5\,Bab^4+Ab^5\right)x^{16}+\frac{5}{14}\left(2\,Ba^2b^3+Aab^4\right)x^{14}+\frac{5}{6}\left(Ba^3b^2+Aa^2b^3\right)x^{12}+\frac{1}{6}\,Aa^5x^6+\frac{1}{2}\left(Ba^4b+2\,Aa^3b^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*b
+ 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Fricas [A] time = 1.27962, size = 302, normalized size = 3.18

$$\frac{1}{18}x^{18}b^5B+\frac{5}{16}x^{16}b^4aB+\frac{1}{16}x^{16}b^5A+\frac{5}{7}x^{14}b^3a^2B+\frac{5}{14}x^{14}b^4aA+\frac{5}{6}x^{12}b^2a^3B+\frac{5}{6}x^{12}b^3a^2A+\frac{1}{2}x^{10}ba^4B+x^{10}b^2a^3A+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 5/7*x^14*b^3*a^2*B
+ 5/14*x^14*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 1/2*x^10*b*
a^4*B + x^10*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*x^8*b*a^4*A + 1/6*x^6*a^5*A
```

Sympy [A] time = 0.083754, size = 133, normalized size = 1.4

$$\frac{Aa^5x^6}{6}+\frac{Bb^5x^{18}}{18}+x^{16}\left(\frac{Ab^5}{16}+\frac{5Bab^4}{16}\right)+x^{14}\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{12}\left(\frac{5Aa^2b^3}{6}+\frac{5Ba^3b^2}{6}\right)+x^{10}\left(Aa^3b^2+\frac{Ba^4b}{2}\right)+x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**18/18 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2
/6) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**8*(5*A*a**4*b/8 + B*a**5/8)
```

Giac [A] time = 1.12368, size = 167, normalized size = 1.76

$$\frac{1}{18}Bb^5x^{18} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 1/2*B*a^4*b
*x^10 + A*a^3*b^2*x^10 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/6*A*a^5*x^6
```

3.28 $\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)$$

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Rubi [A] time = 0.0701615, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```


```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```


Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^5\left(A+Bx^2\right)dx=\int\left(a^5Ax^4+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^8+10a^2b^2(Ab+aB)x^{10}+5ab^3(Ab+2$$
$$=\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{13}ab^3(Ab$$

Mathematica [A] time = 0.0152669, size = 117, normalized size = 1.

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```


```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{17}}{17}+\frac{\left(b^5A+5\,ab^4B\right)x^{15}}{15}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{13}}{13}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{11}}{11}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^9}{9}+\frac{(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/17*b^5*B*x^17+1/15*(A*b^5+5*B*a*b^4)*x^15+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/7*(5*A*a^4*b+B*a^5)*x^7+1/5*a^5*A*x^5
```

Maxima [A] time = 1.01344, size = 161, normalized size = 1.38

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{1}{5}Aa^5x^5+\frac{5}{9}\left(Ba^4b+2\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^4*
b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Fricas [A] time = 1.28002, size = 313, normalized size = 2.68

$$\frac{1}{17}x^{17}b^5B+\frac{1}{3}x^{15}b^4aB+\frac{1}{15}x^{15}b^5A+\frac{10}{13}x^{13}b^3a^2B+\frac{5}{13}x^{13}b^4aA+\frac{10}{11}x^{11}b^2a^3B+\frac{10}{11}x^{11}b^3a^2A+\frac{5}{9}x^9ba^4B+\frac{10}{9}x^9b^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5*B + 1/3*x^15*b^4*a*B + 1/15*x^15*b^5*A + 10/13*x^13*b^3*a^2*B
 + 5/13*x^13*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^11*b^3*a^2*A + 5/9*x^
9*b*a^4*B + 10/9*x^9*b^2*a^3*A + 1/7*x^7*a^5*B + 5/7*x^7*b*a^4*A + 1/5*x^5*
a^5*A
```

Sympy [A] time = 0.083354, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{17}}{17}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{13}\left(\frac{5Aab^4}{13}+\frac{10Ba^2b^3}{13}\right)+x^{11}\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^9\left(\frac{10Aa^3b^2}{9}+\frac{5Ba}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**17/17 + x**15*(A*b**5/15 + B*a*b**4/3) + x**13*(5
*A*a*b**4/13 + 10*B*a**2*b**3/13) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b*
*2/11) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**7*(5*A*a**4*b/7 + B*a*
```

```
*5/7)
```

Giac [A] time = 1.13461, size = 169, normalized size = 1.44

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{3}Bab^4x^{15}+\frac{1}{15}Ab^5x^{15}+\frac{10}{13}Ba^2b^3x^{13}+\frac{5}{13}Aab^4x^{13}+\frac{10}{11}Ba^3b^2x^{11}+\frac{10}{11}Aa^2b^3x^{11}+\frac{5}{9}Ba^4bx^9+\frac{10}{9}Aa^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 10/13*B*a^2*b^3*x^13
 + 5/13*A*a*b^4*x^13 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/9*B*
a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/5*A*a^
5*x^5
```

3.29 $\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=67

$$\frac{\left(a+bx^2\right)^7 (Ab-2aB)}{14b^3}-\frac{a\left(a+bx^2\right)^6 (Ab-aB)}{12b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^6)/(12*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^7)/(14
*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rubi [A] time = 0.143136, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{\left(a+bx^2\right)^7 (Ab-2aB)}{14b^3}-\frac{a\left(a+bx^2\right)^6 (Ab-aB)}{12b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^6)/(12*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^7)/(14
*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx)^7}{b^2}\right) dx,x,x^2\right)\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^6}{12b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^7}{14b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}\end{aligned}$$

Mathematica [A] time = 0.015552, size = 114, normalized size = 1.7

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{6}a^4x^6(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{12}ab^3x^{12}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^16)/16
```

Maple [B] time = 0.001, size = 124, normalized size = 1.9

$$\frac{b^5Bx^{16}}{16}+\frac{\left(b^5A+5\,ab^4B\right)x^{14}}{14}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{12}}{12}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{10}}{10}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^8}{8}+\frac{(5\,a^4b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/16*b^5*B*x^16+1/14*(A*b^5+5*B*a*b^4)*x^14+1/12*(5*A*a*b^4+10*B*a^2*b^3)*x
^12+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/4*a^5*A*x^4
```

Maxima [A] time = 0.981593, size = 159, normalized size = 2.37

$$\frac{1}{16}\,Bb^5x^{16}+\frac{1}{14}\left(5\,Bab^4+Ab^5\right)x^{14}+\frac{5}{12}\left(2\,Ba^2b^3+Aab^4\right)x^{12}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{1}{4}\,Aa^5x^4+\frac{5}{8}\left(Ba^4b+2\,Aa^3b^2\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b + 2*
A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Fricas [A] time = 1.30656, size = 294, normalized size = 4.39

$$\frac{1}{16}x^{16}b^5B+\frac{5}{14}x^{14}b^4aB+\frac{1}{14}x^{14}b^5A+\frac{5}{6}x^{12}b^3a^2B+\frac{5}{12}x^{12}b^4aA+x^{10}b^2a^3B+x^{10}b^3a^2A+\frac{5}{8}x^8ba^4B+\frac{5}{4}x^8b^2a^3A+\frac{1}{6}x^6a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5*B + 5/14*x^14*b^4*a*B + 1/14*x^14*b^5*A + 5/6*x^12*b^3*a^2*B
+ 5/12*x^12*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A + 5/8*x^8*b*a^4*B + 5
/4*x^8*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/4*x^4*a^5*A
```

Sympy [B] time = 0.082481, size = 131, normalized size = 1.96

$$\frac{Aa^5x^4}{4}+\frac{Bb^5x^{16}}{16}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^{10}\left(Aa^2b^3+Ba^3b^2\right)+x^8\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^6\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**16/16 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**12
*(5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x*
*8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Giac [A] time = 1.15456, size = 166, normalized size = 2.48

$$\frac{1}{16}Bb^5x^{16} + \frac{5}{14}Bab^4x^{14} + \frac{1}{14}Ab^5x^{14} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 5/6*B*a^2*b^3*x^12
+ 5/12*A*a*b^4*x^12 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/8*B*a^4*b*x^8 + 5
/4*A*a^3*b^2*x^8 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/4*A*a^5*x^4
```

3.30 $\int x^2 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{3}a^5Ax^3+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+$$

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rubi [A] time = 0.0664618, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{3}a^5Ax^3+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx = \int \left(a^5Ax^2+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^8+5ab^3(Ab+2a\right.$$
$$=\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}ab^3(Ab+$$

Mathematica [A] time = 0.0151001, size = 117, normalized size = 1.

$$\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{3}a^5Ax^3+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{15}}{15}+\frac{\left(b^5A+5\,ab^4B\right)x^{13}}{13}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{11}}{11}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^9}{9}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^7}{7}+\frac{(5\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/15*b^5*B*x^15+1/13*(A*b^5+5*B*a*b^4)*x^13+1/11*(5*A*a*b^4+10*B*a^2*b^3)*x
^11+1/9*(10*A*a^2*b^3+10*B*a^3*b^2)*x^9+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+1/
5*(5*A*a^4*b+B*a^5)*x^5+1/3*a^5*A*x^3
```

Maxima [A] time = 0.985409, size = 161, normalized size = 1.38

$$\frac{1}{15}\,Bb^5x^{15}+\frac{1}{13}\left(5\,Bab^4+Ab^5\right)x^{13}+\frac{5}{11}\left(2\,Ba^2b^3+Aab^4\right)x^{11}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{1}{3}\,Aa^5x^3+\frac{5}{7}\left(Ba^4b+2\,Aa^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*b
+ 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Fricas [A] time = 1.31291, size = 304, normalized size = 2.6

$$\frac{1}{15}x^{15}b^5B+\frac{5}{13}x^{13}b^4aB+\frac{1}{13}x^{13}b^5A+\frac{10}{11}x^{11}b^3a^2B+\frac{5}{11}x^{11}b^4aA+\frac{10}{9}x^9b^2a^3B+\frac{10}{9}x^9b^3a^2A+\frac{5}{7}x^7ba^4B+\frac{10}{7}x^7b^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5*B + 5/13*x^13*b^4*a*B + 1/13*x^13*b^5*A + 10/11*x^11*b^3*a^2*
B + 5/11*x^11*b^4*a*A + 10/9*x^9*b^2*a^3*B + 10/9*x^9*b^3*a^2*A + 5/7*x^7*b
*a^4*B + 10/7*x^7*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/3*x^3*a^5*A
```

Sympy [A] time = 0.082216, size = 134, normalized size = 1.15

$$\frac{Aa^5x^3}{3}+\frac{Bb^5x^{15}}{15}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{11}\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^9\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^7\left(\frac{10Aa^3b^2}{7}+\frac{5Ba}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**15/15 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x**11
*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b
**2/9) + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**5*(A*a**4*b + B*a**5/5
```

)

Giac [A] time = 1.15428, size = 167, normalized size = 1.43

$$\frac{1}{15}Bb^5x^{15} + \frac{5}{13}Bab^4x^{13} + \frac{1}{13}Ab^5x^{13} + \frac{10}{11}Ba^2b^3x^{11} + \frac{5}{11}Aab^4x^{11} + \frac{10}{9}Ba^3b^2x^9 + \frac{10}{9}Aa^2b^3x^9 + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/7*B*a^4
*b*x^7 + 10/7*A*a^3*b^2*x^7 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/3*A*a^5*x^3
```

3.31 $\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rubi [A] time = 0.0661224, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^5(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^6}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}\end{aligned}$$

Mathematica [B] time = 0.0229003, size = 107, normalized size = 2.55

$$\frac{1}{84}x^2\left(105a^2b^2x^6(aB+Ab)+70a^3bx^4(aB+2Ab)+21a^4x^2(aB+5Ab)+42a^5A+7b^4x^{10}(5aB+Ab)+42ab^3x^8(2aB+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^2*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^2 + 70*a^3*b*(2*A*b + a*B)*x^4 + 10
5*a^2*b^2*(A*b + a*B)*x^6 + 42*a*b^3*(A*b + 2*a*B)*x^8 + 7*b^4*(A*b + 5*a*B
)*x^10 + 6*b^5*B*x^12))/84
```

Maple [B] time = 0.002, size = 124, normalized size = 3.

$$\frac{b^5Bx^{14}}{14}+\frac{\left(b^5A+5\,ab^4B\right)x^{12}}{12}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{10}}{10}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^8}{8}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^6}{6}+\frac{\left(5\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/14*b^5*B*x^14+1/12*(A*b^5+5*B*a*b^4)*x^12+1/10*(5*A*a*b^4+10*B*a^2*b^3)*x
^10+1/8*(10*A*a^2*b^3+10*B*a^3*b^2)*x^8+1/6*(10*A*a^3*b^2+5*B*a^4*b)*x^6+1/
4*(5*A*a^4*b+B*a^5)*x^4+1/2*a^5*A*x^2
```

Maxima [B] time = 0.980255, size = 161, normalized size = 3.83

$$\frac{1}{14}\,Bb^5x^{14}+\frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12}+\frac{1}{2}\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\frac{1}{2}\,Aa^5x^2+\frac{5}{6}\left(Ba^4b+2\,Aa^3b^2\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Fricas [B] time = 1.26913, size = 296, normalized size = 7.05

$$\frac{1}{14}x^{14}b^5B+\frac{5}{12}x^{12}b^4aB+\frac{1}{12}x^{12}b^5A+x^{10}b^3a^2B+\frac{1}{2}x^{10}b^4aA+\frac{5}{4}x^8b^2a^3B+\frac{5}{4}x^8b^3a^2A+\frac{5}{6}x^6ba^4B+\frac{5}{3}x^6b^2a^3A+\frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5*B + 5/12*x^12*b^4*a*B + 1/12*x^12*b^5*A + x^10*b^3*a^2*B + 1/
2*x^10*b^4*a*A + 5/4*x^8*b^2*a^3*B + 5/4*x^8*b^3*a^2*A + 5/6*x^6*b*a^4*B +
5/3*x^6*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b*a^4*A + 1/2*x^2*a^5*A
```

Sympy [B] time = 0.081339, size = 133, normalized size = 3.17

$$\frac{Aa^5x^2}{2}+\frac{Bb^5x^{14}}{14}+x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right)+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^8\left(\frac{5Aa^2b^3}{4}+\frac{5Ba^3b^2}{4}\right)+x^6\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4b}{6}\right)+x^4\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**14/14 + x**12*(A*b**5/12 + 5*B*a*b**4/12) + x**10
*(A*a*b**4/2 + B*a**2*b**3) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4) + x*
*6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Giac [B] time = 1.19393, size = 167, normalized size = 3.98

$$\frac{1}{14}Bb^5x^{14} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + Ba^2b^3x^{10} + \frac{1}{2}Aab^4x^{10} + \frac{5}{4}Ba^3b^2x^8 + \frac{5}{4}Aa^2b^3x^8 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + 5/6*B*a^4*b*x^6 +
5/3*A*a^3*b^2*x^6 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + 1/2*A*a^5*x^2
```

3.32 $\int (a+bx^2)^5 (A+Bx^2)\,dx$

Optimal. Leaf size=109

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{3}a^4x^3(aB+5Ab)+a^5Ax+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5B$$

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Rubi [A] time = 0.0600984, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{3}a^4x^3(aB+5Ab)+a^5Ax+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5B$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^5 (A+Bx^2)\,dx &= \int \left(a^5A+a^4(5Ab+aB)x^2+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^6+5ab^3(Ab+2aB)x^8\right.\\ &= a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+2aB)x^9\end{aligned}$$

Mathematica [A] time = 0.0167549, size = 109, normalized size = 1.

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{3}a^4x^3(aB+5Ab)+a^5Ax+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5B$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Maple [A] time = 0.001, size = 121, normalized size = 1.1

$$\frac{b^5Bx^{13}}{13}+\frac{\left(b^5A+5\,ab^4B\right)x^{11}}{11}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^9}{9}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^7}{7}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^5}{5}+\frac{\left(5\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/13*b^5*B*x^13+1/11*(A*b^5+5*B*a*b^4)*x^11+1/9*(5*A*a*b^4+10*B*a^2*b^3)*x^
9+1/7*(10*A*a^2*b^3+10*B*a^3*b^2)*x^7+1/5*(10*A*a^3*b^2+5*B*a^4*b)*x^5+1/3*
(5*A*a^4*b+B*a^5)*x^3+a^5*A*x
```

Maxima [A] time = 0.975597, size = 155, normalized size = 1.42

$$\frac{1}{13}\,Bb^5x^{13}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+Aa^5x+\left(Ba^4b+2\,Aa^3b^2\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*b^
2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Fricas [A] time = 1.28871, size = 286, normalized size = 2.62

$$\frac{1}{13}x^{13}b^5B+\frac{5}{11}x^{11}b^4aB+\frac{1}{11}x^{11}b^5A+\frac{10}{9}x^9b^3a^2B+\frac{5}{9}x^9b^4aA+\frac{10}{7}x^7b^2a^3B+\frac{10}{7}x^7b^3a^2A+x^5ba^4B+2x^5b^2a^3A+\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5*B + 5/11*x^11*b^4*a*B + 1/11*x^11*b^5*A + 10/9*x^9*b^3*a^2*B
+ 5/9*x^9*b^4*a*A + 10/7*x^7*b^2*a^3*B + 10/7*x^7*b^3*a^2*A + x^5*b*a^4*B +
 2*x^5*b^2*a^3*A + 1/3*x^3*a^5*B + 5/3*x^3*b*a^4*A + x*a^5*A
```

Sympy [A] time = 0.079743, size = 129, normalized size = 1.18

$$Aa^5x+\frac{Bb^5x^{13}}{13}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^9\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right)+x^7\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)+x^5\left(2Aa^3b^2+Ba^4b\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**13/13 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**9*(5*A*
a*b**4/9 + 10*B*a**2*b**3/9) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7) +
 x**5*(2*A*a**3*b**2 + B*a**4*b) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Giac [A] time = 1.15942, size = 163, normalized size = 1.5

$$\frac{1}{13} Bb^5x^{13} + \frac{5}{11} Bab^4x^{11} + \frac{1}{11} Ab^5x^{11} + \frac{10}{9} Ba^2b^3x^9 + \frac{5}{9} Aab^4x^9 + \frac{10}{7} Ba^3b^2x^7 + \frac{10}{7} Aa^2b^3x^7 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + B*a^4*b*x^5 +
 2*A*a^3*b^2*x^5 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*x
```

3.33 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx$

Optimal. Leaf size=88

$$\frac{5}{3}a^2Ab^3x^6+\frac{5}{2}a^3Ab^2x^4+\frac{5}{2}a^4Abx^2+a^5A\log(x)+\frac{5}{8}aAb^4x^8+\frac{B(a+bx^2)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (5*a*A*b^4*
x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a^5*A*Log[x]
```

Rubi [A] time = 0.0645913, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 80, 43}

$$\frac{5}{3}a^2Ab^3x^6+\frac{5}{2}a^3Ab^2x^4+\frac{5}{2}a^4Abx^2+a^5A\log(x)+\frac{5}{8}aAb^4x^8+\frac{B(a+bx^2)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (5*a*A*b^4*
x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a^5*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x} \, dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^6}{12b} + \frac{1}{2} A \, \text{Subst}\left(\int \frac{(a+bx)^5}{x} \, dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^6}{12b} + \frac{1}{2} A \, \text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4\right) dx, x, x^2\right)$$
$$= \frac{5}{2}a^4Abx^2 + \frac{5}{2}a^3Ab^2x^4 + \frac{5}{3}a^2Ab^3x^6 + \frac{5}{8}aAb^4x^8 + \frac{1}{10}Ab^5x^{10} + \frac{B(a+bx^2)^6}{12b} + a^5 A \log(x)$$

Mathematica [A] time = 0.028904, size = 113, normalized size = 1.28

$$\frac{5}{3}a^2b^2x^6(aB+Ab) + \frac{5}{4}a^3bx^4(aB+2Ab) + \frac{1}{2}a^4x^2(aB+5Ab) + a^5A\log(x) + \frac{1}{10}b^4x^{10}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) +$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (5*a^2*b^2*(A*b
 + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^10)/1
0 + (b^5*B*x^12)/12 + a^5*A*Log[x]
```

Maple [A] time = 0.003, size = 124, normalized size = 1.4

$$\frac{Bb^5x^{12}}{12} + \frac{Ab^5x^{10}}{10} + \frac{Bx^{10}ab^4}{2} + \frac{5\,aAb^4x^8}{8} + \frac{5\,Bx^8a^2b^3}{4} + \frac{5\,a^2Ab^3x^6}{3} + \frac{5\,Bx^6a^3b^2}{3} + \frac{5\,a^3Ab^2x^4}{2} + \frac{5\,Bx^4a^4b}{4} + \frac{5\,a^4Abx^2}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x,x)
```

```
[Out] 1/12*B*b^5*x^12+1/10*A*b^5*x^10+1/2*B*x^10*a*b^4+5/8*a*A*b^4*x^8+5/4*B*x^8*
a^2*b^3+5/3*a^2*A*b^3*x^6+5/3*B*x^6*a^3*b^2+5/2*a^3*A*b^2*x^4+5/4*B*x^4*a^4
*b+5/2*a^4*A*b*x^2+1/2*B*x^2*a^5+a^5*A*ln(x)
```

Maxima [A] time = 1.02679, size = 162, normalized size = 1.84

$$\frac{1}{12}\,Bb^5x^{12} + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{8}\left(2\,Ba^2b^3 + Aab^4\right)x^8 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \frac{1}{2}\,Aa^5\log\left(x^2\right) + \frac{5}{4}\left(Ba^4b + 2\,Aa^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1/2*A*a^5*log(x^2) + 5/4*(B*a^4*
b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Fricas [A] time = 1.45067, size = 265, normalized size = 3.01

$$\frac{1}{12}Bb^5x^{12} + \frac{1}{10}\left(5Bab^4 + Ab^5\right)x^{10} + \frac{5}{8}\left(2Ba^2b^3 + Aab^4\right)x^8 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + Aa^5\log(x) + \frac{5}{4}\left(Ba^4b + 2Aa^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + A*a^5*log(x) + 5/4*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Sympy [A] time = 0.368107, size = 134, normalized size = 1.52

$$Aa^5\log(x) + \frac{Bb^5x^{12}}{12} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^8\left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^6\left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^4\left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**12/12 + x**10*(A*b**5/10 + B*a*b**4/2) + x**8*(5*
A*a*b**4/8 + 5*B*a**2*b**3/4) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) +
x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Giac [A] time = 1.33182, size = 170, normalized size = 1.93

$$\frac{1}{12}Bb^5x^{12} + \frac{1}{2}Bab^4x^{10} + \frac{1}{10}Ab^5x^{10} + \frac{5}{4}Ba^2b^3x^8 + \frac{5}{8}Aab^4x^8 + \frac{5}{3}Ba^3b^2x^6 + \frac{5}{3}Aa^2b^3x^6 + \frac{5}{4}Ba^4bx^4 + \frac{5}{2}Aa^3b^2x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 5/4*B*a^2*b^3*x^8 +
5/8*A*a*b^4*x^8 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/4*B*a^4*b*x^4 +
 5/2*A*a^3*b^2*x^4 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 + 1/2*A*a^5*log(x^2)
```

3.34 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=108

$$2a^2b^2x^5(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

Rubi [A] time = 0.0619118, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$2a^2b^2x^5(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2}\,dx=\int \left(a^4(5Ab+aB)+\frac{a^5A}{x^2}+5a^3b(2Ab+aB)x^2+10a^2b^2(Ab+aB)x^4+5ab^3(Ab+2aB)x^6\right.$$
$$=-\frac{a^5A}{x}+a^4(5Ab+aB)x+\frac{5}{3}a^3b(2Ab+aB)x^3+2a^2b^2(Ab+aB)x^5+\frac{5}{7}ab^3(Ab+2aB)x^7+$$

Mathematica [A] time = 0.0294418, size = 108, normalized size = 1.

$$2a^2b^2x^5(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
```

```
9)/9 + (b^5*B*x^11)/11
```

Maple [A] time = 0.003, size = 121, normalized size = 1.1

$$\frac{b^5Bx^{11}}{11}+\frac{Ax^9b^5}{9}+\frac{5\,Bx^9ab^4}{9}+\frac{5\,Ax^7ab^4}{7}+\frac{10\,Bx^7a^2b^3}{7}+2\,Ax^5a^2b^3+2\,Bx^5a^3b^2+\frac{10\,Ax^3a^3b^2}{3}+\frac{5\,Bx^3a^4b}{3}+5\,a^4bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^2,x)
```

```
[Out] 1/11*b^5*B*x^11+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7*a^
2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+10/3*A*x^3*a^3*b^2+5/3*B*x^3*a^4*b+5*
a^4*b*A*x+a^5*B*x-a^5*A/x
```

Maxima [A] time = 0.980533, size = 157, normalized size = 1.45

$$\frac{1}{11}\,Bb^5x^{11}+\frac{1}{9}\left(5\,Bab^4+Ab^5\right)x^9+\frac{5}{7}\left(2\,Ba^2b^3+Aab^4\right)x^7+2\left(Ba^3b^2+Aa^2b^3\right)x^5-\frac{Aa^5}{x}+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3+\left(B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/7*(2*B*a^2*b^3 + A*a*b^4)
*x^7 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 - A*a^5/x + 5/3*(B*a^4*b + 2*A*a^3*b^2
)*x^3 + (B*a^5 + 5*A*a^4*b)*x
```

Fricas [A] time = 1.39418, size = 271, normalized size = 2.51

$$\frac{63\,Bb^5x^{12}+77\left(5\,Bab^4+Ab^5\right)x^{10}+495\left(2\,Ba^2b^3+Aab^4\right)x^8+1386\left(Ba^3b^2+Aa^2b^3\right)x^6-693\,Aa^5+1155\left(Ba^4b+2\right.}{693\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^12 + 77*(5*B*a*b^4 + A*b^5)*x^10 + 495*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 693*A*a^5 + 1155*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 693*(B*a^5 + 5*A*a^4*b)*x^2)/x
```

Sympy [A] time = 0.368079, size = 126, normalized size = 1.17

$$-\frac{Aa^5}{x}+\frac{Bb^5x^{11}}{11}+x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right)+x^7\left(\frac{5Aab^4}{7}+\frac{10Ba^2b^3}{7}\right)+x^5\left(2Aa^2b^3+2Ba^3b^2\right)+x^3\left(\frac{10Aa^3b^2}{3}+\frac{5Ba^4b}{3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**11/11 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x**7*(5*A*a*
b**4/7 + 10*B*a**2*b**3/7) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**3*(1
0*A*a**3*b**2/3 + 5*B*a**4*b/3) + x*(5*A*a**4*b + B*a**5)
```

Giac [A] time = 1.15517, size = 162, normalized size = 1.5

$$\frac{1}{11}Bb^5x^{11} + \frac{5}{9}Bab^4x^9 + \frac{1}{9}Ab^5x^9 + \frac{10}{7}Ba^2b^3x^7 + \frac{5}{7}Aab^4x^7 + 2\,Ba^3b^2x^5 + 2\,Aa^2b^3x^5 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + Ba^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 10/7*B*a^2*b^3*x^7 + 5/
7*A*a*b^4*x^7 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/3*B*a^4*b*x^3 + 10/3*
A*a^3*b^2*x^3 + B*a^5*x + 5*A*a^4*b*x - A*a^5/x
```

3.35 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}$$

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.112783, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x+5ab^3(Ab\right.\right.$$
$$=-\frac{a^5A}{2x^2}+\frac{5}{2}a^3b(2Ab+aB)x^2+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{8}b^4(Ab+5a$$

Mathematica [A] time = 0.0418384, size = 115, normalized size = 1.02

$$\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+\log(x)\left(5a^4Ab+a^5B\right)-\frac{a^5A}{2x^2}+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A] time = 0.005, size = 123, normalized size = 1.1

$$\frac{b^5Bx^{10}}{10}+\frac{Ax^8b^5}{8}+\frac{5\,Bx^8ab^4}{8}+\frac{5\,Ax^6ab^4}{6}+\frac{5\,Bx^6a^2b^3}{3}+\frac{5\,Ax^4a^2b^3}{2}+\frac{5\,Bx^4a^3b^2}{2}+5\,Ax^2a^3b^2+\frac{5\,Bx^2a^4b}{2}+5\,A\ln(x)\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^3,x)
```

```
[Out] 1/10*b^5*B*x^10+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+5/6*A*x^6*a*b^4+5/3*B*x^6*a^2
*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b+5*
A*ln(x)*a^4*b+B*ln(x)*a^5-1/2*a^5*A/x^2
```

Maxima [A] time = 1.00614, size = 162, normalized size = 1.43

$$\frac{1}{10}\,Bb^5x^{10}+\frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8+\frac{5}{6}\left(2\,Ba^2b^3+Aab^4\right)x^6+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4-\frac{Aa^5}{2\,x^2}+\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^2+\frac{1}{2}\,(B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 - 1/2*A*a^5/x^2 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2)
```

Fricas [A] time = 1.42959, size = 279, normalized size = 2.47

$$\frac{12\,Bb^5x^{12}+15\left(5\,Bab^4+Ab^5\right)x^{10}+100\left(2\,Ba^2b^3+Aab^4\right)x^8+300\left(Ba^3b^2+Aa^2b^3\right)x^6-60\,Aa^5+300\left(Ba^4b+2\,Aa^3b^2\right)}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(12*B*b^5*x^12 + 15*(5*B*a*b^4 + A*b^5)*x^10 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 60*A*a^5 + 300*(B*a^4*b + 2*A
*a^3*b^2)*x^4 + 120*(B*a^5 + 5*A*a^4*b)*x^2*log(x))/x^2
```

Sympy [A] time = 0.463312, size = 131, normalized size = 1.16

$$-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{10}}{10}+a^4\,(5Ab+Ba)\log(x)+x^8\left(\frac{Ab^5}{8}+\frac{5Bab^4}{8}\right)+x^6\left(\frac{5Aab^4}{6}+\frac{5Ba^2b^3}{3}\right)+x^4\left(\frac{5Aa^2b^3}{2}+\frac{5Ba^3b^2}{2}\right)+x^2\left(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x) + x**8*(A*b*
*5/8 + 5*B*a*b**4/8) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**4*(5*A*a*
*2*b**3/2 + 5*B*a**3*b**2/2) + x**2*(5*A*a**3*b**2 + 5*B*a**4*b/2)
```

Giac [A] time = 1.62579, size = 196, normalized size = 1.73

$$\frac{1}{10}Bb^5x^{10} + \frac{5}{8}Bab^4x^8 + \frac{1}{8}Ab^5x^8 + \frac{5}{3}Ba^2b^3x^6 + \frac{5}{6}Aab^4x^6 + \frac{5}{2}Ba^3b^2x^4 + \frac{5}{2}Aa^2b^3x^4 + \frac{5}{2}Ba^4bx^2 + 5\,Aa^3b^2x^2 + \frac{1}{2}\big(B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2) - 1/2*(B*a^5*x^2 + 5*A*a^4
*b*x^2 + A*a^5)/x^2
```

3.36 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} \, dx$

Optimal. Leaf size=108

$$\frac{10}{3}a^2b^2x^3(aB+Ab)+5a^3bx(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{3x^3}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

```
[Out] -(a^5*A)/(3*x^3) - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a^
2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^7
)/7 + (b^5*B*x^9)/9
```

Rubi [A] time = 0.0610051, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{3}a^2b^2x^3(aB+Ab)+5a^3bx(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{3x^3}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a^
2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^7
)/7 + (b^5*B*x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} \, dx = \int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x^2+5ab^3(Ab+2aB)x^4+\right.$$
$$= -\frac{a^5A}{3x^3}-\frac{a^4(5Ab+aB)}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB)x^5+\frac{1}{7}$$

Mathematica [A] time = 0.0338894, size = 110, normalized size = 1.02

$$\frac{10}{3}a^2b^2x^3(aB+Ab)+5a^3bx(aB+2Ab)+\frac{a^5(-B)-5a^4Ab}{x}-\frac{a^5A}{3x^3}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (-5*a^4*A*b - a^5*B)/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a
^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^
```

```
7)/7 + (b^5*B*x^9)/9
```

Maple [A] time = 0.007, size = 118, normalized size = 1.1

$$\frac{b^5Bx^9}{9}+\frac{Ax^7b^5}{7}+\frac{5\,Bx^7ab^4}{7}+Ax^5ab^4+2\,Bx^5a^2b^3+\frac{10\,Ax^3a^2b^3}{3}+\frac{10\,Bx^3a^3b^2}{3}+10\,a^3b^2Ax+5\,a^4bBx-\frac{Aa^5}{3\,x^3}-\frac{a^4\,(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^4,x)
```

```
[Out] 1/9*b^5*B*x^9+1/7*A*x^7*b^5+5/7*B*x^7*a*b^4+A*x^5*a*b^4+2*B*x^5*a^2*b^3+10/
3*A*x^3*a^2*b^3+10/3*B*x^3*a^3*b^2+10*a^3*b^2*A*x+5*a^4*b*B*x-1/3*a^5*A/x^3
-a^4*(5*A*b+B*a)/x
```

Maxima [A] time = 1.0016, size = 159, normalized size = 1.47

$$\frac{1}{9}\,Bb^5x^9+\frac{1}{7}\left(5\,Bab^4+Ab^5\right)x^7+\left(2\,Ba^2b^3+Aab^4\right)x^5+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+5\left(Ba^4b+2\,Aa^3b^2\right)x-\frac{Aa^5+3\left(Ba^5\right.}{3.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + (2*B*a^2*b^3 + A*a*b^4)*x^5 +
 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/3*(A*a^
5 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Fricas [A] time = 1.41363, size = 263, normalized size = 2.44

$$\frac{7\,Bb^5x^{12}+9\left(5\,Bab^4+Ab^5\right)x^{10}+63\left(2\,Ba^2b^3+Aab^4\right)x^8+210\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5+315\left(Ba^4b+2\,Aa^3b^2\right)x}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^5*x^12 + 9*(5*B*a*b^4 + A*b^5)*x^10 + 63*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 + 315*(B*a^4*b + 2*A*a^3
*b^2)*x^4 - 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Sympy [A] time = 0.481343, size = 126, normalized size = 1.17

$$\frac{Bb^5x^9}{9}+x^7\left(\frac{Ab^5}{7}+\frac{5Bab^4}{7}\right)+x^5\left(Aab^4+2Ba^2b^3\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)+x\left(10Aa^3b^2+5Ba^4b\right)-\frac{Aa^5+x^2\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**5*x**9/9 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**5*(A*a*b**4 + 2*B*a**2*
b**3) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*(10*A*a**3*b**2 + 5*
B*a**4*b) - (A*a**5 + x**2*(15*A*a**4*b + 3*B*a**5))/(3*x**3)
```

Giac [A] time = 1.135, size = 165, normalized size = 1.53

$$\frac{1}{9}Bb^5x^9 + \frac{5}{7}Bab^4x^7 + \frac{1}{7}Ab^5x^7 + 2Ba^2b^3x^5 + Aab^4x^5 + \frac{10}{3}Ba^3b^2x^3 + \frac{10}{3}Aa^2b^3x^3 + 5Ba^4bx + 10Aa^3b^2x - \frac{3Ba^5x^2+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 2*B*a^2*b^3*x^5 + A*a*b^4
*x^5 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*B*a^4*b*x + 10*A*a^3*b^2
*x - 1/3*(3*B*a^5*x^2 + 15*A*a^4*b*x^2 + A*a^5)/x^3
```

3.37 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=112

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{4x^4}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5B$$

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.10434, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{4x^4}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5B$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}+5ab^3(Ab\right.\right.\\ &= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{2x^2}+5a^2b^2(Ab+aB)x^2+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{6}b^4(Ab+5aB)x^6\end{aligned}$$

Mathematica [A] time = 0.0375762, size = 112, normalized size = 1.

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{4x^4}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5B$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Maple [A] time = 0.007, size = 124, normalized size = 1.1

$$\frac{b^5Bx^8}{8}+\frac{Ax^6b^5}{6}+\frac{5\,Bx^6ab^4}{6}+\frac{5\,Ax^4ab^4}{4}+\frac{5\,Bx^4a^2b^3}{2}+5\,Ax^2a^2b^3+5\,Bx^2a^3b^2+10\,A\ln(x)\,a^3b^2+5\,B\ln(x)\,a^4b-\frac{Aa^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^5,x)
```

```
[Out] 1/8*b^5*B*x^8+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2*b
^3+5*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2+10*A*ln(x)*a^3*b^2+5*B*ln(x)*a^4*b-1/4*a
^5*A/x^4-5/2*a^4/x^2*A*b-1/2*a^5/x^2*B
```

Maxima [A] time = 1.00494, size = 165, normalized size = 1.47

$$\frac{1}{8}\,Bb^5x^8+\frac{1}{6}\left(5\,Bab^4+Ab^5\right)x^6+\frac{5}{4}\left(2\,Ba^2b^3+Aab^4\right)x^4+5\left(Ba^3b^2+Aa^2b^3\right)x^2+\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)\log\left(x^2\right)-\frac{Aa^5+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/4*(2*B*a^2*b^3 + A*a*b^4)*x
^4 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*log(x^2) -
 1/4*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

Fricas [A] time = 1.46823, size = 271, normalized size = 2.42

$$\frac{3\,Bb^5x^{12}+4\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6-6\,Aa^5+120\left(Ba^4b+2\,Aa^3b^2\right)x^4\log}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^12 + 4*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 6*A*a^5 + 120*(B*a^4*b + 2*A*a^3*
b^2)*x^4*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

Sympy [A] time = 0.794991, size = 126, normalized size = 1.12

$$\frac{Bb^5x^8}{8}+5a^3b\left(2Ab+Ba\right)\log\left(x\right)+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^4\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right)+x^2\left(5Aa^2b^3+5Ba^3b^2\right)-\frac{Aa^5+x^2\left(10A\right.}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**5*x**8/8 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b**4
/6) + x**4*(5*A*a*b**4/4 + 5*B*a**2*b**3/2) + x**2*(5*A*a**2*b**3 + 5*B*a**
3*b**2) - (A*a**5 + x**2*(10*A*a**4*b + 2*B*a**5))/(4*x**4)
```

Giac [A] time = 1.14906, size = 201, normalized size = 1.79

$$\frac{1}{8}Bb^5x^8 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + \frac{5}{2}Ba^2b^3x^4 + \frac{5}{4}Aab^4x^4 + 5Ba^3b^2x^2 + 5Aa^2b^3x^2 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)\log\left(x^2\right) - \frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 5/2*B*a^2*b^3*x^4 + 5/4*A
*a*b^4*x^4 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2
)*log(x^2) - 1/4*(15*B*a^4*b*x^4 + 30*A*a^3*b^2*x^4 + 2*B*a^5*x^2 + 10*A*a^
4*b*x^2 + A*a^5)/x^4
```

3.38 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=111

$$10a^2b^2x(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{5x^5}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rubi [A] time = 0.0623546, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$10a^2b^2x(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{5x^5}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx &= \int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^2}+5ab^3(Ab+2aB)x^2+b^4\right.\\ &= -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{5}b\end{aligned}$$

Mathematica [A] time = 0.0346202, size = 111, normalized size = 1.

$$10a^2b^2x(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{5x^5}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
```

```
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Maple [A] time = 0.006, size = 113, normalized size = 1.

$$\frac{b^5Bx^7}{7}+\frac{Ax^5b^5}{5}+Bx^5ab^4+\frac{5\,Ax^3ab^4}{3}+\frac{10\,Bx^3a^2b^3}{3}+10\,a^2b^3Ax+10\,a^3b^2Bx-\frac{a^4\,(5\,Ab+Ba)}{3\,x^3}-\frac{Aa^5}{5\,x^5}-5\,\frac{a^3b\,(2\,Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^6,x)
```

```
[Out] 1/7*b^5*B*x^7+1/5*A*x^5*b^5+B*x^5*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^3*a^2*b^3+
10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/3*a^4*(5*A*b+B*a)/x^3-1/5*a^5*A/x^5-5*a^3*b
*(2*A*b+B*a)/x
```

Maxima [A] time = 0.99986, size = 162, normalized size = 1.46

$$\frac{1}{7}\,Bb^5x^7+\frac{1}{5}\left(5\,Bab^4+Ab^5\right)x^5+\frac{5}{3}\left(2\,Ba^2b^3+Aab^4\right)x^3+10\left(Ba^3b^2+Aa^2b^3\right)x-\frac{3\,Aa^5+75\left(Ba^4b+2\,Aa^3b^2\right)x^4+5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/15*(3*A*a^5 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^4 + 5*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

Fricas [A] time = 1.39156, size = 270, normalized size = 2.43

$$\frac{15\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+175\left(2\,Ba^2b^3+Aab^4\right)x^8+1050\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5-525\left(Ba^4b+2\,Aa\right.}{105\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 175*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 - 525*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 35*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

Sympy [A] time = 0.882236, size = 126, normalized size = 1.14

$$\frac{Bb^5x^7}{7}+x^5\left(\frac{Ab^5}{5}+Bab^4\right)+x^3\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right)+x\left(10Aa^2b^3+10Ba^3b^2\right)-\frac{3Aa^5+x^4\left(150Aa^3b^2+75Ba^4b\right)+x}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**5*x**7/7 + x**5*(A*b**5/5 + B*a*b**4) + x**3*(5*A*a*b**4/3 + 10*B*a**2
*b**3/3) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) - (3*A*a**5 + x**4*(150*A*a*
*3*b**2 + 75*B*a**4*b) + x**2*(25*A*a**4*b + 5*B*a**5))/(15*x**5)
```

Giac [A] time = 1.13736, size = 166, normalized size = 1.5

$$\frac{1}{7}\,Bb^5x^7 + Bab^4x^5 + \frac{1}{5}\,Ab^5x^5 + \frac{10}{3}\,Ba^2b^3x^3 + \frac{5}{3}\,Aab^4x^3 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{75\,Ba^4bx^4 + 150\,Aa^3b^2x^4 + 5\,Ba^5x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 10/3*B*a^2*b^3*x^3 + 5/3*A*a*
b^4*x^3 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/15*(75*B*a^4*b*x^4 + 150*A*a^
3*b^2*x^4 + 5*B*a^5*x^2 + 25*A*a^4*b*x^2 + 3*A*a^5)/x^5
```

3.39 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7} dx$

Optimal. Leaf size=114

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{a^5A}{6x^6}+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx$$

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*
x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rubi [A] time = 0.10056, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{a^5A}{6x^6}+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*
x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+\frac{10a^2b^2(Ab}{x}\right.\right.\\ &= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{2x^2}+\frac{5}{2}ab^3(Ab+2aB)x^2+\frac{1}{4}b^4(Ab+5aB)x^4+\end{aligned}$$

Mathematica [A] time = 0.0378388, size = 116, normalized size = 1.02

$$\frac{1}{12}\left(120a^2b^2\log(x)(aB+Ab)-\frac{60a^3Ab^2}{x^2}-\frac{15a^4b\left(A+2Bx^2\right)}{x^4}-\frac{a^5\left(2A+3Bx^2\right)}{x^6}+60a^2b^3Bx^2+15ab^4x^2\left(2A+Bx^2\right)+b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] ((-60*a^3*A*b^2)/x^2 + 60*a^2*b^3*B*x^2 + 15*a*b^4*x^2*(2*A + B*x^2) - (15*
a^4*b*(A + 2*B*x^2))/x^4 + b^5*x^4*(3*A + 2*B*x^2) - (a^5*(2*A + 3*B*x^2))/
x^6 + 120*a^2*b^2*(A*b + a*B)*Log[x])/12
```

Maple [A] time = 0.007, size = 124, normalized size = 1.1

$$\frac{b^5Bx^6}{6}+\frac{Ax^4b^5}{4}+\frac{5\,Bx^4ab^4}{4}+\frac{5\,Ax^2ab^4}{2}+5\,Bx^2a^2b^3+10\,A\ln(x)\,a^2b^3+10\,B\ln(x)\,a^3b^2-\frac{5\,a^4bA}{4\,x^4}-\frac{a^5B}{4\,x^4}-5\,\frac{a^3b^2A}{x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^7,x)
```

```
[Out] 1/6*b^5*B*x^6+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5/2*A*x^2*a*b^4+5*B*x^2*a^2*b^3
+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2-5/4*a^4/x^4*A*b-1/4*a^5/x^4*B-5*a^3*
b^2/x^2*A-5/2*a^4*b/x^2*B-1/6*a^5*A/x^6
```

Maxima [A] time = 1.00007, size = 166, normalized size = 1.46

$$\frac{1}{6}\,Bb^5x^6+\frac{1}{4}\left(5\,Bab^4+Ab^5\right)x^4+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)x^2+5\left(Ba^3b^2+Aa^2b^3\right)\log\left(x^2\right)-\frac{2\,Aa^5+30\left(Ba^4b+2\,Aa^3b^2\right)x^4+}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(2*A*a^5 + 30*(B*a^4*b + 2*A
*a^3*b^2)*x^4 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

Fricas [A] time = 1.42026, size = 269, normalized size = 2.36

$$\frac{2\,Bb^5x^{12}+3\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6\log(x)-2\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right.}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^12 + 3*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6*log(x) - 2*A*a^5 - 30*(B*a^4*b + 2*
```

```
A*a^3*b^2)*x^4 - 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

Sympy [A] time = 1.52849, size = 124, normalized size = 1.09

$$\frac{Bb^5x^6}{6} + 10a^2b^2(Ab + Ba)\log(x) + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x^2\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) - \frac{2Aa^5 + x^4\left(60Aa^3b^2 + 30Ba^4b\right) + :}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**5*x**6/6 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**4*(A*b**5/4 + 5*B*a*b*
*4/4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) - (2*A*a**5 + x**4*(60*A*a**3*b
**2 + 30*B*a**4*b) + x**2*(15*A*a**4*b + 3*B*a**5))/(12*x**6)
```

Giac [A] time = 1.44273, size = 204, normalized size = 1.79

$$\frac{1}{6}Bb^5x^6 + \frac{5}{4}Bab^4x^4 + \frac{1}{4}Ab^5x^4 + 5Ba^2b^3x^2 + \frac{5}{2}Aab^4x^2 + 5\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^2\right) - \frac{110Ba^3b^2x^6 + 110Aa^2b^3x^6 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 5*B*a^2*b^3*x^2 + 5/2*A*a
*b^4*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(110*B*a^3*b^2*x^6 + 1
10*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 3*B*a^5*x^2 + 15*A*a
^4*b*x^2 + 2*A*a^5)/x^6
```

3.40 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=111

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^5A}{7x^7}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rubi [A] time = 0.0617063, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^5A}{7x^7}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx &= \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^8}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^2}+b^4(\right.\\ &= -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x+\frac{1}{3}b^4(\end{aligned}$$

Mathematica [A] time = 0.0365976, size = 111, normalized size = 1.

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^5A}{7x^7}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
```

```
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Maple [A] time = 0.006, size = 108, normalized size = 1.

$$\frac{b^5Bx^5}{5}+\frac{Ax^3b^5}{3}+\frac{5\,Bx^3ab^4}{3}+5\,ab^4Ax+10\,a^2b^3Bx-\frac{5\,a^3b\,(2\,Ab+Ba)}{3\,x^3}-\frac{a^4\,(5\,Ab+Ba)}{5\,x^5}-\frac{Aa^5}{7\,x^7}-10\,\frac{b^2a^2\,(Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^8,x)
```

```
[Out] 1/5*b^5*B*x^5+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-5/3*
a^3*b*(2*A*b+B*a)/x^3-1/5*a^4*(5*A*b+B*a)/x^5-1/7*a^5*A/x^7-10*a^2*b^2*(A*b
+B*a)/x
```

Maxima [A] time = 1.05277, size = 162, normalized size = 1.46

$$\frac{1}{5}\,Bb^5x^5+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)x^3+5\left(2\,Ba^2b^3+Aab^4\right)x-\frac{1050\left(Ba^3b^2+Aa^2b^3\right)x^6+15\,Aa^5+175\left(Ba^4b+2\,Aa^3b^2\right)x^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/105*(1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 15*A*a^5 + 175*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

Fricas [A] time = 1.37307, size = 270, normalized size = 2.43

$$\frac{21\,Bb^5x^{12}+35\left(5\,Bab^4+Ab^5\right)x^{10}+525\left(2\,Ba^2b^3+Aab^4\right)x^8-1050\left(Ba^3b^2+Aa^2b^3\right)x^6-15\,Aa^5-175\left(Ba^4b+2\,Aa\right.}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*B*b^5*x^12 + 35*(5*B*a*b^4 + A*b^5)*x^10 + 525*(2*B*a^2*b^3 + A*a
*b^4)*x^8 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 15*A*a^5 - 175*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

Sympy [A] time = 1.69062, size = 126, normalized size = 1.14

$$\frac{Bb^5x^5}{5}+x^3\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+x\left(5Aab^4+10Ba^2b^3\right)-\frac{15Aa^5+x^6\left(1050Aa^2b^3+1050Ba^3b^2\right)+x^4\left(350Aa^3b^2+175B\right.}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

```
[Out] B*b**5*x**5/5 + x**3*(A*b**5/3 + 5*B*a*b**4/3) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) - (15*A*a**5 + x**6*(1050*A*a**2*b**3 + 1050*B*a**3*b**2) + x**4*(350
*A*a**3*b**2 + 175*B*a**4*b) + x**2*(105*A*a**4*b + 21*B*a**5))/(105*x**7)
```

Giac [A] time = 1.1622, size = 167, normalized size = 1.5

$$\frac{1}{5}\,Bb^5x^5 + \frac{5}{3}\,Bab^4x^3 + \frac{1}{3}\,Ab^5x^3 + 10\,Ba^2b^3x + 5\,Aab^4x - \frac{1050\,Ba^3b^2x^6 + 1050\,Aa^2b^3x^6 + 175\,Ba^4bx^4 + 350\,Aa^3b^2x^4 + 2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/105*(1050*B*a^3*b^2*x^6 + 1050*A*a^2*b^3*x^6 + 175*B*a^4*b*x^4 + 35
0*A*a^3*b^2*x^4 + 21*B*a^5*x^2 + 105*A*a^4*b*x^2 + 15*A*a^5)/x^7
```

3.41 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx$

Optimal. Leaf size=112

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^5A}{8x^8}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(4
*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^
4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rubi [A] time = 0.0965514, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^5A}{8x^8}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(4
*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^
4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{10a^2b^2(Ab+\right.\right.\\ &= -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{5a^2b^2(Ab+aB)}{x^2}+\frac{1}{2}b^4(Ab+5aB)x^2+\frac{1}{4}b\end{aligned}$$

Mathematica [A] time = 0.0505843, size = 116, normalized size = 1.04

$$5ab^3\log(x)(2aB+Ab)-\frac{60a^3b^2x^4(A+2Bx^2)+120a^2Ab^3x^6+10a^4bx^2(2A+3Bx^2)+a^5(3A+4Bx^2)-60ab^4Bx^{10}}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -(120*a^2*A*b^3*x^6 - 60*a*b^4*B*x^10 - 6*b^5*x^10*(2*A + B*x^2) + 60*a^3*b
^2*x^4*(A + 2*B*x^2) + 10*a^4*b*x^2*(2*A + 3*B*x^2) + a^5*(3*A + 4*B*x^2))/
(24*x^8) + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Maple [A] time = 0.006, size = 124, normalized size = 1.1

$$\frac{b^5Bx^4}{4}+\frac{Ax^2b^5}{2}+\frac{5\,Bx^2ab^4}{2}+5\,A\ln(x)\,ab^4+10\,B\ln(x)\,a^2b^3-\frac{5\,a^3b^2A}{2\,x^4}-\frac{5\,a^4bB}{4\,x^4}-\frac{Aa^5}{8\,x^8}-5\,\frac{a^2b^3A}{x^2}-5\,\frac{a^3b^2B}{x^2}-\frac{5\,a^4bA}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^9,x)
```

```
[Out] 1/4*b^5*B*x^4+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4+5*A*ln(x)*a*b^4+10*B*ln(x)*a^2*
b^3-5/2*a^3*b^2/x^4*A-5/4*a^4*b/x^4*B-1/8*a^5*A/x^8-5*b^3*a^2/x^2*A-5*b^2*a
^3/x^2*B-5/6*a^4/x^6*A*b-1/6*a^5/x^6*B
```

Maxima [A] time = 1.00189, size = 166, normalized size = 1.48

$$\frac{1}{4}\,Bb^5x^4+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)x^2+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)\log\left(x^2\right)-\frac{120\left(Ba^3b^2+Aa^2b^3\right)x^6+3\,Aa^5+30\left(Ba^4b+2\,Aa^3b^2\right)x^4+4\left(Ba^5+5\,Aa^4b\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^2) - 1/24*(120*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3*A*a^5 + 30*(B*a^4*b + 2
*A*a^3*b^2)*x^4 + 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

Fricas [A] time = 1.4237, size = 271, normalized size = 2.42

$$\frac{6\,Bb^5x^{12}+12\left(5\,Bab^4+Ab^5\right)x^{10}+120\left(2\,Ba^2b^3+Aab^4\right)x^8\log(x)-120\left(Ba^3b^2+Aa^2b^3\right)x^6-3\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right)x^4-4\left(Ba^5+5\,Aa^4b\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(6*B*b^5*x^12 + 12*(5*B*a*b^4 + A*b^5)*x^10 + 120*(2*B*a^2*b^3 + A*a*b
^4)*x^8*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 3*A*a^5 - 30*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

Sympy [A] time = 3.03705, size = 124, normalized size = 1.11

$$\frac{Bb^5x^4}{4}+5ab^3\left(Ab+2Ba\right)\log(x)+x^2\left(\frac{Ab^5}{2}+\frac{5Bab^4}{2}\right)-\frac{3Aa^5+x^6\left(120Aa^2b^3+120Ba^3b^2\right)+x^4\left(60Aa^3b^2+30Ba^4b\right)+\cdots}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

```
[Out] B*b**5*x**4/4 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**2*(A*b**5/2 + 5*B*a*b**4
/2) - (3*A*a**5 + x**6*(120*A*a**2*b**3 + 120*B*a**3*b**2) + x**4*(60*A*a**
3*b**2 + 30*B*a**4*b) + x**2*(20*A*a**4*b + 4*B*a**5))/(24*x**8)
```

Giac [A] time = 1.20431, size = 203, normalized size = 1.81

$$\frac{1}{4}Bb^5x^4 + \frac{5}{2}Bab^4x^2 + \frac{1}{2}Ab^5x^2 + \frac{5}{2}\left(2Ba^2b^3 + Aab^4\right)\log\left(x^2\right) - \frac{250Ba^2b^3x^8 + 125Aab^4x^8 + 120Ba^3b^2x^6 + 120Aa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^
4)*log(x^2) - 1/24*(250*B*a^2*b^3*x^8 + 125*A*a*b^4*x^8 + 120*B*a^3*b^2*x^6
 + 120*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 4*B*a^5*x^2 + 20
*A*a^4*b*x^2 + 3*A*a^5)/x^8
```

3.42 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=108

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^5A}{9x^9}-\frac{5ab^3(2aB+Ab)}{x}+b^4x(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b + a*B))/x^5
- (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*a*B))/x + b^4*(A*b +
 5*a*B)*x + (b^5*B*x^3)/3
```

Rubi [A] time = 0.0663631, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^5A}{9x^9}-\frac{5ab^3(2aB+Ab)}{x}+b^4x(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b + a*B))/x^5
- (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*a*B))/x + b^4*(A*b +
 5*a*B)*x + (b^5*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx=\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^{10}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab+aB)}{x^4}+\frac{5ab^3(\right.$$

$$=-\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{10a^2b^2(Ab+aB)}{3x^3}-\frac{5ab^3(Ab+2aB)}{x}+b^4(Ab+$$

Mathematica [A] time = 0.0321109, size = 115, normalized size = 1.06

$$-\frac{210a^2b^3x^6\left(A+3Bx^2\right)+42a^3b^2x^4\left(3A+5Bx^2\right)+9a^4bx^2\left(5A+7Bx^2\right)+a^5\left(7A+9Bx^2\right)+315ab^4x^8\left(A-Bx^2\right)-21b^5}{63x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -(315*a*b^4*x^8*(A - B*x^2) - 21*b^5*x^10*(3*A + B*x^2) + 210*a^2*b^3*x^6*(
A + 3*B*x^2) + 42*a^3*b^2*x^4*(3*A + 5*B*x^2) + 9*a^4*b*x^2*(5*A + 7*B*x^2)
```

```
+ a^5*(7*A + 9*B*x^2))/(63*x^9)
```

Maple [A] time = 0.007, size = 102, normalized size = 0.9

$$\frac{b^5Bx^3}{3}+b^5Ax+5\,ab^4Bx-\frac{10\,b^2a^2\,(Ab+Ba)}{3\,x^3}-\frac{a^3b\,(2\,Ab+Ba)}{x^5}-\frac{a^4\,(5\,Ab+Ba)}{7\,x^7}-5\,\frac{ab^3\,(Ab+2\,Ba)}{x}-\frac{Aa^5}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^10,x)
```

```
[Out] 1/3*b^5*B*x^3+b^5*A*x+5*a*b^4*B*x-10/3*a^2*b^2*(A*b+B*a)/x^3-a^3*b*(2*A*b+B
*a)/x^5-1/7*a^4*(5*A*b+B*a)/x^7-5*a*b^3*(A*b+2*B*a)/x-1/9*a^5*A/x^9
```

Maxima [A] time = 1.01553, size = 161, normalized size = 1.49

$$\frac{1}{3}\,Bb^5x^3+\left(5\,Bab^4+Ab^5\right)x-\frac{315\left(2\,Ba^2b^3+Aab^4\right)x^8+210\left(Ba^3b^2+Aa^2b^3\right)x^6+7\,Aa^5+63\left(Ba^4b+2\,Aa^3b^2\right)x^4+9}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*x - 1/63*(315*(2*B*a^2*b^3 + A*a*b^4)*x
^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 7*A*a^5 + 63*(B*a^4*b + 2*A*a^3*b^2)
*x^4 + 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Fricas [A] time = 1.44344, size = 263, normalized size = 2.44

$$\frac{21\,Bb^5x^{12}+63\left(5\,Bab^4+Ab^5\right)x^{10}-315\left(2\,Ba^2b^3+Aab^4\right)x^8-210\left(Ba^3b^2+Aa^2b^3\right)x^6-7\,Aa^5-63\left(Ba^4b+2\,Aa^3b^2\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*B*b^5*x^12 + 63*(5*B*a*b^4 + A*b^5)*x^10 - 315*(2*B*a^2*b^3 + A*a*
b^4)*x^8 - 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 7*A*a^5 - 63*(B*a^4*b + 2*A*a^
3*b^2)*x^4 - 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Sympy [A] time = 3.39189, size = 122, normalized size = 1.13

$$\frac{Bb^5x^3}{3}+x\left(Ab^5+5Bab^4\right)-\frac{7Aa^5+x^8\left(315Aab^4+630Ba^2b^3\right)+x^6\left(210Aa^2b^3+210Ba^3b^2\right)+x^4\left(126Aa^3b^2+63Ba^4\right.}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

```
[Out] B*b**5*x**3/3 + x*(A*b**5 + 5*B*a*b**4) - (7*A*a**5 + x**8*(315*A*a*b**4 +
630*B*a**2*b**3) + x**6*(210*A*a**2*b**3 + 210*B*a**3*b**2) + x**4*(126*A*a
**3*b**2 + 63*B*a**4*b) + x**2*(45*A*a**4*b + 9*B*a**5))/(63*x**9)
```

Giac [A] time = 1.15211, size = 166, normalized size = 1.54

$$\frac{1}{3}Bb^5x^3 + 5\,Bab^4x + Ab^5x - \frac{630\,Ba^2b^3x^8 + 315\,Aab^4x^8 + 210\,Ba^3b^2x^6 + 210\,Aa^2b^3x^6 + 63\,Ba^4bx^4 + 126\,Aa^3b^2x^4 + 9\,B}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + 5*B*a*b^4*x + A*b^5*x - 1/63*(630*B*a^2*b^3*x^8 + 315*A*a*b
^4*x^8 + 210*B*a^3*b^2*x^6 + 210*A*a^2*b^3*x^6 + 63*B*a^4*b*x^4 + 126*A*a^3
*b^2*x^4 + 9*B*a^5*x^2 + 45*A*a^4*b*x^2 + 7*A*a^5)/x^9
```

3.43 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}} dx$

Optimal. Leaf size=113

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^5A}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4\log(x)(5aB+Ab)+\frac{1}{2}b^5Bx^2$$

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi [A] time = 0.0893554, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^5A}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4\log(x)(5aB+Ab)+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6} dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}+\frac{5a\right.\right.\\ &= -\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{2x^2}+\frac{1}{2}b\end{aligned}$$

Mathematica [A] time = 0.0569273, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{300a^2b^3x^6(A+2Bx^2)+100a^3b^2x^4(2A+3Bx^2)+25a^4bx^2(3A+4Bx^2)+3a^5(4A+5Bx^2)+}{120x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -(300*a*A*b^4*x^8 - 60*b^5*B*x^12 + 300*a^2*b^3*x^6*(A + 2*B*x^2) + 100*a^3
*b^2*x^4*(2*A + 3*B*x^2) + 25*a^4*b*x^2*(3*A + 4*B*x^2) + 3*a^5*(4*A + 5*B*
x^2))/(120*x^10) + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A] time = 0.007, size = 123, normalized size = 1.1

$$\frac{b^5Bx^2}{2}+A\ln(x)\,b^5+5\,B\ln(x)\,ab^4-\frac{5\,a^2b^3A}{2\,x^4}-\frac{5\,a^3b^2B}{2\,x^4}-\frac{5\,ab^4A}{2\,x^2}-5\,\frac{a^2b^3B}{x^2}-\frac{5\,a^3b^2A}{3\,x^6}-\frac{5\,a^4bB}{6\,x^6}-\frac{Aa^5}{10\,x^{10}}-\frac{5\,a^4bA}{8\,x^8}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^11,x)
```

```
[Out] 1/2*b^5*B*x^2+A*ln(x)*b^5+5*B*ln(x)*a*b^4-5/2*b^3*a^2/x^4*A-5/2*b^2*a^3/x^4
*B-5/2*a*b^4/x^2*A-5*a^2*b^3/x^2*B-5/3*a^3*b^2/x^6*A-5/6*a^4*b/x^6*B-1/10*a
^5*A/x^10-5/8*a^4/x^8*A*b-1/8*a^5/x^8*B
```

Maxima [A] time = 1.0346, size = 166, normalized size = 1.47

$$\frac{1}{2}\,Bb^5x^2+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)\log\left(x^2\right)-\frac{300\left(2\,Ba^2b^3+Aab^4\right)x^8+300\left(Ba^3b^2+Aa^2b^3\right)x^6+12\,Aa^5+100\left(Ba^4b+2\,Aa^3\cdots\right.}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 12*A*a^5 + 100*(B*a^4*b
+ 2*A*a^3*b^2)*x^4 + 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

Fricas [A] time = 1.49088, size = 281, normalized size = 2.49

$$\frac{60\,Bb^5x^{12}+120\left(5\,Bab^4+Ab^5\right)x^{10}\log(x)-300\left(2\,Ba^2b^3+Aab^4\right)x^8-300\left(Ba^3b^2+Aa^2b^3\right)x^6-12\,Aa^5-100\left(Ba^4b+2\cdots\right.}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(60*B*b^5*x^12 + 120*(5*B*a*b^4 + A*b^5)*x^10*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 12*A*a^5 - 100*(B*a^4
*b + 2*A*a^3*b^2)*x^4 - 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

Sympy [A] time = 6.39761, size = 122, normalized size = 1.08

$$\frac{Bb^5x^2}{2}+b^4\left(Ab+5Ba\right)\log(x)-\frac{12Aa^5+x^8\left(300Aab^4+600Ba^2b^3\right)+x^6\left(300Aa^2b^3+300Ba^3b^2\right)+x^4\left(200Aa^3b^2+100\cdots\right.}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

```
[Out] B*b**5*x**2/2 + b**4*(A*b + 5*B*a)*log(x) - (12*A*a**5 + x**8*(300*A*a*b**4
 + 600*B*a**2*b**3) + x**6*(300*A*a**2*b**3 + 300*B*a**3*b**2) + x**4*(200*
A*a**3*b**2 + 100*B*a**4*b) + x**2*(75*A*a**4*b + 15*B*a**5))/(120*x**10)
```

<u>Giac</u> [A] time = 1.10586, size = 198, normalized size = 1.75

$$\frac{1}{2}Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\log\left(x^2\right) - \frac{685\,Bab^4x^{10} + 137\,Ab^5x^{10} + 600\,Ba^2b^3x^8 + 300\,Aab^4x^8 + 300\,Ba^3b^2x^6 + 300\,\ldots}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(685*B*a*b^4*x^10
+ 137*A*b^5*x^10 + 600*B*a^2*b^3*x^8 + 300*A*a*b^4*x^8 + 300*B*a^3*b^2*x^6
+ 300*A*a^2*b^3*x^6 + 100*B*a^4*b*x^4 + 200*A*a^3*b^2*x^4 + 15*B*a^5*x^2 +
75*A*a^4*b*x^2 + 12*A*a^5)/x^10
```

3.44 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx$

Optimal. Leaf size=108

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{3x^3}-\frac{b^4(5aB+Ab)}{x}+b^5Bx$$

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) - (
b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rubi [A] time = 0.0621864, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{3x^3}-\frac{b^4(5aB+Ab)}{x}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) - (
b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx=\int \left(b^5B+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^{10}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^6}+\frac{5ab^3(Ab+2aB)}{x^4}\right.$$
$$=-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{3x^3}-\frac{b^4(Ab}{}$$

Mathematica [A] time = 0.0414157, size = 122, normalized size = 1.13

$$-\frac{2a^3b^2\left(5A+7Bx^2\right)}{7x^7}-\frac{2a^2b^3\left(3A+5Bx^2\right)}{3x^5}-\frac{5a^4b\left(7A+9Bx^2\right)}{63x^9}-\frac{a^5\left(9A+11Bx^2\right)}{99x^{11}}-\frac{5ab^4\left(A+3Bx^2\right)}{3x^3}-\frac{Ab^5}{x}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -((A*b^5)/x) + b^5*B*x - (5*a*b^4*(A + 3*B*x^2))/(3*x^3) - (2*a^2*b^3*(3*A
+ 5*B*x^2))/(3*x^5) - (2*a^3*b^2*(5*A + 7*B*x^2))/(7*x^7) - (5*a^4*b*(7*A +
```

```
9*B*x^2))/(63*x^9) - (a^5*(9*A + 11*B*x^2))/(99*x^11)
```

Maple [A] time = 0.007, size = 101, normalized size = 0.9

$$-\frac{Aa^5}{11\,x^{11}}-\frac{a^4\,(5\,Ab+Ba)}{9\,x^9}-\frac{5\,a^3b\,(2\,Ab+Ba)}{7\,x^7}-2\,\frac{b^2a^2\,(Ab+Ba)}{x^5}-\frac{5\,ab^3\,(Ab+2\,Ba)}{3\,x^3}-\frac{b^4\,(Ab+5\,Ba)}{x}+b^5Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^12,x)
```

```
[Out] -1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b^
2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

Maxima [A] time = 0.98483, size = 161, normalized size = 1.49

$$Bb^5x-\frac{693\left(5\,Bab^4+Ab^5\right)x^{10}+1155\left(2\,Ba^2b^3+Aab^4\right)x^8+1386\left(Ba^3b^2+Aa^2b^3\right)x^6+63\,Aa^5+495\left(Ba^4b+2\,Aa^3b\right.}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/693*(693*(5*B*a*b^4 + A*b^5)*x^10 + 1155*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 63*A*a^5 + 495*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

Fricas [A] time = 1.3699, size = 275, normalized size = 2.55

$$\frac{693\,Bb^5x^{12}-693\left(5\,Bab^4+Ab^5\right)x^{10}-1155\left(2\,Ba^2b^3+Aab^4\right)x^8-1386\left(Ba^3b^2+Aa^2b^3\right)x^6-63\,Aa^5-495\left(Ba^4b+2\right.}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/693*(693*B*b^5*x^12 - 693*(5*B*a*b^4 + A*b^5)*x^10 - 1155*(2*B*a^2*b^3 +
A*a*b^4)*x^8 - 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 63*A*a^5 - 495*(B*a^4*b +
 2*A*a^3*b^2)*x^4 - 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

Sympy [A] time = 7.75584, size = 122, normalized size = 1.13

$$Bb^5x-\frac{63Aa^5+x^{10}\left(693Ab^5+3465Bab^4\right)+x^8\left(1155Aab^4+2310Ba^2b^3\right)+x^6\left(1386Aa^2b^3+1386Ba^3b^2\right)+x^4\left(990A\right.}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)
```

```
[Out] B*b**5*x - (63*A*a**5 + x**10*(693*A*b**5 + 3465*B*a*b**4) + x**8*(1155*A*a
*b**4 + 2310*B*a**2*b**3) + x**6*(1386*A*a**2*b**3 + 1386*B*a**3*b**2) + x*
*4*(990*A*a**3*b**2 + 495*B*a**4*b) + x**2*(385*A*a**4*b + 77*B*a**5))/(693
*x**11)
```

Giac [A] time = 1.12713, size = 169, normalized size = 1.56

$$Bb^5x - \frac{3465\,Bab^4x^{10} + 693\,Ab^5x^{10} + 2310\,Ba^2b^3x^8 + 1155\,Aab^4x^8 + 1386\,Ba^3b^2x^6 + 1386\,Aa^2b^3x^6 + 495\,Ba^4bx^4 + 990}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/693*(3465*B*a*b^4*x^10 + 693*A*b^5*x^10 + 2310*B*a^2*b^3*x^8 +
1155*A*a*b^4*x^8 + 1386*B*a^3*b^2*x^6 + 1386*A*a^2*b^3*x^6 + 495*B*a^4*b*x^
4 + 990*A*a^3*b^2*x^4 + 77*B*a^5*x^2 + 385*A*a^4*b*x^2 + 63*A*a^5)/x^11
```

3.45 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}} \, dx$

Optimal. Leaf size=91

$$-\frac{5a^2b^3B}{2x^4}-\frac{5a^3b^2B}{3x^6}-\frac{5a^4bB}{8x^8}-\frac{a^5B}{10x^{10}}-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

```
[Out] -(a^5*B)/(10*x^10) - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6) - (5*a^2*b
^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)/(12*a*x^12) + b^5*B
*Log[x]
```

Rubi [A] time = 0.055555, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{2x^4}-\frac{5a^3b^2B}{3x^6}-\frac{5a^4bB}{8x^8}-\frac{a^5B}{10x^{10}}-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -(a^5*B)/(10*x^10) - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6) - (5*a^2*b
^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)/(12*a*x^12) + b^5*B
*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx, x, x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^6}{12ax^{12}} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^6}{12ax^{12}} + \frac{1}{2}B\,\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx, x, x^2\right)$$
$$= -\frac{a^5B}{10x^{10}} - \frac{5a^4bB}{8x^8} - \frac{5a^3b^2B}{3x^6} - \frac{5a^2b^3B}{2x^4} - \frac{5ab^4B}{2x^2} - \frac{A\left(a+bx^2\right)^6}{12ax^{12}} + b^5B\log(x)$$

Mathematica [A] time = 0.0570483, size = 118, normalized size = 1.3

$$b^5B\log(x) - \frac{100a^2b^3x^6\left(2A+3Bx^2\right)+50a^3b^2x^4\left(3A+4Bx^2\right)+15a^4bx^2\left(4A+5Bx^2\right)+2a^5\left(5A+6Bx^2\right)+150ab^4x^8\left(A\right.}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -(60*A*b^5*x^10 + 150*a*b^4*x^8*(A + 2*B*x^2) + 100*a^2*b^3*x^6*(2*A + 3*B*
x^2) + 50*a^3*b^2*x^4*(3*A + 4*B*x^2) + 15*a^4*b*x^2*(4*A + 5*B*x^2) + 2*a^
5*(5*A + 6*B*x^2))/(120*x^12) + b^5*B*Log[x]
```

Maple [A] time = 0.006, size = 124, normalized size = 1.4

$$b^5B\ln(x) - \frac{Aa^5}{12\,x^{12}} - \frac{5\,ab^4A}{4\,x^4} - \frac{5\,a^2b^3B}{2\,x^4} - \frac{b^5A}{2\,x^2} - \frac{5\,ab^4B}{2\,x^2} - \frac{5\,a^2b^3A}{3\,x^6} - \frac{5\,a^3b^2B}{3\,x^6} - \frac{5\,a^3b^2A}{4\,x^8} - \frac{5\,a^4bB}{8\,x^8} - \frac{a^4bA}{2\,x^{10}} - \frac{a^5B}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^13,x)
```

```
[Out] b^5*B*ln(x)-1/12*A*a^5/x^12-5/4*a*b^4/x^4*A-5/2*a^2*b^3*B/x^4-1/2*b^5/x^2*A
-5/2*a*b^4*B/x^2-5/3*b^3*a^2/x^6*A-5/3*a^3*b^2*B/x^6-5/4*a^3*b^2/x^8*A-5/8*
a^4*b*B/x^8-1/2*a^4/x^10*A*b-1/10*a^5*B/x^10
```

Maxima [A] time = 1.01765, size = 166, normalized size = 1.82

$$\frac{1}{2}Bb^5\log\left(x^2\right) - \frac{60\left(5\,Bab^4+Ab^5\right)x^{10}+150\left(2\,Ba^2b^3+Aab^4\right)x^8+200\left(Ba^3b^2+Aa^2b^3\right)x^6+10\,Aa^5+75\left(Ba^4b+2\,Aa^3\right.}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(60*(5*B*a*b^4 + A*b^5)*x^10 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 10*A*a^5 + 75*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

Fricas [A] time = 1.46626, size = 279, normalized size = 3.07

$$\frac{120\,Bb^5x^{12}\log(x) - 60\left(5\,Bab^4 + Ab^5\right)x^{10} - 150\left(2\,Ba^2b^3 + Aab^4\right)x^8 - 200\left(Ba^3b^2 + Aa^2b^3\right)x^6 - 10\,Aa^5 - 75\left(Ba^4b + \right.}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*B*b^5*x^12*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^10 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 10*A*a^5 - 75*(B*a^4*
b + 2*A*a^3*b^2)*x^4 - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

Sympy [A] time = 14.2837, size = 124, normalized size = 1.36

$$Bb^5\log(x) - \frac{10Aa^5 + x^{10}\left(60Ab^5 + 300Bab^4\right) + x^8\left(150Aab^4 + 300Ba^2b^3\right) + x^6\left(200Aa^2b^3 + 200Ba^3b^2\right) + x^4\left(150Aa\right.}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

```
[Out] B*b**5*log(x) - (10*A*a**5 + x**10*(60*A*b**5 + 300*B*a*b**4) + x**8*(150*A
*a*b**4 + 300*B*a**2*b**3) + x**6*(200*A*a**2*b**3 + 200*B*a**3*b**2) + x**
4*(150*A*a**3*b**2 + 75*B*a**4*b) + x**2*(60*A*a**4*b + 12*B*a**5))/(120*x*
*12)
```

Giac [A] time = 1.13902, size = 186, normalized size = 2.04

$$\frac{1}{2}\,Bb^5\log\left(x^2\right) - \frac{147\,Bb^5x^{12} + 300\,Bab^4x^{10} + 60\,Ab^5x^{10} + 300\,Ba^2b^3x^8 + 150\,Aab^4x^8 + 200\,Ba^3b^2x^6 + 200\,Aa^2b^3x^6 + }{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(147*B*b^5*x^12 + 300*B*a*b^4*x^10 + 60*A*b^5*x^
10 + 300*B*a^2*b^3*x^8 + 150*A*a*b^4*x^8 + 200*B*a^3*b^2*x^6 + 200*A*a^2*b^
3*x^6 + 75*B*a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 12*B*a^5*x^2 + 60*A*a^4*b*x^2
+ 10*A*a^5)/x^12
```

3.46 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx$

Optimal. Leaf size=113

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{13x^{13}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{b^5B}{x}$$

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(A*b + 2*a*B))/x^5 -
(b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rubi [A] time = 0.062222, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{13x^{13}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(A*b + 2*a*B))/x^5 -
(b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx &= \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^{10}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab+2aB)}{x^6}+\frac{b^4(\right.\\ &= -\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab}{3}\end{aligned}$$

Mathematica [A] time = 0.0325218, size = 119, normalized size = 1.05

$$-\frac{2574a^2b^3x^6\left(5A+7Bx^2\right)+1430a^3b^2x^4\left(7A+9Bx^2\right)+455a^4bx^2\left(9A+11Bx^2\right)+63a^5\left(11A+13Bx^2\right)+3003ab^4x^8\left(3A\right.}{9009x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -(3003*b^5*x^10*(A + 3*B*x^2) + 3003*a*b^4*x^8*(3*A + 5*B*x^2) + 2574*a^2*b
^3*x^6*(5*A + 7*B*x^2) + 1430*a^3*b^2*x^4*(7*A + 9*B*x^2) + 455*a^4*b*x^2*(
```

```
9*A + 11*B*x^2) + 63*a^5*(11*A + 13*B*x^2))/(9009*x^13)
```

Maple [A] time = 0.006, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{13\,x^{13}}-\frac{a^4\,(5\,Ab+Ba)}{11\,x^{11}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{9\,x^9}-\frac{10\,b^2a^2\,(Ab+Ba)}{7\,x^7}-\frac{ab^3\,(Ab+2\,Ba)}{x^5}-\frac{b^4\,(Ab+5\,Ba)}{3\,x^3}-\frac{Bb^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^14,x)
```

```
[Out] -1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*a
^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/x
```

Maxima [A] time = 1.00434, size = 163, normalized size = 1.44

$$-\frac{9009\,Bb^5x^{12}+3003\left(5\,Bab^4+Ab^5\right)x^{10}+9009\left(2\,Ba^2b^3+Aab^4\right)x^8+12870\left(Ba^3b^2+Aa^2b^3\right)x^6+693\,Aa^5+5005\,(}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

Fricas [A] time = 1.43882, size = 286, normalized size = 2.53

$$-\frac{9009\,Bb^5x^{12}+3003\left(5\,Bab^4+Ab^5\right)x^{10}+9009\left(2\,Ba^2b^3+Aab^4\right)x^8+12870\left(Ba^3b^2+Aa^2b^3\right)x^6+693\,Aa^5+5005\,(}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

Sympy [A] time = 16.9007, size = 128, normalized size = 1.13

$$-\frac{693Aa^5+9009Bb^5x^{12}+x^{10}\left(3003Ab^5+15015Bab^4\right)+x^8\left(9009Aab^4+18018Ba^2b^3\right)+x^6\left(12870Aa^2b^3+12870Ba\right.}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

```
[Out] -(693*A*a**5 + 9009*B*b**5*x**12 + x**10*(3003*A*b**5 + 15015*B*a*b**4) + x
**8*(9009*A*a*b**4 + 18018*B*a**2*b**3) + x**6*(12870*A*a**2*b**3 + 12870*B
*a**3*b**2) + x**4*(10010*A*a**3*b**2 + 5005*B*a**4*b) + x**2*(4095*A*a**4*
b + 819*B*a**5))/(9009*x**13)
```

Giac [A] time = 1.16323, size = 171, normalized size = 1.51

$$-\frac{9009\,Bb^5x^{12} + 15015\,Bab^4x^{10} + 3003\,Ab^5x^{10} + 18018\,Ba^2b^3x^8 + 9009\,Aab^4x^8 + 12870\,Ba^3b^2x^6 + 12870\,Aa^2b^3x^6 + 500}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 15015*B*a*b^4*x^10 + 3003*A*b^5*x^10 + 18018*B*a
^2*b^3*x^8 + 9009*A*a*b^4*x^8 + 12870*B*a^3*b^2*x^6 + 12870*A*a^2*b^3*x^6 +
 5005*B*a^4*b*x^4 + 10010*A*a^3*b^2*x^4 + 819*B*a^5*x^2 + 4095*A*a^4*b*x^2
+ 693*A*a^5)/x^13
```

3.47 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}} dx$

Optimal. Leaf size=48

$$\frac{(a+bx^2)^6 (Ab-7aB)}{84a^2x^{12}} - \frac{A(a+bx^2)^6}{14ax^{14}}$$

```
[Out] -(A*(a + b*x^2)^6)/(14*a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(84*a^2*x^12
)
```

Rubi [A] time = 0.0296584, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 78, 37}

$$\frac{(a+bx^2)^6 (Ab-7aB)}{84a^2x^{12}} - \frac{A(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(14*a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(84*a^2*x^12
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{15}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx,x,x^2\right)$$
$$=-\frac{A\left(a+bx^2\right)^6}{14ax^{14}}+\frac{(-Ab+7aB)\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^2\right)}{14a}$$
$$=-\frac{A\left(a+bx^2\right)^6}{14ax^{14}}+\frac{(Ab-7aB)\left(a+bx^2\right)^6}{84a^2x^{12}}$$

Mathematica [B] time = 0.0318664, size = 118, normalized size = 2.46

$$-\frac{35a^2b^3x^6\left(3A+4Bx^2\right)+21a^3b^2x^4\left(4A+5Bx^2\right)+7a^4bx^2\left(5A+6Bx^2\right)+a^5\left(6A+7Bx^2\right)+35ab^4x^8\left(2A+3Bx^2\right)+21b}{84x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -(21*b^5*x^10*(A + 2*B*x^2) + 35*a*b^4*x^8*(2*A + 3*B*x^2) + 35*a^2*b^3*x^6
*(3*A + 4*B*x^2) + 21*a^3*b^2*x^4*(4*A + 5*B*x^2) + 7*a^4*b*x^2*(5*A + 6*B*
x^2) + a^5*(6*A + 7*B*x^2))/(84*x^14)
```

Maple [B] time = 0.006, size = 104, normalized size = 2.2

$$-\frac{a^3b\,(2\,Ab+Ba)}{2\,x^{10}}-\frac{a^4\,(5\,Ab+Ba)}{12\,x^{12}}-\frac{5\,b^2a^2\,(Ab+Ba)}{4\,x^8}-\frac{b^4\,(Ab+5\,Ba)}{4\,x^4}-\frac{Bb^5}{2\,x^2}-\frac{5\,ab^3\,(Ab+2\,Ba)}{6\,x^6}-\frac{Aa^5}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^15,x)
```

```
[Out] -1/2*a^3*b*(2*A*b+B*a)/x^10-1/12*a^4*(5*A*b+B*a)/x^12-5/4*b^2*a^2*(A*b+B*a)
/x^8-1/4*b^4*(A*b+5*B*a)/x^4-1/2*B*b^5/x^2-5/6*a*b^3*(A*b+2*B*a)/x^6-1/14*A
*a^5/x^14
```

Maxima [B] time = 1.02963, size = 163, normalized size = 3.4

$$-\frac{42\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+70\left(2\,Ba^2b^3+Aab^4\right)x^8+105\left(Ba^3b^2+Aa^2b^3\right)x^6+6\,Aa^5+42\left(Ba^4b+2\,Aa^3b^2\right)x^4}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Fricas [B] time = 1.40334, size = 265, normalized size = 5.52

$$-\frac{42\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+70\left(2\,Ba^2b^3+Aab^4\right)x^8+105\left(Ba^3b^2+Aa^2b^3\right)x^6+6\,Aa^5+42\left(Ba^4b+2\,Aa^3b^2\right.}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Sympy [B] time = 25.9282, size = 128, normalized size = 2.67

$$-\frac{6Aa^5+42Bb^5x^{12}+x^{10}\left(21Ab^5+105Bab^4\right)+x^8\left(70Aab^4+140Ba^2b^3\right)+x^6\left(105Aa^2b^3+105Ba^3b^2\right)+x^4\left(84Aa^3b^2\right.}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

```
[Out] -(6*A*a**5 + 42*B*b**5*x**12 + x**10*(21*A*b**5 + 105*B*a*b**4) + x**8*(70*
A*a*b**4 + 140*B*a**2*b**3) + x**6*(105*A*a**2*b**3 + 105*B*a**3*b**2) + x*
*4*(84*A*a**3*b**2 + 42*B*a**4*b) + x**2*(35*A*a**4*b + 7*B*a**5))/(84*x**1
4)
```

Giac [B] time = 1.15746, size = 171, normalized size = 3.56

$$-\frac{42\,Bb^5x^{12}+105\,Bab^4x^{10}+21\,Ab^5x^{10}+140\,Ba^2b^3x^8+70\,Aab^4x^8+105\,Ba^3b^2x^6+105\,Aa^2b^3x^6+42\,Ba^4bx^4+84\,A}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 105*B*a*b^4*x^10 + 21*A*b^5*x^10 + 140*B*a^2*b^3*x^8
 + 70*A*a*b^4*x^8 + 105*B*a^3*b^2*x^6 + 105*A*a^2*b^3*x^6 + 42*B*a^4*b*x^4
+ 84*A*a^3*b^2*x^4 + 7*B*a^5*x^2 + 35*A*a^4*b*x^2 + 6*A*a^5)/x^14
```

3.48 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx$

Optimal. Leaf size=117

$$-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{3x^3}$$

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/(7
*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(3*x^3)
```

Rubi [A] time = 0.058662, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/(7
*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(3*x^3)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx = \int \left(\frac{a^5A}{x^{16}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^{10}}+\frac{5ab^3(Ab+2aB)}{x^8}+\frac{b^4(\right.$$

$$=-\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{10a^2b^2(Ab+aB)}{9x^9}-\frac{5ab^3(Ab+2aB)}{7x^7}-\frac{b^4(A}{}$$

Mathematica [A] time = 0.0335061, size = 121, normalized size = 1.03

$$-\frac{7150a^2b^3x^6\left(7A+9Bx^2\right)+4550a^3b^2x^4\left(9A+11Bx^2\right)+1575a^4bx^2\left(11A+13Bx^2\right)+231a^5\left(13A+15Bx^2\right)+6435ab^4x^8}{45045x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -(3003*b^5*x^10*(3*A + 5*B*x^2) + 6435*a*b^4*x^8*(5*A + 7*B*x^2) + 7150*a^2
*b^3*x^6*(7*A + 9*B*x^2) + 4550*a^3*b^2*x^4*(9*A + 11*B*x^2) + 1575*a^4*b*x
```

```
^2*(11*A + 13*B*x^2) + 231*a^5*(13*A + 15*B*x^2))/(45045*x^15)
```

Maple [A] time = 0.005, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{15\,x^{15}}-\frac{a^4\,(5\,Ab+Ba)}{13\,x^{13}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{11\,x^{11}}-\frac{10\,b^2a^2\,(Ab+Ba)}{9\,x^9}-\frac{5\,ab^3\,(Ab+2\,Ba)}{7\,x^7}-\frac{b^4\,(Ab+5\,Ba)}{5\,x^5}-\frac{Bb^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^16,x)
```

```
[Out] -1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/9
*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-1/
3*b^5*B/x^3
```

Maxima [A] time = 1.03946, size = 163, normalized size = 1.39

$$-\frac{15015\,Bb^5x^{12}+9009\left(5\,Bab^4+Ab^5\right)x^{10}+32175\left(2\,Ba^2b^3+Aab^4\right)x^8+50050\left(Ba^3b^2+Aa^2b^3\right)x^6+3003\,Aa^5+204}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

Fricas [A] time = 1.49205, size = 294, normalized size = 2.51

$$-\frac{15015\,Bb^5x^{12}+9009\left(5\,Bab^4+Ab^5\right)x^{10}+32175\left(2\,Ba^2b^3+Aab^4\right)x^8+50050\left(Ba^3b^2+Aa^2b^3\right)x^6+3003\,Aa^5+204}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

Sympy [A] time = 37.3983, size = 128, normalized size = 1.09

$$-\frac{3003Aa^5+15015Bb^5x^{12}+x^{10}\left(9009Ab^5+45045Bab^4\right)+x^8\left(32175Aab^4+64350Ba^2b^3\right)+x^6\left(50050Aa^2b^3+5005\right.}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

```
[Out] -(3003*A*a**5 + 15015*B*b**5*x**12 + x**10*(9009*A*b**5 + 45045*B*a*b**4) +
 x**8*(32175*A*a*b**4 + 64350*B*a**2*b**3) + x**6*(50050*A*a**2*b**3 + 5005
0*B*a**3*b**2) + x**4*(40950*A*a**3*b**2 + 20475*B*a**4*b) + x**2*(17325*A*
a**4*b + 3465*B*a**5))/(45045*x**15)
```

Giac [A] time = 1.13318, size = 171, normalized size = 1.46

$$-\frac{15015\,Bb^5x^{12} + 45045\,Bab^4x^{10} + 9009\,Ab^5x^{10} + 64350\,Ba^2b^3x^8 + 32175\,Aab^4x^8 + 50050\,Ba^3b^2x^6 + 50050\,Aa^2b^3x^6 + \cdots}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 45045*B*a*b^4*x^10 + 9009*A*b^5*x^10 + 64350*B
*a^2*b^3*x^8 + 32175*A*a*b^4*x^8 + 50050*B*a^3*b^2*x^6 + 50050*A*a^2*b^3*x^
6 + 20475*B*a^4*b*x^4 + 40950*A*a^3*b^2*x^4 + 3465*B*a^5*x^2 + 17325*A*a^4*
b*x^2 + 3003*A*a^5)/x^15
```

3.49 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}} \, dx$

Optimal. Leaf size=76

$$-\frac{b(a+bx^2)^6 (Ab-4aB)}{336a^3x^{12}}+\frac{(a+bx^2)^6 (Ab-4aB)}{56a^2x^{14}}-\frac{A(a+bx^2)^6}{16ax^{16}}$$

```
[Out] -(A*(a + b*x^2)^6)/(16*a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(56*a^2*x^14
) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rubi [A] time = 0.0513359, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 78, 45, 37}

$$-\frac{b(a+bx^2)^6 (Ab-4aB)}{336a^3x^{12}}+\frac{(a+bx^2)^6 (Ab-4aB)}{56a^2x^{14}}-\frac{A(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(16*a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(56*a^2*x^14
) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx,x,x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}} + \frac{(-2Ab+8aB)\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx,x,x^2\right)}{16a}$$
$$= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}} + \frac{(Ab-4aB)\left(a+bx^2\right)^6}{56a^2x^{14}} + \frac{(b(Ab-4aB))\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^2\right)}{56a^2}$$
$$= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}} + \frac{(Ab-4aB)\left(a+bx^2\right)^6}{56a^2x^{14}} - \frac{b(Ab-4aB)\left(a+bx^2\right)^6}{336a^3x^{12}}$$

Mathematica [A] time = 0.029576, size = 121, normalized size = 1.59

$$-\frac{84a^2b^3x^6\left(4A+5Bx^2\right)+56a^3b^2x^4\left(5A+6Bx^2\right)+20a^4bx^2\left(6A+7Bx^2\right)+3a^5\left(7A+8Bx^2\right)+70ab^4x^8\left(3A+4Bx^2\right)+2}{336x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -(28*b^5*x^10*(2*A + 3*B*x^2) + 70*a*b^4*x^8*(3*A + 4*B*x^2) + 84*a^2*b^3*x
^6*(4*A + 5*B*x^2) + 56*a^3*b^2*x^4*(5*A + 6*B*x^2) + 20*a^4*b*x^2*(6*A + 7
*B*x^2) + 3*a^5*(7*A + 8*B*x^2))/(336*x^16)
```

Maple [A] time = 0.005, size = 104, normalized size = 1.4

$$-\frac{5\,a^3b\,(2\,Ab+Ba)}{12\,x^{12}} - \frac{b^2a^2\,(Ab+Ba)}{x^{10}} - \frac{Aa^5}{16\,x^{16}} - \frac{a^4\,(5\,Ab+Ba)}{14\,x^{14}} - \frac{Bb^5}{4\,x^4} - \frac{b^4\,(Ab+5\,Ba)}{6\,x^6} - \frac{5\,ab^3\,(Ab+2\,Ba)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^17,x)
```

```
[Out] -5/12*a^3*b*(2*A*b+B*a)/x^12-b^2*a^2*(A*b+B*a)/x^10-1/16*A*a^5/x^16-1/14*a^
4*(5*A*b+B*a)/x^14-1/4*B*b^5/x^4-1/6*b^4*(A*b+5*B*a)/x^6-5/8*a*b^3*(A*b+2*B
*a)/x^8
```

Maxima [A] time = 0.997342, size = 163, normalized size = 2.14

$$-\frac{84\,Bb^5x^{12}+56\left(5\,Bab^4+Ab^5\right)x^{10}+210\left(2\,Ba^2b^3+Aab^4\right)x^8+336\left(Ba^3b^2+Aa^2b^3\right)x^6+21\,Aa^5+140\left(Ba^4b+2\,Aa^3b^2\right.}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

Fricas [A] time = 1.42996, size = 271, normalized size = 3.57

$$-\frac{84\,Bb^{5}x^{12}+56\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+210\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+336\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+21\,Aa^{5}+140\left(Ba^{4}b+2\,A\cdots\right.}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

Sympy [A] time = 59.3195, size = 128, normalized size = 1.68

$$-\frac{21Aa^{5}+84Bb^{5}x^{12}+x^{10}\left(56Ab^{5}+280Bab^{4}\right)+x^{8}\left(210Aab^{4}+420Ba^{2}b^{3}\right)+x^{6}\left(336Aa^{2}b^{3}+336Ba^{3}b^{2}\right)+x^{4}\left(280A\cdots\right.}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

```
[Out] -(21*A*a**5 + 84*B*b**5*x**12 + x**10*(56*A*b**5 + 280*B*a*b**4) + x**8*(21
0*A*a*b**4 + 420*B*a**2*b**3) + x**6*(336*A*a**2*b**3 + 336*B*a**3*b**2) +
x**4*(280*A*a**3*b**2 + 140*B*a**4*b) + x**2*(120*A*a**4*b + 24*B*a**5))/(3
36*x**16)
```

Giac [A] time = 1.14475, size = 171, normalized size = 2.25

$$-\frac{84\,Bb^{5}x^{12}+280\,Bab^{4}x^{10}+56\,Ab^{5}x^{10}+420\,Ba^{2}b^{3}x^{8}+210\,Aab^{4}x^{8}+336\,Ba^{3}b^{2}x^{6}+336\,Aa^{2}b^{3}x^{6}+140\,Ba^{4}bx^{4}+2\cdots}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 280*B*a*b^4*x^10 + 56*A*b^5*x^10 + 420*B*a^2*b^3*x^
8 + 210*A*a*b^4*x^8 + 336*B*a^3*b^2*x^6 + 336*A*a^2*b^3*x^6 + 140*B*a^4*b*x
^4 + 280*A*a^3*b^2*x^4 + 24*B*a^5*x^2 + 120*A*a^4*b*x^2 + 21*A*a^5)/x^16
```

3.50 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}} dx$

Optimal. Leaf size=117

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{17x^{17}}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}$$

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Rubi [A] time = 0.0593107, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{17x^{17}}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}} dx = \int \left(\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{16}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab+2aB)}{x^{10}}+\frac{b^4(\right.$$
$$= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{9x^9}-\frac{b^4(A}{}$$

Mathematica [A] time = 0.0470096, size = 117, normalized size = 1.

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{17x^{17}}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
```

```
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Maple [A] time = 0.005, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{17\,x^{17}}-\frac{a^4\,(5\,Ab+Ba)}{15\,x^{15}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{13\,x^{13}}-\frac{10\,b^2a^2\,(Ab+Ba)}{11\,x^{11}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{9\,x^9}-\frac{b^4\,(Ab+5\,Ba)}{7\,x^7}-\frac{Bb^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^7-
1/5*b^5*B/x^5
```

Maxima [A] time = 1.02527, size = 163, normalized size = 1.39

$$-\frac{153153\,Bb^5x^{12}+109395\left(5\,Bab^4+Ab^5\right)x^{10}+425425\left(2\,Ba^2b^3+Aab^4\right)x^8+696150\left(Ba^3b^2+Aa^2b^3\right)x^6+45045\,Aa}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="maxima")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Fricas [A] time = 1.43609, size = 306, normalized size = 2.62

$$-\frac{153153\,Bb^5x^{12}+109395\left(5\,Bab^4+Ab^5\right)x^{10}+425425\left(2\,Ba^2b^3+Aab^4\right)x^8+696150\left(Ba^3b^2+Aa^2b^3\right)x^6+45045\,Aa}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="fricas")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Sympy [A] time = 86.1741, size = 128, normalized size = 1.09

$$-\frac{45045Aa^5+153153Bb^5x^{12}+x^{10}\left(109395Ab^5+546975Bab^4\right)+x^8\left(425425Aab^4+850850Ba^2b^3\right)+x^6\left(696150Aa^2\right.}{765765x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

```
[Out] -(45045*A*a**5 + 153153*B*b**5*x**12 + x**10*(109395*A*b**5 + 546975*B*a*b*
*4) + x**8*(425425*A*a*b**4 + 850850*B*a**2*b**3) + x**6*(696150*A*a**2*b**
3 + 696150*B*a**3*b**2) + x**4*(589050*A*a**3*b**2 + 294525*B*a**4*b) + x**
2*(255255*A*a**4*b + 51051*B*a**5))/(765765*x**17)
```

Giac [A] time = 1.1154, size = 171, normalized size = 1.46

$$-\frac{153153\,Bb^5x^{12} + 546975\,Bab^4x^{10} + 109395\,Ab^5x^{10} + 850850\,Ba^2b^3x^8 + 425425\,Aab^4x^8 + 696150\,Ba^3b^2x^6 + 696150\,A}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="giac")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 546975*B*a*b^4*x^10 + 109395*A*b^5*x^10 + 85
0850*B*a^2*b^3*x^8 + 425425*A*a*b^4*x^8 + 696150*B*a^3*b^2*x^6 + 696150*A*a
^2*b^3*x^6 + 294525*B*a^4*b*x^4 + 589050*A*a^3*b^2*x^4 + 51051*B*a^5*x^2 +
255255*A*a^4*b*x^2 + 45045*A*a^5)/x^17
```

3.51 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} dx$

Optimal. Leaf size=117

$$-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5A}{18x^{18}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{b^5B}{6x^6}$$

```
[Out] -(a^5*A)/(18*x^18) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3*(A*b + 2*a*B))/(2*x
^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/(6*x^6)
```

Rubi [A] time = 0.0838325, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5A}{18x^{18}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{b^5B}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -(a^5*A)/(18*x^18) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3*(A*b + 2*a*B))/(2*x
^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/(6*x^6)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{10}} dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5A}{x^{10}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^7}+\frac{5ab^3(Ab}{x^6}\right.\right. \\ &= -\frac{a^5A}{18x^{18}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{5a^2b^2(Ab+aB)}{6x^{12}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(}{}\end{aligned}$$

Mathematica [A] time = 0.0293235, size = 121, normalized size = 1.03

$$-\frac{168a^2b^3x^6(5A+6Bx^2)+120a^3b^2x^4(6A+7Bx^2)+45a^4bx^2(7A+8Bx^2)+7a^5(8A+9Bx^2)+126ab^4x^8(4A+5B}{1008x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -(42*b^5*x^10*(3*A + 4*B*x^2) + 126*a*b^4*x^8*(4*A + 5*B*x^2) + 168*a^2*b^3
*x^6*(5*A + 6*B*x^2) + 120*a^3*b^2*x^4*(6*A + 7*B*x^2) + 45*a^4*b*x^2*(7*A
+ 8*B*x^2) + 7*a^5*(8*A + 9*B*x^2))/(1008*x^18)
```

Maple [A] time = 0.006, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{18\,x^{18}}-\frac{a^4\,(5\,Ab+Ba)}{16\,x^{16}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{14\,x^{14}}-\frac{5\,b^2a^2\,(Ab+Ba)}{6\,x^{12}}-\frac{ab^3\,(Ab+2\,Ba)}{2\,x^{10}}-\frac{b^4\,(Ab+5\,Ba)}{8\,x^8}-\frac{Bb^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^19,x)
```

```
[Out] -1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6*
a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8-1
/6*b^5*B/x^6
```

Maxima [A] time = 0.994185, size = 163, normalized size = 1.39

$$-\frac{168\,Bb^5x^{12}+126\,\left(5\,Bab^4+Ab^5\right)x^{10}+504\,\left(2\,Ba^2b^3+Aab^4\right)x^8+840\,\left(Ba^3b^2+Aa^2b^3\right)x^6+56\,Aa^5+360\,\left(Ba^4b+2\,Aa\right.}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

Fricas [A] time = 1.45371, size = 275, normalized size = 2.35

$$-\frac{168\,Bb^5x^{12}+126\,\left(5\,Bab^4+Ab^5\right)x^{10}+504\,\left(2\,Ba^2b^3+Aab^4\right)x^8+840\,\left(Ba^3b^2+Aa^2b^3\right)x^6+56\,Aa^5+360\,\left(Ba^4b+2\,Aa\right.}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

Sympy [A] time = 111.735, size = 128, normalized size = 1.09

$$-\frac{56Aa^5+168Bb^5x^{12}+x^{10}\left(126Ab^5+630Bab^4\right)+x^8\left(504Aab^4+1008Ba^2b^3\right)+x^6\left(840Aa^2b^3+840Ba^3b^2\right)+x^4\left(720A\right.}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

```
[Out] -(56*A*a**5 + 168*B*b**5*x**12 + x**10*(126*A*b**5 + 630*B*a*b**4) + x**8*(
504*A*a*b**4 + 1008*B*a**2*b**3) + x**6*(840*A*a**2*b**3 + 840*B*a**3*b**2)
 + x**4*(720*A*a**3*b**2 + 360*B*a**4*b) + x**2*(315*A*a**4*b + 63*B*a**5))
/(1008*x**18)
```

Giac [A] time = 1.12449, size = 171, normalized size = 1.46

$$-\frac{168\,Bb^5x^{12}+630\,Bab^4x^{10}+126\,Ab^5x^{10}+1008\,Ba^2b^3x^8+504\,Aab^4x^8+840\,Ba^3b^2x^6+840\,Aa^2b^3x^6+360\,Ba^4bx^4+}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 630*B*a*b^4*x^10 + 126*A*b^5*x^10 + 1008*B*a^2*b^
3*x^8 + 504*A*a*b^4*x^8 + 840*B*a^3*b^2*x^6 + 840*A*a^2*b^3*x^6 + 360*B*a^4
*b*x^4 + 720*A*a^3*b^2*x^4 + 63*B*a^5*x^2 + 315*A*a^4*b*x^2 + 56*A*a^5)/x^1
8
```

3.52 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}} \, dx$

Optimal. Leaf size=117

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{b^5B}{7x^7}$$

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Rubi [A] time = 0.0586581, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}} \, dx &= \int \left(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{16}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab+2aB)}{x^{12}}+\frac{b^4(\right.\\ &= -\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{a^3b(2Ab+aB)}{3x^{15}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{5ab^3(Ab+2aB)}{11x^{11}}-\frac{b^4(Ab}{9}\end{aligned}$$

Mathematica [A] time = 0.0425499, size = 117, normalized size = 1.

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
```

```
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Maple [A] time = 0.005, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{19\,x^{19}}-\frac{a^4\,(5\,Ab+Ba)}{17\,x^{17}}-\frac{a^3b\,(2\,Ab+Ba)}{3\,x^{15}}-\frac{10\,b^2a^2\,(Ab+Ba)}{13\,x^{13}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{11\,x^{11}}-\frac{b^4\,(Ab+5\,Ba)}{9\,x^9}-\frac{Bb^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/13
*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x^9
-1/7*b^5*B/x^7
```

Maxima [A] time = 1.00315, size = 163, normalized size = 1.39

$$-\frac{415701\,Bb^5x^{12}+323323\left(5\,Bab^4+Ab^5\right)x^{10}+1322685\left(2\,Ba^2b^3+Aab^4\right)x^8+2238390\left(Ba^3b^2+Aa^2b^3\right)x^6+153153}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

Fricas [A] time = 1.38052, size = 313, normalized size = 2.68

$$-\frac{415701\,Bb^5x^{12}+323323\left(5\,Bab^4+Ab^5\right)x^{10}+1322685\left(2\,Ba^2b^3+Aab^4\right)x^8+2238390\left(Ba^3b^2+Aa^2b^3\right)x^6+153153}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

Sympy [A] time = 176.26, size = 128, normalized size = 1.09

$$-\frac{153153Aa^5+415701Bb^5x^{12}+x^{10}\left(323323Ab^5+1616615Bab^4\right)+x^8\left(1322685Aab^4+2645370Ba^2b^3\right)+x^6\left(223839\right.}{2909907x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

```
[Out] -(153153*A*a**5 + 415701*B*b**5*x**12 + x**10*(323323*A*b**5 + 1616615*B*a*
b**4) + x**8*(1322685*A*a*b**4 + 2645370*B*a**2*b**3) + x**6*(2238390*A*a**
2*b**3 + 2238390*B*a**3*b**2) + x**4*(1939938*A*a**3*b**2 + 969969*B*a**4*b
) + x**2*(855855*A*a**4*b + 171171*B*a**5))/(2909907*x**19)
```

Giac [A] time = 1.18392, size = 171, normalized size = 1.46

$$-\frac{415701\,Bb^5x^{12} + 1616615\,Bab^4x^{10} + 323323\,Ab^5x^{10} + 2645370\,Ba^2b^3x^8 + 1322685\,Aab^4x^8 + 2238390\,Ba^3b^2x^6 + 2238}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 1616615*B*a*b^4*x^10 + 323323*A*b^5*x^10 +
2645370*B*a^2*b^3*x^8 + 1322685*A*a*b^4*x^8 + 2238390*B*a^3*b^2*x^6 + 22383
90*A*a^2*b^3*x^6 + 969969*B*a^4*b*x^4 + 1939938*A*a^3*b^2*x^4 + 171171*B*a^
5*x^2 + 855855*A*a^4*b*x^2 + 153153*A*a^5)/x^19
```

3.53 $\int \frac{\left(a+bx^2\right)^5 \left(A+Bx^2\right)}{x^{21}} \, dx$

Optimal. Leaf size=117

$$-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{b^5B}{8x^8}$$

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b^3*(A*b + 2*a*B))/(1
2*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^5*B)/(8*x^8)
```

Rubi [A] time = 0.0825582, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{b^5B}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b^3*(A*b + 2*a*B))/(1
2*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^5*B)/(8*x^8)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5 \left(A+Bx^2\right)}{x^{21}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{11}} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^5A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^{10}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab}{x^7}\right.\right. \\
&= -\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+aB)}{18x^{18}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{5a^2b^2(Ab+aB)}{7x^{14}}-\frac{5ab^3(Ab+2aB)}{12x^{12}}-\frac{b^4}{}
\end{aligned}$$

Mathematica [A] time = 0.0312151, size = 121, normalized size = 1.03

$$-\frac{600a^2b^3x^6\left(6A+7Bx^2\right)+450a^3b^2x^4\left(7A+8Bx^2\right)+175a^4bx^2\left(8A+9Bx^2\right)+28a^5\left(9A+10Bx^2\right)+420ab^4x^8\left(5A+\right.}{5040x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -(126*b^5*x^10*(4*A + 5*B*x^2) + 420*a*b^4*x^8*(5*A + 6*B*x^2) + 600*a^2*b^
3*x^6*(6*A + 7*B*x^2) + 450*a^3*b^2*x^4*(7*A + 8*B*x^2) + 175*a^4*b*x^2*(8*
A + 9*B*x^2) + 28*a^5*(9*A + 10*B*x^2))/(5040*x^20)
```

Maple [A] time = 0.007, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{20\,x^{20}}-\frac{a^4\,(5\,Ab+Ba)}{18\,x^{18}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{16\,x^{16}}-\frac{5\,b^2a^2\,(Ab+Ba)}{7\,x^{14}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{12\,x^{12}}-\frac{b^4\,(Ab+5\,Ba)}{10\,x^{10}}-\frac{Bb^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7*
a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x^1
0-1/8*b^5*B/x^8
```

Maxima [A] time = 1.02775, size = 163, normalized size = 1.39

$$-\frac{630\,Bb^5x^{12}+504\left(5\,Bab^4+Ab^5\right)x^{10}+2100\left(2\,Ba^2b^3+Aab^4\right)x^8+3600\left(Ba^3b^2+Aa^2b^3\right)x^6+252\,Aa^5+1575\left(Ba^4b+\right.}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

Fricas [A] time = 1.39844, size = 282, normalized size = 2.41

$$-\frac{630\,Bb^5x^{12}+504\left(5\,Bab^4+Ab^5\right)x^{10}+2100\left(2\,Ba^2b^3+Aab^4\right)x^8+3600\left(Ba^3b^2+Aa^2b^3\right)x^6+252\,Aa^5+1575\left(Ba^4b+\right.}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11512, size = 171, normalized size = 1.46

$$-\frac{630 Bb^5x^{12} + 2520 Bab^4x^{10} + 504 Ab^5x^{10} + 4200 Ba^2b^3x^8 + 2100 Aab^4x^8 + 3600 Ba^3b^2x^6 + 3600 Aa^2b^3x^6 + 1575 B}{5040 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 2520*B*a*b^4*x^10 + 504*A*b^5*x^10 + 4200*B*a^2*b
^3*x^8 + 2100*A*a*b^4*x^8 + 3600*B*a^3*b^2*x^6 + 3600*A*a^2*b^3*x^6 + 1575*
B*a^4*b*x^4 + 3150*A*a^3*b^2*x^4 + 280*B*a^5*x^2 + 1400*A*a^4*b*x^2 + 252*A
*a^5)/x^20
```

3.54 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx$

Optimal. Leaf size=117

$$-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{a^5A}{21x^{21}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{b^5B}{9x^9}$$

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Rubi [A] time = 0.0620889, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{a^5A}{21x^{21}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx &= \int \left(\frac{a^5A}{x^{22}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{18}}+\frac{10a^2b^2(Ab+aB)}{x^{16}}+\frac{5ab^3(Ab+2aB)}{x^{14}}+\frac{b^4(\right.\\ &= -\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}}-\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB)}{13x^{13}}-\frac{b^4(Ab}{11}\end{aligned}$$

Mathematica [A] time = 0.0422499, size = 117, normalized size = 1.

$$-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{a^5A}{21x^{21}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
```

```
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Maple [A] time = 0.007, size = 104, normalized size = 0.9

$$-\frac{Aa^{5}}{21\,x^{21}}-\frac{a^{4}\left(5\,Ab+Ba\right)}{19\,x^{19}}-\frac{5\,a^{3}b\left(2\,Ab+Ba\right)}{17\,x^{17}}-\frac{2\,b^{2}a^{2}\left(Ab+Ba\right)}{3\,x^{15}}-\frac{5\,ab^{3}\left(Ab+2\,Ba\right)}{13\,x^{13}}-\frac{b^{4}\left(Ab+5\,Ba\right)}{11\,x^{11}}-\frac{Bb^{5}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^22,x)
```

```
[Out] -1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3*
a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x^1
1-1/9*b^5*B/x^9
```

Maxima [A] time = 1.02141, size = 163, normalized size = 1.39

$$-\frac{323323\,Bb^{5}x^{12}+264537\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+1119195\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+1939938\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+138567}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

Fricas [A] time = 1.36854, size = 313, normalized size = 2.68

$$-\frac{323323\,Bb^{5}x^{12}+264537\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+1119195\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+1939938\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+138567}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

[Out] Timed out

Giac [A] time = 1.11926, size = 171, normalized size = 1.46

$$-\frac{323323\,Bb^5x^{12}+1322685\,Bab^4x^{10}+264537\,Ab^5x^{10}+2238390\,Ba^2b^3x^8+1119195\,Aab^4x^8+1939938\,Ba^3b^2x^6+1939\cdots}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 1322685*B*a*b^4*x^10 + 264537*A*b^5*x^10 +
2238390*B*a^2*b^3*x^8 + 1119195*A*a*b^4*x^8 + 1939938*B*a^3*b^2*x^6 + 19399
38*A*a^2*b^3*x^6 + 855855*B*a^4*b*x^4 + 1711710*A*a^3*b^2*x^4 + 153153*B*a^
5*x^2 + 765765*A*a^4*b*x^2 + 138567*A*a^5)/x^21
```

3.55 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx$

Optimal. Leaf size=117

$$-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{a^5A}{22x^{22}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{b^5B}{10x^{10}}$$

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*A*b + a*B)
)/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b^3*(A*b + 2*a*B))/(1
4*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^5*B)/(10*x^10)
```

Rubi [A] time = 0.0848973, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{a^5A}{22x^{22}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{b^5B}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*A*b + a*B)
)/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b^3*(A*b + 2*a*B))/(1
4*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^5*B)/(10*x^10)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^{10}}+\frac{10a^2b^2(Ab+aB)}{x^9}+\frac{5ab^3(Ab}{x^8}\right.\right.\\ &= -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{20x^{20}}-\frac{5a^3b(2Ab+aB)}{18x^{18}}-\frac{5a^2b^2(Ab+aB)}{8x^{16}}-\frac{5ab^3(Ab+2aB)}{14x^{14}}-\frac{b^4}{}\end{aligned}$$

Mathematica [A] time = 0.030633, size = 121, normalized size = 1.03

$$-\frac{2475a^2b^3x^6\left(7A+8Bx^2\right)+1925a^3b^2x^4\left(8A+9Bx^2\right)+770a^4bx^2\left(9A+10Bx^2\right)+126a^5\left(10A+11Bx^2\right)+1650ab^4x^8}{27720x^{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -(462*b^5*x^10*(5*A + 6*B*x^2) + 1650*a*b^4*x^8*(6*A + 7*B*x^2) + 2475*a^2*
b^3*x^6*(7*A + 8*B*x^2) + 1925*a^3*b^2*x^4*(8*A + 9*B*x^2) + 770*a^4*b*x^2*
(9*A + 10*B*x^2) + 126*a^5*(10*A + 11*B*x^2))/(27720*x^22)
```

Maple [A] time = 0.007, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{22\,x^{22}}-\frac{a^4\,(5\,Ab+Ba)}{20\,x^{20}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{18\,x^{18}}-\frac{5\,b^2a^2\,(Ab+Ba)}{8\,x^{16}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{14\,x^{14}}-\frac{b^4\,(Ab+5\,Ba)}{12\,x^{12}}-\frac{Bb^5}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^23,x)
```

```
[Out] -1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8*
a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x^1
2-1/10*b^5*B/x^10
```

Maxima [A] time = 0.983949, size = 163, normalized size = 1.39

$$-\frac{2772\,Bb^5x^{12}+2310\left(5\,Bab^4+Ab^5\right)x^{10}+9900\left(2\,Ba^2b^3+Aab^4\right)x^8+17325\left(Ba^3b^2+Aa^2b^3\right)x^6+1260\,Aa^5+7700\left(Ba\right.}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

Fricas [A] time = 1.17341, size = 290, normalized size = 2.48

$$-\frac{2772\,Bb^5x^{12}+2310\left(5\,Bab^4+Ab^5\right)x^{10}+9900\left(2\,Ba^2b^3+Aab^4\right)x^8+17325\left(Ba^3b^2+Aa^2b^3\right)x^6+1260\,Aa^5+7700\left(Ba\right.}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14536, size = 171, normalized size = 1.46

$$-\frac{2772\,Bb^5x^{12} + 11550\,Bab^4x^{10} + 2310\,Ab^5x^{10} + 19800\,Ba^2b^3x^8 + 9900\,Aab^4x^8 + 17325\,Ba^3b^2x^6 + 17325\,Aa^2b^3x^6 +}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 11550*B*a*b^4*x^10 + 2310*A*b^5*x^10 + 19800*B*
a^2*b^3*x^8 + 9900*A*a*b^4*x^8 + 17325*B*a^3*b^2*x^6 + 17325*A*a^2*b^3*x^6
+ 7700*B*a^4*b*x^4 + 15400*A*a^3*b^2*x^4 + 1386*B*a^5*x^2 + 6930*A*a^4*b*x^
2 + 1260*A*a^5)/x^22
```

3.56 $\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$\frac{a^2x(Ab-aB)}{b^4}-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x^5(Ab-aB)}{5b^2}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{Bx^7}{7b}$$

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a*B)*x^5)/(
5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(9/2)
```

Rubi [A] time = 0.0605939, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {459, 302, 205}

$$\frac{a^2x(Ab-aB)}{b^4}-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x^5(Ab-aB)}{5b^2}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a*B)*x^5)/(
5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(9/2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6 (A+Bx^2)}{a+bx^2} dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^2} dx}{7b}$$
$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3(a+bx^2)}\right) dx}{7b}$$
$$= \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} - \frac{\left(a^3(Ab-aB)\right)\int \frac{1}{a+bx^2} dx}{b^4}$$
$$= \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} - \frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

Mathematica [A] time = 0.0709316, size = 98, normalized size = 1.

$$-\frac{a^2x(aB-Ab)}{b^4} + \frac{a^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}} + \frac{x^5(Ab-aB)}{5b^2} + \frac{ax^3(aB-Ab)}{3b^3} + \frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(A*b) + a*B)*x)/b^4) + (a*(-(A*b) + a*B)*x^3)/(3*b^3) + ((A*b - a*
B)*x^5)/(5*b^2) + (B*x^7)/(7*b) + (a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.003, size = 116, normalized size = 1.2

$$\frac{Bx^7}{7b} + \frac{Ax^5}{5b} - \frac{Bx^5a}{5b^2} - \frac{aAx^3}{3b^2} + \frac{Bx^3a^2}{3b^3} + \frac{a^2Ax}{b^3} - \frac{Ba^3x}{b^4} - \frac{a^3A}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{Ba^4}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/7*B*x^7/b+1/5/b*A*x^5-1/5/b^2*B*x^5*a-1/3/b^2*A*x^3*a+1/3/b^3*B*x^3*a^2+1
/b^3*A*a^2*x-1/b^4*B*a^3*x-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+a^
4/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26916, size = 487, normalized size = 4.97

$$\left[\frac{30\,Bb^3x^7 - 42\left(Bab^2 - Ab^3\right)x^5 + 70\left(Ba^2b - Aab^2\right)x^3 - 105\left(Ba^3 - Aa^2b\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 210\left(Ba^3 - Aa^2b\right)x}{210\,b^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*b^3*x^7 - 42*(B*a*b^2 - A*b^3)*x^5 + 70*(B*a^2*b - A*a*b^2)*x^
3 - 105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*
x^2 + a)) - 210*(B*a^3 - A*a^2*b)*x)/b^4, 1/105*(15*B*b^3*x^7 - 21*(B*a*b^2
 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 + 105*(B*a^3 - A*a^2*b)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 105*(B*a^3 - A*a^2*b)*x)/b^4]
```

Sympy [A] time = 0.501098, size = 173, normalized size = 1.77

$$\frac{Bx^7}{7b} - \frac{\sqrt{-\frac{a^5}{b^9}}\left(-Ab+Ba\right)\log\left(-\frac{b^4\sqrt{-\frac{a^5}{b^9}}\left(-Ab+Ba\right)}{-Aa^2b+Ba^3} + x\right)}{2} + \frac{\sqrt{-\frac{a^5}{b^9}}\left(-Ab+Ba\right)\log\left(\frac{b^4\sqrt{-\frac{a^5}{b^9}}\left(-Ab+Ba\right)}{-Aa^2b+Ba^3} + x\right)}{2} - \frac{x^5\left(-Ab+Ba\right)}{5b^2} + \frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**7/(7*b) - sqrt(-a**5/b**9)*(-A*b + B*a)*log(-b**4*sqrt(-a**5/b**9)*(-A
*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 + sqrt(-a**5/b**9)*(-A*b + B*a)*log(b
**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 - x**5*(-A*b
+ B*a)/(5*b**2) + x**3*(-A*a*b + B*a**2)/(3*b**3) - x*(-A*a**2*b + B*a**3)/
b**4
```

Giac [A] time = 1.12601, size = 146, normalized size = 1.49

$$\frac{\left(Ba^4 - Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Bb^6x^7 - 21\,Bab^5x^5 + 21\,Ab^6x^5 + 35\,Ba^2b^4x^3 - 35\,Aab^5x^3 - 105\,Ba^3b^3x + 105\,Aa^2b^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^6*x
^7 - 21*B*a*b^5*x^5 + 21*A*b^6*x^5 + 35*B*a^2*b^4*x^3 - 35*A*a*b^5*x^3 - 10
5*B*a^3*b^3*x + 105*A*a^2*b^4*x)/b^7
```

3.57 $\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(Ab-aB)}{4b^2}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{Bx^6}{6b}$$

```
[Out] -(a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6)/(6*b) +
(a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0869095, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(Ab-aB)}{4b^2}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6)/(6*b) +
(a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^3}+\frac{(Ab-aB)x}{b^2}+\frac{Bx^2}{b}-\frac{a^2(-Ab+aB)}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{a(Ab-aB)x^2}{2b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^6}{6b}+\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0315902, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2B-3ab\left(2A+Bx^2\right)+b^2x^2\left(3A+2Bx^2\right)\right)+6a^2(Ab-aB)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*B - 3*a*b*(2*A + B*x^2) + b^2*x^2*(3*A + 2*B*x^2)) + 6*a^2*(A
*b - a*B)*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.003, size = 86, normalized size = 1.2

$$\frac{Bx^6}{6b}+\frac{Ax^4}{4b}-\frac{Bx^4a}{4b^2}-\frac{aAx^2}{2b^2}+\frac{Bx^2a^2}{2b^3}+\frac{a^2\ln\left(bx^2+a\right)A}{2b^3}-\frac{a^3\ln\left(bx^2+a\right)B}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/6*B*x^6/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/2/b^2*A*x^2*a+1/2/b^3*B*x^2*a^2+1
/2*a^2/b^3*ln(b*x^2+a)*A-1/2*a^3/b^4*ln(b*x^2+a)*B
```

Maxima [A] time = 0.993539, size = 100, normalized size = 1.33

$$\frac{2\,Bb^2x^6-3\left(Bab-Ab^2\right)x^4+6\left(Ba^2-Aab\right)x^2}{12\,b^3}-\frac{\left(Ba^3-Aa^2b\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*(B*a*b - A*b^2)*x^4 + 6*(B*a^2 - A*a*b)*x^2)/b^3 - 1/
2*(B*a^3 - A*a^2*b)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.1942, size = 155, normalized size = 2.07

$$\frac{2\,Bb^3x^6-3\left(Bab^2-Ab^3\right)x^4+6\left(Ba^2b-Aab^2\right)x^2-6\left(Ba^3-Aa^2b\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^3*x^6 - 3*(B*a*b^2 - A*b^3)*x^4 + 6*(B*a^2*b - A*a*b^2)*x^2 - 6
*(B*a^3 - A*a^2*b)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 0.438359, size = 65, normalized size = 0.87

$$\frac{Bx^6}{6b}-\frac{a^2\left(-Ab+Ba\right)\log\left(a+bx^2\right)}{2b^4}-\frac{x^4\left(-Ab+Ba\right)}{4b^2}+\frac{x^2\left(-Aab+Ba^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**6/(6*b) - a**2*(-A*b + B*a)*log(a + b*x**2)/(2*b**4) - x**4*(-A*b + B*
a)/(4*b**2) + x**2*(-A*a*b + B*a**2)/(2*b**3)
```

Giac [A] time = 1.13074, size = 104, normalized size = 1.39

$$\frac{2\,Bb^2x^6 - 3\,Babx^4 + 3\,Ab^2x^4 + 6\,Ba^2x^2 - 6\,Aabx^2}{12\,b^3} - \frac{\left(Ba^3 - Aa^2b\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*B*a*b*x^4 + 3*A*b^2*x^4 + 6*B*a^2*x^2 - 6*A*a*b*x^2)/
b^3 - 1/2*(B*a^3 - A*a^2*b)*log(abs(b*x^2 + a))/b^4
```

3.58 $\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(Ab-aB)}{3b^2}-\frac{ax(Ab-aB)}{b^3}+\frac{Bx^5}{5b}$$

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) + (a^(
3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0501443, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {459, 302, 205}

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(Ab-aB)}{3b^2}-\frac{ax(Ab-aB)}{b^3}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) + (a^(
3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4 (A+Bx^2)}{a+bx^2} dx &= \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^2} dx}{5b} \\
&= \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right) dx}{5b} \\
&= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^5}{5b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^2} dx}{b^3} \\
&= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^5}{5b} + \frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0484471, size = 77, normalized size = 1.

$$
-\frac{a^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{x^3(Ab-aB)}{3b^2} + \frac{ax(aB-Ab)}{b^3} + \frac{Bx^5}{5b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(A*b) + a*B)*x)/b^3 + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) - (a^(
3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A] time = 0.002, size = 92, normalized size = 1.2

$$
\frac{Bx^5}{5\,b} + \frac{Ax^3}{3\,b} - \frac{Bx^3a}{3\,b^2} - \frac{aAx}{b^2} + \frac{a^2Bx}{b^3} + \frac{a^2A}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{Ba^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/5*B*x^5/b+1/3/b*A*x^3-1/3/b^2*B*x^3*a-1/b^2*a*A*x+1/b^3*a^2*B*x+a^2/b^2/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26106, size = 381, normalized size = 4.95

$$\left[\frac{6\,Bb^2x^5 - 10\left(Bab - Ab^2\right)x^3 - 15\left(Ba^2 - Aab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 30\left(Ba^2 - Aab\right)x}{30\,b^3}, \frac{3\,Bb^2x^5 - 5\left(Bab - Ab^2\right)x^3 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*b^2*x^5 - 10*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(B*a^2 - A*a*b)*x)/b^
3, 1/15*(3*B*b^2*x^5 - 5*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) + 15*(B*a^2 - A*a*b)*x)/b^3]
```

Sympy [B] time = 0.478674, size = 150, normalized size = 1.95

$$\frac{Bx^5}{5b} + \frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab + Ba)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab + Ba)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{x^3\,(-Ab + Ba)}{3b^2} + \frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**5/(5*b) + sqrt(-a**3/b**7)*(-A*b + B*a)*log(-b**3*sqrt(-a**3/b**7)*(-A
*b + B*a)/(-A*a*b + B*a**2) + x)/2 - sqrt(-a**3/b**7)*(-A*b + B*a)*log(b**3
*sqrt(-a**3/b**7)*(-A*b + B*a)/(-A*a*b + B*a**2) + x)/2 - x**3*(-A*b + B*a)
/(3*b**2) + x*(-A*a*b + B*a**2)/b**3
```

Giac [A] time = 1.13787, size = 115, normalized size = 1.49

$$-\frac{\left(Ba^3 - Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Bb^4x^5 - 5\,Bab^3x^3 + 5\,Ab^4x^3 + 15\,Ba^2b^2x - 15\,Aab^3x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^4*x^
5 - 5*B*a*b^3*x^3 + 5*A*b^4*x^3 + 15*B*a^2*b^2*x - 15*A*a*b^3*x)/b^5
```

3.59 $\int \frac{x^3(A+Bx^2)}{a+bx^2} dx$

Optimal. Leaf size=54

$$\frac{x^2(Ab-aB)}{2b^2} - \frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3} + \frac{Bx^4}{4b}$$

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a + b*x^2])/
(2*b^3)
```

Rubi [A] time = 0.0555315, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{x^2(Ab-aB)}{2b^2} - \frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3} + \frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a + b*x^2])/
(2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{a+bx^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2} + \frac{Bx}{b} + \frac{a(-Ab+aB)}{b^2(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^4}{4b} - \frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0193973, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2aB+2Ab+bBx^2\right)+2a(aB-Ab)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*A*b - 2*a*B + b*B*x^2) + 2*a*(-(A*b) + a*B)*Log[a + b*x^2])/(4*b^
3)
```

Maple [A] time = 0.001, size = 62, normalized size = 1.2

$$\frac{Bx^4}{4b} + \frac{Ax^2}{2b} - \frac{Bx^2a}{2b^2} - \frac{a\ln\left(bx^2+a\right)A}{2b^2} + \frac{a^2\ln\left(bx^2+a\right)B}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a-1/2*a/b^2*ln(b*x^2+a)*A+1/2*a^2/b^3
*ln(b*x^2+a)*B
```

Maxima [A] time = 1.01723, size = 68, normalized size = 1.26

$$\frac{Bbx^4 - 2(Ba-Ab)x^2}{4b^2} + \frac{\left(Ba^2-Aab\right)\log\left(bx^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 2*(B*a - A*b)*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(b*x^2 + a)/
b^3
```

Fricas [A] time = 1.21181, size = 108, normalized size = 2.

$$\frac{Bb^2x^4 - 2\left(Bab-Ab^2\right)x^2 + 2\left(Ba^2-Aab\right)\log\left(bx^2+a\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^4 - 2*(B*a*b - A*b^2)*x^2 + 2*(B*a^2 - A*a*b)*log(b*x^2 + a))/
b^3
```

Sympy [A] time = 0.411963, size = 44, normalized size = 0.81

$$\frac{Bx^4}{4b} + \frac{a(-Ab+Ba)\log\left(a+bx^2\right)}{2b^3} - \frac{x^2(-Ab+Ba)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**4/(4*b) + a*(-A*b + B*a)*log(a + b*x**2)/(2*b**3) - x**2*(-A*b + B*a)/
(2*b**2)
```

Giac [A] time = 1.13305, size = 70, normalized size = 1.3

$$\frac{Bbx^4 - 2\,Bax^2 + 2\,Abx^2}{4\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*b*x^4 - 2*B*a*x^2 + 2*A*b*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(abs(b*x
^2 + a))/b^3
```

3.60 $\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{x(Ab-aB)}{b^2}-\frac{\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Bx^3}{3b}$$

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.035743, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {459, 321, 205}

$$\frac{x(Ab-aB)}{b^2}-\frac{\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{a+bx^2}\,dx=\frac{Bx^3}{3b}-\frac{(-3Ab+3aB)\int\frac{x^2}{a+bx^2}\,dx}{3b}$$
$$=\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{(a(Ab-aB))\int\frac{1}{a+bx^2}\,dx}{b^2}$$
$$=\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0460141, size = 57, normalized size = 0.98

$$\frac{x(Ab-aB)}{b^2}+\frac{\sqrt{a}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) + (Sqrt[a]*(-(A*b) + a*B)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A] time = 0.003, size = 68, normalized size = 1.2

$$\frac{Bx^3}{3\,b}+\frac{Ax}{b}-\frac{Bax}{b^2}-\frac{aA}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2B}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/b+1/b*A*x-1/b^2*B*a*x-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+a
^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23557, size = 277, normalized size = 4.78

$$\left[\frac{2\,Bbx^3-3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Ba-Ab)x}{6\,b^2},\frac{Bbx^3+3\,(Ba-Ab)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,(Ba-Ab)x}{3\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*b*x^3 - 3*(B*a - A*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) - 6*(B*a - A*b)*x)/b^2, 1/3*(B*b*x^3 + 3*(B*a - A*b)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) - 3*(B*a - A*b)*x)/b^2]
```

Sympy [A] time = 0.446227, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3b} - \frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2} - \frac{x\,(-Ab+Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**3/(3*b) - sqrt(-a/b**5)*(-A*b + B*a)*log(-b**2*sqrt(-a/b**5) + x)/2 +
sqrt(-a/b**5)*(-A*b + B*a)*log(b**2*sqrt(-a/b**5) + x)/2 - x*(-A*b + B*a)/b
**2
```

Giac [A] time = 1.10586, size = 77, normalized size = 1.33

$$\frac{\left(Ba^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Bb^2x^3 - 3\,Babx + 3\,Ab^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b^2*x^3 - 3*
B*a*b*x + 3*A*b^2*x)/b^3
```

3.61 $\int \frac{x(A+Bx^2)}{a+bx^2} dx$

Optimal. Leaf size=35

$$\frac{(Ab-aB)\log(a+bx^2)}{2b^2}+\frac{Bx^2}{2b}$$

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0299063, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{(Ab-aB)\log(a+bx^2)}{2b^2}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Bx^2}{2b}+\frac{(Ab-aB)\log(a+bx^2)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0108656, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log(a+bx^2)+bBx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2),x]

[Out] (b*B*x^2 + (A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.003, size = 40, normalized size = 1.1

$$\frac{Bx^2}{2b} + \frac{\ln\left(bx^2+a\right)A}{2b} - \frac{\ln\left(bx^2+a\right)Ba}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a),x)

[Out] 1/2*B*x^2/b+1/2/b*ln(b*x^2+a)*A-1/2/b^2*ln(b*x^2+a)*B*a
```

Maxima [A] time = 0.991694, size = 42, normalized size = 1.2

$$\frac{Bx^2}{2b} - \frac{(Ba - Ab)\log\left(bx^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.32872, size = 65, normalized size = 1.86

$$\frac{Bbx^2 - (Ba - Ab)\log\left(bx^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*(B*b*x^2 - (B*a - A*b)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.375242, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2b} - \frac{(-Ab + Ba)\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a),x)

[Out] B*x**2/(2*b) - (-A*b + B*a)*log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 1.16417, size = 43, normalized size = 1.23

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/b^2
```

3.62 $\int \frac{A+Bx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0168032, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a+bx^2}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{Bx}{b}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0250127, size = 40, normalized size = 1.03

$$\frac{Bx}{b}-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.002, size = 45, normalized size = 1.2

$$\frac{Bx}{b}+A\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{Ba}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a),x)
```

```
[Out] B*x/b+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-1/b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.20962, size = 223, normalized size = 5.72

$$\left[\frac{2\,Babx+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{Babx-(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x + (B*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (B*a*b*x - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] time = 0.400456, size = 82, normalized size = 2.1

$$\frac{Bx}{b}+\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)) + x)/2
```

Giac [A] time = 1.19269, size = 46, normalized size = 1.18

$$\frac{Bx}{b} - \frac{(Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.63 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)} dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.0335423, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 72}

$$\frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab+aB}{a(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}
\end{aligned}$$

Mathematica [A] time = 0.0121514, size = 34, normalized size = 1.

$$\frac{(aB-Ab)\log\left(a+bx^2\right)}{2ab} + \frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)),x]
```

[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^2])/(2*a*b)

Maple [A] time = 0.004, size = 37, normalized size = 1.1

$$\frac{A\ln(x)}{a} - \frac{\ln\left(bx^2+a\right)A}{2a} + \frac{\ln\left(bx^2+a\right)B}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x/(b*x^2+a),x)

[Out] A*ln(x)/a-1/2/a*ln(b*x^2+a)*A+1/2/b*ln(b*x^2+a)*B

Maxima [A] time = 1.00515, size = 47, normalized size = 1.38

$$\frac{A\log\left(x^2\right)}{2a} + \frac{(Ba-Ab)\log\left(bx^2+a\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(b*x^2 + a)/(a*b)

Fricas [A] time = 1.21652, size = 74, normalized size = 2.18

$$\frac{2\,Ab\log(x) + (Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*(2*A*b*log(x) + (B*a - A*b)*log(b*x^2 + a))/(a*b)

Sympy [A] time = 0.654914, size = 26, normalized size = 0.76

$$\frac{A\log(x)}{a} + \frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x/(b*x**2+a),x)

[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**2)/(2*a*b)

<u>Giac</u> [A] time = 1.14665, size = 49, normalized size = 1.44

$$\frac{A\log\left(x^2\right)}{2\,a} + \frac{(Ba - Ab)\log\left(\left|bx^2 + a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/(a*b)
```

3.64 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

```
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0209988, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {453, 205}

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx &= -\frac{A}{ax}-\frac{(Ab-aB)\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{A}{ax}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0252587, size = 42, normalized size = 0.98

$$\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.003, size = 48, normalized size = 1.1

$$-\frac{A}{ax}-\frac{Ab}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+B\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a),x)
```

```
[Out] -A/a/x-1/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A*b+1/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22512, size = 228, normalized size = 5.3

$$\left[\frac{(Ba-Ab)\sqrt{-ab}x\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Aab}{2\,a^2bx},\frac{(Ba-Ab)\sqrt{ab}x\arctan\left(\frac{\sqrt{ab}x}{a}\right)-Aab}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*A*a*b)/(a^2*b*x), ((B*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) - A*
a*b)/(a^2*b*x)]
```

Sympy [B] time = 0.46172, size = 82, normalized size = 1.91

$$-\frac{A}{ax}-\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) - sqrt(-1/(a**3*b))*(-A*b + B*a)*log(-a**2*sqrt(-1/(a**3*b)) + x)/
2 + sqrt(-1/(a**3*b))*(-A*b + B*a)*log(a**2*sqrt(-1/(a**3*b)) + x)/2
```

Giac [A] time = 1.12937, size = 49, normalized size = 1.14

$$\frac{(Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

3.65 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[Out] -A/(2*a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/(2*a
^2)
```

Rubi [A] time = 0.0476906, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/(2*a
^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0203936, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/(
2*a^2)
```

Maple [A] time = 0.006, size = 56, normalized size = 1.1

$$-\frac{A}{2\,a x^{2}}-\frac{A\ln\left(x\right)b}{a^{2}}+\frac{\ln\left(x\right)B}{a}+\frac{\ln\left(b x^{2}+a\right)Ab}{2\,a^{2}}-\frac{\ln\left(b x^{2}+a\right)B}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B+1/2/a^2*ln(b*x^2+a)*A*b-1/2/a*ln(b
*x^2+a)*B
```

Maxima [A] time = 1.00942, size = 65, normalized size = 1.3

$$-\frac{(Ba-Ab)\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{(Ba-Ab)\log\left(x^{2}\right)}{2\,a^{2}}-\frac{A}{2\,a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*log(b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*A/
(a*x^2)
```

Fricas [A] time = 1.18832, size = 111, normalized size = 2.22

$$-\frac{(Ba-Ab)x^{2}\log\left(bx^{2}+a\right)-2\,(Ba-Ab)x^{2}\log\left(x\right)+Aa}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*((B*a - A*b)*x^2*log(b*x^2 + a) - 2*(B*a - A*b)*x^2*log(x) + A*a)/(a^2
*x^2)
```

Sympy [A] time = 0.790597, size = 41, normalized size = 0.82

$$-\frac{A}{2ax^{2}}+\frac{(-Ab+Ba)\log\left(x\right)}{a^{2}}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^{2}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**2)/(2*
a**2)
```

Giac [A] time = 1.14143, size = 96, normalized size = 1.92

$$\frac{(Ba - Ab)\log\left(x^2\right)}{2\,a^2} - \frac{\left(Bab - Ab^2\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^2b} - \frac{Bax^2 - Abx^2 + Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*(B*a*b - A*b^2)*log(abs(b*x^2 + a))/(a^2
*b) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

3.66 $\int \frac{A+Bx^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=59

$$\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{A}{3ax^3}$$

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0393545, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {453, 325, 205}

$$\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx = -\frac{A}{3ax^3}-\frac{(3Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{3a}$$
$$= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{(b(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.0523703, size = 60, normalized size = 1.02

$$\frac{Ab-aB}{a^2x}-\frac{\sqrt{b}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[b]*(-(A*b) + a*B)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.005, size = 72, normalized size = 1.2

$$-\frac{A}{3\,ax^3}+\frac{Ab}{a^2x}-\frac{B}{ax}+\frac{b^2A}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bB}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a),x)
```

```
[Out] -1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)*A-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30634, size = 296, normalized size = 5.02

$$\left[-\frac{3\,(Ba-Ab)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,(Ba-Ab)x^2+2\,Aa}{6\,a^2x^3},-\frac{3\,(Ba-Ab)x^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(Ba-Ab)x^2}{3\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 6*(B*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/3*(3*(B*a - A*b)*x^3*sq
rt(b/a)*arctan(x*sqrt(b/a)) + 3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)]
```

Sympy [B] time = 0.562504, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\left(-Ab+Ba\right)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\left(-Ab+Ba\right)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{Aa+x^2\left(-3Ab+3Ba\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(-A*b + B*a)*log(-a**3*sqrt(-b/a**5)*(-A*b + B*a)/(-A*b**2 +
B*a*b) + x)/2 - sqrt(-b/a**5)*(-A*b + B*a)*log(a**3*sqrt(-b/a**5)*(-A*b + B
*a)/(-A*b**2 + B*a*b) + x)/2 - (A*a + x**2*(-3*A*b + 3*B*a))/(3*a**2*x**3)
```

Giac [A] time = 1.14362, size = 77, normalized size = 1.31

$$-\frac{\left(Bab-Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}-\frac{3\,Bax^2-3\,Abx^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*B*a*x^2 - 3
*A*b*x^2 + A*a)/(a^2*x^3)
```

3.67 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=69

$$\frac{Ab-aB}{2a^2x^2}-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{4ax^4}$$

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.0602773, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{Ab-aB}{2a^2x^2}-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{4ax^4}+\frac{Ab-aB}{2a^2x^2}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.027891, size = 70, normalized size = 1.01

$$\frac{-a\left(aA+2aBx^2-2Abx^2\right)+4bx^4\log(x)(Ab-aB)+2bx^4(aB-Ab)\log\left(a+bx^2\right)}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^2 + 2*a*B*x^2)) + 4*b*(A*b - a*B)*x^4*Log[x] + 2*b*(-(A
*b) + a*B)*x^4*Log[a + b*x^2])/(4*a^3*x^4)
```

Maple [A] time = 0.006, size = 81, normalized size = 1.2

$$-\frac{A}{4\,a x^4}+\frac{Ab}{2\,a^2 x^2}-\frac{B}{2\,a x^2}+\frac{A\ln(x)\,b^2}{a^3}-\frac{bB\ln(x)}{a^2}-\frac{b^2\ln\left(bx^2+a\right)A}{2\,a^3}+\frac{b\ln\left(bx^2+a\right)B}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a),x)
```

```
[Out] -1/4*A/a/x^4+1/2/a^2/x^2*A*b-1/2/a/x^2*B+1/a^3*b^2*ln(x)*A-1/a^2*b*ln(x)*B-
1/2*b^2/a^3*ln(b*x^2+a)*A+1/2*b/a^2*ln(b*x^2+a)*B
```

Maxima [A] time = 1.02217, size = 95, normalized size = 1.38

$$\frac{\left(Bab-Ab^2\right)\log\left(bx^2+a\right)}{2\,a^3}-\frac{\left(Bab-Ab^2\right)\log\left(x^2\right)}{2\,a^3}-\frac{2\,(Ba-Ab)x^2+Aa}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b - A*b^2)*log(b*x^2 + a)/a^3 - 1/2*(B*a*b - A*b^2)*log(x^2)/a^3 -
 1/4*(2*(B*a - A*b)*x^2 + A*a)/(a^2*x^4)
```

Fricas [A] time = 1.17974, size = 158, normalized size = 2.29

$$\frac{2\left(Bab-Ab^2\right)x^4\log\left(bx^2+a\right)-4\left(Bab-Ab^2\right)x^4\log(x)-Aa^2-2\left(Ba^2-Aab\right)x^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a*b - A*b^2)*x^4*log(b*x^2 + a) - 4*(B*a*b - A*b^2)*x^4*log(x) -
A*a^2 - 2*(B*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

Sympy [A] time = 0.986181, size = 61, normalized size = 0.88

$$-\frac{Aa+x^2\left(-2Ab+2Ba\right)}{4a^2x^4}-\frac{b\left(-Ab+Ba\right)\log(x)}{a^3}+\frac{b\left(-Ab+Ba\right)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

```
[Out] -(A*a + x**2*(-2*A*b + 2*B*a))/(4*a**2*x**4) - b*(-A*b + B*a)*log(x)/a**3 +
 b*(-A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

Giac [A] time = 1.14773, size = 135, normalized size = 1.96

$$-\frac{\left(Bab-Ab^2\right)\log\left(x^2\right)}{2\,a^3}+\frac{\left(Bab^2-Ab^3\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^3b}+\frac{3\,Babx^4-3\,Ab^2x^4-2\,Ba^2x^2+2\,Aabx^2-Aa^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b - A*b^2)*log(x^2)/a^3 + 1/2*(B*a*b^2 - A*b^3)*log(abs(b*x^2 + a
))/(a^3*b) + 1/4*(3*B*a*b*x^4 - 3*A*b^2*x^4 - 2*B*a^2*x^2 + 2*A*a*b*x^2 - A
*a^2)/(a^3*x^4)
```

3.68 $\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{A}{5ax^5}$$

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x) - (b^(3/2)
*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0500374, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {453, 325, 205}

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```



```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x) - (b^(3/2)
*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx = -\frac{A}{5ax^5} - \frac{(5Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{5a}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{3a^2x^3} + \frac{(b(Ab-aB))\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^2}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{3a^2x^3} - \frac{b(Ab-aB)}{a^3x} - \frac{\left(b^2(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{a^3}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{3a^2x^3} - \frac{b(Ab-aB)}{a^3x} - \frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] time = 0.0518063, size = 78, normalized size = 0.98

$$\frac{b^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}} + \frac{Ab-aB}{3a^2x^3} + \frac{b(aB-Ab)}{a^3x} - \frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (b*(-(A*b) + a*B))/(a^3*x) + (b^(3
/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Maple [A] time = 0.006, size = 96, normalized size = 1.2

$$-\frac{A}{5\,ax^5} + \frac{Ab}{3\,a^2x^3} - \frac{B}{3\,ax^3} - \frac{b^2A}{a^3x} + \frac{bB}{a^2x} - \frac{Ab^3}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{b^2B}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a),x)
```

```
[Out] -1/5*A/a/x^5+1/3/a^2/x^3*A*b-1/3/a/x^3*B-1/a^3*b^2/x*A+1/a^2*b/x*B-b^3/a^3/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18793, size = 398, normalized size = 4.97

$$\left[-\frac{15\left(Bab - Ab^2\right)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 30\left(Bab - Ab^2\right)x^4 + 6\,Aa^2 + 10\left(Ba^2 - Aab\right)x^2}{30\,a^3x^5}, \frac{15\left(Bab - Ab^2\right)x^5\sqrt{\frac{b}{a}}\,\mathrm{arct}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*a*b - A*b^2)*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) - 30*(B*a*b - A*b^2)*x^4 + 6*A*a^2 + 10*(B*a^2 - A*a*b)*x^2)
/(a^3*x^5), 1/15*(15*(B*a*b - A*b^2)*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15
*(B*a*b - A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)]
```

Sympy [B] time = 0.689464, size = 163, normalized size = 2.04

$$-\frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2} + \frac{-3Aa^2 + x^4\left(-15Ab^2 + 1\cdots\right.}{15\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**7)*(-A*b + B*a)*log(-a**4*sqrt(-b**3/a**7)*(-A*b + B*a)/(-A*
b**3 + B*a*b**2) + x)/2 + sqrt(-b**3/a**7)*(-A*b + B*a)*log(a**4*sqrt(-b**3
/a**7)*(-A*b + B*a)/(-A*b**3 + B*a*b**2) + x)/2 + (-3*A*a**2 + x**4*(-15*A*
b**2 + 15*B*a*b) + x**2*(5*A*a*b - 5*B*a**2))/(15*a**3*x**5)
```

Giac [A] time = 1.16381, size = 109, normalized size = 1.36

$$\frac{\left(Bab^2 - Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{15\,Babx^4 - 15\,Ab^2x^4 - 5\,Ba^2x^2 + 5\,Aabx^2 - 3\,Aa^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*B*a*b*x^
4 - 15*A*b^2*x^4 - 5*B*a^2*x^2 + 5*A*a*b*x^2 - 3*A*a^2)/(a^3*x^5)
```

3.69 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=93

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x^2) - (b^2
*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0823365, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x^2) - (b^2
*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x^2])/(2*a^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax^4}+\frac{-Ab+aB}{a^2x^3}-\frac{b(-Ab+aB)}{a^3x^2}+\frac{b^2(-Ab+aB)}{a^4x}-\frac{b^3(-Ab+aB)}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{2a^3x^2}-\frac{b^2(Ab-aB)\log(x)}{a^4}+\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.0359372, size = 96, normalized size = 1.03

$$\frac{\left(Ab^3-ab^2B\right)\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)\left(ab^2B-Ab^3\right)}{a^4}+\frac{b(aB-Ab)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) + (b*(-(A*b) + a*B))/(2*a^3*x^2) + (
(-(A*b^3) + a*b^2*B)*Log[x])/a^4 + ((A*b^3 - a*b^2*B)*Log[a + b*x^2])/(2*a^
4)
```

Maple [A] time = 0.007, size = 107, normalized size = 1.2

$$-\frac{A}{6\,a x^{6}}+\frac{A b}{4\,a^{2} x^{4}}-\frac{B}{4\,a x^{4}}-\frac{b^{2} A}{2\,a^{3} x^{2}}+\frac{b B}{2\,a^{2} x^{2}}-\frac{b^{3} \ln \left(x\right) A}{a^{4}}+\frac{b^{2} B \ln \left(x\right)}{a^{3}}+\frac{b^{3} \ln \left(b x^{2}+a\right) A}{2\,a^{4}}-\frac{b^{2} \ln \left(b x^{2}+a\right) B}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a),x)
```

```
[Out] -1/6*A/a/x^6+1/4/a^2/x^4*A*b-1/4/a/x^4*B-1/2/a^3*b^2/x^2*A+1/2/a^2*b/x^2*B-
1/a^4*b^3*ln(x)*A+1/a^3*b^2*ln(x)*B+1/2*b^3/a^4*ln(b*x^2+a)*A-1/2*b^2/a^3*l
n(b*x^2+a)*B
```

Maxima [A] time = 1.0235, size = 130, normalized size = 1.4

$$-\frac{\left(B a b^{2}-A b^{3}\right) \log \left(b x^{2}+a\right)}{2\,a^{4}}+\frac{\left(B a b^{2}-A b^{3}\right) \log \left(x^{2}\right)}{2\,a^{4}}+\frac{6\,\left(B a b-A b^{2}\right) x^{4}-2\,A a^{2}-3\,\left(B a^{2}-A a b\right) x^{2}}{12\,a^{3} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*b^2 - A*b^3)*log(b*x^2 + a)/a^4 + 1/2*(B*a*b^2 - A*b^3)*log(x^2)/
a^4 + 1/12*(6*(B*a*b - A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 - A*a*b)*x^2)/(a^3*x
^6)
```

Fricas [A] time = 1.22837, size = 211, normalized size = 2.27

$$-\frac{6\,\left(B a b^{2}-A b^{3}\right) x^{6} \log \left(b x^{2}+a\right)-12\,\left(B a b^{2}-A b^{3}\right) x^{6} \log \left(x\right)-6\,\left(B a^{2} b-A a b^{2}\right) x^{4}+2\,A a^{3}+3\,\left(B a^{3}-A a^{2} b\right) x^{2}}{12\,a^{4} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*a*b^2 - A*b^3)*x^6*log(b*x^2 + a) - 12*(B*a*b^2 - A*b^3)*x^6*lo
g(x) - 6*(B*a^2*b - A*a*b^2)*x^4 + 2*A*a^3 + 3*(B*a^3 - A*a^2*b)*x^2)/(a^4*
x^6)
```

Sympy [A] time = 1.16445, size = 88, normalized size = 0.95

$$\frac{-2Aa^{2}+x^{4}\left(-6Ab^{2}+6Bab\right)+x^{2}\left(3Aab-3Ba^{2}\right)}{12a^{3}x^{6}}+\frac{b^{2}\left(-Ab+Ba\right)\log \left(x\right)}{a^{4}}-\frac{b^{2}\left(-Ab+Ba\right)\log \left(\frac{a}{b}+x^{2}\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 6*B*a*b) + x**2*(3*A*a*b - 3*B*a**2))/(12*a*
*3*x**6) + b**2*(-A*b + B*a)*log(x)/a**4 - b**2*(-A*b + B*a)*log(a/b + x**2
)/(2*a**4)
```

Giac [A] time = 1.35631, size = 170, normalized size = 1.83

$$\frac{\left(Bab^2 - Ab^3\right)\log\left(x^2\right)}{2\,a^4} - \frac{\left(Bab^3 - Ab^4\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^4 b} - \frac{11\,Bab^2x^6 - 11\,Ab^3x^6 - 6\,Ba^2bx^4 + 6\,Aab^2x^4 + 3\,Ba^3x^2 - 3\,A}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b^2 - A*b^3)*log(x^2)/a^4 - 1/2*(B*a*b^3 - A*b^4)*log(abs(b*x^2 +
a))/(a^4*b) - 1/12*(11*B*a*b^2*x^6 - 11*A*b^3*x^6 - 6*B*a^2*b*x^4 + 6*A*a*b
^2*x^4 + 3*B*a^3*x^2 - 3*A*a^2*b*x^2 + 2*A*a^3)/(a^4*x^6)
```

3.70 $\int \frac{A+Bx^2}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=99

$$\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x^3) + (b^2
*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a
^(9/2)
```

Rubi [A] time = 0.0681236, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {453, 325, 205}

$$\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x^3) + (b^2
*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a
^(9/2)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx = -\frac{A}{7ax^7} - \frac{(7Ab-7aB)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{7a}$$
$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{5a^2x^5} + \frac{(b(Ab-aB))\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a^2}$$
$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{5a^2x^5} - \frac{b(Ab-aB)}{3a^3x^3} - \frac{\left(b^2(Ab-aB)\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^3}$$
$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{5a^2x^5} - \frac{b(Ab-aB)}{3a^3x^3} + \frac{b^2(Ab-aB)}{a^4x} + \frac{\left(b^3(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{a^4}$$
$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{5a^2x^5} - \frac{b(Ab-aB)}{3a^3x^3} + \frac{b^2(Ab-aB)}{a^4x} + \frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [A] time = 0.0695324, size = 101, normalized size = 1.02

$$-\frac{b^2(aB-Ab)}{a^4x} - \frac{b^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}} + \frac{b(aB-Ab)}{3a^3x^3} + \frac{Ab-aB}{5a^2x^5} - \frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (b*(-(A*b) + a*B))/(3*a^3*x^3) - (
b^2*(-(A*b) + a*B))/(a^4*x) - (b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(9/2)
```

Maple [A] time = 0.005, size = 120, normalized size = 1.2

$$-\frac{A}{7\,ax^7} + \frac{Ab}{5\,a^2x^5} - \frac{B}{5\,ax^5} - \frac{b^2A}{3\,a^3x^3} + \frac{bB}{3\,a^2x^3} + \frac{b^3A}{a^4x} - \frac{b^2B}{a^3x} + \frac{Ab^4}{a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{b^3B}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a),x)
```

```
[Out] -1/7*A/a/x^7+1/5/a^2/x^5*A*b-1/5/a/x^5*B-1/3/a^3*b^2/x^3*A+1/3/a^2*b/x^3*B+
1/a^4*b^3/x*A-1/a^3*b^2/x*B+b^4/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-b
^3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27974, size = 506, normalized size = 5.11

$$\left[-\frac{105\left(Bab^2-Ab^3\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\left(Bab^2-Ab^3\right)x^6-70\left(Ba^2b-Aab^2\right)x^4+30Aa^3+42\left(Ba^3-Aa^2b\right)}{210a^4x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 210*(B*a*b^2 - A*b^3)*x^6 - 70*(B*a^2*b - A*a*b^2)*x^4
 + 30*A*a^3 + 42*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7), -1/105*(105*(B*a*b^2 - A
*b^3)*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(B*a*b^2 - A*b^3)*x^6 - 35*(B
*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)]
```

Sympy [B] time = 0.840096, size = 187, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2}-\frac{15Aa^3+x^6\left(-105Ab^3+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**9)*(-A*b + B*a)*log(-a**5*sqrt(-b**5/a**9)*(-A*b + B*a)/(-A*b
**4 + B*a*b**3) + x)/2 - sqrt(-b**5/a**9)*(-A*b + B*a)*log(a**5*sqrt(-b**5/
a**9)*(-A*b + B*a)/(-A*b**4 + B*a*b**3) + x)/2 - (15*A*a**3 + x**6*(-105*A*
b**3 + 105*B*a*b**2) + x**4*(35*A*a*b**2 - 35*B*a**2*b) + x**2*(-21*A*a**2*
b + 21*B*a**3))/(105*a**4*x**7)
```

Giac [A] time = 1.23837, size = 143, normalized size = 1.44

$$-\frac{\left(Bab^3-Ab^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}-\frac{105\,Bab^2x^6-105\,Ab^3x^6-35\,Ba^2bx^4+35\,Aab^2x^4+21\,Ba^3x^2-21\,Aa^2bx^2+15\,Aa^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*B*a*b
^2*x^6 - 105*A*b^3*x^6 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^4 + 21*B*a^3*x^2 - 2
1*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

3.71 $\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=126

$$\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)\log\left(a+bx^2\right)}{2b^6}+\frac{x^6(Ab-2aB)}{6b^3}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{Bx^8}{8b^2}$$

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^4) + ((A*b
 - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b - a*B))/(2*b^6*(a + b*
x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.16913, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)\log\left(a+bx^2\right)}{2b^6}+\frac{x^6(Ab-2aB)}{6b^3}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{Bx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^4) + ((A*b
 - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b - a*B))/(2*b^6*(a + b*
x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/(2*b^6)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^2(-3Ab+4aB)}{b^5}+\frac{a(-2Ab+3aB)x}{b^4}+\frac{(Ab-2aB)x^2}{b^3}+\frac{Bx^3}{b^2}-\frac{a^4(-Ab+aB)}{b^5(a+bx)^2}+\right.\right.$$

$$=\frac{a^2(3Ab-4aB)x^2}{2b^5}-\frac{a(2Ab-3aB)x^4}{4b^4}+\frac{(Ab-2aB)x^6}{6b^3}+\frac{Bx^8}{8b^2}-\frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)}{2b^6}$$

Mathematica [A] time = 0.0766637, size = 113, normalized size = 0.9

$$\frac{-12a^2bx^2(4aB-3Ab)+\frac{12a^4(aB-Ab)}{a+bx^2}+12a^3(5aB-4Ab)\log\left(a+bx^2\right)+4b^3x^6(Ab-2aB)+6ab^2x^4(3aB-2Ab)+3b^4Bx^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-12*a^2*b*(-3*A*b + 4*a*B)*x^2 + 6*a*b^2*(-2*A*b + 3*a*B)*x^4 + 4*b^3*(A*b
 - 2*a*B)*x^6 + 3*b^4*B*x^8 + (12*a^4*(-(A*b) + a*B))/(a + b*x^2) + 12*a^3*
(-4*A*b + 5*a*B)*Log[a + b*x^2])/(24*b^6)
```

Maple [A] time = 0.009, size = 146, normalized size = 1.2

$$\frac{Bx^8}{8b^2}+\frac{x^6A}{6b^2}-\frac{x^6Ba}{3b^3}-\frac{x^4Aa}{2b^3}+\frac{3x^4Ba^2}{4b^4}+\frac{3a^2Ax^2}{2b^4}-2\frac{Bx^2a^3}{b^5}-2\frac{a^3\ln\left(bx^2+a\right)A}{b^5}+\frac{5a^4\ln\left(bx^2+a\right)B}{2b^6}-\frac{a^4A}{2b^5\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/8*B*x^8/b^2+1/6/b^2*x^6*A-1/3/b^3*x^6*B*a-1/2/b^3*x^4*A*a+3/4/b^4*x^4*B*a
^2+3/2/b^4*x^2*A*a^2-2/b^5*x^2*B*a^3-2*a^3/b^5*ln(b*x^2+a)*A+5/2*a^4/b^6*ln
(b*x^2+a)*B-1/2*a^4/b^5/(b*x^2+a)*A+1/2*a^5/b^6/(b*x^2+a)*B
```

Maxima [A] time = 1.32536, size = 177, normalized size = 1.4

$$\frac{Ba^5-Aa^4b}{2\left(b^7x^2+ab^6\right)}+\frac{3Bb^3x^8-4\left(2Bab^2-Ab^3\right)x^6+6\left(3Ba^2b-2Aab^2\right)x^4-12\left(4Ba^3-3Aa^2b\right)x^2}{24b^5}+\frac{\left(5Ba^4-4Aa^3b\right)\log}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^5 - A*a^4*b)/(b^7*x^2 + a*b^6) + 1/24*(3*B*b^3*x^8 - 4*(2*B*a*b^2
- A*b^3)*x^6 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^4 - 12*(4*B*a^3 - 3*A*a^2*b)*x^2
)/b^5 + 1/2*(5*B*a^4 - 4*A*a^3*b)*log(b*x^2 + a)/b^6
```

Fricas [A] time = 1.23371, size = 365, normalized size = 2.9

$$\frac{3Bb^5x^{10}-\left(5Bab^4-4Ab^5\right)x^8+2\left(5Ba^2b^3-4Aab^4\right)x^6+12Ba^5-12Aa^4b-6\left(5Ba^3b^2-4Aa^2b^3\right)x^4-12\left(4Ba^4b-3A\right.}{24\left(b^7x^2+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^10 - (5*B*a*b^4 - 4*A*b^5)*x^8 + 2*(5*B*a^2*b^3 - 4*A*a*b^4
)*x^6 + 12*B*a^5 - 12*A*a^4*b - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^4 - 12*(4*B
```

```
*a^4*b - 3*A*a^3*b^2)*x^2 + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a^3*
b^2)*x^2)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A] time = 0.905388, size = 126, normalized size = 1.

$$\frac{Bx^8}{8b^2} + \frac{a^3(-4Ab+5Ba)\log(a+bx^2)}{2b^6} + \frac{-Aa^4b+Ba^5}{2ab^6+2b^7x^2} - \frac{x^6(-Ab+2Ba)}{6b^3} + \frac{x^4(-2Aab+3Ba^2)}{4b^4} - \frac{x^2(-3Aa^2b+4Ba^3)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**8/(8*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x**2)/(2*b**6) + (-A*a**4
*b + B*a**5)/(2*a*b**6 + 2*b**7*x**2) - x**6*(-A*b + 2*B*a)/(6*b**3) + x**4
*(-2*A*a*b + 3*B*a**2)/(4*b**4) - x**2*(-3*A*a**2*b + 4*B*a**3)/(2*b**5)
```

Giac [A] time = 1.17899, size = 215, normalized size = 1.71

$$\frac{(5\,Ba^4 - 4\,Aa^3b)\log(|bx^2+a|)}{2\,b^6} - \frac{5\,Ba^4bx^2 - 4\,Aa^3b^2x^2 + 4\,Ba^5 - 3\,Aa^4b}{2\,(bx^2+a)b^6} + \frac{3\,Bb^6x^8 - 8\,Bab^5x^6 + 4\,Ab^6x^6 + 18\,Ba^2b^4x^4 - 12\,Aab^5x^4 - 48\,Ba^3b^3x^2 + 36\,Aa^2b^4x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^4 - 4*A*a^3*b)*log(abs(b*x^2 + a))/b^6 - 1/2*(5*B*a^4*b*x^2 - 4*
A*a^3*b^2*x^2 + 4*B*a^5 - 3*A*a^4*b)/((b*x^2 + a)*b^6) + 1/24*(3*B*b^6*x^8
- 8*B*a*b^5*x^6 + 4*A*b^6*x^6 + 18*B*a^2*b^4*x^4 - 12*A*a*b^5*x^4 - 48*B*a^
3*b^3*x^2 + 36*A*a^2*b^4*x^2)/b^8
```

3.72 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$\frac{a^3x(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x^5(Ab-2aB)}{5b^3}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{Bx^7}{7b^2}$$

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + ((A*b - 2*a
*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x)/(2*b^5*(a + b*x^2)
) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.150482, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 1810, 205}

$$\frac{a^3x(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x^5(Ab-2aB)}{5b^3}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + ((A*b - 2*a
*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x)/(2*b^5*(a + b*x^2)
) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8 (A+Bx^2)}{(a+bx^2)^2} dx = \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{\int \frac{a^3(Ab-aB)-2a^2b(Ab-aB)x^2+2ab^2(Ab-aB)x^4-2b^3(Ab-aB)x^6-2b^4Bx^8}{a+bx^2} dx}{2b^5}$$

$$= \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{\int \left(-2a^2(3Ab-4aB) + 2ab(2Ab-3aB)x^2 - 2b^2(Ab-2aB)x^4 - 2b^3Bx^6 + \frac{7a^3A}{a+}\right.}{2b^5}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{(a^3(7Ab-9aB)}{2b^5}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{a^{5/2}(7Ab-9aB)}{2b^{11/}}$$

Mathematica [A] time = 0.103465, size = 134, normalized size = 1.02

$$\frac{x(a^3Ab-a^4B)}{2b^5(a+bx^2)} - \frac{a^2x(4aB-3Ab)}{b^5} + \frac{a^{5/2}(9aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}} + \frac{x^5(Ab-2aB)}{5b^3} + \frac{ax^3(3aB-2Ab)}{3b^4} + \frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a^2*(-3*A*b + 4*a*B)*x)/b^5) + (a*(-2*A*b + 3*a*B)*x^3)/(3*b^4) + ((A*b
- 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + ((a^3*A*b - a^4*B)*x)/(2*b^5*(a +
 b*x^2)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/
2))
```

Maple [A] time = 0.009, size = 155, normalized size = 1.2

$$\frac{Bx^7}{7b^2} + \frac{Ax^5}{5b^2} - \frac{2Bx^5a}{5b^3} - \frac{2aAx^3}{3b^3} + \frac{Bx^3a^2}{b^4} + 3\frac{a^2Ax}{b^4} - 4\frac{Ba^3x}{b^5} + \frac{a^3xA}{2b^4(bx^2+a)} - \frac{a^4xB}{2b^5(bx^2+a)} - \frac{7Aa^3}{2b^4}\arctan\left(bx\frac{1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*B*x^7/b^2+1/5/b^2*A*x^5-2/5/b^3*B*x^5*a-2/3/b^3*A*x^3*a+1/b^4*B*x^3*a^2
+3/b^4*A*a^2*x-4/b^5*B*a^3*x+1/2*a^3/b^4*x/(b*x^2+a)*A-1/2*a^4/b^5*x/(b*x^2
+a)*B-7/2*a^3/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+9/2*a^4/b^5/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.29378, size = 755, normalized size = 5.76

$$\left[\frac{60\,Bb^4x^9-12\left(9\,Bab^3-7\,Ab^4\right)x^7+28\left(9\,Ba^2b^2-7\,Aab^3\right)x^5-140\left(9\,Ba^3b-7\,Aa^2b^2\right)x^3-105\left(9\,Ba^4-7\,Aa^3b+\left(9\,Ba^3\right.\right.}{420\left(b^6x^2+ab^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*b^4*x^9 - 12*(9*B*a*b^3 - 7*A*b^4)*x^7 + 28*(9*B*a^2*b^2 - 7*A
*a*b^3)*x^5 - 140*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 7*A*a^3*b
+ (9*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) - 210*(9*B*a^4 - 7*A*a^3*b)*x)/(b^6*x^2 + a*b^5), 1/210*(3
0*B*b^4*x^9 - 6*(9*B*a*b^3 - 7*A*b^4)*x^7 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^
5 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 + 105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*
b - 7*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(9*B*a^4 - 7*
A*a^3*b)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 0.90571, size = 233, normalized size = 1.78

$$\frac{Bx^7}{7b^2}-\frac{x\left(-Aa^3b+Ba^4\right)}{2ab^5+2b^6x^2}-\frac{\sqrt{-\frac{a^5}{b^{11}}}\left(-7Ab+9Ba\right)\log\left(-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+9Ba)}{-7Aa^2b+9Ba^3}+x\right)}{4}+\frac{\sqrt{-\frac{a^5}{b^{11}}}\left(-7Ab+9Ba\right)\log\left(\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+\right.}{-7Aa^2b+9Ba\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) - x*(-A*a**3*b + B*a**4)/(2*a*b**5 + 2*b**6*x**2) - sqrt(-a
**5/b**11)*(-7*A*b + 9*B*a)*log(-b**5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-
7*A*a**2*b + 9*B*a**3) + x)/4 + sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)*log(b**5
*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7*A*a**2*b + 9*B*a**3) + x)/4 - x**5*
(-A*b + 2*B*a)/(5*b**3) + x**3*(-2*A*a*b + 3*B*a**2)/(3*b**4) - x*(-3*A*a**
2*b + 4*B*a**3)/b**5
```

Giac [A] time = 1.12032, size = 188, normalized size = 1.44

$$\frac{\left(9\,Ba^4-7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}-\frac{Ba^4x-Aa^3bx}{2\left(bx^2+a\right)b^5}+\frac{15\,Bb^{12}x^7-42\,Bab^{11}x^5+21\,Ab^{12}x^5+105\,Ba^2b^{10}x^3-70\,Aab^{11}x^3-4}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*(B*a^
4*x - A*a^3*b*x)/((b*x^2 + a)*b^5) + 1/105*(15*B*b^12*x^7 - 42*B*a*b^11*x^5
```

```
+ 21*A*b^12*x^5 + 105*B*a^2*b^10*x^3 - 70*A*a*b^11*x^3 - 420*B*a^3*b^9*x +
315*A*a^2*b^10*x)/b^14
```

3.73 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^4(Ab-2aB)}{4b^3}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{Bx^6}{6b^2}$$

```
[Out] -(a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) + (B*x^6)/(6
*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3*A*b - 4*a*B)*Log[a
+ b*x^2])/(2*b^5)
```

Rubi [A] time = 0.126427, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^4(Ab-2aB)}{4b^3}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) + (B*x^6)/(6
*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3*A*b - 4*a*B)*Log[a
+ b*x^2])/(2*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-2Ab+3aB)}{b^4}+\frac{(Ab-2aB)x}{b^3}+\frac{Bx^2}{b^2}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^2}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)}\right)dx, x, x\right.\\
&= -\frac{a(2Ab-3aB)x^2}{2b^4}+\frac{(Ab-2aB)x^4}{4b^3}+\frac{Bx^6}{6b^2}+\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\left(a+bx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.0641138, size = 93, normalized size = 0.89

$$\frac{\frac{6a^3(Ab-aB)}{a+bx^2}+6a^2(3Ab-4aB)\log\left(a+bx^2\right)+3b^2x^4(Ab-2aB)+6abx^2(3aB-2Ab)+2b^3Bx^6}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (6*a*b*(-2*A*b + 3*a*B)*x^2 + 3*b^2*(A*b - 2*a*B)*x^4 + 2*b^3*B*x^6 + (6*a^
3*(A*b - a*B))/(a + b*x^2) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x^2])/(12*b^5)
```

Maple [A] time = 0.01, size = 122, normalized size = 1.2

$$\frac{Bx^6}{6\,b^2}+\frac{Ax^4}{4\,b^2}-\frac{Bx^4a}{2\,b^3}-\frac{aAx^2}{b^3}+\frac{3\,Bx^2a^2}{2\,b^4}+\frac{3\,a^2\ln\left(bx^2+a\right)A}{2\,b^4}-2\,\frac{a^3\ln\left(bx^2+a\right)B}{b^5}+\frac{a^3A}{2\,b^4\left(bx^2+a\right)}-\frac{Ba^4}{2\,b^5\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-1/b^3*A*x^2*a+3/2/b^4*B*x^2*a^2
+3/2*a^2/b^4*ln(b*x^2+a)*A-2*a^3/b^5*ln(b*x^2+a)*B+1/2*a^3/b^4/(b*x^2+a)*A-
1/2*a^4/b^5/(b*x^2+a)*B
```

Maxima [A] time = 0.987453, size = 144, normalized size = 1.38

$$-\frac{Ba^4-Aa^3b}{2\left(b^6x^2+ab^5\right)}+\frac{2\,Bb^2x^6-3\left(2\,Bab-Ab^2\right)x^4+6\left(3\,Ba^2-2\,Aab\right)x^2}{12\,b^4}-\frac{\left(4\,Ba^3-3\,Aa^2b\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)/(b^6*x^2 + a*b^5) + 1/12*(2*B*b^2*x^6 - 3*(2*B*a*b -
 A*b^2)*x^4 + 6*(3*B*a^2 - 2*A*a*b)*x^2)/b^4 - 1/2*(4*B*a^3 - 3*A*a^2*b)*lo
g(b*x^2 + a)/b^5
```

Fricas [A] time = 1.2057, size = 309, normalized size = 2.97

$$\frac{2\,Bb^4x^8-\left(4\,Bab^3-3\,Ab^4\right)x^6-6\,Ba^4+6\,Aa^3b+3\left(4\,Ba^2b^2-3\,Aab^3\right)x^4+6\left(3\,Ba^3b-2\,Aa^2b^2\right)x^2-6\left(4\,Ba^4-3\,Aa^3\right.}{12\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^4*x^8 - (4*B*a*b^3 - 3*A*b^4)*x^6 - 6*B*a^4 + 6*A*a^3*b + 3*(4*
B*a^2*b^2 - 3*A*a*b^3)*x^4 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x^2 - 6*(4*B*a^4 -
 3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^2 + a*b^
```

5)

Sympy [A] time = 0.848989, size = 102, normalized size = 0.98

$$\frac{Bx^6}{6b^2} - \frac{a^2(-3Ab+4Ba)\log\left(a+bx^2\right)}{2b^5} - \frac{-Aa^3b+Ba^4}{2ab^5+2b^6x^2} - \frac{x^4(-Ab+2Ba)}{4b^3} + \frac{x^2\left(-2Aab+3Ba^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x**2)/(2*b**5) - (-A*a**3
*b + B*a**4)/(2*a*b**5 + 2*b**6*x**2) - x**4*(-A*b + 2*B*a)/(4*b**3) + x**2
*(-2*A*a*b + 3*B*a**2)/(2*b**4)
```

Giac [A] time = 1.1271, size = 182, normalized size = 1.75

$$-\frac{\left(4\,Ba^3-3\,Aa^2b\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^5} + \frac{2\,Bb^4x^6-6\,Bab^3x^4+3\,Ab^4x^4+18\,Ba^2b^2x^2-12\,Aab^3x^2}{12\,b^6} + \frac{4\,Ba^3bx^2-3\,Aa^2b^2x^2+\cdots}{2\left(bx^2+a\right)b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^5 + 1/12*(2*B*b^4*x^6 - 6*
B*a*b^3*x^4 + 3*A*b^4*x^4 + 18*B*a^2*b^2*x^2 - 12*A*a*b^3*x^2)/b^6 + 1/2*(4
*B*a^3*b*x^2 - 3*A*a^2*b^2*x^2 + 3*B*a^4 - 2*A*a^3*b)/((b*x^2 + a)*b^5)
```

3.74 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}+\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{ax(2Ab-3aB)}{b^4}+\frac{Bx^5}{5b^2}$$

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2
) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*(5*A*b - 7*a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] time = 0.113795, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 1810, 205}

$$-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}+\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{ax(2Ab-3aB)}{b^4}+\frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2
) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*(5*A*b - 7*a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2} dx = -\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)} - \frac{\int \frac{-a^2(Ab-aB)+2ab(Ab-aB)x^2-2b^2(Ab-aB)x^4-2b^3Bx^6}{a+bx^2} dx}{2b^4}$$

$$= -\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)} - \frac{\int \left(2a(2Ab-3aB)-2b(Ab-2aB)x^2-2b^2Bx^4+\frac{-5a^2Ab+7a^3B}{a+bx^2}\right) dx}{2b^4}$$

$$= -\frac{a(2Ab-3aB)x}{b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^5}{5b^2} - \frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)} + \frac{\left(a^2(5Ab-7aB)\right)\int \frac{1}{a+bx^2} dx}{2b^4}$$

$$= -\frac{a(2Ab-3aB)x}{b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^5}{5b^2} - \frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)} + \frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Mathematica [A] time = 0.0853155, size = 111, normalized size = 1.01

$$-\frac{x\left(a^2Ab-a^3B\right)}{2b^4\left(a+bx^2\right)} - \frac{a^{3/2}(7aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} + \frac{x^3(Ab-2aB)}{3b^3} + \frac{ax(3aB-2Ab)}{b^4} + \frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*A*b + 3*a*B)*x)/b^4 + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2)
- ((a^2*A*b - a^3*B)*x)/(2*b^4*(a + b*x^2)) - (a^(3/2)*(-5*A*b + 7*a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.008, size = 132, normalized size = 1.2

$$\frac{Bx^5}{5b^2} + \frac{Ax^3}{3b^2} - \frac{2Bx^3a}{3b^3} - 2\frac{aAx}{b^3} + 3\frac{a^2Bx}{b^4} - \frac{a^2Ax}{2b^3\left(bx^2+a\right)} + \frac{a^3xB}{2b^4\left(bx^2+a\right)} + \frac{5Aa^2}{2b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{7Ba^3}{2b^4}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*B*x^5/b^2+1/3/b^2*A*x^3-2/3/b^3*B*x^3*a-2/b^3*a*A*x+3/b^4*a^2*B*x-1/2*a
^2/b^3*x/(b*x^2+a)*A+1/2*a^3/b^4*x/(b*x^2+a)*B+5/2*a^2/b^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*A-7/2*a^3/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28166, size = 637, normalized size = 5.79

$$\left[\frac{12\,Bb^3x^7-4\left(7\,Bab^2-5\,Ab^3\right)x^5+20\left(7\,Ba^2b-5\,Aab^2\right)x^3-15\left(7\,Ba^3-5\,Aa^2b+\left(7\,Ba^2b-5\,Aab^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2}{}\right.}{60\left(b^5x^2+ab^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*b^3*x^7 - 4*(7*B*a*b^2 - 5*A*b^3)*x^5 + 20*(7*B*a^2*b - 5*A*a*b
^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(7*B*a^3 - 5*A*a^2*b)
*x)/(b^5*x^2 + a*b^4), 1/30*(6*B*b^3*x^7 - 2*(7*B*a*b^2 - 5*A*b^3)*x^5 + 10
*(7*B*a^2*b - 5*A*a*b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a
*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(7*B*a^3 - 5*A*a^2*b)*x)/
(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 0.853715, size = 206, normalized size = 1.87

$$\frac{Bx^5}{5b^2}+\frac{x\left(-Aa^2b+Ba^3\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab+7Ba)}{-5Aab+7Ba^2}+x\right)}{4}-\frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab-}{-5Aab+7Ba}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x*(-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-a
**3/b**9)*(-5*A*b + 7*B*a)*log(-b**4*sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*
A*a*b + 7*B*a**2) + x)/4 - sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)*log(b**4*sqrt(
-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*b + 7*B*a**2) + x)/4 - x**3*(-A*b + 2*
B*a)/(3*b**3) + x*(-2*A*a*b + 3*B*a**2)/b**4
```

Giac [A] time = 1.15127, size = 155, normalized size = 1.41

$$-\frac{\left(7\,Ba^3-5\,Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}+\frac{Ba^3x-Aa^2bx}{2\left(bx^2+a\right)b^4}+\frac{3\,Bb^8x^5-10\,Bab^7x^3+5\,Ab^8x^3+45\,Ba^2b^6x-30\,Aab^7x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*(B*a
^3*x - A*a^2*b*x)/((b*x^2 + a)*b^4) + 1/15*(3*B*b^8*x^5 - 10*B*a*b^7*x^3 +
5*A*b^8*x^3 + 45*B*a^2*b^6*x - 30*A*a*b^7*x)/b^10
```

3.75 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}+\frac{x^2(Ab-2aB)}{2b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^4}{4b^2}$$

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))/(2*b^4*(a
 + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0873322, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}+\frac{x^2(Ab-2aB)}{2b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))/(2*b^4*(a
 + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^4}{4b^2}-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0645405, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB-Ab)}{a+bx^2}+2bx^2(Ab-2aB)+2a(3aB-2Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^2 + b^2*B*x^4 + (2*a^2*(-(A*b) + a*B))/(a + b*x^2) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(4*b^4)
```

Maple [A] time = 0.01, size = 98, normalized size = 1.2

$$\frac{Bx^4}{4\,b^2}+\frac{Ax^2}{2\,b^2}-\frac{Bx^2a}{b^3}-\frac{a\ln\left(bx^2+a\right)A}{b^3}+\frac{3\,a^2\ln\left(bx^2+a\right)B}{2\,b^4}-\frac{a^2A}{2\,b^3\left(bx^2+a\right)}+\frac{Ba^3}{2\,b^4\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*B*x^4/b^2+1/2/b^2*A*x^2-1/b^3*B*x^2*a-a/b^3*ln(b*x^2+a)*A+3/2*a^2/b^4*l
n(b*x^2+a)*B-1/2*a^2/b^3/(b*x^2+a)*A+1/2*a^3/b^4/(b*x^2+a)*B
```

Maxima [A] time = 1.01722, size = 111, normalized size = 1.35

$$\frac{Ba^3-Aa^2b}{2\left(b^5x^2+ab^4\right)}+\frac{Bbx^4-2\,(2\,Ba-Ab)x^2}{4\,b^3}+\frac{\left(3\,Ba^2-2\,Aab\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)/(b^5*x^2 + a*b^4) + 1/4*(B*b*x^4 - 2*(2*B*a - A*b)*x^
2)/b^3 + 1/2*(3*B*a^2 - 2*A*a*b)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.17782, size = 251, normalized size = 3.06

$$\frac{Bb^3x^6-\left(3\,Bab^2-2\,Ab^3\right)x^4+2\,Ba^3-2\,Aa^2b-2\left(2\,Ba^2b-Aab^2\right)x^2+2\left(3\,Ba^3-2\,Aa^2b+\left(3\,Ba^2b-2\,Aab^2\right)x^2\right)\log\left(\right.}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^3*x^6 - (3*B*a*b^2 - 2*A*b^3)*x^4 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^2 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^2)
*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 0.798151, size = 78, normalized size = 0.95

$$\frac{Bx^4}{4b^2} + \frac{a(-2Ab + 3Ba)\log\left(a + bx^2\right)}{2b^4} + \frac{-Aa^2b + Ba^3}{2ab^4 + 2b^5x^2} - \frac{x^2(-Ab + 2Ba)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**4) + (-A*a**2*b
+ B*a**3)/(2*a*b**4 + 2*b**5*x**2) - x**2*(-A*b + 2*B*a)/(2*b**3)
```

Giac [A] time = 1.16731, size = 143, normalized size = 1.74

$$\frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{Bb^2x^4 - 4\,Babx^2 + 2\,Ab^2x^2}{4\,b^4} - \frac{3\,Ba^2bx^2 - 2\,Aab^2x^2 + 2\,Ba^3 - Aa^2b}{2\left(bx^2 + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(B*b^2*x^4 - 4*B*a*b*
x^2 + 2*A*b^2*x^2)/b^4 - 1/2*(3*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 2*B*a^3 - A*a
^2*b)/((b*x^2 + a)*b^4)
```

3.76 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$\frac{ax(Ab-aB)}{2b^3(a+bx^2)}+\frac{x(Ab-2aB)}{b^3}-\frac{\sqrt{a}(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{Bx^3}{3b^2}$$

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.0702195, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 1153, 205}

$$\frac{ax(Ab-aB)}{2b^3(a+bx^2)}+\frac{x(Ab-2aB)}{b^3}-\frac{\sqrt{a}(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)} - \frac{\int \frac{a(Ab-aB)-2b(Ab-aB)x^2-2b^2Bx^4}{a+bx^2}\,dx}{2b^3}$$

$$= \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)} - \frac{\int \left(-2(Ab-2aB)-2bBx^2+\frac{3aAb-5a^2B}{a+bx^2}\right)dx}{2b^3}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)} - \frac{(a(3Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{2b^3}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)} - \frac{\sqrt{a}(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.0718059, size = 89, normalized size = 1.02

$$\frac{x\left(aAb-a^2B\right)}{2b^3\left(a+bx^2\right)} + \frac{x(Ab-2aB)}{b^3} + \frac{\sqrt{a}(5aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + ((a*A*b - a^2*B)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.008, size = 105, normalized size = 1.2

$$\frac{Bx^3}{3\,b^2} + \frac{Ax}{b^2} - 2\,\frac{Bax}{b^3} + \frac{aAx}{2\,b^2\left(bx^2+a\right)} - \frac{a^2Bx}{2\,b^3\left(bx^2+a\right)} - \frac{3\,Aa}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{5\,a^2B}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/b^2*A*x-2/b^3*B*a*x+1/2*a/b^2*x/(b*x^2+a)*A-1/2*a^2/b^3*x/(
b*x^2+a)*B-3/2*a/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+5/2*a^2/b^3/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22287, size = 513, normalized size = 5.9

$$\left[\frac{4 Bb^2x^5 - 4\left(5 Bab - 3 Ab^2\right)x^3 - 3\left(5 Ba^2 - 3 Aab + \left(5 Bab - 3 Ab^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2 bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 6\left(5 Ba^2 - 3 Aab\right)x}{12\left(b^4x^2 + ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*b^2*x^5 - 4*(5*B*a*b - 3*A*b^2)*x^3 - 3*(5*B*a^2 - 3*A*a*b + (5*
B*a*b - 3*A*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) - 6*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3), 1/6*(2*B*b^2*x^5 - 2*(5
*B*a*b - 3*A*b^2)*x^3 + 3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3)
]
```

Sympy [A] time = 0.779591, size = 128, normalized size = 1.47

$$\frac{Bx^3}{3b^2} - \frac{x\left(-Aab + Ba^2\right)}{2ab^3 + 2b^4x^2} - \frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab + 5Ba)\log\left(-b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab + 5Ba)\log\left(b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4} - \frac{x\,(-A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) - x*(-A*a*b + B*a**2)/(2*a*b**3 + 2*b**4*x**2) - sqrt(-a/b*
*7)*(-3*A*b + 5*B*a)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**7)*(-3*A*b
 + 5*B*a)*log(b**3*sqrt(-a/b**7) + x)/4 - x*(-A*b + 2*B*a)/b**3
```

Giac [A] time = 1.11612, size = 119, normalized size = 1.37

$$\frac{\left(5 Ba^2 - 3 Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^3} - \frac{Ba^2x - Aabx}{2\left(bx^2 + a\right)b^3} + \frac{Bb^4x^3 - 6 Bab^3x + 3 Ab^4x}{3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(B*a^2*
x - A*a*b*x)/((b*x^2 + a)*b^3) + 1/3*(B*b^4*x^3 - 6*B*a*b^3*x + 3*A*b^4*x)/
b^6
```

3.77 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}+\frac{Bx^2}{2b^2}$$

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2*a*B)*Log[
a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.0576699, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}+\frac{Bx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2*a*B)*Log[
a + b*x^2])/(2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{Bx^2}{2b^2}+\frac{a(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0346349, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^2}+(Ab-2aB)\log\left(a+bx^2\right)+bBx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*B*x^2 + (a*(A*b - a*B))/(a + b*x^2) + (A*b - 2*a*B)*Log[a + b*x^2])/(2*b
^3)
```

Maple [A] time = 0.009, size = 74, normalized size = 1.2

$$\frac{Bx^2}{2\,b^2}+\frac{\ln\left(bx^2+a\right)A}{2\,b^2}-\frac{\ln\left(bx^2+a\right)Ba}{b^3}+\frac{aA}{2\,b^2\left(bx^2+a\right)}-\frac{a^2B}{2\,b^3\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2+1/2/b^2*ln(b*x^2+a)*A-1/b^3*ln(b*x^2+a)*B*a+1/2/b^2*a/(b*x^2+
a)*A-1/2/b^3*a^2/(b*x^2+a)*B
```

Maxima [A] time = 0.994239, size = 81, normalized size = 1.35

$$\frac{Bx^2}{2\,b^2}-\frac{Ba^2-Aab}{2\left(b^4x^2+ab^3\right)}-\frac{(2\,Ba-Ab)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b^2 - 1/2*(B*a^2 - A*a*b)/(b^4*x^2 + a*b^3) - 1/2*(2*B*a - A*b)*l
og(b*x^2 + a)/b^3
```

Fricas [A] time = 1.23948, size = 165, normalized size = 2.75

$$\frac{Bb^2x^4+Babx^2-Ba^2+Aab-\left(2\,Ba^2-Aab+\left(2\,Bab-Ab^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b^2*x^4 + B*a*b*x^2 - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b -
A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 0.702217, size = 56, normalized size = 0.93

$$\frac{Bx^2}{2b^2} - \frac{-Aab + Ba^2}{2ab^3 + 2b^4x^2} - \frac{(-Ab + 2Ba)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) - (-A*a*b + B*a**2)/(2*a*b**3 + 2*b**4*x**2) - (-A*b + 2*B*
a)*log(a + b*x**2)/(2*b**3)
```

Giac [A] time = 1.12768, size = 123, normalized size = 2.05

$$\frac{\frac{\left(bx^2+a\right)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^2+a} - \frac{Aab^2}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^2 + a) - A*a*b^2/(b*x^2 + a))/b^3)/b
```

3.78 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=67

$$-\frac{x(Ab-aB)}{2b^2(a+bx^2)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{Bx}{b^2}$$

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0504237, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 388, 205}

$$-\frac{x(Ab-aB)}{2b^2(a+bx^2)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ab-aB)x}{2b^2(a+bx^2)} - \frac{\int \frac{-Ab+aB-2bBx^2}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{Bx}{b^2} - \frac{(Ab-aB)x}{2b^2(a+bx^2)} + \frac{(Ab-3aB)\int \frac{1}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{Bx}{b^2} - \frac{(Ab-aB)x}{2b^2(a+bx^2)} + \frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

Mathematica [A] time = 0.0692784, size = 68, normalized size = 1.01

$$-\frac{x(Ab-aB)}{2b^2(a+bx^2)} - \frac{(3aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) - ((-(A*b) + 3*a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.007, size = 82, normalized size = 1.2

$$\frac{Bx}{b^2} - \frac{xA}{2b(bx^2+a)} + \frac{Bax}{2b^2(bx^2+a)} + \frac{A}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3Ba}{2b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] B*x/b^2-1/2/b*x/(b*x^2+a)*A+1/2/b^2*x/(b*x^2+a)*B*a+1/2/b/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*A-3/2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2665, size = 433, normalized size = 6.46

$$\left[\frac{4Bab^2x^3 + (3Ba^2 - Aab + (3Bab - Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) + 2(3Ba^2b - Aab^2)x}{4(ab^4x^2 + a^2b^3)}, \frac{2Bab^2x^3 - (3Ba^2 - Aab + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*a*b^2*x^3 + (3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^2*b - A*a*b^2)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*B*a*b^2*x^3 - (3*B*a^2 - A*a*b + (3*B*a*b - A
*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*B*a^2*b - A*a*b^2)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

Sympy [A] time = 0.666211, size = 114, normalized size = 1.7

$$\frac{Bx}{b^2} + \frac{x(-Ab + Ba)}{2ab^2 + 2b^3x^2} + \frac{\sqrt{-\frac{1}{ab^5}}\,(-Ab + 3Ba)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^5}}\,(-Ab + 3Ba)\log\left(ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(-A*
b + 3*B*a)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(-A*b +
 3*B*a)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4
```

Giac [A] time = 1.14648, size = 80, normalized size = 1.19

$$\frac{Bx}{b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Bax - Abx}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(B*
a*x - A*b*x)/((b*x^2 + a)*b^2)
```

3.79 $\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=41

$$\frac{B\log(a+bx^2)}{2b^2}-\frac{Ab-aB}{2b^2(a+bx^2)}$$

```
[Out] -(A*b - a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0363941, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{B\log(a+bx^2)}{2b^2}-\frac{Ab-aB}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(A*b - a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{Ab-aB}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0122215, size = 41, normalized size = 1.

$$\frac{aB-Ab}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-(A*b) + a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.008, size = 47, normalized size = 1.2

$$\frac{B \ln \left(b x^{2}+a\right)}{2 b^{2}}-\frac{A}{2 b \left(b x^{2}+a\right)}+\frac{B a}{2 b^{2} \left(b x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*B*ln(b*x^2+a)/b^2-1/2/b/(b*x^2+a)*A+1/2/b^2/(b*x^2+a)*B*a
```

Maxima [A] time = 1.03535, size = 54, normalized size = 1.32

$$\frac{B a - A b}{2 \left(b^{3} x^{2}+a b^{2}\right)}+\frac{B \log \left(b x^{2}+a\right)}{2 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)/(b^3*x^2 + a*b^2) + 1/2*B*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.22481, size = 92, normalized size = 2.24

$$\frac{B a - A b + \left(B b x^{2}+B a\right) \log \left(b x^{2}+a\right)}{2 \left(b^{3} x^{2}+a b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*a - A*b + (B*b*x^2 + B*a)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.507564, size = 36, normalized size = 0.88

$$\frac{B \log \left(a+b x^{2}\right)}{2 b^{2}}+\frac{-A b+B a}{2 a b^{2}+2 b^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**2) + (-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2)
```

Giac [A] time = 1.23471, size = 88, normalized size = 2.15

$$-\frac{B\left(\frac{\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^2+a\right)b}\right)}{2\,b}-\frac{A}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*A/((b*x^2 + a)*b)
```

3.80 $\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab(a+bx^2)}$$

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0220863, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx&=\frac{(Ab-aB)x}{2ab(a+bx^2)}+\frac{(Ab+aB)\int \frac{1}{a+bx^2}\,dx}{2ab}\\&=\frac{(Ab-aB)x}{2ab(a+bx^2)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0435718, size = 63, normalized size = 1.

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(aB-Ab)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(A*b) + a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.007, size = 68, normalized size = 1.1

$$\frac{(Ab-Ba)\,x}{2\,ab\left(bx^2+a\right)}+\frac{A}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+1
/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32335, size = 381, normalized size = 6.05

$$\left[-\frac{\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\left(Ba^2b-Aab^2\right)x}{4\left(a^2b^3x^2+a^3b^2\right)},\ \frac{\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\sqrt{ab}\,\text{arcta}}{2\left(a^2b^3x^2+a^3b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*(B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) - (B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] time = 0.542572, size = 112, normalized size = 1.78

$$-\frac{x\,(-Ab+Ba)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab+Ba)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab+Ba)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] -x*(-A*b + B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(A*b + B*
a)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(A*b + B*
a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A] time = 1.16405, size = 77, normalized size = 1.22

$$\frac{(Ba + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} - \frac{Bax - Abx}{2\left(bx^2 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*x - A*b*x)
/((b*x^2 + a)*a*b)
```

3.81 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/(2*a^
2)
```

Rubi [A] time = 0.0448555, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```


```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/(2*a^
2)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{Ab-aB}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0283369, size = 46, normalized size = 0.9

$$\frac{\frac{a(Ab-aB)}{b(a+bx^2)} - A\log\left(a+bx^2\right) + 2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^2)) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.011, size = 53, normalized size = 1.

$$\frac{A\ln(x)}{a^2} - \frac{A\ln\left(bx^2+a\right)}{2\,a^2} + \frac{A}{2\,a\left(bx^2+a\right)} - \frac{B}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2+1/2/a/(b*x^2+a)*A-1/2/b/(b*x^2+a)*B
```

Maxima [A] time = 1.02756, size = 69, normalized size = 1.35

$$-\frac{Ba-Ab}{2\left(ab^2x^2+a^2b\right)} - \frac{A\log\left(bx^2+a\right)}{2\,a^2} + \frac{A\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)/a^2 + 1/2*A*log
(x^2)/a^2
```

Fricas [A] time = 1.21234, size = 151, normalized size = 2.96

$$-\frac{Ba^2 - Aab + \left(Ab^2x^2 + Aab\right)\log\left(bx^2+a\right) - 2\left(Ab^2x^2 + Aab\right)\log(x)}{2\left(a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*a^2 - A*a*b + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b^2*x^2 + A
*a*b)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 0.55797, size = 46, normalized size = 0.9

$$\frac{A\log{(x)}}{a^2} - \frac{A\log{\left(\frac{a}{b} + x^2\right)}}{2a^2} - \frac{-Ab + Ba}{2a^2b + 2ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**2)/(2*a**2) - (-A*b + B*a)/(2*a**2*b + 2*a*b
**2*x**2)
```

Giac [A] time = 1.12279, size = 85, normalized size = 1.67

$$\frac{A\log\left(x^2\right)}{2\,a^2} - \frac{A\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{Ab^2x^2 - Ba^2 + 2\,Aab}{2\left(bx^2 + a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^2 - 1/2*A*log(abs(b*x^2 + a))/a^2 + 1/2*(A*b^2*x^2 - B*a^2
 + 2*A*a*b)/((b*x^2 + a)*a^2*b)
```

3.82 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}$$

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0511968, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 453, 205}

$$-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}dx = -\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)} - \frac{1}{2}\int \frac{-\frac{2A}{a}+\frac{(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)}dx$$

$$= -\frac{A}{a^2x} - \frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)} - \frac{(3Ab-aB)\int \frac{1}{a+bx^2}dx}{2a^2}$$

$$= -\frac{A}{a^2x} - \frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)} - \frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.0323544, size = 70, normalized size = 0.99

$$\frac{x(aB-Ab)}{2a^2\left(a+bx^2\right)} + \frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} - \frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + ((-(A*b) + a*B)*x)/(2*a^2*(a + b*x^2)) + ((-3*A*b + a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.008, size = 85, normalized size = 1.2

$$-\frac{A}{a^2x} - \frac{Abx}{2\,a^2\left(bx^2+a\right)} + \frac{xB}{2\,a\left(bx^2+a\right)} - \frac{3\,Ab}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{B}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -A/a^2/x-1/2/a^2*x/(b*x^2+a)*A*b+1/2/a*x/(b*x^2+a)*B-3/2/a^2/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))*A*b+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37724, size = 447, normalized size = 6.3

$$\left[-\frac{4\,Aa^2b - 2\left(Ba^2b - 3\,Aab^2\right)x^2 - \left(\left(Bab - 3\,Ab^2\right)x^3 + \left(Ba^2 - 3\,Aab\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(a^3b^2x^3 + a^4bx\right)}, -\frac{2\,Aa^2b - \left(Ba^2b - 3\,A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - ((B*a*b - 3*A*b^2)*x^3 + (
B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^2 - (
(B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

Sympy [A] time = 0.632495, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\left(-3Ab+Ba\right)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\left(-3Ab+Ba\right)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2Aa+x^2\left(-3Ab+Ba\right)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt
(-1/(a**5*b))*(-3*A*b + B*a)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*A*a +
x**2*(-3*A*b + B*a))/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A] time = 1.1373, size = 84, normalized size = 1.18

$$\frac{\left(Ba-3\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}+\frac{Bax^2-3\,Abx^2-2\,Aa}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(B*a*x^2 - 3*
A*b*x^2 - 2*A*a)/((b*x^3 + a*x)*a^2)
```

3.83 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{2a^2x^2}$$

```
[Out] -A/(2*a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.0745697, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{2a^2x^2}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.0462199, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^2}+(2Ab-aB)\log\left(a+bx^2\right)+2\log(x)(aB-2Ab)-\frac{aA}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) + (a*(-(A*b) + a*B))/(a + b*x^2) + 2*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.012, size = 86, normalized size = 1.1

$$-\frac{A}{2a^2x^2}-2\frac{A\ln(x)b}{a^3}+\frac{\ln(x)B}{a^2}+\frac{b\ln\left(bx^2+a\right)A}{a^3}-\frac{\ln\left(bx^2+a\right)B}{2a^2}-\frac{Ab}{2a^2\left(bx^2+a\right)}+\frac{B}{2a\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B+1/a^3*b*ln(b*x^2+a)*A-1/2/a^2*
ln(b*x^2+a)*B-1/2/a^2*b/(b*x^2+a)*A+1/2/a/(b*x^2+a)*B
```

Maxima [A] time = 0.992297, size = 103, normalized size = 1.36

$$\frac{(Ba-2Ab)x^2-Aa}{2\left(a^2bx^4+a^3x^2\right)}-\frac{(Ba-2Ab)\log\left(bx^2+a\right)}{2a^3}+\frac{(Ba-2Ab)\log\left(x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^3*x^2) - 1/2*(B*a - 2*A*b)*log
(b*x^2 + a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

Fricas [A] time = 1.19792, size = 248, normalized size = 3.26

$$-\frac{Aa^2-\left(Ba^2-2Aab\right)x^2+\left(\left(Bab-2Ab^2\right)x^4+\left(Ba^2-2Aab\right)x^2\right)\log\left(bx^2+a\right)-2\left(\left(Bab-2Ab^2\right)x^4+\left(Ba^2-2Aab\right)\right.}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*a^2 - (B*a^2 - 2*A*a*b)*x^2 + ((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A
*a*b)*x^2)*log(b*x^2 + a) - 2*((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A*a*b)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 1.03925, size = 70, normalized size = 0.92

$$\frac{-Aa+x^2\left(-2Ab+Ba\right)}{2a^3x^2+2a^2bx^4}+\frac{\left(-2Ab+Ba\right)\log\left(x\right)}{a^3}-\frac{\left(-2Ab+Ba\right)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-A*a + x**2*(-2*A*b + B*a))/(2*a**3*x**2 + 2*a**2*b*x**4) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

Giac [A] time = 1.1369, size = 111, normalized size = 1.46

$$\frac{\left(Ba-2\,Ab\right)\log\left(x^2\right)}{2\,a^3}+\frac{Bax^2-2\,Abx^2-Aa}{2\left(bx^4+ax^2\right)a^2}-\frac{\left(Bab-2\,Ab^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 2*A*b)*log(x^2)/a^3 + 1/2*(B*a*x^2 - 2*A*b*x^2 - A*a)/((b*x^4 +
a*x^2)*a^2) - 1/2*(B*a*b - 2*A*b^2)*log(abs(b*x^2 + a))/(a^3*b)
```

3.84 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}+\frac{\sqrt{b}(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{3a^2x^3}$$

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.105757, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 1261, 205}

$$\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}+\frac{\sqrt{b}(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```


```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```


Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)} - \frac{1}{2}b\int \frac{-\frac{2A}{ab}+\frac{2(Ab-aB)x^2}{a^2b}-\frac{(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx$$
$$= \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)} - \frac{1}{2}b\int \left(-\frac{2A}{a^2bx^4} - \frac{2(-2Ab+aB)}{a^3bx^2} + \frac{-5Ab+3aB}{a^3\left(a+bx^2\right)}\right)dx$$
$$= -\frac{A}{3a^2x^3} + \frac{2Ab-aB}{a^3x} + \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)} + \frac{(b(5Ab-3aB))\int \frac{1}{a+bx^2}\,dx}{2a^3}$$
$$= -\frac{A}{3a^2x^3} + \frac{2Ab-aB}{a^3x} + \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)} + \frac{\sqrt{b}(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.0732672, size = 90, normalized size = 1.

$$-\frac{bx(aB-Ab)}{2a^3\left(a+bx^2\right)} + \frac{2Ab-aB}{a^3x} - \frac{\sqrt{b}(3aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}} - \frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) - (b*(-(A*b) + a*B)*x)/(2*a^3*(a + b
*x^2)) - (Sqrt[b]*(-5*A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.01, size = 110, normalized size = 1.2

$$-\frac{A}{3\,a^2x^3} + 2\,\frac{Ab}{a^3x} - \frac{B}{a^2x} + \frac{b^2Ax}{2\,a^3\left(bx^2+a\right)} - \frac{bBx}{2\,a^2\left(bx^2+a\right)} + \frac{5\,b^2A}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3\,bB}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*A/a^2/x^3+2/a^3/x*A*b-1/a^2/x*B+1/2/a^3*b^2*x/(b*x^2+a)*A-1/2/a^2*b*x/
(b*x^2+a)*B+5/2/a^3*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-3/2/a^2*b/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32063, size = 532, normalized size = 5.91

$$\left[-\frac{6\left(3\,Bab-5\,Ab^2\right)x^4+4\,Aa^2+4\left(3\,Ba^2-5\,Aab\right)x^2+3\left(\left(3\,Bab-5\,Ab^2\right)x^5+\left(3\,Ba^2-5\,Aab\right)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\ldots}}{bx^2+\ldots}\right.}{12\left(a^3bx^5+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*a*b - 5*A*b^2)*x^4 + 4*A*a^2 + 4*(3*B*a^2 - 5*A*a*b)*x^2 + 3
*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(-b/a)*log((b*x^2
+ 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), -1/6*(3*(3*B*a
*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2 + 3*((3*B*a*b - 5*A
*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*
x^5 + a^4*x^3)]
```

Sympy [B] time = 0.803905, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\left(-5Ab+3Ba\right)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\left(-5Ab+3Ba\right)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4}-\frac{2Aa^2+x^4\left(-1\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(-a**4*sqrt(-b/a**7)*(-5*A*b + 3*B*a)/(-5
*A*b**2 + 3*B*a*b) + x)/4 - sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(a**4*sqrt(-b
/a**7)*(-5*A*b + 3*B*a)/(-5*A*b**2 + 3*B*a*b) + x)/4 - (2*A*a**2 + x**4*(-1
5*A*b**2 + 9*B*a*b) + x**2*(-10*A*a*b + 6*B*a**2))/(6*a**4*x**3 + 6*a**3*b*
x**5)
```

Giac [A] time = 1.14811, size = 115, normalized size = 1.28

$$-\frac{\left(3\,Bab-5\,Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}-\frac{Babx-Ab^2x}{2\left(bx^2+a\right)a^3}-\frac{3\,Bax^2-6\,Abx^2+Aa}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(B*a*b
*x - A*b^2*x)/((b*x^2 + a)*a^3) - 1/3*(3*B*a*x^2 - 6*A*b*x^2 + A*a)/(a^3*x^
3)
```

3.85 $\int \frac{A+Bx^2}{x^5(a+bx^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{4a^2x^4}$$

```
[Out] -A/(4*a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a^3*(a + b*
x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^2])
/(2*a^4)
```

Rubi [A] time = 0.0950933, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -A/(4*a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a^3*(a + b*
x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^2])
/(2*a^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{4a^2x^4}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.0971785, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^4}+\frac{2ab(aB-Ab)}{a+bx^2}+\frac{2a(aB-2Ab)}{x^2}+2b(3Ab-2aB)\log\left(a+bx^2\right)-4b\log(x)(3Ab-2aB)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -((a^2*A)/x^4 + (2*a*(-2*A*b + a*B))/x^2 + (2*a*b*(-(A*b) + a*B))/(a + b*x^
2) - 4*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^2])/(4*a^
4)
```

Maple [A] time = 0.013, size = 114, normalized size = 1.2

$$-\frac{A}{4a^2x^4}+\frac{Ab}{a^3x^2}-\frac{B}{2a^2x^2}+3\frac{A\ln(x)b^2}{a^4}-2\frac{bB\ln(x)}{a^3}-\frac{3b^2\ln\left(bx^2+a\right)A}{2a^4}+\frac{b\ln\left(bx^2+a\right)B}{a^3}+\frac{Ab^2}{2a^3\left(bx^2+a\right)}-\frac{}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^2,x)
```

```
[Out] -1/4*A/a^2/x^4+1/a^3/x^2*A*b-1/2/a^2/x^2*B+3*b^2/a^4*ln(x)*A-2*b/a^3*ln(x)*
B-3/2/a^4*b^2*ln(b*x^2+a)*A+1/a^3*b*ln(b*x^2+a)*B+1/2/a^3*b^2/(b*x^2+a)*A-1
/2/a^2*b/(b*x^2+a)*B
```

Maxima [A] time = 1.03495, size = 143, normalized size = 1.47

$$-\frac{2\left(2Bab-3Ab^2\right)x^4+Aa^2+\left(2Ba^2-3Aab\right)x^2}{4\left(a^3bx^6+a^4x^4\right)}+\frac{\left(2Bab-3Ab^2\right)\log\left(bx^2+a\right)}{2a^4}-\frac{\left(2Bab-3Ab^2\right)\log\left(x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*a*b - 3*A*b^2)*x^4 + A*a^2 + (2*B*a^2 - 3*A*a*b)*x^2)/(a^3*b*x
^6 + a^4*x^4) + 1/2*(2*B*a*b - 3*A*b^2)*log(b*x^2 + a)/a^4 - 1/2*(2*B*a*b -
 3*A*b^2)*log(x^2)/a^4
```

Fricas [A] time = 1.34228, size = 327, normalized size = 3.37

$$-\frac{2\left(2Ba^2b-3Aab^2\right)x^4+Aa^3+\left(2Ba^3-3Aa^2b\right)x^2-2\left(\left(2Bab^2-3Ab^3\right)x^6+\left(2Ba^2b-3Aab^2\right)x^4\right)\log\left(bx^2+a\right)+}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a^2*b - 3*A*a*b^2)*x^4 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^2 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(b*x^2 + a) +
```

```
 4*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(x))/(a^4*b
*x^6 + a^5*x^4)
```

Sympy [A] time = 1.3074, size = 100, normalized size = 1.03

$$-\frac{Aa^2 + x^4\left(-6Ab^2 + 4Bab\right) + x^2\left(-3Aab + 2Ba^2\right)}{4a^4x^4 + 4a^3bx^6} - \frac{b\left(-3Ab + 2Ba\right)\log\left(x\right)}{a^4} + \frac{b\left(-3Ab + 2Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

```
[Out] -(A*a**2 + x**4*(-6*A*b**2 + 4*B*a*b) + x**2*(-3*A*a*b + 2*B*a**2))/(4*a**4
*x**4 + 4*a**3*b*x**6) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a
)*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 1.11282, size = 203, normalized size = 2.09

$$-\frac{\left(2\,Bab - 3\,Ab^2\right)\log\left(x^2\right)}{2\,a^4} + \frac{\left(2\,Bab^2 - 3\,Ab^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^4b} - \frac{2\,Bab^2x^2 - 3\,Ab^3x^2 + 3\,Ba^2b - 4\,Aab^2}{2\,\left(bx^2 + a\right)a^4} + \frac{6\,Babx^4 - 9\,Ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*b - 3*A*b^2)*log(x^2)/a^4 + 1/2*(2*B*a*b^2 - 3*A*b^3)*log(abs(b
*x^2 + a))/(a^4*b) - 1/2*(2*B*a*b^2*x^2 - 3*A*b^3*x^2 + 3*B*a^2*b - 4*A*a*b
^2)/((b*x^2 + a)*a^4) + 1/4*(6*B*a*b*x^4 - 9*A*b^2*x^4 - 2*B*a^2*x^2 + 4*A*
a*b*x^2 - A*a^2)/(a^4*x^4)
```

3.86 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{A}{5a^2x^5}$$

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/(a^4*x) -
(b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A*b - 5*a*B)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] time = 0.18644, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 1802, 205}

$$-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/(a^4*x) -
(b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A*b - 5*a*B)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{1}{2}b^2\int \frac{-\frac{2A}{ab^2}+\frac{2(Ab-aB)x^2}{a^2b^2}-\frac{2(Ab-aB)x^4}{a^3b}+\frac{(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)}\,dx$$

$$= -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{1}{2}b^2\int \left(-\frac{2A}{a^2b^2x^6}-\frac{2(-2Ab+aB)}{a^3b^2x^4}+\frac{2(-3Ab+2aB)}{a^4bx^2}+\frac{7Ab-5aB}{a^4\left(a+bx^2\right)}\right)dx$$

$$= -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{\left(b^2(7Ab-5aB)\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}$$

$$= -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Mathematica [A] time = 0.0754468, size = 112, normalized size = 0.99

$$\frac{b^2x(aB-Ab)}{2a^4\left(a+bx^2\right)}+\frac{b^{3/2}(5aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(2aB-3Ab)}{a^4x}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(-3*A*b + 2*a*B))/(a^4*x) +
 (b^2*(-(A*b) + a*B)*x)/(2*a^4*(a + b*x^2)) + (b^(3/2)*(-7*A*b + 5*a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A] time = 0.011, size = 136, normalized size = 1.2

$$-\frac{A}{5\,a^2x^5}+\frac{2\,Ab}{3\,a^3x^3}-\frac{B}{3\,a^2x^3}-3\,\frac{Ab^2}{a^4x}+2\,\frac{Bb}{a^3x}-\frac{b^3xA}{2\,a^4\left(bx^2+a\right)}+\frac{b^2Bx}{2\,a^3\left(bx^2+a\right)}-\frac{7\,b^3A}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2B}{2\,a^3}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5*A/a^2/x^5+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B-3*b^2/a^4/x*A+2*b/a^3/x*B-1/2/
a^4*b^3*x/(b*x^2+a)*A+1/2/a^3*b^2*x/(b*x^2+a)*B-7/2/a^4*b^3/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*A+5/2/a^3*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31028, size = 653, normalized size = 5.78

$$\left[\frac{30\left(5\,Bab^2-7\,Ab^3\right)x^6+20\left(5\,Ba^2b-7\,Aab^2\right)x^4-12\,Aa^3-4\left(5\,Ba^3-7\,Aa^2b\right)x^2-15\left(\left(5\,Bab^2-7\,Ab^3\right)x^7+\left(5\,Ba^2b\right.\right.}{60\left(a^4bx^7+a^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*a*b^2 - 7*A*b^3)*x^6 + 20*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 12*A
*a^3 - 4*(5*B*a^3 - 7*A*a^2*b)*x^2 - 15*((5*B*a*b^2 - 7*A*b^3)*x^7 + (5*B*a
^2*b - 7*A*a*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^4*b*x^7 + a^5*x^5), 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B
*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2 + 15*((5*B*
a*b^2 - 7*A*b^3)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(b/a)*arctan(x*sqrt
(b/a)))/(a^4*b*x^7 + a^5*x^5)]
```

Sympy [B] time = 1.00349, size = 218, normalized size = 1.93

$$-\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{-6Aa^3+x^6\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log(-a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B
*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log
(a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + (
-6*A*a**3 + x**6*(-105*A*b**3 + 75*B*a*b**2) + x**4*(-70*A*a*b**2 + 50*B*a*
*2*b) + x**2*(14*A*a**2*b - 10*B*a**3))/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A] time = 1.14432, size = 151, normalized size = 1.34

$$\frac{\left(5\,Bab^2-7\,Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}+\frac{Bab^2x-Ab^3x}{2\left(bx^2+a\right)a^4}+\frac{30\,Babx^4-45\,Ab^2x^4-5\,Ba^2x^2+10\,Aabx^2-3\,Aa^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a*b^2 - 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*(B*a*
b^2*x - A*b^3*x)/((b*x^2 + a)*a^4) + 1/15*(30*B*a*b*x^4 - 45*A*b^2*x^4 - 5*
B*a^2*x^2 + 10*A*a*b*x^2 - 3*A*a^2)/(a^4*x^5)
```

3.87 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}+\frac{b^2(4Ab-3aB)\log\left(a+bx^2\right)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/(2*a^4*x^2
) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b - 3*a*B)*Log[x])/a^
5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a^5)
```

Rubi [A] time = 0.129539, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}+\frac{b^2(4Ab-3aB)\log\left(a+bx^2\right)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/(2*a^4*x^2
) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b - 3*a*B)*Log[x])/a^
5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x^4}+\frac{-2Ab+aB}{a^3x^3}-\frac{b(-3Ab+2aB)}{a^4x^2}+\frac{b^2(-4Ab+3aB)}{a^5x}-\frac{b^3(-Ab+aB)}{a^4(a+bx)^2}-\frac{b^3(-4A}{a^5(}\right.\right.\\ &= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{4a^3x^4}-\frac{b(3Ab-2aB)}{2a^4x^2}-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab-3aB)\log}{2a^5}\end{aligned}$$

Mathematica [A] time = 0.0984883, size = 110, normalized size = 0.89

$$\frac{-\frac{3a^2(aB-2Ab)}{x^4}-\frac{2a^3A}{x^6}+\frac{6ab^2(aB-Ab)}{a+bx^2}+6b^2(4Ab-3aB)\log\left(a+bx^2\right)+12b^2\log(x)(3aB-4Ab)+\frac{6ab(2aB-3Ab)}{x^2}}{12a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-2*A*b + a*B))/x^4 + (6*a*b*(-3*A*b + 2*a*B))/x^2
 + (6*a*b^2*(-(A*b) + a*B))/(a + b*x^2) + 12*b^2*(-4*A*b + 3*a*B)*Log[x] +
6*b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^5)
```

Maple [A] time = 0.014, size = 143, normalized size = 1.2

$$-\frac{A}{6\,a^2x^6}+\frac{Ab}{2\,a^3x^4}-\frac{B}{4\,a^2x^4}-\frac{3\,Ab^2}{2\,a^4x^2}+\frac{Bb}{a^3x^2}-4\,\frac{b^3\ln(x)\,A}{a^5}+3\,\frac{b^2B\ln(x)}{a^4}+2\,\frac{b^3\ln\left(bx^2+a\right)A}{a^5}-\frac{3\,b^2\ln\left(bx^2+a\right)B}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/6*A/a^2/x^6+1/2/a^3/x^4*A*b-1/4/a^2/x^4*B-3/2*b^2/a^4/x^2*A+b/a^3/x^2*B-
4*b^3/a^5*ln(x)*A+3*b^2/a^4*ln(x)*B+2/a^5*b^3*ln(b*x^2+a)*A-3/2/a^4*b^2*ln(
b*x^2+a)*B-1/2/a^4*b^3/(b*x^2+a)*A+1/2/a^3*b^2/(b*x^2+a)*B
```

Maxima [A] time = 0.980521, size = 184, normalized size = 1.48

$$\frac{6\left(3\,Bab^2-4\,Ab^3\right)x^6+3\left(3\,Ba^2b-4\,Aab^2\right)x^4-2\,Aa^3-\left(3\,Ba^3-4\,Aa^2b\right)x^2}{12\left(a^4bx^8+a^5x^6\right)}-\frac{\left(3\,Bab^2-4\,Ab^3\right)\log\left(bx^2+a\right)}{2\,a^5}+\frac{(3\,Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(3*B*a*b^2 - 4*A*b^3)*x^6 + 3*(3*B*a^2*b - 4*A*a*b^2)*x^4 - 2*A*a^3
 - (3*B*a^3 - 4*A*a^2*b)*x^2)/(a^4*b*x^8 + a^5*x^6) - 1/2*(3*B*a*b^2 - 4*A*
b^3)*log(b*x^2 + a)/a^5 + 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5
```

Fricas [A] time = 1.41584, size = 385, normalized size = 3.1

$$\frac{6\left(3\,Ba^2b^2-4\,Aab^3\right)x^6-2\,Aa^4+3\left(3\,Ba^3b-4\,Aa^2b^2\right)x^4-\left(3\,Ba^4-4\,Aa^3b\right)x^2-6\left(\left(3\,Bab^3-4\,Ab^4\right)x^8+\left(3\,Ba^2b^2-\right.\right.}{12\left(a^5bx^8+a^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 2*A*a^4 + 3*(3*B*a^3*b - 4*A*a^2*b^
2)*x^4 - (3*B*a^4 - 4*A*a^3*b)*x^2 - 6*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3*B*a^
```

```
2*b^2 - 4*A*a*b^3)*x^6)*log(b*x^2 + a) + 12*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3
*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(x))/(a^5*b*x^8 + a^6*x^6)
```

Sympy [A] time = 1.58456, size = 129, normalized size = 1.04

$$\frac{-2Aa^3 + x^6\left(-24Ab^3 + 18Bab^2\right) + x^4\left(-12Aab^2 + 9Ba^2b\right) + x^2\left(4Aa^2b - 3Ba^3\right)}{12a^5x^6 + 12a^4bx^8} + \frac{b^2\left(-4Ab + 3Ba\right)\log\left(x\right)}{a^5} - \frac{b^2\left(-4Ab + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-2*A*a**3 + x**6*(-24*A*b**3 + 18*B*a*b**2) + x**4*(-12*A*a*b**2 + 9*B*a**
2*b) + x**2*(4*A*a**2*b - 3*B*a**3))/(12*a**5*x**6 + 12*a**4*b*x**8) + b**2
*(-4*A*b + 3*B*a)*log(x)/a**5 - b**2*(-4*A*b + 3*B*a)*log(a/b + x**2)/(2*a*
*5)
```

Giac [A] time = 1.62136, size = 240, normalized size = 1.94

$$\frac{\left(3\,Bab^2 - 4\,Ab^3\right)\log\left(x^2\right)}{2\,a^5} - \frac{\left(3\,Bab^3 - 4\,Ab^4\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^5b} + \frac{3\,Bab^3x^2 - 4\,Ab^4x^2 + 4\,Ba^2b^2 - 5\,Aab^3}{2\left(bx^2 + a\right)a^5} - \frac{33\,Bab^2x^6 - 44}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5 - 1/2*(3*B*a*b^3 - 4*A*b^4)*log(abs(
b*x^2 + a))/(a^5*b) + 1/2*(3*B*a*b^3*x^2 - 4*A*b^4*x^2 + 4*B*a^2*b^2 - 5*A*
a*b^3)/((b*x^2 + a)*a^5) - 1/12*(33*B*a*b^2*x^6 - 44*A*b^3*x^6 - 12*B*a^2*b
*x^4 + 18*A*a*b^2*x^4 + 3*B*a^3*x^2 - 6*A*a^2*b*x^2 + 2*A*a^3)/(a^5*x^6)
```

3.88 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=150

$$\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx^2)}+\frac{a^5(Ab-aB)}{4b^7(a+bx^2)^2}-\frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}+\frac{x^6(Ab-3aB)}{6b^4}-\frac{3ax^4(Ab-2}{4b^5}$$

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) + ((A*b - 3
*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B))/(4*b^7*(a + b*x^2)
^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2)) - (5*a^3*(2*A*b - 3*a*B)*Lo
g[a + b*x^2])/(2*b^7)
```

Rubi [A] time = 0.230432, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx^2)}+\frac{a^5(Ab-aB)}{4b^7(a+bx^2)^2}-\frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}+\frac{x^6(Ab-3aB)}{6b^4}-\frac{3ax^4(Ab-2}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) + ((A*b - 3
*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B))/(4*b^7*(a + b*x^2)
^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2)) - (5*a^3*(2*A*b - 3*a*B)*Lo
g[a + b*x^2])/(2*b^7)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{2a^2(-3Ab+5aB)}{b^6}+\frac{3a(-Ab+2aB)x}{b^5}+\frac{(Ab-3aB)x^2}{b^4}+\frac{Bx^3}{b^3}+\frac{a^5(-Ab+aB)}{b^6(a+bx)^3}-\right.\right.$$
$$= \frac{a^2(3Ab-5aB)x^2}{b^6}-\frac{3a(Ab-2aB)x^4}{4b^5}+\frac{(Ab-3aB)x^6}{6b^4}+\frac{Bx^8}{8b^3}+\frac{a^5(Ab-aB)}{4b^7\left(a+bx^2\right)^2}-\frac{a^4(5Ab-6aB)}{2b^7\left(a+bx^2\right)}-$$

Mathematica [A] time = 0.087402, size = 136, normalized size = 0.91

$$\frac{-24a^2bx^2(5aB-3Ab)+\frac{12a^4(6aB-5Ab)}{a+bx^2}+\frac{6a^5(Ab-aB)}{\left(a+bx^2\right)^2}+60a^3(3aB-2Ab)\log\left(a+bx^2\right)+4b^3x^6(Ab-3aB)+18ab^2x^4(2aB-}{24b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-24*a^2*b*(-3*A*b + 5*a*B)*x^2 + 18*a*b^2*(-(A*b) + 2*a*B)*x^4 + 4*b^3*(A*
b - 3*a*B)*x^6 + 3*b^4*B*x^8 + (6*a^5*(A*b - a*B))/(a + b*x^2)^2 + (12*a^4*
(-5*A*b + 6*a*B))/(a + b*x^2) + 60*a^3*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(24
*b^7)
```

Maple [A] time = 0.012, size = 182, normalized size = 1.2

$$\frac{Bx^8}{8\,b^3}+\frac{x^6A}{6\,b^3}-\frac{x^6Ba}{2\,b^4}-\frac{3\,x^4Aa}{4\,b^4}+\frac{3\,x^4Ba^2}{2\,b^5}+3\,\frac{a^2Ax^2}{b^5}-5\,\frac{Bx^2a^3}{b^6}+\frac{a^5A}{4\,b^6\left(bx^2+a\right)^2}-\frac{Ba^6}{4\,b^7\left(bx^2+a\right)^2}-5\,\frac{a^3\ln\left(bx^2+a\right)A}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/8*B*x^8/b^3+1/6/b^3*x^6*A-1/2/b^4*x^6*B*a-3/4/b^4*x^4*A*a+3/2/b^5*x^4*B*a
^2+3/b^5*A*x^2*a^2-5/b^6*B*x^2*a^3+1/4*a^5/b^6/(b*x^2+a)^2*A-1/4*a^6/b^7/(b
*x^2+a)^2*B-5*a^3/b^6*ln(b*x^2+a)*A+15/2*a^4/b^7*ln(b*x^2+a)*B-5/2*a^4/b^6/
(b*x^2+a)*A+3*a^5/b^7/(b*x^2+a)*B
```

Maxima [A] time = 1.02869, size = 223, normalized size = 1.49

$$\frac{11\,Ba^6-9\,Aa^5b+2\left(6\,Ba^5b-5\,Aa^4b^2\right)x^2}{4\left(b^9x^4+2\,ab^8x^2+a^2b^7\right)}+\frac{3\,Bb^3x^8-4\left(3\,Bab^2-Ab^3\right)x^6+18\left(2\,Ba^2b-Aab^2\right)x^4-24\left(5\,Ba^3-3\,Aa^2b\right)}{24\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(11*B*a^6 - 9*A*a^5*b + 2*(6*B*a^5*b - 5*A*a^4*b^2)*x^2)/(b^9*x^4 + 2*a
*b^8*x^2 + a^2*b^7) + 1/24*(3*B*b^3*x^8 - 4*(3*B*a*b^2 - A*b^3)*x^6 + 18*(2
*B*a^2*b - A*a*b^2)*x^4 - 24*(5*B*a^3 - 3*A*a^2*b)*x^2)/b^6 + 5/2*(3*B*a^4
```

```
- 2*A*a^3*b)*log(b*x^2 + a)/b^7
```

Fricas [A] time = 1.48384, size = 489, normalized size = 3.26

$$\frac{3\,Bb^6x^{12} - 2\left(3\,Bab^5 - 2\,Ab^6\right)x^{10} + 5\left(3\,Ba^2b^4 - 2\,Aab^5\right)x^8 + 66\,Ba^6 - 54\,Aa^5b - 20\left(3\,Ba^3b^3 - 2\,Aa^2b^4\right)x^6 - 6\left(34\,Ba\right.}{24\left(b^9x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^6*x^12 - 2*(3*B*a*b^5 - 2*A*b^6)*x^10 + 5*(3*B*a^2*b^4 - 2*A*a*
b^5)*x^8 + 66*B*a^6 - 54*A*a^5*b - 20*(3*B*a^3*b^3 - 2*A*a^2*b^4)*x^6 - 6*(
34*B*a^4*b^2 - 21*A*a^3*b^3)*x^4 - 12*(4*B*a^5*b - A*a^4*b^2)*x^2 + 60*(3*B
*a^6 - 2*A*a^5*b + (3*B*a^4*b^2 - 2*A*a^3*b^3)*x^4 + 2*(3*B*a^5*b - 2*A*a^4
*b^2)*x^2)*log(b*x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A] time = 1.95257, size = 163, normalized size = 1.09

$$\frac{Bx^8}{8b^3} + \frac{5a^3\left(-2Ab + 3Ba\right)\log\left(a + bx^2\right)}{2b^7} + \frac{-9Aa^5b + 11Ba^6 + x^2\left(-10Aa^4b^2 + 12Ba^5b\right)}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4} - \frac{x^6\left(-Ab + 3Ba\right)}{6b^4} + \frac{x^4\left(-3Aa\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**8/(8*b**3) + 5*a**3*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**7) + (-9*A*
a**5*b + 11*B*a**6 + x**2*(-10*A*a**4*b**2 + 12*B*a**5*b))/(4*a**2*b**7 + 8
*a*b**8*x**2 + 4*b**9*x**4) - x**6*(-A*b + 3*B*a)/(6*b**4) + x**4*(-3*A*a*b
 + 6*B*a**2)/(4*b**5) - x**2*(-3*A*a**2*b + 5*B*a**3)/b**6
```

Giac [A] time = 1.12296, size = 247, normalized size = 1.65

$$\frac{5\left(3\,Ba^4 - 2\,Aa^3b\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^7} - \frac{45\,Ba^4b^2x^4 - 30\,Aa^3b^3x^4 + 78\,Ba^5bx^2 - 50\,Aa^4b^2x^2 + 34\,Ba^6 - 21\,Aa^5b}{4\left(bx^2 + a\right)^2b^7} + \frac{3\,Bb^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/2*(3*B*a^4 - 2*A*a^3*b)*log(abs(b*x^2 + a))/b^7 - 1/4*(45*B*a^4*b^2*x^4 -
 30*A*a^3*b^3*x^4 + 78*B*a^5*b*x^2 - 50*A*a^4*b^2*x^2 + 34*B*a^6 - 21*A*a^5
*b)/((b*x^2 + a)^2*b^7) + 1/24*(3*B*b^9*x^8 - 12*B*a*b^8*x^6 + 4*A*b^9*x^6
+ 36*B*a^2*b^7*x^4 - 18*A*a*b^8*x^4 - 120*B*a^3*b^6*x^2 + 72*A*a^2*b^7*x^2)
/b^12
```

3.89 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=128

$$\frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)}-\frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}+\frac{x^4(Ab-3aB)}{4b^4}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{Bx^6}{6b^3}$$

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) + (B*x^6)/(6
*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*(4*A*b - 5*a*B))/(2*
b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/b^6
```

Rubi [A] time = 0.16556, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)}-\frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}+\frac{x^4(Ab-3aB)}{4b^4}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) + (B*x^6)/(6
*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*(4*A*b - 5*a*B))/(2*
b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/b^6
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{3a(-Ab+2aB)}{b^5}+\frac{(Ab-3aB)x}{b^4}+\frac{Bx^2}{b^3}-\frac{a^4(-Ab+aB)}{b^5(a+bx)^3}+\frac{a^3(-4Ab+5aB)}{b^5(a+bx)^2}-\frac{2a^2(-}{b^5}\right.\right.$$

$$=-\frac{3a(Ab-2aB)x^2}{2b^5}+\frac{(Ab-3aB)x^4}{4b^4}+\frac{Bx^6}{6b^3}-\frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)}+\frac{a^2(3Ab-5aB)\log}{b^6}$$

Mathematica [A] time = 0.0737771, size = 116, normalized size = 0.91

$$\frac{\frac{6a^3(4Ab-5aB)}{a+bx^2}+\frac{3a^4(aB-Ab)}{(a+bx^2)^2}+12a^2(3Ab-5aB)\log\left(a+bx^2\right)+3b^2x^4(Ab-3aB)+18abx^2(2aB-Ab)+2b^3Bx^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (18*a*b*(-(A*b) + 2*a*B)*x^2 + 3*b^2*(A*b - 3*a*B)*x^4 + 2*b^3*B*x^6 + (3*a
^4*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a^3*(4*A*b - 5*a*B))/(a + b*x^2) + 12
*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/(12*b^6)
```

Maple [A] time = 0.011, size = 158, normalized size = 1.2

$$\frac{Bx^6}{6b^3}+\frac{Ax^4}{4b^3}-\frac{3Bx^4a}{4b^4}-\frac{3aAx^2}{2b^4}+3\frac{Bx^2a^2}{b^5}-\frac{a^4A}{4b^5\left(bx^2+a\right)^2}+\frac{Ba^5}{4b^6\left(bx^2+a\right)^2}+3\frac{a^2\ln\left(bx^2+a\right)A}{b^5}-5\frac{a^3\ln\left(bx^2+a\right.}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/4/b^3*A*x^4-3/4/b^4*B*x^4*a-3/2/b^4*A*x^2*a+3/b^5*B*x^2*a^2
-1/4*a^4/b^5/(b*x^2+a)^2*A+1/4*a^5/b^6/(b*x^2+a)^2*B+3*a^2/b^5*ln(b*x^2+a)*
A-5*a^3/b^6*ln(b*x^2+a)*B+2*a^3/b^5/(b*x^2+a)*A-5/2*a^4/b^6/(b*x^2+a)*B
```

Maxima [A] time = 1.02698, size = 190, normalized size = 1.48

$$-\frac{9Ba^5-7Aa^4b+2\left(5Ba^4b-4Aa^3b^2\right)x^2}{4\left(b^8x^4+2ab^7x^2+a^2b^6\right)}+\frac{2Bb^2x^6-3\left(3Bab-Ab^2\right)x^4+18\left(2Ba^2-Aab\right)x^2}{12b^5}-\frac{\left(5Ba^3-3Aa^2b\right)\mathrm{l}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x^2)/(b^8*x^4 + 2*a
*b^7*x^2 + a^2*b^6) + 1/12*(2*B*b^2*x^6 - 3*(3*B*a*b - A*b^2)*x^4 + 18*(2*B
*a^2 - A*a*b)*x^2)/b^5 - (5*B*a^3 - 3*A*a^2*b)*log(b*x^2 + a)/b^6
```

Fricas [A] time = 1.24628, size = 431, normalized size = 3.37

$$\frac{2Bb^5x^{10}-\left(5Bab^4-3Ab^5\right)x^8+4\left(5Ba^2b^3-3Aab^4\right)x^6-27Ba^5+21Aa^4b+3\left(21Ba^3b^2-11Aa^2b^3\right)x^4+6\left(Ba^4b+\right.}{12\left(b^8x^4+2ab^7x^2+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^10 - (5*B*a*b^4 - 3*A*b^5)*x^8 + 4*(5*B*a^2*b^3 - 3*A*a*b^4
)*x^6 - 27*B*a^5 + 21*A*a^4*b + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 6*(B*
```

```
a^4*b + A*a^3*b^2)*x^2 - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^2*b
^3)*x^4 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x^2)*log(b*x^2 + a))/(b^8*x^4 + 2*a*b
^7*x^2 + a^2*b^6)
```

Sympy [A] time = 1.79809, size = 138, normalized size = 1.08

$$\frac{Bx^6}{6b^3} - \frac{a^2(-3Ab+5Ba)\log\left(a+bx^2\right)}{b^6} - \frac{-7Aa^4b+9Ba^5+x^2\left(-8Aa^3b^2+10Ba^4b\right)}{4a^2b^6+8ab^7x^2+4b^8x^4} - \frac{x^4(-Ab+3Ba)}{4b^4} + \frac{x^2\left(-3Aab+6B\right.}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) - a**2*(-3*A*b + 5*B*a)*log(a + b*x**2)/b**6 - (-7*A*a**4*b
 + 9*B*a**5 + x**2*(-8*A*a**3*b**2 + 10*B*a**4*b))/(4*a**2*b**6 + 8*a*b**7*
x**2 + 4*b**8*x**4) - x**4*(-A*b + 3*B*a)/(4*b**4) + x**2*(-3*A*a*b + 6*B*a
**2)/(2*b**5)
```

Giac [A] time = 1.10819, size = 215, normalized size = 1.68

$$-\frac{\left(5\,Ba^3-3\,Aa^2b\right)\log\left(\left|bx^2+a\right|\right)}{b^6} + \frac{30\,Ba^3b^2x^4-18\,Aa^2b^3x^4+50\,Ba^4bx^2-28\,Aa^3b^2x^2+21\,Ba^5-11\,Aa^4b}{4\left(bx^2+a\right)^2b^6} + \frac{2\,Bb^6x^6-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^6 + 1/4*(30*B*a^3*b^2*x^4 - 18
*A*a^2*b^3*x^4 + 50*B*a^4*b*x^2 - 28*A*a^3*b^2*x^2 + 21*B*a^5 - 11*A*a^4*b)
/((b*x^2 + a)^2*b^6) + 1/12*(2*B*b^6*x^6 - 9*B*a*b^5*x^4 + 3*A*b^6*x^4 + 36
*B*a^2*b^4*x^2 - 18*A*a*b^5*x^2)/b^9
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}+\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}+\frac{x^2(Ab-3aB)}{2b^4}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{Bx^4}{4b^3}$$

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))/(4*b^5*(a
 + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)) - (3*a*(A*b - 2*a*
B)*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.121235, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}+\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}+\frac{x^2(Ab-3aB)}{2b^4}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{Bx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))/(4*b^5*(a
 + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)) - (3*a*(A*b - 2*a*
B)*Log[a + b*x^2])/(2*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{(Ab-3aB)x^2}{2b^4}+\frac{Bx^4}{4b^3}+\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.0652748, size = 94, normalized size = 0.86

$$\frac{\frac{2a^2(4aB-3Ab)}{a+bx^2}+\frac{a^3(Ab-aB)}{(a+bx^2)^2}+2bx^2(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^2 + b^2*B*x^4 + (a^3*(A*b - a*B))/(a + b*x^2)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^2) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^2])/(4*
b^5)
```

Maple [A] time = 0.011, size = 134, normalized size = 1.2

$$\frac{Bx^4}{4\,b^3}-\frac{3\,Bx^2a}{2\,b^4}+\frac{Ax^2}{2\,b^3}+\frac{a^3A}{4\,b^4\left(bx^2+a\right)^2}-\frac{Ba^4}{4\,b^5\left(bx^2+a\right)^2}-\frac{3\,a\ln\left(bx^2+a\right)A}{2\,b^4}+3\,\frac{a^2\ln\left(bx^2+a\right)B}{b^5}-\frac{3\,Aa^2}{2\,b^4\left(bx^2+a\right)}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*B*x^4/b^3-3/2/b^4*B*x^2*a+1/2/b^3*A*x^2+1/4*a^3/b^4/(b*x^2+a)^2*A-1/4*a
^4/b^5/(b*x^2+a)^2*B-3/2*a/b^4*ln(b*x^2+a)*A+3*a^2/b^5*ln(b*x^2+a)*B-3/2*a^
2/b^4/(b*x^2+a)*A+2*a^3/b^5/(b*x^2+a)*B
```

Maxima [A] time = 0.99354, size = 157, normalized size = 1.44

$$\frac{7\,Ba^4-5\,Aa^3b+2\left(4\,Ba^3b-3\,Aa^2b^2\right)x^2}{4\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}+\frac{Bbx^4-2\,(3\,Ba-Ab)x^2}{4\,b^4}+\frac{3\left(2\,Ba^2-Aab\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^2)/(b^7*x^4 + 2*a*
b^6*x^2 + a^2*b^5) + 1/4*(B*b*x^4 - 2*(3*B*a - A*b)*x^2)/b^4 + 3/2*(2*B*a^2
 - A*a*b)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 1.24866, size = 360, normalized size = 3.3

$$\frac{Bb^4x^8-2\left(2\,Bab^3-Ab^4\right)x^6+7\,Ba^4-5\,Aa^3b-\left(11\,Ba^2b^2-4\,Aab^3\right)x^4+2\left(Ba^3b-2\,Aa^2b^2\right)x^2+6\left(2\,Ba^4-Aa^3b+(2\,Ba\right.}{4\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^4*x^8 - 2*(2*B*a*b^3 - A*b^4)*x^6 + 7*B*a^4 - 5*A*a^3*b - (11*B*a^
2*b^2 - 4*A*a*b^3)*x^4 + 2*(B*a^3*b - 2*A*a^2*b^2)*x^2 + 6*(2*B*a^4 - A*a^3
```

```
*b + (2*B*a^2*b^2 - A*a*b^3)*x^4 + 2*(2*B*a^3*b - A*a^2*b^2)*x^2)*log(b*x^2
 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

Sympy [A] time = 1.65688, size = 116, normalized size = 1.06

$$\frac{Bx^4}{4b^3} + \frac{3a(-Ab + 2Ba)\log\left(a + bx^2\right)}{2b^5} + \frac{-5Aa^3b + 7Ba^4 + x^2\left(-6Aa^2b^2 + 8Ba^3b\right)}{4a^2b^5 + 8ab^6x^2 + 4b^7x^4} - \frac{x^2(-Ab + 3Ba)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x**2)/(2*b**5) + (-5*A*a**3*
b + 7*B*a**4 + x**2*(-6*A*a**2*b**2 + 8*B*a**3*b))/(4*a**2*b**5 + 8*a*b**6*
x**2 + 4*b**7*x**4) - x**2*(-A*b + 3*B*a)/(2*b**4)
```

Giac [A] time = 1.12029, size = 178, normalized size = 1.63

$$\frac{3\left(2Ba^2 - Aab\right)\log\left(\left|bx^2 + a\right|\right)}{2b^5} + \frac{Bb^3x^4 - 6Bab^2x^2 + 2Ab^3x^2}{4b^6} - \frac{18Ba^2b^2x^4 - 9Aab^3x^4 + 28Ba^3bx^2 - 12Aa^2b^2x^2 +}{4\left(bx^2 + a\right)^2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*a^2 - A*a*b)*log(abs(b*x^2 + a))/b^5 + 1/4*(B*b^3*x^4 - 6*B*a*b^2*
x^2 + 2*A*b^3*x^2)/b^6 - 1/4*(18*B*a^2*b^2*x^4 - 9*A*a*b^3*x^4 + 28*B*a^3*b
*x^2 - 12*A*a^2*b^2*x^2 + 11*B*a^4 - 4*A*a^3*b)/((b*x^2 + a)^2*b^5)
```

3.91 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=88

$$-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2*A*b - 3*a
*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.09024, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2*A*b - 3*a
*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{Bx^2}{2b^3}-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0368467, size = 92, normalized size = 1.05

$$\frac{2aAb-3a^2B}{2b^4\left(a+bx^2\right)}+\frac{a^3B-a^2Ab}{4b^4\left(a+bx^2\right)^2}+\frac{(Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) + (-(a^2*A*b) + a^3*B)/(4*b^4*(a + b*x^2)^2) + (2*a*A*b - 3
*a^2*B)/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Maple [A] time = 0.009, size = 109, normalized size = 1.2

$$\frac{Bx^2}{2\,b^3}-\frac{a^2A}{4\,b^3\left(bx^2+a\right)^2}+\frac{Ba^3}{4\,b^4\left(bx^2+a\right)^2}+\frac{\ln\left(bx^2+a\right)A}{2\,b^3}-\frac{3\,\ln\left(bx^2+a\right)Ba}{2\,b^4}+\frac{aA}{b^3\left(bx^2+a\right)}-\frac{3\,a^2B}{2\,b^4\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/b^3-1/4/b^3*a^2/(b*x^2+a)^2*A+1/4/b^4*a^3/(b*x^2+a)^2*B+1/2/b^3*l
n(b*x^2+a)*A-3/2/b^4*ln(b*x^2+a)*B*a+1/b^3*a/(b*x^2+a)*A-3/2/b^4*a^2/(b*x^2
+a)*B
```

Maxima [A] time = 1.0209, size = 127, normalized size = 1.44

$$-\frac{5\,Ba^3-3\,Aa^2b+2\left(3\,Ba^2b-2\,Aab^2\right)x^2}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{Bx^2}{2\,b^3}-\frac{(3\,Ba-Ab)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^2)/(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4) + 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.16418, size = 289, normalized size = 3.28

$$\frac{2\,Bb^3x^6+4\,Bab^2x^4-5\,Ba^3+3\,Aa^2b-4\left(Ba^2b-Aab^2\right)x^2-2\left(\left(3\,Bab^2-Ab^3\right)x^4+3\,Ba^3-Aa^2b+2\left(3\,Ba^2b-Aab^2\right)\right.\right.}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^3*x^6 + 4*B*a*b^2*x^4 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^2 - 2*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
```

```
*a*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A] time = 1.44463, size = 94, normalized size = 1.07

$$\frac{Bx^2}{2b^3} - \frac{-3Aa^2b + 5Ba^3 + x^2\left(-4Aab^2 + 6Ba^2b\right)}{4a^2b^4 + 8ab^5x^2 + 4b^6x^4} - \frac{(-Ab + 3Ba)\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) - (-3*A*a**2*b + 5*B*a**3 + x**2*(-4*A*a*b**2 + 6*B*a**2*b)
)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - (-A*b + 3*B*a)*log(a + b*x*
*2)/(2*b**4)
```

Giac [A] time = 1.14964, size = 126, normalized size = 1.43

$$\frac{Bx^2}{2\,b^3} - \frac{(3\,Ba - Ab)\log\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{9\,Bab^2x^4 - 3\,Ab^3x^4 + 12\,Ba^2bx^2 - 2\,Aab^2x^2 + 4\,Ba^3}{4\left(bx^2 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(9*B*a*b^2*
x^4 - 3*A*b^3*x^4 + 12*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 4*B*a^3)/((b*x^2 + a)^
2*b^4)
```

3.92 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a + b*x^2)) +
 (B*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.06588, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a + b*x^2)) +
 (B*Log[a + b*x^2])/(2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{B}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0240274, size = 64, normalized size = 0.97

$$\frac{3a^2B-ab\left(A-4Bx^2\right)+2B\left(a+bx^2\right)^2\log\left(a+bx^2\right)-2Ab^2x^2}{4b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^2 - a*b*(A - 4*B*x^2) + 2*B*(a + b*x^2)^2*Log[a + b*x^
2]])/(4*b^3*(a + b*x^2)^2)
```

Maple [A] time = 0.009, size = 80, normalized size = 1.2

$$\frac{Aa}{4\,b^2\left(bx^2+a\right)^2}-\frac{a^2B}{4\,b^3\left(bx^2+a\right)^2}+\frac{B\ln\left(bx^2+a\right)}{2\,b^3}-\frac{A}{2\,b^2\left(bx^2+a\right)}+\frac{Ba}{b^3\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*a/b^2/(b*x^2+a)^2*A-1/4*a^2/b^3/(b*x^2+a)^2*B+1/2*B*ln(b*x^2+a)/b^3-1/2
/b^2/(b*x^2+a)*A+1/b^3/(b*x^2+a)*B*a
```

Maxima [A] time = 0.996254, size = 97, normalized size = 1.47

$$\frac{3\,Ba^2-Aab+2\left(2\,Bab-Ab^2\right)x^2}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{B\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^
2*b^3) + 1/2*B*log(b*x^2 + a)/b^3
```

Fricas [A] time = 1.29813, size = 184, normalized size = 2.79

$$\frac{3\,Ba^2-Aab+2\left(2\,Bab-Ab^2\right)x^2+2\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2 + 2*(B*b^2*x^4 + 2*B*a*b*x^2
 + B*a^2)*log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A] time = 1.09992, size = 70, normalized size = 1.06

$$\frac{B\log\left(a+bx^2\right)}{2b^3}+\frac{-Aab+3Ba^2+x^2\left(-2Ab^2+4Bab\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**3) + (-A*a*b + 3*B*a**2 + x**2*(-2*A*b**2 + 4*B*a*b
))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
```

Giac [A] time = 1.13556, size = 82, normalized size = 1.24

$$\frac{B\log\left(|bx^2+a|\right)}{2\,b^3}+\frac{2\,(2\,Ba-Ab)x^2+\frac{3\,Ba^2-Aab}{b}}{4\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(b*x^2 + a))/b^3 + 1/4*(2*(2*B*a - A*b)*x^2 + (3*B*a^2 - A*a*b
)/b)/((b*x^2 + a)^2*b^2)
```

3.93 $\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{\left(A+Bx^2\right)^2}{4\left(a+bx^2\right)^2(Ab-aB)}$$

```
[Out] -(A + B*x^2)^2/(4*(A*b - a*B)*(a + b*x^2)^2)
```

Rubi [A] time = 0.0195665, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 37}

$$-\frac{\left(A+Bx^2\right)^2}{4\left(a+bx^2\right)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(A + B*x^2)^2/(4*(A*b - a*B)*(a + b*x^2)^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^3}\,dx, x, x^2\right)\\ &= -\frac{\left(A+Bx^2\right)^2}{4(Ab-aB)\left(a+bx^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.0130987, size = 30, normalized size = 0.94

$$-\frac{B\left(a+2bx^2\right)+Ab}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(A*b + B*(a + 2*b*x^2))/(4*b^2*(a + b*x^2)^2)
```

Maple [A] time = 0.008, size = 39, normalized size = 1.2

$$-\frac{Ab - Ba}{4\,b^2\left(bx^2 + a\right)^2} - \frac{B}{2\,b^2\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] -1/4*(A*b-B*a)/b^2/(b*x^2+a)^2-1/2*B/b^2/(b*x^2+a)
```

Maxima [A] time = 0.997769, size = 57, normalized size = 1.78

$$-\frac{2\,Bbx^2 + Ba + Ab}{4\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A] time = 1.18997, size = 86, normalized size = 2.69

$$-\frac{2\,Bbx^2 + Ba + Ab}{4\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [A] time = 0.658565, size = 42, normalized size = 1.31

$$-\frac{Ab + Ba + 2Bbx^2}{4a^2b^2 + 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -(A*b + B*a + 2*B*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A] time = 1.1122, size = 38, normalized size = 1.19

$$-\frac{2\,Bbx^2 + Ba + Ab}{4\left(bx^2 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/((b*x^2 + a)^2*b^2)
```

3.94 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=68

$$\frac{A}{2a^2\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Log[x])/a^3
- (A*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.0604057, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{A}{2a^2\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Log[x])/a^3
- (A*Log[a + b*x^2])/(2*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Ab-aB}{4ab\left(a+bx^2\right)^2}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.0466887, size = 59, normalized size = 0.87

$$\frac{\frac{a(a^2(-B)+3aAb+2Ab^2x^2)}{b(a+bx^2)^2} - 2A\log\left(a+bx^2\right) + 4A\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^2))/(b*(a + b*x^2)^2) + 4*A*Log[x] - 2*A*L
og[a + b*x^2])/(4*a^3)
```

Maple [A] time = 0.011, size = 68, normalized size = 1.

$$\frac{A\ln(x)}{a^3} + \frac{A}{4a\left(bx^2+a\right)^2} - \frac{B}{4b\left(bx^2+a\right)^2} - \frac{A\ln\left(bx^2+a\right)}{2a^3} + \frac{A}{2a^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^3,x)
```

```
[Out] A*ln(x)/a^3+1/4/a/(b*x^2+a)^2*A-1/4/b/(b*x^2+a)^2*B-1/2*A*ln(b*x^2+a)/a^3+1
/2*A/a^2/(b*x^2+a)
```

Maxima [A] time = 1.01256, size = 104, normalized size = 1.53

$$\frac{2Ab^2x^2 - Ba^2 + 3Aab}{4\left(a^2b^3x^4 + 2a^3b^2x^2 + a^4b\right)} - \frac{A\log\left(bx^2+a\right)}{2a^3} + \frac{A\log\left(x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*b^2*x^2 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) -
 1/2*A*log(b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3
```

Fricas [A] time = 1.23087, size = 250, normalized size = 3.68

$$\frac{2Aab^2x^2 - Ba^3 + 3Aa^2b - 2\left(Ab^3x^4 + 2Aab^2x^2 + Aa^2b\right)\log\left(bx^2+a\right) + 4\left(Ab^3x^4 + 2Aab^2x^2 + Aa^2b\right)\log(x)}{4\left(a^3b^3x^4 + 2a^4b^2x^2 + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*a*b^2*x^2 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a
^2*b)*log(b*x^2 + a) + 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [A] time = 0.743284, size = 75, normalized size = 1.1

$$\frac{A\log(x)}{a^3} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^3} + \frac{3Aab + 2Ab^2x^2 - Ba^2}{4a^4b + 8a^3b^2x^2 + 4a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**2)/(2*a**3) + (3*A*a*b + 2*A*b**2*x**2 - B*a
**2)/(4*a**4*b + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**4)
```

Giac [A] time = 1.15929, size = 103, normalized size = 1.51

$$\frac{A\log\left(x^2\right)}{2\,a^3} - \frac{A\log\left(|bx^2 + a|\right)}{2\,a^3} + \frac{3\,Ab^3x^4 + 8\,Aab^2x^2 - Ba^3 + 6\,Aa^2b}{4\left(bx^2 + a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^3 - 1/2*A*log(abs(b*x^2 + a))/a^3 + 1/4*(3*A*b^3*x^4 + 8*A
*a*b^2*x^2 - B*a^3 + 6*A*a^2*b)/((b*x^2 + a)^2*a^3*b)
```

3.95 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{2a^3x^2}$$

```
[Out] -A/(2*a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B)/(2*a^3*(
a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2])/(
2*a^4)
```

Rubi [A] time = 0.104898, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B)/(2*a^3*(
a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2])/(
2*a^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.0569309, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{(a+bx^2)^2}+\frac{2a(aB-2Ab)}{a+bx^2}+2(3Ab-aB)\log\left(a+bx^2\right)+4\log(x)(aB-3Ab)-\frac{2aA}{x^2}}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*A)/x^2 + (a^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^2) + 4*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^2])/(4*a
^4)
```

Maple [A] time = 0.014, size = 118, normalized size = 1.2

$$-\frac{A}{2a^3x^2}-3\frac{A\ln(x)b}{a^4}+\frac{\ln(x)B}{a^3}-\frac{Ab}{4a^2\left(bx^2+a\right)^2}+\frac{B}{4a\left(bx^2+a\right)^2}+\frac{3b\ln\left(bx^2+a\right)A}{2a^4}-\frac{\ln\left(bx^2+a\right)B}{2a^3}-\frac{Ab}{a^3\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*A/a^3/x^2-3/a^4*ln(x)*A*b+1/a^3*ln(x)*B-1/4/a^2*b/(b*x^2+a)^2*A+1/4/a/
(b*x^2+a)^2*B+3/2/a^4*b*ln(b*x^2+a)*A-1/2/a^3*ln(b*x^2+a)*B-1/a^3*b*A/(b*x^
2+a)+1/2/a^2/(b*x^2+a)*B
```

Maxima [A] time = 1.04071, size = 147, normalized size = 1.46

$$\frac{2\left(Bab-3Ab^2\right)x^4-2Aa^2+3\left(Ba^2-3Aab\right)x^2}{4\left(a^3b^2x^6+2a^4bx^4+a^5x^2\right)}-\frac{(Ba-3Ab)\log\left(bx^2+a\right)}{2a^4}+\frac{(Ba-3Ab)\log\left(x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*a*b - 3*A*b^2)*x^4 - 2*A*a^2 + 3*(B*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*
x^6 + 2*a^4*b*x^4 + a^5*x^2) - 1/2*(B*a - 3*A*b)*log(b*x^2 + a)/a^4 + 1/2*(
B*a - 3*A*b)*log(x^2)/a^4
```

Fricas [B] time = 1.17257, size = 412, normalized size = 4.08

$$\frac{2\left(Ba^2b-3Aab^2\right)x^4-2Aa^3+3\left(Ba^3-3Aa^2b\right)x^2-2\left(\left(Bab^2-3Ab^3\right)x^6+2\left(Ba^2b-3Aab^2\right)x^4+\left(Ba^3-3Aa^2b\right)x^2\right)}{4\left(a^4b^2x^6+2a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a^2*b - 3*A*a*b^2)*x^4 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^2 - 2*
((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*A*a^2*b
```

```
)*x^2)*log(b*x^2 + a) + 4*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2
)*x^4 + (B*a^3 - 3*A*a^2*b)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x
^2)
```

Sympy [A] time = 1.33974, size = 107, normalized size = 1.06

$$\frac{-2Aa^2 + x^4\left(-6Ab^2 + 2Bab\right) + x^2\left(-9Aab + 3Ba^2\right)}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} + \frac{(-3Ab + Ba)\log(x)}{a^4} - \frac{(-3Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 2*B*a*b) + x**2*(-9*A*a*b + 3*B*a**2))/(4*a*
*5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + (-3*A*b + B*a)*log(x)/a**4 -
(-3*A*b + B*a)*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 1.12732, size = 186, normalized size = 1.84

$$\frac{(Ba - 3\,Ab)\log\left(x^2\right)}{2\,a^4} - \frac{\left(Bab - 3\,Ab^2\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^4b} + \frac{3\,Bab^2x^4 - 9\,Ab^3x^4 + 8\,Ba^2bx^2 - 22\,Aab^2x^2 + 6\,Ba^3 - 14\,Aa^2b}{4\left(bx^2 + a\right)^2a^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*b - 3*A*b^2)*log(abs(b*x^2 + a))/
(a^4*b) + 1/4*(3*B*a*b^2*x^4 - 9*A*b^3*x^4 + 8*B*a^2*b*x^2 - 22*A*a*b^2*x^2
 + 6*B*a^3 - 14*A*a^2*b)/((b*x^2 + a)^2*a^4) - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

3.96 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=124

$$\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{4a^3x^4}$$

```
[Out] -A/(4*a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a^3*(a + b*
x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/(2*a^5)
```

Rubi [A] time = 0.129733, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -A/(4*a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a^3*(a + b*
x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x^3}+\frac{-3Ab+aB}{a^4x^2}-\frac{3b(-2Ab+aB)}{a^5x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^3}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)^2}+\frac{3b^2}{\ldots}\right.\right.$$

$$=-\frac{A}{4a^3x^4}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{3b(2Ab-aB)\ldots}{2\ldots}$$

Mathematica [A] time = 0.0785861, size = 108, normalized size = 0.87

$$\frac{\frac{a^2b(Ab-aB)}{(a+bx^2)^2}-\frac{a^2A}{x^4}+\frac{2ab(3Ab-2aB)}{a+bx^2}-\frac{2a(aB-3Ab)}{x^2}+6b(aB-2Ab)\log\left(a+bx^2\right)+12b\log(x)(2Ab-aB)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] (-((a^2*A)/x^4) - (2*a*(-3*A*b + a*B))/x^2 + (a^2*b*(A*b - a*B))/(a + b*x^2
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^2) + 12*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^2])/(4*a^5)
```

Maple [A] time = 0.017, size = 150, normalized size = 1.2

$$-\frac{A}{4\,a^3x^4}+\frac{3\,Ab}{2\,a^4x^2}-\frac{B}{2\,a^3x^2}+6\,\frac{A\ln(x)\,b^2}{a^5}-3\,\frac{bB\ln(x)}{a^4}+\frac{Ab^2}{4\,a^3\left(bx^2+a\right)^2}-\frac{Bb}{4\,a^2\left(bx^2+a\right)^2}-3\,\frac{b^2\ln\left(bx^2+a\right)A}{a^5}+\frac{3\,b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^3,x)
```

```
[Out] -1/4*A/a^3/x^4+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B+6*b^2/a^5*ln(x)*A-3*b/a^4*ln(x
)*B+1/4/a^3*b^2/(b*x^2+a)^2*A-1/4/a^2*b/(b*x^2+a)^2*B-3/a^5*b^2*ln(b*x^2+a)
*A+3/2/a^4*b*ln(b*x^2+a)*B+3/2/a^4*b^2*A/(b*x^2+a)-1/a^3*b/(b*x^2+a)*B
```

Maxima [A] time = 1.01524, size = 185, normalized size = 1.49

$$-\frac{6\left(Bab^2-2\,Ab^3\right)x^6+9\left(Ba^2b-2\,Aab^2\right)x^4+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^2}{4\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)}+\frac{3\left(Bab-2\,Ab^2\right)\log\left(bx^2+a\right)}{2\,a^5}-\frac{3\left(Bab-2\,A\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*a*b^2 - 2*A*b^3)*x^6 + 9*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^2)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) + 3/2*(B*a*b
- 2*A*b^2)*log(b*x^2 + a)/a^5 - 3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5
```

Fricas [B] time = 1.30121, size = 474, normalized size = 3.82

$$-\frac{6\left(Ba^2b^2-2\,Aab^3\right)x^6+Aa^4+9\left(Ba^3b-2\,Aa^2b^2\right)x^4+2\left(Ba^4-2\,Aa^3b\right)x^2-6\left(\left(Bab^3-2\,Ab^4\right)x^8+2\left(Ba^2b^2-2\,Aab^3\right)\right.}{4\left(a^5b^2x^8+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + A*a^4 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^2 - 6*((B*a*b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 -
```

```
2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)*log(b*x^2 + a) + 12*((B*a*b^3
 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x
^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A] time = 1.82531, size = 136, normalized size = 1.1

$$-\frac{Aa^3+x^6\left(-12Ab^3+6Bab^2\right)+x^4\left(-18Aab^2+9Ba^2b\right)+x^2\left(-4Aa^2b+2Ba^3\right)}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}-\frac{3b\left(-2Ab+Ba\right)\log\left(x\right)}{a^5}+\frac{3b\left(-2Ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

```
[Out] -(A*a**3 + x**6*(-12*A*b**3 + 6*B*a*b**2) + x**4*(-18*A*a*b**2 + 9*B*a**2*b
) + x**2*(-4*A*a**2*b + 2*B*a**3))/(4*a**6*x**4 + 8*a**5*b*x**6 + 4*a**4*b*
*2*x**8) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + 3*b*(-2*A*b + B*a)*log(a/b + x*
*2)/(2*a**5)
```

Giac [A] time = 1.15009, size = 180, normalized size = 1.45

$$-\frac{3\left(Bab-2\,Ab^2\right)\log\left(x^2\right)}{2\,a^5}+\frac{3\left(Bab^2-2\,Ab^3\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^5b}-\frac{6\,Bab^2x^6-12\,Ab^3x^6+9\,Ba^2bx^4-18\,Aab^2x^4+2\,Ba^3}{4\left(bx^4+ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5 + 3/2*(B*a*b^2 - 2*A*b^3)*log(abs(b*x^2
 + a))/(a^5*b) - 1/4*(6*B*a*b^2*x^6 - 12*A*b^3*x^6 + 9*B*a^2*b*x^4 - 18*A*a
*b^2*x^4 + 2*B*a^3*x^2 - 4*A*a^2*b*x^2 + A*a^3)/((b*x^4 + a*x^2)^2*a^4)
```

3.97 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{3b(2Ab-aB)}{2a^5x^2}+\frac{3Ab-aB}{4a^4x^4}$$

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/(2*a^5*x^2
) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A*b - 3*a*B))/(2*a^5*
(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])/a^6 + (b^2*(5*A*b - 3*a*B)*Lo
g[a + b*x^2])/a^6
```

Rubi [A] time = 0.166051, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{3b(2Ab-aB)}{2a^5x^2}+\frac{3Ab-aB}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/(2*a^5*x^2
) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A*b - 3*a*B))/(2*a^5*
(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])/a^6 + (b^2*(5*A*b - 3*a*B)*Lo
g[a + b*x^2])/a^6
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{a^3x^4} + \frac{-3Ab+aB}{a^4x^3} - \frac{3b(-2Ab+aB)}{a^5x^2} + \frac{2b^2(-5Ab+3aB)}{a^6x} - \frac{b^3(-Ab+aB)}{a^4(a+bx)^3} - \frac{b^3}{}\right.\right.$$
$$= -\frac{A}{6a^3x^6} + \frac{3Ab-aB}{4a^4x^4} - \frac{3b(2Ab-aB)}{2a^5x^2} - \frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2} - \frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)} - \frac{2b^2(5Ab-3aB)\log}{a^6}$$

Mathematica [A] time = 0.118974, size = 135, normalized size = 0.91

$$\frac{\frac{3a^2b^2(aB-Ab)}{\left(a+bx^2\right)^2} - \frac{3a^2(aB-3Ab)}{x^4} - \frac{2a^3A}{x^6} + \frac{6ab^2(3aB-4Ab)}{a+bx^2} + 12b^2(5Ab-3aB)\log\left(a+bx^2\right) + 24b^2\log(x)(3aB-5Ab) + \frac{18ab(aB-2}{x^2}}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-3*A*b + a*B))/x^4 + (18*a*b*(-2*A*b + a*B))/x^2
+ (3*a^2*b^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a*b^2*(-4*A*b + 3*a*B))/(a
+ b*x^2) + 24*b^2*(-5*A*b + 3*a*B)*Log[x] + 12*b^2*(5*A*b - 3*a*B)*Log[a +
b*x^2])/(12*a^6)
```

Maple [A] time = 0.014, size = 180, normalized size = 1.2

$$-\frac{A}{6\,a^3x^6} + \frac{3\,Ab}{4\,a^4x^4} - \frac{B}{4\,a^3x^4} - 3\,\frac{Ab^2}{a^5x^2} + \frac{3\,Bb}{2\,a^4x^2} - 10\,\frac{b^3\ln(x)\,A}{a^6} + 6\,\frac{b^2B\ln(x)}{a^5} - \frac{Ab^3}{4\,a^4\left(bx^2+a\right)^2} + \frac{Bb^2}{4\,a^3\left(bx^2+a\right)^2} + 5\,\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^3,x)
```

```
[Out] -1/6*A/a^3/x^6+3/4/a^4/x^4*A*b-1/4/a^3/x^4*B-3*b^2/a^5/x^2*A+3/2*b/a^4/x^2*
B-10*b^3/a^6*ln(x)*A+6*b^2/a^5*ln(x)*B-1/4/a^4*b^3/(b*x^2+a)^2*A+1/4/a^3*b^
2/(b*x^2+a)^2*B+5/a^6*b^3*ln(b*x^2+a)*A-3/a^5*b^2*ln(b*x^2+a)*B-2/a^5*b^3*A
/(b*x^2+a)+3/2/a^4*b^2/(b*x^2+a)*B
```

Maxima [A] time = 0.999754, size = 230, normalized size = 1.54

$$\frac{12\left(3\,Bab^3-5\,Ab^4\right)x^8 + 18\left(3\,Ba^2b^2-5\,Aab^3\right)x^6 - 2\,Aa^4 + 4\left(3\,Ba^3b-5\,Aa^2b^2\right)x^4 - \left(3\,Ba^4-5\,Aa^3b\right)x^2}{12\left(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6\right)} - \frac{\left(3\,Bab^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*a*b^3 - 5*A*b^4)*x^8 + 18*(3*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 2*A
*a^4 + 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^4 - (3*B*a^4 - 5*A*a^3*b)*x^2)/(a^5*b^
2*x^10 + 2*a^6*b*x^8 + a^7*x^6) - (3*B*a*b^2 - 5*A*b^3)*log(b*x^2 + a)/a^6
```

```
+ (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6
```

Fricas [A] time = 1.28166, size = 560, normalized size = 3.76

$$\frac{12\left(3\,Ba^2b^3-5\,Aab^4\right)x^8+18\left(3\,Ba^3b^2-5\,Aa^2b^3\right)x^6-2\,Aa^5+4\left(3\,Ba^4b-5\,Aa^3b^2\right)x^4-\left(3\,Ba^5-5\,Aa^4b\right)x^2-12\left(\left(3\,Bab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + 18*(3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6
 - 2*A*a^5 + 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^4 - (3*B*a^5 - 5*A*a^4*b)*x^2 -
12*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3
*b^2 - 5*A*a^2*b^3)*x^6)*log(b*x^2 + a) + 24*((3*B*a*b^4 - 5*A*b^5)*x^10 +
2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(x))/
(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6)
```

Sympy [A] time = 2.42316, size = 165, normalized size = 1.11

$$\frac{-2Aa^4+x^8\left(-60Ab^4+36Bab^3\right)+x^6\left(-90Aab^3+54Ba^2b^2\right)+x^4\left(-20Aa^2b^2+12Ba^3b\right)+x^2\left(5Aa^3b-3Ba^4\right)}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}+\frac{2b^2\left(-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**4 + x**8*(-60*A*b**4 + 36*B*a*b**3) + x**6*(-90*A*a*b**3 + 54*B*a*
*2*b**2) + x**4*(-20*A*a**2*b**2 + 12*B*a**3*b) + x**2*(5*A*a**3*b - 3*B*a*
*4))/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) + 2*b**2*(-5*A*b
+ 3*B*a)*log(x)/a**6 - b**2*(-5*A*b + 3*B*a)*log(a/b + x**2)/a**6
```

Giac [A] time = 1.14076, size = 271, normalized size = 1.82

$$\frac{\left(3\,Bab^2-5\,Ab^3\right)\log\left(x^2\right)}{a^6}-\frac{\left(3\,Bab^3-5\,Ab^4\right)\log\left(\left|bx^2+a\right|\right)}{a^6b}+\frac{18\,Bab^4x^4-30\,Ab^5x^4+42\,Ba^2b^3x^2-68\,Aab^4x^2+25\,Ba}{4\left(bx^2+a\right)^2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6 - (3*B*a*b^3 - 5*A*b^4)*log(abs(b*x^2 +
a))/(a^6*b) + 1/4*(18*B*a*b^4*x^4 - 30*A*b^5*x^4 + 42*B*a^2*b^3*x^2 - 68*A*
a*b^4*x^2 + 25*B*a^3*b^2 - 39*A*a^2*b^3)/((b*x^2 + a)^2*a^6) - 1/12*(66*B*a
*b^2*x^6 - 110*A*b^3*x^6 - 18*B*a^2*b*x^4 + 36*A*a*b^2*x^4 + 3*B*a^3*x^2 -
9*A*a^2*b*x^2 + 2*A*a^3)/(a^6*x^6)
```

3.98 $\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=158

$$\frac{a^3x(17Ab-21aB)}{8b^6(a+bx^2)}-\frac{a^4x(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}+\frac{x^5(Ab-3aB)}{5b^4}-\frac{ax^3(Ab}{b}$$

```
[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b - 3*a*B)*
x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4*b^6*(a + b*x^2)^2)
+ (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) - (9*a^(5/2)*(7*A*b - 11*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rubi [A] time = 0.283153, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {455, 1814, 1810, 205}

$$\frac{a^3x(17Ab-21aB)}{8b^6(a+bx^2)}-\frac{a^4x(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}+\frac{x^5(Ab-3aB)}{5b^4}-\frac{ax^3(Ab}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b - 3*a*B)*
x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4*b^6*(a + b*x^2)^2)
+ (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) - (9*a^(5/2)*(7*A*b - 11*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}dx = -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} - \frac{\int \frac{-a^4(Ab-aB)+4a^3b(Ab-aB)x^2-4a^2b^2(Ab-aB)x^4+4ab^3(Ab-aB)x^6-4b^4(Ab-aB)x^8-4b^5Bx^{10}}{\left(a+bx^2\right)^2}dx}{4b^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)} + \frac{\int \frac{-a^4(15Ab-19aB)+8a^3b(3Ab-4aB)x^2-8a^2b^2(2Ab-3aB)x^4+8ab^3(Ab-2aB}{a+bx^2}}{8ab^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)} + \frac{\int \Big(16a^3(3Ab-5aB) - 24a^2b(Ab-2aB)x^2 + 8ab^2(Ab-3}{8ab^6}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0915596, size = 158, normalized size = 1.

$$\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)} + \frac{a^4x(aB-Ab)}{4b^6\left(a+bx^2\right)^2} - \frac{2a^2x(5aB-3Ab)}{b^6} + \frac{9a^{5/2}(11aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}} + \frac{x^5(Ab-3aB)}{5b^4} + \frac{ax^3(2aB-}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-2*a^2*(-3*A*b + 5*a*B)*x)/b^6 + (a*(-(A*b) + 2*a*B)*x^3)/b^5 + ((A*b - 3*
a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) + (a^4*(-(A*b) + a*B)*x)/(4*b^6*(a + b*
x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) + (9*a^(5/2)*(-7*A*
b + 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Maple [A] time = 0.011, size = 198, normalized size = 1.3

$$\frac{Bx^7}{7b^3} + \frac{Ax^5}{5b^3} - \frac{3Bx^5a}{5b^4} - \frac{aAx^3}{b^4} + 2\frac{Bx^3a^2}{b^5} + 6\frac{a^2Ax}{b^5} - 10\frac{Ba^3x}{b^6} + \frac{17Aa^3x^3}{8b^4\left(bx^2+a\right)^2} - \frac{21Ba^4x^3}{8b^5\left(bx^2+a\right)^2} + \frac{15Aa^4x}{8b^5\left(bx^2+a\right)^2} - \frac{}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*B*x^7/b^3+1/5/b^3*A*x^5-3/5/b^4*B*x^5*a-1/b^4*A*x^3*a+2/b^5*B*x^3*a^2+6
/b^5*A*a^2*x-10/b^6*B*a^3*x+17/8*a^3/b^4/(b*x^2+a)^2*A*x^3-21/8*a^4/b^5/(b*
x^2+a)^2*B*x^3+15/8*a^4/b^5/(b*x^2+a)^2*A*x-19/8*a^5/b^6/(b*x^2+a)^2*B*x-63
/8*a^3/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+99/8*a^4/b^6/(a*b)^(1/2)*a
```

```
rctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3479, size = 1025, normalized size = 6.49

$$\left[\frac{80\,Bb^5x^{11}-16\left(11\,Bab^4-7\,Ab^5\right)x^9+48\left(11\,Ba^2b^3-7\,Aab^4\right)x^7-336\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^5-1050\left(11\,Ba^4b-7\,Aa\right.}{560\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*B*b^5*x^11 - 16*(11*B*a*b^4 - 7*A*b^5)*x^9 + 48*(11*B*a^2*b^3 -
7*A*a*b^4)*x^7 - 336*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 1050*(11*B*a^4*b -
7*A*a^3*b^2)*x^3 - 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2 - 7*A*a^2*b^3)
*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7
*x^2 + a^2*b^6), 1/280*(40*B*b^5*x^11 - 8*(11*B*a*b^4 - 7*A*b^5)*x^9 + 24*(
11*B*a^2*b^3 - 7*A*a*b^4)*x^7 - 168*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 525*
(11*B*a^4*b - 7*A*a^3*b^2)*x^3 + 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2
- 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) - 315*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*
b^6)]
```

Sympy [A] time = 1.71509, size = 274, normalized size = 1.73

$$\frac{Bx^7}{7b^3}-\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(-\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16}+\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**7/(7*b**3) - 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(-9*b**6*sqrt(-a
**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3) + x)/16 + 9*sqrt(-a
**5/b**13)*(-7*A*b + 11*B*a)*log(9*b**6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)
/(-63*A*a**2*b + 99*B*a**3) + x)/16 - (x**3*(-17*A*a**3*b**2 + 21*B*a**4*b)
 + x*(-15*A*a**4*b + 19*B*a**5))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**
```

```
4) - x**5*(-A*b + 3*B*a)/(5*b**4) + x**3*(-A*a*b + 2*B*a**2)/b**5 - x*(-6*A
*a**2*b + 10*B*a**3)/b**6
```

Giac [A] time = 1.18908, size = 219, normalized size = 1.39

$$\frac{9\left(11\,Ba^{4}-7\,Aa^{3}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^{6}}-\frac{21\,Ba^{4}bx^{3}-17\,Aa^{3}b^{2}x^{3}+19\,Ba^{5}x-15\,Aa^{4}bx}{8\left(bx^{2}+a\right)^{2}b^{6}}+\frac{5\,Bb^{18}x^{7}-21\,Bab^{17}x^{5}+7\,Ab^{18}x^{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*
B*a^4*b*x^3 - 17*A*a^3*b^2*x^3 + 19*B*a^5*x - 15*A*a^4*b*x)/((b*x^2 + a)^2*
b^6) + 1/35*(5*B*b^18*x^7 - 21*B*a*b^17*x^5 + 7*A*b^18*x^5 + 70*B*a^2*b^16*
x^3 - 35*A*a*b^17*x^3 - 350*B*a^3*b^15*x + 210*A*a^2*b^16*x)/b^21
```

3.99 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=138

$$-\frac{a^2x(13Ab-17aB)}{8b^5(a+bx^2)}+\frac{a^3x(Ab-aB)}{4b^5(a+bx^2)^2}+\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{3ax(Ab-2aB)}{b^5}+\frac{Bx^5}{5b^3}$$

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5)/(5*b^3)
+ (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*A*b - 17*a*B)*x)/(8*
b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*b^(11/2))
```

Rubi [A] time = 0.217875, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {455, 1814, 1810, 205}

$$-\frac{a^2x(13Ab-17aB)}{8b^5(a+bx^2)}+\frac{a^3x(Ab-aB)}{4b^5(a+bx^2)^2}+\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{3ax(Ab-2aB)}{b^5}+\frac{Bx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5)/(5*b^3)
+ (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*A*b - 17*a*B)*x)/(8*
b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*b^(11/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx=\frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2}-\frac{\int \frac{a^3(Ab-aB)-4a^2b(Ab-aB)x^2+4ab^2(Ab-aB)x^4-4b^3(Ab-aB)x^6-4b^4Bx^8}{\left(a+bx^2\right)^2}\,dx}{4b^5}$$

$$=\frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)}+\frac{\int \frac{a^3(11Ab-15aB)-8a^2b(2Ab-3aB)x^2+8ab^2(Ab-2aB)x^4+8ab^3Bx^6}{a+bx^2}\,dx}{8ab^5}$$

$$=\frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)}+\frac{\int \Big(-24a^2(Ab-2aB)+8ab(Ab-3aB)x^2+8ab^2Bx^4-\frac{7(-5a}{}$$

$$=-\frac{3a(Ab-2aB)x}{b^5}+\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^5}{5b^3}+\frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)}+\frac{\left(7a^2(5Ab-9aB\right.}{8b^5}$$

$$=-\frac{3a(Ab-2aB)x}{b^5}+\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^5}{5b^3}+\frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)}+\frac{7a^{3/2}(5Ab-9a}{8b^1}$$

Mathematica [A] time = 0.102357, size = 133, normalized size = 0.96

$$\frac{x\left(7a^2b^2x^2\left(72Bx^2-125A\right)-525a^3b\left(A-3Bx^2\right)+945a^4B-8ab^3x^4\left(35A+9Bx^2\right)+8b^4x^6\left(5A+3Bx^2\right)\right)}{120b^5\left(a+bx^2\right)^2}-\frac{7a^{3/2}(9aB-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*B - 525*a^3*b*(A - 3*B*x^2) + 8*b^4*x^6*(5*A + 3*B*x^2) - 8*a*b
^3*x^4*(35*A + 9*B*x^2) + 7*a^2*b^2*x^2*(-125*A + 72*B*x^2)))/(120*b^5*(a +
 b*x^2)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(11/2))
```

Maple [A] time = 0.01, size = 174, normalized size = 1.3

$$\frac{Bx^5}{5\,b^3}+\frac{Ax^3}{3\,b^3}-\frac{Bx^3a}{b^4}-3\,\frac{aAx}{b^4}+6\,\frac{a^2Bx}{b^5}-\frac{13\,a^2Ax^3}{8\,b^3\left(bx^2+a\right)^2}+\frac{17\,Ba^3x^3}{8\,b^4\left(bx^2+a\right)^2}-\frac{11\,Aa^3x}{8\,b^4\left(bx^2+a\right)^2}+\frac{15\,Ba^4x}{8\,b^5\left(bx^2+a\right)^2}+\frac{35\,Aa^2}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*B*x^5/b^3+1/3/b^3*A*x^3-1/b^4*B*x^3*a-3/b^4*a*A*x+6/b^5*a^2*B*x-13/8*a^
2/b^3/(b*x^2+a)^2*A*x^3+17/8*a^3/b^4/(b*x^2+a)^2*B*x^3-11/8*a^3/b^4/(b*x^2+
a)^2*A*x+15/8*a^4/b^5/(b*x^2+a)^2*B*x+35/8*a^2/b^4/(a*b)^(1/2)*arctan(b*x/(
```

```
a*b)^(1/2))*A-63/8*a^3/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31963, size = 892, normalized size = 6.46

$$\left[\frac{48\,Bb^4x^9 - 16\left(9\,Bab^3 - 5\,Ab^4\right)x^7 + 112\left(9\,Ba^2b^2 - 5\,Aab^3\right)x^5 + 350\left(9\,Ba^3b - 5\,Aa^2b^2\right)x^3 - 105\left(9\,Ba^4 - 5\,Aa^3b + \left(9\right.\right.}{240\left(b^7x^4 + 2\,ab^6x^2 + a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*b^4*x^9 - 16*(9*B*a*b^3 - 5*A*b^4)*x^7 + 112*(9*B*a^2*b^2 - 5*
A*a*b^3)*x^5 + 350*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b
 + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(9*B*a^4 - 5*A*a^
3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/120*(24*B*b^4*x^9 - 8*(9*B*a*b
^3 - 5*A*b^4)*x^7 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^5 + 175*(9*B*a^3*b - 5*A
*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 +
2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(9
*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [A] time = 1.60939, size = 250, normalized size = 1.81

$$\frac{Bx^5}{5b^3} + \frac{7\sqrt{-\frac{a^3}{b^{11}}}\left(-5Ab + 9Ba\right)\log\left(-\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\left(-5Ab+9Ba\right)}{-35Aab+63Ba^2} + x\right)}{16} - \frac{7\sqrt{-\frac{a^3}{b^{11}}}\left(-5Ab + 9Ba\right)\log\left(\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\left(-5Ab+9Ba\right)}{-35Aab+63Ba^2} + x\right)}{16} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(-7*b**5*sqrt(-a*
*3/b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)/16 - 7*sqrt(-a**3/b
**11)*(-5*A*b + 9*B*a)*log(7*b**5*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)/(-35*A
*a*b + 63*B*a**2) + x)/16 + (x**3*(-13*A*a**2*b**2 + 17*B*a**3*b) + x*(-11*
A*a**3*b + 15*B*a**4))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4) - x**3*
(-A*b + 3*B*a)/(3*b**4) + x*(-3*A*a*b + 6*B*a**2)/b**5
```

Giac [A] time = 1.6717, size = 186, normalized size = 1.35

$$-\frac{7\left(9\,Ba^3-5\,Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}+\frac{17\,Ba^3bx^3-13\,Aa^2b^2x^3+15\,Ba^4x-11\,Aa^3bx}{8\left(bx^2+a\right)^2b^5}+\frac{3\,Bb^{12}x^5-15\,Bab^{11}x^3+5\,Ab^{12}x^3}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*
B*a^3*b*x^3 - 13*A*a^2*b^2*x^3 + 15*B*a^4*x - 11*A*a^3*b*x)/((b*x^2 + a)^2*
b^5) + 1/15*(3*B*b^12*x^5 - 15*B*a*b^11*x^3 + 5*A*b^12*x^3 + 90*B*a^2*b^10*
x - 45*A*a*b^11*x)/b^15
```

3.100 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{ax(9Ab-13aB)}{8b^4(a+bx^2)}+\frac{x(Ab-3aB)}{b^4}-\frac{5\sqrt{a}(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{Bx^3}{3b^3}$$

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*b^4*(a + b
*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) - (5*Sqrt[a]*(3*A*b -
 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rubi [A] time = 0.151509, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {455, 1814, 1153, 205}

$$-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{ax(9Ab-13aB)}{8b^4(a+bx^2)}+\frac{x(Ab-3aB)}{b^4}-\frac{5\sqrt{a}(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*b^4*(a + b
*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) - (5*Sqrt[a]*(3*A*b -
 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx &= -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}-\frac{\int \frac{-a^2(Ab-aB)+4ab(Ab-aB)x^2-4b^2(Ab-aB)x^4-4b^3Bx^6}{(a+bx^2)^2}\,dx}{4b^4}\\
&= -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}+\frac{\int \frac{-a^2(7Ab-11aB)+8ab(Ab-2aB)x^2+8ab^2Bx^4}{a+bx^2}\,dx}{8ab^4}\\
&= -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}+\frac{\int \left(8a(Ab-3aB)+8abBx^2+\frac{5(-3a^2Ab+7a^3B)}{a+bx^2}\right)dx}{8ab^4}\\
&= \frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}-\frac{(5a(3Ab-7aB))\int \frac{1}{a+bx^2}\,dx}{8b^4}\\
&= \frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}-\frac{5\sqrt{a}(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0850038, size = 113, normalized size = 0.97

$$\frac{5a^2bx(9A-35Bx^2)-105a^3Bx+ab^2x^3(75A-56Bx^2)+8b^3x^5(3A+Bx^2)}{24b^4(a+bx^2)^2}+\frac{5\sqrt{a}(7aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(75*A - 56*B*x^2) + 5*a^2*b*x*(9*A - 35*B*x^2) +
8*b^3*x^5*(3*A + B*x^2))/(24*b^4*(a + b*x^2)^2) + (5*Sqrt[a]*(-3*A*b + 7*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Maple [A] time = 0.009, size = 147, normalized size = 1.3

$$\frac{Bx^3}{3b^3}+\frac{Ax}{b^3}-3\frac{Bax}{b^4}+\frac{9aAx^3}{8b^2(bx^2+a)^2}-\frac{13a^2Bx^3}{8b^3(bx^2+a)^2}+\frac{7a^2Ax}{8b^3(bx^2+a)^2}-\frac{11Ba^3x}{8b^4(bx^2+a)^2}-\frac{15Aa}{8b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3+1/b^3*A*x-3/b^4*B*a*x+9/8*a/b^2/(b*x^2+a)^2*A*x^3-13/8*a^2/b^
3/(b*x^2+a)^2*B*x^3+7/8*a^2/b^3/(b*x^2+a)^2*A*x-11/8*a^3/b^4/(b*x^2+a)^2*B*
x-15/8*a/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+35/8*a^2/b^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31871, size = 763, normalized size = 6.58

$$\left[\frac{16\,Bb^3x^7 - 16\left(7\,Bab^2 - 3\,Ab^3\right)x^5 - 50\left(7\,Ba^2b - 3\,Aab^2\right)x^3 - 15\left(\left(7\,Bab^2 - 3\,Ab^3\right)x^4 + 7\,Ba^3 - 3\,Aa^2b + 2\left(7\,Ba^2b - \right.\right.}{48\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*b^3*x^7 - 16*(7*B*a*b^2 - 3*A*b^3)*x^5 - 50*(7*B*a^2*b - 3*A*a*
b^2)*x^3 - 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2
*b - 3*A*a*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) - 30*(7*B*a^3 - 3*A*a^2*b)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*
(8*B*b^3*x^7 - 8*(7*B*a*b^2 - 3*A*b^3)*x^5 - 25*(7*B*a^2*b - 3*A*a*b^2)*x^3
 + 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2*b - 3*A
*a*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 15*(7*B*a^3 - 3*A*a^2*b)*x
)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A] time = 1.47809, size = 212, normalized size = 1.83

$$\frac{Bx^3}{3b^3} - \frac{5\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)\log\left(-\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba} + x\right)}{16} + \frac{5\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)\log\left(\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba} + x\right)}{16} - \frac{x^3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) - 5*sqrt(-a/b**9)*(-3*A*b + 7*B*a)*log(-5*b**4*sqrt(-a/b**9
)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16 + 5*sqrt(-a/b**9)*(-3*A*b + 7
*B*a)*log(5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16
- (x**3*(-9*A*a*b**2 + 13*B*a**2*b) + x*(-7*A*a**2*b + 11*B*a**3))/(8*a**2*
b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) - x*(-A*b + 3*B*a)/b**4
```

Giac [A] time = 1.48851, size = 150, normalized size = 1.29

$$\frac{5\left(7\,Ba^2 - 3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}b^4} - \frac{13\,Ba^2bx^3 - 9\,Aab^2x^3 + 11\,Ba^3x - 7\,Aa^2bx}{8\left(bx^2 + a\right)^2b^4} + \frac{Bb^6x^3 - 9\,Bab^5x + 3\,Ab^6x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*B*a
^2*b*x^3 - 9*A*a*b^2*x^3 + 11*B*a^3*x - 7*A*a^2*b*x)/((b*x^2 + a)^2*b^4) +
1/3*(B*b^6*x^3 - 9*B*a*b^5*x + 3*A*b^6*x)/b^9
```

3.101 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=94

$$-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{Bx}{b^3}$$

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9*a*B)*x)/(
8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[
a]*b^(7/2))
```

Rubi [A] time = 0.0862616, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {455, 1157, 388, 205}

$$-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{Bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9*a*B)*x)/(
8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[
a]*b^(7/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4 (A+Bx^2)}{(a+bx^2)^3}\,dx &= \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2} - \frac{\int \frac{a(Ab-aB)-4b(Ab-aB)x^2-4b^2Bx^4}{(a+bx^2)^2}\,dx}{4b^3} \\ &= \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2} - \frac{(5Ab-9aB)x}{8b^3(a+bx^2)} + \frac{\int \frac{a(3Ab-7aB)+8abBx^2}{a+bx^2}\,dx}{8ab^3} \\ &= \frac{Bx}{b^3} + \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2} - \frac{(5Ab-9aB)x}{8b^3(a+bx^2)} + \frac{(3(Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{8b^3} \\ &= \frac{Bx}{b^3} + \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2} - \frac{(5Ab-9aB)x}{8b^3(a+bx^2)} + \frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0751205, size = 91, normalized size = 0.97

$$\frac{x\left(15a^2B+a\left(25bBx^2-3Ab\right)+b^2x^2\left(8Bx^2-5A\right)\right)}{8b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^2*B + b^2*x^2*(-5*A + 8*B*x^2) + a*(-3*A*b + 25*b*B*x^2)))/(8*b^3*
(a + b*x^2)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b
^(7/2))
```

Maple [A] time = 0.009, size = 122, normalized size = 1.3

$$\frac{Bx}{b^3} - \frac{5\,Ax^3}{8\,b\left(bx^2+a\right)^2} + \frac{9\,Bx^3a}{8\,b^2\left(bx^2+a\right)^2} - \frac{3\,aAx}{8\,b^2\left(bx^2+a\right)^2} + \frac{7\,a^2Bx}{8\,b^3\left(bx^2+a\right)^2} + \frac{3\,A}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{15\,Ba}{8\,b^3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] B*x/b^3-5/8/b/(b*x^2+a)^2*A*x^3+9/8/b^2/(b*x^2+a)^2*B*x^3*a-3/8/b^2/(b*x^2+
a)^2*a*A*x+7/8/b^3/(b*x^2+a)^2*a^2*B*x+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))*A-15/8/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26305, size = 672, normalized size = 7.15

$$\left[\frac{16\,Bab^3x^5+10\left(5\,Ba^2b^2-Aab^3\right)x^3+3\left(\left(5\,Bab^2-Ab^3\right)x^4+5\,Ba^3-Aa^2b+2\left(5\,Ba^2b-Aab^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-}}{bx^2+}\right.}{16\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*a*b^3*x^5 + 10*(5*B*a^2*b^2 - A*a*b^3)*x^3 + 3*((5*B*a*b^2 - A*
b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a*b^2)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*B*a^3*b - A*a^2*b^2)*x)/(a
*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4), 1/8*(8*B*a*b^3*x^5 + 5*(5*B*a^2*b^2 -
A*a*b^3)*x^3 - 3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*
b - A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*B*a^3*b - A*a^2*b^
2)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] time = 1.21583, size = 194, normalized size = 2.06

$$\frac{Bx}{b^3}+\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(-\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}+\frac{x^3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x/b**3 + 3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)*log(-3*a*b**3*sqrt(-1/(a*b**7
))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/16 - 3*sqrt(-1/(a*b**7))*(-A*b + 5
*B*a)*log(3*a*b**3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/
16 + (x**3*(-5*A*b**2 + 9*B*a*b) + x*(-3*A*a*b + 7*B*a**2))/(8*a**2*b**3 +
16*a*b**4*x**2 + 8*b**5*x**4)
```

Giac [A] time = 1.16061, size = 108, normalized size = 1.15

$$\frac{Bx}{b^3}-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{9\,Babx^3-5\,Ab^2x^3+7\,Ba^2x-3\,Aabx}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*(9*
B*a*b*x^3 - 5*A*b^2*x^3 + 7*B*a^2*x - 3*A*a*b*x)/((b*x^2 + a)^2*b^3)
```

3.102 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=89

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4b^2\left(a+bx^2\right)^2}$$

```
[Out] -((A*b - a*B)*x)/(4*b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b^2*(a + b*
x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.0619753, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 385, 205}

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -((A*b - a*B)*x)/(4*b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b^2*(a + b*
x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2} - \frac{\int \frac{-Ab+aB-4bBx^2}{\left(a+bx^2\right)^2}\,dx}{4b^2}$$
$$= -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2} + \frac{(Ab-5aB)x}{8ab^2\left(a+bx^2\right)} + \frac{(Ab+3aB)\int \frac{1}{a+bx^2}\,dx}{8ab^2}$$
$$= -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2} + \frac{(Ab-5aB)x}{8ab^2\left(a+bx^2\right)} + \frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.0824406, size = 83, normalized size = 0.93

$$\frac{\frac{\sqrt{b}x\left(-3a^2B-ab\left(A+5Bx^2\right)+Ab^2x^2\right)}{a\left(a+bx^2\right)^2} + \frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - a*b*(A + 5*B*x^2)))/(a*(a + b*x^2)^2) +
 ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(8*b^(5/2))
```

Maple [A] time = 0.009, size = 89, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(Ab-5\,Ba)\,x^3}{8\,ab} - \frac{(Ab+3\,Ba)\,x}{8\,b^2}\right) + \frac{A}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{3\,B}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*B*a)/a/b*x^3-1/8*(A*b+3*B*a)/b^2*x)/(b*x^2+a)^2+1/8/b/a/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*A+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33385, size = 624, normalized size = 7.01

$$\left[-\frac{2\left(5\,Ba^2b^2 - Aab^3\right)x^3 + \left(\left(3\,Bab^2 + Ab^3\right)x^4 + 3\,Ba^3 + Aa^2b + 2\left(3\,Ba^2b + Aab^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) + 2\left(3\right.}{16\left(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*a^2*b^2 - A*a*b^3)*x^3 + ((3*B*a*b^2 + A*b^3)*x^4 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*
b^4*x^2 + a^4*b^3), -1/8*((5*B*a^2*b^2 - A*a*b^3)*x^3 - ((3*B*a*b^2 + A*b^3
)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) + (3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

Sympy [A] time = 0.881778, size = 153, normalized size = 1.72

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}-\frac{x^3\left(-Ab^2+5Bab\right)+x\left(A\right.}{8a^3b^2+16a^2b^3x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 - (x**3*(-A*b**2 + 5*B*a*b) + x*(A*a*b + 3*B*a**2))/(8*a**3*b**2
+ 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

Giac [A] time = 1.42951, size = 105, normalized size = 1.18

$$\frac{(3\,Ba + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}ab^2} - \frac{5\,Babx^3 - Ab^2x^3 + 3\,Ba^2x + Aabx}{8\left(bx^2 + a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*B*a*b*x^
3 - A*b^2*x^3 + 3*B*a^2*x + A*a*b*x)/((b*x^2 + a)^2*a*b^2)
```

3.103 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=92

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\left(a+bx^2\right)}+\frac{x(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0324289, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\left(a+bx^2\right)}+\frac{x(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^3} dx = \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)\int \frac{1}{\left(a+bx^2\right)^2} dx}{4ab}$$
$$= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)} + \frac{(3Ab+aB)\int \frac{1}{a+bx^2} dx}{8a^2b}$$
$$= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)} + \frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.0635813, size = 84, normalized size = 0.91

$$\frac{x\left(a^2(-B)+ab\left(5A+Bx^2\right)+3Ab^2x^2\right)}{8a^2b\left(a+bx^2\right)^2} + \frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Maple [A] time = 0.008, size = 90, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(3\,Ab+Ba)\,x^3}{8\,a^2}+\frac{(5\,Ab-Ba)\,x}{8\,ab}\right)+\frac{3\,A}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+3/8/a^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*A+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28903, size = 621, normalized size = 6.75

$$\left[\frac{2\left(Ba^2b^2+3\,Aab^3\right)x^3-\left(\left(Bab^2+3\,Ab^3\right)x^4+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\left(Ba^3b-\right.}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*a^2*b^2 + 3*A*a*b^3)*x^3 - ((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 + 3
*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((B*a^2*b^2 + 3*A*a*b^3)*x^3 + ((B*a*b^2 + 3*A*b^3)*
x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [A] time = 0.725714, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ba)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ba)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3\left(3Ab^2+Bab\right)+x\left(5Aab-\right.}{8a^4b+16a^3b^2x^2+8a^2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b - B*a**2))/(8*a**4*b + 16*a**3*
b**2*x**2 + 8*a**2*b**3*x**4)
```

Giac [A] time = 1.19365, size = 105, normalized size = 1.14

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}+\frac{Babx^3+3\,Ab^2x^3-Ba^2x+5\,Aabx}{8\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(B*a*b*x^3
+ 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2*b)
```

3.104 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=97

$$-\frac{x(7Ab-3aB)}{8a^3\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4a^2\left(a+bx^2\right)^2}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}$$

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b - 3*a*B)*x)/
(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7
/2)*Sqrt[b])
```

Rubi [A] time = 0.100256, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 453, 205}

$$-\frac{x(7Ab-3aB)}{8a^3\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4a^2\left(a+bx^2\right)^2}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b - 3*a*B)*x)/
(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7
/2)*Sqrt[b])
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2} - \frac{1}{4}\int \frac{-\frac{4A}{a}+\frac{3(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)^2}\,dx$$

$$= -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2} - \frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)} + \frac{1}{8}\int \frac{\frac{8A}{a^2}-\frac{(7Ab-3aB)x^2}{a^3}}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{A}{a^3x} - \frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2} - \frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)} - \frac{(3(5Ab-aB))\int \frac{1}{a+bx^2}\,dx}{8a^3}$$

$$= -\frac{A}{a^3x} - \frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2} - \frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)} - \frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.057687, size = 96, normalized size = 0.99

$$\frac{x(3aB-7Ab)}{8a^3\left(a+bx^2\right)} + \frac{x(aB-Ab)}{4a^2\left(a+bx^2\right)^2} + \frac{3(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} - \frac{A}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + ((-(A*b) + a*B)*x)/(4*a^2*(a + b*x^2)^2) + ((-7*A*b + 3*a*B)
*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.012, size = 125, normalized size = 1.3

$$-\frac{A}{a^3x} - \frac{7\,Ax^3b^2}{8\,a^3\left(bx^2+a\right)^2} + \frac{3\,bBx^3}{8\,a^2\left(bx^2+a\right)^2} - \frac{9\,Abx}{8\,a^2\left(bx^2+a\right)^2} + \frac{5\,Bx}{8\,a\left(bx^2+a\right)^2} - \frac{15\,Ab}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{3\,B}{8\,a^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -A/a^3/x-7/8/a^3/(b*x^2+a)^2*A*x^3*b^2+3/8/a^2/(b*x^2+a)^2*B*x^3*b-9/8/a^2/
(b*x^2+a)^2*b*A*x+5/8/a/(b*x^2+a)^2*B*x-15/8/a^3/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*A*b+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29727, size = 686, normalized size = 7.07

$$\left[-\frac{16\,Aa^3b-6\left(Ba^2b^2-5\,Aab^3\right)x^4-10\left(Ba^3b-5\,Aa^2b^2\right)x^2-3\left(\left(Bab^2-5\,Ab^3\right)x^5+2\left(Ba^2b-5\,Aab^2\right)x^3+\left(Ba^3-5\right.\right.}{16\left(a^4b^3x^5+2\,a^5b^2x^3+a^6bx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*a^3*b - 6*(B*a^2*b^2 - 5*A*a*b^3)*x^4 - 10*(B*a^3*b - 5*A*a^2*
b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^
3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*A*a^3*b - 3*(B*a^2*b^2 -
5*A*a*b^3)*x^4 - 5*(B*a^3*b - 5*A*a^2*b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5
 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

Sympy [B] time = 0.87941, size = 194, normalized size = 2.

$$-\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(-\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{-8Aa^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(-3*a**4*sqrt(-1/(a**7*b))*(-5*A*b +
 B*a)/(-15*A*b + 3*B*a) + x)/16 + 3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(3*
a**4*sqrt(-1/(a**7*b))*(-5*A*b + B*a)/(-15*A*b + 3*B*a) + x)/16 + (-8*A*a**
2 + x**4*(-15*A*b**2 + 3*B*a*b) + x**2*(-25*A*a*b + 5*B*a**2))/(8*a**5*x +
16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A] time = 1.18626, size = 111, normalized size = 1.14

$$\frac{3\left(Ba-5\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{A}{a^3x}+\frac{3\,Babx^3-7\,Ab^2x^3+5\,Ba^2x-9\,Aabx}{8\left(bx^2+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - A/(a^3*x) + 1/8*(
3*B*a*b*x^3 - 7*A*b^2*x^3 + 5*B*a^2*x - 9*A*a*b*x)/((b*x^2 + a)^2*a^3)
```

3.105 $\int \frac{A+Bx^2}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=117

$$\frac{bx(11Ab-7aB)}{8a^4(a+bx^2)}+\frac{bx(Ab-aB)}{4a^3(a+bx^2)^2}+\frac{3Ab-aB}{a^4x}+\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{3a^3x^3}$$

```
[Out] -A/(3*a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3*(a + b*x^
2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] time = 0.165264, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {456, 1259, 1261, 205}

$$\frac{bx(11Ab-7aB)}{8a^4(a+bx^2)}+\frac{bx(Ab-aB)}{4a^3(a+bx^2)^2}+\frac{3Ab-aB}{a^4x}+\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -A/(3*a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3*(a + b*x^
2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx = \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4}b\int \frac{-\frac{4A}{ab}+\frac{4(Ab-aB)x^2}{a^2b}-\frac{3(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx$$

$$= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \frac{-8aAb+8b(2Ab-aB)x^2-\frac{b^2(11Ab-7aB)x^4}{a}}{x^4(a+bx^2)}\,dx}{8a^3b}$$

$$= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \left(-\frac{8Ab}{x^4}-\frac{8b(-3Ab+aB)}{ax^2}+\frac{5b^2(-7Ab+3aB)}{a(a+bx^2)}\right)dx}{8a^3b}$$

$$= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{(5b(7Ab-3aB))\int \frac{1}{a+bx^2}\,dx}{8a^4}$$

$$= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.0893896, size = 116, normalized size = 0.99

$$\frac{a^2bx^2\left(56A-75Bx^2\right)-8a^3\left(A+3Bx^2\right)+5ab^2x^4\left(35A-9Bx^2\right)+105Ab^3x^6}{24a^4x^3\left(a+bx^2\right)^2}+\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (105*A*b^3*x^6 + a^2*b*x^2*(56*A - 75*B*x^2) + 5*a*b^2*x^4*(35*A - 9*B*x^2)
 - 8*a^3*(A + 3*B*x^2))/(24*a^4*x^3*(a + b*x^2)^2) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Maple [A] time = 0.013, size = 152, normalized size = 1.3

$$-\frac{A}{3\,a^3x^3}+3\,\frac{Ab}{a^4x}-\frac{B}{a^3x}+\frac{11\,b^3Ax^3}{8\,a^4\left(bx^2+a\right)^2}-\frac{7\,b^2Bx^3}{8\,a^3\left(bx^2+a\right)^2}+\frac{13\,b^2Ax}{8\,a^3\left(bx^2+a\right)^2}-\frac{9\,bBx}{8\,a^2\left(bx^2+a\right)^2}+\frac{35\,Ab^2}{8\,a^4}\arctan\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3*A/a^3/x^3+3/a^4/x*A*b-1/a^3/x*B+11/8/a^4*b^3/(b*x^2+a)^2*A*x^3-7/8/a^3
*b^2/(b*x^2+a)^2*B*x^3+13/8/a^3*b^2/(b*x^2+a)^2*A*x-9/8/a^2*b/(b*x^2+a)^2*B
```

```
*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-15/8/a^3*b/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32574, size = 782, normalized size = 6.68

$$\left[-\frac{30\left(3\,Bab^2-7\,Ab^3\right)x^6+50\left(3\,Ba^2b-7\,Aab^2\right)x^4+16\,Aa^3+16\left(3\,Ba^3-7\,Aa^2b\right)x^2+15\left(\left(3\,Bab^2-7\,Ab^3\right)x^7+2\left(3\,Ba^2\right.\right.}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*a*b^2 - 7*A*b^3)*x^6 + 50*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 16*
A*a^3 + 16*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(3
*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^
3), -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 +
8*A*a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(
3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [B] time = 1.18524, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(-\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{8Aa^3+x^6\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

```
[Out] 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(-5*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)
/(-35*A*b**2 + 15*B*a*b) + x)/16 - 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(5*a
**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)/(-35*A*b**2 + 15*B*a*b) + x)/16 - (8*A*a
**3 + x**6*(-105*A*b**3 + 45*B*a*b**2) + x**4*(-175*A*a*b**2 + 75*B*a**2*b)
 + x**2*(-56*A*a**2*b + 24*B*a**3))/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**
4*b**2*x**7)
```

Giac [A] time = 1.48622, size = 146, normalized size = 1.25

$$-\frac{5\left(3\,Bab-7\,Ab^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^{4}}-\frac{7\,Bab^{2}x^{3}-11\,Ab^{3}x^{3}+9\,Ba^{2}bx-13\,Aab^{2}x}{8\,\left(bx^{2}+a\right)^{2}a^{4}}-\frac{3\,Bax^{2}-9\,Abx^{2}+Aa}{3\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(7*B*a
*b^2*x^3 - 11*A*b^3*x^3 + 9*B*a^2*b*x - 13*A*a*b^2*x)/((b*x^2 + a)^2*a^4) -
 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

3.106 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{b^2x(15Ab-11aB)}{8a^5\left(a+bx^2\right)}-\frac{b^2x(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{A}{5a^3x^5}$$

```
[Out] -A/(5*a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/(a^5*x) -
(b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*b - 11*a*B)*x)/(8*a^
5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(11/2))
```

Rubi [A] time = 0.329677, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {456, 1805, 1802, 205}

$$-\frac{b^2x(15Ab-11aB)}{8a^5\left(a+bx^2\right)}-\frac{b^2x(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{A}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -A/(5*a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/(a^5*x) -
(b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*b - 11*a*B)*x)/(8*a^
5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(11/2))
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4}b^2\int \frac{-\frac{4A}{ab^2}+\frac{4(Ab-aB)x^2}{a^2b^2}-\frac{4(Ab-aB)x^4}{a^3b}+\frac{3(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)^2}\,dx$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}+\frac{b^2\int \frac{\frac{8A}{ab^2}-\frac{8(2Ab-aB)x^2}{a^2b^2}+\frac{8(3Ab-2aB)x^4}{a^3b}-\frac{(15Ab-11aB)x^6}{a^4}}{x^6\left(a+bx^2\right)}\,dx}{8a}$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}+\frac{b^2\int \left(\frac{8A}{a^2b^2x^6}+\frac{8(-3Ab+aB)}{a^3b^2x^4}-\frac{24(-2Ab+aB)}{a^4bx^2}+\frac{7(-9Ab+5aB)}{a^4\left(a+bx^2\right)}\right) d}{8a}$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{\left(7b^2(9Ab-5aB\right.}{8a^5}$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{7b^{3/2}(9Ab-5aB}{8a^{11}}$$

Mathematica [A] time = 0.100048, size = 139, normalized size = 0.98

$$\frac{7a^2b^2x^4\left(125Bx^2-72A\right)+8a^3bx^2\left(9A+35Bx^2\right)-8a^4\left(3A+5Bx^2\right)+525ab^3x^6\left(Bx^2-3A\right)-945Ab^4x^8}{120a^5x^5\left(a+bx^2\right)^2}+\frac{7b^{3/2}(5aB}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] (-945*A*b^4*x^8 + 525*a*b^3*x^6*(-3*A + B*x^2) - 8*a^4*(3*A + 5*B*x^2) + 8*
a^3*b*x^2*(9*A + 35*B*x^2) + 7*a^2*b^2*x^4*(-72*A + 125*B*x^2))/(120*a^5*x^
5*(a + b*x^2)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(11/2))
```

Maple [A] time = 0.015, size = 177, normalized size = 1.3

$$-\frac{A}{5\,a^3x^5}+\frac{Ab}{a^4x^3}-\frac{B}{3\,a^3x^3}-6\,\frac{Ab^2}{a^5x}+3\,\frac{Bb}{a^4x}-\frac{15\,b^4Ax^3}{8\,a^5\left(bx^2+a\right)^2}+\frac{11\,b^3Bx^3}{8\,a^4\left(bx^2+a\right)^2}-\frac{17\,b^3Ax}{8\,a^4\left(bx^2+a\right)^2}+\frac{13\,b^2Bx}{8\,a^3\left(bx^2+a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5*A/a^3/x^5+1/a^4/x^3*A*b-1/3/a^3/x^3*B-6*b^2/a^5/x*A+3*b/a^4/x*B-15/8/a
^5*b^4/(b*x^2+a)^2*A*x^3+11/8/a^4*b^3/(b*x^2+a)^2*B*x^3-17/8/a^4*b^3/(b*x^2
```

```
+a)^2*A*x+13/8/a^3*b^2/(b*x^2+a)^2*B*x-63/8/a^5*b^3/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*A+35/8/a^4*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34927, size = 909, normalized size = 6.4

$$\left[\frac{210\left(5\,Bab^3-9\,Ab^4\right)x^8+350\left(5\,Ba^2b^2-9\,Aab^3\right)x^6-48\,Aa^4+112\left(5\,Ba^3b-9\,Aa^2b^2\right)x^4-16\left(5\,Ba^4-9\,Aa^3b\right)x^2-105}{240\left(a^5b^2x^9+2\,a^6bx^7+a^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*a*b^3 - 9*A*b^4)*x^8 + 350*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
 48*A*a^4 + 112*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 16*(5*B*a^4 - 9*A*a^3*b)*x^
2 - 105*((5*B*a*b^3 - 9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B
*a^3*b - 9*A*a^2*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), 1/120*(105*(5*B*a*b^3 -
9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 - 24*A*a^4 + 56*(5*B*a^3*b
 - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2 + 105*((5*B*a*b^3 - 9*A*b
^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A*a^2*b^2)*x^5)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A] time = 1.64817, size = 260, normalized size = 1.83

$$-\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(-\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{-24Aa^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -7*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)*log(-7*a**6*sqrt(-b**3/a**11)*(-9*A*b
 + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + 7*sqrt(-b**3/a**11)*(-9*A*b
+ 5*B*a)*log(7*a**6*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)/(-63*A*b**3 + 35*B*a
*b**2) + x)/16 + (-24*A*a**4 + x**8*(-945*A*b**4 + 525*B*a*b**3) + x**6*(-1
575*A*a*b**3 + 875*B*a**2*b**2) + x**4*(-504*A*a**2*b**2 + 280*B*a**3*b) +
x**2*(72*A*a**3*b - 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5
```

```
*b**2*x**9)
```

Giac [A] time = 1.15926, size = 182, normalized size = 1.28

$$\frac{7\left(5\,Bab^2-9\,Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}+\frac{11\,Bab^3x^3-15\,Ab^4x^3+13\,Ba^2b^2x-17\,Aab^3x}{8\left(bx^2+a\right)^2a^5}+\frac{45\,Babx^4-90\,Ab^2x^4-5\,Ba^2x^2}{15\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(11*B
*a*b^3*x^3 - 15*A*b^4*x^3 + 13*B*a^2*b^2*x - 17*A*a*b^3*x)/((b*x^2 + a)^2*a
^5) + 1/15*(45*B*a*b*x^4 - 90*A*b^2*x^4 - 5*B*a^2*x^2 + 15*A*a*b*x^2 - 3*A*
a^2)/(a^5*x^5)
```

3.107 $\int \frac{a+bx^2}{1+x^2}\,dx$

Optimal. Leaf size=12

$$(a-b)\tan^{-1}(x)+bx$$

```
[Out] b*x + (a - b)*ArcTan[x]
```

Rubi [A] time = 0.0063076, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 203}

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] b*x + (a - b)*ArcTan[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2}\,dx &= bx-(-a+b)\int \frac{1}{1+x^2}\,dx\\ &= bx+(a-b)\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0062263, size = 12, normalized size = 1.

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] b*x + (a - b)*ArcTan[x]
```

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$bx + \arctan(x)\, a - \arctan(x)\, b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^2+1),x)
```

```
[Out] b*x+arctan(x)*a-arctan(x)*b
```

Maxima [A] time = 1.51263, size = 16, normalized size = 1.33

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] b*x + (a - b)*arctan(x)
```

Fricas [A] time = 1.20739, size = 34, normalized size = 2.83

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] b*x + (a - b)*arctan(x)
```

Sympy [C] time = 0.288837, size = 26, normalized size = 2.17

$$bx - \frac{i(a - b)\log(x - i)}{2} + \frac{i(a - b)\log(x + i)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**2+1),x)
```

```
[Out] b*x - I*(a - b)*log(x - I)/2 + I*(a - b)*log(x + I)/2
```

Giac [A] time = 1.15347, size = 16, normalized size = 1.33

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] b*x + (a - b)*arctan(x)
```

3.108 $\int \frac{a+bx^2}{1-x^2} \, dx$

Optimal. Leaf size=11

$$(a+b)\tanh^{-1}(x)-bx$$

```
[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rubi [A] time = 0.0066391, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 206}

$$(a+b)\tanh^{-1}(x)-bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1-x^2}\,dx &= -bx-(-a-b)\int \frac{1}{1-x^2}\,dx\\ &= -bx+(a+b)\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0085583, size = 28, normalized size = 2.55

$$\frac{1}{2}(-(a+b)\log(1-x)+(a+b)\log(x+1)-2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] (-2*b*x - (a + b)*Log[1 - x] + (a + b)*Log[1 + x])/2
```

Maple [B] time = 0.003, size = 34, normalized size = 3.1

$$-bx - \frac{\ln(-1+x)\,a}{2} - \frac{\ln(-1+x)\,b}{2} + \frac{\ln(1+x)\,a}{2} + \frac{\ln(1+x)\,b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-x^2+1),x)
```

```
[Out] -b*x-1/2*ln(-1+x)*a-1/2*ln(-1+x)*b+1/2*ln(1+x)*a+1/2*ln(1+x)*b
```

Maxima [B] time = 0.996524, size = 31, normalized size = 2.82

$$-bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Fricas [B] time = 1.27101, size = 76, normalized size = 6.91

$$-bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Sympy [B] time = 0.300267, size = 22, normalized size = 2.

$$-bx - \frac{(a+b)\log(x-1)}{2} + \frac{(a+b)\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-x**2+1),x)
```

```
[Out] -b*x - (a + b)*log(x - 1)/2 + (a + b)*log(x + 1)/2
```

Giac [B] time = 1.14409, size = 34, normalized size = 3.09

$$-bx + \frac{1}{2}(a+b)\log(|x+1|) - \frac{1}{2}(a+b)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(a + b)*log(abs(x + 1)) - 1/2*(a + b)*log(abs(x - 1))
```

3.109 $\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(1 - x^2)
```

Rubi [A] time = 0.0028755, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-1 + x^2)^2,x]

[Out] x/(1 - x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx = \frac{x}{1-x^2}$$

Mathematica [A] time = 0.0037914, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-1 + x^2)^2,x]

[Out] -(x/(-1 + x^2))
```

Maple [A] time = 0.005, size = 16, normalized size = 1.5

$$-\frac{1}{-2+2x}-\frac{1}{2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)^2,x)

[Out] -1/2/(-1+x)-1/2/(1+x)
```

Maxima [A] time = 0.972072, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="maxima")

[Out] -x/(x^2 - 1)
```

Fricas [A] time = 1.2316, size = 19, normalized size = 1.73

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="fricas")

[Out] -x/(x^2 - 1)
```

Sympy [A] time = 0.080466, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)**2,x)

[Out] -x/(x**2 - 1)
```

Giac [A] time = 1.1409, size = 15, normalized size = 1.36

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="giac")

[Out] -1/(x - 1/x)
```

3.110 $\int \frac{1-x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

[Out] x/(1 + x^2)

Rubi [A] time = 0.0030811, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2)

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx = \frac{x}{1+x^2}$$

Mathematica [A] time = 0.0036344, size = 9, normalized size = 1.

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2)

Maple [A] time = 0.004, size = 10, normalized size = 1.1

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)^2,x)
```

```
[Out] x/(x^2+1)
```

Maxima [A] time = 1.01633, size = 12, normalized size = 1.33

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A] time = 1.17403, size = 18, normalized size = 2.

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A] time = 0.082789, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1)
```

Giac [A] time = 1.1526, size = 9, normalized size = 1.

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

3.111 $\int \frac{3+2x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2

Rubi [A] time = 0.0043252, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {385, 203}

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)/(1 + x^2)^2,x]

[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{\left(1+x^2\right)^2} \, dx &= \frac{x}{2\left(1+x^2\right)}+\frac{5}{2}\int \frac{1}{1+x^2} \, dx\\ &= \frac{x}{2\left(1+x^2\right)}+\frac{5}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0071309, size = 19, normalized size = 1.

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x^2)/(1 + x^2)^2,x]

```
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$\frac{x}{2\,x^{2}+2}+\frac{5\,\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^2+1)^2,x)
```

```
[Out] 1/2*x/(x^2+1)+5/2*arctan(x)
```

Maxima [A] time = 1.48689, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^{2}+1\right)}+\frac{5}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

Fricas [A] time = 1.28191, size = 58, normalized size = 3.05

$$\frac{5\left(x^{2}+1\right)\arctan\left(x\right)+x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.099179, size = 14, normalized size = 0.74

$$\frac{x}{2x^{2}+2}+\frac{5\,\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + 5*atan(x)/2
```

Giac [A] time = 1.12801, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

3.112 $\int \frac{-2+x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rubi [A] time = 0.0040069, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 203}

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2}{\left(1+x^2\right)^2} \, dx &= -\frac{3x}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= -\frac{3x}{2\left(1+x^2\right)} - \frac{1}{2}\tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0071582, size = 19, normalized size = 1.

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{3\,x}{2\,x^{2}+2}-\frac{\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/(x^2+1)^2,x)
```

```
[Out] -3/2*x/(x^2+1)-1/2*arctan(x)
```

Maxima [A] time = 1.52571, size = 20, normalized size = 1.05

$$-\frac{3\,x}{2\left(x^{2}+1\right)}-\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

Fricas [A] time = 1.24542, size = 59, normalized size = 3.11

$$-\frac{\left(x^{2}+1\right)\arctan\left(x\right)+3\,x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) + 3*x)/(x^2 + 1)
```

Sympy [A] time = 0.098991, size = 15, normalized size = 0.79

$$-\frac{3x}{2x^{2}+2}-\frac{\operatorname{atan}{\left(x\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/(x**2+1)**2,x)
```

```
[Out] -3*x/(2*x**2 + 2) - atan(x)/2
```

<u>Giac</u> [A] time = 1.11447, size = 20, normalized size = 1.05

$$-\frac{3\,x}{2\left(x^2+1\right)}-\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

3.113 $\int \frac{3+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=14

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Rubi [A] time = 0.0040127, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 203}

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{\left(1+x^2\right)^2} \, dx &= \frac{x}{1+x^2}+2\int \frac{1}{1+x^2} \, dx\\ &= \frac{x}{1+x^2}+2\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0059902, size = 14, normalized size = 1.

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.1

$$\frac{x}{x^2+1}+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2+1)^2,x)
```

```
[Out] x/(x^2+1)+2*arctan(x)
```

Maxima [A] time = 1.49805, size = 19, normalized size = 1.36

$$\frac{x}{x^2+1}+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

Fricas [A] time = 1.30389, size = 53, normalized size = 3.79

$$\frac{2\left(x^2+1\right)\arctan(x)+x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (2*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.092744, size = 10, normalized size = 0.71

$$\frac{x}{x^2+1}+2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1) + 2*atan(x)
```

Giac [A] time = 1.64378, size = 19, normalized size = 1.36

$$\frac{x}{x^2+1}+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="giac")

[Out] x/(x^2 + 1) + 2*arctan(x)
```

3.114 $\int \frac{a+bx^2}{\left(-a+bx^2\right)^2} dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

```
[Out] x/(a - b*x^2)
```

Rubi [A] time = 0.0043831, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {383}

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(-a + b*x^2)^2,x]
```

```
[Out] x/(a - b*x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(-a+bx^2\right)^2} dx = \frac{x}{a-bx^2}$$

Mathematica [A] time = 0.007032, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(-a + b*x^2)^2,x]
```

```
[Out] -(x/(-a + b*x^2))
```

Maple [A] time = 0.006, size = 15, normalized size = 1.3

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(b*x^2-a)^2,x)
```

```
[Out] -x/(b*x^2-a)
```

Maxima [A] time = 0.966137, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

Fricas [A] time = 1.30814, size = 22, normalized size = 1.83

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="fricas")
```

```
[Out] -x/(b*x^2 - a)
```

Sympy [A] time = 0.317147, size = 8, normalized size = 0.67

$$-\frac{x}{-a + bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(b*x**2-a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

Giac [A] time = 1.17046, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="giac")
```

```
[Out] -x/(b*x^2 - a)
```

3.115 $\int \frac{a+bx^2}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

```
[Out] x/(a - b*x^2)
```

Rubi [A] time = 0.004321, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {383}

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] x/(a - b*x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = \frac{x}{a-bx^2}$$

Mathematica [A] time = 0.0051393, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] -(x/(-a + b*x^2))
```

Maple [A] time = 0.005, size = 15, normalized size = 1.3

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-b*x^2+a)^2,x)

[Out] -x/(b*x^2-a)
```

Maxima [A] time = 1.00896, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="maxima")

[Out] -x/(b*x^2 - a)
```

Fricas [A] time = 1.18756, size = 22, normalized size = 1.83

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="fricas")

[Out] -x/(b*x^2 - a)
```

Sympy [A] time = 0.314577, size = 8, normalized size = 0.67

$$-\frac{x}{-a + bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-b*x**2+a)**2,x)

[Out] -x/(-a + b*x**2)
```

Giac [A] time = 1.12682, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="giac")

[Out] -x/(b*x^2 - a)
```

3.116 $\int \frac{A+Bx^2}{a-bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.018227, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {388, 208}

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a-bx^2}\,dx &= -\frac{Bx}{b}+\frac{(Ab+aB)\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{Bx}{b}+\frac{(Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0214197, size = 39, normalized size = 1.

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.003, size = 37, normalized size = 1.

$$-\frac{Bx}{b} - \frac{-Ab - Ba}{b}\mathrm{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-b*x^2+a),x)
```

```
[Out] -B*x/b-(-A*b-B*a)/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.20187, size = 225, normalized size = 5.77

$$\left[-\frac{2\,Babx - (Ba + Ab)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{2\,ab^2}, -\frac{Babx + (Ba + Ab)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a*b*x - (B*a + A*b)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b
*x^2 - a)))/(a*b^2), -(B*a*b*x + (B*a + A*b)*sqrt(-a*b)*arctan(sqrt(-a*b)*x
/a))/(a*b^2)]
```

Sympy [B] time = 0.408331, size = 75, normalized size = 1.92

$$-\frac{Bx}{b} - \frac{\sqrt{\frac{1}{ab^3}}\,(Ab + Ba)\log\left(-ab\sqrt{\frac{1}{ab^3}} + x\right)}{2} + \frac{\sqrt{\frac{1}{ab^3}}\,(Ab + Ba)\log\left(ab\sqrt{\frac{1}{ab^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-b*x**2+a),x)
```

```
[Out] -B*x/b - sqrt(1/(a*b**3))*(A*b + B*a)*log(-a*b*sqrt(1/(a*b**3)) + x)/2 + sq
rt(1/(a*b**3))*(A*b + B*a)*log(a*b*sqrt(1/(a*b**3)) + x)/2
```

Giac [A] time = 1.17008, size = 49, normalized size = 1.26

$$-\frac{Bx}{b}-\frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -B*x/b - (B*a + A*b)*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b)
```

3.117 $\int \frac{1+x^2}{\left(16+x^2\right)^3}\,dx$

Optimal. Leaf size=35

$$\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Rubi [A] time = 0.0068099, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {385, 199, 203}

$$\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^2}{\left(16+x^2\right)^3}\,dx = -\frac{15x}{64\left(16+x^2\right)^2} + \frac{19}{64}\int \frac{1}{\left(16+x^2\right)^2}\,dx$$
$$= -\frac{15x}{64\left(16+x^2\right)^2} + \frac{19x}{2048\left(16+x^2\right)} + \frac{19\int \frac{1}{16+x^2}\,dx}{2048}$$
$$= -\frac{15x}{64\left(16+x^2\right)^2} + \frac{19x}{2048\left(16+x^2\right)} + \frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Mathematica [A] time = 0.0091254, size = 35, normalized size = 1.

$$\frac{19x}{2048\left(x^2+16\right)} - \frac{15x}{64\left(x^2+16\right)^2} + \frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Maple [A] time = 0.007, size = 25, normalized size = 0.7

$$\frac{1}{\left(x^2+16\right)^2}\left(\frac{19\,x^3}{2048} - \frac{11\,x}{128}\right) + \frac{19}{8192}\arctan\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+16)^3,x)
```

```
[Out] (19/2048*x^3-11/128*x)/(x^2+16)^2+19/8192*arctan(1/4*x)
```

Maxima [A] time = 1.50527, size = 41, normalized size = 1.17

$$\frac{19\,x^3 - 176\,x}{2048\left(x^4 + 32\,x^2 + 256\right)} + \frac{19}{8192}\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^4 + 32*x^2 + 256) + 19/8192*arctan(1/4*x)
```

Fricas [A] time = 1.16367, size = 116, normalized size = 3.31

$$\frac{76\,x^3 + 19\left(x^4 + 32\,x^2 + 256\right)\arctan\left(\frac{1}{4}\,x\right) - 704\,x}{8192\left(x^4 + 32\,x^2 + 256\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(76*x^3 + 19*(x^4 + 32*x^2 + 256)*arctan(1/4*x) - 704*x)/(x^4 + 32*x
^2 + 256)
```

Sympy [A] time = 0.124476, size = 27, normalized size = 0.77

$$\frac{19x^3 - 176x}{2048x^4 + 65536x^2 + 524288} + \frac{19\operatorname{atan}\left(\frac{x}{4}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+16)**3,x)
```

```
[Out] (19*x**3 - 176*x)/(2048*x**4 + 65536*x**2 + 524288) + 19*atan(x/4)/8192
```

Giac [A] time = 1.15011, size = 34, normalized size = 0.97

$$\frac{19\,x^3 - 176\,x}{2048\,\left(x^2 + 16\right)^2} + \frac{19}{8192}\,\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^2 + 16)^2 + 19/8192*arctan(1/4*x)
```

3.118 $\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Rubi [A] time = 0.0083808, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{x^3(1+x)^3}\,dx, x, x^2\right)\\ &= -\frac{1}{4x^4\left(1+x^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.0058138, size = 14, normalized size = 1.

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Maple [B] time = 0.011, size = 30, normalized size = 2.1

$$-\frac{1}{4\left(x^{2}+1\right)^{2}}-\frac{1}{2x^{2}+2}-\frac{1}{4x^{4}}+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/x^5/(x^2+1)^3,x)
```

```
[Out] -1/4/(x^2+1)^2-1/2/(x^2+1)-1/4/x^4+1/2/x^2
```

Maxima [A] time = 0.981792, size = 22, normalized size = 1.57

$$-\frac{1}{4\left(x^{8}+2x^{6}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Fricas [A] time = 1.22072, size = 35, normalized size = 2.5

$$-\frac{1}{4\left(x^{8}+2x^{6}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] time = 0.127676, size = 17, normalized size = 1.21

$$-\frac{1}{4x^{8}+8x^{6}+4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/x**5/(x**2+1)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

Giac [A] time = 1.14359, size = 15, normalized size = 1.07

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

3.119 $\int \frac{\left(1-x^2\right)^2}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0004407, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {21, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] x
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(1-x^2\right)^2}{\left(-1+x^2\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0002571, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] x
```

Maple [A] time = 0.002, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2/(x^2-1)^2,x)
```

```
[Out] x
```

Maxima [A] time = 0.993379, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.09399, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.054186, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2/(x**2-1)**2,x)
```

```
[Out] x
```

Giac [A] time = 1.17185, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] x
```

3.120 $\int \frac{x^3(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^4}{4}$$

```
[Out] (c*x^4)/4
```

Rubi [A] time = 0.0015319, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 (ac + bcx^2)}{a + bx^2} dx &= c \int x^3 dx \\ &= \frac{cx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0004125, size = 8, normalized size = 1.

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/4*c*x^4
```

Maxima [A] time = 1.0178, size = 8, normalized size = 1.

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4
```

Fricas [A] time = 1.26015, size = 15, normalized size = 1.88

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4
```

Sympy [A] time = 0.06744, size = 5, normalized size = 0.62

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**4/4
```

Giac [A] time = 1.17708, size = 8, normalized size = 1.

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4
```

3.121 $\int \frac{x^2(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^3}{3}$$

[Out] (c*x^3)/3

Rubi [A] time = 0.0014441, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^3)/3

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 \left(ac + bcx^2\right)}{a + bx^2} dx &= c \int x^2 dx \\ &= \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0003193, size = 8, normalized size = 1.

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^3)/3

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/3*c*x^3
```

Maxima [A] time = 0.989311, size = 8, normalized size = 1.

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3
```

Fricas [A] time = 1.19772, size = 15, normalized size = 1.88

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3
```

Sympy [A] time = 0.065597, size = 5, normalized size = 0.62

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**3/3
```

Giac [A] time = 1.12868, size = 8, normalized size = 1.

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3
```

3.122 $\int \frac{x(ac+bcx^2)}{a+bx^2} \, dx$

Optimal. Leaf size=8

$$\frac{cx^2}{2}$$

```
[Out] (c*x^2)/2
```

Rubi [A] time = 0.0015419, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {21, 30}

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x(ac+bcx^2)}{a+bx^2} \, dx &= c \int x \, dx \\ &= \frac{cx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0002229, size = 8, normalized size = 1.

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/2*c*x^2
```

Maxima [A] time = 0.993909, size = 8, normalized size = 1.

$$\frac{1}{2} cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2
```

Fricas [A] time = 1.18695, size = 15, normalized size = 1.88

$$\frac{1}{2} cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*c*x^2
```

Sympy [A] time = 0.064669, size = 5, normalized size = 0.62

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**2/2
```

Giac [A] time = 1.14358, size = 8, normalized size = 1.

$$\frac{1}{2} cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")

[Out] 1/2*c*x^2
```

3.123 $\int \frac{ac+bcx^2}{a+bx^2} \, dx$

Optimal. Leaf size=3

$$cx$$

```
[Out] c*x
```

Rubi [A] time = 0.0009539, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2),x]
```

```
[Out] c*x
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{ac + bcx^2}{a + bx^2} \, dx &= c \int 1 \, dx \\ &= cx \end{aligned}$$

Mathematica [A] time = 0.0001705, size = 3, normalized size = 1.

$$cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2),x]
```

```
[Out] c*x
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] c*x
```

Maxima [A] time = 0.988227, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*x
```

Fricas [A] time = 1.24767, size = 7, normalized size = 2.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*x
```

Sympy [A] time = 0.061941, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x
```

Giac [A] time = 1.12315, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*x
```

3.124 $\int \frac{ac+bcx^2}{x(a+bx^2)}\,dx$

Optimal. Leaf size=4

$$c\log(x)$$

[Out] c*Log[x]

Rubi [A] time = 0.0010773, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 29}

$$c\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]

[Out] c*Log[x]

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx &= c\int \frac{1}{x}\,dx\\ &= c\log(x)\end{aligned}$$

Mathematica [A] time = 0.0002694, size = 4, normalized size = 1.

$$c\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]

[Out] c*Log[x]

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a),x)
```

```
[Out] c*ln(x)
```

Maxima [A] time = 0.998049, size = 9, normalized size = 2.25

$$\frac{1}{2}c\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)
```

Fricas [A] time = 1.22363, size = 14, normalized size = 3.5

$$c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*log(x)
```

Sympy [A] time = 0.067502, size = 3, normalized size = 0.75

$$c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)
```

Giac [A] time = 1.14353, size = 7, normalized size = 1.75

$$c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))
```

3.125 $\int \frac{ac+bcx^2}{x^2(a+bx^2)} dx$

Optimal. Leaf size=6

$$-\frac{c}{x}$$

[Out] -(c/x)

Rubi [A] time = 0.0014597, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]

[Out] -(c/x)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)} dx &= c \int \frac{1}{x^2} dx \\ &= -\frac{c}{x} \end{aligned}$$

Mathematica [A] time = 0.0003065, size = 6, normalized size = 1.

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]

[Out] -(c/x)

Maple [A] time = 0., size = 7, normalized size = 1.2

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a),x)
```

```
[Out] -c/x
```

Maxima [A] time = 0.995483, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -c/x
```

Fricas [A] time = 1.19108, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -c/x
```

Sympy [A] time = 0.067935, size = 3, normalized size = 0.5

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/x
```

Giac [A] time = 1.11491, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/x
```

3.126 $\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=8

$$-\frac{c}{2x^2}$$

[Out] -c/(2*x^2)

Rubi [A] time = 0.0014944, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]

[Out] -c/(2*x^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx &= c\int \frac{1}{x^3}\,dx \\ &= -\frac{c}{2x^2} \end{aligned}$$

Mathematica [A] time = 0.0002812, size = 8, normalized size = 1.

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]

[Out] -c/(2*x^2)

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*c/x^2
```

Maxima [A] time = 0.996967, size = 8, normalized size = 1.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*c/x^2
```

Fricas [A] time = 1.19666, size = 16, normalized size = 2.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*c/x^2
```

Sympy [A] time = 0.07011, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*x**2)
```

Giac [A] time = 1.16327, size = 8, normalized size = 1.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*c/x^2
```

3.127 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=29

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

```
[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0213888, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 266, 43}

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx &= c\int \frac{x^3}{a+bx^2} dx \\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{x}{a+bx} dx, x, x^2\right) \\
&= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0044315, size = 29, normalized size = 1.

$$c\left(\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] c*(x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$\frac{cx^2}{2b}-\frac{ac\ln\left(bx^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*c*x^2/b-1/2*a*c*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.00552, size = 34, normalized size = 1.17

$$\frac{cx^2}{2b}-\frac{ac\log\left(bx^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/b - 1/2*a*c*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.19585, size = 54, normalized size = 1.86

$$\frac{bcx^2-ac\log\left(bx^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2 - a*c*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.290732, size = 22, normalized size = 0.76

$$c\left(-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b))
```

Giac [A] time = 1.65468, size = 63, normalized size = 2.17

$$\frac{\frac{ac\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}+\frac{(bx^2+a)c}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b + (b*x^2 + a)*c/b)/b
```

3.128 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=33

$$\frac{cx}{b} - \frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.01463, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 321, 205}

$$\frac{cx}{b} - \frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2} dx &= c\int \frac{x^2}{a+bx^2} dx\\
&= \frac{cx}{b} - \frac{(ac)\int \frac{1}{a+bx^2} dx}{b}\\
&= \frac{cx}{b} - \frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0077643, size = 33, normalized size = 1.

$$c\left(\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{cx}{b}-\frac{ac}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] c*x/b-c/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30362, size = 176, normalized size = 5.33

$$\left[\frac{c\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,cx}{2\,b},-\frac{c\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-cx}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*c*x)
/b, -(c*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - c*x)/b]
```

Sympy [A] time = 0.298485, size = 58, normalized size = 1.76

$$c\left(\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/
b**3) + x)/2 + x/b)
```

Giac [A] time = 1.14343, size = 38, normalized size = 1.15

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

3.129 $\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=16

$$\frac{c \log \left(a+bx^2\right)}{2b}$$

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Rubi [A] time = 0.0045227, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {21, 260}

$$\frac{c \log \left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2} \, dx &= c \int \frac{x}{a+bx^2} \, dx \\ &= \frac{c \log \left(a+bx^2\right)}{2b} \end{aligned}$$

Mathematica [A] time = 0.002228, size = 16, normalized size = 1.

$$\frac{c \log \left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$\frac{c \ln \left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*c*ln(b*x^2+a)/b
```

Maxima [A] time = 0.99726, size = 19, normalized size = 1.19

$$\frac{c \log \left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Fricas [A] time = 1.24215, size = 32, normalized size = 2.

$$\frac{c \log \left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Sympy [A] time = 0.111414, size = 12, normalized size = 0.75

$$\frac{c \log \left(a+b x^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*log(a + b*x**2)/(2*b)
```

Giac [B] time = 1.12881, size = 85, normalized size = 5.31

$$-\frac{1}{2}c\left(\frac{\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^2+a\right)b}\right)-\frac{ac}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b)) -
 1/2*a*c/((b*x^2 + a)*b)
```

3.130 $\int \frac{ac+bcx^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=25

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0082619, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^2,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{(a+bx^2)^2} dx &= c\int \frac{1}{a+bx^2} dx\\ &= \frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0039821, size = 25, normalized size = 1.

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$c \arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25022, size = 157, normalized size = 6.28

$$\left[-\frac{\sqrt{-ab}c\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqr
t(a*b)*c*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] time = 0.132129, size = 54, normalized size = 2.16

$$c\left(-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x)/2)
```

Giac [A] time = 1.18793, size = 22, normalized size = 0.88

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.131 $\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=24

$$\frac{c\log(x)}{a} - \frac{c\log\left(a+bx^2\right)}{2a}$$

```
[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)
```

Rubi [A] time = 0.0130802, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {21, 266, 36, 29, 31}

$$\frac{c\log(x)}{a} - \frac{c\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x\left(a+bx^2\right)}\,dx \\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right) \\
&= \frac{c\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a} - \frac{(bc)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a} \\
&= \frac{c\log(x)}{a} - \frac{c\log\left(a+bx^2\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0050465, size = 24, normalized size = 1.

$$
c\left(\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] c*(Log[x]/a - Log[a + b*x^2]/(2*a))
```

Maple [A] time = 0.003, size = 23, normalized size = 1.

$$
\frac{c\ln(x)}{a} - \frac{c\ln\left(bx^2+a\right)}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^2,x)
```

```
[Out] c*ln(x)/a-1/2*c*ln(b*x^2+a)/a
```

Maxima [A] time = 0.999539, size = 34, normalized size = 1.42

$$
-\frac{c\log\left(bx^2+a\right)}{2a} + \frac{c\log\left(x^2\right)}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*c*log(b*x^2 + a)/a + 1/2*c*log(x^2)/a
```

Fricas [A] time = 1.22329, size = 54, normalized size = 2.25

$$
-\frac{c\log\left(bx^2+a\right) - 2c\log(x)}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(c*log(b*x^2 + a) - 2*c*log(x))/a
```

Sympy [A] time = 0.190086, size = 17, normalized size = 0.71

$$c\left(\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)
```

```
[Out] c*(log(x)/a - log(a/b + x**2)/(2*a))
```

Giac [A] time = 1.15705, size = 35, normalized size = 1.46

$$\frac{c\log\left(x^2\right)}{2\,a} - \frac{c\log\left(\left|bx^2 + a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*c*log(abs(b*x^2 + a))/a
```

3.132 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0145887, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 325, 205}

$$-\frac{\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x^2\left(a+bx^2\right)}\,dx\\
&= -\frac{c}{ax}-\frac{(bc)\int \frac{1}{a+bx^2}\,dx}{a}\\
&= -\frac{c}{ax}-\frac{\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.012891, size = 36, normalized size = 1.

$$c\left(-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$-\frac{c}{ax}-\frac{bc}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c/a/x-c*b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21956, size = 181, normalized size = 5.03

$$\left[\frac{cx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,c}{2\,ax},-\frac{cx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+c}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*c)
/(a*x), -(c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) + c)/(a*x)]
```

Sympy [B] time = 0.325169, size = 66, normalized size = 1.83

$$c\left(\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*
sqrt(-b/a**3)/b + x)/2 - 1/(a*x))
```

Giac [A] time = 1.16858, size = 42, normalized size = 1.17

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.133 $\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

```
[Out] -c/(2*a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0237782, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 266, 44}

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c/(2*a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x^3\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c}{2ax^2}-\frac{bc\log(x)}{a^2}+\frac{bc\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0068979, size = 37, normalized size = 0.97

$$c\left(\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] c*(-1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2))
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{c}{2\,ax^2}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c/a/x^2-b*c*ln(x)/a^2+1/2*b*c*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.03755, size = 49, normalized size = 1.29

$$\frac{bc\log\left(bx^2+a\right)}{2\,a^2}-\frac{bc\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*log(b*x^2 + a)/a^2 - 1/2*b*c*log(x^2)/a^2 - 1/2*c/(a*x^2)
```

Fricas [A] time = 1.25337, size = 88, normalized size = 2.32

$$\frac{bcx^2\log\left(bx^2+a\right)-2\,bcx^2\log(x)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log(b*x^2 + a) - 2*b*c*x^2*log(x) - a*c)/(a^2*x^2)
```

Sympy [A] time = 0.408551, size = 32, normalized size = 0.84

$$c\left(-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] c*(-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2))
```

Giac [A] time = 1.18102, size = 63, normalized size = 1.66

$$-\frac{bc\log\left(x^2\right)}{2\,a^2}+\frac{bc\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bcx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*log(x^2)/a^2 + 1/2*b*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 - a*
c)/(a^2*x^2)
```

3.134 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=35

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

```
[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0268245, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 266, 43}

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3 \left(ac + bcx^2\right)}{\left(a + bx^2\right)^3} \, dx = c \int \frac{x^3}{\left(a + bx^2\right)^2} \, dx$$
$$= \frac{1}{2} c \,\text{Subst}\left(\int \frac{x}{(a + bx)^2} \, dx, x, x^2\right)$$
$$= \frac{1}{2} c \,\text{Subst}\left(\int \left(-\frac{a}{b(a + bx)^2} + \frac{1}{b(a + bx)}\right) dx, x, x^2\right)$$
$$= \frac{ac}{2b^2 \left(a + bx^2\right)} + \frac{c \log \left(a + bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.0089612, size = 28, normalized size = 0.8

$$\frac{c \left(\frac{a}{a+bx^2} + \log \left(a + bx^2\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (c*(a/(a + b*x^2) + Log[a + b*x^2]))/(2*b^2)
```

Maple [A] time = 0.007, size = 32, normalized size = 0.9

$$\frac{ac}{2\, b^2 \left(bx^2 + a\right)} + \frac{c \ln \left(bx^2 + a\right)}{2\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*a*c/b^2/(b*x^2+a)+1/2*c*ln(b*x^2+a)/b^2
```

Maxima [A] time = 0.997979, size = 46, normalized size = 1.31

$$\frac{ac}{2\left(b^3x^2 + ab^2\right)} + \frac{c \log \left(bx^2 + a\right)}{2\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c/(b^3*x^2 + a*b^2) + 1/2*c*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.21433, size = 84, normalized size = 2.4

$$\frac{ac + \left(bcx^2 + ac\right) \log \left(bx^2 + a\right)}{2\left(b^3x^2 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c + (b*c*x^2 + a*c)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.340099, size = 31, normalized size = 0.89

$$c\left(\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log\left(a + bx^2\right)}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2))
```

Giac [A] time = 1.23018, size = 43, normalized size = 1.23

$$\frac{c\log\left(\left|bx^2 + a\right|\right)}{2\,b^2} + \frac{ac}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(b*x^2 + a))/b^2 + 1/2*a*c/((b*x^2 + a)*b^2)
```

3.135 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=47

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{cx}{2b\left(a+bx^2\right)}$$

```
[Out] -(c*x)/(2*b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(3/
2))
```

Rubi [A] time = 0.0153651, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 288, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{cx}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(c*x)/(2*b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(3/
2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2 \left(ac + bcx^2\right)}{\left(a + bx^2\right)^3} dx = c \int \frac{x^2}{\left(a + bx^2\right)^2} dx$$
$$= -\frac{cx}{2b\left(a + bx^2\right)} + \frac{c \int \frac{1}{a+bx^2} dx}{2b}$$
$$= -\frac{cx}{2b\left(a + bx^2\right)} + \frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

Mathematica [A] time = 0.0200343, size = 47, normalized size = 1.

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{x}{2b\left(a + bx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] c*(-x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2)))
```

Maple [A] time = 0.006, size = 38, normalized size = 0.8

$$-\frac{cx}{2\,b\left(bx^2 + a\right)} + \frac{c}{2\,b} \arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c*x/b/(b*x^2+a)+1/2*c/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21429, size = 279, normalized size = 5.94

$$\left[-\frac{2\,abcx + \left(bcx^2 + ac\right)\sqrt{-ab}\log\left(\frac{bx^2 - 2\,\sqrt{-ab}x - a}{bx^2 + a}\right)}{4\left(ab^3x^2 + a^2b^2\right)}, -\frac{abcx - \left(bcx^2 + ac\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(ab^3x^2 + a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*c*x + (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*c*x - (b*c*x^2 + a*c)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] time = 0.353539, size = 80, normalized size = 1.7

$$c\left(-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3))
+ x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4)
```

Giac [A] time = 1.16672, size = 50, normalized size = 1.06

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b}-\frac{cx}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*c*x/((b*x^2 + a)*b)
```

3.136 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=17

$$-\frac{c}{2b(a+bx^2)}$$

```
[Out] -c/(2*b*(a + b*x^2))
```

Rubi [A] time = 0.0044847, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {21, 261}

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -c/(2*b*(a + b*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(ac+bcx^2)}{(a+bx^2)^3} dx &= c\int \frac{x}{(a+bx^2)^2} dx\\ &= -\frac{c}{2b(a+bx^2)}\end{aligned}$$

Mathematica [A] time = 0.0020816, size = 17, normalized size = 1.

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -c/(2*b*(a + b*x^2))
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$-\frac{c}{2\,b\left(b x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/b/(b*x^2+a)
```

Maxima [A] time = 0.982895, size = 22, normalized size = 1.29

$$-\frac{c}{2\left(b^2 x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

Fricas [A] time = 1.22489, size = 32, normalized size = 1.88

$$-\frac{c}{2\left(b^2 x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

Sympy [A] time = 0.305678, size = 15, normalized size = 0.88

$$-\frac{c}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] -c/(2*a*b + 2*b**2*x**2)
```

Giac [A] time = 1.67979, size = 20, normalized size = 1.18

$$-\frac{c}{2\left(bx^{2}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*c/((b*x^2 + a)*b)
```

3.137 $\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=47

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{cx}{2a\left(a+bx^2\right)}$$

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b
])
```

Rubi [A] time = 0.0133698, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {21, 199, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{cx}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+bcx^2}{\left(a+bx^2\right)^3}\,dx = c\int \frac{1}{\left(a+bx^2\right)^2}\,dx$$
$$= \frac{cx}{2a\left(a+bx^2\right)} + \frac{c\int \frac{1}{a+bx^2}\,dx}{2a}$$
$$= \frac{cx}{2a\left(a+bx^2\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.0232739, size = 47, normalized size = 1.

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$\frac{cx}{2\,a\left(bx^2+a\right)} + \frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21978, size = 277, normalized size = 5.89

$$\left[\frac{2\,abcx - \left(bcx^2+ac\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)}, \frac{abcx + \left(bcx^2+ac\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*c*x - (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*c*x + (b*c*x^2 + a*c)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] time = 0.361104, size = 80, normalized size = 1.7

$$c\left(\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
+ x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4)
```

Giac [A] time = 1.21171, size = 50, normalized size = 1.06

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}+\frac{cx}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*c*x/((b*x^2 + a)*a)
```

3.138 $\int \frac{ac+bcx^2}{x(a+bx^2)^3}\,dx$

Optimal. Leaf size=41

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0301275, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 266, 44}

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x\left(a+bx^2\right)^2}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{c}{2a\left(a+bx^2\right)}+\frac{c\log(x)}{a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0138165, size = 34, normalized size = 0.83

$$\frac{c\left(\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (c*(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2]))/(2*a^2)
```

Maple [A] time = 0.01, size = 38, normalized size = 0.9

$$\frac{c}{2\,a\left(bx^2+a\right)}+\frac{c\ln(x)}{a^2}-\frac{c\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

Maxima [A] time = 0.978968, size = 54, normalized size = 1.32

$$\frac{c}{2\left(abx^2+a^2\right)}-\frac{c\log\left(bx^2+a\right)}{2\,a^2}+\frac{c\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c/(a*b*x^2 + a^2) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(x^2)/a^2
```

Fricas [A] time = 1.2695, size = 120, normalized size = 2.93

$$\frac{ac-\left(bcx^2+ac\right)\log\left(bx^2+a\right)+2\left(bcx^2+ac\right)\log(x)}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c - (b*c*x^2 + a*c)*log(b*x^2 + a) + 2*(b*c*x^2 + a*c)*log(x))/(a^2*
b*x^2 + a^3)
```

Sympy [A] time = 0.432526, size = 36, normalized size = 0.88

$$c\left(\frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
```

```
[Out] c*(1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2))
```

Giac [A] time = 1.12721, size = 69, normalized size = 1.68

$$\frac{c\log\left(x^2\right)}{2\,a^2}-\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bcx^2+2\,ac}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 + 2*a*c)/
((b*x^2 + a)*a^2)
```

3.139 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0220548, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {21, 290, 325, 205}

$$-\frac{3\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx \\
&= \frac{c}{2ax\left(a+bx^2\right)}+\frac{(3c)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a} \\
&= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{(3bc)\int \frac{1}{a+bx^2}\,dx}{2a^2} \\
&= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0363686, size = 56, normalized size = 0.93

$$
c\left(-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{a^2x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] c*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2)))
```

Maple [A] time = 0.007, size = 49, normalized size = 0.8

$$
-\frac{c}{a^2x}-\frac{bcx}{2\,a^2\left(bx^2+a\right)}-\frac{3\,bc}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -c/a^2/x-1/2*c*b/a^2*x/(b*x^2+a)-3/2*c*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27035, size = 309, normalized size = 5.15

$$\left[-\frac{6\,bcx^{2}-3\,\left(bcx^{3}+acx\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+4\,ac}{4\,\left(a^{2}bx^{3}+a^{3}x\right)},-\frac{3\,bcx^{2}+3\,\left(bcx^{3}+acx\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,ac}{2\,\left(a^{2}bx^{3}+a^{3}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*c*x^2 - 3*(b*c*x^3 + a*c*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-
b/a) - a)/(b*x^2 + a)) + 4*a*c)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*c*x^2 + 3*(b
*c*x^3 + a*c*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a*c)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] time = 0.44569, size = 92, normalized size = 1.53

$$c\left(\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(-\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}-\frac{2a+3bx^{2}}{2a^{3}x+2a^{2}bx^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] c*(3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a
**3*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3))
```

Giac [A] time = 1.1715, size = 68, normalized size = 1.13

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^{2}}-\frac{3\,bcx^{2}+2\,ac}{2\,\left(bx^{3}+ax\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/((
b*x^3 + a*x)*a^2)
```

3.140 $\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=53

$$-\frac{bc}{2a^2\left(a+bx^2\right)}+\frac{bc\log\left(a+bx^2\right)}{a^3}-\frac{2bc\log(x)}{a^3}-\frac{c}{2a^2x^2}$$

```
[Out] -c/(2*a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 + (b*c*Log[
a + b*x^2])/a^3
```

Rubi [A] time = 0.0410684, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {21, 266, 44}

$$-\frac{bc}{2a^2\left(a+bx^2\right)}+\frac{bc\log\left(a+bx^2\right)}{a^3}-\frac{2bc\log(x)}{a^3}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 + (b*c*Log[
a + b*x^2])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx = c\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx$$
$$= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^2\right)$$
$$= -\frac{c}{2a^2x^2}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{2bc\log(x)}{a^3}+\frac{bc\log\left(a+bx^2\right)}{a^3}$$

Mathematica [A] time = 0.0352622, size = 42, normalized size = 0.79

$$-\frac{c\left(a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -(c*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2]))/(2*a^3)
```

Maple [A] time = 0.011, size = 50, normalized size = 0.9

$$-\frac{c}{2\,a^2x^2}-\frac{bc}{2\,a^2\left(bx^2+a\right)}-2\,\frac{bc\ln(x)}{a^3}+\frac{bc\ln\left(bx^2+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/a^2/x^2-1/2*b*c/a^2/(b*x^2+a)-2*b*c*ln(x)/a^3+b*c*ln(b*x^2+a)/a^3
```

Maxima [A] time = 1.02034, size = 77, normalized size = 1.45

$$-\frac{2\,bcx^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{bc\log\left(bx^2+a\right)}{a^3}-\frac{bc\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + b*c*log(b*x^2 + a)/a^3 - b*c
*log(x^2)/a^3
```

Fricas [A] time = 1.24731, size = 173, normalized size = 3.26

$$-\frac{2\,abcx^2+a^2c-2\left(b^2cx^4+abcx^2\right)\log\left(bx^2+a\right)+4\left(b^2cx^4+abcx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c*x^2 + a^2*c - 2*(b^2*c*x^4 + a*b*c*x^2)*log(b*x^2 + a) + 4*(b
^2*c*x^4 + a*b*c*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 0.529049, size = 51, normalized size = 0.96

$$c\left(-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] c*(-(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(
a/b + x**2)/a**3)
```

Giac [A] time = 1.15864, size = 76, normalized size = 1.43

$$-\frac{bc\log\left(x^2\right)}{a^3}+\frac{bc\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bcx^2+ac}{2\,\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -b*c*log(x^2)/a^3 + b*c*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*c*x^2 + a*c)/((b
*x^4 + a*x^2)*a^2)
```

3.141 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right) dx$

Optimal. Leaf size=55

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Rubi [A] time = 0.0346943, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)dx&=\int\left(a^2cx^4+a(2bc+ad)x^6+b(bc+2ad)x^8+b^2dx^{10}\right)dx\\&=\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}\end{aligned}$$

Mathematica [A] time = 0.0089776, size = 55, normalized size = 1.

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{b^2dx^{11}}{11}+\frac{\left(2\,abd+b^2c\right)x^9}{9}+\frac{\left(a^2d+2\,abc\right)x^7}{7}+\frac{a^2cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/11*b^2*d*x^11+1/9*(2*a*b*d+b^2*c)*x^9+1/7*(a^2*d+2*a*b*c)*x^7+1/5*a^2*c*x
^5
```

Maxima [A] time = 1.01054, size = 69, normalized size = 1.25

$$\frac{1}{11}b^2dx^{11} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}a^2cx^5 + \frac{1}{7}\left(2\,abc + a^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
+ a^2*d)*x^7
```

Fricas [A] time = 1.1569, size = 131, normalized size = 2.38

$$\frac{1}{11}x^{11}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{7}x^7cba + \frac{1}{7}x^7da^2 + \frac{1}{5}x^5ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/7*x^7*c*b*a + 1/7*x^7*d
*a^2 + 1/5*x^5*c*a^2
```

Sympy [A] time = 0.067704, size = 56, normalized size = 1.02

$$\frac{a^2cx^5}{5} + \frac{b^2dx^{11}}{11} + x^9\left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^7\left(\frac{a^2d}{7} + \frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + b**2*d*x**11/11 + x**9*(2*a*b*d/9 + b**2*c/9) + x**7*(a**2*
d/7 + 2*a*b*c/7)
```

Giac [A] time = 1.14615, size = 72, normalized size = 1.31

$$\frac{1}{11}b^2dx^{11} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abdx^9 + \frac{2}{7}abcx^7 + \frac{1}{7}a^2dx^7 + \frac{1}{5}a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/7*a*b*c*x^7 + 1/7*a^2*d
*x^7 + 1/5*a^2*c*x^5
```

3.142 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=55

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Rubi [A] time = 0.0610322, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)\, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2cx+a(2bc+ad)x^2+b(bc+2ad)x^3+b^2dx^4\right) dx, x, x^2\right)\\ &= \frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A] time = 0.0081722, size = 55, normalized size = 1.

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Maple [A] time = 0., size = 52, normalized size = 1.

$$\frac{b^2dx^{10}}{10}+\frac{\left(2\,abd+b^2c\right)x^8}{8}+\frac{\left(a^2d+2\,abc\right)x^6}{6}+\frac{a^2cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/8*(2*a*b*d+b^2*c)*x^8+1/6*(a^2*d+2*a*b*c)*x^6+1/4*a^2*c*x
^4
```

Maximа [A] time = 1.01488, size = 69, normalized size = 1.25

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{8}\left(b^2c+2\,abd\right)x^8+\frac{1}{4}\,a^2cx^4+\frac{1}{6}\left(2\,abc+a^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
+ a^2*d)*x^6
```

Fricas [A] time = 1.00007, size = 131, normalized size = 2.38

$$\frac{1}{10}x^{10}db^2+\frac{1}{8}x^8cb^2+\frac{1}{4}x^8dba+\frac{1}{3}x^6cba+\frac{1}{6}x^6da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/8*x^8*c*b^2 + 1/4*x^8*d*b*a + 1/3*x^6*c*b*a + 1/6*x^6*d
*a^2 + 1/4*x^4*c*a^2
```

Sympy [A] time = 0.067256, size = 53, normalized size = 0.96

$$\frac{a^2cx^4}{4}+\frac{b^2dx^{10}}{10}+x^8\left(\frac{abd}{4}+\frac{b^2c}{8}\right)+x^6\left(\frac{a^2d}{6}+\frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + b**2*d*x**10/10 + x**8*(a*b*d/4 + b**2*c/8) + x**6*(a**2*d/
6 + a*b*c/3)
```

Giac [A] time = 1.13532, size = 72, normalized size = 1.31

$$\frac{1}{10}b^2dx^{10} + \frac{1}{8}b^2cx^8 + \frac{1}{4}abdx^8 + \frac{1}{3}abcx^6 + \frac{1}{6}a^2dx^6 + \frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*b^2*c*x^8 + 1/4*a*b*d*x^8 + 1/3*a*b*c*x^6 + 1/6*a^2*d
*x^6 + 1/4*a^2*c*x^4
```

3.143 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right) dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Rubi [A] time = 0.0296691, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)dx&=\int\left(a^2cx^2+a(2bc+ad)x^4+b(bc+2ad)x^6+b^2dx^8\right)dx\\&=\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9\end{aligned}$$

Mathematica [A] time = 0.0074007, size = 55, normalized size = 1.

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{b^2dx^9}{9}+\frac{\left(2\,abd+b^2c\right)x^7}{7}+\frac{\left(a^2d+2\,abc\right)x^5}{5}+\frac{a^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/9*b^2*d*x^9+1/7*(2*a*b*d+b^2*c)*x^7+1/5*(a^2*d+2*a*b*c)*x^5+1/3*a^2*c*x^3
```

Maxima [A] time = 1.00241, size = 69, normalized size = 1.25

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\left(b^2c+2\,abd\right)x^7+\frac{1}{3}\,a^2cx^3+\frac{1}{5}\left(2\,abc+a^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
a^2*d)*x^5
```

Fricas [A] time = 1.1323, size = 128, normalized size = 2.33

$$\frac{1}{9}x^9db^2+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7dba+\frac{2}{5}x^5cba+\frac{1}{5}x^5da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d*a
^2 + 1/3*x^3*c*a^2
```

Sympy [A] time = 0.067026, size = 56, normalized size = 1.02

$$\frac{a^2cx^3}{3}+\frac{b^2dx^9}{9}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + b**2*d*x**9/9 + x**7*(2*a*b*d/7 + b**2*c/7) + x**5*(a**2*d/
5 + 2*a*b*c/5)
```

Giac [A] time = 1.18801, size = 72, normalized size = 1.31

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 2/5*a*b*c*x^5 + 1/5*a^2*d*x
^5 + 1/3*a^2*c*x^3
```

3.144 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rubi [A] time = 0.0592418, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx,x,x^2\right)\\&=\frac{(bc-ad)\left(a+bx^2\right)^3}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.0123594, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2c+4bx^4(2ad+bc)+6ax^2(ad+2bc)+3b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (x^2*(12*a^2*c + 6*a*(2*b*c + a*d)*x^2 + 4*b*(b*c + 2*a*d)*x^4 + 3*b^2*d*x^
6))/24
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{b^{2}d x^{8}}{8}+\frac{\left(2 a b d+b^{2} c\right) x^{6}}{6}+\frac{\left(a^{2} d+2 a b c\right) x^{4}}{4}+\frac{a^{2} c x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/8*b^2*d*x^8+1/6*(2*a*b*d+b^2*c)*x^6+1/4*(a^2*d+2*a*b*c)*x^4+1/2*a^2*c*x^2
```

Maxima [A] time = 1.00039, size = 69, normalized size = 1.64

$$\frac{1}{8} b^{2} d x^{8}+\frac{1}{6}\left(b^{2} c+2 a b d\right) x^{6}+\frac{1}{2} a^{2} c x^{2}+\frac{1}{4}\left(2 a b c+a^{2} d\right) x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
a^2*d)*x^4
```

Fricas [A] time = 1.03948, size = 128, normalized size = 3.05

$$\frac{1}{8} x^{8} d b^{2}+\frac{1}{6} x^{6} c b^{2}+\frac{1}{3} x^{6} d b a+\frac{1}{2} x^{4} c b a+\frac{1}{4} x^{4} d a^{2}+\frac{1}{2} x^{2} c a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/6*x^6*c*b^2 + 1/3*x^6*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d*a
^2 + 1/2*x^2*c*a^2
```

Sympy [A] time = 0.066984, size = 53, normalized size = 1.26

$$\frac{a^{2} c x^{2}}{2}+\frac{b^{2} d x^{8}}{8}+x^{6}\left(\frac{a b d}{3}+\frac{b^{2} c}{6}\right)+x^{4}\left(\frac{a^{2} d}{4}+\frac{a b c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**2/2 + b**2*d*x**8/8 + x**6*(a*b*d/3 + b**2*c/6) + x**4*(a**2*d/4
+ a*b*c/2)
```

Giac [A] time = 1.12865, size = 72, normalized size = 1.71

$$\frac{1}{8}b^2dx^8 + \frac{1}{6}b^2cx^6 + \frac{1}{3}abdx^6 + \frac{1}{2}abcx^4 + \frac{1}{4}a^2dx^4 + \frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*b^2*c*x^6 + 1/3*a*b*d*x^6 + 1/2*a*b*c*x^4 + 1/4*a^2*d*x
^4 + 1/2*a^2*c*x^2
```

3.145 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right) dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rubi [A] time = 0.0250865, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right)dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A] time = 0.0074746, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Maple [A] time = 0., size = 49, normalized size = 1.

$$\frac{b^2dx^7}{7}+\frac{\left(2\,abd+b^2c\right)x^5}{5}+\frac{\left(a^2d+2\,abc\right)x^3}{3}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

Maxima [A] time = 1.00811, size = 65, normalized size = 1.3

$$\frac{1}{7}b^2dx^7 + \frac{1}{5}\left(b^2c + 2\,abd\right)x^5 + a^2cx + \frac{1}{3}\left(2\,abc + a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Fricas [A] time = 1.07312, size = 120, normalized size = 2.4

$$\frac{1}{7}x^7db^2 + \frac{1}{5}x^5cb^2 + \frac{2}{5}x^5dba + \frac{2}{3}x^3cba + \frac{1}{3}x^3da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1/3*x^3*d*a
^2 + x*c*a^2
```

Sympy [A] time = 0.067187, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^7}{7} + x^5\left(\frac{2abd}{5} + \frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3} + \frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

Giac [A] time = 1.14705, size = 68, normalized size = 1.36

$$\frac{1}{7}b^2dx^7 + \frac{1}{5}b^2cx^5 + \frac{2}{5}abdx^5 + \frac{2}{3}abcx^3 + \frac{1}{3}a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

3.146 $\int \frac{(a+bx^2)^2(c+dx^2)}{x} dx$

Optimal. Leaf size=43

$$a^2c\log(x) + abcx^2 + \frac{d(a+bx^2)^3}{6b} + \frac{1}{4}b^2cx^4$$

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x]
```

Rubi [A] time = 0.0312346, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 80, 43}

$$a^2c\log(x) + abcx^2 + \frac{d(a+bx^2)^3}{6b} + \frac{1}{4}b^2cx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x}\,dx, x, x^2\right)$$
$$= \frac{d\left(a+bx^2\right)^3}{6b} + \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^2\right)$$
$$= \frac{d\left(a+bx^2\right)^3}{6b} + \frac{1}{2}c\,\mathrm{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx, x, x^2\right)$$
$$= abcx^2 + \frac{1}{4}b^2cx^4 + \frac{d\left(a+bx^2\right)^3}{6b} + a^2c\log(x)$$

Mathematica [A] time = 0.0144786, size = 51, normalized size = 1.19

$$a^2c\log(x) + \frac{1}{4}bx^4(2ad+bc) + \frac{1}{2}ax^2(ad+2bc) + \frac{1}{6}b^2dx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] (a*(2*b*c + a*d)*x^2)/2 + (b*(b*c + 2*a*d)*x^4)/4 + (b^2*d*x^6)/6 + a^2*c*L
og[x]
```

Maple [A] time = 0.002, size = 51, normalized size = 1.2

$$\frac{b^2dx^6}{6} + \frac{x^4abd}{2} + \frac{b^2cx^4}{4} + \frac{x^2a^2d}{2} + abcx^2 + a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x,x)
```

```
[Out] 1/6*b^2*d*x^6+1/2*x^4*a*b*d+1/4*b^2*c*x^4+1/2*x^2*a^2*d+a*b*c*x^2+a^2*c*ln(
x)
```

Maxima [A] time = 1.00555, size = 70, normalized size = 1.63

$$\frac{1}{6}\,b^2dx^6 + \frac{1}{4}\left(b^2c + 2\,abd\right)x^4 + \frac{1}{2}\,a^2c\log\left(x^2\right) + \frac{1}{2}\left(2\,abc + a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + 1/2*a^2*c*log(x^2) + 1/2*(2*a*b
*c + a^2*d)*x^2
```

Fricas [A] time = 1.2487, size = 116, normalized size = 2.7

$$\frac{1}{6}\,b^2dx^6 + \frac{1}{4}\left(b^2c + 2\,abd\right)x^4 + a^2c\log(x) + \frac{1}{2}\left(2\,abc + a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + a^2*c*log(x) + 1/2*(2*a*b*c + a
^2*d)*x^2
```

Sympy [A] time = 0.282249, size = 49, normalized size = 1.14

$$a^2 c \log(x) + \frac{b^2 d x^6}{6} + x^4 \left(\frac{abd}{2} + \frac{b^2 c}{4}\right) + x^2 \left(\frac{a^2 d}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

```
[Out] a**2*c*log(x) + b**2*d*x**6/6 + x**4*(a*b*d/2 + b**2*c/4) + x**2*(a**2*d/2
+ a*b*c)
```

Giac [A] time = 1.19921, size = 72, normalized size = 1.67

$$\frac{1}{6} b^2 d x^6 + \frac{1}{4} b^2 c x^4 + \frac{1}{2} abd x^4 + abc x^2 + \frac{1}{2} a^2 d x^2 + \frac{1}{2} a^2 c \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*b^2*c*x^4 + 1/2*a*b*d*x^4 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 1/2*a^2*c*log(x^2)
```

3.147 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^2} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{x} + \frac{1}{3}bx^3(2ad + bc) + ax(ad + 2bc) + \frac{1}{5}b^2dx^5$$

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Rubi [A] time = 0.0256699, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2c}{x} + \frac{1}{3}bx^3(2ad + bc) + ax(ad + 2bc) + \frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2)^2 (c + dx^2)}{x^2} \, dx &= \int \left(a(2bc + ad) + \frac{a^2c}{x^2} + b(bc + 2ad)x^2 + b^2dx^4\right) dx \\ &= -\frac{a^2c}{x} + a(2bc + ad)x + \frac{1}{3}b(bc + 2ad)x^3 + \frac{1}{5}b^2dx^5 \end{aligned}$$

Mathematica [A] time = 0.0161062, size = 48, normalized size = 1.

$$-\frac{a^2c}{x} + \frac{1}{3}bx^3(2ad + bc) + ax(ad + 2bc) + \frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Maple [A] time = 0.003, size = 49, normalized size = 1.

$$\frac{b^2dx^5}{5} + \frac{2\,x^3abd}{3} + \frac{x^3b^2c}{3} + a^2dx + 2\,abcx - \frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^2,x)
```

```
[Out] 1/5*b^2*d*x^5+2/3*x^3*a*b*d+1/3*x^3*b^2*c+a^2*d*x+2*a*b*c*x-a^2*c/x
```

Maxima [A] time = 1.00418, size = 65, normalized size = 1.35

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\left(b^2c+2\,abd\right)x^3-\frac{a^2c}{x}+\left(2\,abc+a^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*(b^2*c + 2*a*b*d)*x^3 - a^2*c/x + (2*a*b*c + a^2*d)*x
```

Fricas [A] time = 1.26258, size = 116, normalized size = 2.42

$$\frac{3\,b^2dx^6+5\left(b^2c+2\,abd\right)x^4-15\,a^2c+15\left(2\,abc+a^2d\right)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d*x^6 + 5*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 15*(2*a*b*c + a^2*
d)*x^2)/x
```

Sympy [A] time = 0.281134, size = 48, normalized size = 1.

$$-\frac{a^2c}{x}+\frac{b^2dx^5}{5}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x\left(a^2d+2abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

```
[Out] -a**2*c/x + b**2*d*x**5/5 + x**3*(2*a*b*d/3 + b**2*c/3) + x*(a**2*d + 2*a*b
*c)
```

Giac [A] time = 1.16103, size = 65, normalized size = 1.35

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+2\,abcx+a^2dx-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + 2*a*b*c*x + a^2*d*x - a^2*c
/x
```

3.148 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

```
[Out] -(a^2*c)/(2*x^2) + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2*b*c + a*d
)*Log[x]
```

Rubi [A] time = 0.0421537, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2*b*c + a*d
)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b(bc+2ad)+\frac{a^2c}{x^2}+\frac{a(2bc+ad)}{x}+b^2dx\right)dx,x,x^2\right)\\ &= -\frac{a^2c}{2x^2}+\frac{1}{2}b(bc+2ad)x^2+\frac{1}{4}b^2dx^4+a(2bc+ad)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0224635, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2c}{x^2}+2bx^2(2ad+bc)+4a\log(x)(ad+2bc)+b^2dx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*c)/x^2 + 2*b*(b*c + 2*a*d)*x^2 + b^2*d*x^4 + 4*a*(2*b*c + a*d)*Log
[x])/4
```

Maple [A] time = 0.005, size = 50, normalized size = 1.

$$\frac{b^2dx^4}{4} + x^2abd + \frac{b^2cx^2}{2} + \ln(x)\,a^2d + 2\,\ln(x)\,abc - \frac{a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^3,x)
```

```
[Out] 1/4*b^2*d*x^4+x^2*a*b*d+1/2*b^2*c*x^2+ln(x)*a^2*d+2*ln(x)*a*b*c-1/2*a^2*c/x
^2
```

Maxima [A] time = 0.981489, size = 70, normalized size = 1.37

$$\frac{1}{4}\,b^2dx^4 + \frac{1}{2}\left(b^2c + 2\,abd\right)x^2 + \frac{1}{2}\left(2\,abc + a^2d\right)\log\left(x^2\right) - \frac{a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*(b^2*c + 2*a*b*d)*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*a^2*c/x^2
```

Fricas [A] time = 1.2384, size = 122, normalized size = 2.39

$$\frac{b^2dx^6 + 2\left(b^2c + 2\,abd\right)x^4 + 4\left(2\,abc + a^2d\right)x^2\log(x) - 2\,a^2c}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^6 + 2*(b^2*c + 2*a*b*d)*x^4 + 4*(2*a*b*c + a^2*d)*x^2*log(x) -
 2*a^2*c)/x^2
```

Sympy [A] time = 0.368969, size = 48, normalized size = 0.94

$$-\frac{a^2c}{2x^2} + a\left(ad + 2bc\right)\log(x) + \frac{b^2dx^4}{4} + x^2\left(abd + \frac{b^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x) + b**2*d*x**4/4 + x**2*(a*b*d + b
**2*c/2)
```

Giac [A] time = 1.16662, size = 95, normalized size = 1.86

$$\frac{1}{4}b^2dx^4 + \frac{1}{2}b^2cx^2 + abdx^2 + \frac{1}{2}\left(2\,abc + a^2d\right)\log\left(x^2\right) - \frac{2\,abcx^2 + a^2dx^2 + a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*b^2*c*x^2 + a*b*d*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*(2*a*b*c*x^2 + a^2*d*x^2 + a^2*c)/x^2
```

3.149 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^4} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

```
[Out] -(a^2*c)/(3*x^3) - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Rubi [A] time = 0.0278166, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx &= \int \left(b(bc+2ad)+\frac{a^2c}{x^4}+\frac{a(2bc+ad)}{x^2}+b^2dx^2\right)dx\\ &= -\frac{a^2c}{3x^3}-\frac{a(2bc+ad)}{x}+b(bc+2ad)x+\frac{1}{3}b^2dx^3\end{aligned}$$

Mathematica [A] time = 0.0193895, size = 50, normalized size = 1.04

$$\frac{a^2(-d)-2abc}{x}-\frac{a^2c}{3x^3}+bx(2ad+bc)+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) + (-2*a*b*c - a^2*d)/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Maple [A] time = 0.004, size = 46, normalized size = 1.

$$\frac{b^2dx^3}{3}+2\,abdx+b^2cx-\frac{a^2c}{3\,x^3}-\frac{a\,(ad+2\,bc)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^4,x)
```

```
[Out] 1/3*b^2*d*x^3+2*a*b*d*x+b^2*c*x-1/3*a^2*c/x^3-a*(a*d+2*b*c)/x
```

Maxima [A] time = 1.07129, size = 68, normalized size = 1.42

$$\frac{1}{3}\,b^2dx^3+\left(b^2c+2\,abd\right)x-\frac{a^2c+3\left(2\,abc+a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*d*x^3 + (b^2*c + 2*a*b*d)*x - 1/3*(a^2*c + 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

Fricas [A] time = 1.11661, size = 109, normalized size = 2.27

$$\frac{b^2dx^6+3\left(b^2c+2\,abd\right)x^4-a^2c-3\left(2\,abc+a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d*x^6 + 3*(b^2*c + 2*a*b*d)*x^4 - a^2*c - 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

Sympy [A] time = 0.381036, size = 49, normalized size = 1.02

$$\frac{b^2dx^3}{3}+x\left(2abd+b^2c\right)-\frac{a^2c+x^2\left(3a^2d+6abc\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

```
[Out] b**2*d*x**3/3 + x*(2*a*b*d + b**2*c) - (a**2*c + x**2*(3*a**2*d + 6*a*b*c))
/(3*x**3)
```

Giac [A] time = 1.15255, size = 68, normalized size = 1.42

$$\frac{1}{3}\,b^2dx^3+b^2cx+2\,abdx-\frac{6\,abcx^2+3\,a^2dx^2+a^2c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d*x^3 + b^2*c*x + 2*a*b*d*x - 1/3*(6*a*b*c*x^2 + 3*a^2*d*x^2 + a^2*
c)/x^3
```

3.150 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Rubi [A] time = 0.0618824, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int\left(a^2c^2x^4+2ac(bc+ad)x^6+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{10}+b^2d^2x^{12}\right.\\ &= \frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{11}bd(bc+ad)x^{11}+\frac{1}{13}b^2c\end{aligned}$$

Mathematica [A] time = 0.0154137, size = 87, normalized size = 1.

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Maple [A] time = 0.001, size = 90, normalized size = 1.

$$\frac{b^2d^2x^{13}}{13}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{11}}{11}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^9}{9}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^7}{7}+\frac{a^2c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/11*(2*a*b*d^2+2*b^2*c*d)*x^11+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/7*(2*a^2*c*d+2*a*b*c^2)*x^7+1/5*a^2*c^2*x^5
```

Maxima [A] time = 0.99778, size = 115, normalized size = 1.32

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\left(b^2cd+abd^2\right)x^{11}+\frac{1}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^9+\frac{1}{5}\,a^2c^2x^5+\frac{2}{7}\left(abc^2+a^2cd\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

Fricas [A] time = 1.0882, size = 225, normalized size = 2.59

$$\frac{1}{13}x^{13}d^2b^2+\frac{2}{11}x^{11}dcb^2+\frac{2}{11}x^{11}d^2ba+\frac{1}{9}x^9c^2b^2+\frac{4}{9}x^9dcba+\frac{1}{9}x^9d^2a^2+\frac{2}{7}x^7c^2ba+\frac{2}{7}x^7dca^2+\frac{1}{5}x^5c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 2/11*x^11*d*c*b^2 + 2/11*x^11*d^2*b*a + 1/9*x^9*c^2*b^2
 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/7*x^7*c^2*b*a + 2/7*x^7*d*c*a^2 +
1/5*x^5*c^2*a^2
```

Sympy [A] time = 0.078768, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^5}{5}+\frac{b^2d^2x^{13}}{13}+x^{11}\left(\frac{2abd^2}{11}+\frac{2b^2cd}{11}\right)+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)+x^7\left(\frac{2a^2cd}{7}+\frac{2abc^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**5/5 + b**2*d**2*x**13/13 + x**11*(2*a*b*d**2/11 + 2*b**2*c*d/1
1) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**7*(2*a**2*c*d/7 +
2*a*b*c**2/7)
```

Giac [A] time = 1.13503, size = 127, normalized size = 1.46

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\,b^2cdx^{11}+\frac{2}{11}\,abd^2x^{11}+\frac{1}{9}\,b^2c^2x^9+\frac{4}{9}\,abcdx^9+\frac{1}{9}\,a^2d^2x^9+\frac{2}{7}\,abc^2x^7+\frac{2}{7}\,a^2cdx^7+\frac{1}{5}\,a^2c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*b^2*c*d*x^11 + 2/11*a*b*d^2*x^11 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/7*a*b*c^2*x^7 + 2/7*a^2*c*d*x^7 +
1/5*a^2*c^2*x^5
```

3.151 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rubi [A] time = 0.10562, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2c^2x+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^3+2bd(bc+ad)x^4+b^2c\right.\right.\\
&= \frac{1}{4}a^2c^2x^4+\frac{1}{3}ac(bc+ad)x^6+\frac{1}{8}\left(b^2c^2+4abcd+a^2d^2\right)x^8+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{12}b^2d^2x^{1}
\end{aligned}$$

Mathematica [A] time = 0.0249023, size = 81, normalized size = 0.93

$$\frac{1}{120}x^4\left(15x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+24bdx^6(ad+bc)+40acx^2(ad+bc)+10b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^4*(30*a^2*c^2 + 40*a*c*(b*c + a*d)*x^2 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 24*b*d*(b*c + a*d)*x^6 + 10*b^2*d^2*x^8))/120
```

Maple [A] time = 0., size = 90, normalized size = 1.

$$\frac{b^2d^2x^{12}}{12}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{10}}{10}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^8}{8}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^6}{6}+\frac{a^2c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/12*b^2*d^2*x^12+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/8*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^8+1/6*(2*a^2*c*d+2*a*b*c^2)*x^6+1/4*a^2*c^2*x^4
```

Maxima [A] time = 0.990316, size = 115, normalized size = 1.32

$$\frac{1}{12}\,b^2d^2x^{12}+\frac{1}{5}\left(b^2cd+abd^2\right)x^{10}+\frac{1}{8}\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+\frac{1}{4}\,a^2c^2x^4+\frac{1}{3}\left(abc^2+a^2cd\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

Fricas [A] time = 1.14923, size = 223, normalized size = 2.56

$$\frac{1}{12}x^{12}d^2b^2+\frac{1}{5}x^{10}dcb^2+\frac{1}{5}x^{10}d^2ba+\frac{1}{8}x^8c^2b^2+\frac{1}{2}x^8dcba+\frac{1}{8}x^8d^2a^2+\frac{1}{3}x^6c^2ba+\frac{1}{3}x^6dca^2+\frac{1}{4}x^4c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/8*x^8*c^2*b^2 +
 1/2*x^8*d*c*b*a + 1/8*x^8*d^2*a^2 + 1/3*x^6*c^2*b*a + 1/3*x^6*d*c*a^2 + 1/
4*x^4*c^2*a^2
```

Sympy [A] time = 0.076072, size = 92, normalized size = 1.06

$$\frac{a^2c^2x^4}{4}+\frac{b^2d^2x^{12}}{12}+x^{10}\left(\frac{abd^2}{5}+\frac{b^2cd}{5}\right)+x^8\left(\frac{a^2d^2}{8}+\frac{abcd}{2}+\frac{b^2c^2}{8}\right)+x^6\left(\frac{a^2cd}{3}+\frac{abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**4/4 + b**2*d**2*x**12/12 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x
**8*(a**2*d**2/8 + a*b*c*d/2 + b**2*c**2/8) + x**6*(a**2*c*d/3 + a*b*c**2/3
)
```

Giac [A] time = 1.18127, size = 127, normalized size = 1.46

$$\frac{1}{12}\,b^{2}d^{2}x^{12}+\frac{1}{5}\,b^{2}cdx^{10}+\frac{1}{5}\,abd^{2}x^{10}+\frac{1}{8}\,b^{2}c^{2}x^{8}+\frac{1}{2}\,abcdx^{8}+\frac{1}{8}\,a^{2}d^{2}x^{8}+\frac{1}{3}\,abc^{2}x^{6}+\frac{1}{3}\,a^{2}cdx^{6}+\frac{1}{4}\,a^{2}c^{2}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/8*b^2*c^2*x^8 +
 1/2*a*b*c*d*x^8 + 1/8*a^2*d^2*x^8 + 1/3*a*b*c^2*x^6 + 1/3*a^2*c*d*x^6 + 1/
4*a^2*c^2*x^4
```

3.152 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Rubi [A] time = 0.052869, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2 dx &= \int \left(a^2c^2x^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^8+b^2d^2x^{10}\right)\\ &= \frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\frac{1}{11}b^2d^2\end{aligned}$$

Mathematica [A] time = 0.0176405, size = 87, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Maple [A] time = 0., size = 90, normalized size = 1.

$$\frac{b^2d^2x^{11}}{11}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^9}{9}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^7}{7}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^5}{5}+\frac{a^2c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^2*d^2*x^11+1/9*(2*a*b*d^2+2*b^2*c*d)*x^9+1/7*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^7+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+1/3*a^2*c^2*x^3
```

Maxima [A] time = 1.00862, size = 115, normalized size = 1.32

$$\frac{1}{11}\, b^2 d^2 x^{11} + \frac{2}{9}\left(b^2 cd + abd^2\right)x^9 + \frac{1}{7}\left(b^2 c^2 + 4\, abcd + a^2 d^2\right)x^7 + \frac{1}{3}\, a^2 c^2 x^3 + \frac{2}{5}\left(abc^2 + a^2 cd\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

Fricas [A] time = 1.0268, size = 220, normalized size = 2.53

$$\frac{1}{11}x^{11}d^2b^2 + \frac{2}{9}x^9dcb^2 + \frac{2}{9}x^9d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{2}{5}x^5c^2ba + \frac{2}{5}x^5dca^2 + \frac{1}{3}x^3c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^2 + 2/9*x^9*d*c*b^2 + 2/9*x^9*d^2*b*a + 1/7*x^7*c^2*b^2 + 4
/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 + 1/3*
x^3*c^2*a^2
```

Sympy [A] time = 0.076705, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^3}{3} + \frac{b^2d^2x^{11}}{11} + x^9\left(\frac{2abd^2}{9} + \frac{2b^2cd}{9}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^5\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + b**2*d**2*x**11/11 + x**9*(2*a*b*d**2/9 + 2*b**2*c*d/9)
+ x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**5*(2*a**2*c*d/5 + 2*a
*b*c**2/5)
```

Giac [A] time = 1.14725, size = 127, normalized size = 1.46

$$\frac{1}{11}\, b^2 d^2 x^{11} + \frac{2}{9}\, b^2 cdx^9 + \frac{2}{9}\, abd^2 x^9 + \frac{1}{7}\, b^2 c^2 x^7 + \frac{4}{7}\, abcdx^7 + \frac{1}{7}\, a^2 d^2 x^7 + \frac{2}{5}\, abc^2 x^5 + \frac{2}{5}\, a^2 cdx^5 + \frac{1}{3}\, a^2 c^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*b^2*c*d*x^9 + 2/9*a*b*d^2*x^9 + 1/7*b^2*c^2*x^7 + 4
/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 + 1/3*
a^2*c^2*x^3
```

3.153 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=71

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2)^4)/(4*b^
3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rubi [A] time = 0.105496, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2)^4)/(4*b^
3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^2(c+dx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2\left(a+bx^2\right)^3}{6b^3}+\frac{d(bc-ad)\left(a+bx^2\right)^4}{4b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.0218552, size = 81, normalized size = 1.14

$$\frac{1}{60}x^2\left(10x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+15bdx^6(ad+bc)+30acx^2(ad+bc)+6b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^2*(30*a^2*c^2 + 30*a*c*(b*c + a*d)*x^2 + 10*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 15*b*d*(b*c + a*d)*x^6 + 6*b^2*d^2*x^8))/60
```

Maple [A] time = 0.001, size = 90, normalized size = 1.3

$$\frac{b^2d^2x^{10}}{10}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^8}{8}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^6}{6}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^4}{4}+\frac{a^2c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/10*b^2*d^2*x^10+1/8*(2*a*b*d^2+2*b^2*c*d)*x^8+1/6*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^6+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+1/2*a^2*c^2*x^2
```

Maxima [A] time = 1.0141, size = 115, normalized size = 1.62

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\left(b^2cd+abd^2\right)x^8+\frac{1}{6}\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Fricas [A] time = 1.0144, size = 220, normalized size = 3.1

$$\frac{1}{10}x^{10}d^2b^2+\frac{1}{4}x^8dcb^2+\frac{1}{4}x^8d^2ba+\frac{1}{6}x^6c^2b^2+\frac{2}{3}x^6dcba+\frac{1}{6}x^6d^2a^2+\frac{1}{2}x^4c^2ba+\frac{1}{2}x^4dca^2+\frac{1}{2}x^2c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b^2 + 1/4*x^8*d*c*b^2 + 1/4*x^8*d^2*b*a + 1/6*x^6*c^2*b^2 + 2
/3*x^6*d*c*b*a + 1/6*x^6*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/2*x^4*d*c*a^2 + 1/2*
x^2*c^2*a^2
```

Sympy [A] time = 0.07623, size = 94, normalized size = 1.32

$$\frac{a^2c^2x^2}{2}+\frac{b^2d^2x^{10}}{10}+x^8\left(\frac{abd^2}{4}+\frac{b^2cd}{4}\right)+x^6\left(\frac{a^2d^2}{6}+\frac{2abcd}{3}+\frac{b^2c^2}{6}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**2/2 + b**2*d**2*x**10/10 + x**8*(a*b*d**2/4 + b**2*c*d/4) + x*
*6*(a**2*d**2/6 + 2*a*b*c*d/3 + b**2*c**2/6) + x**4*(a**2*c*d/2 + a*b*c**2/
2)
```

Giac [A] time = 1.14301, size = 127, normalized size = 1.79

$$\frac{1}{10}\,b^2d^2x^{10} + \frac{1}{4}\,b^2cdx^8 + \frac{1}{4}\,abd^2x^8 + \frac{1}{6}\,b^2c^2x^6 + \frac{2}{3}\,abcdx^6 + \frac{1}{6}\,a^2d^2x^6 + \frac{1}{2}\,abc^2x^4 + \frac{1}{2}\,a^2cdx^4 + \frac{1}{2}\,a^2c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*b^2*c*d*x^8 + 1/4*a*b*d^2*x^8 + 1/6*b^2*c^2*x^6 + 2
/3*a*b*c*d*x^6 + 1/6*a^2*d^2*x^6 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + 1/2*
a^2*c^2*x^2
```

3.154 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi [A] time = 0.0387515, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2 dx &= \int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2x^8\right) dx\\ &= a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9\end{aligned}$$

Mathematica [A] time = 0.0158499, size = 82, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A] time = 0., size = 87, normalized size = 1.1

$$\frac{b^2d^2x^9}{9}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^7}{7}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^5}{5}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^3}{3}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A] time = 1.21545, size = 111, normalized size = 1.35

$$\frac{1}{9}b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A] time = 1.14746, size = 209, normalized size = 2.55

$$\frac{1}{9}x^9d^2b^2 + \frac{2}{7}x^7dcb^2 + \frac{2}{7}x^7d^2ba + \frac{1}{5}x^5c^2b^2 + \frac{4}{5}x^5dcba + \frac{1}{5}x^5d^2a^2 + \frac{2}{3}x^3c^2ba + \frac{2}{3}x^3dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2*b^2 + 4/5
*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*x^3*d*c*a^2 + x*c^2*
a^2
```

Sympy [A] time = 0.075106, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^9}{9} + x^7\left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3\left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

Giac [A] time = 1.1322, size = 123, normalized size = 1.5

$$\frac{1}{9}b^2d^2x^9 + \frac{2}{7}b^2cdx^7 + \frac{2}{7}abd^2x^7 + \frac{1}{5}b^2c^2x^5 + \frac{4}{5}abcdx^5 + \frac{1}{5}a^2d^2x^5 + \frac{2}{3}abc^2x^3 + \frac{2}{3}a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

3.155 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx$

Optimal. Leaf size=80

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rubi [A] time = 0.0750831, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(2ac(bc+ad)+\frac{a^2c^2}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+2bd(bc+ad)x^2+b^2d^2x^3\right) d\right.\\ &= ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4+\frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0240972, size = 80, normalized size = 1.

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Maple [A] time = 0.002, size = 90, normalized size = 1.1

$$\frac{b^2d^2x^8}{8}+\frac{x^6abd^2}{3}+\frac{x^6b^2cd}{3}+\frac{x^4a^2d^2}{4}+x^4abcd+\frac{x^4b^2c^2}{4}+x^2a^2cd+ac^2bx^2+a^2c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x,x)
```

```
[Out] 1/8*b^2*d^2*x^8+1/3*x^6*a*b*d^2+1/3*x^6*b^2*c*d+1/4*x^4*a^2*d^2+x^4*a*b*c*d
+1/4*x^4*b^2*c^2+x^2*a^2*c*d+a*c^2*b*x^2+a^2*c^2*ln(x)
```

Maxima [A] time = 0.975723, size = 115, normalized size = 1.44

$$\frac{1}{8}b^2d^2x^8+\frac{1}{3}\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+\frac{1}{2}a^2c^2\log\left(x^2\right)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + 1/2*a^2*c^2*log(x^2) + (a*b*c^2 + a^2*c*d)*x^2
```

Fricas [A] time = 1.28162, size = 178, normalized size = 2.22

$$\frac{1}{8}b^2d^2x^8+\frac{1}{3}\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+a^2c^2\log(x)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + a^2*c^2*log(x) + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A] time = 0.326519, size = 85, normalized size = 1.06

$$a^2c^2\log(x)+\frac{b^2d^2x^8}{8}+x^6\left(\frac{abd^2}{3}+\frac{b^2cd}{3}\right)+x^4\left(\frac{a^2d^2}{4}+abcd+\frac{b^2c^2}{4}\right)+x^2\left(a^2cd+abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

```
[Out] a**2*c**2*log(x) + b**2*d**2*x**8/8 + x**6*(a*b*d**2/3 + b**2*c*d/3) + x**4
*(a**2*d**2/4 + a*b*c*d + b**2*c**2/4) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A] time = 1.17909, size = 124, normalized size = 1.55

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,b^2cdx^6+\frac{1}{3}\,abd^2x^6+\frac{1}{4}\,b^2c^2x^4+abcdx^4+\frac{1}{4}\,a^2d^2x^4+abc^2x^2+a^2cdx^2+\frac{1}{2}\,a^2c^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*b^2*c*d*x^6 + 1/3*a*b*d^2*x^6 + 1/4*b^2*c^2*x^4 + a*b
*c*d*x^4 + 1/4*a^2*d^2*x^4 + a*b*c^2*x^2 + a^2*c*d*x^2 + 1/2*a^2*c^2*log(x^
2)
```

3.156 $\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^2} dx$

Optimal. Leaf size=81

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rubi [A] time = 0.0426554, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx &= \int \left(2ac(bc+ad)+\frac{a^2c^2}{x^2}+\left(b^2c^2+4abcd+a^2d^2\right)x^2+2bd(bc+ad)x^4+b^2d^2x^6\right)dx\\ &= -\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x^7\end{aligned}$$

Mathematica [A] time = 0.0377618, size = 81, normalized size = 1.

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Maple [A] time = 0.003, size = 91, normalized size = 1.1

$$\frac{b^2d^2x^7}{7}+\frac{2\,x^5abd^2}{5}+\frac{2\,x^5b^2cd}{5}+\frac{x^3a^2d^2}{3}+\frac{4\,x^3abcd}{3}+\frac{x^3b^2c^2}{3}+2\,a^2cdx+2\,abc^2x-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^2,x)
```

```
[Out] 1/7*b^2*d^2*x^7+2/5*x^5*a*b*d^2+2/5*x^5*b^2*c*d+1/3*x^3*a^2*d^2+4/3*x^3*a*b
*c*d+1/3*x^3*b^2*c^2+2*a^2*c*d*x+2*a*b*c^2*x-a^2*c^2/x
```

Maxima [A] time = 1.00465, size = 112, normalized size = 1.38

$$\frac{1}{7}\,b^2d^2x^7+\frac{2}{5}\left(b^2cd+abd^2\right)x^5+\frac{1}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^3-\frac{a^2c^2}{x}+2\left(abc^2+a^2cd\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*(b^2*c*d + a*b*d^2)*x^5 + 1/3*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^3 - a^2*c^2/x + 2*(a*b*c^2 + a^2*c*d)*x
```

Fricas [A] time = 1.28043, size = 189, normalized size = 2.33

$$\frac{15\,b^2d^2x^8+42\left(b^2cd+abd^2\right)x^6+35\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-105\,a^2c^2+210\left(abc^2+a^2cd\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^2*x^8 + 42*(b^2*c*d + a*b*d^2)*x^6 + 35*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^4 - 105*a^2*c^2 + 210*(a*b*c^2 + a^2*c*d)*x^2)/x
```

Sympy [A] time = 0.315012, size = 92, normalized size = 1.14

$$-\frac{a^2c^2}{x}+\frac{b^2d^2x^7}{7}+x^5\left(\frac{2abd^2}{5}+\frac{2b^2cd}{5}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x\left(2a^2cd+2abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

```
[Out] -a**2*c**2/x + b**2*d**2*x**7/7 + x**5*(2*a*b*d**2/5 + 2*b**2*c*d/5) + x**3
*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x*(2*a**2*c*d + 2*a*b*c**2)
```

Giac [A] time = 1.3286, size = 122, normalized size = 1.51

$$\frac{1}{7}\,b^2d^2x^7 + \frac{2}{5}\,b^2cdx^5 + \frac{2}{5}\,abd^2x^5 + \frac{1}{3}\,b^2c^2x^3 + \frac{4}{3}\,abcdx^3 + \frac{1}{3}\,a^2d^2x^3 + 2\,abc^2x + 2\,a^2cdx - \frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*b^2*c*d*x^5 + 2/5*a*b*d^2*x^5 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + 2*a*b*c^2*x + 2*a^2*c*d*x - a^2*c^2/x
```

3.157 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

```
[Out] -(a^2*c^2)/(2*x^2) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b*d*(b*c +
a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x]
```

Rubi [A] time = 0.078103, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] -(a^2*c^2)/(2*x^2) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b*d*(b*c +
a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^2}+\frac{2ac(bc+ad)}{x}+2bd(bc+ad)x+b^2d^2x^2\right) d\\ &= -\frac{a^2c^2}{2x^2}+\frac{1}{2}\left(b^2c^2+4abcd+a^2d^2\right)x^2+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{6}b^2d^2x^6+2ac(bc+ad)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0413864, size = 83, normalized size = 0.99

$$\frac{1}{6}\left(\frac{3a^2\left(d^2x^4-c^2\right)}{x^2}+3abdx^2\left(4c+dx^2\right)+12ac\log(x)(ad+bc)+b^2x^2\left(3c^2+3cdx^2+d^2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] (3*a*b*d*x^2*(4*c + d*x^2) + (3*a^2*(-c^2 + d^2*x^4))/x^2 + b^2*x^2*(3*c^2
+ 3*c*d*x^2 + d^2*x^4) + 12*a*c*(b*c + a*d)*Log[x])/6
```

Maple [A] time = 0.007, size = 93, normalized size = 1.1

$$\frac{b^2d^2x^6}{6}+\frac{x^4abd^2}{2}+\frac{x^4b^2cd}{2}+\frac{x^2a^2d^2}{2}+2\,x^2abcd+\frac{x^2b^2c^2}{2}+2\,\ln\left(x\right)a^2cd+2\,\ln\left(x\right)abc^2-\frac{a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^3,x)
```

```
[Out] 1/6*b^2*d^2*x^6+1/2*x^4*a*b*d^2+1/2*x^4*b^2*c*d+1/2*x^2*a^2*d^2+2*x^2*a*b*c
*d+1/2*x^2*b^2*c^2+2*ln(x)*a^2*c*d+2*ln(x)*a*b*c^2-1/2*a^2*c^2/x^2
```

Maxima [A] time = 0.995944, size = 115, normalized size = 1.37

$$\frac{1}{6}\,b^2d^2x^6+\frac{1}{2}\left(b^2cd+abd^2\right)x^4+\frac{1}{2}\left(b^2c^2+4\,abcd+a^2d^2\right)x^2-\frac{a^2c^2}{2\,x^2}+\left(abc^2+a^2cd\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/2*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^2 - 1/2*a^2*c^2/x^2 + (a*b*c^2 + a^2*c*d)*log(x^2)
```

Fricas [A] time = 1.26927, size = 188, normalized size = 2.24

$$\frac{b^2d^2x^8+3\left(b^2cd+abd^2\right)x^6+3\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-3\,a^2c^2+12\left(abc^2+a^2cd\right)x^2\log\left(x\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*d^2*x^8 + 3*(b^2*c*d + a*b*d^2)*x^6 + 3*(b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^4 - 3*a^2*c^2 + 12*(a*b*c^2 + a^2*c*d)*x^2*log(x))/x^2
```

Sympy [A] time = 0.412152, size = 87, normalized size = 1.04

$$-\frac{a^2c^2}{2x^2}+2ac\left(ad+bc\right)\log\left(x\right)+\frac{b^2d^2x^6}{6}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}\right)+x^2\left(\frac{a^2d^2}{2}+2abcd+\frac{b^2c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

```
[Out] -a**2*c**2/(2*x**2) + 2*a*c*(a*d + b*c)*log(x) + b**2*d**2*x**6/6 + x**4*(a
*b*d**2/2 + b**2*c*d/2) + x**2*(a**2*d**2/2 + 2*a*b*c*d + b**2*c**2/2)
```

Giac [A] time = 1.19398, size = 154, normalized size = 1.83

$$\frac{1}{6}b^2d^2x^6 + \frac{1}{2}b^2cdx^4 + \frac{1}{2}abd^2x^4 + \frac{1}{2}b^2c^2x^2 + 2\,abcdx^2 + \frac{1}{2}a^2d^2x^2 + \left(abc^2 + a^2cd\right)\log\left(x^2\right) - \frac{2\,abc^2x^2 + 2\,a^2cdx^2 + a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/2*b^2*c^2*x^2 + 2*a
*b*c*d*x^2 + 1/2*a^2*d^2*x^2 + (a*b*c^2 + a^2*c*d)*log(x^2) - 1/2*(2*a*b*c^
2*x^2 + 2*a^2*c*d*x^2 + a^2*c^2)/x^2
```

3.158 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4} dx$

Optimal. Leaf size=80

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rubi [A] time = 0.0527717, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^4}\,dx&=\int \left(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^4}+\frac{2ac(bc+ad)}{x^2}+2bd(bc+ad)x^2+b^2d^2x^4\right)dx\\&=-\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5\end{aligned}$$

Mathematica [A] time = 0.0409307, size = 80, normalized size = 1.

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Maple [A] time = 0.005, size = 81, normalized size = 1.

$$\frac{b^2d^2x^5}{5}+\frac{2\,x^3abd^2}{3}+\frac{2\,x^3b^2cd}{3}+a^2d^2x+4\,cabdx+b^2c^2x-\frac{a^2c^2}{3\,x^3}-2\,\frac{ac\,(ad+bc)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^4,x)
```

```
[Out] 1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+a^2*d^2*x+4*c*a*b*d*x+b^2*c
^2*x-1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x
```

Maxima [A] time = 0.999957, size = 113, normalized size = 1.41

$$\frac{1}{5}\,b^2d^2x^5+\frac{2}{3}\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x-\frac{a^2c^2+6\left(abc^2+a^2cd\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x - 1/3*(a^2*c^2 + 6*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

Fricas [A] time = 1.17608, size = 185, normalized size = 2.31

$$\frac{3\,b^2d^2x^8+10\left(b^2cd+abd^2\right)x^6+15\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-5\,a^2c^2-30\left(abc^2+a^2cd\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^8 + 10*(b^2*c*d + a*b*d^2)*x^6 + 15*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^4 - 5*a^2*c^2 - 30*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

Sympy [A] time = 0.426533, size = 90, normalized size = 1.12

$$\frac{b^2d^2x^5}{5}+x^3\left(\frac{2abd^2}{3}+\frac{2b^2cd}{3}\right)+x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2+x^2\left(6a^2cd+6abc^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

```
[Out] b**2*d**2*x**5/5 + x**3*(2*a*b*d**2/3 + 2*b**2*c*d/3) + x*(a**2*d**2 + 4*a*
b*c*d + b**2*c**2) - (a**2*c**2 + x**2*(6*a**2*c*d + 6*a*b*c**2))/(3*x**3)
```

Giac [A] time = 1.15307, size = 119, normalized size = 1.49

$$\frac{1}{5}\,b^2d^2x^5+\frac{2}{3}\,b^2cdx^3+\frac{2}{3}\,abd^2x^3+b^2c^2x+4\,abcdx+a^2d^2x-\frac{6\,abc^2x^2+6\,a^2cdx^2+a^2c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*b^2*c*d*x^3 + 2/3*a*b*d^2*x^3 + b^2*c^2*x + 4*a*b*c*d
*x + a^2*d^2*x - 1/3*(6*a*b*c^2*x^2 + 6*a^2*c*d*x^2 + a^2*c^2)/x^3
```

3.159 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+$$

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rubi [A] time = 0.0880314, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^4+ac^2(2bc+3ad)x^6+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6abcd+a^2d^2\right)\right.\\ &= \frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{11}d\left(3b^2c^2+6abcd+a\right.\end{aligned}$$

Mathematica [A] time = 0.0266395, size = 127, normalized size = 1.

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Maple [A] time = 0.001, size = 128, normalized size = 1.

$$\frac{b^2d^3x^{15}}{15}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{11}}{11}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^9}{9}+\frac{\left(3\,a^2c^2d+2\,abc\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/15*b^2*d^3*x^15+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/11*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^11+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/7*(3*a^2*c
^2*d+2*a*b*c^3)*x^7+1/5*a^2*c^3*x^5
```

Maxima [A] time = 1.03304, size = 171, normalized size = 1.35

$$\frac{1}{15}\,b^2d^3x^{15}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}\,a^2c^3x^5+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

Fricas [A] time = 1.12534, size = 323, normalized size = 2.54

$$\frac{1}{15}x^{15}d^3b^2+\frac{3}{13}x^{13}d^2cb^2+\frac{2}{13}x^{13}d^3ba+\frac{3}{11}x^{11}dc^2b^2+\frac{6}{11}x^{11}d^2cba+\frac{1}{11}x^{11}d^3a^2+\frac{1}{9}x^9c^3b^2+\frac{2}{3}x^9dc^2ba+\frac{1}{3}x^9d^2ca^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/11*x^11*d*c
^2*b^2 + 6/11*x^11*d^2*c*b*a + 1/11*x^11*d^3*a^2 + 1/9*x^9*c^3*b^2 + 2/3*x^
9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/7*x^7*c^3*b*a + 3/7*x^7*d*c^2*a^2 + 1/5
*x^5*c^3*a^2
```

Sympy [A] time = 0.086249, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^5}{5}+\frac{b^2d^3x^{15}}{15}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{11}\left(\frac{a^2d^3}{11}+\frac{6abcd^2}{11}+\frac{3b^2c^2d}{11}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^7\left(\frac{3a^2}{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**5/5 + b**2*d**3*x**15/15 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**
2/13) + x**11*(a**2*d**3/11 + 6*a*b*c*d**2/11 + 3*b**2*c**2*d/11) + x**9*(a
**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**7*(3*a**2*c**2*d/7 + 2*a*
```

```
b*c**3/7)
```

Giac [A] time = 1.14775, size = 182, normalized size = 1.43

$$\frac{1}{15}b^2d^3x^{15} + \frac{3}{13}b^2cd^2x^{13} + \frac{2}{13}abd^3x^{13} + \frac{3}{11}b^2c^2dx^{11} + \frac{6}{11}abcd^2x^{11} + \frac{1}{11}a^2d^3x^{11} + \frac{1}{9}b^2c^3x^9 + \frac{2}{3}abc^2dx^9 + \frac{1}{3}a^2cd^2x^9 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^2*d^3*x^15 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/11*b^2*c^2*
d*x^11 + 6/11*a*b*c*d^2*x^11 + 1/11*a^2*d^3*x^11 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/7*a*b*c^3*x^7 + 3/7*a^2*c^2*d*x^7 + 1/5
*a^2*c^3*x^5
```

3.160 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=106

$$-\frac{b\left(c+dx^2\right)^6(3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5(bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4(bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^4)/(8*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/(12*d^4) + (b^2*(c +
 d*x^2)^7)/(14*d^4)
```

Rubi [A] time = 0.224857, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^6(3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5(bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4(bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^4)/(8*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/(12*d^4) + (b^2*(c +
 d*x^2)^7)/(14*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2(c+dx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c(bc-ad)^2(c+dx)^3}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^4}{d^3}-\frac{b(3bc-2ad)(c\right.\right.\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^4}{8d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^5}{10d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^6}{12d^4}+\end{aligned}$$

Mathematica [A] time = 0.0320621, size = 119, normalized size = 1.12

$$\frac{1}{840}x^4\left(84dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+105cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+210a^2c^3+140ac^2x^2(3ad+2bc)+70bd^2x^8(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^4*(210*a^2*c^3 + 140*a*c^2*(2*b*c + 3*a*d)*x^2 + 105*c*(b^2*c^2 + 6*a*b*
c*d + 3*a^2*d^2)*x^4 + 84*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 70*b*d^
2*(3*b*c + 2*a*d)*x^8 + 60*b^2*d^3*x^10))/840
```

Maple [A] time = 0., size = 128, normalized size = 1.2

$$\frac{b^2d^3x^{14}}{14}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{12}}{12}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{10}}{10}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^8}{8}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/14*b^2*d^3*x^14+1/12*(2*a*b*d^3+3*b^2*c*d^2)*x^12+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/8*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^8+1/6*(3*a^2*c
^2*d+2*a*b*c^3)*x^6+1/4*a^2*c^3*x^4
```

Maxima [A] time = 1.0068, size = 171, normalized size = 1.61

$$\frac{1}{14}\,b^2d^3x^{14}+\frac{1}{12}\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{4}\,a^2c^3x^4+\frac{1}{8}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

Fricas [A] time = 1.13755, size = 319, normalized size = 3.01

$$\frac{1}{14}x^{14}d^3b^2+\frac{1}{4}x^{12}d^2cb^2+\frac{1}{6}x^{12}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{8}x^8c^3b^2+\frac{3}{4}x^8dc^2ba+\frac{3}{8}x^8d^2ca^2+\frac{1}{3}x^6c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^3*b^2 + 1/4*x^12*d^2*c*b^2 + 1/6*x^12*d^3*b*a + 3/10*x^10*d*c^2
*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/8*x^8*c^3*b^2 + 3/4*x^8*d
*c^2*b*a + 3/8*x^8*d^2*c*a^2 + 1/3*x^6*c^3*b*a + 1/2*x^6*d*c^2*a^2 + 1/4*x^
4*c^3*a^2
```

Sympy [A] time = 0.084389, size = 138, normalized size = 1.3

$$\frac{a^2c^3x^4}{4}+\frac{b^2d^3x^{14}}{14}+x^{12}\left(\frac{abd^3}{6}+\frac{b^2cd^2}{4}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^8\left(\frac{3a^2cd^2}{8}+\frac{3abc^2d}{4}+\frac{b^2c^3}{8}\right)+x^6\left(\frac{a^2c^2d}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**4/4 + b**2*d**3*x**14/14 + x**12*(a*b*d**3/6 + b**2*c*d**2/4)
+ x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**8*(3*a**2*c
*d**2/8 + 3*a*b*c**2*d/4 + b**2*c**3/8) + x**6*(a**2*c**2*d/2 + a*b*c**3/3)
```

Giac [A] time = 1.13849, size = 182, normalized size = 1.72

$$\frac{1}{14}\,b^2d^3x^{14}+\frac{1}{4}\,b^2cd^2x^{12}+\frac{1}{6}\,abd^3x^{12}+\frac{3}{10}\,b^2c^2dx^{10}+\frac{3}{5}\,abcd^2x^{10}+\frac{1}{10}\,a^2d^3x^{10}+\frac{1}{8}\,b^2c^3x^8+\frac{3}{4}\,abc^2dx^8+\frac{3}{8}\,a^2cd^2x^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/4*b^2*c*d^2*x^12 + 1/6*a*b*d^3*x^12 + 3/10*b^2*c^2*d*
x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/8*b^2*c^3*x^8 + 3/4*a*b*c
^2*d*x^8 + 3/8*a^2*c*d^2*x^8 + 1/3*a*b*c^3*x^6 + 1/2*a^2*c^2*d*x^6 + 1/4*a^
2*c^3*x^4
```

3.161 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{1}$$

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rubi [A] time = 0.0716647, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^2+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abcd+a^2d^2\right)\right.\\ &= \frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6abcd+a^2\right.\end{aligned}$$

Mathematica [A] time = 0.0209268, size = 127, normalized size = 1.

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Maple [A] time = 0., size = 128, normalized size = 1.

$$\frac{b^2d^3x^{13}}{13}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{11}}{11}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^9}{9}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^7}{7}+\frac{\left(3\,a^2c^2d+2\,abc^3\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^2*d^3*x^13+1/11*(2*a*b*d^3+3*b^2*c*d^2)*x^11+1/9*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^9+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/5*(3*a^2*c^2
*d+2*a*b*c^3)*x^5+1/3*a^2*c^3*x^3
```

Maxima [A] time = 0.991218, size = 171, normalized size = 1.35

$$\frac{1}{13}\,b^2d^3x^{13}+\frac{1}{11}\left(3\,b^2cd^2+2\,abd^3\right)x^{11}+\frac{1}{9}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^9+\frac{1}{3}\,a^2c^3x^3+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Fricas [A] time = 1.0419, size = 315, normalized size = 2.48

$$\frac{1}{13}x^{13}d^3b^2+\frac{3}{11}x^{11}d^2cb^2+\frac{2}{11}x^{11}d^3ba+\frac{1}{3}x^9dc^2b^2+\frac{2}{3}x^9d^2cba+\frac{1}{9}x^9d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+\frac{2}{5}x^5c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^2 + 3/11*x^11*d^2*c*b^2 + 2/11*x^11*d^3*b*a + 1/3*x^9*d*c^2
*b^2 + 2/3*x^9*d^2*c*b*a + 1/9*x^9*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7*d*c^
2*b*a + 3/7*x^7*d^2*c*a^2 + 2/5*x^5*c^3*b*a + 3/5*x^5*d*c^2*a^2 + 1/3*x^3*c
^3*a^2
```

Sympy [A] time = 0.083517, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^3}{3}+\frac{b^2d^3x^{13}}{13}+x^{11}\left(\frac{2abd^3}{11}+\frac{3b^2cd^2}{11}\right)+x^9\left(\frac{a^2d^3}{9}+\frac{2abcd^2}{3}+\frac{b^2c^2d}{3}\right)+x^7\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^5\left(\frac{3a^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**3/3 + b**2*d**3*x**13/13 + x**11*(2*a*b*d**3/11 + 3*b**2*c*d**
2/11) + x**9*(a**2*d**3/9 + 2*a*b*c*d**2/3 + b**2*c**2*d/3) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c*
```

```
*3/5)
```

Giac [A] time = 1.16888, size = 182, normalized size = 1.43

$$\frac{1}{13}b^2d^3x^{13} + \frac{3}{11}b^2cd^2x^{11} + \frac{2}{11}abd^3x^{11} + \frac{1}{3}b^2c^2dx^9 + \frac{2}{3}abcd^2x^9 + \frac{1}{9}a^2d^3x^9 + \frac{1}{7}b^2c^3x^7 + \frac{6}{7}abc^2dx^7 + \frac{3}{7}a^2cd^2x^7 + \frac{2}{5}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^3*x^13 + 3/11*b^2*c*d^2*x^11 + 2/11*a*b*d^3*x^11 + 1/3*b^2*c^2*d
*x^9 + 2/3*a*b*c*d^2*x^9 + 1/9*a^2*d^3*x^9 + 1/7*b^2*c^3*x^7 + 6/7*a*b*c^2*
d*x^7 + 3/7*a^2*c*d^2*x^7 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + 1/3*a^2*c
^3*x^3
```

3.162 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=71

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2)^5)/(5*d^
3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rubi [A] time = 0.118306, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2)^5)/(5*d^
3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+bx)^2(c+dx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx,x,x^2\right)\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^4}{8d^3}-\frac{b(bc-ad)\left(c+dx^2\right)^5}{5d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}\end{aligned}$$

Mathematica [A] time = 0.0283017, size = 119, normalized size = 1.68

$$\frac{1}{120}x^2\left(15dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+20cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+60a^2c^3+30ac^2x^2(3ad+2bc)+12bd^2x^8(2ad\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^2*(60*a^2*c^3 + 30*a*c^2*(2*b*c + 3*a*d)*x^2 + 20*c*(b^2*c^2 + 6*a*b*c*d
 + 3*a^2*d^2)*x^4 + 15*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 12*b*d^2*(
3*b*c + 2*a*d)*x^8 + 10*b^2*d^3*x^10))/120
```

Maple [A] time = 0.001, size = 128, normalized size = 1.8

$$\frac{b^2d^3x^{12}}{12}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{10}}{10}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^8}{8}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^6}{6}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/12*b^2*d^3*x^12+1/10*(2*a*b*d^3+3*b^2*c*d^2)*x^10+1/8*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^8+1/6*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^6+1/4*(3*a^2*c^2
*d+2*a*b*c^3)*x^4+1/2*a^2*c^3*x^2
```

Maxima [A] time = 1.006, size = 171, normalized size = 2.41

$$\frac{1}{12}\,b^2d^3x^{12}+\frac{1}{10}\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+\frac{1}{8}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+\frac{1}{2}\,a^2c^3x^2+\frac{1}{6}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Fricas [B] time = 1.054, size = 308, normalized size = 4.34

$$\frac{1}{12}x^{12}d^3b^2+\frac{3}{10}x^{10}d^2cb^2+\frac{1}{5}x^{10}d^3ba+\frac{3}{8}x^8dc^2b^2+\frac{3}{4}x^8d^2cba+\frac{1}{8}x^8d^3a^2+\frac{1}{6}x^6c^3b^2+x^6dc^2ba+\frac{1}{2}x^6d^2ca^2+\frac{1}{2}x^4c^3ba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^3*b^2 + 3/10*x^10*d^2*c*b^2 + 1/5*x^10*d^3*b*a + 3/8*x^8*d*c^2*
b^2 + 3/4*x^8*d^2*c*b*a + 1/8*x^8*d^3*a^2 + 1/6*x^6*c^3*b^2 + x^6*d*c^2*b*a
 + 1/2*x^6*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + 1/2*x^2*c^3*a^
2
```

Sympy [B] time = 0.083065, size = 136, normalized size = 1.92

$$\frac{a^2c^3x^2}{2}+\frac{b^2d^3x^{12}}{12}+x^{10}\left(\frac{abd^3}{5}+\frac{3b^2cd^2}{10}\right)+x^8\left(\frac{a^2d^3}{8}+\frac{3abcd^2}{4}+\frac{3b^2c^2d}{8}\right)+x^6\left(\frac{a^2cd^2}{2}+abc^2d+\frac{b^2c^3}{6}\right)+x^4\left(\frac{3a^2c^2d}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**2/2 + b**2*d**3*x**12/12 + x**10*(a*b*d**3/5 + 3*b**2*c*d**2/1
0) + x**8*(a**2*d**3/8 + 3*a*b*c*d**2/4 + 3*b**2*c**2*d/8) + x**6*(a**2*c*d
**2/2 + a*b*c**2*d + b**2*c**3/6) + x**4*(3*a**2*c**2*d/4 + a*b*c**3/2)
```

Giac [B] time = 1.16478, size = 181, normalized size = 2.55

$$\frac{1}{12} b^2d^3x^{12} + \frac{3}{10} b^2cd^2x^{10} + \frac{1}{5} abd^3x^{10} + \frac{3}{8} b^2c^2dx^8 + \frac{3}{4} abcd^2x^8 + \frac{1}{8} a^2d^3x^8 + \frac{1}{6} b^2c^3x^6 + abc^2dx^6 + \frac{1}{2} a^2cd^2x^6 + \frac{1}{2} ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^3*x^12 + 3/10*b^2*c*d^2*x^10 + 1/5*a*b*d^3*x^10 + 3/8*b^2*c^2*d*
x^8 + 3/4*a*b*c*d^2*x^8 + 1/8*a^2*d^3*x^8 + 1/6*b^2*c^3*x^6 + a*b*c^2*d*x^6
 + 1/2*a^2*c*d^2*x^6 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + 1/2*a^2*c^3*x^
2
```

3.163 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi [A] time = 0.0538846, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+\right.\\ &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7\end{aligned}$$

Mathematica [A] time = 0.0209226, size = 122, normalized size = 1.

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A] time = 0., size = 125, normalized size = 1.

$$\frac{b^2d^3x^{11}}{11}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^9}{9}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^7}{7}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^5}{5}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A] time = 0.989701, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Fricas [A] time = 1.08426, size = 296, normalized size = 2.43

$$\frac{1}{11}x^{11}d^3b^2+\frac{1}{3}x^9d^2cb^2+\frac{2}{9}x^9d^3ba+\frac{3}{7}x^7dc^2b^2+\frac{6}{7}x^7d^2cba+\frac{1}{7}x^7d^3a^2+\frac{1}{5}x^5c^3b^2+\frac{6}{5}x^5dc^2ba+\frac{3}{5}x^5d^2ca^2+\frac{2}{3}x^3c^3ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7*d*c^2*b^2
 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^2 + 6/5*x^5*d*c^2*b*
a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3*d*c^2*a^2 + x*c^3*a^2
```

Sympy [A] time = 0.080935, size = 136, normalized size = 1.11

$$a^2c^3x+\frac{b^2d^3x^{11}}{11}+x^9\left(\frac{2abd^3}{9}+\frac{b^2cd^2}{3}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+x^5\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^3\left(a^2c^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

Giac [A] time = 1.12057, size = 177, normalized size = 1.45

$$\frac{1}{11}b^2d^3x^{11} + \frac{1}{3}b^2cd^2x^9 + \frac{2}{9}abd^3x^9 + \frac{3}{7}b^2c^2dx^7 + \frac{6}{7}abcd^2x^7 + \frac{1}{7}a^2d^3x^7 + \frac{1}{5}b^2c^3x^5 + \frac{6}{5}abc^2dx^5 + \frac{3}{5}a^2cd^2x^5 + \frac{2}{3}abc^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

3.164 $\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x} \, dx$

Optimal. Leaf size=123

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)$$

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rubi [A] time = 0.103883, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(ac^2(2bc+3ad)+\frac{a^2c^3}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+d\left(3b^2c^2+6abcd+\right.\right.\right.\\ &= \frac{1}{2}ac^2(2bc+3ad)x^2+\frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+\frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+\frac{1}{8}\end{aligned}$$

Mathematica [A] time = 0.0298954, size = 123, normalized size = 1.

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Maple [A] time = 0.002, size = 132, normalized size = 1.1

$$\frac{b^2d^3x^{10}}{10}+\frac{x^8abd^3}{4}+\frac{3\,x^8b^2cd^2}{8}+\frac{x^6a^2d^3}{6}+x^6abcd^2+\frac{x^6b^2c^2d}{2}+\frac{3\,x^4a^2cd^2}{4}+\frac{3\,x^4abc^2d}{2}+\frac{x^4b^2c^3}{4}+\frac{3\,x^2a^2c^2d}{2}+x^2abc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x,x)
```

```
[Out] 1/10*b^2*d^3*x^10+1/4*x^8*a*b*d^3+3/8*x^8*b^2*c*d^2+1/6*x^6*a^2*d^3+x^6*a*b
*c*d^2+1/2*x^6*b^2*c^2*d+3/4*x^4*a^2*c*d^2+3/2*x^4*a*b*c^2*d+1/4*x^4*b^2*c^
3+3/2*x^2*a^2*c^2*d+x^2*a*b*c^3+a^2*c^3*ln(x)
```

Maxima [A] time = 0.977729, size = 173, normalized size = 1.41

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+\frac{1}{2}\,a^2c^3\log\left(x^2\right)+\frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + 1/2*a^2*c^3*log(x^2) + 1/4*(b^2*c^3 + 6*a*b*c^
2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [A] time = 1.20148, size = 275, normalized size = 2.24

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+a^2c^3\log\left(x\right)+\frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3*log(x) + 1/4*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [A] time = 0.36256, size = 133, normalized size = 1.08

$$a^2c^3\log\left(x\right)+\frac{b^2d^3x^{10}}{10}+x^8\left(\frac{abd^3}{4}+\frac{3b^2cd^2}{8}\right)+x^6\left(\frac{a^2d^3}{6}+abcd^2+\frac{b^2c^2d}{2}\right)+x^4\left(\frac{3a^2cd^2}{4}+\frac{3abc^2d}{2}+\frac{b^2c^3}{4}\right)+x^2\left(\frac{3a^2c^2d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

```
[Out] a**2*c**3*log(x) + b**2*d**3*x**10/10 + x**8*(a*b*d**3/4 + 3*b**2*c*d**2/8)
 + x**6*(a**2*d**3/6 + a*b*c*d**2 + b**2*c**2*d/2) + x**4*(3*a**2*c*d**2/4
+ 3*a*b*c**2*d/2 + b**2*c**3/4) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [A] time = 1.13317, size = 181, normalized size = 1.47

$$\frac{1}{10}\,b^2d^3x^{10} + \frac{3}{8}\,b^2cd^2x^8 + \frac{1}{4}\,abd^3x^8 + \frac{1}{2}\,b^2c^2dx^6 + abcd^2x^6 + \frac{1}{6}\,a^2d^3x^6 + \frac{1}{4}\,b^2c^3x^4 + \frac{3}{2}\,abc^2dx^4 + \frac{3}{4}\,a^2cd^2x^4 + abc^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^3*x^10 + 3/8*b^2*c*d^2*x^8 + 1/4*a*b*d^3*x^8 + 1/2*b^2*c^2*d*x^6
 + a*b*c*d^2*x^6 + 1/6*a^2*d^3*x^6 + 1/4*b^2*c^3*x^4 + 3/2*a*b*c^2*d*x^4 +
3/4*a^2*c*d^2*x^4 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + 1/2*a^2*c^3*log(x^2)
```

3.165 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx$

Optimal. Leaf size=120

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rubi [A] time = 0.0564373, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx &= \int \left(ac^2(2bc+3ad)+\frac{a^2c^3}{x^2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+b\right.\\ &= -\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{5}d\left(3b^2c^2+6abcd+a^2d^2\right)x^5-\end{aligned}$$

Mathematica [A] time = 0.0381116, size = 120, normalized size = 1.

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
```

```
2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Maple [A] time = 0.003, size = 131, normalized size = 1.1

$$\frac{b^2d^3x^9}{9}+\frac{2\,x^7abd^3}{7}+\frac{3\,x^7b^2cd^2}{7}+\frac{x^5a^2d^3}{5}+\frac{6\,x^5abcd^2}{5}+\frac{3\,x^5b^2c^2d}{5}+x^3a^2cd^2+2\,x^3abc^2d+\frac{x^3b^2c^3}{3}+3\,a^2c^2dx+2\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^2,x)
```

```
[Out] 1/9*b^2*d^3*x^9+2/7*x^7*a*b*d^3+3/7*x^7*b^2*c*d^2+1/5*x^5*a^2*d^3+6/5*x^5*a
*b*c*d^2+3/5*x^5*b^2*c^2*d+x^3*a^2*c*d^2+2*x^3*a*b*c^2*d+1/3*x^3*b^2*c^3+3*
a^2*c^2*d*x+2*a*b*c^3*x-a^2*c^3/x
```

Maxima [A] time = 0.991279, size = 167, normalized size = 1.39

$$\frac{1}{9}\,b^2d^3x^9+\frac{1}{7}\left(3\,b^2cd^2+2\,abd^3\right)x^7+\frac{1}{5}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^5-\frac{a^2c^3}{x}+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\left(2\,abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^3*x^9 + 1/7*(3*b^2*c*d^2 + 2*a*b*d^3)*x^7 + 1/5*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^5 - a^2*c^3/x + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*x
```

Fricas [A] time = 1.21753, size = 279, normalized size = 2.32

$$\frac{35\,b^2d^3x^{10}+45\left(3\,b^2cd^2+2\,abd^3\right)x^8+63\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-315\,a^2c^3+105\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^3*x^10 + 45*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 63*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 315*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x
```

Sympy [A] time = 0.360903, size = 131, normalized size = 1.09

$$-\frac{a^2c^3}{x}+\frac{b^2d^3x^9}{9}+x^7\left(\frac{2abd^3}{7}+\frac{3b^2cd^2}{7}\right)+x^5\left(\frac{a^2d^3}{5}+\frac{6abcd^2}{5}+\frac{3b^2c^2d}{5}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x\left(3a^2c^2d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

```
[Out] -a**2*c**3/x + b**2*d**3*x**9/9 + x**7*(2*a*b*d**3/7 + 3*b**2*c*d**2/7) + x
**5*(a**2*d**3/5 + 6*a*b*c*d**2/5 + 3*b**2*c**2*d/5) + x**3*(a**2*c*d**2 +
2*a*b*c**2*d + b**2*c**3/3) + x*(3*a**2*c**2*d + 2*a*b*c**3)
```

Giac [A] time = 1.12283, size = 176, normalized size = 1.47

$$\frac{1}{9}\,b^{2}d^{3}x^{9}+\frac{3}{7}\,b^{2}cd^{2}x^{7}+\frac{2}{7}\,abd^{3}x^{7}+\frac{3}{5}\,b^{2}c^{2}dx^{5}+\frac{6}{5}\,abcd^{2}x^{5}+\frac{1}{5}\,a^{2}d^{3}x^{5}+\frac{1}{3}\,b^{2}c^{3}x^{3}+2\,abc^{2}dx^{3}+a^{2}cd^{2}x^{3}+2\,abc^{3}x+3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^3*x^9 + 3/7*b^2*c*d^2*x^7 + 2/7*a*b*d^3*x^7 + 3/5*b^2*c^2*d*x^5 +
 6/5*a*b*c*d^2*x^5 + 1/5*a^2*d^3*x^5 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + 2*a*b*c^3*x + 3*a^2*c^2*d*x - a^2*c^3/x
```

3.166 $\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} dx$

Optimal. Leaf size=123

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}$$

```
[Out] -(a^2*c^3)/(2*x^2) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2 + (d*(3*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2*a*d)*x^6)/6 + (b^2*
d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Rubi [A] time = 0.100873, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```


```
[Out] -(a^2*c^3)/(2*x^2) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2 + (d*(3*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2*a*d)*x^6)/6 + (b^2*
d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(c\left(b^2c^2+6abcd+3a^2d^2\right)+\frac{a^2c^3}{x^2}+\frac{ac^2(2bc+3ad)}{x}+d\left(3b^2c^2+6abcd+a\right.\right.\right.\\ &= -\frac{a^2c^3}{2x^2}+\frac{1}{2}c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+\frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+\frac{1}{6}bd^2(3bc+2a\end{aligned}$$

Mathematica [A] time = 0.0476805, size = 120, normalized size = 0.98

$$\frac{6a^2\left(-2c^3+6cd^2x^4+d^3x^6\right)+4abdx^4\left(18c^2+9cdx^2+2d^2x^4\right)+3b^2x^4\left(6c^2dx^2+4c^3+4cd^2x^4+d^3x^6\right)}{24x^2}+ac^2\log(x)(3a$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] (4*a*b*d*x^4*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^2*x^4*(4*c^3 + 6*c^2*d*
x^2 + 4*c*d^2*x^4 + d^3*x^6) + 6*a^2*(-2*c^3 + 6*c*d^2*x^4 + d^3*x^6))/(24*
x^2) + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Maple [A] time = 0.007, size = 134, normalized size = 1.1

$$\frac{b^2d^3x^8}{8}+\frac{x^6abd^3}{3}+\frac{x^6b^2cd^2}{2}+\frac{x^4a^2d^3}{4}+\frac{3\,x^4abcd^2}{2}+\frac{3\,x^4b^2c^2d}{4}+\frac{3\,x^2a^2cd^2}{2}+3\,x^2abc^2d+\frac{x^2b^2c^3}{2}+3\,\ln(x)\,a^2c^2d+2\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^3,x)
```

```
[Out] 1/8*b^2*d^3*x^8+1/3*x^6*a*b*d^3+1/2*x^6*b^2*c*d^2+1/4*x^4*a^2*d^3+3/2*x^4*a
*b*c*d^2+3/4*x^4*b^2*c^2*d+3/2*x^2*a^2*c*d^2+3*x^2*a*b*c^2*d+1/2*x^2*b^2*c^
3+3*ln(x)*a^2*c^2*d+2*ln(x)*a*b*c^3-1/2*a^2*c^3/x^2
```

Maxima [A] time = 0.988683, size = 173, normalized size = 1.41

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{6}\left(3\,b^2cd^2+2\,abd^3\right)x^6+\frac{1}{4}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4-\frac{a^2c^3}{2\,x^2}+\frac{1}{2}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^2+\frac{1}{2}\left(2\,abc^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/6*(3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 1/4*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^4 - 1/2*a^2*c^3/x^2 + 1/2*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2)
```

Fricas [A] time = 1.25571, size = 282, normalized size = 2.29

$$\frac{3\,b^2d^3x^{10}+4\left(3\,b^2cd^2+2\,abd^3\right)x^8+6\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-12\,a^2c^3+12\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+24\left(2\,a\right.}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*d^3*x^10 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 6*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^6 - 12*a^2*c^3 + 12*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*
c*d^2)*x^4 + 24*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2*log(x))/x^2
```

Sympy [A] time = 0.453486, size = 133, normalized size = 1.08

$$-\frac{a^2c^3}{2x^2}+ac^2\,(3ad+2bc)\log{(x)}+\frac{b^2d^3x^8}{8}+x^6\left(\frac{abd^3}{3}+\frac{b^2cd^2}{2}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^2\left(\frac{3a^2cd^2}{2}+3abc^2d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

```
[Out] -a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x) + b**2*d**3*x**8/8 + x*
*6*(a*b*d**3/3 + b**2*c*d**2/2) + x**4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b*
*2*c**2*d/4) + x**2*(3*a**2*c*d**2/2 + 3*a*b*c**2*d + b**2*c**3/2)
```

Giac [A] time = 1.15674, size = 216, normalized size = 1.76

$$\frac{1}{8}b^2d^3x^8 + \frac{1}{2}b^2cd^2x^6 + \frac{1}{3}abd^3x^6 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{2}b^2c^3x^2 + 3\,abc^2dx^2 + \frac{3}{2}a^2cd^2x^2 + \frac{1}{2}\left(2\,abc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/2*b^2*c*d^2*x^6 + 1/3*a*b*d^3*x^6 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/2*b^2*c^3*x^2 + 3*a*b*c^2*d*x^2 +
3/2*a^2*c*d^2*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2) - 1/2*(2*a*b*c^3
*x^2 + 3*a^2*c^2*d*x^2 + a^2*c^3)/x^2
```

3.167 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rubi [A] time = 0.0601588, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx=\int \left(c\left(b^2c^2+6abcd+3a^2d^2\right)+\frac{a^2c^3}{x^4}+\frac{ac^2(2bc+3ad)}{x^2}+d\left(3b^2c^2+6abcd+a^2d^2\right)x^2+bd^2\right.$$
$$=-\frac{a^2c^3}{3x^3}-\frac{ac^2(2bc+3ad)}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+\frac{1}{3}d\left(3b^2c^2+6abcd+a^2d^2\right)x^3+\frac{1}{5}$$

Mathematica [A] time = 0.0420917, size = 120, normalized size = 1.

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
```

```
2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Maple [A] time = 0.005, size = 124, normalized size = 1.

$$\frac{b^2d^3x^7}{7}+\frac{2\,x^5abd^3}{5}+\frac{3\,x^5b^2cd^2}{5}+\frac{x^3a^2d^3}{3}+2\,x^3abcd^2+x^3b^2c^2d+3\,a^2cd^2x+6\,abc^2dx+b^2c^3x-\frac{a^2c^3}{3\,x^3}-\frac{ac^2\,(3\,ad+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^4,x)
```

```
[Out] 1/7*b^2*d^3*x^7+2/5*x^5*a*b*d^3+3/5*x^5*b^2*c*d^2+1/3*x^3*a^2*d^3+2*x^3*a*b
*c*d^2+x^3*b^2*c^2*d+3*a^2*c*d^2*x+6*a*b*c^2*d*x+b^2*c^3*x-1/3*a^2*c^3/x^3-
a*c^2*(3*a*d+2*b*c)/x
```

Maxima [A] time = 0.984019, size = 170, normalized size = 1.42

$$\frac{1}{7}\,b^2d^3x^7+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{3}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x-\frac{a^2c^3+3\left(2\,abc^3\right.}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^3*x^7 + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/3*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x - 1/3*(a
^2*c^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

Fricas [A] time = 1.28312, size = 281, normalized size = 2.34

$$\frac{15\,b^2d^3x^{10}+21\left(3\,b^2cd^2+2\,abd^3\right)x^8+35\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-35\,a^2c^3+105\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4-}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^10 + 21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 35*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 35*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^4 - 105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

Sympy [A] time = 0.472482, size = 129, normalized size = 1.08

$$\frac{b^2d^3x^7}{7}+x^5\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right)+x^3\left(\frac{a^2d^3}{3}+2abcd^2+b^2c^2d\right)+x\left(3a^2cd^2+6abc^2d+b^2c^3\right)-\frac{a^2c^3+x^2\left(9a^2c^2d+6al\right.}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

```
[Out] b**2*d**3*x**7/7 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x**3*(a**2*d**3/
3 + 2*a*b*c*d**2 + b**2*c**2*d) + x*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c*
*3) - (a**2*c**3 + x**2*(9*a**2*c**2*d + 6*a*b*c**3))/(3*x**3)
```

Giac [A] time = 1.14904, size = 174, normalized size = 1.45

$$\frac{1}{7}b^2d^3x^7 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + b^2c^2dx^3 + 2\,abcd^2x^3 + \frac{1}{3}a^2d^3x^3 + b^2c^3x + 6\,abc^2dx + 3\,a^2cd^2x - \frac{6\,abc^3x^2 + 9\,a^2c^2dx}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^3*x^7 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + b^2*c^2*d*x^3 + 2*a
*b*c*d^2*x^3 + 1/3*a^2*d^3*x^3 + b^2*c^3*x + 6*a*b*c^2*d*x + 3*a^2*c*d^2*x
- 1/3*(6*a*b*c^3*x^2 + 9*a^2*c^2*d*x^2 + a^2*c^3)/x^3
```

3.168 $\int \frac{x^4(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=104

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{cx(bc-ad)^2}{d^4}+\frac{b^2x^7}{7d}$$

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a*d)
*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/d^(9/2)
```

Rubi [A] time = 0.0737845, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{cx(bc-ad)^2}{d^4}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a*d)
*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/d^(9/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(bc-ad)^2x^2}{d^3}-\frac{b(bc-2ad)x^4}{d^2}+\frac{b^2x^6}{d}+\frac{b^2c^4-2abc^3d+a^2c^2d^2}{d^4\left(c+dx^2\right)}\right)dx\\ &= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{\left(c^2(bc-ad)^2\right)\int \frac{1}{c+dx^2}\,dx}{d^4}\\ &= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0930724, size = 104, normalized size = 1.

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{x^3(ad-bc)^2}{3d^3}-\frac{cx(bc-ad)^2}{d^4}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((-(b*c) + a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a
*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/d^(9/2)
```

Maple [A] time = 0.003, size = 176, normalized size = 1.7

$$\frac{b^2x^7}{7\,d}+\frac{2\,x^5ab}{5\,d}-\frac{x^5b^2c}{5\,d^2}+\frac{x^3a^2}{3\,d}-\frac{2\,x^3abc}{3\,d^2}+\frac{x^3b^2c^2}{3\,d^3}-\frac{a^2cx}{d^2}+2\,\frac{abc^2x}{d^3}-\frac{b^2c^3x}{d^4}+\frac{a^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\,\frac{abc^3}{d^3\sqrt{cd}}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/7*b^2*x^7/d+2/5/d*x^5*a*b-1/5/d^2*x^5*b^2*c+1/3/d*x^3*a^2-2/3/d^2*x^3*a*b
*c+1/3/d^3*x^3*b^2*c^2-1/d^2*a^2*c*x+2/d^3*a*b*c^2*x-1/d^4*b^2*c^3*x+c^2/d^
2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-2*c^3/d^3/(c*d)^(1/2)*arctan(x*d/
(c*d)^(1/2))*a*b+c^4/d^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30505, size = 649, normalized size = 6.24

$$\left[\frac{30\,b^2d^3x^7-42\left(b^2cd^2-2\,abd^3\right)x^5+70\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^3+105\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-}{dx^2+c}\right.}{210\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^2*d^3*x^7 - 42*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 70*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(-c
/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 210*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*x)/d^4, 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^
3)*x^5 + 35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - 105*(b^2*c^3 - 2*a*b
*c^2*d + a^2*c*d^2)*x)/d^4]
```

Sympy [B] time = 0.65842, size = 240, normalized size = 2.31

$$\frac{b^2x^7}{7d} - \frac{\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2\log\left(-\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3}+x\right)}{2} + \frac{\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2\log\left(\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3}+x\right)}{2} + \frac{x^5(2abd-b^2c)}{5d^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**7/(7*d) - sqrt(-c**3/d**9)*(a*d - b*c)**2*log(-d**4*sqrt(-c**3/d**9
)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2 + sqrt(-c*
*3/d**9)*(a*d - b*c)**2*log(d**4*sqrt(-c**3/d**9)*(a*d - b*c)**2/(a**2*c*d*
*2 - 2*a*b*c**2*d + b**2*c**3) + x)/2 + x**5*(2*a*b*d - b**2*c)/(5*d**2) +
x**3*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**3) - x*(a**2*c*d**2 - 2*a*b*
c**2*d + b**2*c**3)/d**4
```

Giac [A] time = 1.20064, size = 207, normalized size = 1.99

$$\frac{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^4} + \frac{15\,b^2d^6x^7 - 21\,b^2cd^5x^5 + 42\,abd^6x^5 + 35\,b^2c^2d^4x^3 - 70\,abcd^5x^3 + 35\,a^2d^6x^3}{105\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4)
 + 1/105*(15*b^2*d^6*x^7 - 21*b^2*c*d^5*x^5 + 42*a*b*d^6*x^5 + 35*b^2*c^2*d
^4*x^3 - 70*a*b*c*d^5*x^3 + 35*a^2*d^6*x^3 - 105*b^2*c^3*d^3*x + 210*a*b*c^
2*d^4*x - 105*a^2*c*d^5*x)/d^7
```

3.169 $\int \frac{x^3(a+bx^2)^2}{c+dx^2} \, dx$

Optimal. Leaf size=79

$$-\frac{bx^4(bc-2ad)}{4d^2}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^6}{6d}$$

```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^2*x^6)/(6*
d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rubi [A] time = 0.0842049, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{bx^4(bc-2ad)}{4d^2}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^6}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^2*x^6)/(6*
d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3}-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^2}{d}-\frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx, x, x^2\right)\\
&= \frac{(bc-ad)^2x^2}{2d^3}-\frac{b(bc-2ad)x^4}{4d^2}+\frac{b^2x^6}{6d}-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}
\end{aligned}$$

Mathematica [A] time = 0.0409976, size = 82, normalized size = 1.04

$$\frac{dx^2\left(6a^2d^2+6abd\left(dx^2-2c\right)+b^2\left(6c^2-3cdx^2+2d^2x^4\right)\right)-6c(bc-ad)^2\log\left(c+dx^2\right)}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (d*x^2*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^2) + b^2*(6*c^2 - 3*c*d*x^2 + 2*d^2
*x^4)) - 6*c*(b*c - a*d)^2*Log[c + d*x^2])/(12*d^4)
```

Maple [A] time = 0.003, size = 124, normalized size = 1.6

$$\frac{b^2x^6}{6d}+\frac{abx^4}{2d}-\frac{b^2cx^4}{4d^2}+\frac{a^2x^2}{2d}-\frac{abcx^2}{d^2}+\frac{x^2b^2c^2}{2d^3}-\frac{c\ln\left(dx^2+c\right)a^2}{2d^2}+\frac{c^2\ln\left(dx^2+c\right)ab}{d^3}-\frac{c^3\ln\left(dx^2+c\right)b^2}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/6*b^2*x^6/d+1/2/d*x^4*a*b-1/4/d^2*x^4*b^2*c+1/2/d*x^2*a^2-1/d^2*x^2*a*b*c
+1/2/d^3*x^2*b^2*c^2-1/2*c/d^2*ln(d*x^2+c)*a^2+c^2/d^3*ln(d*x^2+c)*a*b-1/2*
c^3/d^4*ln(d*x^2+c)*b^2
```

Maxima [A] time = 0.98049, size = 135, normalized size = 1.71

$$\frac{2\,b^2d^2x^6-3\left(b^2cd-2\,abd^2\right)x^4+6\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{12\,d^3}-\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*(b^2*c*d - 2*a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c
)/d^4
```

Fricas [A] time = 1.16406, size = 212, normalized size = 2.68

$$\frac{2\,b^2d^3x^6-3\left(b^2cd^2-2\,abd^3\right)x^4+6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^2+c\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 6*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*x^2 - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c)
)/d^4
```

Sympy [A] time = 0.553723, size = 83, normalized size = 1.05

$$\frac{b^2x^6}{6d}-\frac{c\left(ad-bc\right)^2\log\left(c+dx^2\right)}{2d^4}+\frac{x^4\left(2abd-b^2c\right)}{4d^2}+\frac{x^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**6/(6*d) - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + x**4*(2*a*b*d
 - b**2*c)/(4*d**2) + x**2*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*d**3)
```

Giac [A] time = 1.16863, size = 144, normalized size = 1.82

$$\frac{2\,b^2d^2x^6 - 3\,b^2cdx^4 + 6\,abd^2x^4 + 6\,b^2c^2x^2 - 12\,abcdx^2 + 6\,a^2d^2x^2}{12\,d^3} - \frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\log\left(\left|dx^2 + c\right|\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*b^2*c*d*x^4 + 6*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*lo
g(abs(d*x^2 + c))/d^4
```

3.170 $\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=83

$$-\frac{bx^3(bc-2ad)}{3d^2}+\frac{x(bc-ad)^2}{d^3}-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{b^2x^5}{5d}$$

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d) - (
Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Rubi [A] time = 0.0644815, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{bx^3(bc-2ad)}{3d^2}+\frac{x(bc-ad)^2}{d^3}-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d) - (
Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(\frac{(bc-ad)^2}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^4}{d}+\frac{-b^2c^3+2abc^2d-a^2cd^2}{d^3\left(c+dx^2\right)}\right)dx\\ &= \frac{(bc-ad)^2x}{d^3}-\frac{b(bc-2ad)x^3}{3d^2}+\frac{b^2x^5}{5d}-\frac{\left(c(bc-ad)^2\right)\int \frac{1}{c+dx^2}\,dx}{d^3}\\ &= \frac{(bc-ad)^2x}{d^3}-\frac{b(bc-2ad)x^3}{3d^2}+\frac{b^2x^5}{5d}-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0693865, size = 83, normalized size = 1.

$$-\frac{bx^3(bc-2ad)}{3d^2}+\frac{x(ad-bc)^2}{d^3}-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((-(b*c) + a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d)
- (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Maple [A] time = 0.003, size = 135, normalized size = 1.6

$$\frac{b^2x^5}{5\,d}+\frac{2\,x^3ab}{3\,d}-\frac{x^3b^2c}{3\,d^2}+\frac{a^2x}{d}-2\,\frac{abcx}{d^2}+\frac{b^2c^2x}{d^3}-\frac{a^2c}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\,\frac{abc^2}{d^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^2c^3}{d^3}\arctan\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2/3/d*x^3*a*b-1/3/d^2*x^3*b^2*c+1/d*a^2*x-2/d^2*c*a*b*x+1/d^3
*b^2*c^2*x-c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+2*c^2/d^2/(c*d)^(1/2
)*arctan(x*d/(c*d)^(1/2))*a*b-c^3/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28082, size = 489, normalized size = 5.89

$$\left[\frac{6\,b^2d^2x^5-10\left(b^2cd-2\,abd^2\right)x^3+15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\left(b^2c^2-2\,abcd+a^2d^2\right)x}{30\,d^3},\ \frac{3\,b^2d\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 - 10*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) +
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3, 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d
 - 2*a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c/d)*arctan(d*x
*sqrt(c/d)/c) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3]
```

Sympy [B] time = 0.614831, size = 192, normalized size = 2.31

$$\frac{b^2x^5}{5d} + \frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(-\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{x^3\left(2abd-b^2c\right)}{3d^2} + \frac{x\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**5/(5*d) + sqrt(-c/d**7)*(a*d - b*c)**2*log(-d**3*sqrt(-c/d**7)*(a*d
 - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-c/d**7)*(a*d
- b*c)**2*log(d**3*sqrt(-c/d**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2) + x)/2 + x**3*(2*a*b*d - b**2*c)/(3*d**2) + x*(a**2*d**2 - 2*a*b*c
*d + b**2*c**2)/d**3
```

Giac [A] time = 1.15543, size = 153, normalized size = 1.84

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^2d^4x^5 - 5\,b^2cd^3x^3 + 10\,abd^4x^3 + 15\,b^2c^2d^2x - 30\,abcd^3x + 15\,a^2d^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
+ 1/15*(3*b^2*d^4*x^5 - 5*b^2*c*d^3*x^3 + 10*a*b*d^4*x^3 + 15*b^2*c^2*d^2*x
 - 30*a*b*c*d^3*x + 15*a^2*d^4*x)/d^5
```

3.171 $\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=61

$$-\frac{bx^2(bc-ad)}{2d^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}+\frac{\left(a+bx^2\right)^2}{4d}$$

```
[Out] -(b*(b*c - a*d)*x^2)/(2*d^2) + (a + b*x^2)^2/(4*d) + ((b*c - a*d)^2*Log[c +
 d*x^2])/(2*d^3)
```

Rubi [A] time = 0.0489539, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$-\frac{bx^2(bc-ad)}{2d^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}+\frac{\left(a+bx^2\right)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -(b*(b*c - a*d)*x^2)/(2*d^2) + (a + b*x^2)^2/(4*d) + ((b*c - a*d)^2*Log[c +
 d*x^2])/(2*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{b(bc-ad)x^2}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}\end{aligned}$$

Mathematica [A] time = 0.0227169, size = 49, normalized size = 0.8

$$\frac{bdx^2\left(4ad-2bc+bdx^2\right)+2(bc-ad)^2\log\left(c+dx^2\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (b*d*x^2*(-2*b*c + 4*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[c + d*x^2])/(4*d^
3)
```

Maple [A] time = 0.003, size = 85, normalized size = 1.4

$$\frac{b^2x^4}{4d}+\frac{abx^2}{d}-\frac{b^2cx^2}{2d^2}+\frac{\ln\left(dx^2+c\right)a^2}{2d}-\frac{\ln\left(dx^2+c\right)cab}{d^2}+\frac{\ln\left(dx^2+c\right)b^2c^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/4*b^2/d*x^4+b/d*a*x^2-1/2*b^2/d^2*x^2*c+1/2/d*ln(d*x^2+c)*a^2-1/d^2*ln(d*
x^2+c)*c*a*b+1/2/d^3*ln(d*x^2+c)*b^2*c^2
```

Maxima [A] time = 0.992624, size = 88, normalized size = 1.44

$$\frac{b^2dx^4-2\left(b^2c-2\,abd\right)x^2}{4\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*(b^2*c - 2*a*b*d)*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(d*x^2 + c)/d^3
```

Fricas [A] time = 1.2664, size = 140, normalized size = 2.3

$$\frac{b^2d^2x^4-2\left(b^2cd-2\,abd^2\right)x^2+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 - 2*(b^2*c*d - 2*a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(d*x^2 + c))/d^3
```

Sympy [A] time = 0.481465, size = 51, normalized size = 0.84

$$\frac{b^2x^4}{4d}+\frac{x^2\left(2abd-b^2c\right)}{2d^2}+\frac{\left(ad-bc\right)^2\log\left(c+dx^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**4/(4*d) + x**2*(2*a*b*d - b**2*c)/(2*d**2) + (a*d - b*c)**2*log(c +
 d*x**2)/(2*d**3)
```

Giac [A] time = 1.17182, size = 90, normalized size = 1.48

$$\frac{b^2dx^4 - 2\,b^2cx^2 + 4\,abdx^2}{4\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|dx^2 + c\right|\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*b^2*c*x^2 + 4*a*b*d*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(d*x^2 + c))/d^3
```

3.172 $\int \frac{(a+bx^2)^2}{c+dx^2}\, dx$

Optimal. Leaf size=63

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}+\frac{b^2x^3}{3d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi [A] time = 0.0392406, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{c+dx^2}\, dx &= \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^2)}\right) dx\\ &= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\int \frac{1}{c+dx^2}\, dx}{d^2}\\ &= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0475764, size = 59, normalized size = 0.94

$$\frac{bx(6ad-3bc+bdx^2)}{3d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Maple [A] time = 0., size = 95, normalized size = 1.5

$$\frac{b^2x^3}{3d}+2\frac{abx}{d}-\frac{b^2cx}{d^2}+a^2\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\frac{abc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{b^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*x*c+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a
^2-2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*c+1/d^2/(c*d)^(1/2)*arctan(x
*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22947, size = 390, normalized size = 6.19

$$\left[\frac{2\,b^2cd^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)-6\left(b^2c^2d-2\,abcd^2\right)x}{6\,cd^3},\frac{b^2cd^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right)\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

Sympy [B] time = 0.540718, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d}-\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{x\left(2abd-b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) - sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(-c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(
c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d
**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + x*(2*a*b*d - b**2*c)/d**2
```

Giac [A] time = 1.14751, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2d^2x^3 - 3\,b^2cdx + 6\,abd^2x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.173 $\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Rubi [A] time = 0.0463171, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{d}+\frac{a^2}{cx}-\frac{(bc-ad)^2}{cd(c+dx)}\right)dx,x,x^2\right)\\&=\frac{b^2x^2}{2d}+\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}\end{aligned}$$

Mathematica [A] time = 0.0215287, size = 50, normalized size = 0.98

$$\frac{2a^2d^2\log(x)-(bc-ad)^2\log\left(c+dx^2\right)+b^2cdx^2}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*c*d*x^2 + 2*a^2*d^2*Log[x] - (b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Maple [A] time = 0.005, size = 69, normalized size = 1.4

$$\frac{b^2x^2}{2d} - \frac{\ln\left(dx^2+c\right)a^2}{2c} + \frac{\ln\left(dx^2+c\right)ab}{d} - \frac{c\ln\left(dx^2+c\right)b^2}{2d^2} + \frac{a^2\ln(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c),x)
```

```
[Out] 1/2*b^2*x^2/d-1/2/c*ln(d*x^2+c)*a^2+1/d*ln(d*x^2+c)*a*b-1/2*c/d^2*ln(d*x^2+
c)*b^2+a^2*ln(x)/c
```

Maxima [A] time = 0.996459, size = 82, normalized size = 1.61

$$\frac{b^2x^2}{2d} + \frac{a^2\log\left(x^2\right)}{2c} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(d*x^2 + c)/(c*d^2)
```

Fricas [A] time = 1.33602, size = 128, normalized size = 2.51

$$\frac{b^2cdx^2 + 2\,a^2d^2\log(x) - \left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*d*x^2 + 2*a^2*d^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d
*x^2 + c))/(c*d^2)
```

Sympy [A] time = 1.29811, size = 41, normalized size = 0.8

$$\frac{a^2\log(x)}{c} + \frac{b^2x^2}{2d} - \frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

```
[Out] a**2*log(x)/c + b**2*x**2/(2*d) - (a*d - b*c)**2*log(c/d + x**2)/(2*c*d**2)
```

Giac [A] time = 1.15437, size = 84, normalized size = 1.65

$$\frac{b^2x^2}{2\,d} + \frac{a^2\log\left(x^2\right)}{2\,c} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|dx^2 + c\right|\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c*d^2)
```

3.174 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)} \, dx$

Optimal. Leaf size=55

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Rubi [A] time = 0.0488449, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^2(c+dx^2)} \, dx &= \int \left(\frac{b^2}{d}+\frac{a^2}{cx^2}-\frac{(bc-ad)^2}{cd(c+dx^2)}\right) dx \\
&= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\int \frac{1}{c+dx^2} \, dx}{cd} \\
&= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0448656, size = 55, normalized size = 1.

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Maple [A] time = 0.006, size = 85, normalized size = 1.6

$$\frac{b^2x}{d}-\frac{a^2d}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\frac{ab}{\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^2c}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c),x)
```

```
[Out] b^2*x/d-1/c*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+2/(c*d)^(1/2)*arctan(
x*d/(c*d)^(1/2))*a*b-c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-a^2/c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32094, size = 344, normalized size = 6.25

$$\left[\frac{2\,b^2c^2dx^2-2\,a^2cd^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}x\log\left(\frac{dx^2+2\,\sqrt{-cd}x-c}{dx^2+c}\right)}{2\,c^2d^2x},\frac{b^2c^2dx^2-a^2cd^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}x\,\text{ar}}{c^2d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b^2*c^2*d*x^2 - 2*a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
-c*d)*x*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^2*d^2*x), (b^2*c^
2*d*x^2 - a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x*arctan(sq
rt(c*d)*x/c))/(c^2*d^2*x)]
```

Sympy [B] time = 0.669569, size = 165, normalized size = 3.

$$-\frac{a^2}{cx}+\frac{b^2x}{d}+\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(-\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

```
[Out] -a**2/(c*x) + b**2*x/d + sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(-c**2*d*sq
rt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/
2 - sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(c**2*d*sqrt(-1/(c**3*d**3))*(a*
d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

Giac [A] time = 1.1792, size = 85, normalized size = 1.55

$$\frac{b^2x}{d} - \frac{a^2}{cx} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c*d)
```

3.175 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

```
[Out] -a^2/(2*c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Log[c + d*x^
2])/(2*c^2*d)
```

Rubi [A] time = 0.0576856, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Log[c + d*x^
2])/(2*c^2*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^2}-\frac{a(-2bc+ad)}{c^2x}+\frac{(bc-ad)^2}{c^2(c+dx)}\right)dx, x, x^2\right)\\ &= -\frac{a^2}{2cx^2}+\frac{a(2bc-ad)\log(x)}{c^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}\end{aligned}$$

Mathematica [A] time = 0.027774, size = 60, normalized size = 1.03

$$\frac{a^2(-c)d-2adx^2\log(x)(ad-2bc)+x^2(bc-ad)^2\log\left(c+dx^2\right)}{2c^2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] (-(a^2*c*d) - 2*a*d*(-2*b*c + a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[c + d
*x^2])/(2*c^2*d*x^2)
```

Maple [A] time = 0.005, size = 81, normalized size = 1.4

$$\frac{d\ln\left(dx^2+c\right)a^2}{2\,c^2}-\frac{\ln\left(dx^2+c\right)ab}{c}+\frac{\ln\left(dx^2+c\right)b^2}{2\,d}-\frac{a^2}{2\,cx^2}-\frac{\ln\left(x\right)a^2d}{c^2}+2\,\frac{a\ln\left(x\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c),x)
```

```
[Out] 1/2/c^2*d*ln(d*x^2+c)*a^2-1/c*ln(d*x^2+c)*a*b+1/2/d*ln(d*x^2+c)*b^2-1/2*a^2
/c/x^2-a^2/c^2*ln(x)*d+2*a/c*ln(x)*b
```

Maxima [A] time = 0.985696, size = 95, normalized size = 1.64

$$\frac{\left(2\,abc-a^2d\right)\log\left(x^2\right)}{2\,c^2}-\frac{a^2}{2\,cx^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 - 1/2*a^2/(c*x^2) + 1/2*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*log(d*x^2 + c)/(c^2*d)
```

Fricas [A] time = 1.33753, size = 159, normalized size = 2.74

$$-\frac{a^2cd-\left(b^2c^2-2\,abcd+a^2d^2\right)x^2\log\left(dx^2+c\right)-2\left(2\,abcd-a^2d^2\right)x^2\log\left(x\right)}{2\,c^2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*d - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(d*x^2 + c) - 2*(2*a
*b*c*d - a^2*d^2)*x^2*log(x))/(c^2*d*x^2)
```

Sympy [A] time = 1.42595, size = 49, normalized size = 0.84

$$-\frac{a^2}{2cx^2}-\frac{a\left(ad-2bc\right)\log\left(x\right)}{c^2}+\frac{\left(ad-bc\right)^2\log\left(\frac{c}{d}+x^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

```
[Out] -a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x)/c**2 + (a*d - b*c)**2*log(c/d + x
**2)/(2*c**2*d)
```

Giac [A] time = 1.14002, size = 123, normalized size = 2.12

$$\frac{\left(2\,abc - a^2d\right)\log\left(x^2\right)}{2\,c^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|dx^2 + c\right|\right)}{2\,c^2d} - \frac{2\,abcx^2 - a^2dx^2 + a^2c}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c^2*d) - 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/(c^2*x^2)
```

3.176 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)} dx$

Optimal. Leaf size=66

$$-\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

```
[Out] -a^2/(3*c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.0547288, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -a^2/(3*c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^4(c+dx^2)} dx &= \int \left(\frac{a^2}{cx^4}-\frac{a(-2bc+ad)}{c^2x^2}+\frac{(bc-ad)^2}{c^2(c+dx^2)}\right) dx\\ &= -\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\int \frac{1}{c+dx^2} dx}{c^2}\\ &= -\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.0520988, size = 64, normalized size = 0.97

$$-\frac{a^2}{3cx^3}+\frac{a(ad-2bc)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -a^2/(3*c*x^3) + (a*(-2*b*c + a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Maple [A] time = 0.006, size = 98, normalized size = 1.5

$$\frac{a^2d^2}{c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\frac{abd}{c\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+b^2\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{3cx^3}+\frac{a^2d}{c^2x}-2\frac{ab}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c),x)
```

```
[Out] 1/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2-2/c/(c*d)^(1/2)*arctan(x*
d/(c*d)^(1/2))*a*b*d+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/3*a^2/c/x^
3+a^2/c^2/x*d-2*a/c/x*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31396, size = 408, normalized size = 6.18

$$\left[-\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-cd}x^3\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2a^2c^2d+6\left(2abc^2d-a^2cd^2\right)x^2}{6c^3dx^3}, \frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{cd}x^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*x^3*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*a^2*c^2*d + 6*(2*a*b*c^2*d - a^2*c*d^2)*x^2)
/(c^3*d*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x^3*arctan(s
qrt(c*d)*x/c) - a^2*c^2*d - 3*(2*a*b*c^2*d - a^2*c*d^2)*x^2)/(c^3*d*x^3)]
```

Sympy [B] time = 0.782587, size = 172, normalized size = 2.61

$$-\frac{\sqrt{-\frac{1}{c^5d}}(ad-bc)^2\log\left(-\frac{c^3\sqrt{-\frac{1}{c^5d}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{c^5d}}(ad-bc)^2\log\left(\frac{c^3\sqrt{-\frac{1}{c^5d}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{-a^2c+x^2\left(3a^2d-6abc\right)}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

```
[Out] -sqrt(-1/(c**5*d))*(a*d - b*c)**2*log(-c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(c**5*d))*(a*d - b*c
)**2*log(c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 + (-a**2*c + x**2*(3*a**2*d - 6*a*b*c))/(3*c**2*x**3)
```

Giac [A] time = 1.15619, size = 96, normalized size = 1.45

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^2} - \frac{6\,abcx^2 - 3\,a^2dx^2 + a^2c}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/3
*(6*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^2*x^3)
```

3.177 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)} dx$

Optimal. Leaf size=75

$$-\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}$$

```
[Out] -a^2/(4*c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*Log[x])/c^3
 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rubi [A] time = 0.0672466, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*Log[x])/c^3
 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{cx^3}-\frac{a(-2bc+ad)}{c^2x^2}+\frac{(bc-ad)^2}{c^3x}-\frac{d(bc-ad)^2}{c^3(c+dx)}\right)dx,x,x^2\right)\\&=-\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}+\frac{(bc-ad)^2\log(x)}{c^3}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.0437776, size = 72, normalized size = 0.96

$$-\frac{ac\left(ac-2adx^2+4bcx^2\right)-4x^4\log(x)(bc-ad)^2+2x^4(bc-ad)^2\log\left(c+dx^2\right)}{4c^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -(a*c*(a*c + 4*b*c*x^2 - 2*a*d*x^2) - 4*(b*c - a*d)^2*x^4*Log[x] + 2*(b*c -
 a*d)^2*x^4*Log[c + d*x^2])/(4*c^3*x^4)
```

Maple [A] time = 0.006, size = 116, normalized size = 1.6

$$-\frac{\ln\left(dx^2+c\right)a^2d^2}{2\,c^3}+\frac{\ln\left(dx^2+c\right)abd}{c^2}-\frac{\ln\left(dx^2+c\right)b^2}{2\,c}-\frac{a^2}{4\,cx^4}+\frac{\ln\left(x\right)a^2d^2}{c^3}-2\,\frac{a\ln\left(x\right)bd}{c^2}+\frac{\ln\left(x\right)b^2}{c}+\frac{a^2d}{2\,c^2x^2}-\frac{a}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c),x)
```

```
[Out] -1/2/c^3*ln(d*x^2+c)*a^2*d^2+1/c^2*ln(d*x^2+c)*a*b*d-1/2/c*ln(d*x^2+c)*b^2-
1/4*a^2/c/x^4+1/c^3*ln(x)*a^2*d^2-2/c^2*ln(x)*a*b*d+1/c*ln(x)*b^2+1/2*a^2/c
^2/x^2*d-a/c/x^2*b
```

Maxima [A] time = 1.00865, size = 130, normalized size = 1.73

$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,c^3}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x^2\right)}{2\,c^3}-\frac{a^2c+2\left(2\,abc-a^2d\right)x^2}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/c^3 + 1/2*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/4*(a^2*c + 2*(2*a*b*c - a^2*d)*x^2)/(c^
2*x^4)
```

Fricas [A] time = 1.26399, size = 213, normalized size = 2.84

$$-\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\log\left(dx^2+c\right)-4\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\log\left(x\right)+a^2c^2+2\left(2\,abc^2-a^2cd\right)x^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(d*x^2 + c) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x^4*log(x) + a^2*c^2 + 2*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^
3*x^4)
```

Sympy [A] time = 1.3518, size = 66, normalized size = 0.88

$$\frac{-a^2c+x^2\left(2a^2d-4abc\right)}{4c^2x^4}+\frac{(ad-bc)^2\log\left(x\right)}{c^3}-\frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

```
[Out] (-a**2*c + x**2*(2*a**2*d - 4*a*b*c))/(4*c**2*x**4) + (a*d - b*c)**2*log(x)
/c**3 - (a*d - b*c)**2*log(c/d + x**2)/(2*c**3)
```

Giac [B] time = 1.15413, size = 188, normalized size = 2.51

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(x^2\right)}{2\,c^3} - \frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(\left|dx^2 + c\right|\right)}{2\,c^3d} - \frac{3\,b^2c^2x^4 - 6\,abcdx^4 + 3\,a^2d^2x^4 + 4\,abc^2x^2 -}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/2*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*log(abs(d*x^2 + c))/(c^3*d) - 1/4*(3*b^2*c^2*x^4 - 6*a*b*c*
d*x^4 + 3*a^2*d^2*x^4 + 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(c^3*x^4)
```

3.178 $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=87

$$-\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}$$

```
[Out] -a^2/(5*c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) - (S
qrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Rubi [A] time = 0.0649296, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -a^2/(5*c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) - (S
qrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)}\,dx &= \int \left(\frac{a^2}{cx^6}-\frac{a(-2bc+ad)}{c^2x^4}+\frac{(bc-ad)^2}{c^3x^2}-\frac{d(bc-ad)^2}{c^3\left(c+dx^2\right)}\right)dx\\
&= -\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\left(d(bc-ad)^2\right)\int\frac{1}{c+dx^2}\,dx}{c^3}\\
&= -\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0695702, size = 86, normalized size = 0.99

$$-\frac{a^2}{5cx^5}+\frac{a(ad-2bc)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -a^2/(5*c*x^5) + (a*(-2*b*c + a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) - (
Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Maple [A] time = 0.006, size = 143, normalized size = 1.6

$$-\frac{a^2d^3}{c^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\frac{abd^2}{c^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^2d}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{5\,cx^5}-\frac{a^2d^2}{c^3x}+2\frac{abd}{c^2x}-\frac{b^2}{cx}+\frac{a^2}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c),x)
```

```
[Out] -d^3/c^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+2*d^2/c^2/(c*d)^(1/2)*arct
an(x*d/(c*d)^(1/2))*a*b-d/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/5*a^2
/c/x^5-1/c^3/x*a^2*d^2+2/c^2/x*a*b*d-1/c/x*b^2+1/3*a^2/c^2/x^3*d-2/3*a/c/x^
3*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2616, size = 506, normalized size = 5.82

$$\left[\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-30\left(b^2c^2-2\,abcd+a^2d^2\right)x^4-6\,a^2c^2-10\left(2\,abc^2-a^2cd\right)x^2}{30\,c^3x^5},-\frac{15(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) - 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 - 6*
a^2*c^2 - 10*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5), -1/15*(15*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5)]
```

Sympy [B] time = 0.950197, size = 207, normalized size = 2.38

$$\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(-\frac{c^4\sqrt{-\frac{d}{c^7}}(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}-\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(\frac{c^4\sqrt{-\frac{d}{c^7}}(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}-\frac{3a^2c^2+x^4\left(15a^2d^2-30ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

```
[Out] sqrt(-d/c**7)*(a*d - b*c)**2*log(-c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d
**3 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - sqrt(-d/c**7)*(a*d - b*c)**2*log
(c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2*d)
 + x)/2 - (3*a**2*c**2 + x**4*(15*a**2*d**2 - 30*a*b*c*d + 15*b**2*c**2) +
x**2*(-5*a**2*c*d + 10*a*b*c**2))/(15*c**3*x**5)
```

Giac [A] time = 1.18369, size = 151, normalized size = 1.74

$$-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^3}-\frac{15\,b^2c^2x^4-30\,abcdx^4+15\,a^2d^2x^4+10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
- 1/15*(15*b^2*c^2*x^4 - 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 + 10*a*b*c^2*x^2 -
 5*a^2*c*d*x^2 + 3*a^2*c^2)/(c^3*x^5)
```

3.179 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}$$

```
[Out] -a^2/(6*c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2*c^3*x^2)
- (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c + d*x^2])/(2*c^4)
```

Rubi [A] time = 0.0833537, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -a^2/(6*c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2*c^3*x^2)
- (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c + d*x^2])/(2*c^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^4}-\frac{a(-2bc+ad)}{c^2x^3}+\frac{(bc-ad)^2}{c^3x^2}-\frac{d(bc-ad)^2}{c^4x}+\frac{d^2(bc-ad)^2}{c^4(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{d(bc-ad)^2\log(x)}{c^4}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.0607518, size = 108, normalized size = 1.1

$$-\frac{c\left(a^2\left(2c^2-3cdx^2+6d^2x^4\right)+6abcx^2\left(c-2dx^2\right)+6b^2c^2x^4\right)+12dx^6\log(x)(bc-ad)^2-6dx^6(bc-ad)^2\log\left(c+dx^2\right)}{12c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -(c*(6*b^2*c^2*x^4 + 6*a*b*c*x^2*(c - 2*d*x^2) + a^2*(2*c^2 - 3*c*d*x^2 + 6
*d^2*x^4)) + 12*d*(b*c - a*d)^2*x^6*Log[x] - 6*d*(b*c - a*d)^2*x^6*Log[c +
d*x^2])/(12*c^4*x^6)
```

Maple [A] time = 0.006, size = 160, normalized size = 1.6

$$\frac{d^3\ln\left(dx^2+c\right)a^2}{2\,c^4}-\frac{d^2\ln\left(dx^2+c\right)ab}{c^3}+\frac{d\ln\left(dx^2+c\right)b^2}{2\,c^2}-\frac{a^2}{6\,cx^6}-\frac{a^2d^2}{2\,c^3x^2}+\frac{abd}{c^2x^2}-\frac{b^2}{2\,cx^2}+\frac{a^2d}{4\,c^2x^4}-\frac{ab}{2\,cx^4}-\frac{d^3\ln(x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c),x)
```

```
[Out] 1/2*d^3/c^4*ln(d*x^2+c)*a^2-d^2/c^3*ln(d*x^2+c)*a*b+1/2*d/c^2*ln(d*x^2+c)*b
^2-1/6*a^2/c/x^6-1/2/c^3/x^2*a^2*d^2+1/c^2/x^2*a*b*d-1/2/c/x^2*b^2+1/4*a^2/
c^2/x^4*d-1/2*a/c/x^4*b-1/c^4*d^3*ln(x)*a^2+2/c^3*d^2*ln(x)*a*b-1/c^2*d*ln(
x)*b^2
```

Maxima [A] time = 0.983237, size = 181, normalized size = 1.85

$$\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(dx^2+c\right)}{2\,c^4}-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(x^2\right)}{2\,c^4}-\frac{6\left(b^2c^2-2\,abcd+a^2d^2\right)x^4+2\,a^2c^2+3\,(}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/c^4 - 1/2*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 - 1/12*(6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*x^4 + 2*a^2*c^2 + 3*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^6)
```

Fricas [A] time = 1.28096, size = 290, normalized size = 2.96

$$\frac{6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^6\log\left(dx^2+c\right)-12\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^6\log\left(x\right)-2\,a^2c^3-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right.}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(d*x^2 + c) - 12*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(x) - 2*a^2*c^3 - 6*(b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*x^4 - 3*(2*a*b*c^3 - a^2*c^2*d)*x^2)/(c^4*x^6)
```

Sympy [A] time = 1.6226, size = 105, normalized size = 1.07

$$-\frac{2a^2c^2 + x^4\left(6a^2d^2 - 12abcd + 6b^2c^2\right) + x^2\left(-3a^2cd + 6abc^2\right)}{12c^3x^6} - \frac{d\left(ad - bc\right)^2\log\left(x\right)}{c^4} + \frac{d\left(ad - bc\right)^2\log\left(\frac{c}{d} + x^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

```
[Out] -(2*a**2*c**2 + x**4*(6*a**2*d**2 - 12*a*b*c*d + 6*b**2*c**2) + x**2*(-3*a*
*2*c*d + 6*a*b*c**2))/(12*c**3*x**6) - d*(a*d - b*c)**2*log(x)/c**4 + d*(a*
d - b*c)**2*log(c/d + x**2)/(2*c**4)
```

Giac [B] time = 1.15046, size = 248, normalized size = 2.53

$$-\frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(x^2\right)}{2\,c^4} + \frac{\left(b^2c^2d^2 - 2\,abcd^3 + a^2d^4\right)\log\left(\left|dx^2 + c\right|\right)}{2\,c^4d} + \frac{11\,b^2c^2dx^6 - 22\,abcd^2x^6 + 11\,a^2d^3x^6 - 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 + 1/2*(b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 + c))/(c^4*d) + 1/12*(11*b^2*c^2*d*x^6
 - 22*a*b*c*d^2*x^6 + 11*a^2*d^3*x^6 - 6*b^2*c^3*x^4 + 12*a*b*c^2*d*x^4 - 6
*a^2*c*d^2*x^4 - 6*a*b*c^3*x^2 + 3*a^2*c^2*d*x^2 - 2*a^2*c^3)/(c^4*x^6)
```

3.180 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=145

$$\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{\sqrt{c}(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{b^2x^5}{5d^2}$$

```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)
/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2))
- (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9
/2))
```

Rubi [A] time = 0.134656, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 302, 205}

$$\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{\sqrt{c}(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)
/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2))
- (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9
/2))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^4\left(-2a^2d^2+5(bc-ad)^2-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{((7bc-3ad)(bc-ad))\int \frac{x^4}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{((7bc-3ad)(bc-ad))\int \left(-\frac{c}{d^2}+\frac{x^2}{d}+\frac{c^2}{d^2\left(c+dx^2\right)}\right)dx}{2cd^2}$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{(c(7bc-3ad)(bc-ad))}{2d^4}$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{\sqrt{c}(7bc-3ad)(bc-ad}{2d^{9/2}}$$

Mathematica [A] time = 0.0915415, size = 138, normalized size = 0.95

$$\frac{x\left(a^2d^2-4abcd+3b^2c^2\right)}{d^4} - \frac{\sqrt{c}\left(3a^2d^2-10abcd+7b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}} - \frac{2bx^3(bc-ad)}{3d^3} + \frac{cx(bc-ad)^2}{2d^4\left(c+dx^2\right)} + \frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x)/d^4 - (2*b*(b*c - a*d)*x^3)/(3*d^3) +
 (b^2*x^5)/(5*d^2) + (c*(b*c - a*d)^2*x)/(2*d^4*(c + d*x^2)) - (Sqrt[c]*(7*
b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

Maple [A] time = 0.01, size = 196, normalized size = 1.4

$$\frac{b^2x^5}{5\,d^2} + \frac{2\,x^3ab}{3\,d^2} - \frac{2\,x^3b^2c}{3\,d^3} + \frac{a^2x}{d^2} - 4\,\frac{abcx}{d^3} + 3\,\frac{b^2c^2x}{d^4} + \frac{a^2cx}{2\,d^2\left(dx^2+c\right)} - \frac{abc^2x}{d^3\left(dx^2+c\right)} + \frac{b^2c^3x}{2\,d^4\left(dx^2+c\right)} - \frac{3\,a^2c}{2\,d^2}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/5*b^2*x^5/d^2+2/3/d^2*x^3*a*b-2/3/d^3*x^3*b^2*c+1/d^2*a^2*x-4/d^3*c*a*b*x
+3/d^4*b^2*c^2*x+1/2*c/d^2*x/(d*x^2+c)*a^2-c^2/d^3*x/(d*x^2+c)*a*b+1/2*c^3/
d^4*x/(d*x^2+c)*b^2-3/2*c/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+5*c^2
/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-7/2*c^3/d^4/(c*d)^(1/2)*arctan
(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30716, size = 851, normalized size = 5.87

$$\left[\frac{12\,b^2d^3x^7 - 4\left(7\,b^2cd^2 - 10\,abd^3\right)x^5 + 20\left(7\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)x^3 + 15\left(7\,b^2c^3 - 10\,abc^2d + 3\,a^2cd^2 + \left(7\,b^2c^2d\right.\right.}{60\left(d^5x^2 + cd^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^2*d^3*x^7 - 4*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 20*(7*b^2*c^2*d
- 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^
2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 30*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4), 1/30*(6*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 10*a*b*d^
3)*x^5 + 10*(7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 15*(7*b^2*c^3 -
10*a*b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*
sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4)]
```

Sympy [B] time = 1.16603, size = 280, normalized size = 1.93

$$\frac{b^2x^5}{5d^2} + \frac{x\left(a^2cd^2 - 2abc^2d + b^2c^3\right)}{2cd^4 + 2d^5x^2} + \frac{\sqrt{-\frac{c}{d^9}}\,(ad - bc)\,(3ad - 7bc)\log\left(-\frac{d^4\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{c}{d^9}}\,(ad - bc)\,(3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**5/(5*d**2) + x*(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3)/(2*c*d**4 +
 2*d**5*x**2) + sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)*log(-d**4*sqrt(-c
/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b*c*d + 7*b**2*c**2)
 + x)/4 - sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)*log(d**4*sqrt(-c/d**9)*
(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b*c*d + 7*b**2*c**2) + x)/4
 + x**3*(2*a*b*d - 2*b**2*c)/(3*d**3) + x*(a**2*d**2 - 4*a*b*c*d + 3*b**2*c
**2)/d**4
```

Giac [A] time = 1.16611, size = 211, normalized size = 1.46

$$-\frac{\left(7 b^{2} c^{3}-10 a b c^{2} d+3 a^{2} c d^{2}\right) \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d} d^{4}}+\frac{b^{2} c^{3} x-2 a b c^{2} d x+a^{2} c d^{2} x}{2\left(d x^{2}+c\right) d^{4}}+\frac{3 b^{2} d^{8} x^{5}-10 b^{2} c d^{7} x^{3}+10 a b d^{8} x^{3}+45 b^{2} c^{2}}{15 d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c
*d)*d^4) + 1/2*(b^2*c^3*x - 2*a*b*c^2*d*x + a^2*c*d^2*x)/((d*x^2 + c)*d^4)
+ 1/15*(3*b^2*d^8*x^5 - 10*b^2*c*d^7*x^3 + 10*a*b*d^8*x^3 + 45*b^2*c^2*d^6*
x - 60*a*b*c*d^7*x + 15*a^2*d^8*x)/d^10
```

3.181 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{bx^2(bc-ad)}{d^3}+\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^4}{4d^2}$$

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2)/(2*d^4*(
c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2])/(2*d^4)
```

Rubi [A] time = 0.0975002, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{bx^2(bc-ad)}{d^3}+\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^4}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2)/(2*d^4*(
c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2])/(2*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{2b(bc-ad)}{d^3}+\frac{b^2x}{d^2}-\frac{c(bc-ad)^2}{d^3(c+dx)^2}+\frac{(bc-ad)(3bc-ad)}{d^3(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{b(bc-ad)x^2}{d^3}+\frac{b^2x^4}{4d^2}+\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}
\end{aligned}$$

Mathematica [A] time = 0.0633547, size = 87, normalized size = 0.97

$$\frac{2\left(a^2d^2-4abcd+3b^2c^2\right)\log\left(c+dx^2\right)+4bdx^2(ad-bc)+\frac{2c(bc-ad)^2}{c+dx^2}+b^2d^2x^4}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (4*b*d*(-(b*c) + a*d)*x^2 + b^2*d^2*x^4 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(4*d^4)
```

Maple [A] time = 0.01, size = 142, normalized size = 1.6

$$\frac{b^2x^4}{4\,d^2}+\frac{abx^2}{d^2}-\frac{b^2cx^2}{d^3}+\frac{\ln\left(dx^2+c\right)a^2}{2\,d^2}-2\,\frac{\ln\left(dx^2+c\right)abc}{d^3}+\frac{3\,\ln\left(dx^2+c\right)b^2c^2}{2\,d^4}+\frac{a^2c}{2\,d^2\left(dx^2+c\right)}-\frac{abc^2}{d^3\left(dx^2+c\right)}+\frac{}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/4*b^2*x^4/d^2+b/d^2*a*x^2-b^2/d^3*x^2*c+1/2/d^2*ln(d*x^2+c)*a^2-2/d^3*ln(
d*x^2+c)*a*b*c+3/2/d^4*ln(d*x^2+c)*b^2*c^2+1/2/d^2*c/(d*x^2+c)*a^2-1/d^3*c^
2/(d*x^2+c)*a*b+1/2/d^4*c^3/(d*x^2+c)*b^2
```

Maxima [A] time = 0.993973, size = 144, normalized size = 1.6

$$\frac{b^2c^3-2\,abc^2d+a^2cd^2}{2\left(d^5x^2+cd^4\right)}+\frac{b^2dx^4-4\left(b^2c-abd\right)x^2}{4\,d^3}+\frac{\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d^5*x^2 + c*d^4) + 1/4*(b^2*d*x^4
- 4*(b^2*c - a*b*d)*x^2)/d^3 + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(d*
x^2 + c)/d^4
```

Fricas [A] time = 1.37787, size = 327, normalized size = 3.63

$$\frac{b^2d^3x^6+2\,b^2c^3-4\,abc^2d+2\,a^2cd^2-\left(3\,b^2cd^2-4\,abd^3\right)x^4-4\left(b^2c^2d-abcd^2\right)x^2+2\left(3\,b^2c^3-4\,abc^2d+a^2cd^2+\left(3\,b^2c^2d\right.\right.}{4\left(d^5x^2+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^3*x^6 + 2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 - (3*b^2*c*d^2 - 4
*a*b*d^3)*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*x^2 + 2*(3*b^2*c^3 - 4*a*b*c^2*d
+ a^2*c*d^2 + (3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2)*log(d*x^2 + c))/(d
```

```
^5*x^2 + c*d^4)
```

Sympy [A] time = 1.10964, size = 97, normalized size = 1.08

$$\frac{b^2x^4}{4d^2} + \frac{a^2cd^2 - 2abc^2d + b^2c^3}{2cd^4 + 2d^5x^2} + \frac{x^2\left(abd - b^2c\right)}{d^3} + \frac{(ad - 3bc)(ad - bc)\log\left(c + dx^2\right)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**4/(4*d**2) + (a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3)/(2*c*d**4 + 2
*d**5*x**2) + x**2*(a*b*d - b**2*c)/d**3 + (a*d - 3*b*c)*(a*d - b*c)*log(c
+ d*x**2)/(2*d**4)
```

Giac [A] time = 1.15326, size = 220, normalized size = 2.44

$$\frac{\frac{\left(dx^2+c\right)^2\left(b^2-\frac{2\left(3b^2cd-2abd^2\right)}{\left(dx^2+c\right)d}\right)}{d^3} - \frac{2\left(3b^2c^2-4abcd+a^2d^2\right)\log\left(\frac{\left|dx^2+c\right|}{\left(dx^2+c\right)^2\left|d\right|}\right)}{d^3} + \frac{2\left(\frac{b^2c^3d^2}{dx^2+c}-\frac{2abc^2d^3}{dx^2+c}+\frac{a^2cd^4}{dx^2+c}\right)}{d^5}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((d*x^2 + c)^2*(b^2 - 2*(3*b^2*c*d - 2*a*b*d^2)/((d*x^2 + c)*d))/d^3 -
2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(abs(d*x^2 + c)/((d*x^2 + c)^2*abs(d
)))/d^3 + 2*(b^2*c^3*d^2/(d*x^2 + c) - 2*a*b*c^2*d^3/(d*x^2 + c) + a^2*c*d^
4/(d*x^2 + c))/d^5)/d
```

3.182 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=118

$$\frac{x^3(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}+\frac{b^2x^3}{3d^2}$$

```
[Out] -((b*c - a*d)*(5*b*c - a*d)*x)/(2*c*d^3) + (b^2*x^3)/(3*d^2) + ((b*c - a*d)
^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rubi [A] time = 0.110162, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 321, 205}

$$\frac{x^3(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*(5*b*c - a*d)*x)/(2*c*d^3) + (b^2*x^3)/(3*d^2) + ((b*c - a*d)
^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx &= \frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)} - \frac{\int \frac{x^2(3b^2c^2-6abcd+a^2d^2-2b^2cdx^2)}{c+dx^2}\,dx}{2cd^2} \\
&= \frac{b^2x^3}{3d^2} + \frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc-ad))\int \frac{x^2}{c+dx^2}\,dx}{2cd^2} \\
&= -\frac{(bc-ad)(5bc-ad)x}{2cd^3} + \frac{b^2x^3}{3d^2} + \frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)} + \frac{((bc-ad)(5bc-ad))\int \frac{1}{c+dx^2}\,dx}{2d^3} \\
&= -\frac{(bc-ad)(5bc-ad)x}{2cd^3} + \frac{b^2x^3}{3d^2} + \frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0713394, size = 105, normalized size = 0.89

$$\frac{(a^2d^2-6abcd+5b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}} - \frac{x(bc-ad)^2}{2d^3(c+dx^2)} - \frac{2bx(bc-ad)}{d^3} + \frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-2*b*(b*c - a*d)*x)/d^3 + (b^2*x^3)/(3*d^2) - ((b*c - a*d)^2*x)/(2*d^3*(c
+ d*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(2*Sqrt[c]*d^(7/2))
```

Maple [A] time = 0.008, size = 156, normalized size = 1.3

$$\frac{b^2x^3}{3\,d^2} + 2\,\frac{abx}{d^2} - 2\,\frac{b^2cx}{d^3} - \frac{xa^2}{2\,d\,(dx^2+c)} + \frac{abcx}{d^2\,(dx^2+c)} - \frac{b^2c^2x}{2\,d^3\,(dx^2+c)} + \frac{a^2}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - 3\,\frac{abc}{d^2\sqrt{cd}}\,\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^2*x^3/d^2+2*b/d^2*a*x-2*b^2/d^3*x*c-1/2/d*x/(d*x^2+c)*a^2+1/d^2*x/(d*
x^2+c)*c*a*b-1/2/d^3*x/(d*x^2+c)*b^2*c^2+1/2/d/(c*d)^(1/2)*arctan(x*d/(c*d)
^(1/2))*a^2-3/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b+5/2/d^3/(c*d)^(
1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50136, size = 713, normalized size = 6.04

$$\left[\frac{4\,b^2cd^3x^5 - 4\left(5\,b^2c^2d^2 - 6\,abcd^3\right)x^3 - 3\left(5\,b^2c^3 - 6\,abc^2d + a^2cd^2 + \left(5\,b^2c^2d - 6\,abcd^2 + a^2d^3\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2 - 2\,\sqrt{-cd}x - }{dx^2 + c}\right.}{12\left(cd^5x^2 + c^2d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*c*d^3*x^5 - 4*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 - 3*(5*b^2*c^3
 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^2*c^3*d - 6*
a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4), 1/6*(2*b^2*c*d^3*x^5 - 2
*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2
 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) - 3*(5*b^2*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4)]
```

Sympy [B] time = 1.00835, size = 245, normalized size = 2.08

$$\frac{b^2x^3}{3d^2} - \frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{2cd^3 + 2d^4x^2} - \frac{\sqrt{-\frac{1}{cd^7}}\,(ad - 5bc)\,(ad - bc)\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{cd^7}}\,(ad - 5bc)\,(ad - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**3/(3*d**2) - x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c*d**3 + 2*d*
*4*x**2) - sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)*log(-c*d**3*sqrt(-1/
(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) +
 x)/4 + sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)*log(c*d**3*sqrt(-1/(c*d
**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/
4 + x*(2*a*b*d - 2*b**2*c)/d**3
```

Giac [A] time = 1.17881, size = 154, normalized size = 1.31

$$\frac{\left(5\,b^2c^2 - 6\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3} - \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(dx^2 + c\right)d^3} + \frac{b^2d^4x^3 - 6\,b^2cd^3x + 6\,abd^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*d^3) + 1/3*(b^2*d
```

```
^4*x^3 - 6*b^2*c*d^3*x + 6*a*b*d^4*x)/d^6
```

3.183 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=62

$$-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}+\frac{b^2x^2}{2d^2}$$

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c - a*d)*Log[
c + d*x^2])/d^3
```

Rubi [A] time = 0.058084, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}+\frac{b^2x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c - a*d)*Log[
c + d*x^2])/d^3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx, x, x^2\right)\\
&= \frac{b^2x^2}{2d^2}-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.0447732, size = 56, normalized size = 0.9

$$\frac{-\frac{(bc-ad)^2}{c+dx^2}+2b(ad-bc)\log\left(c+dx^2\right)+b^2dx^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*d*x^2 - (b*c - a*d)^2/(c + d*x^2) + 2*b*(-(b*c) + a*d)*Log[c + d*x^2])
/(2*d^3)
```

Maple [A] time = 0.008, size = 97, normalized size = 1.6

$$\frac{b^2x^2}{2\,d^2}+\frac{b\ln\left(dx^2+c\right)a}{d^2}-\frac{b^2\ln\left(dx^2+c\right)c}{d^3}-\frac{a^2}{2\,d\left(dx^2+c\right)}+\frac{abc}{d^2\left(dx^2+c\right)}-\frac{b^2c^2}{2\,d^3\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2*x^2/d^2+1/d^2*b*ln(d*x^2+c)*a-1/d^3*b^2*ln(d*x^2+c)*c-1/2/d/(d*x^2+
c)*a^2+1/d^2/(d*x^2+c)*a*b*c-1/2/d^3/(d*x^2+c)*b^2*c^2
```

Maxima [A] time = 0.987961, size = 100, normalized size = 1.61

$$\frac{b^2x^2}{2\,d^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(d^4x^2+cd^3\right)}-\frac{\left(b^2c-abd\right)\log\left(dx^2+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x^2 + c*d^3) - (
b^2*c - a*b*d)*log(d*x^2 + c)/d^3
```

Fricas [A] time = 1.41026, size = 200, normalized size = 3.23

$$\frac{b^2d^2x^4+b^2cdx^2-b^2c^2+2\,abcd-a^2d^2-2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)\log\left(dx^2+c\right)}{2\left(d^4x^2+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + b^2*c*d*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2
 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)*log(d*x^2 + c))/(d^4*x^2 + c*d^3)
```

Sympy [A] time = 0.911734, size = 68, normalized size = 1.1

$$\frac{b^2x^2}{2d^2}+\frac{b\left(ad-bc\right)\log\left(c+dx^2\right)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{2cd^3+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**2/(2*d**2) + b*(a*d - b*c)*log(c + d*x**2)/d**3 - (a**2*d**2 - 2*a*
b*c*d + b**2*c**2)/(2*c*d**3 + 2*d**4*x**2)
```

Giac [A] time = 1.16857, size = 149, normalized size = 2.4

$$\frac{\left(dx^2+c\right)b^2}{2\,d^3}+\frac{\left(b^2c-abd\right)\log\left(\frac{|dx^2+c|}{\left(dx^2+c\right)^2|d|}\right)}{d^3}-\frac{\frac{b^2c^2d}{dx^2+c}-\frac{2\,abcd^2}{dx^2+c}+\frac{a^2d^3}{dx^2+c}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*b^2/d^3 + (b^2*c - a*b*d)*log(abs(d*x^2 + c)/((d*x^2 + c)^2
*abs(d)))/d^3 - 1/2*(b^2*c^2*d/(d*x^2 + c) - 2*a*b*c*d^2/(d*x^2 + c) + a^2*
d^3/(d*x^2 + c))/d^4
```

3.184 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rubi [A] time = 0.107216, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2\left(c+dx^2\right)^2}\right) dx$$

$$= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.0583691, size = 89, normalized size = 1.09

$$-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}} + \frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Maple [A] time = 0., size = 129, normalized size = 1.6

$$\frac{b^2x}{d^2} + \frac{xa^2}{2\,c\left(dx^2+c\right)} - \frac{abx}{d\left(dx^2+c\right)} + \frac{b^2cx}{2\,d^2\left(dx^2+c\right)} + \frac{a^2}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{ab}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{3\,b^2c}{2\,d^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(d*x^2+c)*b
^2+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1/d/(c*d)^(1/2)*arctan(x*d
/(c*d)^(1/2))*a*b-3/2/d^2*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5174, size = 612, normalized size = 7.46

$$\left[\frac{4\,b^2c^2d^2x^3+\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)+2\left(3\,b^2c^3d-2\,abc^2d\right.}{4\left(c^2d^4x^2+c^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

Sympy [B] time = 0.850305, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2c^2d^2+2cd^3x^2}-\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

Giac [A] time = 1.18076, size = 128, normalized size = 1.56

$$\frac{b^2x}{d^2}-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(dx^2+c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

3.185 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^2 - b^2/d^
2)*Log[c + d*x^2])/2
```

Rubi [A] time = 0.0635326, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^2 - b^2/d^
2)*Log[c + d*x^2])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{c^2x}-\frac{(bc-ad)^2}{cd(c+dx)^2}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{a^2\log(x)}{c^2}-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0438614, size = 70, normalized size = 1.04

$$\frac{2a^2\log(x)+\frac{(bc-ad)\left(\left(c+dx^2\right)(ad+bc)\log\left(c+dx^2\right)+c(bc-ad)\right)}{d^2\left(c+dx^2\right)}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (2*a^2*Log[x] + ((b*c - a*d)*(c*(b*c - a*d) + (b*c + a*d)*(c + d*x^2)*Log[c
 + d*x^2]))/(d^2*(c + d*x^2)))/(2*c^2)
```

Maple [A] time = 0.012, size = 94, normalized size = 1.4

$$-\frac{\ln\left(dx^2+c\right)a^2}{2\,c^2}+\frac{\ln\left(dx^2+c\right)b^2}{2\,d^2}+\frac{a^2}{2\,c\left(dx^2+c\right)}-\frac{ab}{d\left(dx^2+c\right)}+\frac{b^2c}{2\,d^2\left(dx^2+c\right)}+\frac{a^2\ln\left(x\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*ln(d*x^2+c)*a^2+1/2/d^2*ln(d*x^2+c)*b^2+1/2/c/(d*x^2+c)*a^2-1/d/(d
*x^2+c)*a*b+1/2*c/d^2/(d*x^2+c)*b^2+a^2*ln(x)/c^2
```

Maxima [A] time = 1.01021, size = 116, normalized size = 1.73

$$\frac{a^2\log\left(x^2\right)}{2\,c^2}+\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(cd^3x^2+c^2d^2\right)}+\frac{\left(b^2c^2-a^2d^2\right)\log\left(dx^2+c\right)}{2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c*d^3*x^2 + c^2
*d^2) + 1/2*(b^2*c^2 - a^2*d^2)*log(d*x^2 + c)/(c^2*d^2)
```

Fricas [A] time = 1.4596, size = 228, normalized size = 3.4

$$\frac{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^3-a^2cd^2+\left(b^2c^2d-a^2d^3\right)x^2\right)\log\left(dx^2+c\right)+2\left(a^2d^3x^2+a^2cd^2\right)\log\left(x\right)}{2\left(c^2d^3x^2+c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^3 - a^2*c*d^2 + (b^2*c^2*d
- a^2*d^3)*x^2)*log(d*x^2 + c) + 2*(a^2*d^3*x^2 + a^2*c*d^2)*log(x))/(c^2*d
^3*x^2 + c^3*d^2)
```

Sympy [A] time = 1.33387, size = 80, normalized size = 1.19

$$\frac{a^2\log\left(x\right)}{c^2}+\frac{a^2d^2-2abcd+b^2c^2}{2c^2d^2+2cd^3x^2}-\frac{\left(ad-bc\right)\left(ad+bc\right)\log\left(\frac{c}{d}+x^2\right)}{2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

```
[Out] a**2*log(x)/c**2 + (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d
**3*x**2) - (a*d - b*c)*(a*d + b*c)*log(c/d + x**2)/(2*c**2*d**2)
```

Giac [A] time = 1.12126, size = 134, normalized size = 2.

$$\frac{a^2\log\left(x^2\right)}{2\,c^2}+\frac{\left(b^2c^2-a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,c^2d^2}-\frac{b^2c^2x^2-a^2d^2x^2+2\,abc^2-2\,a^2cd}{2\left(dx^2+c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - a^2*d^2)*log(abs(d*x^2 + c))/(c^2*d^2
) - 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((d*x^2 + c)*c^
2*d)
```

3.186 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2} dx$

Optimal. Leaf size=106

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

```
[Out] -(a^2/(c*x*(c + d*x^2))) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x)/(2*c^2*d*(
c + d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^
(5/2)*d^(3/2))
```

Rubi [A] time = 0.0756784, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {462, 385, 205}

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2))) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x)/(2*c^2*d*(
c + d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^
(5/2)*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^2}\,dx = -\frac{a^2}{cx\left(c+dx^2\right)} + \frac{\int \frac{a(2bc-3ad)+b^2cx^2}{\left(c+dx^2\right)^2}\,dx}{c}$$

$$= -\frac{a^2}{cx\left(c+dx^2\right)} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)x}{2c^2d\left(c+dx^2\right)} + \frac{((bc-ad)(bc+3ad))\int \frac{1}{c+dx^2}\,dx}{2c^2d}$$

$$= -\frac{a^2}{cx\left(c+dx^2\right)} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)x}{2c^2d\left(c+dx^2\right)} + \frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Mathematica [A] time = 0.0601066, size = 91, normalized size = 0.86

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}} - \frac{a^2}{c^2x} - \frac{x(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c^2*x)) - ((b*c - a*d)^2*x)/(2*c^2*d*(c + d*x^2)) + ((b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

Maple [A] time = 0.009, size = 131, normalized size = 1.2

$$-\frac{a^2dx}{2c^2\left(dx^2+c\right)} + \frac{abx}{c\left(dx^2+c\right)} - \frac{xb^2}{2d\left(dx^2+c\right)} - \frac{3a^2d}{2c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{ab}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{b^2}{2d}\arctan\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x/(d*x^2+c)*a^2+1/c*x/(d*x^2+c)*a*b-1/2/d*x/(d*x^2+c)*b^2-3/2/c^
2*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1/c/(c*d)^(1/2)*arctan(x*d/(c*d
)^(1/2))*a*b+1/2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-a^2/c^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55426, size = 625, normalized size = 5.9

$$\left[-\frac{4\,a^2c^2d^2+2\left(b^2c^3d-2\,abc^2d^2+3\,a^2cd^3\right)x^2-\left(\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)x^3+\left(b^2c^3+2\,abc^2d-3\,a^2cd^2\right)x\right)\sqrt{-cd}\log}{4\left(c^3d^3x^3+c^4d^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*c^2*d^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2 - ((
b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c
*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^3*d^3
*x^3 + c^4*d^2*x), -1/2*(2*a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2
*c*d^3)*x^2 - ((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b
*c^2*d - 3*a^2*c*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^3*d^3*x^3 + c^
4*d^2*x)]
```

Sympy [B] time = 1.01355, size = 238, normalized size = 2.25

$$\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+b}{3a^2d^2-2abcd-b^2c^2}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d*
*3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4
 - sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*
d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)
/4 - (2*a**2*c*d + x**2*(3*a**2*d**2 - 2*a*b*c*d + b**2*c**2))/(2*c**3*d*x
+ 2*c**2*d**2*x**3)
```

Giac [A] time = 1.1602, size = 138, normalized size = 1.3

$$\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^2d}-\frac{b^2c^2x^2-2\,abcdx^2+3\,a^2d^2x^2+2\,a^2cd}{2\left(dx^3+cx\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*
d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 3*a^2*d^2*x^2 + 2*a^2*c*d)/((d*x^3
+ c*x)*c^2*d)
```

3.187 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2} dx$

Optimal. Leaf size=81

$$-\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}$$

```
[Out] -a^2/(2*c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*c - a*d)*L
og[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rubi [A] time = 0.0789378, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -a^2/(2*c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*c - a*d)*L
og[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{c^2x^2}-\frac{2a(-bc+ad)}{c^3x}+\frac{(bc-ad)^2}{c^2(c+dx)^2}+\frac{2ad(-bc+ad)}{c^3(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}+\frac{2a(bc-ad)\log(x)}{c^3}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}\end{aligned}$$

Mathematica [A] time = 0.0927358, size = 72, normalized size = 0.89

$$-\frac{\frac{a^2c}{x^2}+\frac{c(bc-ad)^2}{d(c+dx^2)}-2a(ad-bc)\log\left(c+dx^2\right)+4a\log(x)(ad-bc)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -((a^2*c)/x^2 + (c*(b*c - a*d)^2)/(d*(c + d*x^2)) + 4*a*(-(b*c) + a*d)*Log[
x] - 2*a*(-(b*c) + a*d)*Log[c + d*x^2])/(2*c^3)
```

Maple [A] time = 0.014, size = 114, normalized size = 1.4

$$\frac{a^2\ln\left(dx^2+c\right)d}{c^3}-\frac{a\ln\left(dx^2+c\right)b}{c^2}-\frac{a^2d}{2c^2\left(dx^2+c\right)}+\frac{ab}{c\left(dx^2+c\right)}-\frac{b^2}{2d\left(dx^2+c\right)}-\frac{a^2}{2c^2x^2}-2\frac{\ln(x)a^2d}{c^3}+2\frac{a\ln(x)b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^2,x)
```

```
[Out] 1/c^3*a^2*ln(d*x^2+c)*d-1/c^2*a*ln(d*x^2+c)*b-1/2/c^2/(d*x^2+c)*a^2*d+1/c/(
d*x^2+c)*a*b-1/2/d/(d*x^2+c)*b^2-1/2*a^2/c^2/x^2-2*a^2/c^3*ln(x)*d+2*a/c^2*
ln(x)*b
```

Maxima [A] time = 0.981822, size = 135, normalized size = 1.67

$$-\frac{a^2cd+\left(b^2c^2-2\,abcd+2\,a^2d^2\right)x^2}{2\left(c^2d^2x^4+c^3dx^2\right)}-\frac{\left(abc-a^2d\right)\log\left(dx^2+c\right)}{c^3}+\frac{\left(abc-a^2d\right)\log\left(x^2\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)/(c^2*d^2*x^4 + c^3*d
*x^2) - (a*b*c - a^2*d)*log(d*x^2 + c)/c^3 + (a*b*c - a^2*d)*log(x^2)/c^3
```

Fricas [B] time = 1.46689, size = 316, normalized size = 3.9

$$-\frac{a^2c^2d+\left(b^2c^3-2\,abc^2d+2\,a^2cd^2\right)x^2+2\left(\left(abcd^2-a^2d^3\right)x^4+\left(abc^2d-a^2cd^2\right)x^2\right)\log\left(dx^2+c\right)-4\left(\left(abcd^2-a^2d^3\right)x^4\right.}{2\left(c^3d^2x^4+c^4dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c^2*d + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x^2 + 2*((a*b*c*d^2
 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(d*x^2 + c) - 4*((a*b*c*d
^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(x))/(c^3*d^2*x^4 + c^4
```

```
*d*x^2)
```

Sympy [A] time = 1.52865, size = 92, normalized size = 1.14

$$-\frac{2a\left(ad-bc\right)\log\left(x\right)}{c^3}+\frac{a\left(ad-bc\right)\log\left(\frac{c}{d}+x^2\right)}{c^3}-\frac{a^2cd+x^2\left(2a^2d^2-2abcd+b^2c^2\right)}{2c^3dx^2+2c^2d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

```
[Out] -2*a*(a*d - b*c)*log(x)/c**3 + a*(a*d - b*c)*log(c/d + x**2)/c**3 - (a**2*c
*d + x**2*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2))/(2*c**3*d*x**2 + 2*c**2*d*
*2*x**4)
```

Giac [A] time = 1.17575, size = 147, normalized size = 1.81

$$\frac{\left(abc-a^2d\right)\log\left(x^2\right)}{c^3}-\frac{\left(abcd-a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{c^3d}-\frac{b^2c^2x^2-2\,abcdx^2+2\,a^2d^2x^2+a^2cd}{2\left(dx^4+cx^2\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b*c - a^2*d)*log(x^2)/c^3 - (a*b*c*d - a^2*d^2)*log(abs(d*x^2 + c))/(c^3
*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 2*a^2*d^2*x^2 + a^2*c*d)/((d*x^4 +
 c*x^2)*c^2*d)
```

3.188 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}-\frac{a(6bc-5ad)}{3c^3x}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

```
[Out] -(a*(6*b*c - 5*a*d))/(3*c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*b^2*c^2 -
6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[d])
```

Rubi [A] time = 0.134679, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}-\frac{a(6bc-5ad)}{3c^3x}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -(a*(6*b*c - 5*a*d))/(3*c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*b^2*c^2 -
6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[d])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = -\frac{a^2}{3cx^3(c+dx^2)}+\frac{\int \frac{a(6bc-5ad)+3b^2cx^2}{x^2(c+dx^2)^2}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}-\frac{\int \frac{-\frac{2a(6bc-5ad)}{c}-\left(3b^2-\frac{6abd}{c}+\frac{5a^2d^2}{c^2}\right)x^2}{x^2(c+dx^2)}\,dx}{6c}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}+\frac{((bc-5ad)(bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c^3}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

Mathematica [A] time = 0.0638684, size = 107, normalized size = 0.85

$$\frac{(5a^2d^2-6abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a^2}{3c^2x^3}+\frac{x(bc-ad)^2}{2c^3(c+dx^2)}+\frac{2a(ad-bc)}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -a^2/(3*c^2*x^3) + (2*a*(-(b*c) + a*d))/(c^3*x) + ((b*c - a*d)^2*x)/(2*c^3*
(c + d*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(2*c^(7/2)*Sqrt[d])
```

Maple [A] time = 0.011, size = 161, normalized size = 1.3

$$\frac{a^2d^2x}{2c^3(dx^2+c)}-\frac{abdx}{c^2(dx^2+c)}+\frac{xb^2}{2c(dx^2+c)}+\frac{5a^2d^2}{2c^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-3\frac{abd}{c^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{b^2}{2c}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^2,x)
```

```
[Out] 1/2/c^3*x/(d*x^2+c)*a^2*d^2-1/c^2*x/(d*x^2+c)*a*b*d+1/2/c*x/(d*x^2+c)*b^2+5
/2/c^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2-3/c^2/(c*d)^(1/2)*arctan
(x*d/(c*d)^(1/2))*a*b*d+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/3*a
^2/c^2/x^3+2*a^2/c^3/x*d-2*a/c^2/x*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48407, size = 737, normalized size = 5.85

$$\left[-\frac{4\,a^2c^3d-6\left(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3\right)x^4+4\left(6\,abc^3d-5\,a^2c^2d^2\right)x^2+3\left(\left(b^2c^2d-6\,abcd^2+5\,a^2d^3\right)x^5+\left(b^2c^3-6\right.\right.}{12\left(c^4d^2x^5+c^5dx^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*c^3*d - 6*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 4*(
6*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2 + 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)
*x^5 + (b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^2*x^5 + c^5*d*x^3), -1/6*(2*a^2*c^3
*d - 3*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 2*(6*a*b*c^3*d - 5*a
^2*c^2*d^2)*x^2 - 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^5 + (b^2*c^3 -
 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^4*d^2*
x^5 + c^5*d*x^3)]
```

Sympy [B] time = 1.17905, size = 248, normalized size = 1.97

$$-\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(-c**4*sqrt(-1/(c**7*d))*(a
*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + sqrt
(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(c**4*sqrt(-1/(c**7*d))*(a*d - b
*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(15*a**2*d**2 - 18*a*b*c*d + 3*b**2*c**2) + x**2*(10*a**2*c*d -
12*a*b*c**2))/(6*c**4*x**3 + 6*c**3*d*x**5)
```

Giac [A] time = 1.12711, size = 150, normalized size = 1.19

$$\frac{\left(b^2c^2-6\,abcd+5\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(dx^2+c\right)c^3}-\frac{6\,abcx^2-6\,a^2dx^2+a^2c}{3\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c^3) - 1/3*(6*a*b
*c*x^2 - 6*a^2*d*x^2 + a^2*c)/(c^3*x^3)
```

3.189 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=163

$$-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}+\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{b}{3}$$

```
[Out] -((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(4*c*d^4) + (b^2*x^3)/(3*d^3) + ((
b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c - a*d)*x)/(
8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rubi [A] time = 0.157701, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 455, 1153, 205}

$$-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}+\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{b}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(4*c*d^4) + (b^2*x^3)/(3*d^3) + ((
b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c - a*d)*x)/(
8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^4\left(-4a^2d^2+5(bc-ad)^2-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\int \frac{cd(bc-ad)(9bc-ad)-2d^2(bc-ad)(9bc-ad)x^2+8b^2cd^3x^4}{c+dx^2}\,dx}{8cd^5}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\int \left(-2d\left(13b^2c^2-10abcd+a^2d^2\right)+8b^2cd^2x^2+\frac{35b^2c^3d-30}{c}\right.}{8cd^5}$$

$$= -\frac{\left(13b^2c^2-10abcd+a^2d^2\right)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\left(35b^2c^2-30abc\right.}{}$$

$$= -\frac{\left(13b^2c^2-10abcd+a^2d^2\right)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\left(35b^2c^2-30abc\right.}{8}$$

Mathematica [A] time = 0.0890152, size = 148, normalized size = 0.91

$$-\frac{x\left(5a^2d^2-18abcd+13b^2c^2\right)}{8d^4\left(c+dx^2\right)} + \frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}} + \frac{cx(bc-ad)^2}{4d^4\left(c+dx^2\right)^2} - \frac{bx(3bc-2ad)}{d^4} + \frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*(3*b*c - 2*a*d)*x)/d^4) + (b^2*x^3)/(3*d^3) + (c*(b*c - a*d)^2*x)/(4*d
^4*(c + d*x^2)^2) - ((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*x)/(8*d^4*(c + d
*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*d^(9/2))
```

Maple [A] time = 0.011, size = 223, normalized size = 1.4

$$\frac{b^2x^3}{3d^3} + 2\frac{abx}{d^3} - 3\frac{b^2cx}{d^4} - \frac{5x^3a^2}{8d\left(dx^2+c\right)^2} + \frac{9x^3abc}{4d^2\left(dx^2+c\right)^2} - \frac{13x^3b^2c^2}{8d^3\left(dx^2+c\right)^2} - \frac{3a^2cx}{8d^2\left(dx^2+c\right)^2} + \frac{7abc^2x}{4d^3\left(dx^2+c\right)^2} - \frac{11b}{8d^4(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/3*b^2*x^3/d^3+2*b/d^3*a*x-3*b^2/d^4*x*c-5/8/d/(d*x^2+c)^2*x^3*a^2+9/4/d^2
/(d*x^2+c)^2*x^3*a*b*c-13/8/d^3/(d*x^2+c)^2*x^3*b^2*c^2-3/8/d^2/(d*x^2+c)^2
```

```
*a^2*c*x+7/4/d^3/(d*x^2+c)^2*a*b*c^2*x-11/8/d^4/(d*x^2+c)^2*b^2*c^3*x+3/8/d
^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-15/4/d^3/(c*d)^(1/2)*arctan(x*d/
(c*d)^(1/2))*c*a*b+35/8/d^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50765, size = 1107, normalized size = 6.79

$$\left[\frac{16\,b^2cd^4x^7 - 16\left(7\,b^2c^2d^3 - 6\,abcd^4\right)x^5 - 10\left(35\,b^2c^3d^2 - 30\,abc^2d^3 + 3\,a^2cd^4\right)x^3 - 3\left(35\,b^2c^4 - 30\,abc^3d + 3\,a^2c^2d^2 + \right.}{48\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^2*c*d^4*x^7 - 16*(7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 10*(35*b^2
*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*x^3 - 3*(35*b^2*c^4 - 30*a*b*c^3*d
 + 3*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*
b^2*c^3*d - 30*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sq
rt(-c*d)*x - c)/(d*x^2 + c)) - 6*(35*b^2*c^4*d - 30*a*b*c^3*d^2 + 3*a^2*c^2
*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5), 1/24*(8*b^2*c*d^4*x^7 - 8*(
7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 5*(35*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a
^2*c*d^4)*x^3 + 3*(35*b^2*c^4 - 30*a*b*c^3*d + 3*a^2*c^2*d^2 + (35*b^2*c^2*
d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 30*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(35*b^2*c^4*d - 30*a*b*
c^3*d^2 + 3*a^2*c^2*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5)]
```

Sympy [A] time = 2.25218, size = 238, normalized size = 1.46

$$\frac{b^2x^3}{3d^3} - \frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2 - 30abcd + 35b^2c^2\right)\log\left(-cd^4\sqrt{-\frac{1}{cd^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2 - 30abcd + 35b^2c^2\right)\log\left(cd^4\sqrt{-\frac{1}{cd^9}} + \right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**3/(3*d**3) - sqrt(-1/(c*d**9))*(3*a**2*d**2 - 30*a*b*c*d + 35*b**2*
c**2)*log(-c*d**4*sqrt(-1/(c*d**9)) + x)/16 + sqrt(-1/(c*d**9))*(3*a**2*d**
2 - 30*a*b*c*d + 35*b**2*c**2)*log(c*d**4*sqrt(-1/(c*d**9)) + x)/16 - (x**3
*(5*a**2*d**3 - 18*a*b*c*d**2 + 13*b**2*c**2*d) + x*(3*a**2*c*d**2 - 14*a*b
*c**2*d + 11*b**2*c**3))/(8*c**2*d**4 + 16*c*d**5*x**2 + 8*d**6*x**4) + x*(
```

```
2*a*b*d - 3*b**2*c)/d**4
```

Giac [A] time = 1.13401, size = 208, normalized size = 1.28

$$\frac{\left(35\,b^2c^2 - 30\,abcd + 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}d^4} - \frac{13\,b^2c^2dx^3 - 18\,abcd^2x^3 + 5\,a^2d^3x^3 + 11\,b^2c^3x - 14\,abc^2dx + 3\,a^2cd^2x}{8\,\left(dx^2 + c\right)^2d^4} + \frac{b^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
d^4) - 1/8*(13*b^2*c^2*d*x^3 - 18*a*b*c*d^2*x^3 + 5*a^2*d^3*x^3 + 11*b^2*c^
3*x - 14*a*b*c^2*d*x + 3*a^2*c*d^2*x)/((d*x^2 + c)^2*d^4) + 1/3*(b^2*d^6*x^
3 - 9*b^2*c*d^5*x + 6*a*b*d^6*x)/d^9
```

3.190 $\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=99

$$\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^2}{2d^3}$$

```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b*c - a*d)*
(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*
d^4)
```

Rubi [A] time = 0.100078, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```



```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b*c - a*d)*
(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*
d^4)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{d^3}-\frac{c(bc-ad)^2}{d^3(c+dx)^3}+\frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^2}-\frac{b(3bc-2ad)}{d^3(c+dx)}\right)dx, x, x^2\right)\\
&= \frac{b^2x^2}{2d^3}+\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(bc-ad)(3bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}
\end{aligned}$$

Mathematica [A] time = 0.0504032, size = 114, normalized size = 1.15

$$\frac{-a^2d^2\left(c+2dx^2\right)+2abcd\left(3c+4dx^2\right)-2b\left(c+dx^2\right)^2(3bc-2ad)\log\left(c+dx^2\right)+b^2\left(-4c^2dx^2-5c^3+4cd^2x^4+2d^3x^6\right)}{4d^4\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2*(c + 2*d*x^2)) + 2*a*b*c*d*(3*c + 4*d*x^2) + b^2*(-5*c^3 - 4*c^2
*d*x^2 + 4*c*d^2*x^4 + 2*d^3*x^6) - 2*b*(3*b*c - 2*a*d)*(c + d*x^2)^2*Log[c
 + d*x^2])/(4*d^4*(c + d*x^2)^2)
```

Maple [A] time = 0.011, size = 155, normalized size = 1.6

$$\frac{b^2x^2}{2\,d^3}+\frac{b\ln\left(dx^2+c\right)a}{d^3}-\frac{3\,b^2\ln\left(dx^2+c\right)c}{2\,d^4}+\frac{a^2c}{4\,d^2\left(dx^2+c\right)^2}-\frac{abc^2}{2\,d^3\left(dx^2+c\right)^2}+\frac{b^2c^3}{4\,d^4\left(dx^2+c\right)^2}-\frac{a^2}{2\,d^2\left(dx^2+c\right)}+2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2*x^2/d^3+1/d^3*b*ln(d*x^2+c)*a-3/2/d^4*b^2*ln(d*x^2+c)*c+1/4/d^2*c/(
d*x^2+c)^2*a^2-1/2/d^3*c^2/(d*x^2+c)^2*a*b+1/4/d^4*c^3/(d*x^2+c)^2*b^2-1/2/
d^2/(d*x^2+c)*a^2+2/d^3/(d*x^2+c)*c*a*b-3/2/d^4/(d*x^2+c)*b^2*c^2
```

Maxima [A] time = 0.982362, size = 162, normalized size = 1.64

$$\frac{b^2x^2}{2\,d^3}-\frac{5\,b^2c^3-6\,abc^2d+a^2cd^2+2\left(3\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2}{4\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}-\frac{\left(3\,b^2c-2\,abd\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d
 - 4*a*b*c*d^2 + a^2*d^3)*x^2)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) - 1/2*(3*b
^2*c - 2*a*b*d)*log(d*x^2 + c)/d^4
```

Fricas [A] time = 1.47316, size = 365, normalized size = 3.69

$$\frac{2\,b^2d^3x^6+4\,b^2cd^2x^4-5\,b^2c^3+6\,abc^2d-a^2cd^2-2\left(2\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2-2\left(3\,b^2c^3-2\,abc^2d+\left(3\,b^2cd^2-2\,abd^3\cdots\right.\right.}{4\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^3*x^6 + 4*b^2*c*d^2*x^4 - 5*b^2*c^3 + 6*a*b*c^2*d - a^2*c*d^2
- 2*(2*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2 - 2*(3*b^2*c^3 - 2*a*b*c^2*d
+ (3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 2*(3*b^2*c^2*d - 2*a*b*c*d^2)*x^2)*log(d*
x^2 + c))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [A] time = 2.6553, size = 122, normalized size = 1.23

$$\frac{b^2x^2}{2d^3} + \frac{b(2ad - 3bc)\log\left(c + dx^2\right)}{2d^4} - \frac{a^2cd^2 - 6abc^2d + 5b^2c^3 + x^2\left(2a^2d^3 - 8abcd^2 + 6b^2c^2d\right)}{4c^2d^4 + 8cd^5x^2 + 4d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**2/(2*d**3) + b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) - (a**2*c*d
**2 - 6*a*b*c**2*d + 5*b**2*c**3 + x**2*(2*a**2*d**3 - 8*a*b*c*d**2 + 6*b**
2*c**2*d))/(4*c**2*d**4 + 8*c*d**5*x**2 + 4*d**6*x**4)
```

Giac [A] time = 1.20211, size = 144, normalized size = 1.45

$$\frac{b^2x^2}{2\,d^3} - \frac{\left(3\,b^2c - 2\,abd\right)\log\left(\left|dx^2 + c\right|\right)}{2\,d^4} - \frac{5\,b^2c^3 - 6\,abc^2d + a^2cd^2 + 2\left(3\,b^2c^2d - 4\,abcd^2 + a^2d^3\right)x^2}{4\left(dx^2 + c\right)^2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/2*(3*b^2*c - 2*a*b*d)*log(abs(d*x^2 + c))/d^4 - 1/4*(5*
b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)
*x^2)/((d*x^2 + c)^2*d^4)
```

3.191 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=127

$$-\frac{(-a^2d^2-6abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x^3(bc-ad)^2}{4cd^2(c+dx^2)^2}+\frac{x(bc-ad)(ad+7bc)}{8cd^3(c+dx^2)}+\frac{b^2x}{d^3}$$

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c - a*d)*(7
*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Rubi [A] time = 0.124717, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 455, 388, 205}

$$-\frac{(-a^2d^2-6abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x^3(bc-ad)^2}{4cd^2(c+dx^2)^2}+\frac{x(bc-ad)(ad+7bc)}{8cd^3(c+dx^2)}+\frac{b^2x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c - a*d)*(7
*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^2\left(-4a^2d^2+3(bc-ad)^2-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)} + \frac{\int \frac{-d(bc-ad)(7bc+ad)+8b^2cd^2x^2}{c+dx^2}\,dx}{8cd^4}$$

$$= \frac{b^2x}{d^3} + \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)} - \frac{\left(15b^2c^2-6abcd-a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8cd^3}$$

$$= \frac{b^2x}{d^3} + \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)} - \frac{\left(15b^2c^2-6abcd-a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

Mathematica [A] time = 0.0995385, size = 130, normalized size = 1.02

$$\frac{x\left(a^2d^2\left(dx^2-c\right)-2abcd\left(3c+5dx^2\right)+b^2c\left(15c^2+25cdx^2+8d^2x^4\right)\right)}{8cd^3\left(c+dx^2\right)^2} - \frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(3*c + 5*d*x^2) + b^2*c*(15*c^2 + 25*c
*d*x^2 + 8*d^2*x^4)))/(8*c*d^3*(c + d*x^2)^2) - ((15*b^2*c^2 - 6*a*b*c*d -
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Maple [A] time = 0.008, size = 196, normalized size = 1.5

$$\frac{b^2x}{d^3} + \frac{x^3a^2}{8\left(dx^2+c\right)^2c} - \frac{5x^3ab}{4d\left(dx^2+c\right)^2} + \frac{9x^3b^2c}{8d^2\left(dx^2+c\right)^2} - \frac{a^2x}{8d\left(dx^2+c\right)^2} - \frac{3abcx}{4d^2\left(dx^2+c\right)^2} + \frac{7b^2c^2x}{8d^3\left(dx^2+c\right)^2} + \frac{a^2}{8cd}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] b^2*x/d^3+1/8/(d*x^2+c)^2/c*x^3*a^2-5/4/d/(d*x^2+c)^2*x^3*a*b+9/8/d^2/(d*x^
2+c)^2*x^3*b^2*c-1/8/d/(d*x^2+c)^2*a^2*x-3/4/d^2/(d*x^2+c)^2*c*a*b*x+7/8/d^
3/(d*x^2+c)^2*b^2*c^2*x+1/8/d/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+3/4
/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-15/8/d^3*c/(c*d)^(1/2)*arctan(
x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52891, size = 963, normalized size = 7.58

$$\left[\frac{16\,b^2c^2d^3x^5+2\left(25\,b^2c^3d^2-10\,abc^2d^3+a^2cd^4\right)x^3+\left(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+\left(15\,b^2c^2d^2-6\,abcd^3-a^2d^4\right)x^4+2\left(15\right.\right.}{16\left(c^2d^6x^4+2\,c^3d^5x^2+c^4d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*c^2*d^3*x^5 + 2*(25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)
*x^3 + (15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*
d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^2*c^4*d - 6*
a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4), 1/8*
(8*b^2*c^2*d^3*x^5 + (25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)*x^3 - (1
5*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*d^3 - a^2
*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) + (15*b^2*c^4*d - 6*a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d
^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4)]
```

Sympy [A] time = 1.76807, size = 223, normalized size = 1.76

$$\frac{b^2x}{d^3}-\frac{\sqrt{-\frac{1}{c^3d^7}}\left(a^2d^2+6abcd-15b^2c^2\right)\log\left(-c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^3d^7}}\left(a^2d^2+6abcd-15b^2c^2\right)\log\left(c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x/d**3 - sqrt(-1/(c**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b**2*c**2)*l
og(-c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 + sqrt(-1/(c**3*d**7))*(a**2*d**
2 + 6*a*b*c*d - 15*b**2*c**2)*log(c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 +
(x**3*(a**2*d**3 - 10*a*b*c*d**2 + 9*b**2*c**2*d) + x*(-a**2*c*d**2 - 6*a*b
*c**2*d + 7*b**2*c**3))/(8*c**3*d**3 + 16*c**2*d**4*x**2 + 8*c*d**5*x**4)
```

Giac [A] time = 1.19722, size = 180, normalized size = 1.42

$$\frac{b^2x}{d^3}-\frac{\left(15\,b^2c^2-6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}cd^3}+\frac{9\,b^2c^2dx^3-10\,abcd^2x^3+a^2d^3x^3+7\,b^2c^3x-6\,abc^2dx-a^2cd^2x}{8\left(dx^2+c\right)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^2*x/d^3 - 1/8*(15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(s
qrt(c*d)*c*d^3) + 1/8*(9*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x^3 + a^2*d^3*x^3 + 7
*b^2*c^3*x - 6*a*b*c^2*d*x - a^2*c*d^2*x)/((d*x^2 + c)^2*c*d^3)
```

3.192 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=67

$$\frac{b(bc-ad)}{d^3(c+dx^2)} - \frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b^2\log(c+dx^2)}{2d^3}$$

```
[Out] -(b*c - a*d)^2/(4*d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c + d*x^2)) +
(b^2*Log[c + d*x^2])/(2*d^3)
```

Rubi [A] time = 0.0681163, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{b(bc-ad)}{d^3(c+dx^2)} - \frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b^2\log(c+dx^2)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -(b*c - a*d)^2/(4*d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c + d*x^2)) +
(b^2*Log[c + d*x^2])/(2*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^2)^2}{(c+dx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3} - \frac{2b(bc-ad)}{d^2(c+dx)^2} + \frac{b^2}{d^2(c+dx)}\right) dx, x, x^2\right) \\
&= -\frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b(bc-ad)}{d^3(c+dx^2)} + \frac{b^2\log(c+dx^2)}{2d^3}
\end{aligned}$$

Mathematica [A] time = 0.0240263, size = 75, normalized size = 1.12

$$\frac{-a^2d^2-2abd\left(c+2dx^2\right)+b^2c\left(3c+4dx^2\right)+2b^2\left(c+dx^2\right)^2\log\left(c+dx^2\right)}{4d^3\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(c + 2*d*x^2) + b^2*c*(3*c + 4*d*x^2) + 2*b^2*(c + d*
x^2)^2*Log[c + d*x^2])/(4*d^3*(c + d*x^2)^2)
```

Maple [A] time = 0.009, size = 105, normalized size = 1.6

$$\frac{b^2\ln\left(dx^2+c\right)}{2\,d^3}-\frac{a^2}{4\,d\left(dx^2+c\right)^2}+\frac{abc}{2\,d^2\left(dx^2+c\right)^2}-\frac{b^2c^2}{4\,d^3\left(dx^2+c\right)^2}-\frac{ab}{d^2\left(dx^2+c\right)}+\frac{b^2c}{d^3\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2*ln(d*x^2+c)/d^3-1/4/d/(d*x^2+c)^2*a^2+1/2/d^2/(d*x^2+c)^2*c*a*b-1/4
/d^3/(d*x^2+c)^2*b^2*c^2-b/d^2/(d*x^2+c)*a+b^2/d^3/(d*x^2+c)*c
```

Maxima [A] time = 1.01857, size = 117, normalized size = 1.75

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\left(b^2cd-abd^2\right)x^2}{4\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}+\frac{b^2\log\left(dx^2+c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2)/(d^5*x^4
+ 2*c*d^4*x^2 + c^2*d^3) + 1/2*b^2*log(d*x^2 + c)/d^3
```

Fricas [A] time = 1.45565, size = 216, normalized size = 3.22

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+2\,b^2cdx^2+b^2c^2\right)\log\left(dx^2+c\right)}{4\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(d^5*x^4 + 2*c*d^4*x^2 +
c^2*d^3)
```

Sympy [A] time = 1.66126, size = 87, normalized size = 1.3

$$\frac{b^2\log\left(c+dx^2\right)}{2d^3}-\frac{a^2d^2+2abcd-3b^2c^2+x^2\left(4abd^2-4b^2cd\right)}{4c^2d^3+8cd^4x^2+4d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(c + d*x**2)/(2*d**3) - (a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2 + x**2
*(4*a*b*d**2 - 4*b**2*c*d))/(4*c**2*d**3 + 8*c*d**4*x**2 + 4*d**5*x**4)
```

Giac [A] time = 1.18528, size = 103, normalized size = 1.54

$$\frac{b^2\log\left(\left|dx^2+c\right|\right)}{2\,d^3}+\frac{4\left(b^2c-abd\right)x^2+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{4\left(dx^2+c\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(d*x^2 + c))/d^3 + 1/4*(4*(b^2*c - a*b*d)*x^2 + (3*b^2*c^2 -
 2*a*b*c*d - a^2*d^2)/d)/((d*x^2 + c)^2*d^2)
```

3.193 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0765431, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)x\,(a+bx^2)}{4cd\,(c+dx^2)^2} + \frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{(c+dx^2)^2}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\,(a+bx^2)}{4cd\,(c+dx^2)^2} + \frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)} + \frac{(3b^2c^2+2abcd+3a^2d^2)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2}$$

$$= -\frac{(bc-ad)x\,(a+bx^2)}{4cd\,(c+dx^2)^2} + \frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)} + \frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Mathematica [A] time = 0.0953444, size = 121, normalized size = 1.04

$$\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A] time = 0., size = 147, normalized size = 1.3

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{\left(3\,a^2d^2+2\,cabd-5\,b^2c^2\right)x^3}{8\,c^2d}+\frac{\left(5\,a^2d^2-2\,cabd-3\,b^2c^2\right)x}{8\,d^2c}\right)+\frac{3\,a^2}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{4\,cd}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^
2+1/4/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+3/8/d^2/(c*d)^(1/2)*arcta
n(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.54271, size = 921, normalized size = 7.94

$$\left[-\frac{2\left(5\,b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}-3\,a^{2}cd^{4}\right)x^{3}+\left(3\,b^{2}c^{4}+2\,abc^{3}d+3\,a^{2}c^{2}d^{2}+\left(3\,b^{2}c^{2}d^{2}+2\,abcd^{3}+3\,a^{2}d^{4}\right)x^{4}+2\left(3\,b^{2}c^{3}d+\right.\right.}{16\left(c^{3}d^{5}x^{4}+2\,c^{4}d^{4}x^{2}+c^{5}d^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

Sympy [B] time = 1.19095, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^{5}d^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(-c^{3}d^{2}\sqrt{-\frac{1}{c^{5}d^{5}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^{5}d^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(c^{3}d^{2}\sqrt{-\frac{1}{c^{5}d^{5}}}+x\right)}{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

Giac [A] time = 1.17647, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^{2}d^{2}}-\frac{5\,b^{2}c^{2}dx^{3}-2\,abcd^{2}x^{3}-3\,a^{2}d^{3}x^{3}+3\,b^{2}c^{3}x+2\,abc^{2}dx-5\,a^{2}cd^{2}x}{8\left(dx^{2}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

3.194 $\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=86

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c + d*x^2))
 + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rubi [A] time = 0.0829324, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c + d*x^2))
 + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{c^3x}-\frac{(bc-ad)^2}{cd(c+dx)^3}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)^2}-\frac{a^2d}{c^3(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}+\frac{a^2\log(x)}{c^3}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.0469384, size = 103, normalized size = 1.2

$$\frac{a^2d^2-b^2c^2}{2c^2d^2\left(c+dx^2\right)}+\frac{a^2d^2-2abcd+b^2c^2}{4cd^2\left(c+dx^2\right)^2}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(4*c*d^2*(c + d*x^2)^2) + (-(b^2*c^2) + a^2
*d^2)/(2*c^2*d^2*(c + d*x^2)) + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*
c^3)
```

Maple [A] time = 0.011, size = 112, normalized size = 1.3

$$-\frac{a^2\ln\left(dx^2+c\right)}{2c^3}+\frac{a^2}{4c\left(dx^2+c\right)^2}-\frac{ab}{2d\left(dx^2+c\right)^2}+\frac{b^2c}{4d^2\left(dx^2+c\right)^2}+\frac{a^2}{2c^2\left(dx^2+c\right)}-\frac{b^2}{2d^2\left(dx^2+c\right)}+\frac{a^2\ln(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^3,x)
```

```
[Out] -1/2*a^2*ln(d*x^2+c)/c^3+1/4/c/(d*x^2+c)^2*a^2-1/2/d/(d*x^2+c)^2*a*b+1/4*c/
d^2/(d*x^2+c)^2*b^2+1/2/c^2/(d*x^2+c)*a^2-1/2/d^2/(d*x^2+c)*b^2+a^2*ln(x)/c
^3
```

Maxima [A] time = 1.00211, size = 147, normalized size = 1.71

$$-\frac{b^2c^3+2\,abc^2d-3\,a^2cd^2+2\left(b^2c^2d-a^2d^3\right)x^2}{4\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)}-\frac{a^2\log\left(dx^2+c\right)}{2\,c^3}+\frac{a^2\log\left(x^2\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2 + 2*(b^2*c^2*d - a^2*d^3)*x^2)/(c
^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) - 1/2*a^2*log(d*x^2 + c)/c^3 + 1/2*a^
2*log(x^2)/c^3
```

Fricas [B] time = 1.4595, size = 324, normalized size = 3.77

$$-\frac{b^2c^4+2\,abc^3d-3\,a^2c^2d^2+2\left(b^2c^3d-a^2cd^3\right)x^2+2\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\log\left(dx^2+c\right)-4\left(a^2d^4x^4+2\,a^2cd\right.}{4\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^4 + 2*a*b*c^3*d - 3*a^2*c^2*d^2 + 2*(b^2*c^3*d - a^2*c*d^3)*x^2
 + 2*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(d*x^2 + c) - 4*(a^2*
```

```
d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(x))/(c^3*d^4*x^4 + 2*c^4*d^3*x
^2 + c^5*d^2)
```

Sympy [A] time = 1.48476, size = 107, normalized size = 1.24

$$\frac{a^2\log(x)}{c^3} - \frac{a^2\log\left(\frac{c}{d}+x^2\right)}{2c^3} + \frac{3a^2cd^2 - 2abc^2d - b^2c^3 + x^2\left(2a^2d^3 - 2b^2c^2d\right)}{4c^4d^2 + 8c^3d^3x^2 + 4c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

```
[Out] a**2*log(x)/c**3 - a**2*log(c/d + x**2)/(2*c**3) + (3*a**2*c*d**2 - 2*a*b*c
**2*d - b**2*c**3 + x**2*(2*a**2*d**3 - 2*b**2*c**2*d))/(4*c**4*d**2 + 8*c*
*3*d**3*x**2 + 4*c**2*d**4*x**4)
```

Giac [A] time = 1.14995, size = 149, normalized size = 1.73

$$\frac{a^2\log\left(x^2\right)}{2c^3} - \frac{a^2\log\left(\left|dx^2+c\right|\right)}{2c^3} + \frac{3a^2d^4x^4 - 2b^2c^3dx^2 + 8a^2cd^3x^2 - b^2c^4 - 2abc^3d + 6a^2c^2d^2}{4\left(dx^2+c\right)^2c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^3 - 1/2*a^2*log(abs(d*x^2 + c))/c^3 + 1/4*(3*a^2*d^4*x^4
 - 2*b^2*c^3*d*x^2 + 8*a^2*c*d^3*x^2 - b^2*c^4 - 2*a*b*c^3*d + 6*a^2*c^2*d^
2)/((d*x^2 + c)^2*c^3*d^2)
```

3.195 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(3ad(2bc-5ad)+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}$$

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(4*c^2*d
*(c + d*x^2)^2) + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*x)/(8*c^3*d*(c + d*x^2
)) + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(
7/2)*d^(3/2))
```

Rubi [A] time = 0.108841, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 385, 199, 205}

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(\frac{3a(2bc-5ad)}{c^2}+\frac{b^2}{d}\right)}{8c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(4*c^2*d
*(c + d*x^2)^2) + ((b^2/d + (3*a*(2*b*c - 5*a*d))/c^2)*x)/(8*c*(c + d*x^2))
 + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/
2)*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^3}\,dx = -\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\int \frac{a(2bc-5ad)+b^2cx^2}{\left(c+dx^2\right)^3}\,dx}{c}$$

$$= -\frac{a^2}{cx\left(c+dx^2\right)^2}-\frac{\left(b^2c^2-2abcd+5a^2d^2\right)x}{4c^2d\left(c+dx^2\right)^2}+\frac{1}{4}\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)\int \frac{1}{\left(c+dx^2\right)^2}\,dx$$

$$= -\frac{a^2}{cx\left(c+dx^2\right)^2}-\frac{\left(b^2c^2-2abcd+5a^2d^2\right)x}{4c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c\left(c+dx^2\right)}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)\int \frac{1}{c+dx^2}\,dx}{8c}$$

$$= -\frac{a^2}{cx\left(c+dx^2\right)^2}-\frac{\left(b^2c^2-2abcd+5a^2d^2\right)x}{4c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c\left(c+dx^2\right)}+\frac{\left(b^2c^2+6abcd-15a^2d^2\right)\tan^{-1}}{8c^{7/2}d^{3/2}}$$

Mathematica [A] time = 0.0880691, size = 133, normalized size = 0.88

$$\frac{x\left(-7a^2d^2+6abcd+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}+\frac{\left(-15a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}-\frac{a^2}{c^3x}-\frac{x(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c^3*x)) - ((b*c - a*d)^2*x)/(4*c^2*d*(c + d*x^2)^2) + ((b^2*c^2 + 6*
a*b*c*d - 7*a^2*d^2)*x)/(8*c^3*d*(c + d*x^2)) + ((b^2*c^2 + 6*a*b*c*d - 15*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

Maple [A] time = 0.012, size = 199, normalized size = 1.3

$$-\frac{7x^3a^2d^2}{8c^3\left(dx^2+c\right)^2}+\frac{3x^3abd}{4c^2\left(dx^2+c\right)^2}+\frac{b^2x^3}{8c\left(dx^2+c\right)^2}-\frac{9a^2dx}{8c^2\left(dx^2+c\right)^2}+\frac{5abx}{4c\left(dx^2+c\right)^2}-\frac{xb^2}{8\left(dx^2+c\right)^2d}-\frac{15a^2d}{8c^3}\arctan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^3,x)
```

```
[Out] -7/8/c^3/(d*x^2+c)^2*x^3*a^2*d^2+3/4/c^2/(d*x^2+c)^2*x^3*a*b*d+1/8/c/(d*x^2
+c)^2*x^3*b^2-9/8/c^2/(d*x^2+c)^2*a^2*d*x+5/4/c/(d*x^2+c)^2*a*b*x-1/8/(d*x^
2+c)^2/d*x*b^2-15/8/c^3*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+3/4/c^2/(
c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+1/8/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)
^(1/2))*b^2-a^2/c^3/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53838, size = 976, normalized size = 6.42

$$\left[-\frac{16\,a^2c^3d^2-2\left(b^2c^3d^2+6\,abc^2d^3-15\,a^2cd^4\right)x^4+2\left(b^2c^4d-10\,abc^3d^2+25\,a^2c^2d^3\right)x^2-\left(\left(b^2c^2d^2+6\,abcd^3-15\,a^2d^4\right.\right.}{16\left(c^4d^4x^5+2\,c^5d^3x^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*c^3*d^2 - 2*(b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4
 + 2*(b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*
a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*
x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x),
 -1/8*(8*a^2*c^3*d^2 - (b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4 + (
b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*a*b*c*
d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*x^3 +
(b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
)/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x)]
```

Sympy [A] time = 1.49247, size = 224, normalized size = 1.47

$$\frac{\sqrt{-\frac{1}{c^7d^3}}\left(15a^2d^2-6abcd-b^2c^2\right)\log\left(-c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{c^7d^3}}\left(15a^2d^2-6abcd-b^2c^2\right)\log\left(c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}-\frac{8a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

```
[Out] sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d - b**2*c**2)*log(-c**4*d*sqr
t(-1/(c**7*d**3)) + x)/16 - sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d
- b**2*c**2)*log(c**4*d*sqrt(-1/(c**7*d**3)) + x)/16 - (8*a**2*c**2*d + x**
4*(15*a**2*d**3 - 6*a*b*c*d**2 - b**2*c**2*d) + x**2*(25*a**2*c*d**2 - 10*a
*b*c**2*d + b**2*c**3))/(8*c**5*d*x + 16*c**4*d**2*x**3 + 8*c**3*d**3*x**5)
```

Giac [A] time = 1.16719, size = 182, normalized size = 1.2

$$-\frac{a^2}{c^3x}+\frac{\left(b^2c^2+6\,abcd-15\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}c^3d}+\frac{b^2c^2dx^3+6\,abcd^2x^3-7\,a^2d^3x^3-b^2c^3x+10\,abc^2dx-9\,a^2cd^2x}{8\left(dx^2+c\right)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a^2/(c^3*x) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c^3*d) + 1/8*(b^2*c^2*d*x^3 + 6*a*b*c*d^2*x^3 - 7*a^2*d^3*x^3 -
 b^2*c^3*x + 10*a*b*c^2*d*x - 9*a^2*c*d^2*x)/((d*x^2 + c)^2*c^3*d)
```

3.196 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx$

Optimal. Leaf size=106

$$-\frac{a^2}{2c^3x^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}+\frac{a\log(x)(2bc-3ad)}{c^4}$$

```
[Out] -a^2/(2*c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*c - a*d))/
(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a*(2*b*c - 3*a*d)*Log
[c + d*x^2])/(2*c^4)
```

Rubi [A] time = 0.114206, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2c^3x^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}+\frac{a\log(x)(2bc-3ad)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] -a^2/(2*c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*c - a*d))/
(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a*(2*b*c - 3*a*d)*Log
[c + d*x^2])/(2*c^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{c^3x^2}-\frac{a(-2bc+3ad)}{c^4x}+\frac{(bc-ad)^2}{c^2(c+dx)^3}+\frac{2ad(-bc+ad)}{c^3(c+dx)^2}+\frac{ad(-2bc+3ad)}{c^4(c+dx)}\right)dx, x, x\right.\\ &= -\frac{a^2}{2c^3x^2}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}+\frac{a(2bc-3ad)\log(x)}{c^4}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.089042, size = 99, normalized size = 0.93

$$\frac{-\frac{2a^2c}{x^2}-\frac{c^2(bc-ad)^2}{d(c+dx^2)^2}+\frac{4ac(bc-ad)}{c+dx^2}+2a(3ad-2bc)\log\left(c+dx^2\right)+4a\log(x)(2bc-3ad)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] ((-2*a^2*c)/x^2 - (c^2*(b*c - a*d)^2)/(d*(c + d*x^2)^2) + (4*a*c*(b*c - a*d
))/(c + d*x^2) + 4*a*(2*b*c - 3*a*d)*Log[x] + 2*a*(-2*b*c + 3*a*d)*Log[c +
d*x^2])/(4*c^4)
```

Maple [A] time = 0.014, size = 149, normalized size = 1.4

$$\frac{3\ln\left(dx^2+c\right)a^2d}{2c^4}-\frac{\ln\left(dx^2+c\right)ab}{c^3}-\frac{a^2d}{4c^2\left(dx^2+c\right)^2}+\frac{ab}{2c\left(dx^2+c\right)^2}-\frac{b^2}{4d\left(dx^2+c\right)^2}-\frac{a^2d}{c^3\left(dx^2+c\right)}+\frac{ab}{c^2\left(dx^2+c\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^3,x)
```

```
[Out] 3/2/c^4*ln(d*x^2+c)*a^2*d-1/c^3*ln(d*x^2+c)*a*b-1/4/c^2*d/(d*x^2+c)^2*a^2+1
/2/c/(d*x^2+c)^2*a*b-1/4/d/(d*x^2+c)^2*b^2-1/c^3*a^2/(d*x^2+c)*d+1/c^2*a/(d
*x^2+c)*b-1/2*a^2/c^3/x^2-3*a^2/c^4*ln(x)*d+2*a/c^3*ln(x)*b
```

Maxima [A] time = 1.00577, size = 192, normalized size = 1.81

$$-\frac{2a^2c^2d-2\left(2abcd^2-3a^2d^3\right)x^4+\left(b^2c^3-6abc^2d+9a^2cd^2\right)x^2}{4\left(c^3d^3x^6+2c^4d^2x^4+c^5dx^2\right)}-\frac{\left(2abc-3a^2d\right)\log\left(dx^2+c\right)}{2c^4}+\frac{\left(2abc-3a^2d\right)\log\left(x^2\right.}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*c^2*d - 2*(2*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (b^2*c^3 - 6*a*b*c^2*
d + 9*a^2*c*d^2)*x^2)/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2) - 1/2*(2*a*
b*c - 3*a^2*d)*log(d*x^2 + c)/c^4 + 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4
```

Fricas [B] time = 1.48117, size = 524, normalized size = 4.94

$$-\frac{2a^2c^3d-2\left(2abc^2d^2-3a^2cd^3\right)x^4+\left(b^2c^4-6abc^3d+9a^2c^2d^2\right)x^2+2\left(\left(2abcd^3-3a^2d^4\right)x^6+2\left(2abc^2d^2-3a^2cd^3\right)x^4\right.}{4\left(c^4d^3x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*c^3*d - 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (b^2*c^4 - 6*a*b*
c^3*d + 9*a^2*c^2*d^2)*x^2 + 2*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^
```

```
2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(d*x^2 + c
) - 4*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4
+ (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(x))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 +
 c^6*d*x^2)
```

Sympy [A] time = 2.1941, size = 139, normalized size = 1.31

$$-\frac{a(3ad-2bc)\log(x)}{c^4}+\frac{a(3ad-2bc)\log\left(\frac{c}{d}+x^2\right)}{2c^4}-\frac{2a^2c^2d+x^4\left(6a^2d^3-4abcd^2\right)+x^2\left(9a^2cd^2-6abc^2d+b^2c^3\right)}{4c^5dx^2+8c^4d^2x^4+4c^3d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

```
[Out] -a*(3*a*d - 2*b*c)*log(x)/c**4 + a*(3*a*d - 2*b*c)*log(c/d + x**2)/(2*c**4)
 - (2*a**2*c**2*d + x**4*(6*a**2*d**3 - 4*a*b*c*d**2) + x**2*(9*a**2*c*d**2
 - 6*a*b*c**2*d + b**2*c**3))/(4*c**5*d*x**2 + 8*c**4*d**2*x**4 + 4*c**3*d*
*3*x**6)
```

Giac [A] time = 1.18844, size = 239, normalized size = 2.25

$$\frac{\left(2\,abc-3\,a^2d\right)\log\left(x^2\right)}{2\,c^4}-\frac{\left(2\,abcd-3\,a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,c^4d}-\frac{2\,abcx^2-3\,a^2dx^2+a^2c}{2\,c^4x^2}+\frac{6\,abcd^3x^4-9\,a^2d^4x^4+16\,abc^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4 - 1/2*(2*a*b*c*d - 3*a^2*d^2)*log(abs(
d*x^2 + c))/(c^4*d) - 1/2*(2*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^4*x^2) + 1
/4*(6*a*b*c*d^3*x^4 - 9*a^2*d^4*x^4 + 16*a*b*c^2*d^2*x^2 - 22*a^2*c*d^3*x^2
 - b^2*c^4 + 12*a*b*c^3*d - 14*a^2*c^2*d^2)/((d*x^2 + c)^2*c^4*d)
```

3.197 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$

Optimal. Leaf size=161

$$\frac{x(7a^2d^2-6abcd+3b^2c^2)}{12c^3(c+dx^2)^2}+\frac{(35a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}-\frac{a^2}{3cx^3(c+dx^2)^2}+\frac{x(3bc-7ad)^2}{24c^4(c+dx^2)}-\frac{a(6bc-7ad)}{3c^4x}$$

```
[Out] -(a*(6*b*c - 7*a*d))/(3*c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((3*b^2*c^2
 - 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((3*b*c - 7*a*d)^2*x)/
(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rubi [A] time = 0.191423, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x(7a^2d^2-6abcd+3b^2c^2)}{12c^3(c+dx^2)^2}+\frac{(35a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}-\frac{a^2}{3cx^3(c+dx^2)^2}+\frac{x(3bc-7ad)^2}{24c^4(c+dx^2)}-\frac{a(6bc-7ad)}{3c^4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -(a*(6*b*c - 7*a*d))/(3*c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((3*b^2*c^2
 - 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((3*b*c - 7*a*d)^2*x)/
(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^3}\,dx = -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\int \frac{a(6bc-7ad)+3b^2cx^2}{x^2\left(c+dx^2\right)^3}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}-\frac{\int \frac{-\frac{4a(6bc-7ad)}{c}-3\left(3b^2-\frac{6abd}{c}+\frac{7a^2d^2}{c^2}\right)x^2}{x^2\left(c+dx^2\right)^2}\,dx}{12c}$$

$$= -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\frac{\int \frac{\frac{8a(6bc-7ad)}{c^2}+\frac{(3bc-7ad)^2x^2}{c^3}}{x^2\left(c+dx^2\right)}\,dx}{24c}$$

$$= -\frac{a(6bc-7ad)}{3c^4x}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\frac{\left(3b^2c^2-30a\right.}{}$$

$$= -\frac{a(6bc-7ad)}{3c^4x}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\frac{\left(3b^2c^2-30a\right.}{}$$

Mathematica [A] time = 0.0724567, size = 148, normalized size = 0.92

$$\frac{x\left(11a^2d^2-14abcd+3b^2c^2\right)}{8c^4\left(c+dx^2\right)}+\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}-\frac{a^2}{3c^3x^3}+\frac{x(bc-ad)^2}{4c^3\left(c+dx^2\right)^2}+\frac{a(3ad-2bc)}{c^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -a^2/(3*c^3*x^3) + (a*(-2*b*c + 3*a*d))/(c^4*x) + ((b*c - a*d)^2*x)/(4*c^3*
(c + d*x^2)^2) + ((3*b^2*c^2 - 14*a*b*c*d + 11*a^2*d^2)*x)/(8*c^4*(c + d*x^
2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
8*c^(9/2)*Sqrt[d])
```

Maple [A] time = 0.014, size = 227, normalized size = 1.4

$$\frac{11\,x^3a^2d^3}{8\,c^4\left(dx^2+c\right)^2}-\frac{7\,x^3abd^2}{4\,c^3\left(dx^2+c\right)^2}+\frac{3\,b^2dx^3}{8\,c^2\left(dx^2+c\right)^2}+\frac{13\,a^2d^2x}{8\,c^3\left(dx^2+c\right)^2}-\frac{9\,abdx}{4\,c^2\left(dx^2+c\right)^2}+\frac{5\,b^2x}{8\,c\left(dx^2+c\right)^2}+\frac{35\,a^2d^2}{8\,c^4}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^3,x)
```

```
[Out] 11/8/c^4/(d*x^2+c)^2*x^3*a^2*d^3-7/4/c^3/(d*x^2+c)^2*x^3*a*b*d^2+3/8/c^2/(d
*x^2+c)^2*x^3*b^2*d+13/8/c^3/(d*x^2+c)^2*a^2*d^2*x-9/4/c^2/(d*x^2+c)^2*a*b*
d*x+5/8/c/(d*x^2+c)^2*b^2*x+35/8/c^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^
2*d^2-15/4/c^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*d+3/8/c^2/(c*d)^(1/2
)*arctan(x*d/(c*d)^(1/2))*b^2-1/3*a^2/c^3/x^3+3*a^2/c^4/x*d-2*a/c^3/x*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55762, size = 1131, normalized size = 7.02

$$\left[-\frac{16\,a^2c^4d-6\left(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4\right)x^6-10\left(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3\right)x^4+16\left(6\,abc^4d-7\,a^2c^3d^2\right)x^2 \cdots}{48\left(c \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^2*c^4*d - 6*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^
6 - 10*(3*b^2*c^4*d - 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 16*(6*a*b*c^4*
d - 7*a^2*c^3*d^2)*x^2 + 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7
 + 2*(3*b^2*c^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*
b*c^3*d + 35*a^2*c^2*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/
(d*x^2 + c)))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^7*d*x^3), -1/24*(8*a^2*c^4*d
 - 3*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^6 - 5*(3*b^2*c^4*d -
 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 8*(6*a*b*c^4*d - 7*a^2*c^3*d^2)*x^2
 - 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*b^2*c^3*d - 30
*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^
7*d*x^3)]
```

Sympy [A] time = 1.79279, size = 240, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{c^9d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(-c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^9d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16}+\frac{-8a^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b**2*c**2)*log(-c**5*sqrt
(-1/(c**9*d)) + x)/16 + sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b*
*2*c**2)*log(c**5*sqrt(-1/(c**9*d)) + x)/16 + (-8*a**2*c**3 + x**6*(105*a**
2*d**3 - 90*a*b*c*d**2 + 9*b**2*c**2*d) + x**4*(175*a**2*c*d**2 - 150*a*b*c
**2*d + 15*b**2*c**3) + x**2*(56*a**2*c**2*d - 48*a*b*c**3))/(24*c**6*x**3
+ 48*c**5*d*x**5 + 24*c**4*d**2*x**7)
```

Giac [A] time = 1.18574, size = 204, normalized size = 1.27

$$\frac{\left(3\,b^2c^2 - 30\,abcd + 35\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^4} + \frac{3\,b^2c^2dx^3 - 14\,abcd^2x^3 + 11\,a^2d^3x^3 + 5\,b^2c^3x - 18\,abc^2dx + 13\,a^2cd^2x}{8\left(dx^2 + c\right)^2c^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
c^4) + 1/8*(3*b^2*c^2*d*x^3 - 14*a*b*c*d^2*x^3 + 11*a^2*d^3*x^3 + 5*b^2*c^3
*x - 18*a*b*c^2*d*x + 13*a^2*c*d^2*x)/((d*x^2 + c)^2*c^4) - 1/3*(6*a*b*c*x^
2 - 9*a^2*d*x^2 + a^2*c)/(c^4*x^3)
```

3.198 $\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(bc-ad)}{4b^2}-\frac{ax^2(bc-ad)}{2b^3}+\frac{dx^6}{6b}$$

```
[Out] -(a*(b*c - a*d)*x^2)/(2*b^3) + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6)/(6*b) +
(a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0852302, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(bc-ad)}{4b^2}-\frac{ax^2(bc-ad)}{2b^3}+\frac{dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)*x^2)/(2*b^3) + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6)/(6*b) +
(a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(c+dx)}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-bc+ad)}{b^3}+\frac{(bc-ad)x}{b^2}+\frac{dx^2}{b}-\frac{a^2(-bc+ad)}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{a(bc-ad)x^2}{2b^3}+\frac{(bc-ad)x^4}{4b^2}+\frac{dx^6}{6b}+\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.03034, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2d-3ab\left(2c+dx^2\right)+b^2x^2\left(3c+2dx^2\right)\right)+6a^2(bc-ad)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d - 3*a*b*(2*c + d*x^2) + b^2*x^2*(3*c + 2*d*x^2)) + 6*a^2*(b
*c - a*d)*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.2

$$\frac{dx^6}{6\,b}-\frac{x^4ad}{4\,b^2}+\frac{cx^4}{4\,b}+\frac{x^2a^2d}{2\,b^3}-\frac{ax^2c}{2\,b^2}-\frac{a^3\ln\left(bx^2+a\right)d}{2\,b^4}+\frac{a^2\ln\left(bx^2+a\right)c}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/6*d*x^6/b-1/4/b^2*x^4*a*d+1/4/b*x^4*c+1/2/b^3*x^2*a^2*d-1/2/b^2*x^2*a*c-1
/2*a^3/b^4*ln(b*x^2+a)*d+1/2*a^2/b^3*ln(b*x^2+a)*c
```

Maxima [A] time = 0.971569, size = 100, normalized size = 1.33

$$\frac{2\,b^2dx^6+3\left(b^2c-abd\right)x^4-6\left(abc-a^2d\right)x^2}{12\,b^3}+\frac{\left(a^2bc-a^3d\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*(b^2*c - a*b*d)*x^4 - 6*(a*b*c - a^2*d)*x^2)/b^3 + 1/
2*(a^2*b*c - a^3*d)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.44851, size = 155, normalized size = 2.07

$$\frac{2\,b^3dx^6+3\left(b^3c-ab^2d\right)x^4-6\left(ab^2c-a^2bd\right)x^2+6\left(a^2bc-a^3d\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d*x^6 + 3*(b^3*c - a*b^2*d)*x^4 - 6*(a*b^2*c - a^2*b*d)*x^2 + 6
*(a^2*b*c - a^3*d)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 0.441804, size = 65, normalized size = 0.87

$$-\frac{a^2\left(ad-bc\right)\log\left(a+bx^2\right)}{2b^4}+\frac{dx^6}{6b}-\frac{x^4\left(ad-bc\right)}{4b^2}+\frac{x^2\left(a^2d-abc\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + d*x**6/(6*b) - x**4*(a*d - b*c
)/(4*b**2) + x**2*(a**2*d - a*b*c)/(2*b**3)
```

Giac [A] time = 1.16312, size = 104, normalized size = 1.39

$$\frac{2\,b^2dx^6+3\,b^2cx^4-3\,abdx^4-6\,abcx^2+6\,a^2dx^2}{12\,b^3}+\frac{\left(a^2bc-a^3d\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*b^2*c*x^4 - 3*a*b*d*x^4 - 6*a*b*c*x^2 + 6*a^2*d*x^2)/
b^3 + 1/2*(a^2*b*c - a^3*d)*log(abs(b*x^2 + a))/b^4
```

3.199 $\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(bc-ad)}{3b^2}-\frac{ax(bc-ad)}{b^3}+\frac{dx^5}{5b}$$

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) + (a^(
3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0516394, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {459, 302, 205}

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(bc-ad)}{3b^2}-\frac{ax(bc-ad)}{b^3}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) + (a^(
3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(c+dx^2\right)}{a+bx^2} dx = \frac{dx^5}{5b} - \frac{(-5bc+5ad) \int \frac{x^4}{a+bx^2} dx}{5b}$$
$$= \frac{dx^5}{5b} - \frac{(-5bc+5ad) \int \left(-\frac{a}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2(a+bx^2)}\right) dx}{5b}$$
$$= -\frac{a(bc-ad)x}{b^3} + \frac{(bc-ad)x^3}{3b^2} + \frac{dx^5}{5b} + \frac{\left(a^2(bc-ad)\right) \int \frac{1}{a+bx^2} dx}{b^3}$$
$$= -\frac{a(bc-ad)x}{b^3} + \frac{(bc-ad)x^3}{3b^2} + \frac{dx^5}{5b} + \frac{a^{3/2}(bc-ad) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.050827, size = 77, normalized size = 1.

$$-\frac{a^{3/2}(ad-bc) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{x^3(bc-ad)}{3b^2} + \frac{ax(ad-bc)}{b^3} + \frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)*x)/b^3 + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) - (a^(
3/2)*(-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A] time = 0.003, size = 92, normalized size = 1.2

$$\frac{dx^5}{5\,b} - \frac{x^3ad}{3\,b^2} + \frac{cx^3}{3\,b} + \frac{a^2dx}{b^3} - \frac{acx}{b^2} - \frac{a^3d}{b^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{a^2c}{b^2} \arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*d*x^5/b-1/3/b^2*x^3*a*d+1/3/b*x^3*c+1/b^3*a^2*d*x-1/b^2*a*c*x-a^3/b^3/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48366, size = 381, normalized size = 4.95

$$\left[\frac{6\,b^2dx^5+10\left(b^2c-abd\right)x^3-15\left(abc-a^2d\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\left(abc-a^2d\right)x}{30\,b^3},\frac{3\,b^2dx^5+5\left(b^2c-abd\right)x^3+1\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d*x^5 + 10*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*sqrt(-a/b)
*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(a*b*c - a^2*d)*x)/b^
3, 1/15*(3*b^2*d*x^5 + 5*(b^2*c - a*b*d)*x^3 + 15*(a*b*c - a^2*d)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) - 15*(a*b*c - a^2*d)*x)/b^3]
```

Sympy [B] time = 0.472932, size = 150, normalized size = 1.95

$$\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2}+\frac{dx^5}{5b}-\frac{x^3\,(ad-bc)}{3b^2}+\frac{x\left(a^2d-ab\ldots\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**7)*(a*d - b*c)*log(-b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a**2*d
 - a*b*c) + x)/2 - sqrt(-a**3/b**7)*(a*d - b*c)*log(b**3*sqrt(-a**3/b**7)*(
a*d - b*c)/(a**2*d - a*b*c) + x)/2 + d*x**5/(5*b) - x**3*(a*d - b*c)/(3*b**
2) + x*(a**2*d - a*b*c)/b**3
```

Giac [A] time = 1.17764, size = 113, normalized size = 1.47

$$\frac{\left(a^2bc-a^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{3\,b^4dx^5+5\,b^4cx^3-5\,ab^3dx^3-15\,ab^3cx+15\,a^2b^2dx}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d*x^5
 + 5*b^4*c*x^3 - 5*a*b^3*d*x^3 - 15*a*b^3*c*x + 15*a^2*b^2*d*x)/b^5
```

3.200 $\int \frac{x^3(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=54

$$\frac{x^2(bc-ad)}{2b^2} - \frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3} + \frac{dx^4}{4b}$$

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a + b*x^2])/
(2*b^3)
```

Rubi [A] time = 0.0559814, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{x^2(bc-ad)}{2b^2} - \frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3} + \frac{dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a + b*x^2])/
(2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{bc-ad}{b^2}+\frac{dx}{b}+\frac{a(-bc+ad)}{b^2(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{(bc-ad)x^2}{2b^2}+\frac{dx^4}{4b}-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0184562, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2ad+2bc+bdx^2\right)+2a(ad-bc)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*b*c - 2*a*d + b*d*x^2) + 2*a*(-(b*c) + a*d)*Log[a + b*x^2])/(4*b^
3)
```

Maple [A] time = 0.002, size = 62, normalized size = 1.2

$$\frac{dx^4}{4\,b}-\frac{adx^2}{2\,b^2}+\frac{cx^2}{2\,b}+\frac{a^2\ln\left(bx^2+a\right)d}{2\,b^3}-\frac{ac\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/4*d*x^4/b-1/2/b^2*a*d*x^2+1/2*c*x^2/b+1/2*a^2/b^3*ln(b*x^2+a)*d-1/2*a*c*l
n(b*x^2+a)/b^2
```

Maxima [A] time = 0.990317, size = 68, normalized size = 1.26

$$\frac{bdx^4+2\,(bc-ad)x^2}{4\,b^2}-\frac{\left(abc-a^2d\right)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d*x^4 + 2*(b*c - a*d)*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(b*x^2 + a)/
b^3
```

Fricas [A] time = 1.44177, size = 108, normalized size = 2.

$$\frac{b^2dx^4+2\left(b^2c-abd\right)x^2-2\left(abc-a^2d\right)\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^4 + 2*(b^2*c - a*b*d)*x^2 - 2*(a*b*c - a^2*d)*log(b*x^2 + a))/
b^3
```

Sympy [A] time = 0.412109, size = 44, normalized size = 0.81

$$\frac{a\,(ad-bc)\log\left(a+bx^2\right)}{2b^3}+\frac{dx^4}{4b}-\frac{x^2\,(ad-bc)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) + d*x**4/(4*b) - x**2*(a*d - b*c)/(2
*b**2)
```

Giac [A] time = 1.16664, size = 70, normalized size = 1.3

$$\frac{bdx^4 + 2\,bcx^2 - 2\,adx^2}{4\,b^2} - \frac{\left(abc - a^2d\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x^4 + 2*b*c*x^2 - 2*a*d*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(abs(b*x
^2 + a))/b^3
```

3.201 $\int \frac{x^2(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=58

$$\frac{x(bc-ad)}{b^2} - \frac{\sqrt{a}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{dx^3}{3b}$$

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.034077, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {459, 321, 205}

$$\frac{x(bc-ad)}{b^2} - \frac{\sqrt{a}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2),x]
```


```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)}{a+bx^2}\,dx = \frac{dx^3}{3b} - \frac{(-3bc+3ad)\int \frac{x^2}{a+bx^2}\,dx}{3b}$$
$$= \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} - \frac{(a(bc-ad))\int \frac{1}{a+bx^2}\,dx}{b^2}$$
$$= \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} - \frac{\sqrt{a}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0409964, size = 57, normalized size = 0.98

$$\frac{x(bc-ad)}{b^2} + \frac{\sqrt{a}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) + (Sqrt[a]*(-(b*c) + a*d)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A] time = 0.003, size = 68, normalized size = 1.2

$$\frac{dx^3}{3b} - \frac{adx}{b^2} + \frac{cx}{b} + \frac{a^2d}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{ac}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/3*d*x^3/b-1/b^2*a*d*x+c*x/b+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d
-c/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53942, size = 277, normalized size = 4.78

$$\left[\frac{2\,bdx^3 - 3\,(bc-ad)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(bc-ad)x}{6\,b^2}, \frac{bdx^3 - 3\,(bc-ad)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,(bc-ad)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*d*x^3 - 3*(b*c - a*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*c - a*d)*x)/b^2, 1/3*(b*d*x^3 - 3*(b*c - a*d)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*c - a*d)*x)/b^2]
```

Sympy [A] time = 0.444769, size = 90, normalized size = 1.55

$$-\frac{\sqrt{-\frac{a}{b^5}}\,(ad-bc)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^5}}\,(ad-bc)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{dx^3}{3b}-\frac{x\,(ad-bc)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**5)*(a*d - b*c)*log(-b**2*sqrt(-a/b**5) + x)/2 + sqrt(-a/b**5)*(
a*d - b*c)*log(b**2*sqrt(-a/b**5) + x)/2 + d*x**3/(3*b) - x*(a*d - b*c)/b**
2
```

Giac [A] time = 1.14221, size = 78, normalized size = 1.34

$$-\frac{\left(abc-a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b^2dx^3+3\,b^2cx-3\,abdx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*d*x^3 + 3
*b^2*c*x - 3*a*b*d*x)/b^3
```

3.202 $\int \frac{x(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=35

$$\frac{(bc-ad)\log(a+bx^2)}{2b^2}+\frac{dx^2}{2b}$$

```
[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0311675, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{(bc-ad)\log(a+bx^2)}{2b^2}+\frac{dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(c+dx^2)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{dx^2}{2b}+\frac{(bc-ad)\log(a+bx^2)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0108063, size = 31, normalized size = 0.89

$$\frac{(bc-ad)\log(a+bx^2)+bdx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*d*x^2 + (b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.003, size = 40, normalized size = 1.1

$$\frac{dx^2}{2\,b} - \frac{\ln\left(bx^2 + a\right)ad}{2\,b^2} + \frac{c\ln\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/2*d*x^2/b-1/2/b^2*ln(b*x^2+a)*a*d+1/2*c*ln(b*x^2+a)/b
```

Maxima [A] time = 0.987372, size = 42, normalized size = 1.2

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.42797, size = 65, normalized size = 1.86

$$\frac{bdx^2 + (bc - ad)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x^2 + (b*c - a*d)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.378325, size = 27, normalized size = 0.77

$$\frac{dx^2}{2b} - \frac{(ad - bc)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x**2/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 1.165, size = 43, normalized size = 1.23

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/b^2
```

3.203 $\int \frac{c+dx^2}{a+bx^2} \, dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.014759, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^2}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0255159, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.003, size = 45, normalized size = 1.2

$$\frac{dx}{b} - \frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] d*x/b-1/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*d+c/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50152, size = 223, normalized size = 5.72

$$\left[\frac{2\,abdx + \sqrt{-ab}(bc - ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abdx + \sqrt{ab}(bc - ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] time = 0.40774, size = 82, normalized size = 2.1

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad - bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}\,(ad - bc)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

Giac [A] time = 1.13574, size = 45, normalized size = 1.15

$$\frac{dx}{b} + \frac{(bc - ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.204 $\int \frac{c+dx^2}{x(a+bx^2)} \, dx$

Optimal. Leaf size=34

$$\frac{c\log(x)}{a} - \frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)

Rubi [A] time = 0.0314507, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 72}

$$\frac{c\log(x)}{a} - \frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)/(x*(a + b*x^2)),x]

[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x\left(a+bx^2\right)} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{c+dx}{x(a+bx)} \, dx, x, x^2\right)\\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{c}{ax} + \frac{-bc+ad}{a(a+bx)}\right) \, dx, x, x^2\right)\\ &= \frac{c\log(x)}{a} - \frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}\end{aligned}$$

Mathematica [A] time = 0.0120316, size = 34, normalized size = 1.

$$\frac{(ad-bc)\log\left(a+bx^2\right)}{2ab} + \frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)),x]

```
[Out] (c*Log[x])/a + ((-(b*c) + a*d)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.003, size = 37, normalized size = 1.1

$$\frac{c\ln(x)}{a}+\frac{\ln\left(bx^2+a\right)d}{2b}-\frac{c\ln\left(bx^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a),x)
```

```
[Out] c*ln(x)/a+1/2/b*ln(b*x^2+a)*d-1/2*c*ln(b*x^2+a)/a
```

Maxima [A] time = 0.998298, size = 47, normalized size = 1.38

$$\frac{c\log\left(x^2\right)}{2a}-\frac{(bc-ad)\log\left(bx^2+a\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(b*x^2 + a)/(a*b)
```

Fricas [A] time = 1.4653, size = 74, normalized size = 2.18

$$\frac{2bc\log(x)-(bc-ad)\log\left(bx^2+a\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*log(x) - (b*c - a*d)*log(b*x^2 + a))/(a*b)
```

Sympy [A] time = 0.65127, size = 26, normalized size = 0.76

$$\frac{c\log(x)}{a}+\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)/a + (a*d - b*c)*log(a/b + x**2)/(2*a*b)
```

Giac [A] time = 1.15308, size = 49, normalized size = 1.44

$$\frac{c\log\left(x^2\right)}{2\,a} - \frac{(bc-ad)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/(a*b)
```

3.205 $\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0203093, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {453, 205}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx &= -\frac{c}{ax}-\frac{(bc-ad)\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{c}{ax}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0253388, size = 42, normalized size = 0.98

$$\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.005, size = 48, normalized size = 1.1

$$-\frac{c}{ax}+d\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bc}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] -c/a/x+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-c*b/a/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50154, size = 230, normalized size = 5.35

$$\left[\frac{\sqrt{-ab}(bc-ad)x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,abc}{2\,a^2bx},-\frac{\sqrt{ab}(bc-ad)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+abc}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*c - a*d)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*c)/(a^2*b*x), -(sqrt(a*b)*(b*c - a*d)*x*arctan(sqrt(a*b)*x/a) + a
*b*c)/(a^2*b*x)]
```

Sympy [B] time = 0.459156, size = 82, normalized size = 1.91

$$-\frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*d - b*c)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*d - b*c)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - c/(a*x)
```

Giac [A] time = 1.15237, size = 50, normalized size = 1.16

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.206 $\int \frac{c+dx^2}{x^3(a+bx^2)} dx$

Optimal. Leaf size=50

$$\frac{(bc-ad)\log(a+bx^2)}{2a^2} - \frac{\log(x)(bc-ad)}{a^2} - \frac{c}{2ax^2}$$

```
[Out] -c/(2*a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/(2*a
^2)
```

Rubi [A] time = 0.045265, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{(bc-ad)\log(a+bx^2)}{2a^2} - \frac{\log(x)(bc-ad)}{a^2} - \frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/(2*a
^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{x^3(a+bx^2)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{ax^2} + \frac{-bc+ad}{a^2x} - \frac{b(-bc+ad)}{a^2(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{c}{2ax^2} - \frac{(bc-ad)\log(x)}{a^2} + \frac{(bc-ad)\log(a+bx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0201228, size = 49, normalized size = 0.98

$$\frac{(bc-ad)\log(a+bx^2)}{2a^2} + \frac{\log(x)(ad-bc)}{a^2} - \frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*a*x^2) + ((-(b*c) + a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/(
2*a^2)
```

Maple [A] time = 0.005, size = 56, normalized size = 1.1

$$-\frac{c}{2\,a x^2}+\frac{\ln(x)\,d}{a}-\frac{bc\ln(x)}{a^2}-\frac{\ln\left(bx^2+a\right)d}{2\,a}+\frac{bc\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*c/a/x^2+1/a*ln(x)*d-b*c*ln(x)/a^2-1/2/a*ln(b*x^2+a)*d+1/2*b*c*ln(b*x^2
+a)/a^2
```

Maxima [A] time = 0.976455, size = 65, normalized size = 1.3

$$\frac{(bc-ad)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*log(b*x^2 + a)/a^2 - 1/2*(b*c - a*d)*log(x^2)/a^2 - 1/2*c/(
a*x^2)
```

Fricas [A] time = 1.4719, size = 109, normalized size = 2.18

$$\frac{(bc-ad)x^2\log\left(bx^2+a\right)-2\,(bc-ad)x^2\log(x)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c - a*d)*x^2*log(b*x^2 + a) - 2*(b*c - a*d)*x^2*log(x) - a*c)/(a^2*
x^2)
```

Sympy [A] time = 0.782153, size = 41, normalized size = 0.82

$$-\frac{c}{2ax^2}+\frac{(ad-bc)\log(x)}{a^2}-\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*a*x**2) + (a*d - b*c)*log(x)/a**2 - (a*d - b*c)*log(a/b + x**2)/(2*a*
*2)
```

Giac [A] time = 1.14478, size = 97, normalized size = 1.94

$$-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c-abd\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bcx^2-adx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*log(x^2)/a^2 + 1/2*(b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^
2*b) + 1/2*(b*c*x^2 - a*d*x^2 - a*c)/(a^2*x^2)
```

3.207 $\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=59

$$\frac{bc-ad}{a^2x}+\frac{\sqrt{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{c}{3ax^3}$$

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0359807, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {453, 325, 205}

$$\frac{bc-ad}{a^2x}+\frac{\sqrt{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{c}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx = -\frac{c}{3ax^3} - \frac{(3bc-3ad)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{3a}$$
$$= -\frac{c}{3ax^3} + \frac{bc-ad}{a^2x} + \frac{(b(bc-ad))\int \frac{1}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{c}{3ax^3} + \frac{bc-ad}{a^2x} + \frac{\sqrt{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.050795, size = 60, normalized size = 1.02

$$\frac{bc-ad}{a^2x} - \frac{\sqrt{b}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{c}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) - (Sqrt[b]*(-(b*c) + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.005, size = 72, normalized size = 1.2

$$-\frac{c}{3\,ax^3} - \frac{d}{ax} + \frac{bc}{a^2x} - \frac{bd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{b^2c}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] -1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+
b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51599, size = 294, normalized size = 4.98

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 6\,(bc-ad)x^2 + 2\,ac}{6\,a^2x^3}, \frac{3\,(bc-ad)x^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 3\,(bc-ad)x^2 - ac}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b*c - a*d)*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) - 6*(b*c - a*d)*x^2 + 2*a*c)/(a^2*x^3), 1/3*(3*(b*c - a*d)*x^3*sqr
t(b/a)*arctan(x*sqrt(b/a)) + 3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)]
```

Sympy [B] time = 0.560699, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\left(ad-bc\right)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\left(ad-bc\right)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2}-\frac{ac+x^2\left(3ad-3bc\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(a*d - b*c)*log(-a**3*sqrt(-b/a**5)*(a*d - b*c)/(a*b*d - b**2
*c) + x)/2 - sqrt(-b/a**5)*(a*d - b*c)*log(a**3*sqrt(-b/a**5)*(a*d - b*c)/(
a*b*d - b**2*c) + x)/2 - (a*c + x**2*(3*a*d - 3*b*c))/(3*a**2*x**3)
```

Giac [A] time = 1.20487, size = 77, normalized size = 1.31

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*c*x^2 - 3*
a*d*x^2 - a*c)/(a^2*x^3)
```

3.208 $\int \frac{x^5(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=103

$$\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{x^4(bc-ad)^2}{4b^3}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{d^2x^8}{8b}$$

```
[Out] -(a*(b*c - a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c -
a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^2*Log[a + b*x^2])/(2
*b^5)
```

Rubi [A] time = 0.12215, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{x^4(bc-ad)^2}{4b^3}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c -
a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^2*Log[a + b*x^2])/(2
*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(c+dx)^2}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a(-bc+ad)^2}{b^4}+\frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^2}{b^2}+\frac{d^2x^3}{b}+\frac{a^2(-bc+ad)^2}{b^4(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{a(bc-ad)^2x^2}{2b^4}+\frac{(bc-ad)^2x^4}{4b^3}+\frac{d(2bc-ad)x^6}{6b^2}+\frac{d^2x^8}{8b}+\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.0521949, size = 116, normalized size = 1.13

$$\frac{\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)\log\left(a+bx^2\right)}{2b^5}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{x^4(bc-ad)^2}{4b^3}-\frac{ax^2(ad-bc)^2}{2b^4}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -(a*(-(b*c) + a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c
 - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + ((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2)*Log[a + b*x^2])/(2*b^5)
```

Maple [A] time = 0.003, size = 165, normalized size = 1.6

$$\frac{d^2x^8}{8\,b}-\frac{x^6ad^2}{6\,b^2}+\frac{x^6cd}{3\,b}+\frac{x^4a^2d^2}{4\,b^3}-\frac{x^4acd}{2\,b^2}+\frac{x^4c^2}{4\,b}-\frac{a^3d^2x^2}{2\,b^4}+\frac{x^2a^2cd}{b^3}-\frac{ac^2x^2}{2\,b^2}+\frac{a^4\ln\left(bx^2+a\right)d^2}{2\,b^5}-\frac{a^3\ln\left(bx^2+a\right)cd}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/8*d^2*x^8/b-1/6/b^2*x^6*a*d^2+1/3/b*x^6*c*d+1/4/b^3*x^4*a^2*d^2-1/2/b^2*x
^4*a*c*d+1/4/b*x^4*c^2-1/2/b^4*a^3*d^2*x^2+1/b^3*a^2*c*d*x^2-1/2/b^2*a*c^2*
x^2+1/2*a^4/b^5*ln(b*x^2+a)*d^2-a^3/b^4*ln(b*x^2+a)*c*d+1/2*a^2/b^3*ln(b*x^
2+a)*c^2
```

Maxima [A] time = 0.998989, size = 185, normalized size = 1.8

$$\frac{3\,b^3d^2x^8+4\left(2\,b^3cd-ab^2d^2\right)x^6+6\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^4-12\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)x^2}{24\,b^4}+\frac{\left(a^2b^2c^2-2\,a^3bcd+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 4*(2*b^3*c*d - a*b^2*d^2)*x^6 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^4 - 12*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)/b^4 + 1/
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 1.38708, size = 285, normalized size = 2.77

$$\frac{3\,b^4d^2x^8+4\left(2\,b^4cd-ab^3d^2\right)x^6+6\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4-12\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2+12\left(a^2b^2c^2-2\,a^3\right.}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^2*x^8 + 4*(2*b^4*c*d - a*b^3*d^2)*x^6 + 6*(b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*x^4 - 12*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 + 1
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a))/b^5
```

Sympy [A] time = 0.570223, size = 119, normalized size = 1.16

$$\frac{a^2 (ad-bc)^2 \log \left(a+bx^2\right)}{2b^5}+\frac{d^2x^8}{8b}-\frac{x^6 \left(ad^2-2bcd\right)}{6b^2}+\frac{x^4 \left(a^2d^2-2abcd+b^2c^2\right)}{4b^3}-\frac{x^2 \left(a^3d^2-2a^2bcd+ab^2c^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + d**2*x**8/(8*b) - x**6*(a*d*
*2 - 2*b*c*d)/(6*b**2) + x**4*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*b**3)
- x**2*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*b**4)
```

Giac [A] time = 1.20576, size = 200, normalized size = 1.94

$$\frac{3\,b^3d^2x^8+8\,b^3cdx^6-4\,ab^2d^2x^6+6\,b^3c^2x^4-12\,ab^2cdx^4+6\,a^2bd^2x^4-12\,ab^2c^2x^2+24\,a^2bcdx^2-12\,a^3d^2x^2}{24\,b^4}+\frac{\left(a^2b^2c^2-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 8*b^3*c*d*x^6 - 4*a*b^2*d^2*x^6 + 6*b^3*c^2*x^4 - 12*
a*b^2*c*d*x^4 + 6*a^2*b*d^2*x^4 - 12*a*b^2*c^2*x^2 + 24*a^2*b*c*d*x^2 - 12*
a^3*d^2*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(abs(b*x^2
+ a))/b^5
```

3.209 $\int \frac{x^4(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=105

$$\frac{a^{3/2}(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{x^3(bc-ad)^2}{3b^3}-\frac{ax(bc-ad)^2}{b^4}+\frac{d^2x^7}{7b}$$

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a*d)
*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.069337, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{a^{3/2}(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{x^3(bc-ad)^2}{3b^3}-\frac{ax(bc-ad)^2}{b^4}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a*d)
*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(9/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(c+dx^2\right)^2}{a+bx^2} dx &= \int \left(-\frac{a(bc-ad)^2}{b^4}+\frac{(bc-ad)^2x^2}{b^3}+\frac{d(2bc-ad)x^4}{b^2}+\frac{d^2x^6}{b}+\frac{a^2b^2c^2-2a^3bcd+a^4d^2}{b^4\left(a+bx^2\right)}\right) dx\\ &= -\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{\left(a^2(bc-ad)^2\right)\int \frac{1}{a+bx^2} dx}{b^4}\\ &= -\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{a^{3/2}(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0947547, size = 105, normalized size = 1.

$$\frac{a^{3/2}(ad-bc)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{x^3(bc-ad)^2}{3b^3}-\frac{ax(ad-bc)^2}{b^4}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(-(b*c) + a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a
*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(-(b*c) + a*d)^2*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.003, size = 176, normalized size = 1.7

$$\frac{d^2x^7}{7\,b}-\frac{x^5ad^2}{5\,b^2}+\frac{2\,x^5cd}{5\,b}+\frac{x^3a^2d^2}{3\,b^3}-\frac{2\,x^3acd}{3\,b^2}+\frac{x^3c^2}{3\,b}-\frac{a^3d^2x}{b^4}+2\,\frac{a^2cdx}{b^3}-\frac{ac^2x}{b^2}+\frac{a^4d^2}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{a^3cd}{b^3\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/7*d^2*x^7/b-1/5/b^2*x^5*a*d^2+2/5/b*x^5*c*d+1/3/b^3*x^3*a^2*d^2-2/3/b^2*x
^3*a*c*d+1/3/b*x^3*c^2-1/b^4*a^3*d^2*x+2/b^3*a^2*c*d*x-1/b^2*a*c^2*x+a^4/b^
4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2-2*a^3/b^3/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*c*d+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69026, size = 649, normalized size = 6.18

$$\left[\frac{30\,b^3d^2x^7+42\left(2\,b^3cd-ab^2d^2\right)x^5+70\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^3+105\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-}{bx^2+a}\right.}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^2*x^7 + 42*(2*b^3*c*d - a*b^2*d^2)*x^5 + 70*(b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*x)/b^4, 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^
2)*x^5 + 35*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)*x)/b^4]
```

Sympy [B] time = 0.645278, size = 240, normalized size = 2.29

$$-\frac{\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{d^2x^7}{7b}-\frac{x^5\left(ad^2-2bcd\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-a**3/b**9)*(a*d - b*c)**2*log(-b**4*sqrt(-a**3/b**9)*(a*d - b*c)**2/
(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + sqrt(-a**3/b**9)*(a*d - b
*c)**2*log(b**4*sqrt(-a**3/b**9)*(a*d - b*c)**2/(a**3*d**2 - 2*a**2*b*c*d +
 a*b**2*c**2) + x)/2 + d**2*x**7/(7*b) - x**5*(a*d**2 - 2*b*c*d)/(5*b**2) +
 x**3*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*b**3) - x*(a**3*d**2 - 2*a**2*
b*c*d + a*b**2*c**2)/b**4
```

Giac [A] time = 1.15823, size = 207, normalized size = 1.97

$$\frac{\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{15b^6d^2x^7+42b^6cdx^5-21ab^5d^2x^5+35b^6c^2x^3-70ab^5cdx^3+35a^2b^4d^2x^3}{105b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/105*(15*b^6*d^2*x^7 + 42*b^6*c*d*x^5 - 21*a*b^5*d^2*x^5 + 35*b^6*c^2*x
^3 - 70*a*b^5*c*d*x^3 + 35*a^2*b^4*d^2*x^3 - 105*a*b^5*c^2*x + 210*a^2*b^4*
c*d*x - 105*a^3*b^3*d^2*x)/b^7
```

3.210 $\int \frac{x^3(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=80

$$\frac{dx^4(2bc-ad)}{4b^2}+\frac{x^2(bc-ad)^2}{2b^3}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^6}{6b}$$

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^2*x^6)/(6*
b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0863416, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^4(2bc-ad)}{4b^2}+\frac{x^2(bc-ad)^2}{2b^3}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^2*x^6)/(6*
b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^2}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(bc-ad)^2}{b^3}+\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^2}{b}-\frac{a(-bc+ad)^2}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2x^2}{2b^3}+\frac{d(2bc-ad)x^4}{4b^2}+\frac{d^2x^6}{6b}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.0385701, size = 82, normalized size = 1.02

$$\frac{bx^2\left(6a^2d^2-3abd\left(4c+dx^2\right)+2b^2\left(3c^2+3cdx^2+d^2x^4\right)\right)-6a(bc-ad)^2\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d^2 - 3*a*b*d*(4*c + d*x^2) + 2*b^2*(3*c^2 + 3*c*d*x^2 + d^2*
x^4)) - 6*a*(b*c - a*d)^2*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.004, size = 124, normalized size = 1.6

$$\frac{d^2x^6}{6b} - \frac{x^4ad^2}{4b^2} + \frac{cx^4d}{2b} + \frac{x^2a^2d^2}{2b^3} - \frac{ax^2cd}{b^2} + \frac{x^2c^2}{2b} - \frac{a^3\ln\left(bx^2+a\right)d^2}{2b^4} + \frac{a^2\ln\left(bx^2+a\right)cd}{b^3} - \frac{a\ln\left(bx^2+a\right)c^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/6*d^2*x^6/b-1/4/b^2*x^4*a*d^2+1/2/b*x^4*c*d+1/2/b^3*x^2*a^2*d^2-1/b^2*x^2
*a*c*d+1/2/b*x^2*c^2-1/2*a^3/b^4*ln(b*x^2+a)*d^2+a^2/b^3*ln(b*x^2+a)*c*d-1/
2*a/b^2*ln(b*x^2+a)*c^2
```

Maxima [A] time = 0.982187, size = 136, normalized size = 1.7

$$\frac{2\,b^2d^2x^6 + 3\left(2\,b^2cd - abd^2\right)x^4 + 6\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^2}{12\,b^3} - \frac{\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 3*(2*b^2*c*d - a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a
)/b^4
```

Fricas [A] time = 1.64973, size = 212, normalized size = 2.65

$$\frac{2\,b^3d^2x^6 + 3\left(2\,b^3cd - ab^2d^2\right)x^4 + 6\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x^2 - 6\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^6 + 3*(2*b^3*c*d - a*b^2*d^2)*x^4 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a)
)/b^4
```

Sympy [A] time = 0.536022, size = 83, normalized size = 1.04

$$-\frac{a\left(ad-bc\right)^2\log\left(a+bx^2\right)}{2b^4} + \frac{d^2x^6}{6b} - \frac{x^4\left(ad^2-2bcd\right)}{4b^2} + \frac{x^2\left(a^2d^2-2abcd+b^2c^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + d**2*x**6/(6*b) - x**4*(a*d**2
 - 2*b*c*d)/(4*b**2) + x**2*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*b**3)
```

Giac [A] time = 1.14728, size = 144, normalized size = 1.8

$$\frac{2\,b^2d^2x^6 + 6\,b^2cdx^4 - 3\,abd^2x^4 + 6\,b^2c^2x^2 - 12\,abcdx^2 + 6\,a^2d^2x^2}{12\,b^3} - \frac{\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 6*b^2*c*d*x^4 - 3*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*lo
g(abs(b*x^2 + a))/b^4
```

3.211 $\int \frac{x^2\left(c+dx^2\right)^2}{a+bx^2} dx$

Optimal. Leaf size=84

$$\frac{dx^3(2bc-ad)}{3b^2}+\frac{x(bc-ad)^2}{b^3}-\frac{\sqrt{a}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{d^2x^5}{5b}$$

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.064104, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^3(2bc-ad)}{3b^2}+\frac{x(bc-ad)^2}{b^3}-\frac{\sqrt{a}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```


```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^2}{a+bx^2} dx &= \int \left(\frac{(bc-ad)^2}{b^3}+\frac{d(2bc-ad)x^2}{b^2}+\frac{d^2x^4}{b}+\frac{-ab^2c^2+2a^2bcd-a^3d^2}{b^3\left(a+bx^2\right)}\right) dx\\
&= \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\left(a(bc-ad)^2\right)\int \frac{1}{a+bx^2} dx}{b^3}\\
&= \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\sqrt{a}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0745149, size = 84, normalized size = 1.

$$\frac{dx^3(2bc-ad)}{3b^2}+\frac{x(bc-ad)^2}{b^3}-\frac{\sqrt{a}(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A] time = 0.003, size = 135, normalized size = 1.6

$$\frac{d^2x^5}{5b} - \frac{x^3ad^2}{3b^2} + \frac{2cx^3d}{3b} + \frac{a^2d^2x}{b^3} - 2\frac{acdx}{b^2} + \frac{c^2x}{b} - \frac{a^3d^2}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + 2\frac{a^2cd}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - \frac{ac^2}{b}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/5*d^2*x^5/b-1/3/b^2*x^3*a*d^2+2/3/b*x^3*c*d+1/b^3*a^2*d^2*x-2/b^2*c*a*d*x
+1/b*c^2*x-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+2*a^2/b^2/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*c*d-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72398, size = 489, normalized size = 5.82

$$\left[\frac{6\,b^2d^2x^5 + 10\left(2\,b^2cd - abd^2\right)x^3 + 15\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 30\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{30\,b^3}, \frac{3\,b^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 + 10*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) +
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3, 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c
*d - a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3]
```

Sympy [B] time = 0.598793, size = 192, normalized size = 2.29

$$\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(-\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^5}{5b}-\frac{x^3\left(ad^2-2bcd\right)}{3b^2}+\frac{x(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] sqrt(-a/b**7)*(a*d - b*c)**2*log(-b**3*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d
**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-a/b**7)*(a*d - b*c)**2*log(b**3
*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 +
d**2*x**5/(5*b) - x**3*(a*d**2 - 2*b*c*d)/(3*b**2) + x*(a**2*d**2 - 2*a*b*c
*d + b**2*c**2)/b**3
```

Giac [A] time = 1.1495, size = 153, normalized size = 1.82

$$-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{3\,b^4d^2x^5+10\,b^4cdx^3-5\,ab^3d^2x^3+15\,b^4c^2x-30\,ab^3cdx+15\,a^2b^2d^2x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^4*d^2*x^5 + 10*b^4*c*d*x^3 - 5*a*b^3*d^2*x^3 + 15*b^4*c^2*x - 3
0*a*b^3*c*d*x + 15*a^2*b^2*d^2*x)/b^5
```

3.212 $\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=61

$$\frac{dx^2(bc-ad)}{2b^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}+\frac{\left(c+dx^2\right)^2}{4b}$$

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^2*Log[a +
b*x^2])/(2*b^3)
```

Rubi [A] time = 0.0489202, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{dx^2(bc-ad)}{2b^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}+\frac{\left(c+dx^2\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^2*Log[a +
b*x^2])/(2*b^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx,x,x^2\right)\\ &= \frac{d(bc-ad)x^2}{2b^2}+\frac{\left(c+dx^2\right)^2}{4b}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0230791, size = 49, normalized size = 0.8

$$\frac{bdx^2\left(-2ad+4bc+bdx^2\right)+2(bc-ad)^2\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(4*b*c - 2*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[a + b*x^2])/(4*b^3
)
```

Maple [A] time = 0.002, size = 85, normalized size = 1.4

$$\frac{d^2x^4}{4\,b}-\frac{ad^2x^2}{2\,b^2}+\frac{dx^2c}{b}+\frac{\ln\left(bx^2+a\right)a^2d^2}{2\,b^3}-\frac{\ln\left(bx^2+a\right)cad}{b^2}+\frac{\ln\left(bx^2+a\right)c^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/4*d^2/b*x^4-1/2*d^2/b^2*a*x^2+d/b*x^2*c+1/2/b^3*ln(b*x^2+a)*a^2*d^2-1/b^2
*ln(b*x^2+a)*c*a*d+1/2/b*ln(b*x^2+a)*c^2
```

Maxima [A] time = 0.988173, size = 89, normalized size = 1.46

$$\frac{bd^2x^4+2\left(2\,bcd-ad^2\right)x^2}{4\,b^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 2*(2*b*c*d - a*d^2)*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 1.42728, size = 140, normalized size = 2.3

$$\frac{b^2d^2x^4+2\left(2\,b^2cd-abd^2\right)x^2+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 + 2*(2*b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(b*x^2 + a))/b^3
```

Sympy [A] time = 0.503939, size = 51, normalized size = 0.84

$$\frac{d^2x^4}{4b}-\frac{x^2\left(ad^2-2bcd\right)}{2b^2}+\frac{(ad-bc)^2\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] d**2*x**4/(4*b) - x**2*(a*d**2 - 2*b*c*d)/(2*b**2) + (a*d - b*c)**2*log(a +
 b*x**2)/(2*b**3)
```

Giac [A] time = 1.12555, size = 90, normalized size = 1.48

$$\frac{bd^2x^4 + 4\,bcdx^2 - 2\,ad^2x^2}{4\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*b*c*d*x^2 - 2*a*d^2*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(b*x^2 + a))/b^3
```

3.213 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{d^2x^3}{3b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0422312, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^2}{a+bx^2}\,dx &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^2)}\right)dx\\ &= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\int\frac{1}{a+bx^2}\,dx}{b^2}\\ &= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0517326, size = 59, normalized size = 0.94

$$\frac{dx(-3ad+6bc+bdx^2)}{3b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A] time = 0., size = 95, normalized size = 1.5

$$\frac{d^2x^3}{3b}-\frac{ad^2x}{b^2}+2\frac{dxc}{b}+\frac{a^2d^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\frac{acd}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^2*x^3/b-d^2/b^2*a*x+2*d/b*x*c+1/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*a^2*d^2-2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*c*d+1/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5016, size = 390, normalized size = 6.19

$$\left[\frac{2\,ab^2d^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(2\,ab^2cd-a^2bd^2\right)x}{6\,ab^3},\frac{ab^2d^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

Sympy [B] time = 0.545219, size = 172, normalized size = 2.73

$$-\frac{\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}-\frac{x\left(ad^2-2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a*b**5))*(a*d - b
*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
 b**2*c**2) + x)/2 + d**2*x**3/(3*b) - x*(a*d**2 - 2*b*c*d)/b**2
```

Giac [A] time = 1.17332, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.214 $\int \frac{(c+dx^2)^2}{x(a+bx^2)} \, dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

```
[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

Rubi [A] time = 0.0482164, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{b}+\frac{c^2}{ax}-\frac{(-bc+ad)^2}{ab(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{d^2x^2}{2b}+\frac{c^2\log(x)}{a}-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}
\end{aligned}$$

Mathematica [A] time = 0.0220223, size = 50, normalized size = 0.98

$$\frac{-(bc-ad)^2\log\left(a+bx^2\right)+abd^2x^2+2b^2c^2\log(x)}{2ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2 + 2*b^2*c^2*Log[x] - (b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

Maple [A] time = 0.005, size = 69, normalized size = 1.4

$$\frac{d^2x^2}{2b}+\frac{c^2\ln(x)}{a}-\frac{a\ln\left(bx^2+a\right)d^2}{2b^2}+\frac{\ln\left(bx^2+a\right)cd}{b}-\frac{\ln\left(bx^2+a\right)c^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a),x)
```

```
[Out] 1/2*d^2*x^2/b+c^2*ln(x)/a-1/2*a/b^2*ln(b*x^2+a)*d^2+1/b*ln(b*x^2+a)*c*d-1/2
/a*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.03126, size = 82, normalized size = 1.61

$$\frac{d^2x^2}{2b}+\frac{c^2\log\left(x^2\right)}{2a}-\frac{\left(b^2c^2-2abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(b*x^2 + a)/(a*b^2)
```

Fricas [A] time = 1.49935, size = 128, normalized size = 2.51

$$\frac{abd^2x^2+2b^2c^2\log(x)-\left(b^2c^2-2abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*d^2*x^2 + 2*b^2*c^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(b
*x^2 + a))/(a*b^2)
```

Sympy [A] time = 1.34536, size = 41, normalized size = 0.8

$$\frac{d^2x^2}{2b}+\frac{c^2\log(x)}{a}-\frac{(ad-bc)^2\log\left(\frac{a}{b}+x^2\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

```
[Out] d**2*x**2/(2*b) + c**2*log(x)/a - (a*d - b*c)**2*log(a/b + x**2)/(2*a*b**2)
```

Giac [A] time = 1.1355, size = 84, normalized size = 1.65

$$\frac{d^2x^2}{2\,b}+\frac{c^2\log\left(x^2\right)}{2\,a}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(b*x^2 + a))/(a*b^2)
```

3.215 $\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=55

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(
3/2)*b^(3/2))
```

Rubi [A] time = 0.0498151, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(
3/2)*b^(3/2))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^2}{b}+\frac{c^2}{ax^2}-\frac{(-bc+ad)^2}{ab\left(a+bx^2\right)}\right)dx\\
&= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{ab}\\
&= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0509508, size = 55, normalized size = 1.

$$-\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(3/2)*b^(3/2))
```

Maple [A] time = 0.005, size = 85, normalized size = 1.6

$$\frac{d^2x}{b}-\frac{c^2}{ax}-\frac{ad^2}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+2\frac{cd}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{bc^2}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a),x)
```

```
[Out] d^2*x/b-c^2/a/x-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+2/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*c*d-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46248, size = 344, normalized size = 6.25

$$\left[\frac{2\,a^2bd^2x^2-2\,ab^2c^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}x\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,a^2b^2x},\ \frac{a^2bd^2x^2-ab^2c^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}x\,\mathrm{a}}{a^2b^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*b*d^2*x^2 - 2*a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x), (a^2*b*
d^2*x^2 - a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x*arctan(sq
rt(a*b)*x/a))/(a^2*b^2*x)]
```

Sympy [B] time = 0.691259, size = 165, normalized size = 3.

$$\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad-bc\right)^2\log\left(-\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad-bc\right)^2\log\left(\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x}{b}-\frac{c^2}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(-a**2*b*sqrt(-1/(a**3*b**3))*(a*d -
 b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-1/(a**3*b**3))*
(a*d - b*c)**2*log(a**2*b*sqrt(-1/(a**3*b**3))*(a*d - b*c)**2/(a**2*d**2 -
2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x/b - c**2/(a*x)
```

Giac [A] time = 1.1572, size = 85, normalized size = 1.55

$$\frac{d^2x}{b} - \frac{c^2}{ax} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b)
```

3.216 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{(bc-ad)^2\log(a+bx^2)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

```
[Out] -c^2/(2*a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Log[a + b*x^
2])/(2*a^2*b)
```

Rubi [A] time = 0.0577754, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{(bc-ad)^2\log(a+bx^2)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^2/(2*a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Log[a + b*x^
2])/(2*a^2*b)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{ax^2}+\frac{c(-bc+2ad)}{a^2x}+\frac{(-bc+ad)^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c^2}{2ax^2}-\frac{c(bc-2ad)\log(x)}{a^2}+\frac{(bc-ad)^2\log(a+bx^2)}{2a^2b}\end{aligned}$$

Mathematica [A] time = 0.02763, size = 60, normalized size = 1.03

$$\frac{-abc^2-2bcx^2\log(x)(bc-2ad)+x^2(bc-ad)^2\log(a+bx^2)}{2a^2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*b*c^2) - 2*b*c*(b*c - 2*a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[a + b*
x^2])/(2*a^2*b*x^2)
```

Maple [A] time = 0.006, size = 81, normalized size = 1.4

$$-\frac{c^{2}}{2\,a x^{2}}+2\,\frac{c\ln\left(x\right)d}{a}-\frac{c^{2}\ln\left(x\right)b}{a^{2}}+\frac{\ln\left(b x^{2}+a\right)d^{2}}{2\,b}-\frac{\ln\left(b x^{2}+a\right)c d}{a}+\frac{b\ln\left(b x^{2}+a\right)c^{2}}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*c^2/a/x^2+2*c/a*ln(x)*d-c^2/a^2*ln(x)*b+1/2/b*ln(b*x^2+a)*d^2-1/a*ln(b
*x^2+a)*c*d+1/2/a^2*b*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.01627, size = 93, normalized size = 1.6

$$-\frac{\left(b c^{2}-2\,a c d\right)\log\left(x^{2}\right)}{2\,a^{2}}-\frac{c^{2}}{2\,a x^{2}}+\frac{\left(b^{2} c^{2}-2\,a b c d+a^{2} d^{2}\right)\log\left(b x^{2}+a\right)}{2\,a^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 - 1/2*c^2/(a*x^2) + 1/2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*log(b*x^2 + a)/(a^2*b)
```

Fricas [A] time = 1.51626, size = 159, normalized size = 2.74

$$-\frac{a b c^{2}-\left(b^{2} c^{2}-2\,a b c d+a^{2} d^{2}\right)x^{2}\log\left(b x^{2}+a\right)+2\left(b^{2} c^{2}-2\,a b c d\right)x^{2}\log\left(x\right)}{2\,a^{2} b x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(b*x^2 + a) + 2*(b^2
*c^2 - 2*a*b*c*d)*x^2*log(x))/(a^2*b*x^2)
```

Sympy [A] time = 1.51608, size = 49, normalized size = 0.84

$$-\frac{c^{2}}{2ax^{2}}+\frac{c\left(2ad-bc\right)\log\left(x\right)}{a^{2}}+\frac{\left(ad-bc\right)^{2}\log\left(\frac{a}{b}+x^{2}\right)}{2a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

```
[Out] -c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x)/a**2 + (a*d - b*c)**2*log(a/b + x
**2)/(2*a**2*b)
```

Giac [A] time = 1.17646, size = 122, normalized size = 2.1

$$-\frac{\left(bc^2-2\,acd\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bc^2x^2-2\,acdx^2-ac^2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c^2*x^2 - 2*a*c*d*x^2 - a*c^2)/(a^2*x^2
)
```

3.217 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=64

$$\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c^2}{3ax^3}$$

```
[Out] -c^2/(3*a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0546262, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^2/(3*a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{ax^4}+\frac{c(-bc+2ad)}{a^2x^2}+\frac{(-bc+ad)^2}{a^2\left(a+bx^2\right)}\right)dx\\ &= -\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0618717, size = 66, normalized size = 1.03

$$-\frac{c(2ad-bc)}{a^2x}+\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^2/(3*a*x^3) - (c*(-(b*c) + 2*a*d))/(a^2*x) + ((-(b*c) + a*d)^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.006, size = 98, normalized size = 1.5

$$-\frac{c^2}{3\,ax^3}-2\,\frac{cd}{ax}+\frac{bc^2}{a^2x}+d^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{bcd}{a\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{b^2c^2}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a),x)
```

```
[Out] -1/3*c^2/a/x^3-2*c/a/x*d+c^2/a^2/x*b+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
d^2-2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*b*d+1/a^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5038, size = 408, normalized size = 6.38

$$\left[-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}x^3\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,a^2bc^2-6\left(ab^2c^2-2\,a^2bcd\right)x^2}{6\,a^3bx^3},\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}x^3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*x^3*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*a^2*b*c^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)
/(a^3*b*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x^3*arctan(s
qrt(a*b)*x/a) - a^2*b*c^2 + 3*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)/(a^3*b*x^3)]
```

Sympy [B] time = 0.786519, size = 172, normalized size = 2.69

$$-\frac{\sqrt{-\frac{1}{a^5b}}\left(ad-bc\right)^2\log\left(-\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^5b}}\left(ad-bc\right)^2\log\left(\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{ac^2+x^2\left(6acd-3bc^2\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - b*c)**2*log(-a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a**5*b))*(a*d - b*c
)**2*log(a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 - (a*c**2 + x**2*(6*a*c*d - 3*b*c**2))/(3*a**2*x**3)
```

Giac [A] time = 1.18474, size = 97, normalized size = 1.52

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bc^2x^2 - 6\,acdx^2 - ac^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3
*(3*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^2*x^3)
```

3.218 $\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=138

$$\frac{dx^6(a^2d^2-3abcd+3b^2c^2)}{6b^3}+\frac{a^2(bc-ad)^3\log(a+bx^2)}{2b^6}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{x^4(bc-ad)^3}{4b^4}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{d^3x^{10}}{10b}$$

```
[Out] -(a*(b*c - a*d)^3*x^2)/(2*b^5) + ((b*c - a*d)^3*x^4)/(4*b^4) + (d*(3*b^2*c^
2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a*d)*x^8)/(8*b^2) + (
d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.178359, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{dx^6(a^2d^2-3abcd+3b^2c^2)}{6b^3}+\frac{a^2(bc-ad)^3\log(a+bx^2)}{2b^6}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{x^4(bc-ad)^3}{4b^4}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{d^3x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^3*x^2)/(2*b^5) + ((b*c - a*d)^3*x^4)/(4*b^4) + (d*(3*b^2*c^
2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a*d)*x^8)/(8*b^2) + (
d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^6)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx)^3}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-bc+ad)^3}{b^5}+\frac{(bc-ad)^3x}{b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)x^2}{b^3}+\frac{d^2(3bc-ad)x^3}{b^2}+\frac{d^3x^4}{b}\right.\right.\\ &= -\frac{a(bc-ad)^3x^2}{2b^5}+\frac{(bc-ad)^3x^4}{4b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)x^6}{6b^3}+\frac{d^2(3bc-ad)x^8}{8b^2}+\frac{d^3x^{10}}{10b}+\frac{a^2(bc-\end{aligned}$$

Mathematica [A] time = 0.0765649, size = 128, normalized size = 0.93

$$\frac{20b^3dx^6(a^2d^2-3abcd+3b^2c^2)+60a^2(bc-ad)^3\log(a+bx^2)+15b^4d^2x^8(3bc-ad)+30b^2x^4(bc-ad)^3+60abx^2(ad-b}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (60*a*b*(-(b*c) + a*d)^3*x^2 + 30*b^2*(b*c - a*d)^3*x^4 + 20*b^3*d*(3*b^2*c
^2 - 3*a*b*c*d + a^2*d^2)*x^6 + 15*b^4*d^2*(3*b*c - a*d)*x^8 + 12*b^5*d^3*x
^10 + 60*a^2*(b*c - a*d)^3*Log[a + b*x^2])/(120*b^6)
```

Maple [B] time = 0.005, size = 263, normalized size = 1.9

$$\frac{d^3x^{10}}{10\,b}-\frac{x^8ad^3}{8\,b^2}+\frac{3\,x^8cd^2}{8\,b}+\frac{x^6a^2d^3}{6\,b^3}-\frac{x^6acd^2}{2\,b^2}+\frac{x^6c^2d}{2\,b}-\frac{x^4a^3d^3}{4\,b^4}+\frac{3\,x^4a^2cd^2}{4\,b^3}-\frac{3\,x^4ac^2d}{4\,b^2}+\frac{x^4c^3}{4\,b}+\frac{a^4d^3x^2}{2\,b^5}-\frac{3\,a^3cd^2x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/10*d^3*x^10/b-1/8/b^2*x^8*a*d^3+3/8/b*x^8*c*d^2+1/6/b^3*x^6*a^2*d^3-1/2/b
^2*x^6*a*c*d^2+1/2/b*x^6*c^2*d-1/4/b^4*x^4*a^3*d^3+3/4/b^3*x^4*a^2*c*d^2-3/
4/b^2*x^4*a*c^2*d+1/4/b*x^4*c^3+1/2/b^5*a^4*d^3*x^2-3/2/b^4*a^3*c*d^2*x^2+3
/2/b^3*a^2*c^2*d*x^2-1/2/b^2*a*c^3*x^2-1/2*a^5/b^6*ln(b*x^2+a)*d^3+3/2*a^4/
b^5*ln(b*x^2+a)*c*d^2-3/2*a^3/b^4*ln(b*x^2+a)*c^2*d+1/2*a^2/b^3*ln(b*x^2+a)
*c^3
```

Maxima [A] time = 1.0417, size = 296, normalized size = 2.14

$$\frac{12\,b^4d^3x^{10}+15\left(3\,b^4cd^2-ab^3d^3\right)x^8+20\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^6+30\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^4}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 15*(3*b^4*c*d^2 - a*b^3*d^3)*x^8 + 20*(3*b^4*c^2*d
 - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^6 + 30*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*x^4 - 60*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 - a^4*d^3)*x^2)/b^5 + 1/2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a)/b^6
```

Fricas [A] time = 1.45105, size = 447, normalized size = 3.24

$$\frac{12\,b^5d^3x^{10}+15\left(3\,b^5cd^2-ab^4d^3\right)x^8+20\left(3\,b^5c^2d-3\,ab^4cd^2+a^2b^3d^3\right)x^6+30\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)x^4}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*d^3*x^10 + 15*(3*b^5*c*d^2 - a*b^4*d^3)*x^8 + 20*(3*b^5*c^2*d
 - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^6 + 30*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*x^4 - 60*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c
```

```
*d^2 - a^4*b*d^3)*x^2 + 60*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a))/b^6
```

Sympy [A] time = 0.729092, size = 187, normalized size = 1.36

$$-\frac{a^2(ad-bc)^3\log\left(a+bx^2\right)}{2b^6}+\frac{d^3x^{10}}{10b}-\frac{x^8\left(ad^3-3bcd^2\right)}{8b^2}+\frac{x^6\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{6b^3}-\frac{x^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d\right.}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + d**3*x**10/(10*b) - x**8*(a
*d**3 - 3*b*c*d**2)/(8*b**2) + x**6*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2
*d)/(6*b**3) - x**4*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c
**3)/(4*b**4) + x**2*(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*
b**3*c**3)/(2*b**5)
```

Giac [A] time = 1.15839, size = 321, normalized size = 2.33

$$\frac{12\,b^4d^3x^{10}+45\,b^4cd^2x^8-15\,ab^3d^3x^8+60\,b^4c^2dx^6-60\,ab^3cd^2x^6+20\,a^2b^2d^3x^6+30\,b^4c^3x^4-90\,ab^3c^2dx^4+90\,a^2b^2cd^2x}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 45*b^4*c*d^2*x^8 - 15*a*b^3*d^3*x^8 + 60*b^4*c^2*d
*x^6 - 60*a*b^3*c*d^2*x^6 + 20*a^2*b^2*d^3*x^6 + 30*b^4*c^3*x^4 - 90*a*b^3*
c^2*d*x^4 + 90*a^2*b^2*c*d^2*x^4 - 30*a^3*b*d^3*x^4 - 60*a*b^3*c^3*x^2 + 18
0*a^2*b^2*c^2*d*x^2 - 180*a^3*b*c*d^2*x^2 + 60*a^4*d^3*x^2)/b^5 + 1/2*(a^2*
b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(abs(b*x^2 + a))/b^
6
```

3.219 $\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=140

$$\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{x^3(bc-ad)^3}{3b^4}-\frac{ax(bc-ad)^3}{b^5}+\frac{d^3x^9}{9b}$$

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi [A] time = 0.0971003, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{x^3(bc-ad)^3}{3b^4}-\frac{ax(bc-ad)^3}{b^5}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(-\frac{a(bc-ad)^3}{b^5}+\frac{(bc-ad)^3x^2}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^4}{b^3}+\frac{d^2(3bc-ad)x^6}{b^2}+\frac{d^3x^8}{b}+\frac{a^2b\cdots}{\cdots}\right.\\ &= -\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+\left(a^2(bc\cdots\right.\\ &= -\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+\frac{a^{3/2}(b\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.0481145, size = 140, normalized size = 1.

$$\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}-\frac{a^{3/2}(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{ax(ad-bc)^3}{b^5}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)^3*x)/b^5 + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) - (a^(3/2)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2
)
```

Maple [B] time = 0.003, size = 276, normalized size = 2.

$$\frac{d^3x^9}{9b}-\frac{x^7ad^3}{7b^2}+\frac{3x^7cd^2}{7b}+\frac{x^5a^2d^3}{5b^3}-\frac{3x^5acd^2}{5b^2}+\frac{3x^5c^2d}{5b}-\frac{x^3a^3d^3}{3b^4}+\frac{x^3a^2cd^2}{b^3}-\frac{x^3ac^2d}{b^2}+\frac{x^3c^3}{3b}+\frac{a^4d^3x}{b^5}-3\frac{a^3cd^2x}{b^4}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/7/b^2*x^7*a*d^3+3/7/b*x^7*c*d^2+1/5/b^3*x^5*a^2*d^3-3/5/b^2
*x^5*a*c*d^2+3/5/b*x^5*c^2*d-1/3/b^4*x^3*a^3*d^3+1/b^3*x^3*a^2*c*d^2-1/b^2*
x^3*a*c^2*d+1/3/b*x^3*c^3+1/b^5*a^4*d^3*x-3/b^4*a^3*c*d^2*x+3/b^3*a^2*c^2*d
*x-1/b^2*a*c^3*x-a^5/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3+3*a^4/b^4/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2-3*a^3/b^3/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*c^2*d+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49598, size = 964, normalized size = 6.89

$$\left[\frac{70\,b^4d^3x^9+90\left(3\,b^4cd^2-ab^3d^3\right)x^7+126\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^5+210\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^3}{630\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*d^3*x^9 + 90*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 126*(3*b^4*c^2*
d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 210*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) -
```

```
630*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5, 1/315*
(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 63*(3*b^4*c^2*d - 3*a*
b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3)*x^3 + 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5]
```

Sympy [B] time = 0.814542, size = 338, normalized size = 2.41

$$\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}+\frac{d^3x^9}{9b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**11)*(a*d - b*c)**3*log(-b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3
/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3) + x)/2 -
sqrt(-a**3/b**11)*(a*d - b*c)**3*log(b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/
(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3) + x)/2 + d
**3*x**9/(9*b) - x**7*(a*d**3 - 3*b*c*d**2)/(7*b**2) + x**5*(a**2*d**3 - 3*
a*b*c*d**2 + 3*b**2*c**2*d)/(5*b**3) - x**3*(a**3*d**3 - 3*a**2*b*c*d**2 +
3*a*b**2*c**2*d - b**3*c**3)/(3*b**4) + x*(a**4*d**3 - 3*a**3*b*c*d**2 + 3*
a**2*b**2*c**2*d - a*b**3*c**3)/b**5
```

Giac [A] time = 1.20489, size = 325, normalized size = 2.32

$$\frac{\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}+\frac{35\,b^8d^3x^9+135\,b^8cd^2x^7-45\,ab^7d^3x^7+189\,b^8c^2dx^5-189\,ab^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*d^3*x^9 + 135*b^8*c*d^2*x^7 - 45*a*b^7
*d^3*x^7 + 189*b^8*c^2*d*x^5 - 189*a*b^7*c*d^2*x^5 + 63*a^2*b^6*d^3*x^5 + 1
05*b^8*c^3*x^3 - 315*a*b^7*c^2*d*x^3 + 315*a^2*b^6*c*d^2*x^3 - 105*a^3*b^5*
d^3*x^3 - 315*a*b^7*c^3*x + 945*a^2*b^6*c^2*d*x - 945*a^3*b^5*c*d^2*x + 315
*a^4*b^4*d^3*x)/b^9
```

3.220 $\int \frac{x^3(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=115

$$\frac{dx^4\left(a^2d^2-3abcd+3b^2c^2\right)}{4b^3}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{x^2(bc-ad)^3}{2b^4}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^8}{8b}$$

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^4)/(4*
b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b) - (a*(b*c - a*d)^3
*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.124793, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^4\left(a^2d^2-3abcd+3b^2c^2\right)}{4b^3}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{x^2(bc-ad)^3}{2b^4}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^4)/(4*
b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b) - (a*(b*c - a*d)^3
*Log[a + b*x^2])/(2*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^3}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(bc-ad)^3}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^3}{b}+\frac{a(-bc+ad)^3}{b^4(a+bx)}\right) d\right.\\ &= \frac{(bc-ad)^3x^2}{2b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^2(3bc-ad)x^6}{6b^2}+\frac{d^3x^8}{8b}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}\end{aligned}$$

Mathematica [A] time = 0.0561667, size = 125, normalized size = 1.09

$$\frac{bx^2\left(6a^2bd^2\left(6c+dx^2\right)-12a^3d^3-2ab^2d\left(18c^2+9cdx^2+2d^2x^4\right)+3b^3\left(6c^2dx^2+4c^3+4cd^2x^4+d^3x^6\right)\right)+12a(ad-bc}{24b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(-12*a^3*d^3 + 6*a^2*b*d^2*(6*c + d*x^2) - 2*a*b^2*d*(18*c^2 + 9*c*d
*x^2 + 2*d^2*x^4) + 3*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6)) +
12*a*(-(b*c) + a*d)^3*Log[a + b*x^2])/(24*b^5)
```

Maple [A] time = 0.003, size = 205, normalized size = 1.8

$$\frac{d^3x^8}{8\,b}-\frac{x^6ad^3}{6\,b^2}+\frac{x^6cd^2}{2\,b}+\frac{x^4a^2d^3}{4\,b^3}-\frac{3\,x^4acd^2}{4\,b^2}+\frac{3\,x^4c^2d}{4\,b}-\frac{a^3d^3x^2}{2\,b^4}+\frac{3\,x^2a^2cd^2}{2\,b^3}-\frac{3\,ac^2dx^2}{2\,b^2}+\frac{c^3x^2}{2\,b}+\frac{a^4\ln\left(bx^2+a\right)d^3}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/8*d^3*x^8/b-1/6/b^2*x^6*a*d^3+1/2/b*x^6*c*d^2+1/4/b^3*x^4*a^2*d^3-3/4/b^2
*x^4*a*c*d^2+3/4/b*x^4*c^2*d-1/2/b^4*a^3*d^3*x^2+3/2/b^3*a^2*c*d^2*x^2-3/2/
b^2*a*c^2*d*x^2+1/2/b*c^3*x^2+1/2*a^4/b^5*ln(b*x^2+a)*d^3-3/2*a^3/b^4*ln(b*
x^2+a)*c*d^2+3/2*a^2/b^3*ln(b*x^2+a)*c^2*d-1/2*a/b^2*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.02363, size = 227, normalized size = 1.97

$$\frac{3\,b^3d^3x^8+4\left(3\,b^3cd^2-ab^2d^3\right)x^6+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^4+12\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^2}{24\,b^4}-\frac{(ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 4*(3*b^3*c*d^2 - a*b^2*d^3)*x^6 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^4 + 12*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 1.40042, size = 342, normalized size = 2.97

$$\frac{3\,b^4d^3x^8+4\left(3\,b^4cd^2-ab^3d^3\right)x^6+6\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^4+12\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^2-1}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^3*x^8 + 4*(3*b^4*c*d^2 - a*b^3*d^3)*x^6 + 6*(3*b^4*c^2*d - 3*
a*b^3*c*d^2 + a^2*b^2*d^3)*x^4 + 12*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
```

```
d^2 - a^3*b*d^3)*x^2 - 12*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a))/b^5
```

Sympy [A] time = 0.67253, size = 136, normalized size = 1.18

$$\frac{a(ad-bc)^3\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^8}{8b}-\frac{x^6\left(ad^3-3bcd^2\right)}{6b^2}+\frac{x^4\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{4b^3}-\frac{x^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3\right.}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) + d**3*x**8/(8*b) - x**6*(a*d**3
- 3*b*c*d**2)/(6*b**2) + x**4*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*d)/(4
*b**3) - x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(
2*b**4)
```

Giac [A] time = 1.14939, size = 243, normalized size = 2.11

$$\frac{3\,b^3d^3x^8+12\,b^3cd^2x^6-4\,ab^2d^3x^6+18\,b^3c^2dx^4-18\,ab^2cd^2x^4+6\,a^2bd^3x^4+12\,b^3c^3x^2-36\,ab^2c^2dx^2+36\,a^2bcd^2x^2-12\,a}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 12*b^3*c*d^2*x^6 - 4*a*b^2*d^3*x^6 + 18*b^3*c^2*d*x^4
 - 18*a*b^2*c*d^2*x^4 + 6*a^2*b*d^3*x^4 + 12*b^3*c^3*x^2 - 36*a*b^2*c^2*d*x
^2 + 36*a^2*b*c*d^2*x^2 - 12*a^3*d^3*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*
c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(abs(b*x^2 + a))/b^5
```

3.221 $\int \frac{x^2\left(c+dx^2\right)^3}{a+bx^2} \, dx$

Optimal. Leaf size=119

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{x(bc-ad)^3}{b^4}-\frac{\sqrt{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{d^3x^7}{7b}$$

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.0847938, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{x(bc-ad)^3}{b^4}-\frac{\sqrt{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^3}{a+bx^2} \, dx &= \int \left(\frac{(bc-ad)^3}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^2}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^6}{b}+\frac{-ab^3c^3+3a^2b^2c^2d-}{b^4\left(a+b\right.}\right.\\
&= \frac{(bc-ad)^3x}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^7}{7b}-\frac{\left(a(bc-ad)^3\right)\int \frac{1}{a+bx^2} \, dx}{b^4}\\
&= \frac{(bc-ad)^3x}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^7}{7b}-\frac{\sqrt{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0390418, size = 118, normalized size = 0.99

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{x(bc-ad)^3}{b^4}+\frac{\sqrt{a}(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) + (Sqrt[a]*(-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [B] time = 0.003, size = 218, normalized size = 1.8

$$\frac{d^3x^7}{7b} - \frac{x^5ad^3}{5b^2} + \frac{3x^5cd^2}{5b} + \frac{x^3a^2d^3}{3b^3} - \frac{x^3acd^2}{b^2} + \frac{x^3c^2d}{b} - \frac{a^3d^3x}{b^4} + 3\frac{a^2cd^2x}{b^3} - 3\frac{ac^2dx}{b^2} + \frac{c^3x}{b} + \frac{a^4d^3}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/5/b^2*x^5*a*d^3+3/5/b*x^5*c*d^2+1/3/b^3*x^3*a^2*d^3-1/b^2*x
^3*a*c*d^2+1/b*x^3*c^2*d-1/b^4*a^3*d^3*x+3/b^3*a^2*c*d^2*x-3/b^2*a*c^2*d*x+
1/b*c^3*x+a^4/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3-3*a^3/b^3/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2+3*a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*c^2*d-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55906, size = 757, normalized size = 6.36

$$\left[\frac{30\,b^3d^3x^7 + 42\left(3\,b^3cd^2 - ab^2d^3\right)x^5 + 70\left(3\,b^3c^2d - 3\,ab^2cd^2 + a^2bd^3\right)x^3 - 105\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{-\frac{a}{b}}\,\mathrm{l}}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^3*x^7 + 42*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 70*(3*b^3*c^2*d
 - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
 + 210*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4, 1/105*(1
5*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 35*(3*b^3*c^2*d - 3*a*b^
2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
```

```
^3*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4]
```

Sympy [B] time = 0.746042, size = 275, normalized size = 2.31

$$-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^7}{7b}-\frac{x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**9)*(a*d - b*c)**3*log(-b**4*sqrt(-a/b**9)*(a*d - b*c)**3/(a**3*
d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + sqrt(-a/b**9
)*(a*d - b*c)**3*log(b**4*sqrt(-a/b**9)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*
b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**7/(7*b) - x**5*(a*
d**3 - 3*b*c*d**2)/(5*b**2) + x**3*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*
d)/(3*b**3) - x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)
/b**4
```

Giac [A] time = 1.22829, size = 248, normalized size = 2.08

$$-\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{15\,b^6d^3x^7+63\,b^6cd^2x^5-21\,ab^5d^3x^5+105\,b^6c^2dx^3-105\,ab^5c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^3*x^7 + 63*b^6*c*d^2*x^5 - 21*a*b^5*d
^3*x^5 + 105*b^6*c^2*d*x^3 - 105*a*b^5*c*d^2*x^3 + 35*a^2*b^4*d^3*x^3 + 105
*b^6*c^3*x - 315*a*b^5*c^2*d*x + 315*a^2*b^4*c*d^2*x - 105*a^3*b^3*d^3*x)/b
^7
```

3.222 $\int \frac{x(c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=87

$$\frac{dx^2(bc-ad)^2}{2b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4b^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}+\frac{\left(c+dx^2\right)^3}{6b}$$

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^2) + (c +
d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0811352, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{dx^2(bc-ad)^2}{2b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4b^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}+\frac{\left(c+dx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^2) + (c +
d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx,x,x^2\right)\\
&= \frac{d(bc-ad)^2x^2}{2b^3}+\frac{(bc-ad)\left(c+dx^2\right)^2}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0303811, size = 82, normalized size = 0.94

$$\frac{bdx^2\left(6a^2d^2-3abd\left(6c+dx^2\right)+b^2\left(18c^2+9cdx^2+2d^2x^4\right)\right)+6(bc-ad)^3\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x^2) + b^2*(18*c^2 + 9*c*d*x^2 + 2*d
^2*x^4)) + 6*(b*c - a*d)^3*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.003, size = 149, normalized size = 1.7

$$\frac{d^3x^6}{6b}-\frac{d^3x^4a}{4b^2}+\frac{3d^2x^4c}{4b}+\frac{d^3x^2a^2}{2b^3}-\frac{3d^2x^2ac}{2b^2}+\frac{3dx^2c^2}{2b}-\frac{\ln\left(bx^2+a\right)a^3d^3}{2b^4}+\frac{3\ln\left(bx^2+a\right)a^2cd^2}{2b^3}-\frac{3\ln\left(bx^2+a\right)a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/6*d^3/b*x^6-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+1/2*d^3/b^3*x^2*a^2-3/2*d^2
/b^2*x^2*a*c+3/2*d/b*x^2*c^2-1/2/b^4*ln(b*x^2+a)*a^3*d^3+3/2/b^3*ln(b*x^2+a
)*a^2*c*d^2-3/2/b^2*ln(b*x^2+a)*a*c^2*d+1/2/b*ln(b*x^2+a)*c^3
```

Maxima [A] time = 0.9848, size = 161, normalized size = 1.85

$$\frac{2b^2d^3x^6+3\left(3b^2cd^2-abd^3\right)x^4+6\left(3b^2c^2d-3abcd^2+a^2d^3\right)x^2}{12b^3}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 6*(3*b^2*c^2*d - 3*a*
b*c*d^2 + a^2*d^3)*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.44448, size = 244, normalized size = 2.8

$$\frac{2b^3d^3x^6+3\left(3b^3cd^2-ab^2d^3\right)x^4+6\left(3b^3c^2d-3ab^2cd^2+a^2bd^3\right)x^2+6\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(bx^2+\right.}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^3*x^6 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 0.625289, size = 88, normalized size = 1.01

$$\frac{d^3x^6}{6b}-\frac{x^4\left(ad^3-3bcd^2\right)}{4b^2}+\frac{x^2\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{2b^3}-\frac{(ad-bc)^3\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] d**3*x**6/(6*b) - x**4*(a*d**3 - 3*b*c*d**2)/(4*b**2) + x**2*(a**2*d**3 - 3
*a*b*c*d**2 + 3*b**2*c**2*d)/(2*b**3) - (a*d - b*c)**3*log(a + b*x**2)/(2*b
**4)
```

Giac [A] time = 1.16644, size = 167, normalized size = 1.92

$$\frac{2\,b^2d^3x^6+9\,b^2cd^2x^4-3\,abd^3x^4+18\,b^2c^2dx^2-18\,abcd^2x^2+6\,a^2d^3x^2}{12\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 9*b^2*c*d^2*x^4 - 3*a*b*d^3*x^4 + 18*b^2*c^2*d*x^2 -
18*a*b*c*d^2*x^2 + 6*a^2*d^3*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/b^4
```

3.223 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^3x^5}{5b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rubi [A] time = 0.0555838, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.066009, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

Maple [A] time = 0., size = 161, normalized size = 1.6

$$\frac{d^3x^5}{5b} - \frac{d^3x^3a}{3b^2} + \frac{d^2x^3c}{b} + \frac{a^2d^3x}{b^3} - 3\frac{ad^2cx}{b^2} + 3\frac{dc^2x}{b} - \frac{a^3d^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + 3\frac{a^2cd^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3\frac{ac^2d}{b\sqrt{ab}}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5*d^3*x^5/b-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c*x+3
*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*d^3+3/b^2/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*a^2*c*d^2-3/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*a*c^2*d+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5064, size = 613, normalized size = 6.26

$$\left[\frac{6\,ab^3d^3x^5 + 10\left(3\,ab^3cd^2 - a^2b^2d^3\right)x^3 + 15\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right) + 30\left(3\,ab^3c^2d - \right.}{30\,ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

Sympy [B] time = 0.719599, size = 240, normalized size = 2.45

$$\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}-\frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*
*3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 - sqr
t(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)**3/(
a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x*
*5/(5*b) - x**3*(a*d**3 - 3*b*c*d**2)/(3*b**2) + x*(a**2*d**3 - 3*a*b*c*d**
2 + 3*b**2*c**2*d)/b**3
```

Giac [A] time = 1.17654, size = 174, normalized size = 1.78

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{3\,b^4d^3x^5+15\,b^4cd^2x^3-5\,ab^3d^3x^3+45\,b^4c^2dx-45\,ab^3cd^2x+15}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

3.224 $\int \frac{(c+dx^2)^3}{x(a+bx^2)} \, dx$

Optimal. Leaf size=73

$$\frac{d^2x^2(3bc-ad)}{2b^2}-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/a - ((b*c
- a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rubi [A] time = 0.0772613, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{d^2x^2(3bc-ad)}{2b^2}-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/a - ((b*c
- a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^3}{x(a+bx^2)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax}+\frac{d^3x}{b}+\frac{(-bc+ad)^3}{ab^2(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{d^2(3bc-ad)x^2}{2b^2}+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}
\end{aligned}$$

Mathematica [A] time = 0.0302298, size = 65, normalized size = 0.89

$$\frac{abd^2x^2\left(-2ad+6bc+bdx^2\right)-2(bc-ad)^3\log\left(a+bx^2\right)+4b^3c^3\log(x)}{4ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2*(6*b*c - 2*a*d + b*d*x^2) + 4*b^3*c^3*Log[x] - 2*(b*c - a*d)^3
*Log[a + b*x^2])/(4*a*b^3)
```

Maple [A] time = 0.004, size = 116, normalized size = 1.6

$$\frac{d^3x^4}{4b}-\frac{d^3ax^2}{2b^2}+\frac{3d^2x^2c}{2b}+\frac{c^3\ln(x)}{a}+\frac{a^2\ln\left(bx^2+a\right)d^3}{2b^3}-\frac{3a\ln\left(bx^2+a\right)cd^2}{2b^2}+\frac{3\ln\left(bx^2+a\right)c^2d}{2b}-\frac{\ln\left(bx^2+a\right)c^3}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a),x)
```

```
[Out] 1/4*d^3*x^4/b-1/2*d^3/b^2*a*x^2+3/2*d^2/b*x^2*c+c^3*ln(x)/a+1/2*a^2/b^3*ln(
b*x^2+a)*d^3-3/2*a/b^2*ln(b*x^2+a)*c*d^2+3/2/b*ln(b*x^2+a)*c^2*d-1/2/a*ln(b
*x^2+a)*c^3
```

Maxima [A] time = 1.01955, size = 132, normalized size = 1.81

$$\frac{c^3\log\left(x^2\right)}{2a}+\frac{bd^3x^4+2\left(3bcd^2-ad^3\right)x^2}{4b^2}-\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{2ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - a*d^3)*x^2)/b^2 - 1/2*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a*b^3)
```

Fricas [A] time = 1.52583, size = 209, normalized size = 2.86

$$\frac{ab^2d^3x^4+4b^3c^3\log(x)+2\left(3ab^2cd^2-a^2bd^3\right)x^2-2\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(a*b^2*d^3*x^4 + 4*b^3*c^3*log(x) + 2*(3*a*b^2*c*d^2 - a^2*b*d^3)*x^2 -
 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a))/(a*b
^3)
```

Sympy [A] time = 1.91193, size = 63, normalized size = 0.86

$$\frac{d^3x^4}{4b}-\frac{x^2\left(ad^3-3bcd^2\right)}{2b^2}+\frac{c^3\log(x)}{a}+\frac{(ad-bc)^3\log\left(\frac{a}{b}+x^2\right)}{2ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

```
[Out] d**3*x**4/(4*b) - x**2*(a*d**3 - 3*b*c*d**2)/(2*b**2) + c**3*log(x)/a + (a*
d - b*c)**3*log(a/b + x**2)/(2*a*b**3)
```

Giac [A] time = 1.15656, size = 134, normalized size = 1.84

$$\frac{c^3 \log\left(x^2\right)}{2\,a} + \frac{bd^3x^4 + 6\,bcd^2x^2 - 2\,ad^3x^2}{4\,b^2} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 6*b*c*d^2*x^2 - 2*a*d^3*x^2)/b^2 - 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(
a*b^3)
```

3.225 $\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=77

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.064137, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax^2}+\frac{d^3x^2}{b}+\frac{(-bc+ad)^3}{ab^2\left(a+bx^2\right)}\right)dx\\
&= -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{ab^2}\\
&= -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0322889, size = 76, normalized size = 0.99

$$\frac{(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) + ((-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A] time = 0.005, size = 135, normalized size = 1.8

$$\frac{d^3x^3}{3b} - \frac{d^3ax}{b^2} + 3\frac{d^2xc}{b} - \frac{c^3}{ax} + \frac{a^2d^3}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - 3\frac{acd^2}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3\frac{c^2d}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - \frac{bc^3}{a}\operatorname{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^3*x^3/b-d^3/b^2*a*x+3*d^2/b*x*c-c^3/a/x+a^2/b^2/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2))*d^3-3*a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2+3/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d-1/a*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51257, size = 520, normalized size = 6.75

$$\left[\frac{2\,a^2b^2d^3x^4 - 6\,ab^3c^3 + 3\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{-ab}x\log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) + 6\left(3\,a^2b^2cd^2 - a^3bd^3\right)x^2}{6\,a^2b^3x}, \frac{a^2b^2d^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*d^3*x^4 - 6*a*b^3*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) + 6*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*d^3*x^4
 - 3*a*b^3*c^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b
^3*x)]
```

Sympy [B] time = 0.931007, size = 221, normalized size = 2.87

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^3}{3b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(-a**2*b**2*sqrt(-1/(a**3*b**5))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**
3) + x)/2 + d**3*x**3/(3*b) - x*(a*d**3 - 3*b*c*d**2)/b**2 - c**3/(a*x)
```

Giac [A] time = 1.13286, size = 140, normalized size = 1.82

$$-\frac{c^3}{ax}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}+\frac{b^2d^3x^3+9\,b^2cd^2x-3\,abd^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^3/(a*x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*d^3*x^3 + 9*b^2*c*d^2*x - 3*a*b*d^
3*x)/b^3
```

3.226 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)} dx$

Optimal. Leaf size=73

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

```
[Out] -c^3/(2*a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2 + ((b*c -
 a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rubi [A] time = 0.0759506, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^3/(2*a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2 + ((b*c -
 a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^2}+\frac{c^2(-bc+3ad)}{a^2x}-\frac{(-bc+ad)^3}{a^2b(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}-\frac{c^2(bc-3ad)\log(x)}{a^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}\end{aligned}$$

Mathematica [A] time = 0.0367246, size = 75, normalized size = 1.03

$$\frac{-2b^2c^2x^2\log(x)(bc-3ad)+ab\left(ad^3x^4-bc^3\right)+x^2(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] (a*b*(-(b*c^3) + a*d^3*x^4) - 2*b^2*c^2*(b*c - 3*a*d)*x^2*Log[x] + (b*c - a
*d)^3*x^2*Log[a + b*x^2])/(2*a^2*b^2*x^2)
```

Maple [A] time = 0.007, size = 114, normalized size = 1.6

$$\frac{d^3x^2}{2b}-\frac{c^3}{2ax^2}+3\,\frac{c^2\ln(x)\,d}{a}-\frac{c^3\ln(x)\,b}{a^2}-\frac{a\ln\left(bx^2+a\right)d^3}{2b^2}+\frac{3\,\ln\left(bx^2+a\right)cd^2}{2b}-\frac{3\,\ln\left(bx^2+a\right)c^2d}{2a}+\frac{b\ln\left(bx^2+a\right.}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a),x)
```

```
[Out] 1/2*d^3*x^2/b-1/2*c^3/a/x^2+3*c^2/a*ln(x)*d-c^3/a^2*ln(x)*b-1/2*a/b^2*ln(b*
x^2+a)*d^3+3/2/b*ln(b*x^2+a)*c*d^2-3/2/a*ln(b*x^2+a)*c^2*d+1/2/a^2*b*ln(b*x
^2+a)*c^3
```

Maxima [A] time = 0.992942, size = 131, normalized size = 1.79

$$\frac{d^3x^2}{2b}-\frac{c^3}{2ax^2}-\frac{\left(bc^3-3ac^2d\right)\log\left(x^2\right)}{2a^2}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*c^3/(a*x^2) - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a^2*b
^2)
```

Fricas [A] time = 1.5159, size = 217, normalized size = 2.97

$$\frac{a^2bd^3x^4-ab^2c^3+\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)x^2\log\left(bx^2+a\right)-2\left(b^3c^3-3ab^2c^2d\right)x^2\log(x)}{2a^2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*d^3*x^4 - a*b^2*c^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*x^2*log(b*x^2 + a) - 2*(b^3*c^3 - 3*a*b^2*c^2*d)*x^2*log(x))/(a^2
*b^2*x^2)
```

Sympy [A] time = 2.38139, size = 63, normalized size = 0.86

$$\frac{d^3x^2}{2b}-\frac{c^3}{2ax^2}+\frac{c^2\,(3ad-bc)\log(x)}{a^2}-\frac{(ad-bc)^3\log\left(\frac{a}{b}+x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

```
[Out] d**3*x**2/(2*b) - c**3/(2*a*x**2) + c**2*(3*a*d - b*c)*log(x)/a**2 - (a*d -
 b*c)**3*log(a/b + x**2)/(2*a**2*b**2)
```

Giac [A] time = 1.14639, size = 162, normalized size = 2.22

$$\frac{d^3x^2}{2b} - \frac{\left(bc^3 - 3ac^2d\right)\log\left(x^2\right)}{2a^2} + \frac{\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2a^2b^2} + \frac{bc^3x^2 - 3ac^2dx^2 - ac^3}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^2) + 1/2*(b*
c^3*x^2 - 3*a*c^2*d*x^2 - a*c^3)/(a^2*x^2)
```

3.227 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0657843, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^3}{b}+\frac{c^3}{ax^4}+\frac{c^2(-bc+3ad)}{a^2x^2}-\frac{(-bc+ad)^3}{a^2b\left(a+bx^2\right)}\right)dx\\
&= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\int \frac{1}{a+bx^2}\,dx}{a^2b}\\
&= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0390994, size = 74, normalized size = 1.

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [B] time = 0.007, size = 135, normalized size = 1.8

$$\frac{d^3x}{b} - \frac{c^3}{3\,ax^3} - 3\,\frac{c^2d}{ax} + \frac{bc^3}{a^2x} - \frac{ad^3}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + 3\,\frac{cd^2}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3\,\frac{bc^2d}{a\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) + \frac{b^2c^3}{a^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a),x)
```

```
[Out] d^3*x/b-1/3*c^3/a/x^3-3*c^2/a/x*d+c^3/a^2/x*b-a/b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*d^3+3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2-3/a*b/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))*c^2*d+1/a^2*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4524, size = 531, normalized size = 7.18

$$\left[\frac{6\,a^3bd^3x^4 - 2\,a^2b^2c^3 + 3\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{-ab}x^3\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 6\left(ab^3c^3 - 3\,a^2b^2c^2d\right)x^2}{6\,a^3b^2x^3}, \frac{3\,a^3b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*d^3*x^4 - 2*a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x^3*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 6*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*d^3
*x^4 - a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/
(a^3*b^2*x^3)]
```

Sympy [B] time = 1.23825, size = 221, normalized size = 2.99

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x}{b}-\frac{ac\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(-a**3*b*sqrt(-1/(a**5*b**3))*(a*d -
 b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2
 - sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(a**3*b*sqrt(-1/(a**5*b**3))*(a*d
 - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 + d**3*x/b - (a*c**3 + x**2*(9*a*c**2*d - 3*b*c**3))/(3*a**2*x**3)
```

Giac [A] time = 1.15046, size = 135, normalized size = 1.82

$$\frac{d^3x}{b}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b}+\frac{3\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^3)/(a^2
*x^3)
```

3.228 $\int \frac{x^5}{(a+bx^2)(c+dx^2)} dx$

Optimal. Leaf size=70

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^2(bc-ad)} - \frac{c^2 \log\left(c+dx^2\right)}{2d^2(bc-ad)} + \frac{x^2}{2bd}$$

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^2
])/(2*d^2*(b*c - a*d))
```

Rubi [A] time = 0.0668481, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^2(bc-ad)} - \frac{c^2 \log\left(c+dx^2\right)}{2d^2(bc-ad)} + \frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^2
])/(2*d^2*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{\left(a+bx^2\right)\left(c+dx^2\right)} dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)} dx, x, x^2\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^2\right) \\ &= \frac{x^2}{2bd} + \frac{a^2 \log\left(a+bx^2\right)}{2b^2(bc-ad)} - \frac{c^2 \log\left(c+dx^2\right)}{2d^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.0302386, size = 66, normalized size = 0.94

$$\frac{a^2d^2 \log\left(a+bx^2\right) - b\left(dx^2(ad-bc) + bc^2 \log\left(c+dx^2\right)\right)}{2b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^2] - b*(d*(-(b*c) + a*d)*x^2 + b*c^2*Log[c + d*x^2]))/
(2*b^2*d^2*(b*c - a*d))
```

Maple [A] time = 0.007, size = 65, normalized size = 0.9

$$\frac{x^2}{2\,bd}+\frac{c^2\ln\left(dx^2+c\right)}{(2\,ad-2\,bc)\,d^2}-\frac{a^2\ln\left(bx^2+a\right)}{(2\,ad-2\,bc)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*x^2/b/d+1/2*c^2/(a*d-b*c)/d^2*ln(d*x^2+c)-1/2*a^2/(a*d-b*c)/b^2*ln(b*x^
2+a)
```

Maxima [A] time = 0.985118, size = 92, normalized size = 1.31

$$\frac{a^2\log\left(bx^2+a\right)}{2\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^2+c\right)}{2\left(bcd^2-ad^3\right)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/(b^3*c - a*b^2*d) - 1/2*c^2*log(d*x^2 + c)/(b*c*d^2
- a*d^3) + 1/2*x^2/(b*d)
```

Fricas [A] time = 1.57231, size = 142, normalized size = 2.03

$$\frac{a^2d^2\log\left(bx^2+a\right)-b^2c^2\log\left(dx^2+c\right)+\left(b^2cd-abd^2\right)x^2}{2\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*log(b*x^2 + a) - b^2*c^2*log(d*x^2 + c) + (b^2*c*d - a*b*d^2)*
x^2)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [B] time = 3.12081, size = 201, normalized size = 2.87

$$-\frac{a^2\log\left(x^2+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{2b^2\,(ad-bc)}+\frac{c^2\log\left(x^2+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{2d^2\,(ad-bc)}+\frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -a**2*log(x**2 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a
**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(2
*b**2*(a*d - b*c)) + c**2*log(x**2 + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d
 + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*
d**2 + b**2*c**2))/(2*d**2*(a*d - b*c)) + x**2/(2*b*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.229 $\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=78

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a*d)) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d))
```

Rubi [A] time = 0.0834722, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {479, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a*d)) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{bd}$$
$$= \frac{x}{bd} + \frac{a^2 \int \frac{1}{a+bx^2}\,dx}{b(bc-ad)} - \frac{c^2 \int \frac{1}{c+dx^2}\,dx}{d(bc-ad)}$$
$$= \frac{x}{bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}$$

Mathematica [A] time = 0.0899106, size = 74, normalized size = 0.95

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{ax}{b} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}} + \frac{cx}{d}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((a*x)/b) + (c*x)/d + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2))/(b*c - a*d)
```

Maple [A] time = 0.007, size = 73, normalized size = 0.9

$$\frac{x}{bd} + \frac{c^2}{(ad-bc)\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{a^2}{(ad-bc)\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] x/b/d+1/d*c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/b*a^2/(a*d-b*
c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64524, size = 810, normalized size = 10.38

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right) - 2\,(bc-ad)x}{2\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\cdots}{dx^2+\cdots}\right)\cdots}{2\,(b^2cd-abd^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), 1/2*(2*a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), -1/2*(2*b*c*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + a*
d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b*c - a*d
)*x)/(b^2*c*d - a*b*d^2), (a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c*sqrt
(c/d)*arctan(d*x*sqrt(c/d)/c) + (b*c - a*d)*x)/(b^2*c*d - a*b*d^2)]
```

Sympy [B] time = 4.57926, size = 921, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**3*b
**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b**3)
**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c)**
3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-a**
3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2*(a*d -
 b*c)) + sqrt(-a**3/b**3)*log(x + (a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) +
 a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*(-a**
3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d -
 b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c**4*sq
rt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2
*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x + (-a**4*d**4*sqrt(-c**3/d**3)/(a*d
- b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d*
*5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d**3)**(3/2
)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - b**4
*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c
**3))/(2*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x + (a**4*d**4*sqrt(-c**3/d**3
)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a**2*b*
*4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-c**3/d**3
)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3
 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a
*b**2*c**3))/(2*(a*d - b*c)) + x/(b*d)
```

Giac [B] time = 1.30069, size = 576, normalized size = 7.38

$$-\frac{\left(\sqrt{ab}b^{3}c^{2}d|b| + \sqrt{ab}a^{2}bd^{3}|b| + \sqrt{ab}bc\left|-b^{2}cd + abd^{2}\right||b| + \sqrt{ab}ad\left|-b^{2}cd + abd^{2}\right||b|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b^{2}cd+abd^{2}+\sqrt{-4ab^{3}cd^{3}+(b^{2}cd+abd^{2}}}{b^{2}d^{2}}}}\right.}{b^{4}cd\left|-b^{2}cd + abd^{2}\right| + ab^{3}d^{2}\left|-b^{2}cd + abd^{2}\right| + \left(b^{2}cd - abd^{2}\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(sqrt(a*b)*b^3*c^2*d*abs(b) + sqrt(a*b)*a^2*b*d^3*abs(b) + sqrt(a*b)*b*c*a
bs(-b^2*c*d + a*b*d^2)*abs(b) + sqrt(a*b)*a*d*abs(-b^2*c*d + a*b*d^2)*abs(b
))*arctan(2*sqrt(1/2)*x/sqrt((b^2*c*d + a*b*d^2 + sqrt(-4*a*b^3*c*d^3 + (b^
2*c*d + a*b*d^2)^2))/(b^2*d^2)))/(b^4*c*d*abs(-b^2*c*d + a*b*d^2) + a*b^3*d
^2*abs(-b^2*c*d + a*b*d^2) + (b^2*c*d - a*b*d^2)^2*b^2) + (sqrt(c*d)*b^3*c^
2*d*abs(d) + sqrt(c*d)*a^2*b*d^3*abs(d) - sqrt(c*d)*b*c*abs(-b^2*c*d + a*b*
d^2)*abs(d) - sqrt(c*d)*a*d*abs(-b^2*c*d + a*b*d^2)*abs(d))*arctan(2*sqrt(1
/2)*x/sqrt((b^2*c*d + a*b*d^2 - sqrt(-4*a*b^3*c*d^3 + (b^2*c*d + a*b*d^2)^2
))/(b^2*d^2)))/(b^2*c*d^3*abs(-b^2*c*d + a*b*d^2) + a*b*d^4*abs(-b^2*c*d +
a*b*d^2) - (b^2*c*d - a*b*d^2)^2*d^2) + x/(b*d)
```

3.230 $\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

```
[Out] -(a*Log[a + b*x^2])/(2*b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(b*c - a*d)
)
```

Rubi [A] time = 0.0490427, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(a*Log[a + b*x^2])/(2*b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(b*c - a*d)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}+\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.020067, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^2\right)-bc\log\left(c+dx^2\right)}{2b^2cd-2abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((a*d*Log[a + b*x^2] - b*c*Log[c + d*x^2])/(2*b^2*c*d - 2*a*b*d^2))
```

Maple [A] time = 0.006, size = 50, normalized size = 0.9

$$-\frac{c\ln\left(dx^2+c\right)}{(2\,ad-2\,bc)\,d}+\frac{a\ln\left(bx^2+a\right)}{(2\,ad-2\,bc)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2*c/(a*d-b*c)/d*ln(d*x^2+c)+1/2*a/(a*d-b*c)/b*ln(b*x^2+a)
```

Maxima [A] time = 1.02311, size = 66, normalized size = 1.25

$$-\frac{a\log\left(bx^2+a\right)}{2\left(b^2c-abd\right)}+\frac{c\log\left(dx^2+c\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c - a*b*d) + 1/2*c*log(d*x^2 + c)/(b*c*d - a*d^2
)
```

Fricas [A] time = 1.54338, size = 92, normalized size = 1.74

$$-\frac{ad\log\left(bx^2+a\right)-bc\log\left(dx^2+c\right)}{2\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*log(b*x^2 + a) - b*c*log(d*x^2 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] time = 1.74429, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^2+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{2b\,(ad-bc)}-\frac{c\log\left(x^2+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{2d\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*log(x**2 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(2*b*(a*d - b*c)) - c*log(x**2 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(2*d*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.231 $\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[c]*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rubi [A] time = 0.0340432, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {481, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[c]*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{a\int \frac{1}{a+bx^2}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0422226, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/Sqrt[d])/(b*c - a*d)
```

Maple [A] time = 0.006, size = 55, normalized size = 0.8

$$-\frac{c}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{a}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -c/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))+a/(a*d-b*c)/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59029, size = 630, normalized size = 9.

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},-\frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\frac{2\,\sqrt{\frac{c}{d}}\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + sqrt(-c
/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(a/b)*arctan(b*x*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d)
 - c)/(d*x^2 + c)))/(b*c - a*d), 1/2*(2*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) -
 sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b*c - a*d), -
(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x*sqrt(c/d)/c))/(b*
c - a*d)]
```

Sympy [B] time = 2.11334, size = 570, normalized size = 8.14

$$\frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}\right.}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*b**2*c*d**
2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d - b*c) - 2*b**3*c**2*d
*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a*d - b
*c)) - sqrt(-a/b)*log(2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*b**2
*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*b**3*
c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a
*d - b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)**3 + 4
*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)/(a*d - b*c) -
2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-c/d)/(a*d - b*c) + x
)/(2*(a*d - b*c)) - sqrt(-c/d)*log(2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)*
*3 - 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/d)/(a*d - b
*c) + 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/d)/(a*d - b*
c) + x)/(2*(a*d - b*c))
```

Giac [B] time = 1.17806, size = 176, normalized size = 2.51

$$\frac{\sqrt{cd}|d|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{d^2|bc-ad|}-\frac{\sqrt{ab}|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2|bc-ad|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] sqrt(c*d)*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt(-4*a*b*c*d + (
b*c + a*d)^2))/(b*d)))/(d^2*abs(b*c - a*d)) - sqrt(a*b)*abs(b)*arctan(2*sqr
t(1/2)*x/sqrt((b*c + a*d - sqrt(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b^2*a
bs(b*c - a*d))
```

3.232 $\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^2)}{2(bc-ad)}-\frac{\log(c+dx^2)}{2(bc-ad)}$$

```
[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))
```

Rubi [A] time = 0.0262701, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^2)}{2(bc-ad)}-\frac{\log(c+dx^2)}{2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^2\right)\\ &= \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2(bc-ad)}-\frac{d\operatorname{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^2\right)}{2(bc-ad)}\\ &= \frac{\log(a+bx^2)}{2(bc-ad)}-\frac{\log(c+dx^2)}{2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0162048, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^2)-\log(c+dx^2)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Log[a + b*x^2] - Log[c + d*x^2])/(2*b*c - 2*a*d)
```

Maple [A] time = 0.006, size = 42, normalized size = 0.9

$$\frac{\ln\left(dx^2+c\right)}{2\,ad-2\,bc}-\frac{\ln\left(bx^2+a\right)}{2\,ad-2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2/(a*d-b*c)*ln(d*x^2+c)-1/2/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.06689, size = 55, normalized size = 1.22

$$\frac{\log\left(bx^2+a\right)}{2\,(bc-ad)}-\frac{\log\left(dx^2+c\right)}{2\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/(b*c - a*d) - 1/2*log(d*x^2 + c)/(b*c - a*d)
```

Fricas [A] time = 1.54995, size = 69, normalized size = 1.53

$$\frac{\log\left(bx^2+a\right)-\log\left(dx^2+c\right)}{2\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x^2 + a) - log(d*x^2 + c))/(b*c - a*d)
```

Sympy [B] time = 0.87919, size = 138, normalized size = 3.07

$$\frac{\log\left(x^2+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\,(ad-bc)}-\frac{\log\left(x^2+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] log(x**2 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c)) - log(x**2 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(2*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.233 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi [A] time = 0.0268952, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0418382, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A] time = 0.001, size = 55, normalized size = 0.8

$$\frac{d}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] d/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-b/(a*d-b*c)/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63959, size = 608, normalized size = 8.69

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},-\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)},\frac{2\sqrt{\frac{b}{a}}\operatorname{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

Sympy [B] time = 2.23409, size = 712, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

Giac [B] time = 1.20458, size = 257, normalized size = 3.67

$$-\frac{2\sqrt{cd}b|d|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{2\sqrt{ab}d|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -2*sqrt(c*d)*b*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt(-4*a*b*c*
d + (b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d^2*abs(b*c - a*d) +
(b*c - a*d)^2*d) + 2*sqrt(a*b)*d*abs(b)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*
d - sqrt(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b*d
*abs(b*c - a*d) - (b*c - a*d)^2*b)
```

3.234 $\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c + d*x^2])/(2
*c*(b*c - a*d))
```

Rubi [A] time = 0.0611619, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c + d*x^2])/(2
*c*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0273714, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^2\right)+ad\log\left(c+dx^2\right)-2ad\log(x)+2bc\log(x)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*b*c*Log[x] - 2*a*d*Log[x] - b*c*Log[a + b*x^2] + a*d*Log[c + d*x^2])/(2*
a*b*c^2 - 2*a^2*c*d)
```

Maple [A] time = 0.008, size = 59, normalized size = 1.

$$-\frac{d\ln\left(dx^2+c\right)}{2c\left(ad-bc\right)}+\frac{\ln\left(x\right)}{ac}+\frac{b\ln\left(bx^2+a\right)}{2a\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2*d/c/(a*d-b*c)*ln(d*x^2+c)+ln(x)/a/c+1/2*b/a/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.07348, size = 82, normalized size = 1.32

$$-\frac{b\log\left(bx^2+a\right)}{2\left(abc-a^2d\right)}+\frac{d\log\left(dx^2+c\right)}{2\left(bc^2-acd\right)}+\frac{\log\left(x^2\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 + a)/(a*b*c - a^2*d) + 1/2*d*log(d*x^2 + c)/(b*c^2 - a*c*d
) + 1/2*log(x^2)/(a*c)
```

Fricas [A] time = 1.86139, size = 123, normalized size = 1.98

$$-\frac{bc\log\left(bx^2+a\right)-ad\log\left(dx^2+c\right)-2\left(bc-ad\right)\log\left(x\right)}{2\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*log(b*x^2 + a) - a*d*log(d*x^2 + c) - 2*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.235 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=81

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d))
+ (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d))
```

Rubi [A] time = 0.0855364, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {480, 522, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d))
+ (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{-bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{ac} \\
&= -\frac{1}{acx}-\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)} \\
&= -\frac{1}{acx}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.0830965, size = 76, normalized size = 0.94

$$
\frac{-\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a}+\frac{d^{3/2}x\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{d}{c}}{bcx-adx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-(b/a) + d/c - (b^(3/2)*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + (d^(3/2)*
x*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c*x - a*d*x)
```

Maple [A] time = 0.008, size = 76, normalized size = 0.9

$$
-\frac{d^2}{c\,(ad-bc)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{1}{acx}+\frac{b^2}{a\,(ad-bc)}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/c*d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/a/c/x+1/a*b^2/(a*d
-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65387, size = 819, normalized size = 10.11

$$
\left[-\frac{bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\left(abc^2-a^2cd\right)x},\frac{2\,adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-bcx\sqrt{-\frac{b}{a}}\log\left(\cdots\right.}{2\left(abc^2-a^2cd\right)\cdots}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + a
*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*c - 2
*a*d)/((a*b*c^2 - a^2*c*d)*x), 1/2*(2*a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) -
 b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b*c +
 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -1/2*(2*b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)
) + a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*
c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -(b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) + b*c - a*d)/((a*b*c^2 - a^2*c*d)*x)]
```

Sympy [B] time = 4.76874, size = 1093, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)*
*3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2*c*
*5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/(a*
d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**
4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(a*d
 - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c))
 + sqrt(-b**3/a**3)*log(x + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)
**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**2*c
**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a**3)/(a
*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**3*b*
*4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**3)/(a*
d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)
) - sqrt(-d**3/c**3)*log(x + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d**3/c**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**3/c**3)/(
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*
c)) + sqrt(-d**3/c**3)*log(x + (a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b
*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**
2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-d**3/c**3)
/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**3
*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-d**3/c**3)/
(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b
*c)) - 1/(a*c*x)
```

Giac [B] time = 1.65034, size = 520, normalized size = 6.42

$$\frac{\left(\sqrt{cd}ab^2c^2|d| + \sqrt{cd}a^2bcd|d| - \sqrt{cd}b\left|abc^2 - a^2cd\right||d|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{abc^2+a^2cd+\sqrt{-4a^3bc^3d+\left(abc^2+a^2cd\right)^2}}{abcd}}}\right)}{abc^2d\left|abc^2 - a^2cd\right| + a^2cd^2\left|abc^2 - a^2cd\right| + \left(abc^2 - a^2cd\right)^2d} - \frac{\left(\sqrt{ab}abc^2d|b| + \sqrt{ab}a^2cd^2|b|\right.}{ab^2c^2\left|abc^2 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (sqrt(c*d)*a*b^2*c^2*abs(d) + sqrt(c*d)*a^2*b*c*d*abs(d) - sqrt(c*d)*b*abs(
a*b*c^2 - a^2*c*d)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((a*b*c^2 + a^2*c*d + s
qrt(-4*a^3*b*c^3*d + (a*b*c^2 + a^2*c*d)^2))/(a*b*c*d)))/(a*b*c^2*d*abs(a*b
*c^2 - a^2*c*d) + a^2*c*d^2*abs(a*b*c^2 - a^2*c*d) + (a*b*c^2 - a^2*c*d)^2*
d) - (sqrt(a*b)*a*b*c^2*d*abs(b) + sqrt(a*b)*a^2*c*d^2*abs(b) + sqrt(a*b)*d
*abs(a*b*c^2 - a^2*c*d)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a*b*c^2 + a^2*c*
d - sqrt(-4*a^3*b*c^3*d + (a*b*c^2 + a^2*c*d)^2))/(a*b*c*d)))/(a*b^2*c^2*ab
s(a*b*c^2 - a^2*c*d) + a^2*b*c*d*abs(a*b*c^2 - a^2*c*d) - (a*b*c^2 - a^2*c*
d)^2*b) - 1/(a*c*x)
```

3.236 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

```
[Out] -1/(2*a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^2])/(2*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Rubi [A] time = 0.0905256, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(2*a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^2])/(2*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2acx^2}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0392652, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(2*a*c*x^2) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^2])/(
2*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Maple [A] time = 0.01, size = 87, normalized size = 1.

$$\frac{d^2 \ln\left(dx^2+c\right)}{2\,c^2\,(ad-bc)}-\frac{1}{2\,acx^2}-\frac{\ln(x)\,d}{ac^2}-\frac{\ln(x)\,b}{a^2c}-\frac{b^2\ln\left(bx^2+a\right)}{2\,a^2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*d^2/c^2/(a*d-b*c)*ln(d*x^2+c)-1/2/a/c/x^2-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)
*b-1/2*b^2/a^2/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.01999, size = 117, normalized size = 1.34

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^2+c\right)}{2\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(a^2*b*c - a^3*d) - 1/2*d^2*log(d*x^2 + c)/(b*c^3 -
a*c^2*d) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) - 1/2/(a*c*x^2)
```

Fricas [A] time = 2.85336, size = 200, normalized size = 2.3

$$\frac{b^2c^2x^2\log\left(bx^2+a\right)-a^2d^2x^2\log\left(dx^2+c\right)-abc^2+a^2cd-2\left(b^2c^2-a^2d^2\right)x^2\log(x)}{2\left(a^2bc^3-a^3c^2d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log(b*x^2 + a) - a^2*d^2*x^2*log(d*x^2 + c) - a*b*c^2 + a^
2*c*d - 2*(b^2*c^2 - a^2*d^2)*x^2*log(x))/((a^2*b*c^3 - a^3*c^2*d)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.237 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=100

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2
)*(b*c - a*d))
```

Rubi [A] time = 0.176916, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {480, 583, 522, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2
)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{3ac}\\ &= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{3a^2c^2}\\ &= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^2}\,dx}{c^2(bc-ad)}\\ &= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.122165, size = 101, normalized size = 1.01

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(-(b*c) + a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
5/2)*(b*c - a*d))
```

Maple [A] time = 0.011, size = 98, normalized size = 1.

$$\frac{d^3}{c^2\,(ad-bc)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{1}{3\,acx^3}+\frac{d}{ac^2x}+\frac{b}{a^2cx}-\frac{b^3}{a^2\,(ad-bc)}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/c^2*d^3/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/3/a/c/x^3+1/a/c^2
/x*d+1/a^2/c/x*b-1/a^2*b^3/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7505, size = 1131, normalized size = 11.31

$$\left[-\frac{3\,b^2c^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+3\,a^2d^2x^3\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,abc^2-2\,a^2cd-6\left(b^2c^2-a^2d^2\right)x^2}{6\left(a^2bc^3-a^3c^2d\right)x^3},-\frac{6\,a^2d^2\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*b^2*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + 3*a^2*d^2*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*a*b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3
*c^2*d)*x^3), -1/6*(6*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*b^2*c^2
*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*a*b*c^2
 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/
6*(6*b^2*c^2*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(-d/c)*l
og((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*a*b*c^2 + 2*a^2*c*d + 6*
(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/3*(3*b^2*c^2*x^3*
sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c))
 - a*b*c^2 + a^2*c*d + 3*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*
x^3)]
```

Sympy [B] time = 9.50538, size = 1353, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b**5/a**5)*log(x + (-a**10*c**5*d**5*(-b**5/a**5)**(3/2)/(a*d - b*c)*
*3 + 2*a**9*b*c**6*d**4*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**8*b**2*c**7
*d**3*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**8*d**8*sqrt(-b**5/a**5)/(a*d
- b*c) - a**7*b**3*c**8*d**2*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b*
*4*c**9*d*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-b**5/a**5)
**(3/2)/(a*d - b*c)**3 - b**8*c**8*sqrt(-b**5/a**5)/(a*d - b*c))/(a**4*b**3
*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b**7*c*
*4*d**3))/(2*(a*d - b*c)) - sqrt(-b**5/a**5)*log(x + (a**10*c**5*d**5*(-b**
5/a**5)**(3/2)/(a*d - b*c)**3 - 2*a**9*b*c**6*d**4*(-b**5/a**5)**(3/2)/(a*d
 - b*c)**3 + a**8*b**2*c**7*d**3*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a**8*
d**8*sqrt(-b**5/a**5)/(a*d - b*c) + a**7*b**3*c**8*d**2*(-b**5/a**5)**(3/2)
/(a*d - b*c)**3 - 2*a**6*b**4*c**9*d*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a
**5*b**5*c**10*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-b**5/a*
*5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 +
 a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d - b*c)) + sqrt(-d**5/c**5)*log
(x + (-a**10*c**5*d**5*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + 2*a**9*b*c**6*d
**4*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - a**8*b**2*c**7*d**3*(-d**5/c**5)**
(3/2)/(a*d - b*c)**3 - a**8*d**8*sqrt(-d**5/c**5)/(a*d - b*c) - a**7*b**3*c
**8*d**2*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b**4*c**9*d*(-d**5/c**
5)**(3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-d**5/c**5)**(3/2)/(a*d - b*c)*
```

```
*3 - b**8*c**8*sqrt(-d**5/c**5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*
d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d -
b*c)) - sqrt(-d**5/c**5)*log(x + (a**10*c**5*d**5*(-d**5/c**5)**(3/2)/(a*d
- b*c)**3 - 2*a**9*b*c**6*d**4*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**8*b*
*2*c**7*d**3*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**8*d**8*sqrt(-d**5/c**5
)/(a*d - b*c) + a**7*b**3*c**8*d**2*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - 2*
a**6*b**4*c**9*d*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**5*b**5*c**10*(-d**
5/c**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-d**5/c**5)/(a*d - b*c))/(a*
*4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 +
b**7*c**4*d**3))/(2*(a*d - b*c)) + (-a*c + x**2*(3*a*d + 3*b*c))/(3*a**2*c*
*2*x**3)
```

Giac [B] time = 1.28634, size = 728, normalized size = 7.28

$$-\frac{\left(\sqrt{cd}a^2b^3c^4|d| + \sqrt{cd}a^4bc^2d^2|d| - \sqrt{cd}b^2c\left|a^2bc^3 - a^3c^2d\right||d| - \sqrt{cd}abd\left|a^2bc^3 - a^3c^2d\right||d|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{a^2bc^3+a^3c^2d+\sqrt{-4a^5bc^5d+(a^2bc}}{a^2bc^2d}}}\right)}{a^2bc^3d\left|a^2bc^3 - a^3c^2d\right| + a^3c^2d^2\left|a^2bc^3 - a^3c^2d\right| + \left(a^2bc^3 - a^3c^2d\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(sqrt(c*d)*a^2*b^3*c^4*abs(d) + sqrt(c*d)*a^4*b*c^2*d^2*abs(d) - sqrt(c*d)
*b^2*c*abs(a^2*b*c^3 - a^3*c^2*d)*abs(d) - sqrt(c*d)*a*b*d*abs(a^2*b*c^3 -
a^3*c^2*d)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c^3 + a^3*c^2*d + sqrt(
-4*a^5*b*c^5*d + (a^2*b*c^3 + a^3*c^2*d)^2))/(a^2*b*c^2*d)))/(a^2*b*c^3*d*a
bs(a^2*b*c^3 - a^3*c^2*d) + a^3*c^2*d^2*abs(a^2*b*c^3 - a^3*c^2*d) + (a^2*b
*c^3 - a^3*c^2*d)^2*d) + (sqrt(a*b)*a^2*b^2*c^4*d*abs(b) + sqrt(a*b)*a^4*c^
2*d^3*abs(b) + sqrt(a*b)*b*c*d*abs(a^2*b*c^3 - a^3*c^2*d)*abs(b) + sqrt(a*b
)*a*d^2*abs(a^2*b*c^3 - a^3*c^2*d)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b
*c^3 + a^3*c^2*d - sqrt(-4*a^5*b*c^5*d + (a^2*b*c^3 + a^3*c^2*d)^2))/(a^2*b
*c^2*d)))/(a^2*b^2*c^3*abs(a^2*b*c^3 - a^3*c^2*d) + a^3*b*c^2*d*abs(a^2*b*c
^3 - a^3*c^2*d) - (a^2*b*c^3 - a^3*c^2*d)^2*b) + 1/3*(3*b*c*x^2 + 3*a*d*x^2
 - a*c)/(a^2*c^2*x^3)
```

3.238 $\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rubi [A] time = 0.127446, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)}+\frac{}{c^3(bc-}\right.\right.\\ &= -\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3}-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{d^3\log\left(c+dx^2\right.}{2c^3(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0536383, size = 119, normalized size = 1.

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{b^3\log\left(a+bx^2\right)}{2a^3(ad-bc)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^2])/(2*a^3*(-(b*c) + a*d)) + (d^3*L
og[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Maple [A] time = 0.013, size = 124, normalized size = 1.

$$-\frac{d^3\ln\left(dx^2+c\right)}{2\,c^3\,(ad-bc)}-\frac{1}{4\,acx^4}+\frac{d}{2\,ac^2x^2}+\frac{b}{2\,a^2cx^2}+\frac{\ln(x)\,d^2}{ac^3}+\frac{\ln(x)\,bd}{a^2c^2}+\frac{\ln(x)\,b^2}{a^3c}+\frac{b^3\ln\left(bx^2+a\right)}{2\,a^3\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2*d^3/c^3/(a*d-b*c)*ln(d*x^2+c)-1/4/a/c/x^4+1/2/a/c^2/x^2*d+1/2/a^2/c/x^
2*b+1/a/c^3*ln(x)*d^2+1/a^2/c^2*ln(x)*b*d+1/a^3/c*ln(x)*b^2+1/2*b^3/a^3/(a*
d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.10021, size = 158, normalized size = 1.33

$$-\frac{b^3\log\left(bx^2+a\right)}{2\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^2+c\right)}{2\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}+\frac{2\,(bc+ad)x^2-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a^3*b*c - a^4*d) + 1/2*d^3*log(d*x^2 + c)/(b*c^4 -
 a*c^3*d) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a^3*c^3) + 1/4*(2*(
b*c + a*d)*x^2 - a*c)/(a^2*c^2*x^4)
```

Fricas [A] time = 7.42589, size = 255, normalized size = 2.14

$$-\frac{2\,b^3c^3x^4\log\left(bx^2+a\right)-2\,a^3d^3x^4\log\left(dx^2+c\right)+a^2bc^3-a^3c^2d-4\left(b^3c^3-a^3d^3\right)x^4\log(x)-2\left(ab^2c^3-a^3cd^2\right)x^2}{4\left(a^3bc^4-a^4c^3d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3*x^4*log(b*x^2 + a) - 2*a^3*d^3*x^4*log(d*x^2 + c) + a^2*b*c
^3 - a^3*c^2*d - 4*(b^3*c^3 - a^3*d^3)*x^4*log(x) - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.239 $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d))
 + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Rubi [A] time = 0.227002, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {480, 583, 522, 205}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d))
 + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{\int \frac{-15\left(b^2c^2+abcd+a^2d^2\right)-15bd(bc+ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{15a^2c^2}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{-15(bc+ad)\left(b^2c^2+a^2d^2\right)-15bd\left(b^2c^2+abcd+a^2d^2\right)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,d}{15a^3c^3}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{1}{c+dx^2}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}$$

Mathematica [A] time = 0.122789, size = 135, normalized size = 1.01

$$\frac{-a^2d^2-abcd-b^2c^2}{a^3c^3x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) + (-(b^2*c^2) - a*b*c*d - a^2*
d^2)/(a^3*c^3*x) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c) +
 a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Maple [A] time = 0.012, size = 141, normalized size = 1.1

$$-\frac{d^4}{c^3(ad-bc)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{1}{5\,acx^5}+\frac{d}{3\,ac^2x^3}+\frac{b}{3\,a^2cx^3}-\frac{d^2}{ac^3x}-\frac{bd}{a^2c^2x}-\frac{b^2}{a^3cx}+\frac{b^4}{a^3(ad-bc)}\arctan\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/c^3*d^4/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/5/a/c/x^5+1/3/a/
c^2/x^3*d+1/3/a^2/c/x^3*b-1/a/c^3/x*d^2-1/a^2/c^2/x*b*d-1/a^3/c/x*b^2+1/a^3
*b^4/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99941, size = 1358, normalized size = 10.13

$$\left[-\frac{15\, b^3c^3x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+15\, a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\, a^2bc^3-6\, a^3c^2d+30\left(b^3c^3-a^3d^3\right)x^4-10\left(a\right.}{30\left(a^3bc^4-a^4c^3d\right)x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^
2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5), 1/30*(30*a^3*d^3*x^5
*sqrt(d/c)*arctan(x*sqrt(d/c)) - 15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6*a^2*b*c^3 + 6*a^3*c^2*d - 30*(b^3*c^3 -
 a^3*d^3)*x^4 + 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^
5), -1/30*(30*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15*a^3*d^3*x^5*sq
rt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 6*a^2*b*c^3 - 6*
a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((
a^3*b*c^4 - a^4*c^3*d)*x^5), -1/15*(15*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 15*a^3*d^3*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*a^2*b*c^3 - 3*a^3*
c^2*d + 15*(b^3*c^3 - a^3*d^3)*x^4 - 5*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b
*c^4 - a^4*c^3*d)*x^5)]
```

Sympy [B] time = 21.3415, size = 1504, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**7/a**7)*log(x + (-a**13*c**7*d**6*(-b**7/a**7)**(3/2)/(a*d - b*c)
**3 + 2*a**12*b*c**8*d**5*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 - a**11*b**2*c
**9*d**4*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 - a**11*d**11*sqrt(-b**7/a**7)/
(a*d - b*c) - a**9*b**4*c**11*d**2*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + 2*a
**8*b**5*c**12*d*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 - a**7*b**6*c**13*(-b**
7/a**7)**(3/2)/(a*d - b*c)**3 - b**11*c**11*sqrt(-b**7/a**7)/(a*d - b*c))/(
a**6*b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8 + a**3*b**7*c**3*d
**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d**4))/(2*(a*d -
b*c)) + sqrt(-b**7/a**7)*log(x + (a**13*c**7*d**6*(-b**7/a**7)**(3/2)/(a*d
- b*c)**3 - 2*a**12*b*c**8*d**5*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + a**11*
b**2*c**9*d**4*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + a**11*d**11*sqrt(-b**7/
a**7)/(a*d - b*c) + a**9*b**4*c**11*d**2*(-b**7/a**7)**(3/2)/(a*d - b*c)**3
 - 2*a**8*b**5*c**12*d*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + a**7*b**6*c**13
```

```
*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + b**11*c**11*sqrt(-b**7/a**7)/(a*d - b
*c))/(a**6*b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8 + a**3*b**7*
c**3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d**4))/(2*(
a*d - b*c)) - sqrt(-d**7/c**7)*log(x + (-a**13*c**7*d**6*(-d**7/c**7)**(3/2
)/(a*d - b*c)**3 + 2*a**12*b*c**8*d**5*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 -
 a**11*b**2*c**9*d**4*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - a**11*d**11*sqrt
(-d**7/c**7)/(a*d - b*c) - a**9*b**4*c**11*d**2*(-d**7/c**7)**(3/2)/(a*d -
b*c)**3 + 2*a**8*b**5*c**12*d*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - a**7*b**
6*c**13*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - b**11*c**11*sqrt(-d**7/c**7)/(
a*d - b*c))/(a**6*b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8 + a**
3*b**7*c**3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d**4
))/(2*(a*d - b*c)) + sqrt(-d**7/c**7)*log(x + (a**13*c**7*d**6*(-d**7/c**7)
**(3/2)/(a*d - b*c)**3 - 2*a**12*b*c**8*d**5*(-d**7/c**7)**(3/2)/(a*d - b*c
)**3 + a**11*b**2*c**9*d**4*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 + a**11*d**1
1*sqrt(-d**7/c**7)/(a*d - b*c) + a**9*b**4*c**11*d**2*(-d**7/c**7)**(3/2)/(
a*d - b*c)**3 - 2*a**8*b**5*c**12*d*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 + a*
*7*b**6*c**13*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 + b**11*c**11*sqrt(-d**7/c
**7)/(a*d - b*c))/(a**6*b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8
 + a**3*b**7*c**3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**
6*d**4))/(2*(a*d - b*c)) - (3*a**2*c**2 + x**4*(15*a**2*d**2 + 15*a*b*c*d +
 15*b**2*c**2) + x**2*(-5*a**2*c*d - 5*a*b*c**2))/(15*a**3*c**3*x**5)
```

Giac [B] time = 1.30495, size = 890, normalized size = 6.64

$$\frac{\left(\sqrt{cd}a^3b^4c^6|d| + \sqrt{cd}a^6bc^3d^3|d| - \sqrt{cd}b^3c^2\left|a^3bc^4 - a^4c^3d\right||d| - \sqrt{cd}ab^2cd\left|a^3bc^4 - a^4c^3d\right||d| - \sqrt{cd}a^2bd^2\left|a^3bc^4 - a^4c^3d\right||d|}{a^3bc^4d\left|a^3bc^4 - a^4c^3d\right| + a^4c^3d^2\left|a^3bc^4 - a^4c^3d\right| + \left(a^3bc^4 - a^4c^3d\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (sqrt(c*d)*a^3*b^4*c^6*abs(d) + sqrt(c*d)*a^6*b*c^3*d^3*abs(d) - sqrt(c*d)*
b^3*c^2*abs(a^3*b*c^4 - a^4*c^3*d)*abs(d) - sqrt(c*d)*a*b^2*c*d*abs(a^3*b*c
^4 - a^4*c^3*d)*abs(d) - sqrt(c*d)*a^2*b*d^2*abs(a^3*b*c^4 - a^4*c^3*d)*abs
(d))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b*c^4 + a^4*c^3*d + sqrt(-4*a^7*b*c^7*d
 + (a^3*b*c^4 + a^4*c^3*d)^2))/(a^3*b*c^3*d)))/(a^3*b*c^4*d*abs(a^3*b*c^4 -
 a^4*c^3*d) + a^4*c^3*d^2*abs(a^3*b*c^4 - a^4*c^3*d) + (a^3*b*c^4 - a^4*c^3
*d)^2*d) - (sqrt(a*b)*a^3*b^3*c^6*d*abs(b) + sqrt(a*b)*a^6*c^3*d^4*abs(b) +
 sqrt(a*b)*b^2*c^2*d*abs(a^3*b*c^4 - a^4*c^3*d)*abs(b) + sqrt(a*b)*a*b*c*d^
2*abs(a^3*b*c^4 - a^4*c^3*d)*abs(b) + sqrt(a*b)*a^2*d^3*abs(a^3*b*c^4 - a^4
*c^3*d)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b*c^4 + a^4*c^3*d - sqrt(-4*
a^7*b*c^7*d + (a^3*b*c^4 + a^4*c^3*d)^2))/(a^3*b*c^3*d)))/(a^3*b^2*c^4*abs(
a^3*b*c^4 - a^4*c^3*d) + a^4*b*c^3*d*abs(a^3*b*c^4 - a^4*c^3*d) - (a^3*b*c^
4 - a^4*c^3*d)^2*b) - 1/15*(15*b^2*c^2*x^4 + 15*a*b*c*d*x^4 + 15*a^2*d^2*x^
4 - 5*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^3*c^3*x^5)
```

3.240 $\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=155

$$-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Log[x])/(a^4*c^4) + (b
^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4*Log[c + d*x^2])/(2*c^4*(b*c -
 a*d))
```

Rubi [A] time = 0.169009, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Log[x])/(a^4*c^4) + (b
^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4*Log[c + d*x^2])/(2*c^4*(b*c -
 a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)(c+dx)}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^4}+\frac{-bc-ad}{a^2c^2x^3}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x^2}-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)}{a^4c^4x}-\frac{}{a^4(-bc}\right.\right.$$

$$=-\frac{1}{6acx^6}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{2a^3c^3x^2}-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\log(x)}{a^4c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad}$$

Mathematica [A] time = 0.0643291, size = 147, normalized size = 0.95

$$\frac{12x^6\log(x)\left(b^4c^4-a^4d^4\right)+a\left(2a^2bc^4+a^3cd\left(-2c^2+3cdx^2-6d^2x^4\right)+6a^3d^4x^6\log\left(c+dx^2\right)-3ab^2c^4x^2+6b^3c^4x^4\right)-}{12a^4c^4x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (12*(b^4*c^4 - a^4*d^4)*x^6*Log[x] - 6*b^4*c^4*x^6*Log[a + b*x^2] + a*(2*a^
2*b*c^4 - 3*a*b^2*c^4*x^2 + 6*b^3*c^4*x^4 + a^3*c*d*(-2*c^2 + 3*c*d*x^2 - 6
*d^2*x^4) + 6*a^3*d^4*x^6*Log[c + d*x^2]))/(12*a^4*c^4*(-(b*c) + a*d)*x^6)
```

Maple [A] time = 0.015, size = 184, normalized size = 1.2

$$\frac{d^4\ln\left(dx^2+c\right)}{2c^4(ad-bc)}-\frac{1}{6acx^6}+\frac{d}{4ac^2x^4}+\frac{b}{4a^2cx^4}-\frac{d^2}{2ac^3x^2}-\frac{bd}{2a^2c^2x^2}-\frac{b^2}{2a^3cx^2}-\frac{\ln(x)d^3}{ac^4}-\frac{\ln(x)bd^2}{a^2c^3}-\frac{\ln(x)b^2d}{a^3c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*d^4/c^4/(a*d-b*c)*ln(d*x^2+c)-1/6/a/c/x^6+1/4/a/c^2/x^4*d+1/4/a^2/c/x^4
*b-1/2/a/c^3/x^2*d^2-1/2/a^2/c^2/x^2*b*d-1/2/a^3/c/x^2*b^2-1/a/c^4*ln(x)*d^
3-1/a^2/c^3*ln(x)*b*d^2-1/a^3/c^2*ln(x)*b^2*d-1/a^4/c*ln(x)*b^3-1/2*b^4/a^4
/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.03975, size = 223, normalized size = 1.44

$$\frac{b^4\log\left(bx^2+a\right)}{2\left(a^4bc-a^5d\right)}-\frac{d^4\log\left(dx^2+c\right)}{2\left(bc^5-ac^4d\right)}-\frac{\left(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2a^4c^4}-\frac{6\left(b^2c^2+abcd+a^2d^2\right)x^4+2a^2c^2-}{12a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^4*b*c - a^5*d) - 1/2*d^4*log(d*x^2 + c)/(b*c^5 -
a*c^4*d) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^
4*c^4) - 1/12*(6*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 - 3*(a*b*c^2
 + a^2*c*d)*x^2)/(a^3*c^3*x^6)
```

Fricas [A] time = 10.5954, size = 311, normalized size = 2.01

$$\frac{6b^4c^4x^6\log\left(bx^2+a\right)-6a^4d^4x^6\log\left(dx^2+c\right)-2a^3bc^4+2a^4c^3d-12\left(b^4c^4-a^4d^4\right)x^6\log(x)-6\left(ab^3c^4-a^4cd^3\right)x^4+}{12\left(a^4bc^5-a^5c^4d\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*c^4*x^6*log(b*x^2 + a) - 6*a^4*d^4*x^6*log(d*x^2 + c) - 2*a^3*b
*c^4 + 2*a^4*c^3*d - 12*(b^4*c^4 - a^4*d^4)*x^6*log(x) - 6*(a*b^3*c^4 - a^4
*c*d^3)*x^4 + 3*(a^2*b^2*c^4 - a^4*c^2*d^2)*x^2)/((a^4*b*c^5 - a^5*c^4*d)*x
^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.241 $\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

```
[Out] -a^2/(2*b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b*x^2])/(2*
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a*d)^2)
```

Rubi [A] time = 0.0865865, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b*x^2])/(2*
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{2b^2(bc-ad)\left(a+bx^2\right)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0460986, size = 91, normalized size = 0.98

$$\frac{a^2d(ad-bc)+b^2c^2\left(a+bx^2\right)\log\left(c+dx^2\right)+ad\left(a+bx^2\right)(ad-2bc)\log\left(a+bx^2\right)}{2b^2d\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a^2*d*(-(b*c) + a*d) + a*d*(-2*b*c + a*d)*(a + b*x^2)*Log[a + b*x^2] + b^2
*c^2*(a + b*x^2)*Log[c + d*x^2])/(2*b^2*d*(b*c - a*d)^2*(a + b*x^2))
```

Maple [A] time = 0.012, size = 136, normalized size = 1.5

$$\frac{c^2\ln\left(dx^2+c\right)}{2\left(ad-bc\right)^2d}+\frac{a^2\ln\left(bx^2+a\right)d}{2\left(ad-bc\right)^2b^2}-\frac{a\ln\left(bx^2+a\right)c}{\left(ad-bc\right)^2b}+\frac{a^3d}{2\left(ad-bc\right)^2b^2\left(bx^2+a\right)}-\frac{a^2c}{2\left(ad-bc\right)^2b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*c^2/(a*d-b*c)^2/d*ln(d*x^2+c)+1/2*a^2/(a*d-b*c)^2/b^2*ln(b*x^2+a)*d-a/(
a*d-b*c)^2/b*ln(b*x^2+a)*c+1/2*a^3/(a*d-b*c)^2/b^2/(b*x^2+a)*d-1/2*a^2/(a*d
-b*c)^2/b/(b*x^2+a)*c
```

Maxima [A] time = 1.08163, size = 176, normalized size = 1.89

$$\frac{c^2\log\left(dx^2+c\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{a^2}{2\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^2\right)}-\frac{\left(2\,abc-a^2d\right)\log\left(bx^2+a\right)}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(d*x^2 + c)/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*a^2/(a*b^3
*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2) - 1/2*(2*a*b*c - a^2*d)*log(b*x^2 +
 a)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

Fricas [A] time = 1.7337, size = 321, normalized size = 3.45

$$-\frac{a^2bcd-a^3d^2+\left(2\,a^2bcd-a^3d^2+\left(2\,ab^2cd-a^2bd^2\right)x^2\right)\log\left(bx^2+a\right)-\left(b^3c^2x^2+ab^2c^2\right)\log\left(dx^2+c\right)}{2\left(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3+\left(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d
^2)*x^2)*log(b*x^2 + a) - (b^3*c^2*x^2 + a*b^2*c^2)*log(d*x^2 + c))/(a*b^4*
c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^
3*d^3)*x^2)
```

Sympy [B] time = 5.72102, size = 348, normalized size = 3.74

$$\frac{a^2}{2a^2b^2d - 2ab^3c + x^2\left(2ab^3d - 2b^4c\right)} + \frac{a\left(ad - 2bc\right)\log\left(x^2 + \frac{\frac{a^4d^3(ad-2bc)}{b(ad-bc)^2} - \frac{3a^3cd^2(ad-2bc)}{(ad-bc)^2} + \frac{3a^2bc^2d(ad-2bc)}{(ad-bc)^2} + a^2cd - \frac{ab^2c^3(ad-2bc)}{(ad-bc)^2} - 3abc^2}{a^2d^2 - 2abcd - b^2c^2}\right)}{2b^2\left(ad - bc\right)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2/(2*a**2*b**2*d - 2*a*b**3*c + x**2*(2*a*b**3*d - 2*b**4*c)) + a*(a*d -
 2*b*c)*log(x**2 + (a**4*d**3*(a*d - 2*b*c)/(b*(a*d - b*c)**2) - 3*a**3*c*d
**2*(a*d - 2*b*c)/(a*d - b*c)**2 + 3*a**2*b*c**2*d*(a*d - 2*b*c)/(a*d - b*c
)**2 + a**2*c*d - a*b**2*c**3*(a*d - 2*b*c)/(a*d - b*c)**2 - 3*a*b*c**2)/(a
**2*d**2 - 2*a*b*c*d - b**2*c**2))/(2*b**2*(a*d - b*c)**2) + c**2*log(x**2
+ (a**3*b*c**2*d**2/(a*d - b*c)**2 - 3*a**2*b**2*c**3*d/(a*d - b*c)**2 + a*
*2*c*d + 3*a*b**3*c**4/(a*d - b*c)**2 - 3*a*b*c**2 - b**4*c**5/(d*(a*d - b*
c)**2))/(a**2*d**2 - 2*a*b*c*d - b**2*c**2))/(2*d*(a*d - b*c)**2)
```

Giac [A] time = 1.16598, size = 205, normalized size = 2.2

$$\frac{c^2\log\left(\left|dx^2 + c\right|\right)}{2\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)} - \frac{\left(2\,abc - a^2d\right)\log\left(\left|bx^2 + a\right|\right)}{2\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)} + \frac{2\,abcx^2 - a^2dx^2 + a^2c}{2\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(abs(d*x^2 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*(2*a*
b*c - a^2*d)*log(abs(b*x^2 + a))/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2) + 1/
2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(b
*x^2 + a))
```

3.242 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=108

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2}+\frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(c*x)/(2*d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.0846266, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {470, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2}+\frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*x)/(2*d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\int \frac{ac+(bc-2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2d(bc-ad)}$$

$$= -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2} + \frac{(c(bc-3ad))\int \frac{1}{c+dx^2}\,dx}{2d(bc-ad)^2}$$

$$= -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2} + \frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}$$

Mathematica [A] time = 0.128526, size = 108, normalized size = 1.

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad-bc)^2} + \frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2} + \frac{cx}{2d\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*x)/(2*d*(-(b*c) + a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[b]*(-(b*c) + a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.009, size = 144, normalized size = 1.3

$$\frac{cxa}{2\,(ad-bc)^2\left(dx^2+c\right)} - \frac{c^2xb}{2\,d\,(ad-bc)^2\left(dx^2+c\right)} - \frac{3\,ac}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{c^2b}{2\,d\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x/(d*x^2+c)*b-3/2*c/(
a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+1/2*c^2/(a*d-b*c)^2/d/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04322, size = 1472, normalized size = 13.63

$$\left[\frac{2\left(ad^2x^2+acd\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-\left(bc^2-3acd+\left(bcd-3ad^2\right)x^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\left(bc^2-acd\right)x}{4\left(b^2c^3d-2abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2abcd^3+a^2d^4\right)x^2\right)},\ \frac{4(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*d^2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x
^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d -
 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/
4*(4*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - (b*c^2 - 3*a*c
*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 +
 (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*((b*c^2 - 3*a*c*d + (b*c*d
 - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + (a*d^2*x^2 + a*c*d)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - (b*c^2 - a*c*d)*
x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*x^2), 1/2*(2*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c
) - (b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)]
```

Sympy [B] time = 13.8954, size = 1850, normalized size = 17.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] c*x/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2)) + sqrt(-a**3/b
)*log(x + (-20*a**5*b*d**8*(-a**3/b)**(3/2)/(a*d - b*c)**6 + 84*a**4*b**2*c
*d**7*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 8*a**4*d**4*sqrt(-a**3/b)/(a*d - b*
c)**2 - 136*a**3*b**3*c**2*d**6*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 27*a**3*b
*c*d**3*sqrt(-a**3/b)/(a*d - b*c)**2 + 104*a**2*b**4*c**3*d**5*(-a**3/b)**(
3/2)/(a*d - b*c)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-a**3/b)/(a*d - b*c)**2 -
 36*a*b**5*c**4*d**4*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 9*a*b**3*c**3*d*sqrt
(-a**3/b)/(a*d - b*c)**2 + 4*b**6*c**5*d**3*(-a**3/b)**(3/2)/(a*d - b*c)**6
 + b**4*c**4*sqrt(-a**3/b)/(a*d - b*c)**2)/(12*a**3*d**2 - 7*a**2*b*c*d + a
*b**2*c**2))/(2*(a*d - b*c)**2) - sqrt(-a**3/b)*log(x + (20*a**5*b*d**8*(-a
**3/b)**(3/2)/(a*d - b*c)**6 - 84*a**4*b**2*c*d**7*(-a**3/b)**(3/2)/(a*d -
b*c)**6 + 8*a**4*d**4*sqrt(-a**3/b)/(a*d - b*c)**2 + 136*a**3*b**3*c**2*d**
6*(-a**3/b)**(3/2)/(a*d - b*c)**6 + 27*a**3*b*c*d**3*sqrt(-a**3/b)/(a*d - b
*c)**2 - 104*a**2*b**4*c**3*d**5*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 27*a**2*
b**2*c**2*d**2*sqrt(-a**3/b)/(a*d - b*c)**2 + 36*a*b**5*c**4*d**4*(-a**3/b)
**(3/2)/(a*d - b*c)**6 + 9*a*b**3*c**3*d*sqrt(-a**3/b)/(a*d - b*c)**2 - 4*b
**6*c**5*d**3*(-a**3/b)**(3/2)/(a*d - b*c)**6 - b**4*c**4*sqrt(-a**3/b)/(a*
d - b*c)**2)/(12*a**3*d**2 - 7*a**2*b*c*d + a*b**2*c**2))/(2*(a*d - b*c)**2
) + sqrt(-c/d**3)*(3*a*d - b*c)*log(x + (-5*a**5*b*d**8*(-c/d**3)**(3/2)*(3
*a*d - b*c)**3/(2*(a*d - b*c)**6) + 21*a**4*b**2*c*d**7*(-c/d**3)**(3/2)*(3
*a*d - b*c)**3/(2*(a*d - b*c)**6) - 4*a**4*d**4*sqrt(-c/d**3)*(3*a*d - b*c)
```

```
/(a*d - b*c)**2 - 17*a**3*b**3*c**2*d**6*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/
(a*d - b*c)**6 - 27*a**3*b*c*d**3*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c
)**2) + 13*a**2*b**4*c**3*d**5*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c
)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2
) - 9*a*b**5*c**4*d**4*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6)
 - 9*a*b**3*c**3*d*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + b**6*c*
*5*d**3*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + b**4*c**4*sq
rt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))/(12*a**3*d**2 - 7*a**2*b*c*d
+ a*b**2*c**2))/(4*(a*d - b*c)**2) - sqrt(-c/d**3)*(3*a*d - b*c)*log(x + (5
*a**5*b*d**8*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 21*a**4
*b**2*c*d**7*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 4*a**4*
d**4*sqrt(-c/d**3)*(3*a*d - b*c)/(a*d - b*c)**2 + 17*a**3*b**3*c**2*d**6*(-
c/d**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 27*a**3*b*c*d**3*sqrt(-c/d
**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 13*a**2*b**4*c**3*d**5*(-c/d**3)**(
3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 27*a**2*b**2*c**2*d**2*sqrt(-c/d**3)
*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 9*a*b**5*c**4*d**4*(-c/d**3)**(3/2)*(3*
a*d - b*c)**3/(2*(a*d - b*c)**6) + 9*a*b**3*c**3*d*sqrt(-c/d**3)*(3*a*d - b
*c)/(2*(a*d - b*c)**2) - b**6*c**5*d**3*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(
2*(a*d - b*c)**6) - b**4*c**4*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2
))/(12*a**3*d**2 - 7*a**2*b*c*d + a*b**2*c**2))/(4*(a*d - b*c)**2)
```

Giac [A] time = 1.21317, size = 163, normalized size = 1.51

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{ab}} + \frac{\left(bc^2 - 3\,acd\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\sqrt{cd}} - \frac{cx}{2\left(bcd - ad^2\right)\left(dx^2 + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/2
*(b*c^2 - 3*a*c*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*sqrt(c*d)) - 1/2*c*x/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.243 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] -c/(2*d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (
a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.0642443, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -c/(2*d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (
a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)}+\frac{c}{(bc-ad)(c+dx)^2}+\frac{ad}{(-bc+ad)^2(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{c}{2d(bc-ad)\left(c+dx^2\right)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.0341205, size = 74, normalized size = 1.

$$\frac{c}{2d\left(c+dx^2\right)(ad-bc)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] c/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) +
 (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.01, size = 95, normalized size = 1.3

$$\frac{a\ln\left(dx^2+c\right)}{2\,(ad-bc)^2}+\frac{ac}{2\,(ad-bc)^2\left(dx^2+c\right)}-\frac{bc^2}{2\,(ad-bc)^2\,d\left(dx^2+c\right)}-\frac{a\ln\left(bx^2+a\right)}{2\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2/(a*d-b*c)^2*a*ln(d*x^2+c)+1/2/(a*d-b*c)^2*c/(d*x^2+c)*a-1/2/(a*d-b*c)^2
*c^2/d/(d*x^2+c)*b-1/2*a/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.05796, size = 142, normalized size = 1.92

$$-\frac{a\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{a\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{c}{2\left(bc^2d-acd^2+\left(bcd^2-ad^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a
*d^3)*x^2)
```

Fricas [A] time = 1.54888, size = 243, normalized size = 3.28

$$-\frac{bc^2-acd+\left(ad^2x^2+acd\right)\log\left(bx^2+a\right)-\left(ad^2x^2+acd\right)\log\left(dx^2+c\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c^2 - a*c*d + (a*d^2*x^2 + a*c*d)*log(b*x^2 + a) - (a*d^2*x^2 + a*c
*d)*log(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*x^2)
```

Sympy [B] time = 2.37832, size = 253, normalized size = 3.42

$$\frac{a\log\left(x^2+\frac{-\frac{a^4d^3}{(ad-bc)^2}+\frac{3a^3bcd^2}{(ad-bc)^2}-\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d+\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2(ad-bc)^2}-\frac{a\log\left(x^2+\frac{\frac{a^4d^3}{(ad-bc)^2}-\frac{3a^3bcd^2}{(ad-bc)^2}+\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d-\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2(ad-bc)^2}+\frac{}{2acd^2-2bc^2d+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] a*log(x**2 + (-a**4*d**3/(a*d - b*c)**2 + 3*a**3*b*c*d**2/(a*d - b*c)**2 -
3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d + a*b**3*c**3/(a*d - b*c)**2 + a
*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) - a*log(x**2 + (a**4*d**3/(a*d - b*c)**
2 - 3*a**3*b*c*d**2/(a*d - b*c)**2 + 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a*
*2*d - a*b**3*c**3/(a*d - b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) +
c/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2))
```

Giac [A] time = 1.17782, size = 123, normalized size = 1.66

$$-\frac{\frac{ad^2\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{cd}{(bcd-ad^2)(dx^2+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3) + c*d/((b*c*d - a*d^2)*(d*x^2 + c)))/d
```

3.244 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{x}{2\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqr
t[d]*(b*c - a*d)^2)
```

Rubi [A] time = 0.0631776, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {471, 522, 205}

$$\frac{x}{2\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqr
t[d]*(b*c - a*d)^2)
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{x}{2(bc-ad)(c+dx^2)} - \frac{\int \frac{a-bx^2}{(a+bx^2)(c+dx^2)}\,dx}{2(bc-ad)}$$

$$= \frac{x}{2(bc-ad)(c+dx^2)} - \frac{(ab)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2} + \frac{(bc+ad)\int \frac{1}{c+dx^2}\,dx}{2(bc-ad)^2}$$

$$= \frac{x}{2(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

Mathematica [A] time = 0.13119, size = 90, normalized size = 0.87

$$\frac{\frac{x(bc-ad)}{c+dx^2} + \frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}} - 2\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (((b*c - a*d)*x)/(c + d*x^2) - 2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
] + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*(b*c -
a*d)^2)
```

Maple [A] time = 0.009, size = 134, normalized size = 1.3

$$-\frac{axd}{2\,(ad-bc)^2\,(dx^2+c)} + \frac{bcx}{2\,(ad-bc)^2\,(dx^2+c)} + \frac{ad}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{bc}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x/(d*x^2+c)*b*c+1/2/(a*d-b
*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*d+1/2/(a*d-b*c)^2/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))*b*c-b/(a*d-b*c)^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12539, size = 1485, normalized size = 14.28

$$\left[\frac{2\left(cd^2x^2+c^2d\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2\left(bc^2d-acd^2\right)x}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a
*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2),
1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
+ (c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)) + (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^
2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2), -1/4*(4*(c*d^2*x^2 + c^2*d)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(
-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b*c^2*d - a*c*d^2)
*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/2*(2*(c*d^2*x^2 + c^2*d)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*
c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

Sympy [B] time = 7.07189, size = 1530, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -x/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d)) + sqrt(-a*b)*log(x + (-
4*a**5*c*d**6*(-a*b)**(3/2)/(a*d - b*c)**6 + 4*a**4*b*c**2*d**5*(-a*b)**(3/
2)/(a*d - b*c)**6 + 24*a**3*b**2*c**3*d**4*(-a*b)**(3/2)/(a*d - b*c)**6 - a
**3*d**3*sqrt(-a*b)/(a*d - b*c)**2 - 56*a**2*b**3*c**4*d**3*(-a*b)**(3/2)/(
a*d - b*c)**6 - 3*a**2*b*c*d**2*sqrt(-a*b)/(a*d - b*c)**2 + 44*a*b**4*c**5*
d**2*(-a*b)**(3/2)/(a*d - b*c)**6 - 11*a*b**2*c**2*d*sqrt(-a*b)/(a*d - b*c)
**2 - 12*b**5*c**6*d*(-a*b)**(3/2)/(a*d - b*c)**6 - b**3*c**3*sqrt(-a*b)/(a
*d - b*c)**2)/(a*b*d + b**2*c))/(2*(a*d - b*c)**2) - sqrt(-a*b)*log(x + (4*
a**5*c*d**6*(-a*b)**(3/2)/(a*d - b*c)**6 - 4*a**4*b*c**2*d**5*(-a*b)**(3/2)
/(a*d - b*c)**6 - 24*a**3*b**2*c**3*d**4*(-a*b)**(3/2)/(a*d - b*c)**6 + a**
3*d**3*sqrt(-a*b)/(a*d - b*c)**2 + 56*a**2*b**3*c**4*d**3*(-a*b)**(3/2)/(a*
d - b*c)**6 + 3*a**2*b*c*d**2*sqrt(-a*b)/(a*d - b*c)**2 - 44*a*b**4*c**5*d*
*2*(-a*b)**(3/2)/(a*d - b*c)**6 + 11*a*b**2*c**2*d*sqrt(-a*b)/(a*d - b*c)**
2 + 12*b**5*c**6*d*(-a*b)**(3/2)/(a*d - b*c)**6 + b**3*c**3*sqrt(-a*b)/(a*d
 - b*c)**2)/(a*b*d + b**2*c))/(2*(a*d - b*c)**2) + sqrt(-1/(c*d))*(a*d + b*
c)*log(x + (-a**5*c*d**6*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6
) + a**4*b*c**2*d**5*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) +
3*a**3*b**2*c**3*d**4*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - a**
3*d**3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) - 7*a**2*b**3*c**4*d**
3*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - 3*a**2*b*c*d**2*sqrt(-1
/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) + 11*a*b**4*c**5*d**2*(-1/(c*d))**(3
/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 11*a*b**2*c**2*d*sqrt(-1/(c*d))*(a*
```

```
d + b*c)/(2*(a*d - b*c)**2) - 3*b**5*c**6*d*(-1/(c*d))**(3/2)*(a*d + b*c)**
3/(2*(a*d - b*c)**6) - b**3*c**3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)*
*2))/(a*b*d + b**2*c))/(4*(a*d - b*c)**2) - sqrt(-1/(c*d))*(a*d + b*c)*log(
x + (a**5*c*d**6*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - a**4
*b*c**2*d**5*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 3*a**3*b
**2*c**3*d**4*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 + a**3*d**3*s
qrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) + 7*a**2*b**3*c**4*d**3*(-1/(c
*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 + 3*a**2*b*c*d**2*sqrt(-1/(c*d))*
(a*d + b*c)/(2*(a*d - b*c)**2) - 11*a*b**4*c**5*d**2*(-1/(c*d))**(3/2)*(a*d
 + b*c)**3/(2*(a*d - b*c)**6) + 11*a*b**2*c**2*d*sqrt(-1/(c*d))*(a*d + b*c)
/(2*(a*d - b*c)**2) + 3*b**5*c**6*d*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*
d - b*c)**6) + b**3*c**3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2))/(a*
b*d + b**2*c))/(4*(a*d - b*c)**2)
```

Giac [A] time = 1.16073, size = 149, normalized size = 1.43

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{x}{2\left(dx^2+c\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*x/((d*x^2 + c)*(b*c - a*d))
```

3.245 $\int \frac{x}{(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=70

$$\frac{1}{2(c+dx^2)(bc-ad)}+\frac{b\log(a+bx^2)}{2(bc-ad)^2}-\frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (b*L
og[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.0541384, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 44}

$$\frac{1}{2(c+dx^2)(bc-ad)}+\frac{b\log(a+bx^2)}{2(bc-ad)^2}-\frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (b*L
og[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)(c+dx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right) dx, x, x^2\right)\\
&= \frac{1}{2(bc-ad)(c+dx^2)}+\frac{b\log(a+bx^2)}{2(bc-ad)^2}-\frac{b\log(c+dx^2)}{2(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.02779, size = 66, normalized size = 0.94

$$\frac{b(c+dx^2)\log(a+bx^2)-ad-b(c+dx^2)\log(c+dx^2)+bc}{2(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x^2)*Log[a + b*x^2] - b*(c + d*x^2)*Log[c + d*x^2])/(
2*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.01, size = 90, normalized size = 1.3

$$-\frac{b\ln\left(dx^2+c\right)}{2\left(ad-bc\right)^2}-\frac{ad}{2\left(ad-bc\right)^2\left(dx^2+c\right)}+\frac{bc}{2\left(ad-bc\right)^2\left(dx^2+c\right)}+\frac{b\ln\left(bx^2+a\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*b*ln(d*x^2+c)-1/2*d/(a*d-b*c)^2/(d*x^2+c)*a+1/2/(a*d-b*c)^
2/(d*x^2+c)*b*c+1/2*b/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.05275, size = 134, normalized size = 1.91

$$\frac{b\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{b\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{1}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*b*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
)
```

Fricas [A] time = 1.58463, size = 217, normalized size = 3.1

$$\frac{bc-ad+\left(bdx^2+bc\right)\log\left(bx^2+a\right)-\left(bdx^2+bc\right)\log\left(dx^2+c\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c - a*d + (b*d*x^2 + b*c)*log(b*x^2 + a) - (b*d*x^2 + b*c)*log(d*x^2
 + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*x^2)
```

Sympy [B] time = 2.18226, size = 248, normalized size = 3.54

$$-\frac{b\log\left(x^2+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2(ad-bc)^2}+\frac{b\log\left(x^2+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2(ad-bc)^2}-\frac{1}{2acd-2bc^2 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -b*log(x**2 + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)
**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b
**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) + b*log(x**2 + (a**3*b*d**3/(a*d - b*
c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2
+ a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2)
 - 1/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d))
```

Giac [A] time = 1.15498, size = 115, normalized size = 1.64

$$\frac{bd\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}+\frac{d}{2\left(bcd-ad^2\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) + 1/2*d/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.246 $\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.0790209, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(c+dx^2)} + \frac{\int \frac{2bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx}{2c(bc-ad)(c+dx^2)} + \frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2} - \frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}$$

$$= -\frac{dx}{2c(bc-ad)(c+dx^2)} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2} - \frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

Mathematica [A] time = 0.163413, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{d}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}} + \frac{dx(ad-bc)}{c(c+dx^2)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.009, size = 144, normalized size = 1.3

$$\frac{d^2xa}{2\,(ad-bc)^2\,c\,(dx^2+c)} - \frac{bdx}{2\,(ad-bc)^2\,(dx^2+c)} + \frac{ad^2}{2\,(ad-bc)^2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)*b+1/2*d^2
/(a*d-b*c)^2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-3/2*d/(a*d-b*c)^2/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23271, size = 1451, normalized size = 13.31

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)},\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

Sympy [B] time = 15.1212, size = 2033, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*a*c**2*d - 2*b*c**3 + x**2*(2*a*c*d**2 - 2*b*c**2*d)) - sqrt(-b**3/a
)*log(x + (-4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 28*a**6*b*c*
*4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**2*c**5*d**4*(-b**3/a)*
*(3/2)/(a*d - b*c)**6 - a**5*d**5*sqrt(-b**3/a)/(a*d - b*c)**2 + 56*a**4*b*
*3*c**6*d**3*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 9*a**4*b*c*d**4*sqrt(-b**3/a
)/(a*d - b*c)**2 - 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 -
27*a**3*b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 - 20*a**2*b**5*c**8*d*(
-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**3*c**3*d**2*sqrt(-b**3/a)/(a*d
- b*c)**2 + 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 8*b**5*c**5*sqr
t(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2
*d))/(2*(a*d - b*c)**2) + sqrt(-b**3/a)*log(x + (4*a**7*c**3*d**6*(-b**3/a)
**(3/2)/(a*d - b*c)**6 - 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**
6 + 64*a**5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 + a**5*d**5*sqrt
(-b**3/a)/(a*d - b*c)**2 - 56*a**4*b**3*c**6*d**3*(-b**3/a)**(3/2)/(a*d - b
*c)**6 - 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**2 + 4*a**3*b**4*c**7*d*
*2*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sqrt(-b**3/a)/(
a*d - b*c)**2 + 20*a**2*b**5*c**8*d*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a*
*2*b**3*c**3*d**2*sqrt(-b**3/a)/(a*d - b*c)**2 - 8*a*b**6*c**9*(-b**3/a)**(
3/2)/(a*d - b*c)**6 - 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*
d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) - sqrt(-d/c**3
)*(a*d - 3*b*c)*log(x + (-a**7*c**3*d**6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/
```

```
(2*(a*d - b*c)**6) + 7*a**6*b*c**4*d**5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(
2*(a*d - b*c)**6) - 8*a**5*b**2*c**5*d**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3
/(a*d - b*c)**6 - a**5*d**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2)
+ 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 +
9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a**3*b**4*
c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 27*a**3*b*
*2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 5*a**2*b**5*c
**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 27*a**2*b**3*c
**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + a*b**6*c**9*(-d/c
**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + 4*b**5*c**5*sqrt(-d/c**3)*(a*
d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2
*d))/(4*(a*d - b*c)**2) + sqrt(-d/c**3)*(a*d - 3*b*c)*log(x + (a**7*c**3*d*
*6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 7*a**6*b*c**4*d**
5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 8*a**5*b**2*c**5*d
**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + a**5*d**5*sqrt(-d/c*
*3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/
2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3
*b*c)/(2*(a*d - b*c)**2) + a**3*b**4*c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*
c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*
c)/(2*(a*d - b*c)**2) + 5*a**2*b**5*c**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**
3/(2*(a*d - b*c)**6) - 27*a**2*b**3*c**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c)/(
2*(a*d - b*c)**2) - a*b**6*c**9*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*
c)**6 - 4*b**5*c**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*
d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(4*(a*d - b*c)**2)
```

Giac [A] time = 1.1655, size = 165, normalized size = 1.51

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{ab}} - \frac{\left(3\,bcd - ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\sqrt{cd}} - \frac{dx}{2\left(bc^2 - acd\right)\left(dx^2 + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.247 $\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

```
[Out] -d/(2*c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a + b*x^2])/(2
*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rubi [A] time = 0.101341, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -d/(2*c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a + b*x^2])/(2
*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^2x}-\frac{b^3}{a(-bc+ad)^2(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^2}+\frac{d^2(2bc-ad)}{c^2(bc-ad)^2(c+dx)}\right)\right.\\ &= -\frac{d}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\log(x)}{ac^2}-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0945337, size = 98, normalized size = 0.98

$$\frac{1}{2}\left(-\frac{b^2\log\left(a+bx^2\right)}{a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{c^2(bc-ad)^2}-\frac{d}{c\left(c+dx^2\right)(bc-ad)}+\frac{2\log(x)}{ac^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-(d/(c*(b*c - a*d)*(c + d*x^2))) + (2*Log[x])/(a*c^2) - (b^2*Log[a + b*x^2
])/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^2)
)/2
```

Maple [A] time = 0.014, size = 139, normalized size = 1.4

$$-\frac{d^2\ln\left(dx^2+c\right)a}{2c^2(ad-bc)^2}+\frac{d\ln\left(dx^2+c\right)b}{c(ad-bc)^2}+\frac{ad^2}{2c(ad-bc)^2\left(dx^2+c\right)}-\frac{bd}{2(ad-bc)^2\left(dx^2+c\right)}+\frac{\ln(x)}{ac^2}-\frac{b^2\ln\left(bx^2+a\right)}{2a(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*a+d/c/(a*d-b*c)^2*ln(d*x^2+c)*b+1/2*d^
2/c/(a*d-b*c)^2/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2/(d*x^2+c)*b+ln(x)/a/c^2-1/2*b
^2/a/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.19167, size = 186, normalized size = 1.86

$$-\frac{b^2\log\left(bx^2+a\right)}{2\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(2bcd-ad^2\right)\log\left(dx^2+c\right)}{2\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}-\frac{d}{2\left(bc^3-ac^2d+\left(bc^2d-acd^2\right)x^2\right)}+\frac{\log\left(x^2\right)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/2*(2*b*c*d
- a*d^2)*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/2*d/(b*c^
3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/2*log(x^2)/(a*c^2)
```

Fricas [B] time = 6.3035, size = 444, normalized size = 4.44

$$-\frac{abc^2d-a^2cd^2+\left(b^2c^2dx^2+b^2c^3\right)\log\left(bx^2+a\right)-\left(2abc^2d-a^2cd^2+\left(2abcd^2-a^2d^3\right)x^2\right)\log\left(dx^2+c\right)-2\left(b^2c^3-2a\right.}{2\left(ab^2c^5-2a^2bc^4d+a^3c^3d^2+\left(ab^2c^4d-2a^2bc^3d^2+a^3c^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2*d - a^2*c*d^2 + (b^2*c^2*d*x^2 + b^2*c^3)*log(b*x^2 + a) - (2
*a*b*c^2*d - a^2*c*d^2 + (2*a*b*c*d^2 - a^2*d^3)*x^2)*log(d*x^2 + c) - 2*(b
^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)
*log(x))/(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2 + (a*b^2*c^4*d - 2*a^2*b*
c^3*d^2 + a^3*c^2*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21068, size = 250, normalized size = 2.5

$$-\frac{b^{3}\log\left(\left|bx^{2}+a\right|\right)}{2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)}+\frac{\left(2\,bcd^{2}-ad^{3}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3}\right)}-\frac{2\,bcd^{2}x^{2}-ad^{3}x^{2}+3\,bc^{2}d-2\,acd^{2}}{2\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\left(dx^{2}+c\right)}+\frac{\log\left(x^{2}\right)}{2\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(b*x^2 + a))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2) + 1/2*
(2*b*c*d^2 - a*d^3)*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3) - 1/2*(2*b*c*d^2*x^2 - a*d^3*x^2 + 3*b*c^2*d - 2*a*c*d^2)/((b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)) + 1/2*log(x^2)/(a*c^2)
```

3.248 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=144

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(2*b*c - 3*a*d)/(2*a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(c + d*x^2)
) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^2) + (d^(3/2
)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.198374, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(2*b*c - 3*a*d)/(2*a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(c + d*x^2)
) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^2) + (d^(3/2
)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx = -\frac{d}{2c(bc-ad)x(c+dx^2)} + \frac{\int \frac{2bc-3ad-3bdx^2}{x^2(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x} - \frac{d}{2c(bc-ad)x(c+dx^2)} - \frac{\int \frac{2b^2c^2+2abcd-3a^2d^2+bd(2bc-3ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{2ac^2(bc-ad)}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x} - \frac{d}{2c(bc-ad)x(c+dx^2)} - \frac{b^3\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)^2} + \frac{(d^2(5bc-3ad))\int \frac{1}{c+dx^2}\,dx}{2c^2(bc-ad)^2}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x} - \frac{d}{2c(bc-ad)x(c+dx^2)} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2} + \frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2c^{5/2}(bc-ad)^2}$$

Mathematica [A] time = 0.249047, size = 123, normalized size = 0.85

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^2} + \frac{d^2x}{2c^2(c+dx^2)(bc-ad)} + \frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2} - \frac{1}{ac^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(1/(a*c^2*x)) + (d^2*x)/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(5/2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*(5*b*c - 3*a*d)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.013, size = 169, normalized size = 1.2

$$-\frac{d^3xa}{2c^2(ad-bc)^2(dx^2+c)} + \frac{bd^2x}{2c(ad-bc)^2(dx^2+c)} - \frac{3ad^3}{2c^2(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{5bd^2}{2c(ad-bc)^2}\arctan\left(dx\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^2*x/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x/(d*x^2+c)*b-
3/2*d^3/c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+5/2*d^2/c/(a*
d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b-1/a/c^2/x-1/a*b^3/(a*d-b*c)^
2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.6902, size = 2049, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2
 + 3*a^2*d^3)*x^2 - 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5*a*b
*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d
^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 - ((5*a*b*c*d^2 - 3*a^2*d^
3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b^
2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d
^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 4*(b^2*c^2*d*x^3 + b^2
*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5
*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c
^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2
*c*d^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 2*(b^2*c^2*d*x^3 + b
^2*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 +
(5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a*b^2*c^4*d
 - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^
3*d^2)*x)]
```

Sympy [B] time = 71.0031, size = 2526, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-b**5/a**3)*log(x + (-12*a**10*c**5*d**7*(-b**5/a**3)**(3/2)/(a*d - b*
c)**6 + 68*a**9*b*c**6*d**6*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 152*a**8*b
**2*c**7*d**5*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 27*a**8*d**8*sqrt(-b**5/
a**3)/(a*d - b*c)**2 + 176*a**7*b**3*c**8*d**4*(-b**5/a**3)**(3/2)/(a*d - b
*c)**6 + 135*a**7*b*c*d**7*sqrt(-b**5/a**3)/(a*d - b*c)**2 - 124*a**6*b**4*
c**9*d**3*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 225*a**6*b**2*c**2*d**6*sqrt
(-b**5/a**3)/(a*d - b*c)**2 + 68*a**5*b**5*c**10*d**2*(-b**5/a**3)**(3/2)/(
a*d - b*c)**6 + 125*a**5*b**3*c**3*d**5*sqrt(-b**5/a**3)/(a*d - b*c)**2 - 3
2*a**4*b**6*c**11*d*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 + 8*a**3*b**7*c**12*
(-b**5/a**3)**(3/2)/(a*d - b*c)**6 + 8*b**8*c**8*sqrt(-b**5/a**3)/(a*d - b*
```

```
c)**2)/(27*a**4*b**3*d**6 - 81*a**3*b**4*c*d**5 + 36*a**2*b**5*c**2*d**4 +
28*a*b**6*c**3*d**3 + 20*b**7*c**4*d**2))/(2*(a*d - b*c)**2) - sqrt(-b**5/a
**3)*log(x + (12*a**10*c**5*d**7*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 68*a*
*9*b*c**6*d**6*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 + 152*a**8*b**2*c**7*d**5
*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 + 27*a**8*d**8*sqrt(-b**5/a**3)/(a*d -
b*c)**2 - 176*a**7*b**3*c**8*d**4*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 135*
a**7*b*c*d**7*sqrt(-b**5/a**3)/(a*d - b*c)**2 + 124*a**6*b**4*c**9*d**3*(-b
**5/a**3)**(3/2)/(a*d - b*c)**6 + 225*a**6*b**2*c**2*d**6*sqrt(-b**5/a**3)/
(a*d - b*c)**2 - 68*a**5*b**5*c**10*d**2*(-b**5/a**3)**(3/2)/(a*d - b*c)**6
 - 125*a**5*b**3*c**3*d**5*sqrt(-b**5/a**3)/(a*d - b*c)**2 + 32*a**4*b**6*c
**11*d*(-b**5/a**3)**(3/2)/(a*d - b*c)**6 - 8*a**3*b**7*c**12*(-b**5/a**3)*
*(3/2)/(a*d - b*c)**6 - 8*b**8*c**8*sqrt(-b**5/a**3)/(a*d - b*c)**2)/(27*a*
*4*b**3*d**6 - 81*a**3*b**4*c*d**5 + 36*a**2*b**5*c**2*d**4 + 28*a*b**6*c**
3*d**3 + 20*b**7*c**4*d**2))/(2*(a*d - b*c)**2) + sqrt(-d**3/c**5)*(3*a*d -
 5*b*c)*log(x + (-3*a**10*c**5*d**7*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/
(2*(a*d - b*c)**6) + 17*a**9*b*c**6*d**6*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c
)**3/(2*(a*d - b*c)**6) - 19*a**8*b**2*c**7*d**5*(-d**3/c**5)**(3/2)*(3*a*d
 - 5*b*c)**3/(a*d - b*c)**6 - 27*a**8*d**8*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)
/(2*(a*d - b*c)**2) + 22*a**7*b**3*c**8*d**4*(-d**3/c**5)**(3/2)*(3*a*d - 5
*b*c)**3/(a*d - b*c)**6 + 135*a**7*b*c*d**7*sqrt(-d**3/c**5)*(3*a*d - 5*b*c
)/(2*(a*d - b*c)**2) - 31*a**6*b**4*c**9*d**3*(-d**3/c**5)**(3/2)*(3*a*d -
5*b*c)**3/(2*(a*d - b*c)**6) - 225*a**6*b**2*c**2*d**6*sqrt(-d**3/c**5)*(3*
a*d - 5*b*c)/(2*(a*d - b*c)**2) + 17*a**5*b**5*c**10*d**2*(-d**3/c**5)**(3/
2)*(3*a*d - 5*b*c)**3/(2*(a*d - b*c)**6) + 125*a**5*b**3*c**3*d**5*sqrt(-d*
*3/c**5)*(3*a*d - 5*b*c)/(2*(a*d - b*c)**2) - 4*a**4*b**6*c**11*d*(-d**3/c*
*5)**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c)**6 + a**3*b**7*c**12*(-d**3/c**5)
**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c)**6 + 4*b**8*c**8*sqrt(-d**3/c**5)*(3
*a*d - 5*b*c)/(a*d - b*c)**2)/(27*a**4*b**3*d**6 - 81*a**3*b**4*c*d**5 + 36
*a**2*b**5*c**2*d**4 + 28*a*b**6*c**3*d**3 + 20*b**7*c**4*d**2))/(4*(a*d -
b*c)**2) - sqrt(-d**3/c**5)*(3*a*d - 5*b*c)*log(x + (3*a**10*c**5*d**7*(-d*
*3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(2*(a*d - b*c)**6) - 17*a**9*b*c**6*d**6
*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(2*(a*d - b*c)**6) + 19*a**8*b**2*c
**7*d**5*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c)**6 + 27*a**8*d*
*8*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)/(2*(a*d - b*c)**2) - 22*a**7*b**3*c**8*
d**4*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c)**6 - 135*a**7*b*c*d
**7*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)/(2*(a*d - b*c)**2) + 31*a**6*b**4*c**9
*d**3*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(2*(a*d - b*c)**6) + 225*a**6*
b**2*c**2*d**6*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)/(2*(a*d - b*c)**2) - 17*a**
5*b**5*c**10*d**2*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(2*(a*d - b*c)**6)
 - 125*a**5*b**3*c**3*d**5*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)/(2*(a*d - b*c)*
*2) + 4*a**4*b**6*c**11*d*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c
)**6 - a**3*b**7*c**12*(-d**3/c**5)**(3/2)*(3*a*d - 5*b*c)**3/(a*d - b*c)**
6 - 4*b**8*c**8*sqrt(-d**3/c**5)*(3*a*d - 5*b*c)/(a*d - b*c)**2)/(27*a**4*b
**3*d**6 - 81*a**3*b**4*c*d**5 + 36*a**2*b**5*c**2*d**4 + 28*a*b**6*c**3*d*
*3 + 20*b**7*c**4*d**2))/(4*(a*d - b*c)**2) - (2*a*c*d - 2*b*c**2 + x**2*(3
*a*d**2 - 2*b*c*d))/(x**3*(2*a**2*c**2*d**2 - 2*a*b*c**3*d) + x*(2*a**2*c**
3*d - 2*a*b*c**4))
```

Giac [A] time = 1.13505, size = 221, normalized size = 1.53

$$-\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{ab}}+\frac{\left(5\,bcd^2-3\,ad^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\sqrt{cd}}-\frac{2\,bcdx^2-3\,ad^2x^2+2\,bc^2-2\,acd}{2\left(abc^3-a^2c^2d\right)\left(dx^3+cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
+ 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c*d*x^2 - 3*a*d^2*x^2 + 2*b*c^2 - 2*a*c
*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^3 + c*x))
```

3.249 $\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=126

$$\frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}+\frac{d^2}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{d^2(3bc-2ad)\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}-\frac{1}{2ac^2x^2}$$

```
[Out] -1/(2*a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c + 2*a*d)*Log
[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*(3*b*c -
 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rubi [A] time = 0.144465, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}+\frac{d^2}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{d^2(3bc-2ad)\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}-\frac{1}{2ac^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/(2*a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c + 2*a*d)*Log
[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*(3*b*c -
 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^2}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^2x^2}+\frac{-bc-2ad}{a^2c^3x}+\frac{b^4}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)^2}-\frac{d}{c^3(b}\right.\right.$$

$$=-\frac{1}{2ac^2x^2}+\frac{d^2}{2c^2(bc-ad)\left(c+dx^2\right)}-\frac{(bc+2ad)\log(x)}{a^2c^3}+\frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{d^2(3bc-2ad}{2c^3(bc}$$

Mathematica [A] time = 0.236739, size = 117, normalized size = 0.93

$$\frac{1}{2}\left(\frac{b^3\log\left(a+bx^2\right)}{a^2(bc-ad)^2}-\frac{2\log(x)(2ad+bc)}{a^2c^3}+\frac{\frac{cd^2}{(c+dx^2)(bc-ad)}+\frac{d^2(2ad-3bc)\log\left(c+dx^2\right)}{(bc-ad)^2}-\frac{c}{ax^2}}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((-2*(b*c + 2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*(b*c - a*d
)^2) + (-(c/(a*x^2)) + (c*d^2)/((b*c - a*d)*(c + d*x^2)) + (d^2*(-3*b*c + 2
*a*d)*Log[c + d*x^2])/(b*c - a*d)^2)/c^3)/2
```

Maple [A] time = 0.018, size = 170, normalized size = 1.4

$$\frac{d^3\ln\left(dx^2+c\right)a}{c^3\left(ad-bc\right)^2}-\frac{3\,d^2\ln\left(dx^2+c\right)b}{2\,c^2\left(ad-bc\right)^2}-\frac{d^3a}{2\,c^2\left(ad-bc\right)^2\left(dx^2+c\right)}+\frac{bd^2}{2\,c\left(ad-bc\right)^2\left(dx^2+c\right)}-\frac{1}{2\,ac^2x^2}-2\,\frac{\ln\left(x\right)d}{ac^3}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] d^3/c^3/(a*d-b*c)^2*ln(d*x^2+c)*a-3/2*d^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*b-1/2
*d^3/c^2/(a*d-b*c)^2/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2/(d*x^2+c)*b-1/2/a/c^
2/x^2-2/a/c^3*ln(x)*d-1/a^2/c^2*ln(x)*b+1/2*b^3/a^2/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.17474, size = 254, normalized size = 2.02

$$\frac{b^3\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}-\frac{\left(3\,bcd^2-2\,ad^3\right)\log\left(dx^2+c\right)}{2\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)}-\frac{bc^2-acd+\left(bcd-2\,ad^2\right)x^2}{2\left(\left(abc^3d-a^2c^2d^2\right)x^4+\left(abc^4-a^2c^3d\right)x^2\right)}-\frac{(bc+2\,a}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/2*(3*b*c*d
^2 - 2*a*d^3)*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(b
*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b
*c^4 - a^2*c^3*d)*x^2) - 1/2*(b*c + 2*a*d)*log(x^2)/(a^2*c^3)
```

Fricas [B] time = 14.2487, size = 601, normalized size = 4.77

$$-\frac{ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+\left(ab^2c^3d-3\,a^2bc^2d^2+2\,a^3cd^3\right)x^2-\left(b^3c^3dx^4+b^3c^4x^2\right)\log\left(bx^2+a\right)+\left(\left(3\,a^2bcd^3-2\,a^3\right.\right.}{2\left(\left(a^2b^2c^5d-2\,a^3bc^4d^2+a^4c^3d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 3*a^2*b*c^2*
d^2 + 2*a^3*c*d^3)*x^2 - (b^3*c^3*d*x^4 + b^3*c^4*x^2)*log(b*x^2 + a) + ((3
*a^2*b*c*d^3 - 2*a^3*d^4)*x^4 + (3*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*log(d*
x^2 + c) + 2*((b^3*c^3*d - 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^4 + (b^3*c^4 - 3*a^
2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*log(x))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 +
 a^4*c^3*d^3)*x^4 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15844, size = 347, normalized size = 2.75

$$\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}-\frac{\left(3\,bcd^3-2\,ad^4\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)}+\frac{b^3c^2dx^4+b^3c^3x^2-2\,ab^2c^2dx^2+6\,a^2bcd^2x^2-4\,a^3d^3x^2}{4\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2) - 1/2
*(3*b*c*d^3 - 2*a*d^4)*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2
*c^3*d^3) + 1/4*(b^3*c^2*d*x^4 + b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 + 6*a^2*b*
c*d^2*x^2 - 4*a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(d*x^4 + c*x^2)) - 1/2*(b*c + 2*a*
d)*log(x^2)/(a^2*c^3)
```

3.250 $\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-a}$$

```
[Out] -(2*b*c - 5*a*d)/(6*a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c*d - 5*a^2
*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^3*(c + d*x^2)) + (b^
(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*(7*b*
c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.272061, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(2*b*c - 5*a*d)/(6*a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c*d - 5*a^2
*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^3*(c + d*x^2)) + (b^
(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*(7*b*
c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = -\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{\int \frac{2bc-5ad-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}-\frac{\int \frac{3\left(2b^2c^2+2abcd-5a^2d^2\right)+3bd(2bc-5ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{6ac^2(bc-ad)}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{\int \frac{3\left(2b^3c^3+2ab^2c^2d+\right.}{}}{}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^2(bc-ad)^2}-$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}$$

Mathematica [A] time = 0.372498, size = 142, normalized size = 0.75

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^2}+\frac{2ad+bc}{a^2c^3x}-\frac{d^3x}{2c^3\left(c+dx^2\right)(bc-ad)}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{1}{3ac^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/(3*a*c^2*x^3) + (b*c + 2*a*d)/(a^2*c^3*x) - (d^3*x)/(2*c^3*(b*c - a*d)*(
c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)
^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c
 - a*d)^2)
```

Maple [A] time = 0.016, size = 191, normalized size = 1.

$$\frac{d^4xa}{2\,c^3\,(ad-bc)^2\left(dx^2+c\right)}-\frac{d^3xb}{2\,c^2\,(ad-bc)^2\left(dx^2+c\right)}+\frac{5\,d^4a}{2\,c^3\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,d^3b}{2\,c^2\,(ad-bc)^2}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/c^3/(a*d-b*c)^2*x/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)^2*x/(d*x^2+c)*b
+5/2*d^4/c^3/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-7/2*d^3/c^2/
```

```
(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b-1/3/a/c^2/x^3+2/a/c^3/x*d
+1/a^2/c^2/x*b+1/a^2*b^4/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.90387, size = 2558, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^
2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 +
 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 +
(7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5
+ (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a
^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^
4 - 2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 + 3*((7
*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d
/c)*arctan(x*sqrt(d/c)) - 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b
*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^
2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1
/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^2*b
*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*
a^3*c*d^3)*x^2 - 12*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(b/a)*arctan(x*sqrt(b
/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^2
*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d
+ a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*
(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 2*(3*b^3*c^4 - a*b^2*c^3*d
- 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c
^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^2*c^5*d -
 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^
4*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17051, size = 223, normalized size = 1.18

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\sqrt{ab}} - \frac{d^3x}{2\left(bc^4 - ac^3d\right)\left(dx^2 + c\right)} - \frac{\left(7\,bcd^3 - 5\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^5 - 2\,abc^4d + a^2c^3d^2\right)\sqrt{cd}} + \frac{3\,bcx^2 + 6\,adx^2 - ac}{3\,a^2c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b))
 - 1/2*d^3*x/((b*c^4 - a*c^3*d)*(d*x^2 + c)) - 1/2*(7*b*c*d^3 - 5*a*d^4)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)) + 1/3
*(3*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^2*c^3*x^3)
```

3.251 $\int \frac{x^5}{(a+bx^2)^3 (c+dx^2)} \, dx$

Optimal. Leaf size=116

$$-\frac{a^2}{4b^2 (a+bx^2)^2 (bc-ad)}+\frac{a(2bc-ad)}{2b^2 (a+bx^2) (bc-ad)^2}+\frac{c^2 \log (a+bx^2)}{2(bc-ad)^3}-\frac{c^2 \log (c+dx^2)}{2(bc-ad)^3}$$

```
[Out] -a^2/(4*b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2*(b*c - a*
d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (c^2*Log[c + d
*x^2])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.113149, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{4b^2 (a+bx^2)^2 (bc-ad)}+\frac{a(2bc-ad)}{2b^2 (a+bx^2) (bc-ad)^2}+\frac{c^2 \log (a+bx^2)}{2(bc-ad)^3}-\frac{c^2 \log (c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2*(b*c - a*
d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (c^2*Log[c + d
*x^2])/(2*(b*c - a*d)^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^2)^3 (c+dx^2)} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2}+\frac{bc^2}{(bc-ad)^3(a+bx)}-\frac{}{(bc-a}\right.\right. \\
&= -\frac{a^2}{4b^2(bc-ad)(a+bx^2)^2}+\frac{a(2bc-ad)}{2b^2(bc-ad)^2(a+bx^2)}+\frac{c^2 \log (a+bx^2)}{2(bc-ad)^3}-\frac{c^2 \log (c+dx^2)}{2(bc-ad)^3}
\end{aligned}$$

Mathematica [A] time = 0.0999876, size = 99, normalized size = 0.85

$$\frac{-\frac{a^2(bc-ad)^2}{b^2(a+bx^2)^2}+\frac{2a(ad-2bc)(ad-bc)}{b^2(a+bx^2)}+2c^2\log\left(a+bx^2\right)-2c^2\log\left(c+dx^2\right)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (-((a^2*(b*c - a*d)^2)/(b^2*(a + b*x^2)^2)) + (2*a*(-2*b*c + a*d)*(-(b*c) +
 a*d))/(b^2*(a + b*x^2)) + 2*c^2*Log[a + b*x^2] - 2*c^2*Log[c + d*x^2])/(4*
(b*c - a*d)^3)
```

Maple [B] time = 0.011, size = 218, normalized size = 1.9

$$\frac{c^2\ln\left(dx^2+c\right)}{2\left(ad-bc\right)^3}+\frac{a^4d^2}{4\left(ad-bc\right)^3b^2\left(bx^2+a\right)^2}-\frac{a^3cd}{2\left(ad-bc\right)^3b\left(bx^2+a\right)^2}+\frac{a^2c^2}{4\left(ad-bc\right)^3\left(bx^2+a\right)^2}-\frac{c^2\ln\left(bx^2+a\right)}{2\left(ad-bc\right)^3}-\frac{}{2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] 1/2*c^2/(a*d-b*c)^3*ln(d*x^2+c)+1/4/(a*d-b*c)^3*a^4/b^2/(b*x^2+a)^2*d^2-1/2
/(a*d-b*c)^3*a^3/b/(b*x^2+a)^2*c*d+1/4/(a*d-b*c)^3*a^2/(b*x^2+a)^2*c^2-1/2/
(a*d-b*c)^3*c^2*ln(b*x^2+a)-1/2/(a*d-b*c)^3/b^2*a^3/(b*x^2+a)*d^2+3/2/(a*d-
b*c)^3/b*a^2/(b*x^2+a)*c*d-1/(a*d-b*c)^3*a/(b*x^2+a)*c^2
```

Maxima [B] time = 1.244, size = 319, normalized size = 2.75

$$\frac{c^2\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{c^2\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{3\,a^2bc-a}{4\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+\left(b^6c^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*c^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^2)/(a^2*b^4*c^2 - 2*
a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(
a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2)
```

Fricas [B] time = 1.60294, size = 571, normalized size = 4.92

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\left(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2\right)x^2+2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(bx^2+a\right)-2\left(b^4c^2x^4\right.}{4\left(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3+\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)x^4+2\left(ab^6c^3-3\,a^2b^5c^2d+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^2 + 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(b*x
^2 + a) - 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(d*x^2 + c))/(
a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3 + (b^7*c^3 -
3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 2*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2)
```

Sympy [B] time = 4.60615, size = 418, normalized size = 3.6

$$\frac{c^2\log\left(x^2+\frac{-\frac{a^4c^2d^4}{(ad-bc)^3}+\frac{4a^3bc^3d^3}{(ad-bc)^3}-\frac{6a^2b^2c^4d^2}{(ad-bc)^3}+\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d-\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2(ad-bc)^3}-\frac{c^2\log\left(x^2+\frac{\frac{a^4c^2d^4}{(ad-bc)^3}-\frac{4a^3bc^3d^3}{(ad-bc)^3}+\frac{6a^2b^2c^4d^2}{(ad-bc)^3}-\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d+\frac{b^4c^6}{(ad-bc)^3}+\cdots}{2bc^2d}\right)}{2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] c**2*log(x**2 + (-a**4*c**2*d**4/(a*d - b*c)**3 + 4*a**3*b*c**3*d**3/(a*d -
 b*c)**3 - 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 + 4*a*b**3*c**5*d/(a*d - b*
c)**3 + a*c**2*d - b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**2*d))/(2*(a*d
 - b*c)**3) - c**2*log(x**2 + (a**4*c**2*d**4/(a*d - b*c)**3 - 4*a**3*b*c**
3*d**3/(a*d - b*c)**3 + 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 - 4*a*b**3*c**
5*d/(a*d - b*c)**3 + a*c**2*d + b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**
2*d))/(2*(a*d - b*c)**3) - (a**3*d - 3*a**2*b*c + x**2*(2*a**2*b*d - 4*a*b*
*2*c))/(4*a**4*b**2*d**2 - 8*a**3*b**3*c*d + 4*a**2*b**4*c**2 + x**4*(4*a**
2*b**4*d**2 - 8*a*b**5*c*d + 4*b**6*c**2) + x**2*(8*a**3*b**3*d**2 - 16*a**
2*b**4*c*d + 8*a*b**5*c**2))
```

Giac [B] time = 1.19138, size = 313, normalized size = 2.7

$$\frac{bc^2\log\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{c^2d\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{3\,b^4c^2x^4+2\,ab^3c^2x^2+6\,a^2b^2cdx^2\cdots}{4\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*c^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 1/2*c^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4) - 1/4*(3*b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + 6*a^2*b^2*
c*d*x^2 - 2*a^3*b*d^2*x^2 + 4*a^3*b*c*d - a^4*d^2)/((b^5*c^3 - 3*a*b^4*c^2*
d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(b*x^2 + a)^2)
```

3.252 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=157

$$\frac{(-3a^2d^2-6abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(bc-5ad)}{8d(c+dx^2)(bc-ad)^2}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

```
[Out] -(c*x)/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(b*c - a*d)
^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)
^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqr
t[c]*d^(3/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.172041, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{(-3a^2d^2-6abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(bc-5ad)}{8d(c+dx^2)(bc-ad)^2}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*x)/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(b*c - a*d)
^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)
^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqr
t[c]*d^(3/2)*(b*c - a*d)^3)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{ac+(bc-4ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4d(bc-ad)}$$

$$= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{ac(bc+3ad)+bc(bc-5ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8cd(bc-ad)^2}$$

$$= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(a^2b\right)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}+\frac{\left(b^2c^2-6abcd-\right.}{8d(bc}$$

$$= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(b^2c^2-6abc\right.}{8\sqrt{c}}$$

Mathematica [A] time = 0.247804, size = 154, normalized size = 0.98

$$\frac{1}{8}\left(\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{8a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(bc-5ad)}{d\left(c+dx^2\right)(bc-ad)^2}+\frac{2cx}{d\left(c+dx^2\right)^2(ad-bc)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*c*x)/(d*(-(b*c) + a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(d*(b*c - a*d
)^2*(c + d*x^2)) + (8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a
*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sq
rt[c]*d^(3/2)*(b*c - a*d)^3))/8
```

Maple [B] time = 0.009, size = 299, normalized size = 1.9

$$-\frac{5\,x^3a^2d^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{3\,x^3abcd}{4\,(ad-bc)^3\left(dx^2+c\right)^2}-\frac{x^3b^2c^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2}-\frac{3\,a^2cdx}{8\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{abc^2}{4\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -5/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2*d^2+3/4/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*
b*c*d-1/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2*c^2-3/8/(a*d-b*c)^3/(d*x^2+c)^2*a
^2*c*d*x+1/4/(a*d-b*c)^3/(d*x^2+c)^2*a*b*c^2*x+1/8/(a*d-b*c)^3/(d*x^2+c)^2*
c^3/d*x*b^2+3/8/(a*d-b*c)^3*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+3/4/(
a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b-1/8/(a*d-b*c)^3/d/(c*d
```

```
)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2-b/(a*d-b*c)^3*a^2/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.63587, size = 3168, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [1/16*(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 - 8*(a*c*d^4*x^4 +
 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b
*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2
*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c
^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a
^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^
2*d^6)*x^2), 1/8*((b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + (b^2*c^
4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a
^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*
(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/16*
(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 16*(a*c*d^4*x^4 + 2*a*
c^2*d^3*x^2 + a*c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (b^2*c^4 - 6*a*b
*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b
^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*
x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^
4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3
 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/8*((b^2*c^3*d^2
 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 8*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*
c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c
^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b
*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - (b^2*c^4*d +
 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b
*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 -
 a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*
c^2*d^6)*x^2)]
```

Sympy [B] time = 65.0699, size = 3390, normalized size = 21.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] sqrt(-a**3*b)*log(x + (-192*a**8*c*d**11*(-a**3*b)**(3/2)/(a*d - b*c)**9 +
256*a**7*b*c**2*d**10*(-a**3*b)**(3/2)/(a*d - b*c)**9 + 2560*a**6*b**2*c**3
*d**9*(-a**3*b)**(3/2)/(a*d - b*c)**9 - 27*a**6*d**6*sqrt(-a**3*b)/(a*d - b
*c)**3 - 9984*a**5*b**3*c**4*d**8*(-a**3*b)**(3/2)/(a*d - b*c)**9 - 162*a**
5*b*c*d**5*sqrt(-a**3*b)/(a*d - b*c)**3 + 16000*a**4*b**4*c**5*d**7*(-a**3*
b)**(3/2)/(a*d - b*c)**9 - 809*a**4*b**2*c**2*d**4*sqrt(-a**3*b)/(a*d - b*c
)**3 - 13568*a**3*b**5*c**6*d**6*(-a**3*b)**(3/2)/(a*d - b*c)**9 - 108*a**3
*b**3*c**3*d**3*sqrt(-a**3*b)/(a*d - b*c)**3 + 6144*a**2*b**6*c**7*d**5*(-a
**3*b)**(3/2)/(a*d - b*c)**9 + 99*a**2*b**4*c**4*d**2*sqrt(-a**3*b)/(a*d -
b*c)**3 - 1280*a*b**7*c**8*d**4*(-a**3*b)**(3/2)/(a*d - b*c)**9 - 18*a*b**5
*c**5*d*sqrt(-a**3*b)/(a*d - b*c)**3 + 64*b**8*c**9*d**3*(-a**3*b)**(3/2)/(
a*d - b*c)**9 + b**6*c**6*sqrt(-a**3*b)/(a*d - b*c)**3)/(27*a**4*b*d**3 + 5
1*a**3*b**2*c*d**2 - 15*a**2*b**3*c**2*d + a*b**4*c**3))/(2*(a*d - b*c)**3)
 - sqrt(-a**3*b)*log(x + (192*a**8*c*d**11*(-a**3*b)**(3/2)/(a*d - b*c)**9
- 256*a**7*b*c**2*d**10*(-a**3*b)**(3/2)/(a*d - b*c)**9 - 2560*a**6*b**2*c*
*3*d**9*(-a**3*b)**(3/2)/(a*d - b*c)**9 + 27*a**6*d**6*sqrt(-a**3*b)/(a*d -
 b*c)**3 + 9984*a**5*b**3*c**4*d**8*(-a**3*b)**(3/2)/(a*d - b*c)**9 + 162*a
**5*b*c*d**5*sqrt(-a**3*b)/(a*d - b*c)**3 - 16000*a**4*b**4*c**5*d**7*(-a**
3*b)**(3/2)/(a*d - b*c)**9 + 809*a**4*b**2*c**2*d**4*sqrt(-a**3*b)/(a*d - b
*c)**3 + 13568*a**3*b**5*c**6*d**6*(-a**3*b)**(3/2)/(a*d - b*c)**9 + 108*a*
*3*b**3*c**3*d**3*sqrt(-a**3*b)/(a*d - b*c)**3 - 6144*a**2*b**6*c**7*d**5*(
-a**3*b)**(3/2)/(a*d - b*c)**9 - 99*a**2*b**4*c**4*d**2*sqrt(-a**3*b)/(a*d
- b*c)**3 + 1280*a*b**7*c**8*d**4*(-a**3*b)**(3/2)/(a*d - b*c)**9 + 18*a*b*
*5*c**5*d*sqrt(-a**3*b)/(a*d - b*c)**3 - 64*b**8*c**9*d**3*(-a**3*b)**(3/2)
/(a*d - b*c)**9 - b**6*c**6*sqrt(-a**3*b)/(a*d - b*c)**3)/(27*a**4*b*d**3 +
 51*a**3*b**2*c*d**2 - 15*a**2*b**3*c**2*d + a*b**4*c**3))/(2*(a*d - b*c)**
3) + sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)*log(x + (-3*a*
*8*c*d**11*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(8
*(a*d - b*c)**9) + a**7*b*c**2*d**10*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*
a*b*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) + 5*a**6*b**2*c**3*d**9*(-1/(c*d
**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(a*d - b*c)**9 - 27*a
**6*d**6*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(8*(a*d -
b*c)**3) - 39*a**5*b**3*c**4*d**8*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b
*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) - 81*a**5*b*c*d**5*sqrt(-1/(c*d**3)
)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(4*(a*d - b*c)**3) + 125*a**4*b**4*
c**5*d**7*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(4*
(a*d - b*c)**9) - 809*a**4*b**2*c**2*d**4*sqrt(-1/(c*d**3))*(3*a**2*d**2 +
6*a*b*c*d - b**2*c**2)/(8*(a*d - b*c)**3) - 53*a**3*b**5*c**6*d**6*(-1/(c*d
**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) -
27*a**3*b**3*c**3*d**3*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c*
*2)/(2*(a*d - b*c)**3) + 12*a**2*b**6*c**7*d**5*(-1/(c*d**3))**(3/2)*(3*a**
2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(a*d - b*c)**9 + 99*a**2*b**4*c**4*d**2*
sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(8*(a*d - b*c)**3)
- 5*a*b**7*c**8*d**4*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c
**2)**3/(2*(a*d - b*c)**9) - 9*a*b**5*c**5*d*sqrt(-1/(c*d**3))*(3*a**2*d**2
 + 6*a*b*c*d - b**2*c**2)/(4*(a*d - b*c)**3) + b**8*c**9*d**3*(-1/(c*d**3))
**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(8*(a*d - b*c)**9) + b**6*
c**6*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(8*(a*d - b*c)
**3))/(27*a**4*b*d**3 + 51*a**3*b**2*c*d**2 - 15*a**2*b**3*c**2*d + a*b**4*
c**3))/(16*(a*d - b*c)**3) - sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b
```

```
**2*c**2)*log(x + (3*a**8*c*d**11*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b
*c*d - b**2*c**2)**3/(8*(a*d - b*c)**9) - a**7*b*c**2*d**10*(-1/(c*d**3))**
(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) - 5*a**6*
b**2*c**3*d**9*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**
3/(a*d - b*c)**9 + 27*a**6*d**6*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d
- b**2*c**2)/(8*(a*d - b*c)**3) + 39*a**5*b**3*c**4*d**8*(-1/(c*d**3))**(3/
2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) + 81*a**5*b*
c*d**5*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(4*(a*d - b*
c)**3) - 125*a**4*b**4*c**5*d**7*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*
c*d - b**2*c**2)**3/(4*(a*d - b*c)**9) + 809*a**4*b**2*c**2*d**4*sqrt(-1/(c
*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(8*(a*d - b*c)**3) + 53*a**3*
b**5*c**6*d**6*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**
3/(2*(a*d - b*c)**9) + 27*a**3*b**3*c**3*d**3*sqrt(-1/(c*d**3))*(3*a**2*d**
2 + 6*a*b*c*d - b**2*c**2)/(2*(a*d - b*c)**3) - 12*a**2*b**6*c**7*d**5*(-1/
(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(a*d - b*c)**9 -
99*a**2*b**4*c**4*d**2*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c*
*2)/(8*(a*d - b*c)**3) + 5*a*b**7*c**8*d**4*(-1/(c*d**3))**(3/2)*(3*a**2*d*
*2 + 6*a*b*c*d - b**2*c**2)**3/(2*(a*d - b*c)**9) + 9*a*b**5*c**5*d*sqrt(-1
/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)/(4*(a*d - b*c)**3) - b**8*
c**9*d**3*(-1/(c*d**3))**(3/2)*(3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)**3/(8*
(a*d - b*c)**9) - b**6*c**6*sqrt(-1/(c*d**3))*(3*a**2*d**2 + 6*a*b*c*d - b*
*2*c**2)/(8*(a*d - b*c)**3))/(27*a**4*b*d**3 + 51*a**3*b**2*c*d**2 - 15*a**
2*b**3*c**2*d + a*b**4*c**3))/(16*(a*d - b*c)**3) - (x**3*(5*a*d**2 - b*c*d
) + x*(3*a*c*d + b*c**2))/(8*a**2*c**2*d**3 - 16*a*b*c**3*d**2 + 8*b**2*c**
4*d + x**4*(8*a**2*d**5 - 16*a*b*c*d**4 + 8*b**2*c**2*d**3) + x**2*(16*a**2
*c*d**4 - 32*a*b*c**2*d**3 + 16*b**2*c**3*d**2))
```

Giac [A] time = 1.14527, size = 275, normalized size = 1.75

$$\frac{a^2 b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} + \frac{\left(b^2c^2 - 6\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4\right)\sqrt{cd}} + \frac{bcdx^3 - 5\,ad^2x^3 - bc^2x - 3\,ac}{8\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\left(dx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d)) + 1
/8*(b*c*d*x^3 - 5*a*d^2*x^3 - b*c^2*x - 3*a*c*d*x)/((b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(d*x^2 + c)^2)
```

3.253 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^3} \, dx$

Optimal. Leaf size=100

$$-\frac{a}{2(c+dx^2)(bc-ad)^2}-\frac{c}{4d(c+dx^2)^2(bc-ad)}-\frac{ab\log(a+bx^2)}{2(bc-ad)^3}+\frac{ab\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] -c/(4*d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^2)) - (a*b
*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.0924123, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a}{2(c+dx^2)(bc-ad)^2}-\frac{c}{4d(c+dx^2)^2(bc-ad)}-\frac{ab\log(a+bx^2)}{2(bc-ad)^3}+\frac{ab\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -c/(4*d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^2)) - (a*b
*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)(c+dx^2)^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^3} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab^2}{(bc-ad)^3(a+bx)}+\frac{c}{(bc-ad)(c+dx)^3}+\frac{ad}{(-bc+ad)^2(c+dx)^2}-\frac{}{(-bc+\right.\right. \\
&= -\frac{c}{4d(bc-ad)(c+dx^2)^2}-\frac{a}{2(bc-ad)^2(c+dx^2)}-\frac{ab\log(a+bx^2)}{2(bc-ad)^3}+\frac{ab\log(c+dx^2)}{2(bc-ad)^3}
\end{aligned}$$

Mathematica [A] time = 0.114326, size = 77, normalized size = 0.77

$$\frac{\frac{(ad-bc)\left(ad\left(c+2dx^2\right)+bc^2\right)}{d\left(c+dx^2\right)^2}+2ab\log\left(c+dx^2\right)-2ab\log\left(a+bx^2\right)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (((-(b*c) + a*d)*(b*c^2 + a*d*(c + 2*d*x^2)))/(d*(c + d*x^2)^2) - 2*a*b*Log
[a + b*x^2] + 2*a*b*Log[c + d*x^2])/(4*(b*c - a*d)^3)
```

Maple [A] time = 0.01, size = 177, normalized size = 1.8

$$-\frac{ab\ln\left(dx^2+c\right)}{2\,(ad-bc)^3}+\frac{a^2cd}{4\,(ad-bc)^3\left(dx^2+c\right)^2}-\frac{abc^2}{2\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{b^2c^3}{4\,(ad-bc)^3\,d\left(dx^2+c\right)^2}-\frac{a^2d}{2\,(ad-bc)^3\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2/(a*d-b*c)^3*a*b*ln(d*x^2+c)+1/4/(a*d-b*c)^3*c*d/(d*x^2+c)^2*a^2-1/2/(a
*d-b*c)^3*c^2/(d*x^2+c)^2*a*b+1/4/(a*d-b*c)^3*c^3/d/(d*x^2+c)^2*b^2-1/2/(a*
d-b*c)^3*a^2/(d*x^2+c)*d+1/2/(a*d-b*c)^3*a/(d*x^2+c)*b*c+1/2*b*a/(a*d-b*c)^
3*ln(b*x^2+a)
```

Maxima [B] time = 1.09098, size = 293, normalized size = 2.93

$$-\frac{ab\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{ab\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{2}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*b*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
 + 1/2*a*b*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) - 1/4*(2*a*d^2*x^2 + b*c^2 + a*c*d)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2
*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c
^2*d^3 + a^2*c*d^4)*x^2)
```

Fricas [B] time = 1.65148, size = 513, normalized size = 5.13

$$-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2+2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\log\left(bx^2+a\right)-2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\mathrm{lo}}{4\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^4+2\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2 + 2*(a*b*d^3*x^4 +
2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(b*x^2 + a) - 2*(a*b*d^3*x^4 + 2*a*b*c*d^2*
x^2 + a*b*c^2*d)*log(d*x^2 + c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3
*d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d
^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x
^2)
```

Sympy [B] time = 4.31335, size = 410, normalized size = 4.1

$$-\frac{ab\log\left(x^2+\frac{-\frac{a^5bd^4}{(ad-bc)^3}+\frac{4a^4b^2cd^3}{(ad-bc)^3}-\frac{6a^3b^3c^2d^2}{(ad-bc)^3}+\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd-\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2(ad-bc)^3}+\frac{ab\log\left(x^2+\frac{\frac{a^5bd^4}{(ad-bc)^3}-\frac{4a^4b^2cd^3}{(ad-bc)^3}+\frac{6a^3b^3c^2d^2}{(ad-bc)^3}-\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd+\cdots}{2ab^2d}\right.}{2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -a*b*log(x**2 + (-a**5*b*d**4/(a*d - b*c)**3 + 4*a**4*b**2*c*d**3/(a*d - b*
c)**3 - 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 + 4*a**2*b**4*c**3*d/(a*d - b*
c)**3 + a**2*b*d - a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(2*a*b**2*d))/(2*
(a*d - b*c)**3) + a*b*log(x**2 + (a**5*b*d**4/(a*d - b*c)**3 - 4*a**4*b**2*
c*d**3/(a*d - b*c)**3 + 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 - 4*a**2*b**4*
c**3*d/(a*d - b*c)**3 + a**2*b*d + a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(
2*a*b**2*d))/(2*(a*d - b*c)**3) - (a*c*d + 2*a*d**2*x**2 + b*c**2)/(4*a**2*
c**2*d**3 - 8*a*b*c**3*d**2 + 4*b**2*c**4*d + x**4*(4*a**2*d**5 - 8*a*b*c*d
**4 + 4*b**2*c**2*d**3) + x**2*(8*a**2*c*d**4 - 16*a*b*c**2*d**3 + 8*b**2*c
**3*d**2))
```

Giac [A] time = 1.17873, size = 235, normalized size = 2.35

$$-\frac{ab^2\log\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{abd\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)\cdots}{4\left(dx^2+c\right)^2(bc-ad)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*a*b^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) + 1/2*a*b*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^
3)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3*d)
```

3.254 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=155

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}(bc-ad)^3}-\frac{\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a*d)^2*(c +
 d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)^3 + ((
3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sq
rt[d]*(b*c - a*d)^3)
```

Rubi [A] time = 0.140216, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}(bc-ad)^3}-\frac{\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a*d)^2*(c +
 d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)^3 + ((
3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sq
rt[d]*(b*c - a*d)^3)
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{a-3bx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4(bc-ad)}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\int \frac{a(5bc-ad)-b(3bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8c(bc-ad)^2}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(ab^2\right)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3} + \frac{\left(3b^2c^2+6abcd-a^2\right.}{8c(bc-a}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3} + \frac{\left(3b^2c^2+6abcd\right.}{8c^{3/2}\sqrt{a}}$$

Mathematica [A] time = 0.227899, size = 151, normalized size = 0.97

$$\frac{1}{8}\left(\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}(bc-ad)^3} + \frac{8\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(ad-bc)^3} + \frac{x(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)^2} + \frac{2x}{\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*x)/((b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(c*(b*c - a*d)^2*(c
+ d*x^2)) + (8*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(-(b*c) + a*d)^
3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2
)*Sqrt[d]*(b*c - a*d)^3))/8
```

Maple [B] time = 0.012, size = 298, normalized size = 1.9

$$\frac{x^3a^2d^3}{8\,(ad-bc)^3\left(dx^2+c\right)^2c} + \frac{x^3abd^2}{4\,(ad-bc)^3\left(dx^2+c\right)^2} - \frac{3\,x^3b^2cd}{8\,(ad-bc)^3\left(dx^2+c\right)^2} + \frac{3\,cabdx}{4\,(ad-bc)^3\left(dx^2+c\right)^2} - \frac{5\,b^2c}{8\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/8/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^3*a^2+1/4/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a
*b*d^2-3/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2*c*d+3/4/(a*d-b*c)^3/(d*x^2+c)^2*
c*a*b*d*x-5/8/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c^2*x-1/8/(a*d-b*c)^3/(d*x^2+c)^2
*a^2*d^2*x+1/8/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2-3/
4/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*d-3/8/(a*d-b*c)^3*c/(
```

```
c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2+b^2*a/(a*d-b*c)^3/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.83158, size = 3168, normalized size = 20.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [1/16*(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4
 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a
*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d -
 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5
*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3
*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^
3*d^5)*x^2), 1/8*((3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 + (3*b^2*
c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x
^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((
b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + (5*b^2*c^4*d - 6*a*b*c^3*d^2 + a
^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4
 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*
(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/16*
(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 16*(b*c^2*d^3*x^4 + 2*
b*c^3*d^2*x^2 + b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (3*b^2*c^4 + 6*a
*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3
*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*
x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2
 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/8*((3*b^2*c^3*d
^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 +
b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c
^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a
*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (5*b^2*c^4*d
 - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c
^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a
^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*
c^3*d^5)*x^2)]
```

Sympy [B] time = 58.7035, size = 3386, normalized size = 21.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-a*b**3)*log(x + (-64*a**8*c**3*d**9*(-a*b**3)**(3/2)/(a*d - b*c)**9
+ 768*a**7*b*c**4*d**8*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 2560*a**6*b**2*c**
5*d**7*(-a*b**3)**(3/2)/(a*d - b*c)**9 - a**6*d**6*sqrt(-a*b**3)/(a*d - b*c
)**3 + 2816*a**5*b**3*c**6*d**6*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 18*a**5*b
*c*d**5*sqrt(-a*b**3)/(a*d - b*c)**3 + 1920*a**4*b**4*c**7*d**5*(-a*b**3)**
(3/2)/(a*d - b*c)**9 - 99*a**4*b**2*c**2*d**4*sqrt(-a*b**3)/(a*d - b*c)**3
- 7936*a**3*b**5*c**8*d**4*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 108*a**3*b**3*
c**3*d**3*sqrt(-a*b**3)/(a*d - b*c)**3 + 8192*a**2*b**6*c**9*d**3*(-a*b**3)
**(3/2)/(a*d - b*c)**9 + 297*a**2*b**4*c**4*d**2*sqrt(-a*b**3)/(a*d - b*c)*
*3 - 3840*a*b**7*c**10*d**2*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 674*a*b**5*c*
*5*d*sqrt(-a*b**3)/(a*d - b*c)**3 + 704*b**8*c**11*d*(-a*b**3)**(3/2)/(a*d
- b*c)**9 + 27*b**6*c**6*sqrt(-a*b**3)/(a*d - b*c)**3)/(a**3*b**2*d**3 - 15
*a**2*b**3*c*d**2 + 51*a*b**4*c**2*d + 27*b**5*c**3))/(2*(a*d - b*c)**3) +
sqrt(-a*b**3)*log(x + (64*a**8*c**3*d**9*(-a*b**3)**(3/2)/(a*d - b*c)**9 -
768*a**7*b*c**4*d**8*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 2560*a**6*b**2*c**5*
d**7*(-a*b**3)**(3/2)/(a*d - b*c)**9 + a**6*d**6*sqrt(-a*b**3)/(a*d - b*c)*
*3 - 2816*a**5*b**3*c**6*d**6*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 18*a**5*b*c
*d**5*sqrt(-a*b**3)/(a*d - b*c)**3 - 1920*a**4*b**4*c**7*d**5*(-a*b**3)**(3
/2)/(a*d - b*c)**9 + 99*a**4*b**2*c**2*d**4*sqrt(-a*b**3)/(a*d - b*c)**3 +
7936*a**3*b**5*c**8*d**4*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 108*a**3*b**3*c*
*3*d**3*sqrt(-a*b**3)/(a*d - b*c)**3 - 8192*a**2*b**6*c**9*d**3*(-a*b**3)**
(3/2)/(a*d - b*c)**9 - 297*a**2*b**4*c**4*d**2*sqrt(-a*b**3)/(a*d - b*c)**3
 + 3840*a*b**7*c**10*d**2*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 674*a*b**5*c**5
*d*sqrt(-a*b**3)/(a*d - b*c)**3 - 704*b**8*c**11*d*(-a*b**3)**(3/2)/(a*d -
b*c)**9 - 27*b**6*c**6*sqrt(-a*b**3)/(a*d - b*c)**3)/(a**3*b**2*d**3 - 15*a
**2*b**3*c*d**2 + 51*a*b**4*c**2*d + 27*b**5*c**3))/(2*(a*d - b*c)**3) - sq
rt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)*log(x + (-a**8*c**3*d
**9*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(8*(a*d -
 b*c)**9) + 3*a**7*b*c**4*d**8*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d
- 3*b**2*c**2)**3/(2*(a*d - b*c)**9) - 5*a**6*b**2*c**5*d**7*(-1/(c**3*d))*
*(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 - a**6*d**6*
sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3)
+ 11*a**5*b**3*c**6*d**6*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b*
*2*c**2)**3/(2*(a*d - b*c)**9) + 9*a**5*b*c*d**5*sqrt(-1/(c**3*d))*(a**2*d*
*2 - 6*a*b*c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) + 15*a**4*b**4*c**7*d**5*(
-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(4*(a*d - b*c)
**9) - 99*a**4*b**2*c**2*d**4*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*
b**2*c**2)/(8*(a*d - b*c)**3) - 31*a**3*b**5*c**8*d**4*(-1/(c**3*d))**(3/2)
*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) + 27*a**3*b**3
*c**3*d**3*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(2*(a*d
- b*c)**3) + 16*a**2*b**6*c**9*d**3*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b
*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 + 297*a**2*b**4*c**4*d**2*sqrt(-1/(c*
*3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3) - 15*a*b**7
*c**10*d**2*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(
2*(a*d - b*c)**9) + 337*a*b**5*c**5*d*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*
c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) + 11*b**8*c**11*d*(-1/(c**3*d))**(3/2
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(8*(a*d - b*c)**9) + 27*b**6*c**
6*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3
))/(a**3*b**2*d**3 - 15*a**2*b**3*c*d**2 + 51*a*b**4*c**2*d + 27*b**5*c**3)
)/(16*(a*d - b*c)**3) + sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c
```

```
**2)*log(x + (a**8*c**3*d**9*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d -
3*b**2*c**2)**3/(8*(a*d - b*c)**9) - 3*a**7*b*c**4*d**8*(-1/(c**3*d))**(3/2
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) + 5*a**6*b**2
*c**5*d**7*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(a
*d - b*c)**9 + a**6*d**6*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*
c**2)/(8*(a*d - b*c)**3) - 11*a**5*b**3*c**6*d**6*(-1/(c**3*d))**(3/2)*(a**
2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) - 9*a**5*b*c*d**5*s
qrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) -
 15*a**4*b**4*c**7*d**5*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**
2*c**2)**3/(4*(a*d - b*c)**9) + 99*a**4*b**2*c**2*d**4*sqrt(-1/(c**3*d))*(a
**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3) + 31*a**3*b**5*c**8*
d**4*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d
- b*c)**9) - 27*a**3*b**3*c**3*d**3*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*
d - 3*b**2*c**2)/(2*(a*d - b*c)**3) - 16*a**2*b**6*c**9*d**3*(-1/(c**3*d))*
*(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 - 297*a**2*b
**4*c**4*d**2*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a
*d - b*c)**3) + 15*a*b**7*c**10*d**2*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*
b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) - 337*a*b**5*c**5*d*sqrt(-1/(c**
3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) - 11*b**8*c*
*11*d*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(8*(a*d
 - b*c)**9) - 27*b**6*c**6*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**
2*c**2)/(8*(a*d - b*c)**3))/(a**3*b**2*d**3 - 15*a**2*b**3*c*d**2 + 51*a*b*
*4*c**2*d + 27*b**5*c**3))/(16*(a*d - b*c)**3) + (x**3*(a*d**2 + 3*b*c*d) +
 x*(-a*c*d + 5*b*c**2))/(8*a**2*c**3*d**2 - 16*a*b*c**4*d + 8*b**2*c**5 + x
**4*(8*a**2*c*d**4 - 16*a*b*c**2*d**3 + 8*b**2*c**3*d**2) + x**2*(16*a**2*c
**2*d**3 - 32*a*b*c**3*d**2 + 16*b**2*c**4*d))
```

Giac [A] time = 1.18821, size = 278, normalized size = 1.79

$$-\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}}+\frac{\left(3\,b^2c^2+6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\sqrt{cd}}+\frac{3\,bcdx^3+ad^2x^3+5\,bc^2x-}{8\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(dx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c
*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d)) +
1/8*(3*b*c*d*x^3 + a*d^2*x^3 + 5*b*c^2*x - a*c*d*x)/((b^2*c^3 - 2*a*b*c^2*d
 + a^2*c*d^2)*(d*x^2 + c)^2)
```

3.255 $\int \frac{x}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=98

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3}+\frac{b}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (b^2*Lo
g[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.0749927, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 44}

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3}+\frac{b}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (b^2*Lo
g[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^3} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2 d}{(bc-ad)^3 \right.\right.\\ &= \frac{1}{4(bc-ad)\left(c+dx^2\right)^2}+\frac{b}{2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2\log\left(c+dx^2\right)}{2(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.0468274, size = 98, normalized size = 1.

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3}+\frac{b}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{1}{4\left(c+dx^2\right)^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(4*(-(b*c) + a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (b^
2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)^3
)
```

Maple [A] time = 0.01, size = 176, normalized size = 1.8

$$\frac{\ln\left(dx^2+c\right)b^2}{2\left(ad-bc\right)^3}-\frac{a^2d^2}{4\left(ad-bc\right)^3\left(dx^2+c\right)^2}+\frac{cabd}{2\left(ad-bc\right)^3\left(dx^2+c\right)^2}-\frac{b^2c^2}{4\left(ad-bc\right)^3\left(dx^2+c\right)^2}+\frac{bda}{2\left(ad-bc\right)^3\left(dx^2+c\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/2/(a*d-b*c)^3*ln(d*x^2+c)*b^2-1/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a^2+1/2*d/(
a*d-b*c)^3/(d*x^2+c)^2*c*a*b-1/4/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c^2+1/2*d/(a*d
-b*c)^3*b/(d*x^2+c)*a-1/2/(a*d-b*c)^3*b^2/(d*x^2+c)*c-1/2*b^2/(a*d-b*c)^3*l
n(b*x^2+a)
```

Maxima [B] time = 1.03754, size = 285, normalized size = 2.91

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{b^2\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{2\,bd}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*b^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(2*b*d*x^2 + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^
2*c*d^3)*x^2)
```

Fricas [B] time = 1.56677, size = 506, normalized size = 5.16

$$\frac{3\,b^2c^2-4\,abcd+a^2d^2+2\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+2\,b^2cdx^2+b^2c^2\right)\log\left(bx^2+a\right)-2\left(b^2d^2x^4+2\,b^2cdx^2+b^2c^2\right)\mathrm{lo}}{4\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3+\left(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5\right)x^4+2\left(b^3c^4d-3\,ab^2c^3d^2+3\,a^2bc^2d^3-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + 2*b^2*c
*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^
2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)
```

```
*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)
```

Sympy [B] time = 4.26234, size = 391, normalized size = 3.99

$$\frac{b^2\log\left(x^2+\frac{-\frac{a^4b^2d^4}{(ad-bc)^3}+\frac{4a^3b^3cd^3}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2}{(ad-bc)^3}+\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d-\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{2(ad-bc)^3}-\frac{b^2\log\left(x^2+\frac{\frac{a^4b^2d^4}{(ad-bc)^3}-\frac{4a^3b^3cd^3}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2}{(ad-bc)^3}-\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d+\frac{b^6c^4}{(ad-bc)^3}+\dots}{2b^3d}\right)}{2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(x**2 + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d -
 b*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*
c)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(2*(a*d -
 b*c)**3) - b**2*log(x**2 + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*
d**3/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*
d/(a*d - b*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d)
)/(2*(a*d - b*c)**3) + (-a*d + 3*b*c + 2*b*d*x**2)/(4*a**2*c**2*d**2 - 8*a*
b*c**3*d + 4*b**2*c**4 + x**4*(4*a**2*d**4 - 8*a*b*c*d**3 + 4*b**2*c**2*d**
2) + x**2*(8*a**2*c*d**3 - 16*a*b*c**2*d**2 + 8*b**2*c**3*d))
```

Giac [A] time = 1.15733, size = 235, normalized size = 2.4

$$\frac{b^3\log\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{b^2d\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}+\frac{3\,b^2c^2-4\,abcd+a^2d^2+2\left(b^2cd\dots\right.}{4\left(dx^2+c\right)^2(bc-ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3) - 1/2*b^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4) + 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d -
 a*b*d^2)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3)
```

3.256 $\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.188846, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3ad)x^2}{(a+bx^2)(c+dx^2)}dx}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^3\int \frac{1}{a+bx^2}dx}{(bc-ad)^3} - \frac{(d(15b^2c^2-10abcd}{8c^2(b}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(15b^2c^2-10}{8c}$$

Mathematica [A] time = 0.230987, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}(3a^2d^2-10abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3} - \frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3} + \frac{dx(3ad-7bc)}{c^2(c+dx^2)(bc-ad)^2} - \frac{2dx}{c(c+dx^2)^2(bc-ad)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

Maple [B] time = 0., size = 310, normalized size = 1.9

$$\frac{3d^4x^3a^2}{8(ad-bc)^3(dx^2+c)^2c^2} - \frac{5d^3x^3ab}{4(ad-bc)^3(dx^2+c)^2c} + \frac{7d^2x^3b^2}{8(ad-bc)^3(dx^2+c)^2} + \frac{5d^3xa^2}{8(ad-bc)^3(dx^2+c)^2c} - \frac{7}{4(ad-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(d*x^2+c)^2
/c*x^3*a*b+7/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+5/8*d^3/(a*d-b*c)^3/(d*x
^2+c)^2/c*x*a^2-7/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x*a*b+9/8*d/(a*d-b*c)^3/(d*
x^2+c)^2*c*x*b^2+3/8*d^3/(a*d-b*c)^3/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)
)*a^2-5/4*d^2/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+15/8*d/
```

```
(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-b^3/(a*d-b*c)^3/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 6.45061, size = 3217, normalized size = 20.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14651, size = 293, normalized size = 1.83

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3\right)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2d}{8\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.257 $\int \frac{1}{x(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=149

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

```
[Out] -d/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*c - a*d)^2
*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(2*a*(b*c - a*d)^3) +
 (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^3)
```

Rubi [A] time = 0.152368, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -d/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*c - a*d)^2
*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(2*a*(b*c - a*d)^3) +
 (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^3} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^3x}+\frac{b^4}{a(-bc+ad)^3(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^3}+\frac{d^2(2bc-ad)}{c^2(bc-ad)^2(c+dx)^2}\right.\right.\\ &= -\frac{d}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(2bc-ad)}{2c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\log(x)}{ac^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}+\frac{d\left(3b^2c^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.28008, size = 141, normalized size = 0.95

$$\frac{\frac{d\left(\frac{c\left(a^2d^2(3c+2dx^2)-2abcd(4c+3dx^2)+b^2c^2(5c+4dx^2)\right)}{(c+dx^2)^2}-2\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)\right)}{c^3}+\frac{2b^3\log\left(a+bx^2\right)}{a}}{4(ad-bc)^3}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Log[x]/(a*c^3) + ((2*b^3*Log[a + b*x^2])/a + (d*((c*(a^2*d^2*(3*c + 2*d*x^2
) - 2*a*b*c*d*(4*c + 3*d*x^2) + b^2*c^2*(5*c + 4*d*x^2)))/(c + d*x^2)^2 - 2
*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2]))/c^3)/(4*(-(b*c) + a*d)^
3)
```

Maple [B] time = 0.016, size = 286, normalized size = 1.9

$$-\frac{d^3\ln\left(dx^2+c\right)a^2}{2c^3(ad-bc)^3}+\frac{3d^2\ln\left(dx^2+c\right)ab}{2c^2(ad-bc)^3}-\frac{3d\ln\left(dx^2+c\right)b^2}{2c(ad-bc)^3}+\frac{a^2d^3}{4c(ad-bc)^3\left(dx^2+c\right)^2}-\frac{ad^2b}{2(ad-bc)^3\left(dx^2+c\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2*d^3/c^3/(a*d-b*c)^3*ln(d*x^2+c)*a^2+3/2*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c
)*a*b-3/2*d/c/(a*d-b*c)^3*ln(d*x^2+c)*b^2+1/4*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2
*a^2-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b+1/4*d*c/(a*d-b*c)^3/(d*x^2+c)^2*b^
2+1/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a^2-3/2*d^2/c/(a*d-b*c)^3/(d*x^2+c)*a*b
+d/(a*d-b*c)^3/(d*x^2+c)*b^2+ln(x)/a/c^3+1/2*b^3/a/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [A] time = 1.19304, size = 375, normalized size = 2.52

$$-\frac{b^3\log\left(bx^2+a\right)}{2\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}+\frac{\left(3b^2c^2d-3abcd^2+a^2d^3\right)\log\left(dx^2+c\right)}{2\left(b^3c^6-3ab^2c^5d+3a^2bc^4d^2-a^3c^3d^3\right)}-\frac{5}{4\left(b^2c^6-2abc^5d+a^2c^4d^2+(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3) + 1/2*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/(b^3*c^6 -
3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/4*(5*b*c^2*d - 3*a*c*d^2
 + 2*(2*b*c*d^2 - a*d^3)*x^2)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c
^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 +
a^2*c^3*d^3)*x^2) + 1/2*log(x^2)/(a*c^3)
```

Fricas [B] time = 23.2272, size = 1027, normalized size = 6.89

$$-\frac{5ab^2c^4d-8a^2bc^3d^2+3a^3c^2d^3+2\left(2ab^2c^3d^2-3a^2bc^2d^3+a^3cd^4\right)x^2+2\left(b^3c^3d^2x^4+2b^3c^4dx^2+b^3c^5\right)\log\left(bx^2+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(5*a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 3*a^3*c^2*d^3 + 2*(2*a*b^2*c^3*d^2
- 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2 + 2*(b^3*c^3*d^2*x^4 + 2*b^3*c^4*d*x^2 +
 b^3*c^5)*log(b*x^2 + a) - 2*(3*a*b^2*c^4*d - 3*a^2*b*c^3*d^2 + a^3*c^2*d^3
 + (3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*x^4 + 2*(3*a*b^2*c^3*d^2 - 3
*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2)*log(d*x^2 + c) - 4*(b^3*c^5 - 3*a*b^2*c^4*
d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*
b*c*d^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 -
 a^3*c*d^4)*x^2)*log(x))/(a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a
^4*c^5*d^3 + (a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3
*d^5)*x^4 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*
d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13324, size = 425, normalized size = 2.85

$$-\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\left(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3\right)}+\frac{\left(3\,b^2c^2d^2-3\,abcd^3+a^2d^4\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)}-\frac{9\,b^2c^2d^3x^4-9\,abcd^4x^4+3\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^4*log(abs(b*x^2 + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
 - a^4*b*d^3) + 1/2*(3*b^2*c^2*d^2 - 3*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 +
 c))/(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/4*(9
*b^2*c^2*d^3*x^4 - 9*a*b*c*d^4*x^4 + 3*a^2*d^5*x^4 + 22*b^2*c^3*d^2*x^2 - 2
4*a*b*c^2*d^3*x^2 + 8*a^2*c*d^4*x^2 + 14*b^2*c^4*d - 17*a*b*c^3*d^2 + 6*a^2
*c^2*d^3)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2
 + c)^2) + 1/2*log(x^2)/(a*c^3)
```

3.258 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=211

$$-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-}$$

```
[Out] -(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(8*a*c^3*(b*c - a*d)^2*x) - d/(4*c*(
b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^2*(b*c - a*d)^2*x*(c
 + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^3)
+ (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(8*c^(7/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.309268, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```


```
[Out] -(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(8*a*c^3*(b*c - a*d)^2*x) - d/(4*c*(
b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^2*(b*c - a*d)^2*x*(c
 + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^3)
+ (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(8*c^(7/2)*(b*c - a*d)^3)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}dx = -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-5ad-5bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}dx}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-27abcd+15a^2d^2-3bd(9bc-5ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}}{8c^2(bc-ad)^2}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{\int \frac{8b^3c^3+8ab^2}{}}{}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{b^4\int \frac{1}{a+bx^2}}{a(bc-ad)}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{b^{7/2}\tan^{-1}\left(\right.}{a^{3/2}(bc-}$$

Mathematica [A] time = 0.389394, size = 172, normalized size = 0.82

$$\frac{1}{8}\left(\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}+\frac{8b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{d^2x(11bc-7ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{2d^2x}{c^2\left(c+dx^2\right)^2(bc-ad)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a*c^3*x) + (2*d^2*x)/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(11*b*c -
7*a*d)*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (8*b^(7/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*(-(b*c) + a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 1
5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^3))/8
```

Maple [A] time = 0.017, size = 335, normalized size = 1.6

$$-\frac{7 d^5 x^3 a^2}{8 c^3 (ad-bc)^3 (dx^2+c)^2}+\frac{9 d^4 x^3 ab}{4 c^2 (ad-bc)^3 (dx^2+c)^2}-\frac{11 d^3 x^3 b^2}{8 c (ad-bc)^3 (dx^2+c)^2}-\frac{9 a^2 d^4 x}{8 c^2 (ad-bc)^3 (dx^2+c)^2}+\frac{}{4 c (a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -7/8*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2+9/4*d^4/c^2/(a*d-b*c)^3/(d*x^2
+c)^2*x^3*a*b-11/8*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2-9/8*d^4/c^2/(a*d-b
*c)^3/(d*x^2+c)^2*a^2*x+11/4*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a*b*x-13/8*d^2/(
a*d-b*c)^3/(d*x^2+c)^2*b^2*x-15/8*d^4/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*
d/(c*d)^(1/2))*a^2+21/4*d^3/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1
/2))*a*b-35/8*d^2/c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/a
/c^3/x+1/a*b^4/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.5784, size = 4054, normalized size = 19.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c^2*d^3 + 2
*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + 2*(
16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 + 8*
(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*
a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 +
 (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*
x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d
^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^
2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a
^3*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b
*c^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3
- 25*a^3*c*d^4)*x^2 - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5)*x^5
 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*b^2*c
^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ 4*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 +
```

```
3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3
*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*
c^6*d^2 - a^4*c^5*d^3)*x), -1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^
3*d^2 - 16*a^3*c^2*d^3 + 2*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + 2*(16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^
3 - 25*a^3*c*d^4)*x^2 + 16*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*
sqrt(b/a)*arctan(x*sqrt(b/a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^
3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (
35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3
 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2
+ 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3
*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b*c
^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4
 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 -
25*a^3*c*d^4)*x^2 + 8*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(
b/a)*arctan(x*sqrt(b/a)) - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5
)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*
b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d
/c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*
x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x
^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17243, size = 319, normalized size = 1.51

$$-\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{ab}}+\frac{\left(35\,b^2c^2d^2-42\,abcd^3+15\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\sqrt{cd}}+\frac{11\,bcd^3x^3-7\,ad^4x^3+13}{8\left(b^2c^5-2\,abc^4d+a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) + 1/8*(11*b*c*d^3*x^3 - 7*a*d^4*x^3 + 13*b*c^2*d^2*x - 9*a*c*d
^3*x)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 1/(a*c^3*x)
```

3.259 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=178

$$-\frac{d^2(3a^2d^2-8abcd+6b^2c^2)\log(c+dx^2)}{2c^4(bc-ad)^3}+\frac{b^4\log(a+bx^2)}{2a^2(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3(c+dx^2)(bc-ad)^2}+\frac{}{4c^2(c}$$

```
[Out] -1/(2*a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(3*b*c - 2*
a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d)*Log[x])/(a^2*c^4)
+ (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3) - (d^2*(6*b^2*c^2 - 8*a*b*c*d
+ 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^3)
```

Rubi [A] time = 0.213417, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{d^2(3a^2d^2-8abcd+6b^2c^2)\log(c+dx^2)}{2c^4(bc-ad)^3}+\frac{b^4\log(a+bx^2)}{2a^2(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3(c+dx^2)(bc-ad)^2}+\frac{}{4c^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(3*b*c - 2*
a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d)*Log[x])/(a^2*c^4)
+ (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3) - (d^2*(6*b^2*c^2 - 8*a*b*c*d
+ 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^3x^2}+\frac{-bc-3ad}{a^2c^4x}-\frac{b^5}{a^2(-bc+ad)^3(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)^3}-\frac{}{c}\right.\right. \\
&= -\frac{1}{2ac^3x^2}+\frac{d^2}{4c^2(bc-ad)(c+dx^2)^2}+\frac{d^2(3bc-2ad)}{2c^3(bc-ad)^2(c+dx^2)}-\frac{(bc+3ad)\log(x)}{a^2c^4}+\frac{b}{}
\end{aligned}$$

Mathematica [A] time = 0.442588, size = 171, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^3}-\frac{2b^4\log\left(a+bx^2\right)}{a^2(ad-bc)^3}-\frac{4\log(x)(3ad+bc)}{a^2c^4}+\frac{2d^2(3bc-2ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{}{c^2(c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a*c^3*x^2) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (2*d^2*(3*b*c - 2*a
*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + 3*a*d)*Log[x])/(a^2*c^4) -
 (2*b^4*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)^3) - (2*d^2*(6*b^2*c^2 - 8*a*b*
c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^3))/4
```

Maple [A] time = 0.018, size = 322, normalized size = 1.8

$$\frac{3\,d^4\ln\left(dx^2+c\right)a^2}{2\,c^4\,(ad-bc)^3}-4\,\frac{d^3\ln\left(dx^2+c\right)ab}{c^3\,(ad-bc)^3}+3\,\frac{d^2\ln\left(dx^2+c\right)b^2}{c^2\,(ad-bc)^3}-\frac{a^2d^4}{4\,c^2\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{ad^3b}{2\,c\,(ad-bc)^3\left(dx^2+c\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/2*d^4/c^4/(a*d-b*c)^3*ln(d*x^2+c)*a^2-4*d^3/c^3/(a*d-b*c)^3*ln(d*x^2+c)*a
*b+3*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b^2-1/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^
2*a^2+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a*b-1/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2
*b^2-d^4/c^3/(a*d-b*c)^3/(d*x^2+c)*a^2+5/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a*
b-3/2*d^2/c/(a*d-b*c)^3/(d*x^2+c)*b^2-1/2/a/c^3/x^2-3/a/c^4*ln(x)*d-1/a^2/c
^3*ln(x)*b-1/2*b^4/a^2/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [B] time = 1.13732, size = 491, normalized size = 2.76

$$\frac{b^4\log\left(bx^2+a\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)}-\frac{\left(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4\right)\log\left(dx^2+c\right)}{2\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3\right)}-\frac{2\,b^2c^4-4\,abc^3d+2\,a^2c^2d^2}{4\left(\left(ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5
*d^3) - 1/2*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*log(d*x^2 + c)/(b^3*c
^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3) - 1/4*(2*b^2*c^4 - 4*a*
b*c^3*d + 2*a^2*c^2*d^2 + 2*(b^2*c^2*d^2 - 5*a*b*c*d^3 + 3*a^2*d^4)*x^4 + (
4*b^2*c^3*d - 15*a*b*c^2*d^2 + 9*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*
c^4*d^3 + a^3*c^3*d^4)*x^6 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3
)*x^4 + (a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x^2) - 1/2*(b*c + 3*a*d)*
log(x^2)/(a^2*c^4)
```

Fricas [B] time = 52.7, size = 1257, normalized size = 7.06

$$-\frac{2\,ab^3c^6 - 6\,a^2b^2c^5d + 6\,a^3bc^4d^2 - 2\,a^4c^3d^3 + 2\left(ab^3c^4d^2 - 6\,a^2b^2c^3d^3 + 8\,a^3bc^2d^4 - 3\,a^4cd^5\right)x^4 + \left(4\,ab^3c^5d - 19\,a^2b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b^3*c^6 - 6*a^2*b^2*c^5*d + 6*a^3*b*c^4*d^2 - 2*a^4*c^3*d^3 + 2*(
a*b^3*c^4*d^2 - 6*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (4
*a*b^3*c^5*d - 19*a^2*b^2*c^4*d^2 + 24*a^3*b*c^3*d^3 - 9*a^4*c^2*d^4)*x^2 -
 2*(b^4*c^4*d^2*x^6 + 2*b^4*c^5*d*x^4 + b^4*c^6*x^2)*log(b*x^2 + a) + 2*((6
*a^2*b^2*c^2*d^4 - 8*a^3*b*c*d^5 + 3*a^4*d^6)*x^6 + 2*(6*a^2*b^2*c^3*d^3 -
8*a^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^4 + (6*a^2*b^2*c^4*d^2 - 8*a^3*b*c^3*d^3 +
 3*a^4*c^2*d^4)*x^2)*log(d*x^2 + c) + 4*((b^4*c^4*d^2 - 6*a^2*b^2*c^2*d^4 +
 8*a^3*b*c*d^5 - 3*a^4*d^6)*x^6 + 2*(b^4*c^5*d - 6*a^2*b^2*c^3*d^3 + 8*a^3*
b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (b^4*c^6 - 6*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d
^3 - 3*a^4*c^2*d^4)*x^2)*log(x))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*
a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^6 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3
*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^4 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*
b*c^7*d^2 - a^5*c^6*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18444, size = 482, normalized size = 2.71

$$\frac{b^5\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^4c^3 - 3\,a^3b^3c^2d + 3\,a^4b^2cd^2 - a^5bd^3\right)} - \frac{\left(6\,b^2c^2d^3 - 8\,abcd^4 + 3\,a^2d^5\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^7d - 3\,ab^2c^6d^2 + 3\,a^2bc^5d^3 - a^3c^4d^4\right)} + \frac{18\,b^2c^2d^4x^4 - 24\,abcd^5x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^
2 - a^5*b*d^3) - 1/2*(6*b^2*c^2*d^3 - 8*a*b*c*d^4 + 3*a^2*d^5)*log(abs(d*x^
2 + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4) + 1/4
*(18*b^2*c^2*d^4*x^4 - 24*a*b*c*d^5*x^4 + 9*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^
2 - 58*a*b*c^2*d^4*x^2 + 22*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 36*a*b*c^3*d^3
 + 14*a^2*c^2*d^4)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*(d*x^2 + c)^2) - 1/2*(b*c + 3*a*d)*log(x^2)/(a^2*c^4) + 1/2*(b*c*x^2 + 3
*a*d*x^2 - a*c)/(a^2*c^4*x^2)
```

3.260 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=270

$$-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{-55a^2bcd^2+35a^3d^3+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}+\ldots$$

```
[Out] -(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(24*a*c^3*(b*c - a*d)^2*x^3) + (8*b^
3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8*a^2*c^4*(b*c - a*d)
^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2) - (d*(11*b*c - 7*a*d))/(8*c^2
*(b*c - a*d)^2*x^3*(c + d*x^2)) + (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(5/2)*(b*c - a*d)^3) - (d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.433726, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{-55a^2bcd^2+35a^3d^3+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(24*a*c^3*(b*c - a*d)^2*x^3) + (8*b^
3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8*a^2*c^4*(b*c - a*d)
^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2) - (d*(11*b*c - 7*a*d))/(8*c^2
*(b*c - a*d)^2*x^3*(c + d*x^2)) + (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(5/2)*(b*c - a*d)^3) - (d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-7ad-7bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2}-\frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-55abcd+35a^2d^2-5bd(11bc-}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}}{8c^2(bc-ad)^2}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3}-\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2}-\frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3\left(c+dx^2\right)}-\frac{\int \frac{3(8b}{}}{}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3}+\frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x}-\frac{d}{4c(bc-ad)x^3\left(c+dx\right.}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3}+\frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x}-\frac{d}{4c(bc-ad)x^3\left(c+dx\right.}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3}+\frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x}-\frac{d}{4c(bc-ad)x^3\left(c+dx\right.}$$

Mathematica [A] time = 0.442556, size = 196, normalized size = 0.73

$$-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3}+\frac{3ad+bc}{a^2c^4x}-\frac{d^3x(15bc-11ad)}{8c^4\left(c+dx^2\right)(bc-ad)^2}-\frac{}{4c^3\left(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(3*a*c^3*x^3) + (b*c + 3*a*d)/(a^2*c^4*x) - (d^3*x)/(4*c^3*(b*c - a*d)*(
c + d*x^2)^2) - (d^3*(15*b*c - 11*a*d)*x)/(8*c^4*(b*c - a*d)^2*(c + d*x^2))
```

```
 - (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (d^(5
/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*
c^(9/2)*(b*c - a*d)^3)
```

Maple [A] time = 0.017, size = 362, normalized size = 1.3

$$\frac{11\,d^6x^3a^2}{8\,c^4\,(ad-bc)^3\left(dx^2+c\right)^2}-\frac{13\,d^5x^3ab}{4\,c^3\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{15\,d^4x^3b^2}{8\,c^2\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{13\,a^2d^5x}{8\,c^3\,(ad-bc)^3\left(dx^2+c\right)^2}-\frac{1}{4\,c^2\,(ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 11/8*d^6/c^4/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2-13/4*d^5/c^3/(a*d-b*c)^3/(d*x^
2+c)^2*x^3*a*b+15/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+13/8*d^5/c^3/(a
*d-b*c)^3/(d*x^2+c)^2*a^2*x-15/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b*x+17/8
*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*b^2*x+35/8*d^5/c^4/(a*d-b*c)^3/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))*a^2-45/4*d^4/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/
(c*d)^(1/2))*a*b+63/8*d^3/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2
))*b^2-1/3/a/c^3/x^3+3/a/c^4/x*d+1/a^2/c^3/x*b-1/a^2*b^5/(a*d-b*c)^3/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 32.6831, size = 4841, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 - 16*a^4*c^3*d^3
 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35*a^4*d^6)*x^6
 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*
d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d
^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 24*(b^4*c^4*d^2*x^7 + 2*b^4*c^
5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*
a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^
2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*s
qrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*
b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4
```

```
*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^
7*d^2 - a^5*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b
*c^4*d^2 - 8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b
*c*d^5 - 35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^
3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5
*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 3*((63*a^
2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 -
90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d
^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 12*(b^4*c^4*d^2*x
^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^
4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d
^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 -
a^5*c^6*d^3)*x^3), -1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^
2 - 16*a^4*c^3*d^3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5
 - 35*a^4*d^6)*x^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^
3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d
- 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 48*(b^4*c^4*
d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3
*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*
b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) -
 c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 -
 a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3
- a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5
*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b*c^4*d^2 -
8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35
*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450
*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2
*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 24*(b^4*c^4*d^2*x^7
+ 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((63*a^2
*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 - 9
0*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d^
3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^7*d^2 -
 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d
- 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3
*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15645, size = 346, normalized size = 1.28

$$\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3\right)\sqrt{ab}} - \frac{\left(63\,b^2c^2d^3 - 90\,abcd^4 + 35\,a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3\right)\sqrt{cd}} - \frac{15\,bcd^4x^3 - 11\,ad^5x^3 +}{8\left(b^2c^6 - 2\,abc^5d +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*arc
tan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*sqrt(c*d)) - 1/8*(15*b*c*d^4*x^3 - 11*a*d^5*x^3 + 17*b*c^2*d^3*x - 13*a*
c*d^4*x)/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 1/3*(3*b*c
*x^2 + 9*a*d*x^2 - a*c)/(a^2*c^4*x^3)
```

3.261 $\int \frac{x}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi [A] time = 0.0126227, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0045045, size = 21, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] time = 0.005, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+4),x)
```

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A] time = 1.29568, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 1.46517, size = 51, normalized size = 2.43

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.111042, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Giac [A] time = 1.14752, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.262 $\int \frac{x^4(c+dx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=87

$$\frac{ax(bc-ad)}{2b^3(a+bx^2)}+\frac{x(bc-2ad)}{b^3}-\frac{\sqrt{a}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{dx^3}{3b^2}$$

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.071397, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 1153, 205}

$$\frac{ax(bc-ad)}{2b^3(a+bx^2)}+\frac{x(bc-2ad)}{b^3}-\frac{\sqrt{a}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(c+dx^2\right)}{\left(a+bx^2\right)^2} dx = \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)} - \frac{\int \frac{a(bc-ad)-2b(bc-ad)x^2-2b^2dx^4}{a+bx^2} dx}{2b^3}$$
$$= \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)} - \frac{\int \left(-2(bc-2ad)-2bdx^2+\frac{3abc-5a^2d}{a+bx^2}\right) dx}{2b^3}$$
$$= \frac{(bc-2ad)x}{b^3} + \frac{dx^3}{3b^2} + \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)} - \frac{(a(3bc-5ad)) \int \frac{1}{a+bx^2} dx}{2b^3}$$
$$= \frac{(bc-2ad)x}{b^3} + \frac{dx^3}{3b^2} + \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)} - \frac{\sqrt{a}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.0753117, size = 89, normalized size = 1.02

$$\frac{x\left(abc-a^2d\right)}{2b^3\left(a+bx^2\right)} + \frac{x(bc-2ad)}{b^3} + \frac{\sqrt{a}(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + ((a*b*c - a^2*d)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.008, size = 105, normalized size = 1.2

$$\frac{dx^3}{3\,b^2} - 2\,\frac{adx}{b^3} + \frac{cx}{b^2} - \frac{a^2dx}{2\,b^3\left(bx^2+a\right)} + \frac{axc}{2\,b^2\left(bx^2+a\right)} + \frac{5\,a^2d}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3\,ac}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d*x^3/b^2-2/b^3*a*d*x+1/b^2*x*c-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b^2*x/(
b*x^2+a)*c+5/2*a^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-3/2*a/b^2/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55801, size = 513, normalized size = 5.9

$$\left[\frac{4 b^2 d x^5 + 4 \left(3 b^2 c - 5 abd\right) x^3 - 3 \left(3 abc - 5 a^2 d + \left(3 b^2 c - 5 abd\right) x^2\right) \sqrt{-\frac{a}{b}} \log\left(\frac{b x^2 + 2 bx \sqrt{-\frac{a}{b}} - a}{b x^2 + a}\right) + 6 \left(3 abc - 5 a^2 d\right) x}{12 \left(b^4 x^2 + a b^3\right)}, \frac{2 b^2 d x \cdots}{\cdots}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*d*x^5 + 4*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*
b^2*c - 5*a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) + 6*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3), 1/6*(2*b^2*d*x^5 + 2*(3
*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*b^2*c - 5*a*b*d)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3)
]
```

Sympy [A] time = 0.766974, size = 128, normalized size = 1.47

$$-\frac{x \left(a^2 d - abc\right)}{2ab^3 + 2b^4 x^2} - \frac{\sqrt{-\frac{a}{b^7}}\,(5ad - 3bc) \log\left(-b^3 \sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}\,(5ad - 3bc) \log\left(b^3 \sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{dx^3}{3b^2} - \frac{x\,(2ad - bc)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**2*d - a*b*c)/(2*a*b**3 + 2*b**4*x**2) - sqrt(-a/b**7)*(5*a*d - 3*b*c
)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**7)*(5*a*d - 3*b*c)*log(b**3*s
qrt(-a/b**7) + x)/4 + d*x**3/(3*b**2) - x*(2*a*d - b*c)/b**3
```

Giac [A] time = 1.11273, size = 119, normalized size = 1.37

$$-\frac{\left(3 abc - 5 a^2 d\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} b^3} + \frac{abcx - a^2 dx}{2 \left(bx^2 + a\right) b^3} + \frac{b^4 d x^3 + 3 b^4 cx - 6 ab^3 dx}{3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c - 5*a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(a*b*c
*x - a^2*d*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d*x^3 + 3*b^4*c*x - 6*a*b^3*d*x)
/b^6
```

3.263 $\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2*a*d)*Log[
a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.0604614, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2*a*d)*Log[
a + b*x^2])/(2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)}{(a+bx)^2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{b^2}+\frac{a(-bc+ad)}{b^2(a+bx)^2}+\frac{bc-2ad}{b^2(a+bx)}\right)dx,x,x^2\right)\\&=\frac{dx^2}{2b^2}+\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0352775, size = 50, normalized size = 0.83

$$\frac{\frac{a(bc-ad)}{a+bx^2}+(bc-2ad)\log\left(a+bx^2\right)+bdx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*d*x^2 + (a*(b*c - a*d))/(a + b*x^2) + (b*c - 2*a*d)*Log[a + b*x^2])/(2*b
^3)
```

Maple [A] time = 0.008, size = 74, normalized size = 1.2

$$\frac{dx^2}{2\,b^2}-\frac{\ln\left(bx^2+a\right)ad}{b^3}+\frac{c\ln\left(bx^2+a\right)}{2\,b^2}-\frac{a^2d}{2\,b^3\left(bx^2+a\right)}+\frac{ac}{2\,b^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d*x^2/b^2-1/b^3*ln(b*x^2+a)*a*d+1/2*c*ln(b*x^2+a)/b^2-1/2/b^3*a^2/(b*x^
2+a)*d+1/2*a*c/b^2/(b*x^2+a)
```

Maxima [A] time = 1.46424, size = 80, normalized size = 1.33

$$\frac{dx^2}{2\,b^2}+\frac{abc-a^2d}{2\left(b^4x^2+ab^3\right)}+\frac{(bc-2\,ad)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b^2 + 1/2*(a*b*c - a^2*d)/(b^4*x^2 + a*b^3) + 1/2*(b*c - 2*a*d)*l
og(b*x^2 + a)/b^3
```

Fricas [A] time = 1.53953, size = 165, normalized size = 2.75

$$\frac{b^2dx^4+abdx^2+abc-a^2d+\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^4 + a*b*d*x^2 + a*b*c - a^2*d + (a*b*c - 2*a^2*d + (b^2*c - 2*
a*b*d)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 0.690072, size = 56, normalized size = 0.93

$$-\frac{a^2d - abc}{2ab^3 + 2b^4x^2} + \frac{dx^2}{2b^2} - \frac{(2ad - bc)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -(a**2*d - a*b*c)/(2*a*b**3 + 2*b**4*x**2) + d*x**2/(2*b**2) - (2*a*d - b*c
)*log(a + b*x**2)/(2*b**3)
```

Giac [A] time = 1.13277, size = 122, normalized size = 2.03

$$\frac{\frac{\left(bx^2+a\right)d}{b^2} - \frac{(bc-2\,ad)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^2} + \frac{\frac{ab^2c}{bx^2+a} - \frac{a^2bd}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*d/b^2 - (b*c - 2*a*d)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 + (a*b^2*c/(b*x^2 + a) - a^2*b*d/(b*x^2 + a))/b^3)/b
```

3.264 $\int \frac{x^2(c+dx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=67

$$-\frac{x(bc-ad)}{2b^2(a+bx^2)}+\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{dx}{b^2}$$

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0507307, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {455, 388, 205}

$$-\frac{x(bc-ad)}{2b^2(a+bx^2)}+\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} - \frac{\int \frac{-bc+ad-2bdx^2}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} + \frac{(bc-3ad)\int \frac{1}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} + \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

Mathematica [A] time = 0.0682567, size = 68, normalized size = 1.01

$$-\frac{x(bc-ad)}{2b^2\left(a+bx^2\right)} - \frac{(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) - ((-(b*c) + 3*a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.009, size = 82, normalized size = 1.2

$$\frac{dx}{b^2} + \frac{axd}{2\,b^2\left(bx^2+a\right)} - \frac{cx}{2\,b\left(bx^2+a\right)} - \frac{3\,ad}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{c}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] d*x/b^2+1/2/b^2*x/(b*x^2+a)*a*d-1/2*c*x/b/(b*x^2+a)-3/2/b^2/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*a*d+1/2*c/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51835, size = 433, normalized size = 6.46

$$\left[\frac{4\,ab^2dx^3 + \left(abc - 3\,a^2d + \left(b^2c - 3\,abd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right) - 2\left(ab^2c - 3\,a^2bd\right)x}{4\left(ab^4x^2 + a^2b^3\right)}, \frac{2\,ab^2dx^3 + \left(abc - 3\,a^2d + \right(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-a*b)*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - 3*a^2*b*d)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a
*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a*b^2*c - 3*a^2*b*d)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

Sympy [A] time = 0.646734, size = 114, normalized size = 1.7

$$\frac{x(ad-bc)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(3ad-bc)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\,(3ad-bc)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}+\frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(3*a*d - b*c)*lo
g(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(3*a*d - b*c)*log(a*
b**2*sqrt(-1/(a*b**5)) + x)/4 + d*x/b**2
```

Giac [A] time = 1.17406, size = 78, normalized size = 1.16

$$\frac{dx}{b^2}+\frac{(bc-3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^2}-\frac{bcx-adx}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(b*
c*x - a*d*x)/((b*x^2 + a)*b^2)
```

3.265 $\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=41

$$\frac{d\log(a+bx^2)}{2b^2} - \frac{bc-ad}{2b^2(a+bx^2)}$$

```
[Out] -(b*c - a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0358749, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{d\log(a+bx^2)}{2b^2} - \frac{bc-ad}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(b*c - a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{(a+bx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{bc-ad}{b(a+bx)^2} + \frac{d}{b(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{bc-ad}{2b^2(a+bx^2)} + \frac{d\log(a+bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0121703, size = 41, normalized size = 1.

$$\frac{ad-bc}{2b^2(a+bx^2)} + \frac{d\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-(b*c) + a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.008, size = 47, normalized size = 1.2

$$\frac{d\ln\left(bx^2+a\right)}{2\,b^2}+\frac{ad}{2\,b^2\left(bx^2+a\right)}-\frac{c}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b^2+1/2/b^2/(b*x^2+a)*a*d-1/2*c/b/(b*x^2+a)
```

Maxima [A] time = 1.0378, size = 54, normalized size = 1.32

$$-\frac{bc-ad}{2\left(b^3x^2+ab^2\right)}+\frac{d\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)/(b^3*x^2 + a*b^2) + 1/2*d*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.51702, size = 93, normalized size = 2.27

$$-\frac{bc-ad-\left(bdx^2+ad\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d - (b*d*x^2 + a*d)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.496225, size = 36, normalized size = 0.88

$$\frac{ad-bc}{2ab^2+2b^3x^2}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + d*log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 1.11657, size = 88, normalized size = 2.15

$$-\frac{d\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2b}-\frac{c}{2(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*c/((b*x^2 + a)*b)
```

3.266 $\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0222988, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0437442, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(ad-bc)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(b*c) + a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0., size = 68, normalized size = 1.1

$$-\frac{(ad-bc)\,x}{2\,ab\left(bx^{2}+a\right)}+\frac{d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+
1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54424, size = 381, normalized size = 6.05

$$\left[-\frac{\left(abc+a^{2}d+\left(b^{2}c+abd\right)x^{2}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\sqrt{-ab}x-a}{bx^{2}+a}\right)-2\left(ab^{2}c-a^{2}bd\right)x}{4\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)},\frac{\left(abc+a^{2}d+\left(b^{2}c+abd\right)x^{2}\right)\sqrt{ab}\arctan\left(\right.}{2\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] time = 0.53018, size = 112, normalized size = 1.78

$$-\frac{x\,(ad-bc)}{2a^{2}b+2ab^{2}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\,(ad+bc)\log\left(-a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\,(ad+bc)\log\left(a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A] time = 1.19489, size = 77, normalized size = 1.22

$$\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}+\frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

3.267 $\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab\left(a+bx^2\right)}$$

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^
2)
```

Rubi [A] time = 0.0441073, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^
2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{a^2x}+\frac{-bc+ad}{a(a+bx)^2}-\frac{bc}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{bc-ad}{2ab\left(a+bx^2\right)}+\frac{c\log(x)}{a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0284382, size = 46, normalized size = 0.9

$$\frac{\frac{a(bc-ad)}{b(a+bx^2)}-c\log\left(a+bx^2\right)+2c\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^2)) + 2*c*Log[x] - c*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.012, size = 53, normalized size = 1.

$$\frac{c\ln(x)}{a^2}-\frac{c\ln\left(bx^2+a\right)}{2a^2}-\frac{d}{2b\left(bx^2+a\right)}+\frac{c}{2a\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a)^2,x)
```

```
[Out] c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2-1/2/b/(b*x^2+a)*d+1/2*c/a/(b*x^2+a)
```

Maxima [A] time = 1.11155, size = 69, normalized size = 1.35

$$\frac{bc-ad}{2\left(ab^2x^2+a^2b\right)}-\frac{c\log\left(bx^2+a\right)}{2a^2}+\frac{c\log\left(x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)/(a*b^2*x^2 + a^2*b) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(
x^2)/a^2
```

Fricas [A] time = 1.49893, size = 150, normalized size = 2.94

$$\frac{abc-a^2d-\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)+2\left(b^2cx^2+abc\right)\log(x)}{2\left(a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d - (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) + 2*(b^2*c*x^2 + a*
b*c)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 0.550187, size = 46, normalized size = 0.9

$$-\frac{ad-bc}{2a^2b+2ab^2x^2}+\frac{c\log(x)}{a^2}-\frac{c\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

```
[Out] -(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) + c*log(x)/a**2 - c*log(a/b + x**2)
/(2*a**2)
```

Giac [A] time = 1.12414, size = 85, normalized size = 1.67

$$\frac{c\log\left(x^2\right)}{2\,a^2}-\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{b^2cx^2+2\,abc-a^2d}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b^2*c*x^2 + 2*a*b
*c - a^2*d)/((b*x^2 + a)*a^2*b)
```

3.268 $\int \frac{c+dx^2}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=71

$$-\frac{x(bc-ad)}{2a^2(a+bx^2)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{c}{a^2x}$$

```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0529184, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 453, 205}

$$-\frac{x(bc-ad)}{2a^2(a+bx^2)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```


```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```


Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)} - \frac{1}{2}\int \frac{-\frac{2c}{a}+\frac{(bc-ad)x^2}{a^2}}{x^2\left(a+bx^2\right)}\,dx$$
$$= -\frac{c}{a^2x} - \frac{(bc-ad)x}{2a^2\left(a+bx^2\right)} - \frac{(3bc-ad)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$
$$= -\frac{c}{a^2x} - \frac{(bc-ad)x}{2a^2\left(a+bx^2\right)} - \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.03296, size = 70, normalized size = 0.99

$$\frac{x(ad-bc)}{2a^2\left(a+bx^2\right)} + \frac{(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} - \frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + ((-(b*c) + a*d)*x)/(2*a^2*(a + b*x^2)) + ((-3*b*c + a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.01, size = 85, normalized size = 1.2

$$-\frac{c}{a^2x} + \frac{dx}{2\,a\left(bx^2+a\right)} - \frac{bcx}{2\,a^2\left(bx^2+a\right)} + \frac{d}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3\,bc}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c/a^2/x+1/2/a*x/(b*x^2+a)*d-1/2*c*b/a^2*x/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*d-3/2*c*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56468, size = 447, normalized size = 6.3

$$\left[-\frac{4\,a^2bc + 2\left(3\,ab^2c - a^2bd\right)x^2 - \left(\left(3\,b^2c - abd\right)x^3 + \left(3\,abc - a^2d\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(a^3b^2x^3 + a^4bx\right)}, -\frac{2\,a^2bc + \left(3\,ab^2c - a^2bc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b*c + 2*(3*a*b^2*c - a^2*b*d)*x^2 - ((3*b^2*c - a*b*d)*x^3 + (
3*a*b*c - a^2*d)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*a^2*b*c + (3*a*b^2*c - a^2*b*d)*x^2 + (
(3*b^2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

Sympy [A] time = 0.626656, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2ac+x^2\,(ad-3bc)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt(
-1/(a**5*b))*(a*d - 3*b*c)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*a*c + x*
*2*(a*d - 3*b*c))/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A] time = 1.15805, size = 86, normalized size = 1.21

$$-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{3\,bcx^2-adx^2+2\,ac}{2\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 -
 a*d*x^2 + 2*a*c)/((b*x^3 + a*x)*a^2)
```

3.269 $\int \frac{c+dx^2}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{bc-ad}{2a^2\left(a+bx^2\right)}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{2a^2x^2}$$

```
[Out] -c/(2*a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)*Log[x])/a
^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.0740186, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{bc-ad}{2a^2\left(a+bx^2\right)}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)*Log[x])/a
^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}-\frac{b(-bc+ad)}{a^2(a+bx)^2}-\frac{b(-2bc+ad)}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c}{2a^2x^2}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.0458944, size = 64, normalized size = 0.84

$$\frac{\frac{a(ad-bc)}{a+bx^2}+(2bc-ad)\log\left(a+bx^2\right)+2\log(x)(ad-2bc)-\frac{ac}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c)/x^2) + (a*(-(b*c) + a*d))/(a + b*x^2) + 2*(-2*b*c + a*d)*Log[x] +
(2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.013, size = 86, normalized size = 1.1

$$-\frac{c}{2\,a^2x^2}+\frac{\ln(x)\,d}{a^2}-2\,\frac{bc\ln(x)}{a^3}-\frac{\ln\left(bx^2+a\right)d}{2\,a^2}+\frac{bc\ln\left(bx^2+a\right)}{a^3}+\frac{d}{2\,a\left(bx^2+a\right)}-\frac{bc}{2\,a^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c/a^2/x^2+1/a^2*ln(x)*d-2*b*c*ln(x)/a^3-1/2/a^2*ln(b*x^2+a)*d+b*c*ln(b
*x^2+a)/a^3+1/2/a/(b*x^2+a)*d-1/2*b*c/a^2/(b*x^2+a)
```

Maxima [A] time = 1.09233, size = 105, normalized size = 1.38

$$-\frac{(2\,bc-ad)x^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{(2\,bc-ad)\log\left(bx^2+a\right)}{2\,a^3}-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*c - a*d)*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + 1/2*(2*b*c - a*d)*lo
g(b*x^2 + a)/a^3 - 1/2*(2*b*c - a*d)*log(x^2)/a^3
```

Fricas [A] time = 1.55478, size = 248, normalized size = 3.26

$$-\frac{a^2c+\left(2\,abc-a^2d\right)x^2-\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log\left(bx^2+a\right)+2\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log(x}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c + (2*a*b*c - a^2*d)*x^2 - ((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a
^2*d)*x^2)*log(b*x^2 + a) + 2*((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a^2*d)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 1.00116, size = 70, normalized size = 0.92

$$\frac{-ac + x^2(ad - 2bc)}{2a^3x^2 + 2a^2bx^4} + \frac{(ad - 2bc)\log(x)}{a^3} - \frac{(ad - 2bc)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*c + x**2*(a*d - 2*b*c))/(2*a**3*x**2 + 2*a**2*b*x**4) + (a*d - 2*b*c)*l
og(x)/a**3 - (a*d - 2*b*c)*log(a/b + x**2)/(2*a**3)
```

Giac [A] time = 1.16426, size = 113, normalized size = 1.49

$$-\frac{(2\,bc - ad)\log\left(x^2\right)}{2\,a^3} - \frac{2\,bcx^2 - adx^2 + ac}{2\left(bx^4 + ax^2\right)a^2} + \frac{\left(2\,b^2c - abd\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c - a*d)*log(x^2)/a^3 - 1/2*(2*b*c*x^2 - a*d*x^2 + a*c)/((b*x^4 +
 a*x^2)*a^2) + 1/2*(2*b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^3*b)
```

3.270 $\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{bx(bc-ad)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}+\frac{\sqrt{b}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{c}{3a^2x^3}$$

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.106381, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {456, 1261, 205}

$$\frac{bx(bc-ad)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}+\frac{\sqrt{b}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```


```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```


Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)} - \frac{1}{2}b\int \frac{-\frac{2c}{ab}+\frac{2(bc-ad)x^2}{a^2b}-\frac{(bc-ad)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx$$
$$= \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)} - \frac{1}{2}b\int \left(-\frac{2c}{a^2bx^4}-\frac{2(-2bc+ad)}{a^3bx^2}+\frac{-5bc+3ad}{a^3\left(a+bx^2\right)}\right)dx$$
$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{(b(5bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a^3}$$
$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.070273, size = 90, normalized size = 1.

$$-\frac{bx(ad-bc)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}-\frac{\sqrt{b}(3ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) - (b*(-(b*c) + a*d)*x)/(2*a^3*(a + b
*x^2)) - (Sqrt[b]*(-5*b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.011, size = 110, normalized size = 1.2

$$-\frac{c}{3\,a^2x^3}-\frac{d}{a^2x}+2\,\frac{bc}{a^3x}-\frac{bdx}{2\,a^2\left(bx^2+a\right)}+\frac{b^2cx}{2\,a^3\left(bx^2+a\right)}-\frac{3\,bd}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2c}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c/a^2/x^3-1/a^2/x*d+2/a^3/x*b*c-1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^2*x/
(b*x^2+a)*c-3/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+5/2/a^3*b^2/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52035, size = 529, normalized size = 5.88

$$\left[\frac{6\left(5\,b^2c-3\,abd\right)x^4-4\,a^2c+4\left(5\,abc-3\,a^2d\right)x^2-3\left(\left(5\,b^2c-3\,abd\right)x^5+\left(5\,abc-3\,a^2d\right)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{12\left(a^3bx^5+a^4x^3\right)},\ \frac{3\,(5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(5*b^2*c - 3*a*b*d)*x^4 - 4*a^2*c + 4*(5*a*b*c - 3*a^2*d)*x^2 - 3*
((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), 1/6*(3*(5*b^2*c
 - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2 + 3*((5*b^2*c - 3*a*b
*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*x^
5 + a^4*x^3)]
```

Sympy [B] time = 0.795143, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{2a^2c+x^4\left(9abd-15b^2c\right)+}{6a^4x^3+6a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(-a**4*sqrt(-b/a**7)*(3*a*d - 5*b*c)/(3*a*
b*d - 5*b**2*c) + x)/4 - sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(a**4*sqrt(-b/a**
7)*(3*a*d - 5*b*c)/(3*a*b*d - 5*b**2*c) + x)/4 - (2*a**2*c + x**4*(9*a*b*d
- 15*b**2*c) + x**2*(6*a**2*d - 10*a*b*c))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A] time = 1.16169, size = 116, normalized size = 1.29

$$\frac{\left(5\,b^2c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3}+\frac{b^2cx-abdx}{2\left(bx^2+a\right)a^3}+\frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*(b^2*c*
x - a*b*d*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3
)
```

3.271 $\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=145

$$\frac{x^5(bc-ad)^2}{2ab^2(a+bx^2)}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{\sqrt{a}(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{d^2x^5}{5b^2}$$

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c - a*d)*x^3)
/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2*a*b^2*(a + b*x^2))
- (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rubi [A] time = 0.13212, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 302, 205}

$$\frac{x^5(bc-ad)^2}{2ab^2(a+bx^2)}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{\sqrt{a}(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c - a*d)*x^3)
/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2*a*b^2*(a + b*x^2))
- (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(-2b^2c^2+5(bc-ad)^2-2abd^2x^2\right)}{a+bx^2}\,dx}{2ab^2}$$

$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{((3bc-7ad)(bc-ad))\int \frac{x^4}{a+bx^2}\,dx}{2ab^2}$$

$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{((3bc-7ad)(bc-ad))\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{2ab^2}$$

$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{(a(3bc-7ad)(bc-ad)}{2b^4}$$

$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{\sqrt{a}(3bc-7ad)(bc-ad}{2b^{9/2}}$$

Mathematica [A] time = 0.0892508, size = 138, normalized size = 0.95

$$\frac{x\left(3a^2d^2-4abcd+b^2c^2\right)}{b^4} - \frac{\sqrt{a}\left(7a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} + \frac{2dx^3(bc-ad)}{3b^3} + \frac{ax(bc-ad)^2}{2b^4\left(a+bx^2\right)} + \frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d*(b*c - a*d)*x^3)/(3*b^3) +
 (d^2*x^5)/(5*b^2) + (a*(b*c - a*d)^2*x)/(2*b^4*(a + b*x^2)) - (Sqrt[a]*(3*
b^2*c^2 - 10*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.009, size = 196, normalized size = 1.4

$$\frac{d^2x^5}{5\,b^2} - \frac{2\,x^3ad^2}{3\,b^3} + \frac{2\,cx^3d}{3\,b^2} + 3\,\frac{a^2d^2x}{b^4} - 4\,\frac{acdx}{b^3} + \frac{c^2x}{b^2} + \frac{a^3xd^2}{2\,b^4\left(bx^2+a\right)} - \frac{a^2cdx}{b^3\left(bx^2+a\right)} + \frac{axc^2}{2\,b^2\left(bx^2+a\right)} - \frac{7\,a^3d^2}{2\,b^4}\arctan\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^2*x^5/b^2-2/3/b^3*x^3*a*d^2+2/3/b^2*x^3*c*d+3/b^4*a^2*d^2*x-4/b^3*c*a
*d*x+1/b^2*c^2*x+1/2*a^3/b^4*x/(b*x^2+a)*d^2-a^2/b^3*x/(b*x^2+a)*c*d+1/2*a/
b^2*x/(b*x^2+a)*c^2-7/2*a^3/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+5*a
^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d-3/2*a/b^2/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64322, size = 851, normalized size = 5.87

$$\left[\frac{12 b^3d^2x^7+4\left(10 b^3cd-7 ab^2d^2\right)x^5+20\left(3 b^3c^2-10 ab^2cd+7 a^2bd^2\right)x^3+15\left(3 ab^2c^2-10 a^2bcd+7 a^3d^2+\left(3 b^3c^2-\right.\right.}{60\left(b^5x^2+ab^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*d^2*x^7 + 4*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 20*(3*b^3*c^2 -
10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^
2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2
*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4), 1/30*(6*b^3*d^2*x^7 + 2*(10*b^3*c*d - 7*a*b^2*d^
2)*x^5 + 10*(3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 - 15*(3*a*b^2*c^2
- 10*a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*
sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4)]
```

Sympy [B] time = 1.12611, size = 280, normalized size = 1.93

$$\frac{x\left(a^3d^2-2a^2bcd+ab^2c^2\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc)\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 + 2*b**5*x**2) + sqrt(
-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(-b**4*sqrt(-a/b**9)*(a*d - b*c)*(7
*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2) + x)/4 - sqrt(-a/b**
9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(b**4*sqrt(-a/b**9)*(a*d - b*c)*(7*a*d -
3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2) + x)/4 + d**2*x**5/(5*b**2)
 - x**3*(2*a*d**2 - 2*b*c*d)/(3*b**3) + x*(3*a**2*d**2 - 4*a*b*c*d + b**2*c
**2)/b**4
```

Giac [A] time = 1.1463, size = 211, normalized size = 1.46

$$-\frac{\left(3\,ab^{2}c^{2}-10\,a^{2}bcd+7\,a^{3}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{4}}+\frac{ab^{2}c^{2}x-2\,a^{2}bcdx+a^{3}d^{2}x}{2\,\left(bx^{2}+a\right)b^{4}}+\frac{3\,b^{8}d^{2}x^{5}+10\,b^{8}cdx^{3}-10\,ab^{7}d^{2}x^{3}+15\,b^{8}c^{2}}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) + 1/2*(a*b^2*c^2*x - 2*a^2*b*c*d*x + a^3*d^2*x)/((b*x^2 + a)*b^4)
+ 1/15*(3*b^8*d^2*x^5 + 10*b^8*c*d*x^3 - 10*a*b^7*d^2*x^3 + 15*b^8*c^2*x -
60*a*b^7*c*d*x + 45*a^2*b^6*d^2*x)/b^10
```

3.272 $\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{dx^2(bc-ad)}{b^3}+\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^4}{4b^2}$$

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(2*b^4*(a +
 b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.105801, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^2(bc-ad)}{b^3}+\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(2*b^4*(a +
 b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^2}{(a+bx)^2}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{2d(bc-ad)}{b^3}+\frac{d^2x}{b^2}-\frac{a(-bc+ad)^2}{b^3(a+bx)^2}+\frac{(bc-3ad)(bc-ad)}{b^3(a+bx)}\right)dx,x,x^2\right)\\&=\frac{d(bc-ad)x^2}{b^3}+\frac{d^2x^4}{4b^2}+\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.0601013, size = 87, normalized size = 0.99

$$\frac{2\left(3a^2d^2-4abcd+b^2c^2\right)\log\left(a+bx^2\right)+4bdx^2(bc-ad)+\frac{2a(bc-ad)^2}{a+bx^2}+b^2d^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (4*b*d*(b*c - a*d)*x^2 + b^2*d^2*x^4 + (2*a*(b*c - a*d)^2)/(a + b*x^2) + 2*
(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Log[a + b*x^2])/(4*b^4)
```

Maple [A] time = 0.01, size = 142, normalized size = 1.6

$$\frac{d^2x^4}{4b^2}-\frac{ad^2x^2}{b^3}+\frac{dx^2c}{b^2}+\frac{3\ln\left(bx^2+a\right)a^2d^2}{2b^4}-2\frac{\ln\left(bx^2+a\right)adc}{b^3}+\frac{\ln\left(bx^2+a\right)c^2}{2b^2}+\frac{a^3d^2}{2b^4\left(bx^2+a\right)}-\frac{a^2cd}{b^3\left(bx^2+a\right)}+\frac{}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*x^4/b^2-d^2/b^3*a*x^2+d/b^2*x^2*c+3/2/b^4*ln(b*x^2+a)*a^2*d^2-2/b^3
*ln(b*x^2+a)*a*d*c+1/2/b^2*ln(b*x^2+a)*c^2+1/2/b^4*a^3/(b*x^2+a)*d^2-1/b^3*
a^2/(b*x^2+a)*d*c+1/2/b^2*a/(b*x^2+a)*c^2
```

Maxima [A] time = 1.1633, size = 144, normalized size = 1.64

$$\frac{ab^2c^2-2a^2bcd+a^3d^2}{2\left(b^5x^2+ab^4\right)}+\frac{bd^2x^4+4\left(bcd-ad^2\right)x^2}{4b^3}+\frac{\left(b^2c^2-4abcd+3a^2d^2\right)\log\left(bx^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^5*x^2 + a*b^4) + 1/4*(b*d^2*x^4
+ 4*(b*c*d - a*d^2)*x^2)/b^3 + 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(b*
x^2 + a)/b^4
```

Fricas [A] time = 1.48316, size = 327, normalized size = 3.72

$$\frac{b^3d^2x^6+2ab^2c^2-4a^2bcd+2a^3d^2+\left(4b^3cd-3ab^2d^2\right)x^4+4\left(ab^2cd-a^2bd^2\right)x^2+2\left(ab^2c^2-4a^2bcd+3a^3d^2+\left(b^3c^2-\right.\right.}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^2*x^6 + 2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (4*b^3*c*d - 3*a
*b^2*d^2)*x^4 + 4*(a*b^2*c*d - a^2*b*d^2)*x^2 + 2*(a*b^2*c^2 - 4*a^2*b*c*d
+ 3*a^3*d^2 + (b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x^2)*log(b*x^2 + a))/(b
```

```
^5*x^2 + a*b^4)
```

Sympy [A] time = 1.09655, size = 97, normalized size = 1.1

$$\frac{a^3d^2 - 2a^2bcd + ab^2c^2}{2ab^4 + 2b^5x^2} + \frac{d^2x^4}{4b^2} - \frac{x^2\left(ad^2 - bcd\right)}{b^3} + \frac{(ad - bc)\,(3ad - bc)\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] (a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 + 2*b**5*x**2) + d**2*x*
*4/(4*b**2) - x**2*(a*d**2 - b*c*d)/b**3 + (a*d - b*c)*(3*a*d - b*c)*log(a
+ b*x**2)/(2*b**4)
```

Giac [A] time = 1.14321, size = 220, normalized size = 2.5

$$\frac{\frac{\left(bx^2+a\right)^2\left(d^2+\frac{2\left(2\,b^2cd-3\,abd^2\right)}{\left(bx^2+a\right)b}\right)}{b^3} - \frac{2\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^3} + \frac{2\left(\frac{ab^4c^2}{bx^2+a}-\frac{2\,a^2b^3cd}{bx^2+a}+\frac{a^3b^2d^2}{bx^2+a}\right)}{b^5}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)^2*(d^2 + 2*(2*b^2*c*d - 3*a*b*d^2)/((b*x^2 + a)*b))/b^3 -
2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b
)))/b^3 + 2*(a*b^4*c^2/(b*x^2 + a) - 2*a^2*b^3*c*d/(b*x^2 + a) + a^3*b^2*d^
2/(b*x^2 + a))/b^5)/b
```

3.273 $\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=116

$$\frac{x^3(bc-ad)^2}{2ab^2(a+bx^2)}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}+\frac{d^2x^3}{3b^2}$$

```
[Out] -((b*c - 5*a*d)*(b*c - a*d)*x)/(2*a*b^3) + (d^2*x^3)/(3*b^2) + ((b*c - a*d)
^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.111412, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 321, 205}

$$\frac{x^3(bc-ad)^2}{2ab^2(a+bx^2)}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] -((b*c - 5*a*d)*(b*c - a*d)*x)/(2*a*b^3) + (d^2*x^3)/(3*b^2) + ((b*c - a*d)
^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}-\frac{\int \frac{x^2\left(b^2c^2-6abcd+3a^2d^2-2abd^2x^2\right)}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}-\frac{((bc-5ad)(bc-ad))\int \frac{x^2}{a+bx^2}\,dx}{2ab^2}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3}+\frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}+\frac{((bc-5ad)(bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3}+\frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0738896, size = 105, normalized size = 0.91

$$
\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}-\frac{x(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{2dx(bc-ad)}{b^3}+\frac{d^2x^3}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (2*d*(b*c - a*d)*x)/b^3 + (d^2*x^3)/(3*b^2) - ((b*c - a*d)^2*x)/(2*b^3*(a +
 b*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.008, size = 156, normalized size = 1.3

$$
\frac{d^2x^3}{3\,b^2}-2\,\frac{ad^2x}{b^3}+2\,\frac{dxc}{b^2}-\frac{a^2d^2x}{2\,b^3\left(bx^2+a\right)}+\frac{cxad}{b^2\left(bx^2+a\right)}-\frac{xc^2}{2\,b\left(bx^2+a\right)}+\frac{5\,a^2d^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\,\frac{acd}{b^2\sqrt{ab}}\mathrm{arc}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^2*x^3/b^2-2*d^2/b^3*a*x+2*d/b^2*x*c-1/2/b^3*x/(b*x^2+a)*a^2*d^2+1/b^2
*x/(b*x^2+a)*c*a*d-1/2/b*x/(b*x^2+a)*c^2+5/2/b^3/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*a^2*d^2-3/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*a*d+1/2/b/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51846, size = 713, normalized size = 6.15

$$\left[\frac{4\,ab^3d^2x^5 + 4\left(6\,ab^3cd - 5\,a^2b^2d^2\right)x^3 - 3\left(ab^2c^2 - 6\,a^2bcd + 5\,a^3d^2 + \left(b^3c^2 - 6\,ab^2cd + 5\,a^2bd^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-}{bx^2+a}\right.}{12\left(ab^5x^2 + a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a*b^3*d^2*x^5 + 4*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 - 3*(a*b^2*c^2
 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a*b^3*c^2 - 6*a^
2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4), 1/6*(2*a*b^3*d^2*x^5 + 2
*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 + 3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2
 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) - 3*(a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [B] time = 1.02119, size = 245, normalized size = 2.11

$$-\frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{2ab^3 + 2b^4x^2} - \frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) - sqrt(-1/(
a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(-a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c
)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + sqrt(-1/(a*b
**7))*(a*d - b*c)*(5*a*d - b*c)*log(a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5
*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + d**2*x**3/(3*b**
2) - x*(2*a*d**2 - 2*b*c*d)/b**3
```

Giac [A] time = 1.1219, size = 154, normalized size = 1.33

$$\frac{\left(b^2c^2 - 6\,abcd + 5\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^3} - \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(bx^2 + a\right)b^3} + \frac{b^4d^2x^3 + 6\,b^4cdx - 6\,ab^3d^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d
```

```
^2*x^3 + 6*b^4*c*d*x - 6*a*b^3*d^2*x)/b^6
```

3.274 $\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}+\frac{d^2x^2}{2b^2}$$

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c - a*d)*Log[
a + b*x^2])/b^3
```

Rubi [A] time = 0.0580752, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}+\frac{d^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c - a*d)*Log[
a + b*x^2])/b^3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^2}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{d^2x^2}{2b^2}-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.046709, size = 56, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx^2}+2d(bc-ad)\log\left(a+bx^2\right)+bd^2x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (b*d^2*x^2 - (b*c - a*d)^2/(a + b*x^2) + 2*d*(b*c - a*d)*Log[a + b*x^2])/(2
*b^3)
```

Maple [A] time = 0.01, size = 97, normalized size = 1.6

$$\frac{d^2x^2}{2\,b^2}-\frac{\ln\left(bx^2+a\right)d^2a}{b^3}+\frac{\ln\left(bx^2+a\right)dc}{b^2}-\frac{a^2d^2}{2\,b^3\left(bx^2+a\right)}+\frac{acd}{b^2\left(bx^2+a\right)}-\frac{c^2}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^2*x^2/b^2-1/b^3*ln(b*x^2+a)*d^2*a+1/b^2*ln(b*x^2+a)*d*c-1/2/b^3/(b*x^
2+a)*a^2*d^2+1/b^2/(b*x^2+a)*a*d*c-1/2/b/(b*x^2+a)*c^2
```

Maxima [A] time = 1.06481, size = 99, normalized size = 1.62

$$\frac{d^2x^2}{2\,b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(b^4x^2+ab^3\right)}+\frac{\left(bcd-ad^2\right)\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x^2 + a*b^3) + (
b*c*d - a*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 1.43955, size = 200, normalized size = 3.28

$$\frac{b^2d^2x^4+abd^2x^2-b^2c^2+2\,abcd-a^2d^2+2\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + a*b*d^2*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d
 - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 0.906413, size = 68, normalized size = 1.11

$$-\frac{a^2d^2-2abcd+b^2c^2}{2ab^3+2b^4x^2}+\frac{d^2x^2}{2b^2}-\frac{d\,(ad-bc)\log\left(a+bx^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] -(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) + d**2*x**2/(
2*b**2) - d*(a*d - b*c)*log(a + b*x**2)/b**3
```

Giac [A] time = 1.13733, size = 150, normalized size = 2.46

$$\frac{(bx^2+a)d^2}{2\,b^3} - \frac{(bcd-ad^2)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^3} - \frac{\frac{b^3c^2}{bx^2+a} - \frac{2\,ab^2cd}{bx^2+a} + \frac{a^2bd^2}{bx^2+a}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)*d^2/b^3 - (b*c*d - a*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2
*abs(b)))/b^3 - 1/2*(b^3*c^2/(b*x^2 + a) - 2*a*b^2*c*d/(b*x^2 + a) + a^2*b*
d^2/(b*x^2 + a))/b^4
```

3.275 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.0991134, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2}{b^2} + \frac{b^2c^2 - a^2d^2 + 2bd(bc-ad)x^2}{b^2\left(a+bx^2\right)^2}\right) dx$$
$$= \frac{d^2x}{b^2} + \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{b^2}$$
$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)} + \frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2}$$
$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)} + \frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.0615265, size = 88, normalized size = 1.07

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}} + \frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)} + \frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Maple [A] time = 0.009, size = 129, normalized size = 1.6

$$\frac{d^2x}{b^2} + \frac{axd^2}{2\,b^2\left(bx^2+a\right)} - \frac{cxd}{b\left(bx^2+a\right)} + \frac{xc^2}{2\,a\left(bx^2+a\right)} - \frac{3\,ad^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{cd}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{c^2}{2\,a}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] d^2*x/b^2+1/2/b^2*x*a/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2*x/a/(b*x^2+a)*c
^2-3/2/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+1/b/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))*c*d+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47759, size = 612, normalized size = 7.46

$$\left[\frac{4\,a^2b^2d^2x^3+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\left(ab^3c^2-2\,a^2b^2cd\right.}{4\left(a^2b^4x^2+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] time = 0.88934, size = 236, normalized size = 2.88

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2a^2b^2+2ab^3x^2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

Giac [A] time = 1.12377, size = 127, normalized size = 1.55

$$\frac{d^2x}{b^2}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(bx^2+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

3.276 $\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^2 - d^2/b^
2)*Log[a + b*x^2])/2
```

Rubi [A] time = 0.0646807, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^2 - d^2/b^
2)*Log[a + b*x^2])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{a^2x}-\frac{(-bc+ad)^2}{ab(a+bx)^2}+\frac{-b^2c^2+a^2d^2}{a^2b(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{c^2\log(x)}{a^2}-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0428798, size = 70, normalized size = 1.04

$$\frac{\frac{(ad-bc)\left(\left(a+bx^2\right)(ad+bc)\log\left(a+bx^2\right)+a(ad-bc)\right)}{b^2\left(a+bx^2\right)}+2c^2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^2*Log[x] + ((-(b*c) + a*d)*(a*(-(b*c) + a*d) + (b*c + a*d)*(a + b*x^2)
*Log[a + b*x^2]))/(b^2*(a + b*x^2)))/(2*a^2)
```

Maple [A] time = 0.011, size = 94, normalized size = 1.4

$$\frac{c^2\ln(x)}{a^2}+\frac{\ln\left(bx^2+a\right)d^2}{2\,b^2}-\frac{\ln\left(bx^2+a\right)c^2}{2\,a^2}+\frac{ad^2}{2\,b^2\left(bx^2+a\right)}-\frac{cd}{b\left(bx^2+a\right)}+\frac{c^2}{2\,a\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a)^2,x)
```

```
[Out] c^2*ln(x)/a^2+1/2/b^2*ln(b*x^2+a)*d^2-1/2/a^2*ln(b*x^2+a)*c^2+1/2*a/b^2/(b*
x^2+a)*d^2-1/b/(b*x^2+a)*d*c+1/2/a/(b*x^2+a)*c^2
```

Maxima [A] time = 1.04862, size = 116, normalized size = 1.73

$$\frac{c^2\log\left(x^2\right)}{2\,a^2}+\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(ab^3x^2+a^2b^2\right)}-\frac{\left(b^2c^2-a^2d^2\right)\log\left(bx^2+a\right)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*b^3*x^2 + a^2
*b^2) - 1/2*(b^2*c^2 - a^2*d^2)*log(b*x^2 + a)/(a^2*b^2)
```

Fricas [A] time = 1.55434, size = 228, normalized size = 3.4

$$\frac{ab^2c^2-2\,a^2bcd+a^3d^2-\left(ab^2c^2-a^3d^2+\left(b^3c^2-a^2bd^2\right)x^2\right)\log\left(bx^2+a\right)+2\left(b^3c^2x^2+ab^2c^2\right)\log(x)}{2\left(a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 - (a*b^2*c^2 - a^3*d^2 + (b^3*c^2 -
a^2*b*d^2)*x^2)*log(b*x^2 + a) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*log(x))/(a^2*b
^3*x^2 + a^3*b^2)
```

Sympy [A] time = 1.37925, size = 80, normalized size = 1.19

$$\frac{a^2d^2-2abcd+b^2c^2}{2a^2b^2+2ab^3x^2}+\frac{c^2\log(x)}{a^2}+\frac{(ad-bc)(ad+bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

```
[Out] (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + c**2*lo
g(x)/a**2 + (a*d - b*c)*(a*d + b*c)*log(a/b + x**2)/(2*a**2*b**2)
```

Giac [A] time = 1.15932, size = 134, normalized size = 2.

$$\frac{c^2\log\left(x^2\right)}{2\,a^2}-\frac{\left(b^2c^2-a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b^2}+\frac{b^2c^2x^2-a^2d^2x^2+2\,abc^2-2\,a^2cd}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 - 1/2*(b^2*c^2 - a^2*d^2)*log(abs(b*x^2 + a))/(a^2*b^2
) + 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((b*x^2 + a)*a^
2*b)
```

3.277 $\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=103

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a\left(a+bx^2\right)}-\frac{c^2}{ax\left(a+bx^2\right)}$$

```
[Out] -(c^2/(a*x*(a + b*x^2))) - (((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/(2*a*(a +
b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2
)*b^(3/2))
```

Rubi [A] time = 0.0777147, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {462, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a\left(a+bx^2\right)}-\frac{c^2}{ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a*x*(a + b*x^2))) - (((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/(2*a*(a +
b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2
)*b^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{c^2}{ax\left(a+bx^2\right)} + \frac{\int \frac{-c(3bc-2ad)+ad^2x^2}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= -\frac{c^2}{ax\left(a+bx^2\right)} - \frac{\left(\frac{3bc^2}{a}-2cd+\frac{ad^2}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{\left((bc-ad)(3bc+ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b}$$

$$= -\frac{c^2}{ax\left(a+bx^2\right)} - \frac{\left(\frac{3bc^2}{a}-2cd+\frac{ad^2}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.0687163, size = 91, normalized size = 0.88

$$\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}} - \frac{x(ad-bc)^2}{2a^2b\left(a+bx^2\right)} - \frac{c^2}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a^2*x)) - ((-(b*c) + a*d)^2*x)/(2*a^2*b*(a + b*x^2)) + ((-3*b^2*c^2
+ 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

Maple [A] time = 0.01, size = 131, normalized size = 1.3

$$-\frac{c^2}{a^2x} - \frac{xd^2}{2\,b\left(bx^2+a\right)} + \frac{cxd}{a\left(bx^2+a\right)} - \frac{bxc^2}{2\,a^2\left(bx^2+a\right)} + \frac{d^2}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{cd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3\,bc^2}{2\,a^2}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c^2/a^2/x-1/2/b*x/(b*x^2+a)*d^2+1/a*x/(b*x^2+a)*c*d-1/2/a^2*b*x/(b*x^2+a)*
c^2+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+1/a/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2))*c*d-3/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5931, size = 625, normalized size = 6.07

$$\left[-\frac{4\,a^2b^2c^2 + 2\left(3\,ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x^2 - \left(\left(3\,b^3c^2 - 2\,ab^2cd - a^2bd^2\right)x^3 + \left(3\,ab^2c^2 - 2\,a^2bcd - a^3d^2\right)x\right)\sqrt{-ab}\log}{4\left(a^3b^3x^3 + a^4b^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b^2*c^2 + 2*(3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 - ((
3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2*a^2*b*c*d - a^3
*d^2)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3
*x^3 + a^4*b^2*x), -1/2*(2*a^2*b^2*c^2 + (3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*x^2 + ((3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2
*a^2*b*c*d - a^3*d^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^3 + a^
4*b^2*x)]
```

Sympy [B] time = 1.01676, size = 238, normalized size = 2.31

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+}{a^2d^2+2abcd-3b^2c^2}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b
**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/
4 + sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5
*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x
)/4 - (2*a*b*c**2 + x**2*(a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2))/(2*a**3*b*x
 + 2*a**2*b**2*x**3)
```

Giac [A] time = 1.15639, size = 139, normalized size = 1.35

$$-\frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b} - \frac{3\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + 2\,abc^2}{2\left(bx^3 + ax\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
*b) - 1/2*(3*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + 2*a*b*c^2)/((b*x^3
 + a*x)*a^2*b)
```

3.278 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2} dx$

Optimal. Leaf size=80

$$-\frac{(bc-ad)^2}{2a^2b(a+bx^2)}+\frac{c(bc-ad)\log(a+bx^2)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{c^2}{2a^2x^2}$$

```
[Out] -c^2/(2*a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*c - a*d)*L
og[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rubi [A] time = 0.0814483, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^2}{2a^2b(a+bx^2)}+\frac{c(bc-ad)\log(a+bx^2)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{c^2}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^2/(2*a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*c - a*d)*L
og[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{a^2x^2}+\frac{2c(-bc+ad)}{a^3x}+\frac{(-bc+ad)^2}{a^2(a+bx)^2}-\frac{2bc(-bc+ad)}{a^3(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{c^2}{2a^2x^2}-\frac{(bc-ad)^2}{2a^2b(a+bx^2)}-\frac{2c(bc-ad)\log(x)}{a^3}+\frac{c(bc-ad)\log(a+bx^2)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.0972196, size = 72, normalized size = 0.9

$$-\frac{\frac{a(bc-ad)^2}{b(a+bx^2)}-2c(bc-ad)\log\left(a+bx^2\right)+4c\log(x)(bc-ad)+\frac{ac^2}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((a*c^2)/x^2 + (a*(b*c - a*d)^2)/(b*(a + b*x^2)) + 4*c*(b*c - a*d)*Log[x]
- 2*c*(b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.014, size = 114, normalized size = 1.4

$$-\frac{c^2}{2\,a^2x^2}+2\,\frac{c\ln(x)\,d}{a^2}-2\,\frac{c^2\ln(x)\,b}{a^3}-\frac{c\ln\left(bx^2+a\right)d}{a^2}+\frac{c^2\ln\left(bx^2+a\right)b}{a^3}-\frac{d^2}{2\,b\left(bx^2+a\right)}+\frac{cd}{a\left(bx^2+a\right)}-\frac{bc^2}{2\,a^2\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c^2/a^2/x^2+2*c/a^2*ln(x)*d-2*c^2/a^3*ln(x)*b-1/a^2*c*ln(b*x^2+a)*d+1/
a^3*c^2*ln(b*x^2+a)*b-1/2/b/(b*x^2+a)*d^2+1/a/(b*x^2+a)*d*c-1/2/a^2/(b*x^2+
a)*c^2*b
```

Maxima [A] time = 1.17934, size = 135, normalized size = 1.69

$$-\frac{abc^2+\left(2\,b^2c^2-2\,abcd+a^2d^2\right)x^2}{2\left(a^2b^2x^4+a^3bx^2\right)}+\frac{\left(bc^2-acd\right)\log\left(bx^2+a\right)}{a^3}-\frac{\left(bc^2-acd\right)\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^4 + a^3*b
*x^2) + (b*c^2 - a*c*d)*log(b*x^2 + a)/a^3 - (b*c^2 - a*c*d)*log(x^2)/a^3
```

Fricas [B] time = 1.52795, size = 316, normalized size = 3.95

$$-\frac{a^2bc^2+\left(2\,ab^2c^2-2\,a^2bcd+a^3d^2\right)x^2-2\left(\left(b^3c^2-ab^2cd\right)x^4+\left(ab^2c^2-a^2bcd\right)x^2\right)\log\left(bx^2+a\right)+4\left(\left(b^3c^2-ab^2cd\right)x^4\right.}{2\left(a^3b^2x^4+a^4bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2 - 2*((b^3*c^2 -
 a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(b*x^2 + a) + 4*((b^3*c^2
 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(x))/(a^3*b^2*x^4 + a^4
```

```
*b*x^2)
```

Sympy [A] time = 1.61149, size = 92, normalized size = 1.15

$$-\frac{abc^2 + x^2\left(a^2d^2 - 2abcd + 2b^2c^2\right)}{2a^3bx^2 + 2a^2b^2x^4} + \frac{2c\,(ad - bc)\log(x)}{a^3} - \frac{c\,(ad - bc)\log\left(\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

```
[Out] -(a*b*c**2 + x**2*(a**2*d**2 - 2*a*b*c*d + 2*b**2*c**2))/(2*a**3*b*x**2 + 2
*a**2*b**2*x**4) + 2*c*(a*d - b*c)*log(x)/a**3 - c*(a*d - b*c)*log(a/b + x*
*2)/a**3
```

Giac [A] time = 1.12221, size = 147, normalized size = 1.84

$$-\frac{\left(bc^2 - acd\right)\log\left(x^2\right)}{a^3} + \frac{\left(b^2c^2 - abcd\right)\log\left(\left|bx^2 + a\right|\right)}{a^3b} - \frac{2\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + abc^2}{2\left(bx^4 + ax^2\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d)*log(x^2)/a^3 + (b^2*c^2 - a*b*c*d)*log(abs(b*x^2 + a))/(a^
3*b) - 1/2*(2*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + a*b*c^2)/((b*x^4
+ a*x^2)*a^2*b)
```

3.279 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=127

$$\frac{x\left(3a^2d^2-6abcd+5b^2c^2\right)}{6a^3\left(a+bx^2\right)}+\frac{c(5bc-6ad)}{3a^3x}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((5*b^2*c^2 - 6
*a*b*c*d + 3*a^2*d^2)*x)/(6*a^3*(a + b*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.142764, antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{6a\left(a+bx^2\right)}+\frac{c(5bc-6ad)}{3a^3x}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((3*d^2 + (b*c*
(5*b*c - 6*a*d))/a^2)*x)/(6*a*(a + b*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx &= -\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\int \frac{-c(5bc-6ad)+3ad^2x^2}{x^2\left(a+bx^2\right)^2}\,dx}{3a}\\
&= -\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}-\frac{\int \frac{\frac{2c(5bc-6ad)}{a}-\left(\frac{5b^2c^2}{a^2}-\frac{6bcd}{a}+3d^2\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{6a}\\
&= \frac{c(5bc-6ad)}{3a^3x}-\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}+\frac{((bc-ad)(5bc-ad))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\
&= \frac{c(5bc-6ad)}{3a^3x}-\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0695077, size = 107, normalized size = 0.84

$$
\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}+\frac{x(ad-bc)^2}{2a^3\left(a+bx^2\right)}-\frac{2c(ad-bc)}{a^3x}-\frac{c^2}{3a^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c^2/(3*a^2*x^3) - (2*c*(-(b*c) + a*d))/(a^3*x) + ((-(b*c) + a*d)^2*x)/(2*a
^3*(a + b*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.011, size = 161, normalized size = 1.3

$$
-\frac{c^2}{3\,a^2x^3}-2\,\frac{cd}{a^2x}+2\,\frac{bc^2}{a^3x}+\frac{xd^2}{2\,a\left(bx^2+a\right)}-\frac{bcxd}{a^2\left(bx^2+a\right)}+\frac{b^2c^2x}{2\,a^3\left(bx^2+a\right)}+\frac{d^2}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\,\frac{bcd}{a^2\sqrt{ab}}\arctan
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c^2/a^2/x^3-2*c/a^2/x*d+2*c^2/a^3/x*b+1/2/a*x/(b*x^2+a)*d^2-1/a^2*x/(b
*x^2+a)*c*b*d+1/2/a^3*x/(b*x^2+a)*b^2*c^2+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*d^2-3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*b*d+5/2/a^3/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56387, size = 737, normalized size = 5.8

$$\left[-\frac{4\,a^3bc^2 - 6\left(5\,ab^3c^2 - 6\,a^2b^2cd + a^3bd^2\right)x^4 - 4\left(5\,a^2b^2c^2 - 6\,a^3bcd\right)x^2 + 3\left(\left(5\,b^3c^2 - 6\,ab^2cd + a^2bd^2\right)x^5 + \left(5\,ab^2c^2 - \right.\right.}{12\left(a^4b^2x^5 + a^5bx^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b*c^2 - 6*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 4*(
5*a^2*b^2*c^2 - 6*a^3*b*c*d)*x^2 + 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)
*x^5 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^2*x^5 + a^5*b*x^3), -1/6*(2*a^3*b*c
^2 - 3*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6
*a^3*b*c*d)*x^2 - 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^5 + (5*a*b^2*c
^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^2*
x^5 + a^5*b*x^3)]
```

Sympy [B] time = 1.18685, size = 248, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{a^7b}}\,(ad - 5bc)\,(ad - bc)\log\left(-\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^7b}}\,(ad - 5bc)\,(ad - bc)\log\left(\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + \right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(-a**4*sqrt(-1/(a**7*b))*(a
*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt
(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(a**4*sqrt(-1/(a**7*b))*(a*d - 5
*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(3*a**2*d**2 - 18*a*b*c*d + 15*b**2*c**2) + x**2*(-12*a**2*c*d +
 10*a*b*c**2))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A] time = 1.16318, size = 151, normalized size = 1.19

$$\frac{\left(5\,b^2c^2 - 6\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(bx^2 + a\right)a^3} + \frac{6\,bc^2x^2 - 6\,acdx^2 - ac^2}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c
^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^3*x^3)
```

3.280 $\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=169

$$\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}+\frac{3d^2x^5(7bc-3ad)}{10b^3}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}-\frac{3\sqrt{a}(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-$$

```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b*c*d + 3*a
^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*b^3) + (9*d^3*x^7)/(
14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 3*a*d)*
(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.156172, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {467, 570, 205}

$$\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}+\frac{3d^2x^5(7bc-3ad)}{10b^3}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}-\frac{3\sqrt{a}(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b*c*d + 3*a
^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*b^3) + (9*d^3*x^7)/(
14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 3*a*d)*
(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```


Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(c+dx^2\right)^2\left(3c+9dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\int \left(\frac{3(bc-3ad)(bc-ad)^2}{b^4} + \frac{3d\left(5b^2c^2-7abcd+3a^2d^2\right)x^2}{b^3} + \frac{3d^2(7bc-3ad)x^4}{b^2} + \frac{9d^3x^6}{b} + \frac{3\left(-ab^3c^3+5a^2b^2c^2\right.}{b^4(a}\right.}{2b}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5} + \frac{d\left(5b^2c^2-7abcd+3a^2d^2\right)x^3}{2b^4} + \frac{3d^2(7bc-3ad)x^5}{10b^3} + \frac{9d^3x^7}{14b^2} - \frac{x^3\left(c+dx^2\right.}{2b\left(a+bx^2\right.}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5} + \frac{d\left(5b^2c^2-7abcd+3a^2d^2\right)x^3}{2b^4} + \frac{3d^2(7bc-3ad)x^5}{10b^3} + \frac{9d^3x^7}{14b^2} - \frac{x^3\left(c+dx^2\right.}{2b\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0843146, size = 151, normalized size = 0.89

$$\frac{d^2x^5(3bc-2ad)}{5b^3} + \frac{dx^3(bc-ad)^2}{b^4} + \frac{ax(bc-ad)^3}{2b^5\left(a+bx^2\right)} + \frac{x(bc-4ad)(bc-ad)^2}{b^5} + \frac{3\sqrt{a}(bc-ad)^2(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}} + \frac{d^3x}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 4*a*d)*(b*c - a*d)^2*x)/b^5 + (d*(b*c - a*d)^2*x^3)/b^4 + (d^2*(3*b
*c - 2*a*d)*x^5)/(5*b^3) + (d^3*x^7)/(7*b^2) + (a*(b*c - a*d)^3*x)/(2*b^5*(
a + b*x^2)) + (3*Sqrt[a]*(b*c - a*d)^2*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*b^(11/2))
```

Maple [B] time = 0.01, size = 302, normalized size = 1.8

$$\frac{d^3x^7}{7\,b^2} - \frac{2\,x^5ad^3}{5\,b^3} + \frac{3\,x^5cd^2}{5\,b^2} + \frac{x^3a^2d^3}{b^4} - 2\,\frac{x^3acd^2}{b^3} + \frac{x^3c^2d}{b^2} - 4\,\frac{a^3d^3x}{b^5} + 9\,\frac{a^2cd^2x}{b^4} - 6\,\frac{ac^2dx}{b^3} + \frac{c^3x}{b^2} - \frac{a^4xd^3}{2\,b^5\left(bx^2+a\right)} + \frac{3\,a}{2\,b^4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^3*x^7/b^2-2/5/b^3*x^5*a*d^3+3/5/b^2*x^5*c*d^2+1/b^4*x^3*a^2*d^3-2/b^3
*x^3*a*c*d^2+1/b^2*x^3*c^2*d-4/b^5*a^3*d^3*x+9/b^4*a^2*c*d^2*x-6/b^3*a*c^2*
d*x+1/b^2*c^3*x-1/2*a^4/b^5*x/(b*x^2+a)*d^3+3/2*a^3/b^4*x/(b*x^2+a)*c*d^2-3
/2*a^2/b^3*x/(b*x^2+a)*c^2*d+1/2*a/b^2*x/(b*x^2+a)*c^3+9/2*a^4/b^5/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*d^3-21/2*a^3/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*c*d^2+15/2*a^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d-3/2*a/b^
2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57987, size = 1196, normalized size = 7.08

$$\left[\frac{20\,b^4d^3x^9+12\left(7\,b^4cd^2-3\,ab^3d^3\right)x^7+28\left(5\,b^4c^2d-7\,ab^3cd^2+3\,a^2b^2d^3\right)x^5+140\left(b^4c^3-5\,ab^3c^2d+7\,a^2b^2cd^2-3\,a^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*d^3*x^9 + 12*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 28*(5*b^4*c^2
*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 140*(b^4*c^3 - 5*a*b^3*c^2*d + 7*
a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3
*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b
*d^3)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210
*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*
b^5), 1/70*(10*b^4*d^3*x^9 + 6*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 14*(5*b^4*
c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 70*(b^4*c^3 - 5*a*b^3*c^2*d +
7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a
^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3
*b*d^3)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(a*b^3*c^3 - 5*a^2*b^2
*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [B] time = 1.59973, size = 382, normalized size = 2.26

$$-\frac{x\left(a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3\right)}{2ab^5+2b^6x^2}-\frac{3\sqrt{-\frac{a}{b^{11}}}\left(ad-bc\right)^2\left(3ad-bc\right)\log\left(-\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{3\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3)/(2*a*b*
*5 + 2*b**6*x**2) - 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)*log(-3*b*
*5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)/(9*a**3*d**3 - 21*a**2*b*c*d
**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) + x)/4 + 3*sqrt(-a/b**11)*(a*d - b*c)
**2*(3*a*d - b*c)*log(3*b**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)/(9
*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) + x)/4 + d*
*3*x**7/(7*b**2) - x**5*(2*a*d**3 - 3*b*c*d**2)/(5*b**3) + x**3*(a**2*d**3
- 2*a*b*c*d**2 + b**2*c**2*d)/b**4 - x*(4*a**3*d**3 - 9*a**2*b*c*d**2 + 6*a
*b**2*c**2*d - b**3*c**3)/b**5
```

Giac [A] time = 1.16649, size = 325, normalized size = 1.92

$$-\frac{3\left(ab^3c^3-5\,a^2b^2c^2d+7\,a^3bcd^2-3\,a^4d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}+\frac{ab^3c^3x-3\,a^2b^2c^2dx+3\,a^3bcd^2x-a^4d^3x}{2\left(bx^2+a\right)b^5}+\frac{5\,b^{12}d^3x^7+21}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c^3*x - 3*a^2*b^2*c^2*d*x + 3*a^3*b*
c*d^2*x - a^4*d^3*x)/((b*x^2 + a)*b^5) + 1/35*(5*b^12*d^3*x^7 + 21*b^12*c*d
^2*x^5 - 14*a*b^11*d^3*x^5 + 35*b^12*c^2*d*x^3 - 70*a*b^11*c*d^2*x^3 + 35*a
^2*b^10*d^3*x^3 + 35*b^12*c^3*x - 210*a*b^11*c^2*d*x + 315*a^2*b^10*c*d^2*x
 - 140*a^3*b^9*d^3*x)/b^14
```

3.281 $\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=117

$$\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{a(bc-ad)^3}{2b^5\left(a+bx^2\right)}+\frac{(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^6}{6b^2}$$

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^3) + (d^3*
x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) + ((b*c - 4*a*d)*(b*c
- a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.148403, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{a(bc-ad)^3}{2b^5\left(a+bx^2\right)}+\frac{(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^3) + (d^3*
x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) + ((b*c - 4*a*d)*(b*c
- a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^3}{(a+bx)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{3d(bc-ad)^2}{b^4}+\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{b^2}+\frac{a(-bc+ad)^3}{b^4(a+bx)^2}+\frac{(bc-4ad)(bc-ad)^2}{b^4(a+bx)}\right)d$$
$$=\frac{3d(bc-ad)^2x^2}{2b^4}+\frac{d^2(3bc-2ad)x^4}{4b^3}+\frac{d^3x^6}{6b^2}+\frac{a(bc-ad)^3}{2b^5\left(a+bx^2\right)}+\frac{(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}$$

Mathematica [A] time = 0.0922064, size = 106, normalized size = 0.91

$$\frac{3b^2d^2x^4(3bc-2ad)+18bdx^2(bc-ad)^2-\frac{6a(ad-bc)^3}{a+bx^2}+6(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)+2b^3d^3x^6}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (18*b*d*(b*c - a*d)^2*x^2 + 3*b^2*d^2*(3*b*c - 2*a*d)*x^4 + 2*b^3*d^3*x^6 -
 (6*a*(-(b*c) + a*d)^3)/(a + b*x^2) + 6*(b*c - 4*a*d)*(b*c - a*d)^2*Log[a +
 b*x^2])/(12*b^5)
```

Maple [B] time = 0.011, size = 229, normalized size = 2.

$$\frac{d^3x^6}{6\,b^2}-\frac{d^3x^4a}{2\,b^3}+\frac{3\,d^2x^4c}{4\,b^2}+\frac{3\,d^3x^2a^2}{2\,b^4}-3\,\frac{d^2x^2ac}{b^3}+\frac{3\,dx^2c^2}{2\,b^2}-2\,\frac{\ln\left(bx^2+a\right)a^3d^3}{b^5}+\frac{9\,\ln\left(bx^2+a\right)a^2d^2c}{2\,b^4}-3\,\frac{\ln\left(bx^2+a\right)a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/6*d^3*x^6/b^2-1/2*d^3/b^3*x^4*a+3/4*d^2/b^2*x^4*c+3/2*d^3/b^4*x^2*a^2-3*d
^2/b^3*x^2*a*c+3/2*d/b^2*x^2*c^2-2/b^5*ln(b*x^2+a)*a^3*d^3+9/2/b^4*ln(b*x^2
+a)*a^2*d^2*c-3/b^3*ln(b*x^2+a)*a*d*c^2+1/2/b^2*ln(b*x^2+a)*c^3-1/2/b^5*a^4
/(b*x^2+a)*d^3+3/2/b^4*a^3/(b*x^2+a)*d^2*c-3/2/b^3*a^2/(b*x^2+a)*d*c^2+1/2/
b^2*a/(b*x^2+a)*c^3
```

Maxima [A] time = 0.981821, size = 235, normalized size = 2.01

$$\frac{ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3}{2\left(b^6x^2+ab^5\right)}+\frac{2\,b^2d^3x^6+3\left(3\,b^2cd^2-2\,abd^3\right)x^4+18\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2}{12\,b^4}+\frac{\left(b^3c^3-6\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)/(b^6*x^2 + a*b^
5) + 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*x^2)/b^4 + 1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*
c*d^2 - 4*a^3*d^3)*log(b*x^2 + a)/b^5
```

Fricas [B] time = 1.42786, size = 518, normalized size = 4.43

$$\frac{2\,b^4d^3x^8+6\,ab^3c^3-18\,a^2b^2c^2d+18\,a^3bcd^2-6\,a^4d^3+\left(9\,b^4cd^2-4\,ab^3d^3\right)x^6+3\left(6\,b^4c^2d-9\,ab^3cd^2+4\,a^2b^2d^3\right)x^4+18\left(\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^4*d^3*x^8 + 6*a*b^3*c^3 - 18*a^2*b^2*c^2*d + 18*a^3*b*c*d^2 - 6*a
^4*d^3 + (9*b^4*c*d^2 - 4*a*b^3*d^3)*x^6 + 3*(6*b^4*c^2*d - 9*a*b^3*c*d^2 +
 4*a^2*b^2*d^3)*x^4 + 18*(a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + a^3*b*d^3)*x^2 +
6*(a*b^3*c^3 - 6*a^2*b^2*c^2*d + 9*a^3*b*c*d^2 - 4*a^4*d^3 + (b^4*c^3 - 6*a
*b^3*c^2*d + 9*a^2*b^2*c*d^2 - 4*a^3*b*d^3)*x^2)*log(b*x^2 + a))/(b^6*x^2 +
 a*b^5)
```

Sympy [A] time = 1.69272, size = 158, normalized size = 1.35

$$-\frac{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}{2ab^5+2b^6x^2}+\frac{d^3x^6}{6b^2}-\frac{x^4\left(2ad^3-3bcd^2\right)}{4b^3}+\frac{x^2\left(3a^2d^3-6abcd^2+3b^2c^2d\right)}{2b^4}-\frac{(ad-bc)^2\,(4ad-\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3)/(2*a*b**5
 + 2*b**6*x**2) + d**3*x**6/(6*b**2) - x**4*(2*a*d**3 - 3*b*c*d**2)/(4*b**3
) + x**2*(3*a**2*d**3 - 6*a*b*c*d**2 + 3*b**2*c**2*d)/(2*b**4) - (a*d - b*c
)**2*(4*a*d - b*c)*log(a + b*x**2)/(2*b**5)
```

Giac [B] time = 1.13976, size = 336, normalized size = 2.87

$$\frac{\frac{\left(2\,d^3+\frac{3\left(3\,b^2cd^2-4\,abd^3\right)}{\left(bx^2+a\right)b}+\frac{18\left(b^4c^2d-3\,ab^3cd^2+2\,a^2b^2d^3\right)}{\left(bx^2+a\right)^2b^2}\right)\left(bx^2+a\right)^3}{b^4}-\frac{6\left(b^3c^3-6\,ab^2c^2d+9\,a^2bcd^2-4\,a^3d^3\right)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^4}+\frac{6\left(\frac{ab^6c^3}{bx^2+a}-\frac{3\,a^2b^5c^2d}{bx^2+a}+\frac{3\,a^3b^4cd^2}{bx^2+a}-\frac{a^4b\cdots}{bx^2\cdots}\right.}{b^7}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*d^3 + 3*(3*b^2*c*d^2 - 4*a*b*d^3)/((b*x^2 + a)*b) + 18*(b^4*c^2*d
- 3*a*b^3*c*d^2 + 2*a^2*b^2*d^3)/((b*x^2 + a)^2*b^2))*(b*x^2 + a)^3/b^4 - 6
*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*log(abs(b*x^2 + a)/(
(b*x^2 + a)^2*abs(b)))/b^4 + 6*(a*b^6*c^3/(b*x^2 + a) - 3*a^2*b^5*c^2*d/(b*
x^2 + a) + 3*a^3*b^4*c*d^2/(b*x^2 + a) - a^4*b^3*d^3/(b*x^2 + a))/b^7)/b
```

3.282 $\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{dx(105a^2d^2-190abcd+81b^2c^2)}{30b^4}+\frac{dx(c+dx^2)(33bc-35ad)}{30b^3}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}+\frac{7dx}{}$$

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*b*c - 35*a
*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10*b^2) - (x*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rubi [A] time = 0.190167, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {467, 528, 388, 205}

$$\frac{dx(105a^2d^2-190abcd+81b^2c^2)}{30b^4}+\frac{dx(c+dx^2)(33bc-35ad)}{30b^3}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}+\frac{7dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*b*c - 35*a
*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10*b^2) - (x*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = -\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\int \frac{\left(c+dx^2\right)^2\left(c+7dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= \frac{7dx\left(c+dx^2\right)^2}{10b^2} - \frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\int \frac{\left(c+dx^2\right)\left(c(5bc-7ad)+d(33bc-35ad)x^2\right)}{a+bx^2}\,dx}{10b^2}$$

$$= \frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3} + \frac{7dx\left(c+dx^2\right)^2}{10b^2} - \frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\int \frac{c\left(15b^2c^2-54abcd+35a^2d^2\right)+d\left(81b^2c^2-1\right.}{a+bx^2}}{30b^3}$$

$$= \frac{d\left(81b^2c^2-190abcd+105a^2d^2\right)x}{30b^4} + \frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3} + \frac{7dx\left(c+dx^2\right)^2}{10b^2} - \frac{x\left(c+dx^2\right)}{2b\left(a+bx^2\right.}$$

$$= \frac{d\left(81b^2c^2-190abcd+105a^2d^2\right)x}{30b^4} + \frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3} + \frac{7dx\left(c+dx^2\right)^2}{10b^2} - \frac{x\left(c+dx^2\right)}{2b\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0669277, size = 125, normalized size = 0.85

$$\frac{d^2x^3(3bc-2ad)}{3b^3} - \frac{x(bc-ad)^3}{2b^4\left(a+bx^2\right)} + \frac{3dx(bc-ad)^2}{b^4} + \frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}} + \frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x)/b^4 + (d^2*(3*b*c - 2*a*d)*x^3)/(3*b^3) + (d^3*x^5)/(
5*b^2) - ((b*c - a*d)^3*x)/(2*b^4*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)
^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Maple [A] time = 0.01, size = 247, normalized size = 1.7

$$\frac{d^3x^5}{5\,b^2} - \frac{2\,d^3x^3a}{3\,b^3} + \frac{d^2x^3c}{b^2} + 3\,\frac{a^2d^3x}{b^4} - 6\,\frac{acd^2x}{b^3} + 3\,\frac{c^2dx}{b^2} + \frac{xa^3d^3}{2\,b^4\left(bx^2+a\right)} - \frac{3\,a^2cd^2x}{2\,b^3\left(bx^2+a\right)} + \frac{3\,axc^2d}{2\,b^2\left(bx^2+a\right)} - \frac{xc^3}{2\,b\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^3/b^2*x^5-2/3*d^3/b^3*x^3*a+d^2/b^2*x^3*c+3*d^3/b^4*a^2*x-6*d^2/b^3*c
*a*x+3*d/b^2*c^2*x+1/2/b^4*x/(b*x^2+a)*a^3*d^3-3/2/b^3*x/(b*x^2+a)*a^2*c*d^
2+3/2/b^2*x/(b*x^2+a)*a*c^2*d-1/2/b*x/(b*x^2+a)*c^3-7/2/b^4/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*a^3*d^3+15/2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a
```

```
^2*c*d^2-9/2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*c^2*d+1/2/b/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59148, size = 1061, normalized size = 7.22

$$\left[\frac{12\,ab^4d^3x^7+4\left(15\,ab^4cd^2-7\,a^2b^3d^3\right)x^5+20\left(9\,ab^4c^2d-15\,a^2b^3cd^2+7\,a^3b^2d^3\right)x^3+15\left(ab^3c^3-9\,a^2b^2c^2d+15\,a^3bcd^2-\right.}{60\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*d^3*x^7 + 4*(15*a*b^4*c*d^2 - 7*a^2*b^3*d^3)*x^5 + 20*(9*a*
b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x^3 + 15*(a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b
^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) - 30*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*b*d
^3)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*d^3*x^7 + 2*(15*a*b^4*c*d^2 - 7
*a^2*b^3*d^3)*x^5 + 10*(9*a*b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x
^3 + 15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^
3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) - 15*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*
b*d^3)*x)/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [B] time = 1.44394, size = 337, normalized size = 2.29

$$\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\left(ad-bc\right)^2\left(7ad-bc\right)\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^2(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^9}}\left(ad-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a*b**4 + 2
*b**5*x**2) + sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)*log(-a*b**4*sq
rt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7*a**3*d**3 - 15*a**2*b*c*d**
2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 - sqrt(-1/(a*b**9))*(a*d - b*c)**2*
(7*a*d - b*c)*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7*
```

```
a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 + d**3*x
**5/(5*b**2) - x**3*(2*a*d**3 - 3*b*c*d**2)/(3*b**3) + x*(3*a**2*d**3 - 6*a
*b*c*d**2 + 3*b**2*c**2*d)/b**4
```

Giac [A] time = 1.12739, size = 248, normalized size = 1.69

$$\frac{\left(b^3c^3 - 9\,ab^2c^2d + 15\,a^2bcd^2 - 7\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} - \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\left(bx^2 + a\right)b^4} + \frac{3\,b^8d^3x^5 + 15\,b^8cd^2x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^4) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
- a^3*d^3*x)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*d^3*x^5 + 15*b^8*c*d^2*x^3 - 1
0*a*b^7*d^3*x^3 + 45*b^8*c^2*d*x - 90*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^1
0
```

3.283 $\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=88

$$\frac{d^2x^2(3bc-2ad)}{2b^3}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^3x^4}{4b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*d)^3/(2*b^
4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.0933621, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{d^2x^2(3bc-2ad)}{2b^3}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*d)^3/(2*b^
4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{d^2(3bc-2ad)x^2}{2b^3}+\frac{d^3x^4}{4b^2}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0440927, size = 127, normalized size = 1.44

$$\frac{-3a^2bcd^2+a^3d^3+3ab^2c^2d-b^3c^3}{2b^4(a+bx^2)}+\frac{3(a^2d^3-2abcd^2+b^2c^2d)\log(a+bx^2)}{2b^4}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) + (-(b^3*c^3) + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)/(2*b^4*(a + b*x^2)) + (3*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*Log[a + b*x^2])/(2*b^4)
```

Maple [B] time = 0.01, size = 168, normalized size = 1.9

$$\frac{d^3x^4}{4\,b^2}-\frac{ad^3x^2}{b^3}+\frac{3\,d^2x^2c}{2\,b^2}+\frac{3\,\ln\left(bx^2+a\right)d^3a^2}{2\,b^4}-3\,\frac{\ln\left(bx^2+a\right)d^2ca}{b^3}+\frac{3\,\ln\left(bx^2+a\right)dc^2}{2\,b^2}+\frac{a^3d^3}{2\,b^4\left(bx^2+a\right)}-\frac{3\,a^2c\cdots}{2\,b^3\left(bx^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-d^3/b^3*a*x^2+3/2*d^2/b^2*x^2*c+3/2/b^4*ln(b*x^2+a)*d^3*a^2
-3/b^3*ln(b*x^2+a)*d^2*c*a+3/2/b^2*ln(b*x^2+a)*d*c^2+1/2/b^4/(b*x^2+a)*a^3*
d^3-3/2/b^3/(b*x^2+a)*a^2*d^2*c+3/2/b^2/(b*x^2+a)*a*d*c^2-1/2/b/(b*x^2+a)*c
^3
```

Maxima [A] time = 1.0022, size = 167, normalized size = 1.9

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\left(b^5x^2+ab^4\right)}+\frac{bd^3x^4+2\left(3\,bcd^2-2\,ad^3\right)x^2}{4\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x^2 + a*b^4)
+ 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - 2*a*d^3)*x^2)/b^3 + 3/2*(b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*log(b*x^2 + a)/b^4
```

Fricas [B] time = 1.4785, size = 365, normalized size = 4.15

$$\frac{b^3d^3x^6-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^4+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x^2+6\left(ab^2c^2d-2\,a^2bc\cdots\right.}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^3*x^6 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^4 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x^2 + 6*(
a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*
d^3)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 1.37117, size = 112, normalized size = 1.27

$$\frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{2ab^4 + 2b^5x^2} + \frac{d^3x^4}{4b^2} - \frac{x^2\left(2ad^3 - 3bcd^2\right)}{2b^3} + \frac{3d\left(ad - bc\right)^2\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a*b**4 + 2*b
**5*x**2) + d**3*x**4/(4*b**2) - x**2*(2*a*d**3 - 3*b*c*d**2)/(2*b**3) + 3*
d*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4)
```

Giac [B] time = 1.15526, size = 247, normalized size = 2.81

$$\frac{\left(d^3 + \frac{6\left(b^2cd^2 - abd^3\right)}{\left(bx^2+a\right)b}\right)\left(bx^2 + a\right)^2}{4\,b^4} - \frac{3\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{2\,b^4} - \frac{\frac{b^5c^3}{bx^2+a} - \frac{3\,ab^4c^2d}{bx^2+a} + \frac{3\,a^2b^3cd^2}{bx^2+a} - \frac{a^3b^2d^3}{bx^2+a}}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x^2 + a)*b))*(b*x^2 + a)^2/b^4 - 3/2
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(
b)))/b^4 - 1/2*(b^5*c^3/(b*x^2 + a) - 3*a*b^4*c^2*d/(b*x^2 + a) + 3*a^2*b^3
*c*d^2/(b*x^2 + a) - a^3*b^2*d^3/(b*x^2 + a))/b^6
```

3.284 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.0910083, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\left(a+bx^2\right)^2}\right)dx$$
$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{\left(a+bx^2\right)^2}\,dx}{b^3}$$
$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}$$
$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

Mathematica [A] time = 0.0597556, size = 106, normalized size = 1.

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Maple [B] time = 0.007, size = 205, normalized size = 1.9

$$\frac{d^3x^3}{3\,b^2}-2\,\frac{ad^3x}{b^3}+3\,\frac{d^2xc}{b^2}-\frac{xa^2d^3}{2\,b^3\left(bx^2+a\right)}+\frac{3\,axcd^2}{2\,b^2\left(bx^2+a\right)}-\frac{3\,xc^2d}{2\,b\left(bx^2+a\right)}+\frac{xc^3}{2\,a\left(bx^2+a\right)}+\frac{5\,a^2d^3}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^3*x^3/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/2/b^3*x*a^2/(b*x^2+a)*d^3+3/2
/b^2*x*a/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2*x/a/(b*x^2+a)*c^3+5/2/
b^3*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3-9/2/b^2*a/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*c*d^2+3/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d+1/
2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.54082, size = 896, normalized size = 8.45

$$\left[\frac{4\,a^2b^3d^3x^5+4\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3-3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\right.\right.}{12\left(a^2b^5x^2+a^3b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] time = 1.24935, size = 313, normalized size = 2.95

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\,(ad-bc)^2\,(5ad+bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\,(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a**2*b**3
 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(-
a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 -
9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7)
)*(a*d - b*c)**2*(5*a*d + b*c)*log(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*
c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3
*c**3) + x)/4 + d**3*x**3/(3*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

Giac [A] time = 1.15621, size = 205, normalized size = 1.93

$$\frac{\left(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\left(bx^2+a\right)ab^3}+\frac{b^4d^3x^3+9\,b^4cd^2x-6\ldots}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

3.285 $\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Log[x])/a^2
- ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b^3)
```

Rubi [A] time = 0.0844912, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Log[x])/a^2
- ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^3}{b^2}+\frac{c^3}{a^2x}+\frac{(-bc+ad)^3}{ab^2(a+bx)^2}-\frac{(-bc+ad)^2(bc+2ad)}{a^2b^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{d^3x^2}{2b^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{c^3\log(x)}{a^2}-\frac{(bc-ad)^2(bc+2ad)\log\left(a+bx^2\right)}{2a^2b^3}\end{aligned}$$

Mathematica [A] time = 0.0990077, size = 111, normalized size = 1.26

$$\frac{\frac{\frac{a(a^2bd^2(3c+dx^2)-a^3d^3+ab^2(d^3x^4-3c^2d)+b^3c^3)}{a+bx^2}-(bc-ad)^2(2ad+bc)\log(a+bx^2)}{b^3}+2c^3\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^3*Log[x] + ((a*(b^3*c^3 - a^3*d^3 + a^2*b*d^2*(3*c + d*x^2) + a*b^2*(-
3*c^2*d + d^3*x^4)))/(a + b*x^2) - (b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^
2])/b^3)/(2*a^2)
```

Maple [A] time = 0.017, size = 146, normalized size = 1.7

$$\frac{d^3x^2}{2b^2}+\frac{c^3\ln(x)}{a^2}-\frac{a\ln(bx^2+a)d^3}{b^3}+\frac{3\ln(bx^2+a)d^2c}{2b^2}-\frac{\ln(bx^2+a)c^3}{2a^2}-\frac{a^2d^3}{2b^3(bx^2+a)}+\frac{3ad^2c}{2b^2(bx^2+a)}-\frac{3dc^2}{2b(bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2+c^3*ln(x)/a^2-1/b^3*a*ln(b*x^2+a)*d^3+3/2/b^2*ln(b*x^2+a)*d
^2*c-1/2/a^2*ln(b*x^2+a)*c^3-1/2/b^3*a^2/(b*x^2+a)*d^3+3/2/b^2*a/(b*x^2+a)*
d^2*c-3/2/b/(b*x^2+a)*d*c^2+1/2/a/(b*x^2+a)*c^3
```

Maxima [A] time = 0.993607, size = 165, normalized size = 1.88

$$\frac{d^3x^2}{2b^2}+\frac{c^3\log(x^2)}{2a^2}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{2(ab^4x^2+a^2b^3)}-\frac{(b^3c^3-3a^2bcd^2+2a^3d^3)\log(bx^2+a)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)/(a*b^4*x^2 + a^2*b^3) - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*log(b*x^2 + a)/(a^2*b^3)
```

Fricas [B] time = 1.57065, size = 352, normalized size = 4.

$$\frac{a^2b^2d^3x^4+a^3bd^3x^2+ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3-(ab^3c^3-3a^3bcd^2+2a^4d^3+(b^4c^3-3a^2b^2cd^2+2a^3bd^3)x^2)}{2(a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2*d^3*x^4 + a^3*b*d^3*x^2 + a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*
b*c*d^2 - a^4*d^3 - (a*b^3*c^3 - 3*a^3*b*c*d^2 + 2*a^4*d^3 + (b^4*c^3 - 3*a
```

```
^2*b^2*c*d^2 + 2*a^3*b*d^3)*x^2)*log(b*x^2 + a) + 2*(b^4*c^3*x^2 + a*b^3*c^
3)*log(x))/(a^2*b^4*x^2 + a^3*b^3)
```

Sympy [A] time = 2.8078, size = 110, normalized size = 1.25

$$-\frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{2a^2b^3 + 2ab^4x^2} + \frac{d^3x^2}{2b^2} + \frac{c^3\log(x)}{a^2} - \frac{(ad - bc)^2\,(2ad + bc)\log\left(\frac{a}{b} + x^2\right)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

```
[Out] -(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a**2*b**3 +
 2*a*b**4*x**2) + d**3*x**2/(2*b**2) + c**3*log(x)/a**2 - (a*d - b*c)**2*(2
*a*d + b*c)*log(a/b + x**2)/(2*a**2*b**3)
```

Giac [A] time = 1.13824, size = 203, normalized size = 2.31

$$\frac{d^3x^2}{2\,b^2} + \frac{c^3\log\left(x^2\right)}{2\,a^2} - \frac{\left(b^3c^3 - 3\,a^2bcd^2 + 2\,a^3d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^2b^3} + \frac{b^4c^3x^2 - 3\,a^2b^2cd^2x^2 + 2\,a^3bd^3x^2 + 2\,ab^3c^3 - 3\,a^2b^2c}{2\,\left(bx^2 + a\right)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*a
^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^3) + 1/2*(b^4*c^3*x^2 - 3*a^2*b^2*c*d^2*
x^2 + 2*a^3*b*d^3*x^2 + 2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3)/((b*x^2 +
a)*a^2*b^3)
```

3.286 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx(a+bx^2)}$$

```
[Out] -(c^2*(3*b*c - a*d))/(2*a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2) + ((b*c
- a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rubi [A] time = 0.133165, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {468, 570, 205}

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(3*b*c - a*d))/(2*a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2) + ((b*c
- a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)} - \frac{\int \frac{\left(c+dx^2\right)\left(-c(3bc-ad)+d(bc-3ad)x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)} - \frac{\int \left(\frac{d^2(bc-3ad)}{b} + \frac{c^2(-3bc+ad)}{ax^2} + \frac{3(-bc+ad)^2(bc+ad)}{ab\left(a+bx^2\right)}\right)dx}{2ab}$$

$$= -\frac{c^2(3bc-ad)}{2a^2bx} - \frac{d^2(bc-3ad)x}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)} - \frac{\left(3(bc-ad)^2(bc+ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}$$

$$= -\frac{c^2(3bc-ad)}{2a^2bx} - \frac{d^2(bc-3ad)x}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)} - \frac{3(bc-ad)^2(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

Mathematica [A] time = 0.0595166, size = 94, normalized size = 0.72

$$\frac{x(ad-bc)^3}{2a^2b^2\left(a+bx^2\right)} - \frac{3(ad-bc)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}} - \frac{c^3}{a^2x} + \frac{d^3x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^3/(a^2*x)) + (d^3*x)/b^2 + ((-(b*c) + a*d)^3*x)/(2*a^2*b^2*(a + b*x^2))
 - (3*(-(b*c) + a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*
b^(5/2))
```

Maple [A] time = 0.013, size = 189, normalized size = 1.4

$$\frac{d^3x}{b^2} - \frac{c^3}{a^2x} + \frac{axd^3}{2\,b^2\left(bx^2+a\right)} - \frac{3\,cxd^2}{2\,b\left(bx^2+a\right)} + \frac{3\,xc^2d}{2\,a\left(bx^2+a\right)} - \frac{bxc^3}{2\,a^2\left(bx^2+a\right)} - \frac{3\,ad^3}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{3\,cd^2}{2\,b}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^2/(b*x^2+a)^2,x)
```

```
[Out] d^3/b^2*x-c^3/a^2/x+1/2*a/b^2*x/(b*x^2+a)*d^3-3/2/b*x/(b*x^2+a)*c*d^2+3/2/a
*x/(b*x^2+a)*c^2*d-1/2/a^2*b*x/(b*x^2+a)*c^3-3/2*a/b^2/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2))*d^3+3/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2+3/2/a/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d-3/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67409, size = 798, normalized size = 6.09

$$\left[\frac{4\,a^{3}b^{2}d^{3}x^{4}-4\,a^{2}b^{3}c^{3}-6\left(ab^{4}c^{3}-a^{2}b^{3}c^{2}d+a^{3}b^{2}cd^{2}-a^{4}bd^{3}\right)x^{2}-3\left(\left(b^{4}c^{3}-ab^{3}c^{2}d-a^{2}b^{2}cd^{2}+a^{3}bd^{3}\right)x^{3}+\left(ab^{3}c^{3}-a^{2}\right.\right.}{4\left(a^{3}b^{4}x^{3}+a^{4}b^{3}x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*d^3*x^4 - 4*a^2*b^3*c^3 - 6*(a*b^4*c^3 - a^2*b^3*c^2*d + a^
3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 +
a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*sqr
t(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b
^3*x), 1/2*(2*a^3*b^2*d^3*x^4 - 2*a^2*b^3*c^3 - 3*(a*b^4*c^3 - a^2*b^3*c^2*
d + a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*
d^2 + a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*
x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

Sympy [B] time = 1.84766, size = 309, normalized size = 2.36

$$\frac{3\sqrt{-\frac{1}{a^{5}b^{5}}}\,(ad-bc)^{2}\,(ad+bc)\log\left(-\frac{3a^{3}b^{2}\sqrt{-\frac{1}{a^{5}b^{5}}}(ad-bc)^{2}(ad+bc)}{3a^{3}d^{3}-3a^{2}bcd^{2}-3ab^{2}c^{2}d+3b^{3}c^{3}}+x\right)}{4}-\frac{3\sqrt{-\frac{1}{a^{5}b^{5}}}\,(ad-bc)^{2}\,(ad+bc)\log\left(\frac{3a^{3}b^{2}\sqrt{-\frac{1}{a^{5}b^{5}}}(ad-bc)^{2}(a}{3a^{3}d^{3}-3a^{2}bcd^{2}-3ab^{2}c^{2}d\cdot}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(-3*a**3*b**2*sqrt(-1/
(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*
a*b**2*c**2*d + 3*b**3*c**3) + x)/4 - 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2
*(a*d + b*c)*log(3*a**3*b**2*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c
)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 +
(-2*a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 3*b
**3*c**3))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + d**3*x/b**2
```

Giac [A] time = 1.12446, size = 193, normalized size = 1.47

$$\frac{d^{3}x}{b^{2}}-\frac{3\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^{2}b^{2}}-\frac{3\,b^{3}c^{3}x^{2}-3\,ab^{2}c^{2}dx^{2}+3\,a^{2}bcd^{2}x^{2}-a^{3}d^{3}x^{2}+2\,ab^{2}c^{3}}{2\left(bx^{3}+ax\right)a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^3*x/b^2 - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3
*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x^3 + a*x)*a^2*b^2)
```

3.287 $\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}+\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{c^3}{2a^2x^2}$$

```
[Out] -c^3/(2*a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(2*b*c - 3*
a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/(2*a^3*b^2)
```

Rubi [A] time = 0.105636, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}+\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{c^3}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^3/(2*a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(2*b*c - 3*
a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/(2*a^3*b^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c^3}{a^2x^2}+\frac{c^2(-2bc+3ad)}{a^3x}-\frac{(-bc+ad)^3}{a^2b(a+bx)^2}+\frac{(-bc+ad)^2(2bc+ad)}{a^3b(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{c^3}{2a^2x^2}-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^2(2bc-3ad)\log(x)}{a^3}+\frac{(bc-ad)^2(2bc+ad)\log\left(a+bx^2\right)}{2a^3b^2}
\end{aligned}$$

Mathematica [A] time = 0.0944897, size = 87, normalized size = 0.89

$$\frac{\frac{a(ad-bc)^3}{b^2\left(a+bx^2\right)}+\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{b^2}+2c^2\log(x)(3ad-2bc)-\frac{ac^3}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c^3)/x^2) + (a*(-(b*c) + a*d)^3)/(b^2*(a + b*x^2)) + 2*c^2*(-2*b*c +
3*a*d)*Log[x] + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/b^2)/(2*a^3)
```

Maple [A] time = 0.014, size = 156, normalized size = 1.6

$$-\frac{c^3}{2a^2x^2}+3\frac{c^2\ln(x)d}{a^2}-2\frac{c^3\ln(x)b}{a^3}+\frac{\ln\left(bx^2+a\right)d^3}{2b^2}-\frac{3\ln\left(bx^2+a\right)dc^2}{2a^2}+\frac{b\ln\left(bx^2+a\right)c^3}{a^3}+\frac{ad^3}{2b^2\left(bx^2+a\right)}-\frac{3}{2b\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c^3/a^2/x^2+3*c^2/a^2*ln(x)*d-2*c^3/a^3*ln(x)*b+1/2/b^2*ln(b*x^2+a)*d^
3-3/2/a^2*ln(b*x^2+a)*d*c^2+1/a^3*b*ln(b*x^2+a)*c^3+1/2*a/b^2/(b*x^2+a)*d^3
-3/2/b/(b*x^2+a)*d^2*c+3/2/a/(b*x^2+a)*d*c^2-1/2/a^2*b/(b*x^2+a)*c^3
```

Maxima [A] time = 1.01254, size = 190, normalized size = 1.94

$$-\frac{ab^2c^3+\left(2b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)x^2}{2\left(a^2b^3x^4+a^3b^2x^2\right)}-\frac{\left(2bc^3-3ac^2d\right)\log\left(x^2\right)}{2a^3}+\frac{\left(2b^3c^3-3ab^2c^2d+a^3d^3\right)\log\left(bx^2+a\right)}{2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^2*c^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2
)/(a^2*b^3*x^4 + a^3*b^2*x^2) - 1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/
2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^3*d^3)*log(b*x^2 + a)/(a^3*b^2)
```

Fricas [B] time = 1.57421, size = 413, normalized size = 4.21

$$-\frac{a^2b^2c^3+\left(2ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)x^2-\left(\left(2b^4c^3-3ab^3c^2d+a^3bd^3\right)x^4+\left(2ab^3c^3-3a^2b^2c^2d+a^4d^3\right)x^2\right)\mathrm{l}}{2\left(a^3b^3x^4+a^4b^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - 3*a^
2*b^2*c^2*d + a^4*d^3)*x^2)*log(b*x^2 + a) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d)
*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)*log(x))/(a^3*b^3*x^4 + a^4*b^2*
x^2)
```

Sympy [A] time = 3.80767, size = 128, normalized size = 1.31

$$\frac{-ab^2c^3 + x^2\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - 2b^3c^3\right)}{2a^3b^2x^2 + 2a^2b^3x^4} + \frac{c^2(3ad - 2bc)\log(x)}{a^3} + \frac{(ad - bc)^2(ad + 2bc)\log\left(\frac{a}{b} + x^2\right)}{2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 2*b**
3*c**3))/(2*a**3*b**2*x**2 + 2*a**2*b**3*x**4) + c**2*(3*a*d - 2*b*c)*log(x
)/a**3 + (a*d - b*c)**2*(a*d + 2*b*c)*log(a/b + x**2)/(2*a**3*b**2)
```

Giac [A] time = 1.13921, size = 212, normalized size = 2.16

$$-\frac{\left(2\,bc^3 - 3\,ac^2d\right)\log\left(x^2\right)}{2\,a^3} + \frac{\left(2\,b^3c^3 - 3\,ab^2c^2d + a^3d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^3b^2} - \frac{a^2bd^3x^4 + 4\,b^3c^3x^2 - 6\,ab^2c^2dx^2 + 6\,a^2bcd^2x^2}{4\left(bx^4 + ax^2\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d +
a^3*d^3)*log(abs(b*x^2 + a))/(a^3*b^2) - 1/4*(a^2*b*d^3*x^4 + 4*b^3*c^3*x^2
 - 6*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x
^4 + a*x^2)*a^2*b^2)
```

3.288 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

```
[Out] -(c^2*(5*b*c - 3*a*d))/(6*a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^
2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^3*(a + b*x^2)) + ((b
*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.140974, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {468, 570, 205}

$$\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(5*b*c - 3*a*d))/(6*a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^
2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^3*(a + b*x^2)) + ((b
*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} - \frac{\int \frac{(c+dx^2)(-c(5bc-3ad)-d(bc+ad)x^2)}{x^4(a+bx^2)}\,dx}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} - \frac{\int \left(\frac{c^2(-5bc+3ad)}{ax^4} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{a^2x^2} - \frac{(-bc+ad)^2(5bc+ad)}{a^2(a+bx^2)}\right) dx}{2ab}$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx} + \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} + \frac{((bc-ad)^2(5bc+ad)) \int}{2a^3b}$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx} + \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} + \frac{(bc-ad)^2(5bc+ad)\tan}{2a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.0622429, size = 109, normalized size = 0.74

$$\frac{(ad-bc)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}} - \frac{c^2(3ad-2bc)}{a^3x} - \frac{x(ad-bc)^3}{2a^3b(a+bx^2)} - \frac{c^3}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c^3/(3*a^2*x^3) - (c^2*(-2*b*c + 3*a*d))/(a^3*x) - ((-(b*c) + a*d)^3*x)/(2
*a^3*b*(a + b*x^2)) + ((-(b*c) + a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A] time = 0.011, size = 209, normalized size = 1.4

$$-\frac{c^3}{3a^2x^3} - 3\frac{c^2d}{a^2x} + 2\frac{c^3b}{a^3x} - \frac{xd^3}{2b(bx^2+a)} + \frac{3cxd^2}{2a(bx^2+a)} - \frac{3bxc^2d}{2a^2(bx^2+a)} + \frac{b^2c^3x}{2a^3(bx^2+a)} + \frac{d^3}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c^3/a^2/x^3-3*c^2/a^2/x*d+2*c^3/a^3/x*b-1/2/b*x/(b*x^2+a)*d^3+3/2/a*x/
(b*x^2+a)*c*d^2-3/2/a^2*b*x/(b*x^2+a)*c^2*d+1/2/a^3*b^2*x/(b*x^2+a)*c^3+1/2
/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3+3/2/a/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*c*d^2-9/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d+5/2/a^3
*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68138, size = 921, normalized size = 6.27

$$\left[-\frac{4\,a^3b^2c^3-6\left(5\,ab^4c^3-9\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3\right)x^4-4\left(5\,a^2b^3c^3-9\,a^3b^2c^2d\right)x^2+3\left(\left(5\,b^4c^3-9\,ab^3c^2d+3\,a^2b^2cd\right.\right.}{12\left(a^4b^3x^5+a^5b^2x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c^3 - 6*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
- a^4*b*d^3)*x^4 - 4*(5*a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 + 3*((5*b^4*c^3
- 9*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3)*x^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3), -1/6*(2*a^3*b^2*c^3 -
3*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x^4 - 2*(5*
a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 - 3*((5*b^4*c^3 - 9*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 + a^4*d^3)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^
3)]
```

Sympy [B] time = 2.2596, size = 321, normalized size = 2.18

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad\cdots}{a^3d^3+3a^2bcd^2-9ab^2c^2d\cdots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-a**4*b*sqrt(-1/(a**
7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b*
*2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d
+ 5*b*c)*log(a**4*b*sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3
*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 - (2*a**2*b
*c**3 + x**4*(3*a**3*d**3 - 9*a**2*b*c*d**2 + 27*a*b**2*c**2*d - 15*b**3*c*
*3) + x**2*(18*a**2*b*c**2*d - 10*a*b**2*c**3))/(6*a**4*b*x**3 + 6*a**3*b**
2*x**5)
```

Giac [A] time = 1.15098, size = 203, normalized size = 1.38

$$\frac{\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\left(bx^2+a\right)a^3b}+\frac{6\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^
3)/(a^3*x^3)
```

3.289 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rubi [A] time = 0.0854561, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {470, 522, 205}

$$-\frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{ac+(-2bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2b(bc-ad)}$$

$$= \frac{ax}{2b(bc-ad)\left(a+bx^2\right)} + \frac{c^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} - \frac{(a(3bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b(bc-ad)^2}$$

$$= \frac{ax}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}$$

Mathematica [A] time = 0.148255, size = 95, normalized size = 0.87

$$\frac{\frac{\sqrt{a}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + \frac{ax(bc-ad)}{b\left(a+bx^2\right)} + \frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] ((a*(b*c - a*d)*x)/(b*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(3/2) + (2*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(
2*(b*c - a*d)^2)
```

Maple [A] time = 0.009, size = 144, normalized size = 1.3

$$\frac{c^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{a^2dx}{2\,(ad-bc)^2\,b\left(bx^2+a\right)} + \frac{axc}{2\,(ad-bc)^2\left(bx^2+a\right)} + \frac{a^2d}{2\,(ad-bc)^2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*a^2/(a*d-b*c)^2/b*x
/(b*x^2+a)*d+1/2*a/(a*d-b*c)^2*x*c/(b*x^2+a)+1/2*a^2/(a*d-b*c)^2/b/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*d-3/2*a/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40994, size = 1476, normalized size = 13.54

$$\left[-\frac{\left(3\,abc - a^2d + \left(3\,b^2c - abd\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 2\left(b^2cx^2 + abc\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right) - 2\left(abc - a^2d\right)x}{4\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2 + \left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)x^2\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*
x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(a*b*c - a^2*d)*x)/(a*b^3*c^2
- 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), -
1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/
b)/a) - (b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (
b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*(4*(b^2*c*x^2 + a*b*c)*sqrt(
c/d)*arctan(d*x*sqrt(c/d)/c) - (3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*(a*b*c - a^2*d
)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*x^2), -1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*ar
ctan(b*x*sqrt(a/b)/a) - 2*(b^2*c*x^2 + a*b*c)*sqrt(c/d)*arctan(d*x*sqrt(c/d
)/c) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

Sympy [B] time = 14.4067, size = 1850, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a*x/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - sqrt(-a/b**
3)*(a*d - 3*b*c)*log(x + (-a**5*b**3*d**6*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3
/(2*(a*d - b*c)**6) + 9*a**4*b**4*c*d**5*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/
(2*(a*d - b*c)**6) - a**4*d**4*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**
2) - 13*a**3*b**5*c**2*d**4*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**
6 + 9*a**3*b*c*d**3*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 17*a**
2*b**6*c**3*d**3*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 27*a**2
*b**2*c**2*d**2*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 21*a*b**7*
c**4*d**2*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 27*a*b**3*
c**3*d*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 5*b**8*c**5*d*(-a/b
**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 4*b**4*c**4*sqrt(-a/b**3)
*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3))
/(4*(a*d - b*c)**2) + sqrt(-a/b**3)*(a*d - 3*b*c)*log(x + (a**5*b**3*d**6*(
-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 9*a**4*b**4*c*d**5*(-
a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + a**4*d**4*sqrt(-a/b**3
)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 13*a**3*b**5*c**2*d**4*(-a/b**3)**(3/2
)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 9*a**3*b*c*d**3*sqrt(-a/b**3)*(a*d - 3*
b*c)/(2*(a*d - b*c)**2) - 17*a**2*b**6*c**3*d**3*(-a/b**3)**(3/2)*(a*d - 3*
b*c)**3/(a*d - b*c)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-a/b**3)*(a*d - 3*b*c)
/(2*(a*d - b*c)**2) + 21*a*b**7*c**4*d**2*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3
/(2*(a*d - b*c)**6) - 27*a*b**3*c**3*d*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d
```

```
- b*c)**2) - 5*b**8*c**5*d*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)
**6) - 4*b**4*c**4*sqrt(-a/b**3)*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*c*d**2
 - 7*a*b*c**2*d + 12*b**2*c**3))/(4*(a*d - b*c)**2) - sqrt(-c**3/d)*log(x +
 (-4*a**5*b**3*d**6*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 36*a**4*b**4*c*d**5*(
-c**3/d)**(3/2)/(a*d - b*c)**6 - a**4*d**4*sqrt(-c**3/d)/(a*d - b*c)**2 - 1
04*a**3*b**5*c**2*d**4*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 9*a**3*b*c*d**3*sq
rt(-c**3/d)/(a*d - b*c)**2 + 136*a**2*b**6*c**3*d**3*(-c**3/d)**(3/2)/(a*d
- b*c)**6 - 27*a**2*b**2*c**2*d**2*sqrt(-c**3/d)/(a*d - b*c)**2 - 84*a*b**7
*c**4*d**2*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 27*a*b**3*c**3*d*sqrt(-c**3/d)
/(a*d - b*c)**2 + 20*b**8*c**5*d*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 8*b**4*c
**4*sqrt(-c**3/d)/(a*d - b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**
3))/(2*(a*d - b*c)**2) + sqrt(-c**3/d)*log(x + (4*a**5*b**3*d**6*(-c**3/d)*
*(3/2)/(a*d - b*c)**6 - 36*a**4*b**4*c*d**5*(-c**3/d)**(3/2)/(a*d - b*c)**6
 + a**4*d**4*sqrt(-c**3/d)/(a*d - b*c)**2 + 104*a**3*b**5*c**2*d**4*(-c**3/
d)**(3/2)/(a*d - b*c)**6 - 9*a**3*b*c*d**3*sqrt(-c**3/d)/(a*d - b*c)**2 - 1
36*a**2*b**6*c**3*d**3*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**2*c**2*
d**2*sqrt(-c**3/d)/(a*d - b*c)**2 + 84*a*b**7*c**4*d**2*(-c**3/d)**(3/2)/(a
*d - b*c)**6 - 27*a*b**3*c**3*d*sqrt(-c**3/d)/(a*d - b*c)**2 - 20*b**8*c**5
*d*(-c**3/d)**(3/2)/(a*d - b*c)**6 - 8*b**4*c**4*sqrt(-c**3/d)/(a*d - b*c)*
*2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3))/(2*(a*d - b*c)**2)
```

Giac [A] time = 1.17898, size = 165, normalized size = 1.51

$$\frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{cd}} - \frac{\left(3\,abc - a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\sqrt{ab}} + \frac{ax}{2\left(b^2c - abd\right)\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) - 1/2
*(3*a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^
2)*sqrt(a*b)) + 1/2*a*x/((b^2*c - a*b*d)*(b*x^2 + a))
```

3.290 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.0653426, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right)dx,x,x^2\right)\\
&= \frac{a}{2b(bc-ad)\left(a+bx^2\right)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.0331597, size = 74, normalized size = 1.

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.01, size = 95, normalized size = 1.3

$$-\frac{c\ln\left(dx^2+c\right)}{2\,(ad-bc)^2}+\frac{c\ln\left(bx^2+a\right)}{2\,(ad-bc)^2}-\frac{a^2d}{2\,(ad-bc)^2\,b\left(bx^2+a\right)}+\frac{ac}{2\,(ad-bc)^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*c/(a*d-b*c)^2*ln(d*x^2+c)+1/2/(a*d-b*c)^2*c*ln(b*x^2+a)-1/2/(a*d-b*c)^
2/b*a^2/(b*x^2+a)*d+1/2/(a*d-b*c)^2*a/(b*x^2+a)*c
```

Maxima [A] time = 0.981009, size = 142, normalized size = 1.92

$$\frac{c\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{c\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{a}{2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a/(a*b^2*c - a^2*b*d + (b^3*c - a*b^
2*d)*x^2)
```

Fricas [A] time = 1.80083, size = 242, normalized size = 3.27

$$\frac{abc-a^2d+\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)-\left(b^2cx^2+abc\right)\log\left(dx^2+c\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d + (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) - (b^2*c*x^2 + a*b*
c)*log(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*
b^3*c*d + a^2*b^2*d^2)*x^2)
```

Sympy [B] time = 2.24498, size = 253, normalized size = 3.42

$$-\frac{a}{2a^2bd - 2ab^2c + x^2\left(2ab^2d - 2b^3c\right)} - \frac{c\log\left(x^2 + \frac{-\frac{a^3cd^3}{(ad-bc)^2} + \frac{3a^2bc^2d^2}{(ad-bc)^2} - \frac{3ab^2c^3d}{(ad-bc)^2} + acd + \frac{b^3c^4}{(ad-bc)^2} + bc^2}{2bcd}\right)}{2\left(ad-bc\right)^2} + \frac{c\log\left(x^2 + \frac{\frac{a^3cd^3}{(ad-bc)^2} - \frac{3a^2bc^2d^2}{(ad-bc)^2} + \frac{3ab}{(ad-}}{2b}\right.}{2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - c*log(x**2 +
(-a**3*c*d**3/(a*d - b*c)**2 + 3*a**2*b*c**2*d**2/(a*d - b*c)**2 - 3*a*b**2
*c**3*d/(a*d - b*c)**2 + a*c*d + b**3*c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*
d))/(2*(a*d - b*c)**2) + c*log(x**2 + (a**3*c*d**3/(a*d - b*c)**2 - 3*a**2*
b*c**2*d**2/(a*d - b*c)**2 + 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d - b**3*
c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2)
```

Giac [A] time = 1.19154, size = 124, normalized size = 1.68

$$-\frac{\frac{b^2c\log\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{b^3c^2 - 2\,ab^2cd + a^2bd^2} - \frac{ab}{\left(b^2c - abd\right)\left(bx^2+a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2) - a*b/((b^2*c - a*b*d)*(b*x^2 + a)))/b
```

3.291 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=104

$$-\frac{x}{2\left(a+bx^2\right)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2}-\frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

```
[Out] -x/(2*(b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(b*c - a*d)^2
```

Rubi [A] time = 0.0646264, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {471, 522, 205}

$$-\frac{x}{2\left(a+bx^2\right)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2}-\frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -x/(2*(b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(b*c - a*d)^2
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)} + \frac{\int \frac{c-dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2(bc-ad)}$$

$$= -\frac{x}{2(bc-ad)\left(a+bx^2\right)} - \frac{(cd)\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2(bc-ad)^2}$$

$$= -\frac{x}{2(bc-ad)\left(a+bx^2\right)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2} - \frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Mathematica [A] time = 0.135417, size = 104, normalized size = 1.

$$\frac{x}{2\left(a+bx^2\right)(ad-bc)} + \frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(ad-bc)^2} - \frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] x/(2*(-(b*c) + a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]*(-(b*c) + a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(b*c - a*d)^2
```

Maple [A] time = 0.009, size = 134, normalized size = 1.3

$$-\frac{cd}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{axd}{2\,(ad-bc)^2\left(bx^2+a\right)} - \frac{bcx}{2\,(ad-bc)^2\left(bx^2+a\right)} + \frac{ad}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -c*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))+1/2/(a*d-b*c)^2*x/(b*x
^2+a)*a*d-1/2/(a*d-b*c)^2*x/(b*x^2+a)*b*c+1/2/(a*d-b*c)^2/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*a*d+1/2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99292, size = 1485, normalized size = 14.28

$$\left[-\frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\left(ab^2x^2+a^2b\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3c^2-2a^3b^2cd+a^4bd^2+\left(ab^4c^2-2a^2b^3cd+a^3b^2d^2\right)x^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 -
2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 -
2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2),
 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
 + (a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 +
 c)) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a
*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2), -1/4*(4*(a*b^2*x^2 + a^2*b)*s
qrt(c*d)*arctan(sqrt(c*d)*x/c) + (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(a*b^2*c - a^2*b*d
)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*x^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) - 2*(a*b^2*x^2 + a^2*b)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2)]
```

Sympy [B] time = 7.14263, size = 1530, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] x/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c)) + sqrt(-1/(a*b))*(a*d +
b*c)*log(x + (-3*a**6*b*d**5*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c
)**6) + 11*a**5*b**2*c*d**4*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)
**6) - 7*a**4*b**3*c**2*d**3*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**
6 + 3*a**3*b**4*c**3*d**2*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 -
 a**3*d**3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) + a**2*b**5*c**4*d
*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 11*a**2*b*c*d**2*sqr
t(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) - a*b**6*c**5*(-1/(a*b))**(3/2)*
(a*d + b*c)**3/(2*(a*d - b*c)**6) - 3*a*b**2*c**2*d*sqrt(-1/(a*b))*(a*d + b
*c)/(2*(a*d - b*c)**2) - b**3*c**3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c
)**2))/(a*d**2 + b*c*d))/(4*(a*d - b*c)**2) - sqrt(-1/(a*b))*(a*d + b*c)*lo
g(x + (3*a**6*b*d**5*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) -
11*a**5*b**2*c*d**4*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) + 7
*a**4*b**3*c**2*d**3*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - 3*a*
*3*b**4*c**3*d**2*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 + a**3*d*
*3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) - a**2*b**5*c**4*d*(-1/(a*
b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) + 11*a**2*b*c*d**2*sqrt(-1/(a*
b))*(a*d + b*c)/(2*(a*d - b*c)**2) + a*b**6*c**5*(-1/(a*b))**(3/2)*(a*d + b
*c)**3/(2*(a*d - b*c)**6) + 3*a*b**2*c**2*d*sqrt(-1/(a*b))*(a*d + b*c)/(2*(
a*d - b*c)**2) + b**3*c**3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2))/(
a*d**2 + b*c*d))/(4*(a*d - b*c)**2) + sqrt(-c*d)*log(x + (-12*a**6*b*d**5*(
-c*d)**(3/2)/(a*d - b*c)**6 + 44*a**5*b**2*c*d**4*(-c*d)**(3/2)/(a*d - b*c)
```

```
**6 - 56*a**4*b**3*c**2*d**3*(-c*d)**(3/2)/(a*d - b*c)**6 + 24*a**3*b**4*c*
*3*d**2*(-c*d)**(3/2)/(a*d - b*c)**6 - a**3*d**3*sqrt(-c*d)/(a*d - b*c)**2
+ 4*a**2*b**5*c**4*d*(-c*d)**(3/2)/(a*d - b*c)**6 - 11*a**2*b*c*d**2*sqrt(-
c*d)/(a*d - b*c)**2 - 4*a*b**6*c**5*(-c*d)**(3/2)/(a*d - b*c)**6 - 3*a*b**2
*c**2*d*sqrt(-c*d)/(a*d - b*c)**2 - b**3*c**3*sqrt(-c*d)/(a*d - b*c)**2)/(a
*d**2 + b*c*d))/(2*(a*d - b*c)**2) - sqrt(-c*d)*log(x + (12*a**6*b*d**5*(-c
*d)**(3/2)/(a*d - b*c)**6 - 44*a**5*b**2*c*d**4*(-c*d)**(3/2)/(a*d - b*c)**
6 + 56*a**4*b**3*c**2*d**3*(-c*d)**(3/2)/(a*d - b*c)**6 - 24*a**3*b**4*c**3
*d**2*(-c*d)**(3/2)/(a*d - b*c)**6 + a**3*d**3*sqrt(-c*d)/(a*d - b*c)**2 -
4*a**2*b**5*c**4*d*(-c*d)**(3/2)/(a*d - b*c)**6 + 11*a**2*b*c*d**2*sqrt(-c*
d)/(a*d - b*c)**2 + 4*a*b**6*c**5*(-c*d)**(3/2)/(a*d - b*c)**6 + 3*a*b**2*c
**2*d*sqrt(-c*d)/(a*d - b*c)**2 + b**3*c**3*sqrt(-c*d)/(a*d - b*c)**2)/(a*d
**2 + b*c*d))/(2*(a*d - b*c)**2)
```

Giac [A] time = 1.14471, size = 149, normalized size = 1.43

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{x}{2\left(bx^2+a\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*x/((b*x^2 + a)*(b*c - a*d))
```

3.292 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] -1/(2*(b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (d*
Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.0516852, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 44}

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (d*
Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.0280295, size = 66, normalized size = 0.94

$$\frac{d\left(a+bx^2\right)\log\left(c+dx^2\right)-d\left(a+bx^2\right)\log\left(a+bx^2\right)+ad-bc}{2\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x^2)*Log[a + b*x^2] + d*(a + b*x^2)*Log[c + d*x^2]
)/(2*(b*c - a*d)^2*(a + b*x^2))
```

Maple [A] time = 0.01, size = 90, normalized size = 1.3

$$\frac{d\ln\left(dx^2+c\right)}{2\,(ad-bc)^2}-\frac{\ln\left(bx^2+a\right)d}{2\,(ad-bc)^2}+\frac{ad}{2\,(ad-bc)^2\left(bx^2+a\right)}-\frac{bc}{2\,(ad-bc)^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*d/(a*d-b*c)^2*ln(d*x^2+c)-1/2/(a*d-b*c)^2*ln(b*x^2+a)*d+1/2/(a*d-b*c)^2
/(b*x^2+a)*a*d-1/2*b/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 0.971497, size = 134, normalized size = 1.91

$$-\frac{d\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{d\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{1}{2\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*d*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^
2)
```

Fricas [A] time = 1.60039, size = 219, normalized size = 3.13

$$-\frac{bc-ad+\left(bdx^2+ad\right)\log\left(bx^2+a\right)-\left(bdx^2+ad\right)\log\left(dx^2+c\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d + (b*d*x^2 + a*d)*log(b*x^2 + a) - (b*d*x^2 + a*d)*log(d*x^
2 + c))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*x^2)
```

Sympy [B] time = 2.18057, size = 248, normalized size = 3.54

$$\frac{d\log\left(x^2+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\left(ad-bc\right)^2}-\frac{d\log\left(x^2+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\left(ad-bc\right)^2}+\frac{}{2a^2d-2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d*log(x**2 + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 -
3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b
*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) - d*log(x**2 + (a**3*d**4/(a*d - b*c)*
*2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a
*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2)
+ 1/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c))
```

Giac [A] time = 1.14854, size = 115, normalized size = 1.64

$$\frac{bd\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{b}{2\left(b^2c-abd\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2) - 1/2*b/((b^2*c - a*b*d)*(b*x^2 + a))
```

3.293 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi [A] time = 0.0804604, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{-bc+2ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} + \frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2}$$

$$= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

Mathematica [A] time = 0.14189, size = 109, normalized size = 1.01

$$-\frac{\sqrt{b}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2} - \frac{bx}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(b*x)/(2*a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A] time = 0.002, size = 144, normalized size = 1.3

$$\frac{d^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{bdx}{2\,(ad-bc)^2\left(bx^2+a\right)} + \frac{b^2cx}{2\,(ad-bc)^2\,a\left(bx^2+a\right)} - \frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*b/(a*d-b*c)^2*x/(b*
x^2+a)*d+1/2*b^2/(a*d-b*c)^2/a*x/(b*x^2+a)*c-3/2*b/(a*d-b*c)^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*d+1/2*b^2/(a*d-b*c)^2/a/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22051, size = 1451, normalized size = 13.44

$$\left[-\frac{\left(abc - 3\,a^2d + \left(b^2c - 3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 2\left(abdx^2 + a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right) - 2\left(b^2c - abd\right)x}{4\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + \left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x^2\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

Sympy [B] time = 15.6387, size = 2033, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -b*x/(2*a**3*d - 2*a**2*b*c + x**2*(2*a**2*b*d - 2*a*b**2*c)) + sqrt(-b/a**
3)*(3*a*d - b*c)*log(x + (-a**9*c*d**6*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a
*d - b*c)**6 + 5*a**8*b*c**2*d**5*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d
 - b*c)**6) + a**7*b**2*c**3*d**4*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d
 - b*c)**6) - 7*a**6*b**3*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d
- b*c)**6 + 8*a**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d -
b*c)**6 - 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d - b*c)/(a*d - b*c)**2 - 7*a**4*b
**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + a**3*b**6
*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 27*a**3*b**2*c
**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 27*a**2*b**3*c**3
*d**2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 9*a*b**4*c**4*d*sqrt
(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + b**5*c**5*sqrt(-b/a**3)*(3*a*d
 - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(4*(a*d - b*c)**2) - sqrt(-b/a**3)*(3*a*d - b*c)*log(x + (a**9*c*d**6
*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 5*a**8*b*c**2*d**5*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - a**7*b**2*c**3*d**4*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 7*a**6*b**3*c**4*d**3*(-
b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 8*a**5*b**4*c**5*d**2*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 4*a**5*d**5*sqrt(-b/a**3)*(3
*a*d - b*c)/(a*d - b*c)**2 + 7*a**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b
*c)**3/(2*(a*d - b*c)**6) - a**3*b**6*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**
```

```
3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(
2*(a*d - b*c)**2) - 27*a**2*b**3*c**3*d**2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(
a*d - b*c)**2) + 9*a*b**4*c**4*d*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)
**2) - b**5*c**5*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b
*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d**2))/(4*(a*d - b*c)**2) + sqrt(-d**3/
c)*log(x + (-8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 20*a**8*b*c**2
*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**7*b**2*c**3*d**4*(-d**3/c)**(3
/2)/(a*d - b*c)**6 - 56*a**6*b**3*c**4*d**3*(-d**3/c)**(3/2)/(a*d - b*c)**6
 + 64*a**5*b**4*c**5*d**2*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 8*a**5*d**5*sqr
t(-d**3/c)/(a*d - b*c)**2 - 28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c
)**6 + 4*a**3*b**6*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 27*a**3*b**2*c**2
*d**3*sqrt(-d**3/c)/(a*d - b*c)**2 + 27*a**2*b**3*c**3*d**2*sqrt(-d**3/c)/(
a*d - b*c)**2 - 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2 + b**5*c**5*sq
rt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(2*(a*d - b*c)**2) - sqrt(-d**3/c)*log(x + (8*a**9*c*d**6*(-d**3/c)**
(3/2)/(a*d - b*c)**6 - 20*a**8*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6
- 4*a**7*b**2*c**3*d**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 56*a**6*b**3*c**4
*d**3*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**4*c**5*d**2*(-d**3/c)**(
3/2)/(a*d - b*c)**6 + 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2 + 28*a**4*b*
*5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**3*b**6*c**7*(-d**3/c)**(3/
2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sqrt(-d**3/c)/(a*d - b*c)**2 - 2
7*a**2*b**3*c**3*d**2*sqrt(-d**3/c)/(a*d - b*c)**2 + 9*a*b**4*c**4*d*sqrt(-
d**3/c)/(a*d - b*c)**2 - b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b
*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2)
```

Giac [A] time = 1.14282, size = 163, normalized size = 1.51

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{cd}} + \frac{\left(b^2c - 3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\sqrt{ab}} + \frac{bx}{2\left(abc - a^2d\right)\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

3.294 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(2*c*(b*c - a*d)^2)
```

Rubi [A] time = 0.105449, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(2*c*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2(c+dx)}\right.\right.\\ &= \frac{b}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.107655, size = 97, normalized size = 0.98

$$\frac{2\log(x)-\frac{a\left(ad^2\left(a+bx^2\right)\log\left(c+dx^2\right)+bc(ad-bc)\right)+bc\left(a+bx^2\right)(bc-2ad)\log\left(a+bx^2\right)}{\left(a+bx^2\right)(bc-ad)^2}}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (2*Log[x] - (b*c*(b*c - 2*a*d)*(a + b*x^2)*Log[a + b*x^2] + a*(b*c*(-(b*c)
+ a*d) + a*d^2*(a + b*x^2)*Log[c + d*x^2]))/((b*c - a*d)^2*(a + b*x^2)))/(2
*a^2*c)
```

Maple [A] time = 0.016, size = 139, normalized size = 1.4

$$-\frac{d^{2}\ln\left(dx^{2}+c\right)}{2c\left(ad-bc\right)^{2}}+\frac{\ln\left(x\right)}{a^{2}c}+\frac{b\ln\left(bx^{2}+a\right)d}{a\left(ad-bc\right)^{2}}-\frac{b^{2}\ln\left(bx^{2}+a\right)c}{2a^{2}\left(ad-bc\right)^{2}}-\frac{bd}{2\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}+\frac{b^{2}c}{2a\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*d^2/c/(a*d-b*c)^2*ln(d*x^2+c)+ln(x)/a^2/c+b/a/(a*d-b*c)^2*ln(b*x^2+a)*
d-1/2*b^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*c-1/2*b/(a*d-b*c)^2/(b*x^2+a)*d+1/2*b
^2/a/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.01521, size = 185, normalized size = 1.87

$$-\frac{d^{2}\log\left(dx^{2}+c\right)}{2\left(b^{2}c^{3}-2abc^{2}d+a^{2}cd^{2}\right)}-\frac{\left(b^{2}c-2abd\right)\log\left(bx^{2}+a\right)}{2\left(a^{2}b^{2}c^{2}-2a^{3}bcd+a^{4}d^{2}\right)}+\frac{b}{2\left(a^{2}bc-a^{3}d+\left(ab^{2}c-a^{2}bd\right)x^{2}\right)}+\frac{\log\left(x^{2}\right)}{2a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*log(d*x^2 + c)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/2*(b^2*c -
2*a*b*d)*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/2*b/(a^2*
b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*log(x^2)/(a^2*c)
```

Fricas [B] time = 6.41115, size = 443, normalized size = 4.47

$$\frac{ab^{2}c^{2}-a^{2}bcd-\left(ab^{2}c^{2}-2a^{2}bcd+\left(b^{3}c^{2}-2ab^{2}cd\right)x^{2}\right)\log\left(bx^{2}+a\right)-\left(a^{2}bd^{2}x^{2}+a^{3}d^{2}\right)\log\left(dx^{2}+c\right)+2\left(ab^{2}c^{2}-2a\right.}{2\left(a^{3}b^{2}c^{3}-2a^{4}bc^{2}d+a^{5}cd^{2}+\left(a^{2}b^{3}c^{3}-2a^{3}b^{2}c^{2}d+a^{4}bcd^{2}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - a^2*b*c*d - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2 - 2*a*b^2*
c*d)*x^2)*log(b*x^2 + a) - (a^2*b*d^2*x^2 + a^3*d^2)*log(d*x^2 + c) + 2*(a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*
log(x))/(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2 + (a^2*b^3*c^3 - 2*a^3*b^2
*c^2*d + a^4*b*c*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19523, size = 247, normalized size = 2.49

$$-\frac{d^3\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)}-\frac{\left(b^3c-2\,ab^2d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}+\frac{b^3cx^2-2\,ab^2dx^2+2\,ab^2c-3\,a^2bd}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\left(bx^2+a\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3) - 1/2*
(b^3*c - 2*a*b^2*d)*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2) + 1/2*(b^3*c*x^2 - 2*a*b^2*d*x^2 + 2*a*b^2*c - 3*a^2*b*d)/((a^2*b^2*
c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)) + 1/2*log(x^2)/(a^2*c)
```

3.295 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=144

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -(3*b*c - 2*a*d)/(2*a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(a + b*x^2)
) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c -
 a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.206279, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(3*b*c - 2*a*d)/(2*a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(a + b*x^2)
) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c -
 a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{-3bc+2ad-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} + \frac{\int \frac{-3b^2c^2+2abcd+2a^2d^2-bd(3bc-2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a^2c(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)^2} - \frac{\left(b^2(3bc-5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a^2(bc-ad)^2}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{c^{3/2}(bc-ad}$$

Mathematica [A] time = 0.188553, size = 123, normalized size = 0.85

$$\frac{b^2x}{2a^2\left(a+bx^2\right)(ad-bc)} + \frac{b^{3/2}(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(ad-bc)^2} - \frac{1}{a^2cx} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(1/(a^2*c*x)) + (b^2*x)/(2*a^2*(-(b*c) + a*d)*(a + b*x^2)) + (b^(3/2)*(-3*
b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(-(b*c) + a*d)^2) - (d
^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.015, size = 169, normalized size = 1.2

$$-\frac{d^3}{c\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{1}{a^2cx} + \frac{b^2xd}{2\,a\,(ad-bc)^2\left(bx^2+a\right)} - \frac{b^3xc}{2\,a^2\,(ad-bc)^2\left(bx^2+a\right)} + \frac{5\,b^2d}{2\,a\,(ad-bc)^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/c*d^3/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/a^2/c/x+1/2*b^2/
a/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*b^3/a^2/(a*d-b*c)^2*x/(b*x^2+a)*c+5/2*b^2/a
/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-3/2*b^3/a^2/(a*d-b*c)^2/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.67996, size = 2049, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d +
 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b
*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a^
2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3
 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d
^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + 4*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3
*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 (a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) +
 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^3
 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*
c*d^2)*x)]
```

Sympy [B] time = 127.24, size = 2526, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**3/a**5)*(5*a*d - 3*b*c)*log(x + (-a**12*c**3*d**7*(-b**3/a**5)**(
3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6 + 4*a**11*b*c**4*d**6*(-b**3/a**5)**
(3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6 - 17*a**10*b**2*c**5*d**5*(-b**3/a*
*5)**(3/2)*(5*a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 31*a**9*b**3*c**6*d**4*(
-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 22*a**8*b**4*c**
7*d**3*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6 - 4*a**8*d**8*
sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(a*d - b*c)**2 + 19*a**7*b**5*c**8*d**2*(-
b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6 - 17*a**6*b**6*c**9*d*(
-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 3*a**5*b**7*c**1
0*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 125*a**3*b**5
```

```
*c**5*d**3*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(2*(a*d - b*c)**2) + 225*a**2*b
**6*c**6*d**2*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(2*(a*d - b*c)**2) - 135*a*b
**7*c**7*d*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(2*(a*d - b*c)**2) + 27*b**8*c*
*8*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(2*(a*d - b*c)**2))/(20*a**4*b**2*d**7
+ 28*a**3*b**3*c*d**6 + 36*a**2*b**4*c**2*d**5 - 81*a*b**5*c**3*d**4 + 27*b
**6*c**4*d**3))/(4*(a*d - b*c)**2) + sqrt(-b**3/a**5)*(5*a*d - 3*b*c)*log(x
 + (a**12*c**3*d**7*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6 -
 4*a**11*b*c**4*d**6*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(a*d - b*c)**6
+ 17*a**10*b**2*c**5*d**5*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2*(a*d -
b*c)**6) - 31*a**9*b**3*c**6*d**4*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2
*(a*d - b*c)**6) + 22*a**8*b**4*c**7*d**3*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*
c)**3/(a*d - b*c)**6 + 4*a**8*d**8*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(a*d -
b*c)**2 - 19*a**7*b**5*c**8*d**2*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(a*
d - b*c)**6 + 17*a**6*b**6*c**9*d*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3/(2
*(a*d - b*c)**6) - 3*a**5*b**7*c**10*(-b**3/a**5)**(3/2)*(5*a*d - 3*b*c)**3
/(2*(a*d - b*c)**6) + 125*a**3*b**5*c**5*d**3*sqrt(-b**3/a**5)*(5*a*d - 3*b
*c)/(2*(a*d - b*c)**2) - 225*a**2*b**6*c**6*d**2*sqrt(-b**3/a**5)*(5*a*d -
3*b*c)/(2*(a*d - b*c)**2) + 135*a*b**7*c**7*d*sqrt(-b**3/a**5)*(5*a*d - 3*b
*c)/(2*(a*d - b*c)**2) - 27*b**8*c**8*sqrt(-b**3/a**5)*(5*a*d - 3*b*c)/(2*(
a*d - b*c)**2))/(20*a**4*b**2*d**7 + 28*a**3*b**3*c*d**6 + 36*a**2*b**4*c**
2*d**5 - 81*a*b**5*c**3*d**4 + 27*b**6*c**4*d**3))/(4*(a*d - b*c)**2) - sqr
t(-d**5/c**3)*log(x + (-8*a**12*c**3*d**7*(-d**5/c**3)**(3/2)/(a*d - b*c)**
6 + 32*a**11*b*c**4*d**6*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 - 68*a**10*b**2
*c**5*d**5*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 + 124*a**9*b**3*c**6*d**4*(-d
**5/c**3)**(3/2)/(a*d - b*c)**6 - 176*a**8*b**4*c**7*d**3*(-d**5/c**3)**(3/
2)/(a*d - b*c)**6 - 8*a**8*d**8*sqrt(-d**5/c**3)/(a*d - b*c)**2 + 152*a**7*
b**5*c**8*d**2*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 - 68*a**6*b**6*c**9*d*(-d
**5/c**3)**(3/2)/(a*d - b*c)**6 + 12*a**5*b**7*c**10*(-d**5/c**3)**(3/2)/(a
*d - b*c)**6 - 125*a**3*b**5*c**5*d**3*sqrt(-d**5/c**3)/(a*d - b*c)**2 + 22
5*a**2*b**6*c**6*d**2*sqrt(-d**5/c**3)/(a*d - b*c)**2 - 135*a*b**7*c**7*d*s
qrt(-d**5/c**3)/(a*d - b*c)**2 + 27*b**8*c**8*sqrt(-d**5/c**3)/(a*d - b*c)*
*2)/(20*a**4*b**2*d**7 + 28*a**3*b**3*c*d**6 + 36*a**2*b**4*c**2*d**5 - 81*
a*b**5*c**3*d**4 + 27*b**6*c**4*d**3))/(2*(a*d - b*c)**2) + sqrt(-d**5/c**3
)*log(x + (8*a**12*c**3*d**7*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 - 32*a**11*
b*c**4*d**6*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 + 68*a**10*b**2*c**5*d**5*(-
d**5/c**3)**(3/2)/(a*d - b*c)**6 - 124*a**9*b**3*c**6*d**4*(-d**5/c**3)**(3
/2)/(a*d - b*c)**6 + 176*a**8*b**4*c**7*d**3*(-d**5/c**3)**(3/2)/(a*d - b*c
)**6 + 8*a**8*d**8*sqrt(-d**5/c**3)/(a*d - b*c)**2 - 152*a**7*b**5*c**8*d**
2*(-d**5/c**3)**(3/2)/(a*d - b*c)**6 + 68*a**6*b**6*c**9*d*(-d**5/c**3)**(3
/2)/(a*d - b*c)**6 - 12*a**5*b**7*c**10*(-d**5/c**3)**(3/2)/(a*d - b*c)**6
+ 125*a**3*b**5*c**5*d**3*sqrt(-d**5/c**3)/(a*d - b*c)**2 - 225*a**2*b**6*c
**6*d**2*sqrt(-d**5/c**3)/(a*d - b*c)**2 + 135*a*b**7*c**7*d*sqrt(-d**5/c**
3)/(a*d - b*c)**2 - 27*b**8*c**8*sqrt(-d**5/c**3)/(a*d - b*c)**2)/(20*a**4*
b**2*d**7 + 28*a**3*b**3*c*d**6 + 36*a**2*b**4*c**2*d**5 - 81*a*b**5*c**3*d
**4 + 27*b**6*c**4*d**3))/(2*(a*d - b*c)**2) - (2*a**2*d - 2*a*b*c + x**2*(
2*a*b*d - 3*b**2*c))/(x**3*(2*a**3*b*c*d - 2*a**2*b**2*c**2) + x*(2*a**4*c*
d - 2*a**3*b*c**2))
```

Giac [A] time = 1.13578, size = 221, normalized size = 1.53

$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{cd}}-\frac{\left(3\,b^3c-5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{ab}}-\frac{3\,b^2cx^2-2\,abdx^2+2\,abc-2\,a^2d}{2\left(a^2bc^2-a^3cd\right)\left(bx^3+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
- 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2)*sqrt(a*b)) - 1/2*(3*b^2*c*x^2 - 2*a*b*d*x^2 + 2*a*b*c - 2*a^2
*d)/((a^2*b*c^2 - a^3*c*d)*(b*x^3 + a*x))
```

3.296 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)} dx$

Optimal. Leaf size=126

$$-\frac{b^2}{2a^2(a+bx^2)(bc-ad)}+\frac{b^2(2bc-3ad)\log(a+bx^2)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{1}{2a^2cx^2}+\frac{d^3\log(c+dx^2)}{2c^2(bc-ad)^2}$$

```
[Out] -1/(2*a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c + a*d)*Log
[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^2)
 + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rubi [A] time = 0.145819, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^2}{2a^2(a+bx^2)(bc-ad)}+\frac{b^2(2bc-3ad)\log(a+bx^2)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{1}{2a^2cx^2}+\frac{d^3\log(c+dx^2)}{2c^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c + a*d)*Log
[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^2)
 + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)}dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx^2}+\frac{-2bc-ad}{a^3c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)^2}-\frac{b^3(-2bc+3ad)}{a^3(-bc+ad)^2(a+bx)}+\frac{}{c^2(}\right.\right.\\ &= -\frac{1}{2a^2cx^2}-\frac{b^2}{2a^2(bc-ad)(a+bx^2)}-\frac{(2bc+ad)\log(x)}{a^3c^2}+\frac{b^2(2bc-3ad)\log(a+bx^2)}{2a^3(bc-ad)^2}+\frac{d}{}\end{aligned}$$

Mathematica [A] time = 0.152716, size = 119, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2}{a^2\left(a+bx^2\right)(ad-bc)}+\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^2}-\frac{2\log(x)(ad+2bc)}{a^3c^2}-\frac{1}{a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{c^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^2)) + b^2/(a^2*(-(b*c) + a*d)*(a + b*x^2)) - (2*(2*b*c + a*d)
*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^
2) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2))/2
```

Maple [A] time = 0.018, size = 170, normalized size = 1.4

$$\frac{d^3\ln\left(dx^2+c\right)}{2c^2(ad-bc)^2}-\frac{1}{2a^2cx^2}-\frac{\ln(x)d}{a^2c^2}-2\frac{\ln(x)b}{a^3c}-\frac{3b^2\ln\left(bx^2+a\right)d}{2a^2(ad-bc)^2}+\frac{b^3\ln\left(bx^2+a\right)c}{a^3(ad-bc)^2}+\frac{b^2d}{2a(ad-bc)^2\left(bx^2+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*d^3/c^2/(a*d-b*c)^2*ln(d*x^2+c)-1/2/a^2/c/x^2-1/a^2/c^2*ln(x)*d-2/a^3/c
*ln(x)*b-3/2*b^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*d+b^3/a^3/(a*d-b*c)^2*ln(b*x^2
+a)*c+1/2*b^2/a/(a*d-b*c)^2/(b*x^2+a)*d-1/2*b^3/a^2/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.01463, size = 255, normalized size = 2.02

$$\frac{d^3\log\left(dx^2+c\right)}{2\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}+\frac{\left(2b^3c-3ab^2d\right)\log\left(bx^2+a\right)}{2\left(a^3b^2c^2-2a^4bcd+a^5d^2\right)}-\frac{abc-a^2d+\left(2b^2c-abd\right)x^2}{2\left(\left(a^2b^2c^2-a^3bcd\right)x^4+\left(a^3bc^2-a^4cd\right)x^2\right)}-\frac{(2bc+ad)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + 1/2*(2*b^3*c
 - 3*a*b^2*d)*log(b*x^2 + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2) - 1/2*(a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^4 + (a^3
*b*c^2 - a^4*c*d)*x^2) - 1/2*(2*b*c + a*d)*log(x^2)/(a^3*c^2)
```

Fricas [B] time = 14.4974, size = 601, normalized size = 4.77

$$-\frac{a^2b^2c^3-2a^3bc^2d+a^4cd^2+\left(2ab^3c^3-3a^2b^2c^2d+a^3bcd^2\right)x^2-\left(\left(2b^4c^3-3ab^3c^2d\right)x^4+\left(2ab^3c^3-3a^2b^2c^2d\right)x^2\right)\log}{2\left(\left(a^3b^3c^4-2a^4b^2c^3d+a^5bc^2d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^3 - 3*a^2*b^2*c^
2*d + a^3*b*c*d^2)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d)*x^4 + (2*a*b^3*c^3 -
3*a^2*b^2*c^2*d)*x^2)*log(b*x^2 + a) - (a^3*b*d^3*x^4 + a^4*d^3*x^2)*log(d*
x^2 + c) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 -
3*a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(x))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a
^5*b*c^2*d^2)*x^4 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15396, size = 347, normalized size = 2.75

$$\frac{d^4 \log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)}+\frac{\left(2\,b^4c-3\,ab^3d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2\right)}+\frac{a^2bd^3x^4-4\,b^3c^3x^2+6\,ab^2c^2dx^2-2\,a^2bcd^2x^2+a^3d^3x^2}{4\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3) + 1/2
*(2*b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a
^5*b*d^2) + 1/4*(a^2*b*d^3*x^4 - 4*b^3*c^3*x^2 + 6*a*b^2*c^2*d*x^2 - 2*a^2*
b*c*d^2*x^2 + a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*x^4 + a*x^2)) - 1/2*(2*b*c + a*
d)*log(x^2)/(a^3*c^2)
```

3.297 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=189

$$\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3(a+bx^2)(bc-a$$

```
[Out] -(5*b*c - 2*a*d)/(6*a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c*d - 2*a^2
*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^
(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c - a*d)^2
) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.277482, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3(a+bx^2)(bc-a$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```



```
[Out] -(5*b*c - 2*a*d)/(6*a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c*d - 2*a^2
*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^
(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c - a*d)^2
) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^2)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} - \frac{\int \frac{-5bc+2ad-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{\int \frac{-3\left(5b^2c^2-2abcd-2a^2d^2\right)-3bd(5bc-2ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{6a^2c(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} - \frac{\int \frac{-3\left(5b^3c^3-2ab^2c^2\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{d^4\int \frac{1}{c+dx^2}\,dx}{c^2(bc-ad)^2} + \ldots$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{b^{5/2}(5bc-7ad)\ldots}{2a^{7/2}(bc-\ldots}$$

Mathematica [A] time = 0.270661, size = 142, normalized size = 0.75

$$-\frac{b^3x}{2a^3\left(a+bx^2\right)(ad-bc)} - \frac{b^{5/2}(7ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(ad-bc)^2} + \frac{ad+2bc}{a^3c^2x} - \frac{1}{3a^2cx^3} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(3*a^2*c*x^3) + (2*b*c + a*d)/(a^3*c^2*x) - (b^3*x)/(2*a^3*(-(b*c) + a*d
)*(a + b*x^2)) - (b^(5/2)*(-5*b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(7/2)*(-(b*c) + a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*
(b*c - a*d)^2)
```

Maple [A] time = 0.015, size = 191, normalized size = 1.

$$\frac{d^4}{c^2(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{1}{3a^2cx^3} + \frac{d}{a^2c^2x} + 2\frac{b}{a^3cx} - \frac{b^3xd}{2a^2(ad-bc)^2\left(bx^2+a\right)} + \frac{b^4xc}{2a^3(ad-bc)^2\left(bx^2+a\right)} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/c^2*d^4/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/3/a^2/c/x^3+1/a
^2/c^2/x*d+2/a^3/c/x*b-1/2*b^3/a^2/(a*d-b*c)^2*x/(b*x^2+a)*d+1/2*b^4/a^3/(a
```

```
*d-b*c)^2*x/(b*x^2+a)*c-7/2*b^3/a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*d+5/2*b^4/a^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.85329, size = 2558, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^2*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^
3*c^2*d + 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2
 + 3*a^4*d^3)*x^2 + 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a
^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) - 6*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5
+ (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/12*(4*a^2*b^2*c^3 -
8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x
^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 12*(
a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*((5*b^4*c^3
- 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log(
(b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*
d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3),
-1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*(5*b^4*c^3 - 7*a*b^3*
c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 +
 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2
*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(a^3*b*d^3*x^5 + a^4*d^3
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^3
*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d
+ a^6*c^2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*
(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*
c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 +
 (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 6*(a^
3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^3*b^3*c^4 - 2
*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^
2*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16141, size = 223, normalized size = 1.18

$$\frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\sqrt{cd}} + \frac{b^3x}{2\left(a^3bc - a^4d\right)\left(bx^2 + a\right)} + \frac{\left(5\,b^4c - 7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2\right)\sqrt{ab}} + \frac{6\,bcx^2 + 3\,adx^2 - ac}{3\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d))
 + 1/2*b^3*x/((a^3*b*c - a^4*d)*(b*x^2 + a)) + 1/2*(5*b^4*c - 7*a*b^3*d)*ar
ctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(a*b)) + 1/3
*(6*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^3*c^2*x^3)
```

3.298 $\int \frac{1}{x^5(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=160

$$\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}-\frac{d^4\log}{2c^3(b}$$

```
[Out] -1/(4*a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*c - a*d)*(
a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3) - (b^3*(
3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c - a*d)^2) - (d^4*Log[c + d*x^2])
/(2*c^3*(b*c - a*d)^2)
```

Rubi [A] time = 0.193524, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}-\frac{d^4\log}{2c^3(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(4*a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*c - a*d)*(
a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3) - (b^3*(
3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c - a*d)^2) - (d^4*Log[c + d*x^2])
/(2*c^3*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^2(c+dx)}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{a^2cx^3}+\frac{-2bc-ad}{a^3c^2x^2}+\frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)^2}+\right.\right.$$

$$=-\frac{1}{4a^2cx^4}+\frac{2bc+ad}{2a^3c^2x^2}+\frac{b^3}{2a^3(bc-ad)\left(a+bx^2\right)}+\frac{\left(3b^2c^2+2abcd+a^2d^2\right)\log(x)}{a^4c^3}-\frac{b^3(}{}$$

Mathematica [A] time = 0.192717, size = 155, normalized size = 0.97

$$\frac{1}{4}\left(\frac{4\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{2b^3}{a^3\left(a+bx^2\right)(ad-bc)}+\frac{2b^3(4ad-3bc)\log\left(a+bx^2\right)}{a^4(bc-ad)^2}+\frac{2ad+4bc}{a^3c^2x^2}-\frac{1}{a^2cx^4}-\frac{2d^4\mathrm{l}}{c^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^4)) + (4*b*c + 2*a*d)/(a^3*c^2*x^2) - (2*b^3)/(a^3*(-(b*c) +
a*d)*(a + b*x^2)) + (4*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3)
+ (2*b^3*(-3*b*c + 4*a*d)*Log[a + b*x^2])/(a^4*(b*c - a*d)^2) - (2*d^4*Log[
c + d*x^2])/(c^3*(b*c - a*d)^2))/4
```

Maple [A] time = 0.018, size = 209, normalized size = 1.3

$$-\frac{d^4\ln\left(dx^2+c\right)}{2c^3(ad-bc)^2}-\frac{1}{4a^2cx^4}+\frac{d}{2a^2c^2x^2}+\frac{b}{a^3cx^2}+\frac{\ln(x)d^2}{a^2c^3}+2\frac{\ln(x)bd}{a^3c^2}+3\frac{\ln(x)b^2}{a^4c}+2\frac{b^3\ln\left(bx^2+a\right)d}{a^3(ad-bc)^2}-\frac{3b^4\ln\left(bx^2\right.}{2a^4(ad-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*d^4/c^3/(a*d-b*c)^2*ln(d*x^2+c)-1/4/a^2/c/x^4+1/2/a^2/c^2/x^2*d+1/a^3/
c/x^2*b+1/a^2/c^3*ln(x)*d^2+2/a^3/c^2*ln(x)*b*d+3/a^4/c*ln(x)*b^2+2*b^3/a^3
/(a*d-b*c)^2*ln(b*x^2+a)*d-3/2*b^4/a^4/(a*d-b*c)^2*ln(b*x^2+a)*c-1/2*b^3/a^
2/(a*d-b*c)^2/(b*x^2+a)*d+1/2*b^4/a^3/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.02167, size = 348, normalized size = 2.17

$$-\frac{d^4\log\left(dx^2+c\right)}{2\left(b^2c^5-2abc^4d+a^2c^3d^2\right)}-\frac{\left(3b^4c-4ab^3d\right)\log\left(bx^2+a\right)}{2\left(a^4b^2c^2-2a^5bcd+a^6d^2\right)}-\frac{a^2bc^2-a^3cd-2\left(3b^3c^2-ab^2cd-a^2bd^2\right)x^4-\left(3ab^2c^2-\right.}{4\left(\left(a^3b^2c^3-a^4bc^2d\right)x^6+\left(a^4bc^3-a^5c^2d\right)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^4*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(3*b^4*
c - 4*a*b^3*d)*log(b*x^2 + a)/(a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2) - 1/4*(
a^2*b*c^2 - a^3*c*d - 2*(3*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^4 - (3*a*b^2*
c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b
*c^3 - a^5*c^2*d)*x^4) + 1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(x^2)/(a^
4*c^3)
```

Fricas [B] time = 35.3027, size = 706, normalized size = 4.41

$$-\frac{a^3b^2c^4-2a^4bc^3d+a^5c^2d^2-2\left(3ab^4c^4-4a^2b^3c^3d+a^4bcd^3\right)x^4-\left(3a^2b^3c^4-4a^3b^2c^3d-a^4bc^2d^2+2a^5cd^3\right)x^2+2\left(\left(3b\right.\right.}{4\left(\left(a^4b^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 - 2*(3*a*b^4*c^4 - 4*a^2*b^
3*c^3*d + a^4*b*c*d^3)*x^4 - (3*a^2*b^3*c^4 - 4*a^3*b^2*c^3*d - a^4*b*c^2*d
^2 + 2*a^5*c*d^3)*x^2 + 2*((3*b^5*c^4 - 4*a*b^4*c^3*d)*x^6 + (3*a*b^4*c^4 -
 4*a^2*b^3*c^3*d)*x^4)*log(b*x^2 + a) + 2*(a^4*b*d^4*x^6 + a^5*d^4*x^4)*log
(d*x^2 + c) - 4*((3*b^5*c^4 - 4*a*b^4*c^3*d + a^4*b*d^4)*x^6 + (3*a*b^4*c^4
 - 4*a^2*b^3*c^3*d + a^5*d^4)*x^4)*log(x))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d
+ a^6*b*c^3*d^2)*x^6 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18213, size = 379, normalized size = 2.37

$$-\frac{d^5\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)}-\frac{\left(3\,b^5c-4\,ab^4d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^4b^3c^2-2\,a^5b^2cd+a^6bd^2\right)}+\frac{3\,b^5cx^2-4\,ab^4dx^2+4\,ab^4c-5\,a^2b^3d}{2\left(a^4b^2c^2-2\,a^5bcd+a^6d^2\right)\left(bx^2+a\right)}+\frac{\left(3\,b^2c^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*d^5*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3) - 1/
2*(3*b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^4*b^3*c^2 - 2*a^5*b^2*c*d +
a^6*b*d^2) + 1/2*(3*b^5*c*x^2 - 4*a*b^4*d*x^2 + 4*a*b^4*c - 5*a^2*b^3*d)/((
a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*(b*x^2 + a)) + 1/2*(3*b^2*c^2 + 2*a*b*
c*d + a^2*d^2)*log(x^2)/(a^4*c^3) - 1/4*(9*b^2*c^2*x^4 + 6*a*b*c*d*x^4 + 3*
a^2*d^2*x^4 - 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^4*c^3*x^4)
```

3.299 $\int \frac{1}{x^6(a+bx^2)^2(c+dx^2)} \, dx$

Optimal. Leaf size=250

$$\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^2bcd^2-2a^3d^3-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}-\frac{d^{9/2}\cdots}{c^{7\cdots}}$$

```
[Out] -(7*b*c - 2*a*d)/(10*a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*c*d - 2*a^
2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c
*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^5*(a + b
*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(
b*c - a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^
2)
```

Rubi [A] time = 0.408762, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^2bcd^2-2a^3d^3-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}-\frac{d^{9/2}\cdots}{c^{7\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(7*b*c - 2*a*d)/(10*a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*c*d - 2*a^
2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c
*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^5*(a + b
*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(
b*c - a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^
2)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)^2 \left(c+dx^2\right)} dx = \frac{b}{2a(bc-ad)x^5 \left(a+bx^2\right)} - \frac{\int \frac{-7bc+2ad-7bdx^2}{x^6\left(a+bx^2\right)\left(c+dx^2\right)} dx}{2a(bc-ad)}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{b}{2a(bc-ad)x^5 \left(a+bx^2\right)} + \frac{\int \frac{-5\left(7b^2c^2-2abcd-2a^2d^2\right)-5bd(7bc-2ad)x^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)} dx}{10a^2c(bc-ad)}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^5 \left(a+bx^2\right)} - \frac{\int \frac{-15\left(7b^3c^3-2\right.}{}}{}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3d^3}{2a^4c^3(bc-ad)x} + $$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3d^3}{2a^4c^3(bc-ad)x} + $$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3d^3}{2a^4c^3(bc-ad)x} + $$

Mathematica [A] time = 0.304151, size = 179, normalized size = 0.72

$$\frac{-a^2d^2-2abcd-3b^2c^2}{a^4c^3x} + \frac{b^4x}{2a^4\left(a+bx^2\right)(ad-bc)} + \frac{b^{7/2}(9ad-7bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(ad-bc)^2} + \frac{ad+2bc}{3a^3c^2x^3} - \frac{1}{5a^2cx^5} - \frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{c^{7/2}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(5*a^2*c*x^5) + (2*b*c + a*d)/(3*a^3*c^2*x^3) + (-3*b^2*c^2 - 2*a*b*c*d
- a^2*d^2)/(a^4*c^3*x) + (b^4*x)/(2*a^4*(-(b*c) + a*d)*(a + b*x^2)) + (b^(7
/2)*(-7*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(-(b*c) + a*d)
^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.019, size = 234, normalized size = 0.9

$$-\frac{d^5}{c^3(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{1}{5a^2cx^5} + \frac{d}{3a^2c^2x^3} + \frac{2b}{3a^3cx^3} - \frac{d^2}{a^2c^3x} - 2\frac{bd}{a^3c^2x} - 3\frac{b^2}{a^4cx} + \frac{b^4xd}{2a^3(ad-bc)^2\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/c^3*d^5/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/5/a^2/c/x^5+1/
3/a^2/c^2/x^3*d+2/3/a^3/c/x^3*b-1/a^2/c^3/x*d^2-2/a^3/c^2/x*b*d-3/a^4/c/x*b
^2+1/2*b^4/a^3/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*b^5/a^4/(a*d-b*c)^2*x/(b*x^2+a
)*c+9/2*b^4/a^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-7/2*b^5/a
^4/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.3758, size = 3002, normalized size = 12.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 -
9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*
b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2
*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^
4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) - 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*
sqrt(-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d
^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/60*(12*a^3*b
^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 - 9*a*b^4*c^3*d +
2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*
d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d
^3)*x^2 + 60*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*sqrt(d/c)) +
15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^4*b^3*c^5 -
 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*
c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*c^3*d + 6*a^5*c^2*d^2 + 15*(
7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 10*(7*a*b^4*c^4 - 9*a^2*b^3*
c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d -
 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 +
 (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 15*(a
^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(
d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^
2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*
c^3*d + 6*a^5*c^2*d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 +
10*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2
*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^
5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*
```

```
sqrt(d/c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2
*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15089, size = 279, normalized size = 1.12

$$-\frac{d^5 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^5 - 2\,abc^4d + a^2c^3d^2\right)\sqrt{cd}} - \frac{b^4x}{2\left(a^4bc - a^5d\right)\left(bx^2 + a\right)} - \frac{\left(7\,b^5c - 9\,ab^4d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^4b^2c^2 - 2\,a^5bcd + a^6d^2\right)\sqrt{ab}} - \frac{45\,b^2c^2x^4 + 30\,abcdx^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)
) - 1/2*b^4*x/((a^4*b*c - a^5*d)*(b*x^2 + a)) - 1/2*(7*b^5*c - 9*a*b^4*d)*a
rctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(a*b)) - 1/
15*(45*b^2*c^2*x^4 + 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 - 10*a*b*c^2*x^2 - 5*a
^2*c*d*x^2 + 3*a^2*c^2)/(a^4*c^3*x^5)
```

3.300 $\int \frac{1}{x^7(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=210

$$-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(2a^2bcd^2+a^3d^3+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{b^4(4bc-5ad)\log\left(a+b\right.}{2a^5(bc-ad)^2}$$

```
[Out] -1/(6*a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a + b*x^2)) - ((4*b^3*c
^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) + (b^4*(4*b
*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/(2
*c^4*(b*c - a*d)^2)
```

Rubi [A] time = 0.248792, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(2a^2bcd^2+a^3d^3+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{b^4(4bc-5ad)\log\left(a+b\right.}{2a^5(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(6*a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a + b*x^2)) - ((4*b^3*c
^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) + (b^4*(4*b
*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/(2
*c^4*(b*c - a*d)^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx^4}+\frac{-2bc-ad}{a^3c^2x^3}+\frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x^2}+\frac{-4b^3c^3-3ab^2c^2d-2a^2bcd^2}{a^5c^4x}\right.\right.\\ &= -\frac{1}{6a^2cx^6}+\frac{2bc+ad}{4a^3c^2x^4}-\frac{3b^2c^2+2abcd+a^2d^2}{2a^4c^3x^2}-\frac{b^4}{2a^4(bc-ad)\left(a+bx^2\right)}-\frac{\left(4b^3c^3+3ab^2c^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.265752, size = 202, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3x^2}-\frac{12\log(x)\left(2a^2bcd^2+a^3d^3+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{6b^4}{a^4\left(a+bx^2\right)(ad-bc)}+\frac{6b^4(4bc-}{a^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-2/(a^2*c*x^6) + (6*b*c + 3*a*d)/(a^3*c^2*x^4) - (6*(3*b^2*c^2 + 2*a*b*c*d
 + a^2*d^2))/(a^4*c^3*x^2) + (6*b^4)/(a^4*(-(b*c) + a*d)*(a + b*x^2)) - (12
*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) +
(6*b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(a^5*(b*c - a*d)^2) + (6*d^5*Log[c +
 d*x^2])/(c^4*(b*c - a*d)^2))/12
```

Maple [A] time = 0.021, size = 268, normalized size = 1.3

$$\frac{d^5\ln\left(dx^2+c\right)}{2\,c^4\,(ad-bc)^2}-\frac{1}{6\,a^2cx^6}+\frac{d}{4\,a^2c^2x^4}+\frac{b}{2\,a^3cx^4}-\frac{d^2}{2\,a^2c^3x^2}-\frac{bd}{a^3c^2x^2}-\frac{3\,b^2}{2\,a^4cx^2}-\frac{\ln(x)\,d^3}{a^2c^4}-2\,\frac{\ln(x)\,d^2b}{a^3c^3}-3\,\frac{\ln(x)\,db^2}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*d^5/c^4/(a*d-b*c)^2*ln(d*x^2+c)-1/6/a^2/c/x^6+1/4/a^2/c^2/x^4*d+1/2/a^3
/c/x^4*b-1/2/a^2/c^3/x^2*d^2-1/a^3/c^2/x^2*b*d-3/2/a^4/c/x^2*b^2-1/a^2/c^4*
ln(x)*d^3-2/a^3/c^3*ln(x)*d^2*b-3/a^4/c^2*ln(x)*d*b^2-4/a^5/c*ln(x)*b^3-5/2
*b^4/a^4/(a*d-b*c)^2*ln(b*x^2+a)*d+2*b^5/a^5/(a*d-b*c)^2*ln(b*x^2+a)*c+1/2*
b^4/a^3/(a*d-b*c)^2/(b*x^2+a)*d-1/2*b^5/a^4/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.04319, size = 458, normalized size = 2.18

$$\frac{d^5\log\left(dx^2+c\right)}{2\left(b^2c^6-2\,abc^5d+a^2c^4d^2\right)}+\frac{\left(4\,b^5c-5\,ab^4d\right)\log\left(bx^2+a\right)}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)}-\frac{2\,a^3bc^3-2\,a^4c^2d+6\left(4\,b^4c^3-ab^3c^2d-a^2b^2cd^2-a^3bd}{12\left(\left(a^4b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^5*log(d*x^2 + c)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2) + 1/2*(4*b^5*c
 - 5*a*b^4*d)*log(b*x^2 + a)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2) - 1/12*(
2*a^3*b*c^3 - 2*a^4*c^2*d + 6*(4*b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 - a^
3*b*d^3)*x^6 + 3*(4*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 - 2*a^4*d^3)*x^
4 - (4*a^2*b^2*c^3 - a^3*b*c^2*d - 3*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*
c^3*d)*x^8 + (a^5*b*c^4 - a^6*c^3*d)*x^6) - 1/2*(4*b^3*c^3 + 3*a*b^2*c^2*d
+ 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4)
```

Fricas [B] time = 54.1615, size = 817, normalized size = 3.89

$$-\frac{2\,a^4b^2c^5-4\,a^5bc^4d+2\,a^6c^3d^2+6\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^5bcd^4\right)x^6+3\left(4\,a^2b^4c^5-5\,a^3b^3c^4d-a^5bc^2d^3+2\,a^6cd^4\right)x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*a^4*b^2*c^5 - 4*a^5*b*c^4*d + 2*a^6*c^3*d^2 + 6*(4*a*b^5*c^5 - 5*a
^2*b^4*c^4*d + a^5*b*c*d^4)*x^6 + 3*(4*a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - a^5*
b*c^2*d^3 + 2*a^6*c*d^4)*x^4 - (4*a^3*b^3*c^5 - 5*a^4*b^2*c^4*d - 2*a^5*b*c
^3*d^2 + 3*a^6*c^2*d^3)*x^2 - 6*((4*b^6*c^5 - 5*a*b^5*c^4*d)*x^8 + (4*a*b^5
*c^5 - 5*a^2*b^4*c^4*d)*x^6)*log(b*x^2 + a) - 6*(a^5*b*d^5*x^8 + a^6*d^5*x^
6)*log(d*x^2 + c) + 12*((4*b^6*c^5 - 5*a*b^5*c^4*d + a^5*b*d^5)*x^8 + (4*a*
b^5*c^5 - 5*a^2*b^4*c^4*d + a^6*d^5)*x^6)*log(x))/((a^5*b^3*c^6 - 2*a^6*b^2
*c^5*d + a^7*b*c^4*d^2)*x^8 + (a^6*b^2*c^6 - 2*a^7*b*c^5*d + a^8*c^4*d^2)*x
^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12494, size = 478, normalized size = 2.28

$$\frac{d^6 \log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^6d-2\,abc^5d^2+a^2c^4d^3\right)}+\frac{\left(4\,b^6c-5\,ab^5d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^5b^3c^2-2\,a^6b^2cd+a^7bd^2\right)}-\frac{4\,b^6cx^2-5\,ab^5dx^2+5\,ab^5c-6\,a^2b^4d}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)\left(bx^2+a\right)}-\frac{\left(4\,b^3c^3+3\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^6*log(abs(d*x^2 + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3) + 1/2
*(4*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^5*b^3*c^2 - 2*a^6*b^2*c*d + a
^7*b*d^2) - 1/2*(4*b^6*c*x^2 - 5*a*b^5*d*x^2 + 5*a*b^5*c - 6*a^2*b^4*d)/((a
^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*(b*x^2 + a)) - 1/2*(4*b^3*c^3 + 3*a*b^2
*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4) + 1/12*(44*b^3*c^3*x^6
 + 33*a*b^2*c^2*d*x^6 + 22*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 18*a*b^2*c^3*
x^4 - 12*a^2*b*c^2*d*x^4 - 6*a^3*c*d^2*x^4 + 6*a^2*b*c^3*x^2 + 3*a^3*c^2*d*
x^2 - 2*a^3*c^3)/(a^5*c^4*x^6)
```

3.301 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2} dx$

Optimal. Leaf size=162

$$\frac{x(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3}+\frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)^3}$$

```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c - a*d)*(a
 + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```

Rubi [A] time = 0.163833, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{x(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3}+\frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c - a*d)*(a
 + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```


Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{ac+(-2bc-ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2b(bc-ad)}$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{4abc^2-2bc(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{4bc(bc-ad)^2}$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(a(3bc+ad))\int \frac{1}{a+bx^2}\,dx}{2(bc-ad)^3} + \frac{(c(b$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\sqrt{a}(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3} + \sqrt{}$$

Mathematica [A] time = 0.189935, size = 133, normalized size = 0.82

$$\frac{1}{2}\left(\frac{ax}{\left(a+bx^2\right)(bc-ad)^2} + \frac{cx}{\left(c+dx^2\right)(bc-ad)^2} + \frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad-bc)^3} + \frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*x)/((b*c - a*d)^2*(a + b*x^2)) + (c*x)/((b*c - a*d)^2*(c + d*x^2)) + (S
qrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(-(b*c) + a*d)^3
) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d
)^3))/2
```

Maple [A] time = 0.013, size = 222, normalized size = 1.4

$$\frac{cxad}{2\,(ad-bc)^3\left(dx^2+c\right)} - \frac{c^2xb}{2\,(ad-bc)^3\left(dx^2+c\right)} - \frac{3\,acd}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{bc^2}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^3*x/(d*x^2+c)*a*d-1/2*c^2/(a*d-b*c)^3*x/(d*x^2+c)*b-3/2*c/(
a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*d-1/2*c^2/(a*d-b*c)^3/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*a^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*a/(
a*d-b*c)^3*x/(b*x^2+a)*b*c+1/2*a^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))*d+3/2*a/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63473, size = 2822, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 +
a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3
*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2
 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*
d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - 2*((3*b^2*c*d + a*b*d
^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^
2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*
x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^
2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 +
3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 + 2*((b^2*c*d + 3*a*b*d^2)
*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/
d)*arctan(d*x*sqrt(c/d)/c) - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c
*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b
^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
a^4*d^4)*x^2), 1/2*((b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 +
3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(a/b)*arct
an(b*x*sqrt(a/b)/a) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b
^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 2*
(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4
*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 +
(b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

Sympy [B] time = 28.3417, size = 2378, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-a/b)*(a*d + 3*b*c)*log(x + (-4*a**7*b*d**8*(-a/b)**(3/2)*(a*d + 3*b*c
)**3/(a*d - b*c)**9 + 20*a**6*b**2*c*d**7*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a
*d - b*c)**9 - 36*a**5*b**3*c**2*d**6*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d -
 b*c)**9 + 20*a**4*b**4*c**3*d**5*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c
)**9 - a**4*d**4*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 20*a**3*b**5*c**
4*d**4*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 36*a**3*b*c*d**3*sqr
t(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 36*a**2*b**6*c**5*d**3*(-a/b)**(3/2)
*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 54*a**2*b**2*c**2*d**2*sqrt(-a/b)*(a*d +
 3*b*c)/(a*d - b*c)**3 + 20*a*b**7*c**6*d**2*(-a/b)**(3/2)*(a*d + 3*b*c)**3
/(a*d - b*c)**9 - 36*a*b**3*c**3*d*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3
- 4*b**8*c**7*d*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - b**4*c**4*s
qrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3)/(3*a**2*d**2 + 10*a*b*c*d + 3*b**2*
c**2))/(4*(a*d - b*c)**3) - sqrt(-a/b)*(a*d + 3*b*c)*log(x + (4*a**7*b*d**8
*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 20*a**6*b**2*c*d**7*(-a/b)
**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 36*a**5*b**3*c**2*d**6*(-a/b)**(3
/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 20*a**4*b**4*c**3*d**5*(-a/b)**(3/2)*
(a*d + 3*b*c)**3/(a*d - b*c)**9 + a**4*d**4*sqrt(-a/b)*(a*d + 3*b*c)/(a*d -
 b*c)**3 - 20*a**3*b**5*c**4*d**4*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c
)**9 + 36*a**3*b*c*d**3*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 36*a**2*b
**6*c**5*d**3*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 54*a**2*b**2*
c**2*d**2*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 20*a*b**7*c**6*d**2*(-a
/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 36*a*b**3*c**3*d*sqrt(-a/b)*(a
*d + 3*b*c)/(a*d - b*c)**3 + 4*b**8*c**7*d*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(
a*d - b*c)**9 + b**4*c**4*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3)/(3*a**2*
d**2 + 10*a*b*c*d + 3*b**2*c**2))/(4*(a*d - b*c)**3) + sqrt(-c/d)*(3*a*d +
b*c)*log(x + (-4*a**7*b*d**8*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9
+ 20*a**6*b**2*c*d**7*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 36*a*
*5*b**3*c**2*d**6*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 20*a**4*b
**4*c**3*d**5*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - a**4*d**4*sqr
t(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3 + 20*a**3*b**5*c**4*d**4*(-c/d)**(3/2)
*(3*a*d + b*c)**3/(a*d - b*c)**9 - 36*a**3*b*c*d**3*sqrt(-c/d)*(3*a*d + b*c
)/(a*d - b*c)**3 - 36*a**2*b**6*c**5*d**3*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a
*d - b*c)**9 - 54*a**2*b**2*c**2*d**2*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)*
*3 + 20*a*b**7*c**6*d**2*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 36
*a*b**3*c**3*d*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3 - 4*b**8*c**7*d*(-c/
d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - b**4*c**4*sqrt(-c/d)*(3*a*d + b
*c)/(a*d - b*c)**3)/(3*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2))/(4*(a*d - b*c
)**3) - sqrt(-c/d)*(3*a*d + b*c)*log(x + (4*a**7*b*d**8*(-c/d)**(3/2)*(3*a*
d + b*c)**3/(a*d - b*c)**9 - 20*a**6*b**2*c*d**7*(-c/d)**(3/2)*(3*a*d + b*c
)**3/(a*d - b*c)**9 + 36*a**5*b**3*c**2*d**6*(-c/d)**(3/2)*(3*a*d + b*c)**3
/(a*d - b*c)**9 - 20*a**4*b**4*c**3*d**5*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*
d - b*c)**9 + a**4*d**4*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3 - 20*a**3*b
**5*c**4*d**4*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 36*a**3*b*c*d
**3*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3 + 36*a**2*b**6*c**5*d**3*(-c/d)
**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 54*a**2*b**2*c**2*d**2*sqrt(-c/d)
*(3*a*d + b*c)/(a*d - b*c)**3 - 20*a*b**7*c**6*d**2*(-c/d)**(3/2)*(3*a*d +
b*c)**3/(a*d - b*c)**9 + 36*a*b**3*c**3*d*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b
*c)**3 + 4*b**8*c**7*d*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + b**4
*c**4*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3)/(3*a**2*d**2 + 10*a*b*c*d +
3*b**2*c**2))/(4*(a*d - b*c)**3) + (2*a*c*x + x**3*(a*d + b*c))/(2*a**3*c*d
**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d*
*2 + 2*b**3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d
 + 2*b**3*c**3))
```

Giac [B] time = 1.46013, size = 1670, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*d)*b^4*c^4*abs(d) + 4*sqrt(c*d)*a*b^3*c^3*d*abs(d) - 10*sqrt(c*
d)*a^2*b^2*c^2*d^2*abs(d) + 4*sqrt(c*d)*a^3*b*c*d^3*abs(d) + sqrt(c*d)*a^4*
d^4*abs(d) + sqrt(c*d)*b*c*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*abs(d) + sqrt(c*d)*a*d*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - a*b^2*c^2*d - a^2*b*
c*d^2 + a^3*d^3 + sqrt((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)^2 -
4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)))/(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)))/(b^3*c^3*d^2*abs(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a*b^2*c^2*d^3*abs(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a^2*b*c*d^4*abs(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + a^3*d^5*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)^2*d^2) - 1/2*(sqrt(a*b)*b^4*c^4*abs(b) + 4*sqrt(a*b)*a*b^3*c^3*d*abs(b)
- 10*sqrt(a*b)*a^2*b^2*c^2*d^2*abs(b) + 4*sqrt(a*b)*a^3*b*c*d^3*abs(b) + sq
rt(a*b)*a^4*d^4*abs(b) - sqrt(a*b)*b*c*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*abs(b) - sqrt(a*b)*a*d*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - a*b^2*c^
2*d - a^2*b*c*d^2 + a^3*d^3 - sqrt((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)^2 - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b^3*c^2*d - 2*a*b^2*
c*d^2 + a^2*b*d^3)))/(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)))/(b^5*c^3*abs
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a*b^4*c^2*d*abs(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a^2*b^3*c*d^2*abs(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + a^3*b^2*d^3*abs(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)^2*b^2) + 1/2*(b*c*x^3 + a*d*x^3 + 2*a*c*x)/((b*d*x^4 + b*c*x
^2 + a*d*x^2 + a*c)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

3.302 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=107

$$\frac{a}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{c}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{(ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2)) + ((b*c +
 a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*Log[c + d*x^2])/(2*(
b*c - a*d)^3)
```

<u>**Rubi [A]**</u> time = 0.107765, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{c}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{(ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2)) + ((b*c +
 a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*Log[c + d*x^2])/(2*(
b*c - a*d)^3)
```

<u>Rule 446</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

<u>Rule 77</u>

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

<u>Rubi steps</u>

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)^2}+\frac{b(bc+ad)}{(bc-ad)^3(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)^2}-\frac{d(bc-}{(bc-ad)}\right.\right.\\ &= \frac{a}{2(bc-ad)^2\left(a+bx^2\right)}+\frac{c}{2(bc-ad)^2\left(c+dx^2\right)}+\frac{(bc+ad)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(bc+ad)\log\left(c\right.}{2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0667347, size = 86, normalized size = 0.8

$$\frac{\frac{a(bc-ad)}{a+bx^2}+\frac{c(bc-ad)}{c+dx^2}+(ad+bc)\log\left(a+bx^2\right)-(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d))/(c + d*x^2) + (b*c + a*d)*Lo
g[a + b*x^2] - (b*c + a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.017, size = 188, normalized size = 1.8

$$\frac{d\ln\left(dx^2+c\right)a}{2\left(ad-bc\right)^3}+\frac{\ln\left(dx^2+c\right)bc}{2\left(ad-bc\right)^3}+\frac{acd}{2\left(ad-bc\right)^3\left(dx^2+c\right)}-\frac{bc^2}{2\left(ad-bc\right)^3\left(dx^2+c\right)}-\frac{\ln\left(bx^2+a\right)ad}{2\left(ad-bc\right)^3}-\frac{b\ln\left(bx^2+a\right)}{2\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d/(a*d-b*c)^3*ln(d*x^2+c)*a+1/2/(a*d-b*c)^3*ln(d*x^2+c)*b*c+1/2*d/(a*d-
b*c)^3*c/(d*x^2+c)*a-1/2/(a*d-b*c)^3*c^2/(d*x^2+c)*b-1/2/(a*d-b*c)^3*ln(b*x
^2+a)*a*d-1/2*b/(a*d-b*c)^3*ln(b*x^2+a)*c+1/2/(a*d-b*c)^3*a^2/(b*x^2+a)*d-1
/2*b/(a*d-b*c)^3*a/(b*x^2+a)*c
```

Maxima [B] time = 0.987347, size = 308, normalized size = 2.88

$$\frac{(bc+ad)\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{(bc+ad)\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c + a*d)*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3) - 1/2*(b*c + a*d)*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^2 + 2*a*c)/(a*b^2*c^3 - 2*a^2*b*c^2*
d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*
b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Fricas [B] time = 1.60208, size = 595, normalized size = 5.56

$$\frac{2\,abc^2-2\,a^2cd+\left(b^2c^2-a^2d^2\right)x^2+\left(\left(b^2cd+abd^2\right)x^4+abc^2+a^2cd+\left(b^2c^2+2\,abcd+a^2d^2\right)x^2\right)\log\left(bx^2+a\right)-\left(\left(b^2c\right.\right.}{2\left(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2 - 2*a^2*c*d + (b^2*c^2 - a^2*d^2)*x^2 + ((b^2*c*d + a*b*d^2)
*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^2)*log(b*x^2 +
 a) - ((b^2*c*d + a*b*d^2)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d
^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4
)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [B] time = 4.39294, size = 507, normalized size = 4.74

$$\frac{2ac + x^2(ad+bc)}{2a^3cd^2 - 4a^2bc^2d + 2ab^2c^3 + x^4\left(2a^2bd^3 - 4ab^2cd^2 + 2b^3c^2d\right) + x^2\left(2a^3d^3 - 2a^2bcd^2 - 2ab^2c^2d + 2b^3c^3\right)} + \frac{(ad+bc)\log\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] (2*a*c + x**2*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**3
 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d*
*3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*c**3)) + (a*d + b*c)*log(x*
*2 + (-a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 + 4*a**3*b*c*d**3*(a*d + b*c)/(
a*d - b*c)**3 - 6*a**2*b**2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**2*d**
2 + 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d - b**4*c**4*(a*d
 + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a*d - b*
c)**3) - (a*d + b*c)*log(x**2 + (a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 - 4*a
**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 + 6*a**2*b**2*c**2*d**2*(a*d + b*c)
/(a*d - b*c)**3 + a**2*d**2 - 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 +
2*a*b*c*d + b**4*c**4*(a*d + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 +
 2*b**2*c*d))/(2*(a*d - b*c)**3)
```

Giac [A] time = 1.22421, size = 240, normalized size = 2.24

$$\frac{\frac{ab^3}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\left(bx^2+a\right)} - \frac{\left(b^3c+ab^2d\right)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{b^2cd}{(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) - (b^3*c + a
*b^2*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^2*c*d/((b*c - a*d)^3*(b*c/(b*x^2 +
 a) - a*d/(b*x^2 + a) + d)))/b
```

3.303 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2} dx$

Optimal. Leaf size=147

$$-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}-\frac{dx}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)^3}$$

```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^2)*(c + d*
x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]
*(b*c - a*d)^3)
```

Rubi [A] time = 0.133562, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}-\frac{dx}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```



```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^2)*(c + d*
x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]
*(b*c - a*d)^3)
```


Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\int \frac{c-3dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2(bc-ad)}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\int \frac{2c(bc+ad)-4bcdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{(d(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2(bc-ad)^3}+\frac{(b(bc\cdots}{\cdots}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\sqrt{b}(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}(\cdots}{\cdots}$$

Mathematica [A] time = 0.178376, size = 137, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{bx}{\left(a+bx^2\right)(bc-ad)^2}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}-\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-((b*x)/((b*c - a*d)^2*(a + b*x^2))) - (d*x)/((b*c - a*d)^2*(c + d*x^2)) -
 (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d
)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c -
a*d)^3))/2
```

Maple [A] time = 0.017, size = 222, normalized size = 1.5

$$-\frac{d^2xa}{2\,(ad-bc)^3\left(dx^2+c\right)}+\frac{bdxc}{2\,(ad-bc)^3\left(dx^2+c\right)}+\frac{ad^2}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,bcd}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x/(d*x^2+c)*b*c+1/2*d^
2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+3/2*d/(a*d-b*c)^3/(c*d)
^(1/2)*arctan(x*d/(c*d)^(1/2))*b*c-1/2*b/(a*d-b*c)^3*x/(b*x^2+a)*a*d+1/2*b^
2/(a*d-b*c)^3*x/(b*x^2+a)*c-3/2*b/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))*a*d-1/2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.32104, size = 2805, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2
+ a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 +
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 -
3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^
2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + 2*((3*b^2*c*d + a*b
*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sq
rt(d/c)*arctan(x*sqrt(d/c)) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*
c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*
sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*
b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*
a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
 a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 - 2*((b^2*c*d + 3*a*b*d^2)*
x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a
)*arctan(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d +
(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^
3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^
2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d
^4)*x^2), -1/2*(2*(b^2*c*d - a*b*d^2)*x^3 - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*
b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a)*arctan
(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c^2 - a
^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^
4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2
*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

Sympy [B] time = 34.427, size = 2399, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-b/a)*(3*a*d + b*c)*log(x + (-a**9*c*d**8*(-b/a)**(3/2)*(3*a*d + b*c)*
*3/(a*d - b*c)**9 + 20*a**7*b**2*c**3*d**6*(-b/a)**(3/2)*(3*a*d + b*c)**3/(
a*d - b*c)**9 - 64*a**6*b**3*c**4*d**5*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d
- b*c)**9 + 90*a**5*b**4*c**5*d**4*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*
c)**9 - a**5*d**5*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - 64*a**4*b**5*c*
*6*d**3*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 9*a**4*b*c*d**4*sqr
t(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + 20*a**3*b**6*c**7*d**2*(-b/a)**(3/2)
*(3*a*d + b*c)**3/(a*d - b*c)**9 - 54*a**3*b**2*c**2*d**3*sqrt(-b/a)*(3*a*d
 + b*c)/(a*d - b*c)**3 - 54*a**2*b**3*c**3*d**2*sqrt(-b/a)*(3*a*d + b*c)/(a
*d - b*c)**3 - a*b**8*c**9*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 -
9*a*b**4*c**4*d*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b
/a)*(3*a*d + b*c)/(a*d - b*c)**3)/(3*a**2*b*d**3 + 10*a*b**2*c*d**2 + 3*b**
3*c**2*d))/(4*(a*d - b*c)**3) - sqrt(-b/a)*(3*a*d + b*c)*log(x + (a**9*c*d*
*8*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 20*a**7*b**2*c**3*d**6*(
-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 64*a**6*b**3*c**4*d**5*(-b/a
)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 90*a**5*b**4*c**5*d**4*(-b/a)**(
3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + a**5*d**5*sqrt(-b/a)*(3*a*d + b*c)/(
a*d - b*c)**3 + 64*a**4*b**5*c**6*d**3*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d
- b*c)**9 + 9*a**4*b*c*d**4*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - 20*a*
*3*b**6*c**7*d**2*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 54*a**3*b
**2*c**2*d**3*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + 54*a**2*b**3*c**3*d
**2*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + a*b**8*c**9*(-b/a)**(3/2)*(3*
a*d + b*c)**3/(a*d - b*c)**9 + 9*a*b**4*c**4*d*sqrt(-b/a)*(3*a*d + b*c)/(a*
d - b*c)**3 + b**5*c**5*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3)/(3*a**2*b*
d**3 + 10*a*b**2*c*d**2 + 3*b**3*c**2*d))/(4*(a*d - b*c)**3) + sqrt(-d/c)*(
a*d + 3*b*c)*log(x + (-a**9*c*d**8*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*
c)**9 + 20*a**7*b**2*c**3*d**6*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**
9 - 64*a**6*b**3*c**4*d**5*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 +
90*a**5*b**4*c**5*d**4*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - a**5
*d**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 - 64*a**4*b**5*c**6*d**3*(-d/
c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 9*a**4*b*c*d**4*sqrt(-d/c)*(a*d
 + 3*b*c)/(a*d - b*c)**3 + 20*a**3*b**6*c**7*d**2*(-d/c)**(3/2)*(a*d + 3*b*
c)**3/(a*d - b*c)**9 - 54*a**3*b**2*c**2*d**3*sqrt(-d/c)*(a*d + 3*b*c)/(a*d
 - b*c)**3 - 54*a**2*b**3*c**3*d**2*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3
 - a*b**8*c**9*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 9*a*b**4*c**
4*d*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d/c)*(a*d + 3
*b*c)/(a*d - b*c)**3)/(3*a**2*b*d**3 + 10*a*b**2*c*d**2 + 3*b**3*c**2*d))/(
4*(a*d - b*c)**3) - sqrt(-d/c)*(a*d + 3*b*c)*log(x + (a**9*c*d**8*(-d/c)**(
3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 20*a**7*b**2*c**3*d**6*(-d/c)**(3/2)
*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 64*a**6*b**3*c**4*d**5*(-d/c)**(3/2)*(a*
d + 3*b*c)**3/(a*d - b*c)**9 - 90*a**5*b**4*c**5*d**4*(-d/c)**(3/2)*(a*d +
3*b*c)**3/(a*d - b*c)**9 + a**5*d**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**
3 + 64*a**4*b**5*c**6*d**3*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 +
9*a**4*b*c*d**4*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 - 20*a**3*b**6*c**7
*d**2*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 54*a**3*b**2*c**2*d**
3*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 + 54*a**2*b**3*c**3*d**2*sqrt(-d/
c)*(a*d + 3*b*c)/(a*d - b*c)**3 + a*b**8*c**9*(-d/c)**(3/2)*(a*d + 3*b*c)**
3/(a*d - b*c)**9 + 9*a*b**4*c**4*d*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3
+ b**5*c**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3)/(3*a**2*b*d**3 + 10*a*
b**2*c*d**2 + 3*b**3*c**2*d))/(4*(a*d - b*c)**3) - (2*b*d*x**3 + x*(a*d + b
*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3
 - 4*a*b**2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 -
 2*a*b**2*c**2*d + 2*b**3*c**3))
```

Giac [B] time = 1.40361, size = 1481, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(2*sqrt(c*d)*b^4*c^3*abs(d) - 2*sqrt(c*d)*a*b^3*c^2*d*abs(d) - 2*sqrt(c*d)
*a^2*b^2*c*d^2*abs(d) + 2*sqrt(c*d)*a^3*b*d^3*abs(d) + sqrt(c*d)*b*abs(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*abs(d))*arctan(2*sqrt(1/2)*x
/sqrt((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3 + sqrt((b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)^2 - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d
^2)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)))/(b^3*c^2*d - 2*a*b^2*c*d^2 +
a^2*b*d^3)))/(b^3*c^3*d*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3) - a*b^2*c^2*d^2*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- a^2*b*c*d^3*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + a^3*
d^4*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)^2*d) + (2*sqrt(a*b)*b^3*c^3*d*abs(b)
- 2*sqrt(a*b)*a*b^2*c^2*d^2*abs(b) - 2*sqrt(a*b)*a^2*b*c*d^3*abs(b) + 2*sqr
t(a*b)*a^3*d^4*abs(b) - sqrt(a*b)*d*abs(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - a*b^2*c^2*d -
a^2*b*c*d^2 + a^3*d^3 - sqrt((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)^2 - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)))/(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)))/(b^4*c^3*abs(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a*b^3*c^2*d*abs(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - a^2*b^2*c*d^2*abs(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + a^3*b*d^3*abs(b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)^2*b) - 1/2*(2*b*d*x^3 + b*c*x + a*d*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2
 + a*c)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

3.304 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{b}{2\left(a+bx^2\right)(bc-ad)^2}-\frac{d}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\frac{bd\log\left(c+dx^2\right)}{(bc-ad)^3}$$

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)) - (b*d*L
og[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*c - a*d)^3
```

Rubi [A] time = 0.0792166, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 44}

$$-\frac{b}{2\left(a+bx^2\right)(bc-ad)^2}-\frac{d}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\frac{bd\log\left(c+dx^2\right)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)) - (b*d*L
og[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*c - a*d)^3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{2bd}{(bc-ad)^3(\dots}\right.\right.\\ &= -\frac{b}{2(bc-ad)^2\left(a+bx^2\right)}-\frac{d}{2(bc-ad)^2\left(c+dx^2\right)}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\frac{bd\log\left(c+dx^2\right)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.0679679, size = 77, normalized size = 0.84

$$\frac{\frac{b(ad-bc)}{a+bx^2}+\frac{d(ad-bc)}{c+dx^2}-2bd\log\left(a+bx^2\right)+2bd\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x^2) + (d*(-(b*c) + a*d))/(c + d*x^2) - 2*b*d*Lo
g[a + b*x^2] + 2*b*d*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.016, size = 143, normalized size = 1.6

$$-\frac{bd\ln\left(dx^2+c\right)}{(ad-bc)^3}-\frac{ad^2}{2\,(ad-bc)^3\left(dx^2+c\right)}+\frac{bdc}{2\,(ad-bc)^3\left(dx^2+c\right)}+\frac{b\ln\left(bx^2+a\right)d}{(ad-bc)^3}-\frac{abd}{2\,(ad-bc)^3\left(bx^2+a\right)}+\frac{}{2\,(ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -d/(a*d-b*c)^3*b*ln(d*x^2+c)-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)*a+1/2*d/(a*d-b*c
)^3/(d*x^2+c)*b*c+b/(a*d-b*c)^3*ln(b*x^2+a)*d-1/2*b/(a*d-b*c)^3/(b*x^2+a)*a
*d+1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*c
```

Maxima [B] time = 0.988063, size = 290, normalized size = 3.15

$$-\frac{bd\log\left(bx^2+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{bd\log\left(dx^2+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,b}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -b*d*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + b
*d*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2
*(2*b*d*x^2 + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*
d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
 a^3*d^3)*x^2)
```

Fricas [B] time = 1.6321, size = 508, normalized size = 5.52

$$-\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)\log\left(bx^2+a\right)-2\left(b^2d^2x^4+abcd+\left(b^2cd+a\right.\right.}{2\left(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^4+\left(b^4c^4-2\,ab^3c^3d+2\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a
^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 -
 a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [B] time = 3.84365, size = 408, normalized size = 4.43

$$-\frac{bd\log\left(x^2+\frac{-\frac{a^4bd^5}{(ad-bc)^3}+\frac{4a^3b^2cd^4}{(ad-bc)^3}-\frac{6a^2b^3c^2d^3}{(ad-bc)^3}+\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2-\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}+\frac{bd\log\left(x^2+\frac{\frac{a^4bd^5}{(ad-bc)^3}-\frac{4a^3b^2cd^4}{(ad-bc)^3}+\frac{6a^2b^3c^2d^3}{(ad-bc)^3}-\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2+\frac{b^5c^4d}{(ad-bc)^3}}{2b^2d^2}\right.}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -b*d*log(x**2 + (-a**4*b*d**5/(a*d - b*c)**3 + 4*a**3*b**2*c*d**4/(a*d - b*
c)**3 - 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 4*a*b**4*c**3*d**2/(a*d - b*
c)**3 + a*b*d**2 - b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b**2*d**2))/(a
*d - b*c)**3 + b*d*log(x**2 + (a**4*b*d**5/(a*d - b*c)**3 - 4*a**3*b**2*c*d
**4/(a*d - b*c)**3 + 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 4*a*b**4*c**3*d
**2/(a*d - b*c)**3 + a*b*d**2 + b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b
**2*d**2))/(a*d - b*c)**3 - (a*d + b*c + 2*b*d*x**2)/(2*a**3*c*d**2 - 4*a**
2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3
*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*c
**3))
```

Giac [A] time = 1.15689, size = 220, normalized size = 2.39

$$\frac{b^2d\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^3}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\left(bx^2+a\right)}+\frac{bd^2}{2\,(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*(b*x^2 + a)) + 1/2*b*d^2/((b*c - a*d)^3*(b*c/(b*x^2 + a) - a*d/(b*
x^2 + a) + d))
```

3.305 $\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2} dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.198204, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-bc+2ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2\left(b^2c^2-4abcd+a^2d^2\right)-2bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}}{4ac(bc-ad)^2}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(bc-5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^3} + \ldots$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} + \ldots$$

Mathematica [A] time = 0.317483, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}+\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A] time = 0., size = 238, normalized size = 1.4

$$\frac{d^3xa}{2\,(ad-bc)^3\,c\left(dx^2+c\right)} - \frac{bd^2x}{2\,(ad-bc)^3\left(dx^2+c\right)} + \frac{d^3a}{2\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{5\,bd^2}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*b+1/2*d
^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-5/2*d^2/(a*d-b*c)^3/
(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2
*b^3/(a*d-b*c)^3/a*x/(b*x^2+a)*c+5/2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*d-1/2*b^3/(a*d-b*c)^3/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.88019, size = 3294, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [B] time = 143.969, size = 3662, normalized size = 21.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-b**3/a**3)*(5*a*d - b*c)*log(x + (-a**12*c**3*d**9*(-b**3/a**3)**(3/
2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 11*a**11*b*c**4*d**8*(-b**3/a**3)**(3/
2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 40*a**10*b**2*c**5*d**7*(-b**3/a**3)**
(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 64*a**9*b**3*c**6*d**6*(-b**3/a**3)
**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 34*a**8*b**4*c**7*d**5*(-b**3/a**
3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - a**8*d**8*sqrt(-b**3/a**3)*(5*a
*d - b*c)/(a*d - b*c)**3 - 34*a**7*b**5*c**8*d**4*(-b**3/a**3)**(3/2)*(5*a*
d - b*c)**3/(a*d - b*c)**9 + 15*a**7*b*c*d**7*sqrt(-b**3/a**3)*(5*a*d - b*c
)/(a*d - b*c)**3 + 64*a**6*b**6*c**9*d**3*(-b**3/a**3)**(3/2)*(5*a*d - b*c)
**3/(a*d - b*c)**9 - 75*a**6*b**2*c**2*d**6*sqrt(-b**3/a**3)*(5*a*d - b*c)/
(a*d - b*c)**3 - 40*a**5*b**7*c**10*d**2*(-b**3/a**3)**(3/2)*(5*a*d - b*c)*
*3/(a*d - b*c)**9 + 125*a**5*b**3*c**3*d**5*sqrt(-b**3/a**3)*(5*a*d - b*c)/
(a*d - b*c)**3 + 11*a**4*b**8*c**11*d*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/
(a*d - b*c)**9 - a**3*b**9*c**12*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d
- b*c)**9 + 125*a**3*b**5*c**5*d**3*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b
*c)**3 - 75*a**2*b**6*c**6*d**2*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)*
*3 + 15*a*b**7*c**7*d*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - b**8*
c**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3)/(5*a**4*b**2*d**6 - 61*
a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3*d**3 + 5*b**6*c
**4*d**2))/(4*(a*d - b*c)**3) + sqrt(-b**3/a**3)*(5*a*d - b*c)*log(x + (a**
12*c**3*d**9*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 11*a**11
*b*c**4*d**8*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 40*a**10
*b**2*c**5*d**7*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 64*a*
*9*b**3*c**6*d**6*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 34*
a**8*b**4*c**7*d**5*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + a
**8*d**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 34*a**7*b**5*c**8*
d**4*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 15*a**7*b*c*d**7
*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - 64*a**6*b**6*c**9*d**3*(-b
**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 75*a**6*b**2*c**2*d**6*s
qrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 40*a**5*b**7*c**10*d**2*(-b*
*3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 125*a**5*b**3*c**3*d**5*s
qrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - 11*a**4*b**8*c**11*d*(-b**3/
a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + a**3*b**9*c**12*(-b**3/a**3)
**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 125*a**3*b**5*c**5*d**3*sqrt(-b**
3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 75*a**2*b**6*c**6*d**2*sqrt(-b**3/a*
*3)*(5*a*d - b*c)/(a*d - b*c)**3 - 15*a*b**7*c**7*d*sqrt(-b**3/a**3)*(5*a*d
 - b*c)/(a*d - b*c)**3 + b**8*c**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*
c)**3)/(5*a**4*b**2*d**6 - 61*a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 -
61*a*b**5*c**3*d**3 + 5*b**6*c**4*d**2))/(4*(a*d - b*c)**3) - sqrt(-d**3/c*
*3)*(a*d - 5*b*c)*log(x + (-a**12*c**3*d**9*(-d**3/c**3)**(3/2)*(a*d - 5*b*
c)**3/(a*d - b*c)**9 + 11*a**11*b*c**4*d**8*(-d**3/c**3)**(3/2)*(a*d - 5*b*
c)**3/(a*d - b*c)**9 - 40*a**10*b**2*c**5*d**7*(-d**3/c**3)**(3/2)*(a*d - 5
*b*c)**3/(a*d - b*c)**9 + 64*a**9*b**3*c**6*d**6*(-d**3/c**3)**(3/2)*(a*d -
 5*b*c)**3/(a*d - b*c)**9 - 34*a**8*b**4*c**7*d**5*(-d**3/c**3)**(3/2)*(a*d
 - 5*b*c)**3/(a*d - b*c)**9 - a**8*d**8*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d
 - b*c)**3 - 34*a**7*b**5*c**8*d**4*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a
*d - b*c)**9 + 15*a**7*b*c*d**7*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)*
*3 + 64*a**6*b**6*c**9*d**3*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c
)**9 - 75*a**6*b**2*c**2*d**6*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3
 - 40*a**5*b**7*c**10*d**2*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)
**9 + 125*a**5*b**3*c**3*d**5*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3
 + 11*a**4*b**8*c**11*d*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9
 - a**3*b**9*c**12*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 12
5*a**3*b**5*c**5*d**3*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 - 75*a*
*2*b**6*c**6*d**2*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 + 15*a*b**7
```

```
*c**7*d*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 - b**8*c**8*sqrt(-d**
3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3)/(5*a**4*b**2*d**6 - 61*a**3*b**3*c*d*
*5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3*d**3 + 5*b**6*c**4*d**2))/(4*
(a*d - b*c)**3) + sqrt(-d**3/c**3)*(a*d - 5*b*c)*log(x + (a**12*c**3*d**9*(
-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 11*a**11*b*c**4*d**8*(
-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 40*a**10*b**2*c**5*d**
7*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 64*a**9*b**3*c**6*d
**6*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 34*a**8*b**4*c**7
*d**5*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + a**8*d**8*sqrt(
-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 + 34*a**7*b**5*c**8*d**4*(-d**3/c*
*3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 15*a**7*b*c*d**7*sqrt(-d**3/c*
*3)*(a*d - 5*b*c)/(a*d - b*c)**3 - 64*a**6*b**6*c**9*d**3*(-d**3/c**3)**(3/
2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 75*a**6*b**2*c**2*d**6*sqrt(-d**3/c**3
)*(a*d - 5*b*c)/(a*d - b*c)**3 + 40*a**5*b**7*c**10*d**2*(-d**3/c**3)**(3/2
)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 125*a**5*b**3*c**3*d**5*sqrt(-d**3/c**3
)*(a*d - 5*b*c)/(a*d - b*c)**3 - 11*a**4*b**8*c**11*d*(-d**3/c**3)**(3/2)*(
a*d - 5*b*c)**3/(a*d - b*c)**9 + a**3*b**9*c**12*(-d**3/c**3)**(3/2)*(a*d -
 5*b*c)**3/(a*d - b*c)**9 - 125*a**3*b**5*c**5*d**3*sqrt(-d**3/c**3)*(a*d -
 5*b*c)/(a*d - b*c)**3 + 75*a**2*b**6*c**6*d**2*sqrt(-d**3/c**3)*(a*d - 5*b
*c)/(a*d - b*c)**3 - 15*a*b**7*c**7*d*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d -
 b*c)**3 + b**8*c**8*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3)/(5*a**4
*b**2*d**6 - 61*a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3
*d**3 + 5*b**6*c**4*d**2))/(4*(a*d - b*c)**3) + (x**3*(a*b*d**2 + b**2*c*d)
 + x*(a**2*d**2 + b**2*c**2))/(2*a**4*c**2*d**2 - 4*a**3*b*c**3*d + 2*a**2*
b**2*c**4 + x**4*(2*a**3*b*c*d**3 - 4*a**2*b**2*c**2*d**2 + 2*a*b**3*c**3*d
) + x**2*(2*a**4*c*d**3 - 2*a**3*b*c**2*d**2 - 2*a**2*b**2*c**3*d + 2*a*b**
3*c**4))
```

Giac [B] time = 1.47755, size = 1928, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(c*d)*a*b^5*c^5*abs(d) - 12*sqrt(c*d)*a^2*b^4*c^4*d*abs(d) + 22*s
qrt(c*d)*a^3*b^3*c^3*d^2*abs(d) - 12*sqrt(c*d)*a^4*b^2*c^2*d^3*abs(d) + sqr
t(c*d)*a^5*b*c*d^4*abs(d) - sqrt(c*d)*b^2*c*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d
 + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*abs(d) - sqrt(c*d)*a*b*d*abs(a*b^3*c^4 - 3*
a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*abs(d))*arctan(2*sqrt(1/2)*x/s
qrt((a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3 + sqrt((a*b^3*c^
4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)^2 - 4*(a^2*b^2*c^4 - 2*a^3*b
*c^3*d + a^4*c^2*d^2)*(a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*
b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*b^3*c^4*d*abs(a*b^3*c^4 -
 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - a^2*b^2*c^3*d^2*abs(a*b^3
*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - a^3*b*c^2*d^3*abs(a
*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + a^4*c*d^4*abs(a
*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + (a*b^3*c^4 - 3*
a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)^2*d) + 1/2*(sqrt(a*b)*a*b^4*c^
5*d*abs(b) - 12*sqrt(a*b)*a^2*b^3*c^4*d^2*abs(b) + 22*sqrt(a*b)*a^3*b^2*c^3
*d^3*abs(b) - 12*sqrt(a*b)*a^4*b*c^2*d^4*abs(b) + sqrt(a*b)*a^5*c*d^5*abs(b
) + sqrt(a*b)*b*c*d*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4
*c*d^3)*abs(b) + sqrt(a*b)*a*d^2*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*
c^2*d^2 - a^4*c*d^3)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c^4 - a^2*b^2
*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3 - sqrt((a*b^3*c^4 - a^2*b^2*c^3*d - a^3*
b*c^2*d^2 + a^4*c*d^3)^2 - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(a
```

```
*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*b^3*c^3*d - 2*a^2*b^2*c^
2*d^2 + a^3*b*c*d^3)))/(a*b^4*c^4*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b
*c^2*d^2 - a^4*c*d^3) - a^2*b^3*c^3*d*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a
^3*b*c^2*d^2 - a^4*c*d^3) - a^3*b^2*c^2*d^2*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d
 + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + a^4*b*c*d^3*abs(a*b^3*c^4 - 3*a^2*b^2*c^3
*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*
c^2*d^2 - a^4*c*d^3)^2*b) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^
2*d^2*x)/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*
x^2 + a*c))
```

3.306 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c + d*x^2)) +
 Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3
) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^3)
```

Rubi [A] time = 0.16858, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c + d*x^2)) +
 Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3
) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^2x}-\frac{b^3}{a(-bc+ad)^2(a+bx)^2}-\frac{b^3(-bc+3ad)}{a^2(-bc+ad)^3(a+bx)}-\frac{d^3}{c(bc-ad)^2(c\right.\right.\\
&= \frac{b^2}{2a(bc-ad)^2\left(a+bx^2\right)}+\frac{d^2}{2c(bc-ad)^2\left(c+dx^2\right)}+\frac{\log(x)}{a^2c^2}-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}
\end{aligned}$$

Mathematica [A] time = 0.23333, size = 133, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{a^2(ad-bc)^3}+\frac{2\log(x)}{a^2c^2}+\frac{b^2}{a\left(a+bx^2\right)(bc-ad)^2}+\frac{d^2(ad-3bc)\log\left(c+dx^2\right)}{c^2(bc-ad)^3}+\frac{d^2}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^2)) + (2
*Log[x])/(a^2*c^2) + (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)
^3) + (d^2*(-3*b*c + a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^3))/2
```

Maple [A] time = 0.021, size = 225, normalized size = 1.6

$$-\frac{d^3\ln\left(dx^2+c\right)a}{2c^2(ad-bc)^3}+\frac{3d^2\ln\left(dx^2+c\right)b}{2c(ad-bc)^3}+\frac{d^3a}{2c(ad-bc)^3\left(dx^2+c\right)}-\frac{d^2b}{2(ad-bc)^3\left(dx^2+c\right)}+\frac{\ln(x)}{a^2c^2}-\frac{3b^2\ln\left(bx^2+a\right)d}{2a(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^3*ln(d*x^2+c)*a+3/2*d^2/c/(a*d-b*c)^3*ln(d*x^2+c)*b+
1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)*b+ln(x)/a^2
/c^2-3/2*b^2/a/(a*d-b*c)^3*ln(b*x^2+a)*d+1/2*b^3/a^2/(a*d-b*c)^3*ln(b*x^2+a
)*c+1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*d-1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*c
```

Maxima [B] time = 1.05212, size = 398, normalized size = 2.82

$$-\frac{\left(b^3c-3ab^2d\right)\log\left(bx^2+a\right)}{2\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}-\frac{\left(3bcd^2-ad^3\right)\log\left(dx^2+c\right)}{2\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}+\frac{}{2\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*
a^4*b*c*d^2 - a^5*d^3) - 1/2*(3*b*c*d^2 - a*d^3)*log(d*x^2 + c)/(b^3*c^5 -
3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/2*(b^2*c^2 + a^2*d^2 + (
b^2*c*d + a*b*d^2)*x^2)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3
*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d
- a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/2*log(x^2)/(a^2*c^2)
```

Fricas [B] time = 25.7502, size = 1042, normalized size = 7.39

$$\frac{ab^3c^4-a^2b^2c^3d+a^3bc^2d^2-a^4cd^3+\left(ab^3c^3d-a^3bcd^3\right)x^2-\left(ab^3c^4-3a^2b^2c^3d+\left(b^4c^3d-3ab^3c^2d^2\right)x^4+\left(b^4c^4-2ab^3c^3d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^3*c^4 - a^2*b^2*c^3*d + a^3*b*c^2*d^2 - a^4*c*d^3 + (a*b^3*c^3*d -
 a^3*b*c*d^3)*x^2 - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2
*d^2)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2)*x^2)*log(b*x^2 +
a) - (3*a^3*b*c^2*d^2 - a^4*c*d^3 + (3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (3*
a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(d*x^2 + c) + 2*(a*b^3*c
^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^
3*b*c*d^3 - a^4*d^4)*x^2)*log(x))/(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*
c^4*d^2 - a^6*c^3*d^3 + (a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*
d^3 - a^5*b*c^2*d^4)*x^4 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3
 - a^6*c^2*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.307 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad}$$

```
[Out] -(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*x) + (d*(b*c
+ a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x*(a + b*x
^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(5/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.311306, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*x) + (d*(b*c
+ a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x*(a + b*x
^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(5/2)*(b*c - a*d)^3)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-3bc+2ad-5bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2\left(3b^2c^2-4abcd+3a^2d^2\right)\cdots}{x^2\left(a+bx^2\right)\left(c+\cdots\right.}}{4ac(bc-a\cdots}$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+d\cdots\right.}$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+d\cdots\right.}$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+d\cdots\right.}$$

Mathematica [A] time = 0.294245, size = 158, normalized size = 0.72

$$\frac{1}{2}\left(-\frac{b^3x}{a^2\left(a+bx^2\right)(bc-ad)^2} + \frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3} - \frac{2}{a^2c^2x} - \frac{d^3x}{c^2\left(c+dx^2\right)(bc-ad)^2} + \frac{d^{5/2}(3ad-7bc)\tan^{-1}\left(\cdots\right.}{c^{5/2}(bc-ad)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x) - (b^3*x)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (d^3*x)/(c^2*(b
*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(-(b*c) + a*d)^3) + (d^(5/2)*(-7*b*c + 3*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/2
```

Maple [A] time = 0.017, size = 261, normalized size = 1.2

$$-\frac{d^4xa}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}+\frac{d^3xb}{2\,c\,(ad-bc)^3\,(dx^2+c)}-\frac{3\,d^4a}{2\,c^2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{7\,d^3b}{2\,c\,(ad-bc)^3}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^4/c^2/(a*d-b*c)^3*x/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3*x/(d*x^2+c)*b-
3/2*d^4/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+7/2*d^3/c/(a*
d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b-1/a^2/c^2/x-1/2*b^3/a/(a*d-b
*c)^3*x/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x/(b*x^2+a)*c-7/2*b^3/a/(a*d-b*
c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+3/2*b^4/a^2/(a*d-b*c)^3/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.5628, size = 4182, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*
(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*
b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 - ((3*b^4*c^3*d -
7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3
+ (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^
2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)
*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*
d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c
^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 -
3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b^3*c^4 - 12
*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*
c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*d
 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5
+ (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 -
3*a^4*c*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - ((3*b^4*c^3*d - 7*a*b^3*c^2
*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*
c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*
c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d
```

```
^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x
), -1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 +
2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(
3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((3*b^4*c^3*
d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*
x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((
7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3
*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3
*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*
a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b
*c^4*d^2 - a^6*c^3*d^3)*x), -1/2*(2*a*b^3*c^4 - 6*a^2*b^2*c^3*d + 6*a^3*b*c
^2*d^2 - 2*a^4*c*d^3 + (3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3
*a^3*b*d^4)*x^4 + (3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x
^2 + ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*
a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(
x*sqrt(b/a)) + ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 +
4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(d/
c)*arctan(x*sqrt(d/c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3
*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^
3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a
^6*c^3*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53009, size = 2430, normalized size = 11.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(c*d)*a^2*b^6*c^7*abs(d) - 13*sqrt(c*d)*a^3*b^5*c^6*d*abs(d) + 1
0*sqrt(c*d)*a^4*b^4*c^5*d^2*abs(d) + 10*sqrt(c*d)*a^5*b^3*c^4*d^3*abs(d) -
13*sqrt(c*d)*a^6*b^2*c^3*d^4*abs(d) + 3*sqrt(c*d)*a^7*b*c^2*d^5*abs(d) - 3*
sqrt(c*d)*b^3*c^2*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5
*c^2*d^3)*abs(d) + 4*sqrt(c*d)*a*b^2*c*d*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d
+ 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*abs(d) - 3*sqrt(c*d)*a^2*b*d^2*abs(a^2*b^3
*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*abs(d))*arctan(2*sq
rt(1/2)*x/sqrt((a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3 +
 sqrt((a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)^2 - 4*(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2
 + a^4*b*c^2*d^3)))/(a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)))/(
a^2*b^3*c^5*d*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2
*d^3) - a^3*b^2*c^4*d^2*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2
```

```
 - a^5*c^2*d^3) - a^4*b*c^3*d^3*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b
*c^3*d^2 - a^5*c^2*d^3) + a^5*c^2*d^4*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3
*a^4*b*c^3*d^2 - a^5*c^2*d^3) + (a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^
3*d^2 - a^5*c^2*d^3)^2*d) - 1/2*(3*sqrt(a*b)*a^2*b^5*c^7*d*abs(b) - 13*sqrt
(a*b)*a^3*b^4*c^6*d^2*abs(b) + 10*sqrt(a*b)*a^4*b^3*c^5*d^3*abs(b) + 10*sqr
t(a*b)*a^5*b^2*c^4*d^4*abs(b) - 13*sqrt(a*b)*a^6*b*c^3*d^5*abs(b) + 3*sqrt(
a*b)*a^7*c^2*d^6*abs(b) + 3*sqrt(a*b)*b^2*c^2*d*abs(a^2*b^3*c^5 - 3*a^3*b^2
*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*abs(b) - 4*sqrt(a*b)*a*b*c*d^2*abs(
a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*abs(b) + 3*s
qrt(a*b)*a^2*d^3*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*
c^2*d^3)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3*c^5 - a^3*b^2*c^4*d - a
^4*b*c^3*d^2 + a^5*c^2*d^3 - sqrt((a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*
d^2 + a^5*c^2*d^3)^2 - 4*(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(a^2*b
^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)))/(a^2*b^3*c^4*d - 2*a^3*b^2*
c^3*d^2 + a^4*b*c^2*d^3)))/(a^2*b^4*c^5*abs(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d +
 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) - a^3*b^3*c^4*d*abs(a^2*b^3*c^5 - 3*a^3*b^2
*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) - a^4*b^2*c^3*d^2*abs(a^2*b^3*c^5 -
 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) + a^5*b*c^2*d^3*abs(a^2*b
^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) - (a^2*b^3*c^5 -
3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)^2*b) - 1/2*(3*b^3*c^2*d*x^
4 - 4*a*b^2*c*d^2*x^4 + 3*a^2*b*d^3*x^4 + 3*b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2
 - 2*a^2*b*c*d^2*x^2 + 3*a^3*d^3*x^2 + 2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*
c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^5 + b*c*x^3 + a*
d*x^3 + a*c*x))
```

3.308 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{b^3}{2a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{1}{2a^2c^2x^2}-\frac{d^3}{2c^2\left(c+dx^2\right)(bc-ad)^2}+\frac{d^3 \ldots$$

```
[Out] -1/(2*a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c^3) + (b^3*(b*c - 2*
a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d^3*(2*b*c - a*d)*Log[c + d*x^2
])/(c^3*(b*c - a*d)^3)
```

Rubi [A] time = 0.20896, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^3}{2a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{1}{2a^2c^2x^2}-\frac{d^3}{2c^2\left(c+dx^2\right)(bc-ad)^2}+\frac{d^3 \ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] -1/(2*a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c^3) + (b^3*(b*c - 2*
a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d^3*(2*b*c - a*d)*Log[c + d*x^2
])/(c^3*(b*c - a*d)^3)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^2x^2}-\frac{2(bc+ad)}{a^3c^3x}+\frac{b^4}{a^2(-bc+ad)^2(a+bx)^2}+\frac{2b^4(-bc+2ad)}{a^3(-bc+ad)^3(a+bx \ldots\\ &= -\frac{1}{2a^2c^2x^2}-\frac{b^3}{2a^2(bc-ad)^2\left(a+bx^2\right)}-\frac{d^3}{2c^2(bc-ad)^2\left(c+dx^2\right)}-\frac{2(bc+ad)\log(x)}{a^3c^3}+ \ldots\end{aligned}$$

Mathematica [A] time = 0.200367, size = 157, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{b^3}{a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{2b^3(2ad-bc)\log\left(a+bx^2\right)}{a^3(ad-bc)^3}-\frac{4\log(x)(ad+bc)}{a^3c^3}-\frac{1}{a^2c^2x^2}-\frac{d^3}{c^2\left(c+dx^2\right)(bc-ad)^2}+\frac{2d^3(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(c^2*(b*c -
 a*d)^2*(c + d*x^2)) - (4*(b*c + a*d)*Log[x])/(a^3*c^3) + (2*b^3*(-(b*c) +
2*a*d)*Log[a + b*x^2])/(a^3*(-(b*c) + a*d)^3) + (2*d^3*(2*b*c - a*d)*Log[c
+ d*x^2])/(c^3*(b*c - a*d)^3))/2
```

Maple [A] time = 0.024, size = 254, normalized size = 1.6

$$\frac{d^4\ln\left(dx^2+c\right)a}{c^3\left(ad-bc\right)^3}-2\frac{d^3\ln\left(dx^2+c\right)b}{c^2\left(ad-bc\right)^3}-\frac{d^4a}{2c^2\left(ad-bc\right)^3\left(dx^2+c\right)}+\frac{d^3b}{2c\left(ad-bc\right)^3\left(dx^2+c\right)}-\frac{1}{2a^2c^2x^2}-2\frac{\ln(x)d}{a^2c^3}-2\frac{\ln}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] d^4/c^3/(a*d-b*c)^3*ln(d*x^2+c)*a-2*d^3/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b-1/2*d
^4/c^2/(a*d-b*c)^3/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)*b-1/2/a^2/c^
2/x^2-2/a^2/c^3*ln(x)*d-2/a^3/c^2*ln(x)*b+2*b^3/a^2/(a*d-b*c)^3*ln(b*x^2+a)
*d-b^4/a^3/(a*d-b*c)^3*ln(b*x^2+a)*c-1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*d+1/2*
b^4/a^2/(a*d-b*c)^3/(b*x^2+a)*c
```

Maxima [B] time = 1.06165, size = 514, normalized size = 3.29

$$\frac{\left(b^4c-2ab^3d\right)\log\left(bx^2+a\right)}{a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3}+\frac{\left(2bcd^3-ad^4\right)\log\left(dx^2+c\right)}{b^3c^6-3ab^2c^5d+3a^2bc^4d^2-a^3c^3d^3}-\frac{ab^2c^3-2a^2bc^2d+a^3cd^2+2\left(b^3\right.}{2\left(\left(a^2b^3c^4d-2a^3b^2c^3d^2+a^4bc^2d^3\right)x^6+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^4*c - 2*a*b^3*d)*log(b*x^2 + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b
*c*d^2 - a^6*d^3) + (2*b*c*d^3 - a*d^4)*log(d*x^2 + c)/(b^3*c^6 - 3*a*b^2*c
^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^
3*c*d^2 + 2*(b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (2*b^3*c^3 - a*b^2*
c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 +
 a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^
2*d^3)*x^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x^2) - (b*c + a*d)
*log(x^2)/(a^3*c^3)
```

Fricas [B] time = 53.814, size = 1287, normalized size = 8.25

$$-\frac{a^2b^3c^5-3a^3b^2c^4d+3a^4bc^3d^2-a^5c^2d^3+2\left(ab^4c^4d-2a^2b^3c^3d^2+2a^3b^2c^2d^3-a^4bcd^4\right)x^4+\left(2ab^4c^5-3a^2b^3c^4d+3a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + 2*(a*
b^4*c^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (2*a
*b^4*c^5 - 3*a^2*b^3*c^4*d + 3*a^4*b*c^2*d^3 - 2*a^5*c*d^4)*x^2 - 2*((b^5*c
^4*d - 2*a*b^4*c^3*d^2)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2)*x
^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d)*x^2)*log(b*x^2 + a) - 2*((2*a^3*b^2*c*d^
4 - a^4*b*d^5)*x^6 + (2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (2*a
^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(d*x^2 + c) + 4*((b^5*c^4*d - 2*a*b^4*c^3
*d^2 + 2*a^3*b^2*c*d^4 - a^4*b*d^5)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^
3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (a*b^4*c^5 - 2
*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(x))/((a^3*b^4*c^6*d
- 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^6 + (a^3*b^4*c^7
 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^4 + (a^4*b^3*c^7 - 3*
a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.309 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=271

$$-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}$$

```
[Out] -(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((b*c
+ a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*x) + (
d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x
^3*(a + b*x^2)*(c + d*x^2)) + (b^(7/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2)*(b*c - a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.448272, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((b*c
+ a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*x) + (
d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x
^3*(a + b*x^2)*(c + d*x^2)) + (b^(7/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2)*(b*c - a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^3)
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-5bc+2ad-7bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2\left(5b^2c^2-4abcd+5a^2d\right.}{x^4\left(a+bx^2\right)(}}{4ac(bc-}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+\right.}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+\right.}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+\right.}$$

Mathematica [A] time = 0.371645, size = 178, normalized size = 0.66

$$\frac{1}{6}\left(\frac{3b^4x}{a^3\left(a+bx^2\right)(bc-ad)^2} + \frac{3b^{7/2}(9ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)^3} + \frac{12(ad+bc)}{a^3c^3x} - \frac{2}{a^2c^2x^3} + \frac{3d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2} + \frac{3d^{7/2}(9}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x^3) + (12*(b*c + a*d))/(a^3*c^3*x) + (3*b^4*x)/(a^3*(b*c - a*
d)^2*(a + b*x^2)) + (3*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (3*b^(7/2)*
```

```
(-5*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c) + a*d)^3) +
(3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d
)^3))/6
```

Maple [A] time = 0.02, size = 285, normalized size = 1.1

$$\frac{d^5xa}{2\,c^3\,(ad-bc)^3\,(dx^2+c)}-\frac{d^4xb}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}+\frac{5\,d^5a}{2\,c^3\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{9\,d^4b}{2\,c^2\,(ad-bc)^3}\arctan\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^5/c^3/(a*d-b*c)^3*x/(d*x^2+c)*a-1/2*d^4/c^2/(a*d-b*c)^3*x/(d*x^2+c)*b
+5/2*d^5/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-9/2*d^4/c^2/
(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b-1/3/a^2/c^2/x^3+2/a^2/c^3
/x*d+2/a^3/c^2/x*b+1/2*b^4/a^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*b^5/a^3/(a*d-b
*c)^3*x/(b*x^2+a)*c+9/2*b^4/a^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*d-5/2*b^5/a^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 27.7042, size = 4880, normalized size = 18.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3
 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 -
2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 +
17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^
5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c
^4*d)*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(
(9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 -
5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log((d*x^2
 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2
+ 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d +
 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^
6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d +
12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3
```

```
*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3
*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^
4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((9*a^3*b^2*
c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 - 5*a^5*d^5)*
x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) -
3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2
*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d
^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*
d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3
*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d
 + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*
b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a
*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((5*b^5*c
^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^
2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a))
 - 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*
d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log(
(d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^
5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c
^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d
+ 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/6*(2*a^2*b^3*c^5 - 6*a^3*b^2*c^4*
d + 6*a^4*b*c^3*d^2 - 2*a^5*c^2*d^3 - 3*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - (15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^
3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 10*(a*b
^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4
*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)
*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^
4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(
x*sqrt(d/c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6
*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^
3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3
)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64454, size = 2608, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(c*d)*a^3*b^7*c^9*abs(d) - 19*sqrt(c*d)*a^4*b^6*c^8*d*abs(d) +
23*sqrt(c*d)*a^5*b^5*c^7*d^2*abs(d) - 18*sqrt(c*d)*a^6*b^4*c^6*d^3*abs(d) +
 23*sqrt(c*d)*a^7*b^3*c^5*d^4*abs(d) - 19*sqrt(c*d)*a^8*b^2*c^4*d^5*abs(d)
+ 5*sqrt(c*d)*a^9*b*c^3*d^6*abs(d) - 5*sqrt(c*d)*b^4*c^3*abs(a^3*b^3*c^6 -
3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*abs(d) + 4*sqrt(c*d)*a*b^3
*c^2*d*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*a
bs(d) + 4*sqrt(c*d)*a^2*b^2*c*d^2*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5
*b*c^4*d^2 - a^6*c^3*d^3)*abs(d) - 5*sqrt(c*d)*a^3*b*d^3*abs(a^3*b^3*c^6 -
3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*abs(d))*arctan(2*sqrt(1/2)
*x/sqrt((a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3 + sqrt((
a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)^2 - 4*(a^4*b^2*c
^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*
b*c^3*d^3)))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)))/(a^3*b^3
*c^6*d*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3) -
 a^4*b^2*c^5*d^2*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*
c^3*d^3) - a^5*b*c^4*d^3*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^
2 - a^6*c^3*d^3) + a^6*c^3*d^4*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*
c^4*d^2 - a^6*c^3*d^3) + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 -
 a^6*c^3*d^3)^2*d) + 1/2*(5*sqrt(a*b)*a^3*b^6*c^9*d*abs(b) - 19*sqrt(a*b)*a
^4*b^5*c^8*d^2*abs(b) + 23*sqrt(a*b)*a^5*b^4*c^7*d^3*abs(b) - 18*sqrt(a*b)*
a^6*b^3*c^6*d^4*abs(b) + 23*sqrt(a*b)*a^7*b^2*c^5*d^5*abs(b) - 19*sqrt(a*b)
*a^8*b*c^4*d^6*abs(b) + 5*sqrt(a*b)*a^9*c^3*d^7*abs(b) + 5*sqrt(a*b)*b^3*c^
3*d*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*abs(
b) - 4*sqrt(a*b)*a*b^2*c^2*d^2*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*
c^4*d^2 - a^6*c^3*d^3)*abs(b) - 4*sqrt(a*b)*a^2*b*c*d^3*abs(a^3*b^3*c^6 - 3
*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*abs(b) + 5*sqrt(a*b)*a^3*d^
4*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*abs(b)
)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 +
a^6*c^3*d^3 - sqrt((a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d
^3)^2 - 4*(a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*(a^3*b^3*c^5*d - 2*a^
4*b^2*c^4*d^2 + a^5*b*c^3*d^3)))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3)))/(a^3*b^4*c^6*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^
2 - a^6*c^3*d^3) - a^4*b^3*c^5*d*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*
b*c^4*d^2 - a^6*c^3*d^3) - a^5*b^2*c^4*d^2*abs(a^3*b^3*c^6 - 3*a^4*b^2*c^5*
d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3) + a^6*b*c^3*d^3*abs(a^3*b^3*c^6 - 3*a^4*
b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3) - (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d
 + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)^2*b) + 1/2*(b^4*c^3*d*x^3 + a^3*b*d^4*x^3
 + b^4*c^4*x + a^4*d^4*x)/((a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(b*d
*x^4 + b*c*x^2 + a*d*x^2 + a*c)) + 1/3*(6*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^3*c
^3*x^3)
```

3.310 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc}$$

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(b*c - a*d)^3*(c + d*
x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c
 - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*Sqrt[d]*(b*c - a*d)^4)
```

Rubi [A] time = 0.276969, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(b*c - a*d)^3*(c + d*
x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c
 - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*Sqrt[d]*(b*c - a*d)^4)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{ac+(-2bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{6abc^2-6bc(bc+2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{8bc(bc-ad)^2}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3\left(c+dx^2\right)} - \int 6ab$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3\left(c+dx^2\right)} - (3ab$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3\left(c+dx^2\right)} - 3\sqrt{a}$$

Mathematica [A] time = 0.353174, size = 166, normalized size = 0.8

$$\frac{\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}} + \frac{2cx(bc-ad)^2}{\left(c+dx^2\right)^2} + \frac{4abx(bc-ad)}{a+bx^2} + \frac{x(5ad+3bc)(bc-ad)}{c+dx^2} - 12\sqrt{a}\sqrt{b}(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*a*b*(b*c - a*d)*x)/(a + b*x^2) + (2*c*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
((b*c - a*d)*(3*b*c + 5*a*d)*x)/(c + d*x^2) - 12*Sqrt[a]*Sqrt[b]*(b*c + a*d
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(8*(b*c - a*d)^4)
```

Maple [B] time = 0.013, size = 388, normalized size = 1.9

$$-\frac{5x^3a^2d^3}{8(ad-bc)^4\left(dx^2+c\right)^2} + \frac{x^3abcd^2}{4(ad-bc)^4\left(dx^2+c\right)^2} + \frac{3x^3b^2c^2d}{8(ad-bc)^4\left(dx^2+c\right)^2} - \frac{3a^2cd^2x}{8(ad-bc)^4\left(dx^2+c\right)^2} - \frac{abc^2dx}{4(ad-bc)^4(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -5/8/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a^2*d^3+1/4/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*
b*c*d^2+3/8/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2*c^2*d-3/8/(a*d-b*c)^4/(d*x^2+c)
^2*a^2*c*d^2*x-1/4/(a*d-b*c)^4/(d*x^2+c)^2*a*b*c^2*d*x+5/8/(a*d-b*c)^4/(d*x
^2+c)^2*b^2*c^3*x+3/8/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d
^2+9/4/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b*d+3/8/(a*d-b*c
)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2-1/2*a^2*b/(a*d-b*c)^4*x/(b*
x^2+a)*d+1/2*a*b^2/(a*d-b*c)^4*x/(b*x^2+a)*c-3/2*a^2*b/(a*d-b*c)^4/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*d-3/2*a*b^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.2699, size = 5736, normalized size = 27.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4
*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 + 12*(a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d
^3 + a^2*c*d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 3*(a*b^2*c^4 + 6*a^2
*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2
*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8
*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 -
 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2 - a^
3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*a^4*
b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^3*d^
5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5*d^3
 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)*x^4
 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4 - 7
*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3
 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 -
5*a^3*c*d^4)*x^3 + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^
2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^
2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*
a^3*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 6*(a*b*c^4*d + a^2*c^3*d^
2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*
d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*
d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3
- 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^
3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4
*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*
```

```
d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4
*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/16*(6*(b^3*c^3*d^2 + 2*a*b^
2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b
*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 24*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 +
a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + (b^2*c^4
*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 +
 a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4
)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d -
2*a^2*b*c^3*d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*
b^2*c^5*d^3 - 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^
4 + 6*a^2*b^3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d
^2 - 7*a*b^4*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*
d^6 + a^5*c*d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8
*a^3*b^2*c^4*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d
^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2
- 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 12*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c
^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 +
 (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^
2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3
+ a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)
*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2
- a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*
a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^
3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5
*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)
*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4
 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2)]
```

Sympy [B] time = 146.596, size = 4041, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] 3*sqrt(-a*b)*(a*d + b*c)*log(x + (-432*a**10*c*d**11*(-a*b)**(3/2)*(a*d + b
*c)**3/(a*d - b*c)**12 - 864*a**9*b*c**2*d**10*(-a*b)**(3/2)*(a*d + b*c)**3
/(a*d - b*c)**12 + 20304*a**8*b**2*c**3*d**9*(-a*b)**(3/2)*(a*d + b*c)**3/(
a*d - b*c)**12 - 79488*a**7*b**3*c**4*d**8*(-a*b)**(3/2)*(a*d + b*c)**3/(a*
d - b*c)**12 + 151200*a**6*b**4*c**5*d**7*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d
 - b*c)**12 - 27*a**6*d**6*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 - 157248*a
**5*b**5*c**6*d**6*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 - 486*a**5*
b*c*d**5*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 + 78624*a**4*b**6*c**7*d**5*
(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 - 4725*a**4*b**2*c**2*d**4*sqr
t(-a*b)*(a*d + b*c)/(a*d - b*c)**4 + 3456*a**3*b**7*c**8*d**4*(-a*b)**(3/2)
*(a*d + b*c)**3/(a*d - b*c)**12 - 11988*a**3*b**3*c**3*d**3*sqrt(-a*b)*(a*d
 + b*c)/(a*d - b*c)**4 - 26352*a**2*b**8*c**9*d**3*(-a*b)**(3/2)*(a*d + b*c
)**3/(a*d - b*c)**12 - 8181*a**2*b**4*c**4*d**2*sqrt(-a*b)*(a*d + b*c)/(a*d
 - b*c)**4 + 12960*a*b**9*c**10*d**2*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*
c)**12 - 2214*a*b**5*c**5*d*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 - 2160*b*
*10*c**11*d*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 - 27*b**6*c**6*sqr
t(-a*b)*(a*d + b*c)/(a*d - b*c)**4)/(27*a**3*b*d**3 + 189*a**2*b**2*c*d**2
+ 189*a*b**3*c**2*d + 27*b**4*c**3))/(4*(a*d - b*c)**4) - 3*sqrt(-a*b)*(a*d
```

```
 + b*c)*log(x + (432*a**10*c*d**11*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)
**12 + 864*a**9*b*c**2*d**10*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 -
 20304*a**8*b**2*c**3*d**9*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 + 7
9488*a**7*b**3*c**4*d**8*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 - 151
200*a**6*b**4*c**5*d**7*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 + 27*a
**6*d**6*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 + 157248*a**5*b**5*c**6*d**6
*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 + 486*a**5*b*c*d**5*sqrt(-a*b
)*(a*d + b*c)/(a*d - b*c)**4 - 78624*a**4*b**6*c**7*d**5*(-a*b)**(3/2)*(a*d
 + b*c)**3/(a*d - b*c)**12 + 4725*a**4*b**2*c**2*d**4*sqrt(-a*b)*(a*d + b*c
)/(a*d - b*c)**4 - 3456*a**3*b**7*c**8*d**4*(-a*b)**(3/2)*(a*d + b*c)**3/(a
*d - b*c)**12 + 11988*a**3*b**3*c**3*d**3*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c
)**4 + 26352*a**2*b**8*c**9*d**3*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**
12 + 8181*a**2*b**4*c**4*d**2*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 - 12960
*a*b**9*c**10*d**2*(-a*b)**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 + 2214*a*b*
*5*c**5*d*sqrt(-a*b)*(a*d + b*c)/(a*d - b*c)**4 + 2160*b**10*c**11*d*(-a*b)
**(3/2)*(a*d + b*c)**3/(a*d - b*c)**12 + 27*b**6*c**6*sqrt(-a*b)*(a*d + b*c
)/(a*d - b*c)**4)/(27*a**3*b*d**3 + 189*a**2*b**2*c*d**2 + 189*a*b**3*c**2*
d + 27*b**4*c**3))/(4*(a*d - b*c)**4) + 3*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b
*c*d + b**2*c**2)*log(x + (-27*a**10*c*d**11*(-1/(c*d))**(3/2)*(a**2*d**2 +
 6*a*b*c*d + b**2*c**2)**3/(4*(a*d - b*c)**12) - 27*a**9*b*c**2*d**10*(-1/(
c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(2*(a*d - b*c)**12) + 1
269*a**8*b**2*c**3*d**9*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**
2)**3/(4*(a*d - b*c)**12) - 1242*a**7*b**3*c**4*d**8*(-1/(c*d))**(3/2)*(a**
2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(a*d - b*c)**12 + 4725*a**6*b**4*c**5*d*
*7*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(2*(a*d - b*c)*
*12) - 27*a**6*d**6*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(4*(
a*d - b*c)**4) - 2457*a**5*b**5*c**6*d**6*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*
a*b*c*d + b**2*c**2)**3/(a*d - b*c)**12 - 243*a**5*b*c*d**5*sqrt(-1/(c*d))*
(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(2*(a*d - b*c)**4) + 2457*a**4*b**6*c**
7*d**5*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(2*(a*d - b
*c)**12) - 4725*a**4*b**2*c**2*d**4*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d +
 b**2*c**2)/(4*(a*d - b*c)**4) + 54*a**3*b**7*c**8*d**4*(-1/(c*d))**(3/2)*(
a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(a*d - b*c)**12 - 2997*a**3*b**3*c**3
*d**3*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(a*d - b*c)**4 - 1
647*a**2*b**8*c**9*d**3*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**
2)**3/(4*(a*d - b*c)**12) - 8181*a**2*b**4*c**4*d**2*sqrt(-1/(c*d))*(a**2*d
**2 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**4) + 405*a*b**9*c**10*d**2*(-1
/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(2*(a*d - b*c)**12) -
 1107*a*b**5*c**5*d*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(2*(
a*d - b*c)**4) - 135*b**10*c**11*d*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d
 + b**2*c**2)**3/(4*(a*d - b*c)**12) - 27*b**6*c**6*sqrt(-1/(c*d))*(a**2*d*
*2 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**4))/(27*a**3*b*d**3 + 189*a**2*
b**2*c*d**2 + 189*a*b**3*c**2*d + 27*b**4*c**3))/(16*(a*d - b*c)**4) - 3*sq
rt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)*log(x + (27*a**10*c*d**11*
(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(4*(a*d - b*c)**12
) + 27*a**9*b*c**2*d**10*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c*
*2)**3/(2*(a*d - b*c)**12) - 1269*a**8*b**2*c**3*d**9*(-1/(c*d))**(3/2)*(a*
*2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(4*(a*d - b*c)**12) + 1242*a**7*b**3*c*
*4*d**8*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(a*d - b*c
)**12 - 4725*a**6*b**4*c**5*d**7*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d +
 b**2*c**2)**3/(2*(a*d - b*c)**12) + 27*a**6*d**6*sqrt(-1/(c*d))*(a**2*d**2
 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**4) + 2457*a**5*b**5*c**6*d**6*(-1
/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(a*d - b*c)**12 + 243
*a**5*b*c*d**5*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(2*(a*d -
 b*c)**4) - 2457*a**4*b**6*c**7*d**5*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c
*d + b**2*c**2)**3/(2*(a*d - b*c)**12) + 4725*a**4*b**2*c**2*d**4*sqrt(-1/(
c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**4) - 54*a**3*b**7
*c**8*d**4*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(a*d -
b*c)**12 + 2997*a**3*b**3*c**3*d**3*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d +
```

```
 b**2*c**2)/(a*d - b*c)**4 + 1647*a**2*b**8*c**9*d**3*(-1/(c*d))**(3/2)*(a*
*2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(4*(a*d - b*c)**12) + 8181*a**2*b**4*c*
*4*d**2*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**
4) - 405*a*b**9*c**10*d**2*(-1/(c*d))**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*
c**2)**3/(2*(a*d - b*c)**12) + 1107*a*b**5*c**5*d*sqrt(-1/(c*d))*(a**2*d**2
 + 6*a*b*c*d + b**2*c**2)/(2*(a*d - b*c)**4) + 135*b**10*c**11*d*(-1/(c*d))
**(3/2)*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)**3/(4*(a*d - b*c)**12) + 27*b**
6*c**6*sqrt(-1/(c*d))*(a**2*d**2 + 6*a*b*c*d + b**2*c**2)/(4*(a*d - b*c)**4
))/(27*a**3*b*d**3 + 189*a**2*b**2*c*d**2 + 189*a*b**3*c**2*d + 27*b**4*c**
3))/(16*(a*d - b*c)**4) - (x**5*(9*a*b*d**2 + 3*b**2*c*d) + x**3*(5*a**2*d*
*2 + 14*a*b*c*d + 5*b**2*c**2) + x*(3*a**2*c*d + 9*a*b*c**2))/(8*a**4*c**2*
d**3 - 24*a**3*b*c**3*d**2 + 24*a**2*b**2*c**4*d - 8*a*b**3*c**5 + x**6*(8*
a**3*b*d**5 - 24*a**2*b**2*c*d**4 + 24*a*b**3*c**2*d**3 - 8*b**4*c**3*d**2)
 + x**4*(8*a**4*d**5 - 8*a**3*b*c*d**4 - 24*a**2*b**2*c**2*d**3 + 40*a*b**3
*c**3*d**2 - 16*b**4*c**4*d) + x**2*(16*a**4*c*d**4 - 40*a**3*b*c**2*d**3 +
 24*a**2*b**2*c**3*d**2 + 8*a*b**3*c**4*d - 8*b**4*c**5))
```

Giac [A] time = 1.16043, size = 406, normalized size = 1.96

$$\frac{abx}{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\left(bx^2 + a\right)} - \frac{3\left(ab^2c + a^2bd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\sqrt{ab}} + \frac{3\left(b^2c\right.}{8\left(b^4c^4 - 4\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a*b*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a))
 - 3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 +
6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3 + 5*
a*d^2*x^3 + 5*b*c^2*x + 3*a*c*d*x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*(d*x^2 + c)^2)
```

3.311 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=142

$$\frac{ab}{2\left(a+bx^2\right)(bc-ad)^3}+\frac{ad+bc}{2\left(c+dx^2\right)(bc-ad)^3}+\frac{c}{4\left(c+dx^2\right)^2(bc-ad)^2}+\frac{b(2ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log}{2(bc-}$$

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*a*d)*Log[a + b*x^2])
/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rubi [A] time = 0.152556, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{ab}{2\left(a+bx^2\right)(bc-ad)^3}+\frac{ad+bc}{2\left(c+dx^2\right)(bc-ad)^3}+\frac{c}{4\left(c+dx^2\right)^2(bc-ad)^2}+\frac{b(2ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log}{2(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*a*d)*Log[a + b*x^2])
/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab^2}{(bc-ad)^3(a+bx)^2}+\frac{b^2(bc+2ad)}{(bc-ad)^4(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)^3}-\frac{d}{(bc-}\right.\right.\\ &= \frac{ab}{2(bc-ad)^3\left(a+bx^2\right)}+\frac{c}{4(bc-ad)^2\left(c+dx^2\right)^2}+\frac{bc+ad}{2(bc-ad)^3\left(c+dx^2\right)}+\frac{b(bc+2ad)\log}{2(bc-}\end{aligned}$$

Mathematica [A] time = 0.0965977, size = 121, normalized size = 0.85

$$\frac{\frac{c(bc-ad)^2}{(c+dx^2)^2}+\frac{2ab(bc-ad)}{a+bx^2}+\frac{2(ad+bc)(bc-ad)}{c+dx^2}+2b(2ad+bc)\log\left(a+bx^2\right)-2b(2ad+bc)\log\left(c+dx^2\right)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((2*a*b*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d)^2)/(c + d*x^2)^2 + (2*(b*
c - a*d)*(b*c + a*d))/(c + d*x^2) + 2*b*(b*c + 2*a*d)*Log[a + b*x^2] - 2*b*
(b*c + 2*a*d)*Log[c + d*x^2])/(4*(b*c - a*d)^4)
```

Maple [B] time = 0.017, size = 283, normalized size = 2.

$$-\frac{bd\ln\left(dx^2+c\right)a}{(ad-bc)^4}-\frac{b^2\ln\left(dx^2+c\right)c}{2\,(ad-bc)^4}+\frac{a^2cd^2}{4\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{abc^2d}{2\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{b^2c^3}{4\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{}{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -d/(a*d-b*c)^4*b*ln(d*x^2+c)*a-1/2/(a*d-b*c)^4*b^2*ln(d*x^2+c)*c+1/4*d^2/(a
*d-b*c)^4*c/(d*x^2+c)^2*a^2-1/2*d/(a*d-b*c)^4*c^2/(d*x^2+c)^2*a*b+1/4/(a*d-
b*c)^4*c^3/(d*x^2+c)^2*b^2-1/2*d^2/(a*d-b*c)^4/(d*x^2+c)*a^2+1/2/(a*d-b*c)^
4/(d*x^2+c)*b^2*c^2+b/(a*d-b*c)^4*ln(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^4*ln(b*
x^2+a)*c-1/2*b/(a*d-b*c)^4*a^2/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^4*a/(b*x^2+a)*
c
```

Maxima [B] time = 1.06748, size = 560, normalized size = 3.94

$$\frac{\left(b^2c+2\,abd\right)\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}-\frac{\left(b^2c+2\,abd\right)\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}+\frac{}{4\left(ab^3c^5-3\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 2*a*b*d)*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c
^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(b^2*c + 2*a*b*d)*log(d*x^2 + c)/(b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/4
*(2*(b^2*c*d + 2*a*b*d^2)*x^4 + 5*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 7*a*b*c*
d + 2*a^2*d^2)*x^2)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^
2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
 (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5
)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^
4*c*d^4)*x^2)
```

Fricas [B] time = 1.47233, size = 1200, normalized size = 8.45

$$\frac{5\,ab^2c^3 - 4\,a^2bc^2d - a^3cd^2 + 2\left(b^3c^2d + ab^2cd^2 - 2\,a^2bd^3\right)x^4 + \left(3\,b^3c^3 + 4\,ab^2c^2d - 5\,a^2bcd^2 - 2\,a^3d^3\right)x^2 + 2\left(\left(b^3cd^2 + \right.\right.}{4\left(ab^4c^6 - 4\,a^2b^3c^5d + 6\,a^3b^2c^4d^2 - 4\,a^4bc^3d^3 + a^5c^2d^4 + \left(b^5c^4d^2 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a*b^2*c^3 - 4*a^2*b*c^2*d - a^3*c*d^2 + 2*(b^3*c^2*d + a*b^2*c*d^2 -
 2*a^2*b*d^3)*x^4 + (3*b^3*c^3 + 4*a*b^2*c^2*d - 5*a^2*b*c*d^2 - 2*a^3*d^3)
*x^2 + 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*c^3 + 2*a^2*b*c^2*d + (2*b^
3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^3*c^3 + 4*a*b^2*c^2*d + 4*a
^2*b*c*d^2)*x^2)*log(b*x^2 + a) - 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*
c^3 + 2*a^2*b*c^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^
3*c^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c^6 - 4*
a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^
4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*
x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^
4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4
*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [B] time = 15.8017, size = 780, normalized size = 5.49

$$-\frac{b\left(2ad+bc\right)\log\left(x^2+\frac{-\frac{a^5bd^5(2ad+bc)}{(ad-bc)^4}+\frac{5a^4b^2cd^4(2ad+bc)}{(ad-bc)^4}-\frac{10a^3b^3c^2d^3(2ad+bc)}{(ad-bc)^4}+\frac{10a^2b^4c^3d^2(2ad+bc)}{(ad-bc)^4}+2a^2bd^2-\frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4}+3ab^2cd+\frac{b^6c^5(2ad+bc)}{(ad-bc)^4}+b^3c^2}{4ab^2d^2+2b^3cd}\right.}{2\left(ad-bc\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -b*(2*a*d + b*c)*log(x**2 + (-a**5*b*d**5*(2*a*d + b*c)/(a*d - b*c)**4 + 5*
a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**3*b**3*c**2*d**3*(2*a
*d + b*c)/(a*d - b*c)**4 + 10*a**2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b*c)
**4 + 2*a**2*b*d**2 - 5*a*b**5*c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b*
*2*c*d + b**6*c**5*(2*a*d + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2*d**2
 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + b*(2*a*d + b*c)*log(x**2 + (a**5*b*d**
5*(2*a*d + b*c)/(a*d - b*c)**4 - 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*
c)**4 + 10*a**3*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**2*b**4*
c**3*d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*d**2 + 5*a*b**5*c**4*d*(2
*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*d - b**6*c**5*(2*a*d + b*c)/(a*d -
b*c)**4 + b**3*c**2)/(4*a*b**2*d**2 + 2*b**3*c*d))/(2*(a*d - b*c)**4) - (a*
*2*c*d + 5*a*b*c**2 + x**4*(4*a*b*d**2 + 2*b**2*c*d) + x**2*(2*a**2*d**2 +
7*a*b*c*d + 3*b**2*c**2))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a**2
*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4 +
12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c*d*
*4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**2*(
8*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*c**
4*d - 4*b**4*c**5))
```

Giac [B] time = 1.18758, size = 360, normalized size = 2.54

$$\frac{\frac{2\,ab^5}{\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)\left(bx^2+a\right)}-\frac{2\left(b^4c+2\,ab^3d\right)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{3\,b^3cd^2+2\,ab^2d^3+\frac{2\left(2\,b^5c^2d-ab^4cd^2-a^2b^3d^3\right)}{\left(bx^2+a\right)b}}{(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*
x^2 + a)) - 2*(b^4*c + 2*a*b^3*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
 + d))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4) - (3*b^3*c*d^2 + 2*a*b^2*d^3 + 2*(2*b^5*c^2*d - a*b^4*c*d^2 - a^2*b
^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
+ d)^2))/b
```

3.312 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(b}$$

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a + b*x^2)*(c
+ d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(c + d*x^2)) + (b^(3/
2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)^4) - (
Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8
*c^(3/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.252424, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a + b*x^2)*(c
+ d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(c + d*x^2)) + (b^(3/
2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)^4) - (
Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8
*c^(3/2)*(b*c - a*d)^4)
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\int \frac{c-5dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2(bc-ad)}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\int \frac{2c(2bc+ad)-18bcdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{8c(bc-ad)^2}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(11bc+ad)x}{8c(bc-ad)^3\left(c+dx^2\right)} + \frac{\int \frac{2c(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(11bc+ad)x}{8c(bc-ad)^3\left(c+dx^2\right)} + \frac{\left(b^2(\ldots\right.}{\ldots}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(11bc+ad)x}{8c(bc-ad)^3\left(c+dx^2\right)} + \frac{b^{3/2}(\ldots}{\ldots}$$

Mathematica [A] time = 0.401621, size = 171, normalized size = 0.86

$$\frac{\frac{\sqrt{d}\left(a^2d^2-10abcd-15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}} - \frac{4b^2x(bc-ad)}{a+bx^2} + \frac{4b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{dx(ad-bc)(ad+7bc)}{c\left(c+dx^2\right)} - \frac{2dx(bc-ad)^2}{\left(c+dx^2\right)^2}}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^2*(b*c - a*d)*x)/(a + b*x^2) - (2*d*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
 (d*(-(b*c) + a*d)*(7*b*c + a*d)*x)/(c*(c + d*x^2)) + (4*b^(3/2)*(b*c + 5*a
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-15*b^2*c^2 - 10*a*b*c
*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(8*(b*c - a*d)^4)
```

Maple [B] time = 0.015, size = 391, normalized size = 2.

$$\frac{d^4x^3a^2}{8\,(ad-bc)^4\left(dx^2+c\right)^2c} + \frac{3\,d^3x^3ab}{4\,(ad-bc)^4\left(dx^2+c\right)^2} - \frac{7\,d^2x^3b^2c}{8\,(ad-bc)^4\left(dx^2+c\right)^2} + \frac{5\,abcd^2x}{4\,(ad-bc)^4\left(dx^2+c\right)^2} - \frac{9\,b^2c^2dx}{8\,(ad-bc)^4\left(dx\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/8*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^3*a^2+3/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x
^3*a*b-7/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2*c+5/4*d^2/(a*d-b*c)^4/(d*x^2
+c)^2*c*a*b*x-9/8*d/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c^2*x-1/8*d^3/(a*d-b*c)^4/(
d*x^2+c)^2*a^2*x+1/8*d^3/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*
a^2-5/4*d^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-15/8*d/(a*d
-b*c)^4*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2+1/2*b^2/(a*d-b*c)^4*x/(b*
x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x/(b*x^2+a)*c+5/2*b^2/(a*d-b*c)^4/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))*a*d+1/2*b^3/(a*d-b*c)^4/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.21907, size = 5824, normalized size = 29.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^3
*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2
*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*
d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2
)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*a*b^2*c^
4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b
*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4
 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt
(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a
*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d
+ 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^
4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b
^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*
b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 +
 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c
^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2
 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*
d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 3
5*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3
*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2
*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^
3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10
*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4
*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4
+ (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 +
a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a
```

```
^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*
d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5
)*x^2), -1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*
b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 8*(a*b^2*c^4
+ 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b
^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d
^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a
^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^
3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*
b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*
c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*
c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 -
4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*
c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5
*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)
*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7
*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^
3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3
*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^
6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b
^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2
*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^
2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*
x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*s
qrt(d/c)*arctan(x*sqrt(d/c)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^
2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*
b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^
4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 +
 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 +
 (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b
*c^3*d^4 + 2*a^5*c^2*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19951, size = 428, normalized size = 2.14

$$-\frac{b^2x}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(bx^2+a\right)}+\frac{\left(b^3c+5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{ab}}-\frac{\left(15\,b\right.}{8\left(b^4c^5-4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)
) + 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
```

```
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^
2*d + 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*(7*b
*c*d^2*x^3 + a*d^3*x^3 + 9*b*c^2*d*x - a*c*d^2*x)/((b^3*c^4 - 3*a*b^2*c^3*d
 + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

3.313 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=126

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^2)^2) - (b
*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^2])/(2*(b*c - a*d)^4
) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rubi [A] time = 0.109223, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 44}

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^2)^2) - (b
*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^2])/(2*(b*c - a*d)^4
) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2bd}{(bc-ad)^3(\right.\right.\\ &= -\frac{b^2}{2(bc-ad)^3\left(a+bx^2\right)}-\frac{d}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{bd}{(bc-ad)^3\left(c+dx^2\right)}-\frac{3b^2d\log\left(a+bx^2\right.}{2(bc-ad)^4}\end{aligned}$$

Mathematica [A] time = 0.133882, size = 107, normalized size = 0.85

$$-\frac{\frac{2b^2(bc-ad)}{a+bx^2}+6b^2d\log\left(a+bx^2\right)+\frac{4bd(bc-ad)}{c+dx^2}+\frac{d(bc-ad)^2}{\left(c+dx^2\right)^2}-6b^2d\log\left(c+dx^2\right)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -((2*b^2*(b*c - a*d))/(a + b*x^2) + (d*(b*c - a*d)^2)/(c + d*x^2)^2 + (4*b*
d*(b*c - a*d))/(c + d*x^2) + 6*b^2*d*Log[a + b*x^2] - 6*b^2*d*Log[c + d*x^2
])/(4*(b*c - a*d)^4)
```

Maple [A] time = 0.016, size = 234, normalized size = 1.9

$$\frac{3\,d\ln\left(dx^2+c\right)b^2}{2\,(ad-bc)^4}-\frac{a^2d^3}{4\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{abcd^2}{2\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{b^2c^2d}{4\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{abd^2}{(ad-bc)^4\left(dx^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/2*d/(a*d-b*c)^4*ln(d*x^2+c)*b^2-1/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a^2+1/2*d
^2/(a*d-b*c)^4/(d*x^2+c)^2*c*a*b-1/4*d/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c^2+d^2/
(a*d-b*c)^4*b/(d*x^2+c)*a-d/(a*d-b*c)^4*b^2/(d*x^2+c)*c-3/2*b^2/(a*d-b*c)^4
*ln(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^4/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4/(b*x^2
+a)*c
```

Maxima [B] time = 1.08609, size = 532, normalized size = 4.22

$$-\frac{3\,b^2d\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}+\frac{3\,b^2d\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}-\frac{}{4\left(ab^3c^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b^2*d*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4) + 3/2*b^2*d*log(d*x^2 + c)/(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/4*(6*b^2*d^2*x^4 + 2*b^2
*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x^2)/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

Fricas [B] time = 1.37767, size = 1007, normalized size = 7.99

$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\left(b^3cd^2-ab^2d^3\right)x^4+3\left(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3\right)x^2+6\left(b^3d^3x^6+ab^2c^2d+\right.}{4\left(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+\left(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5+a^4bd^6\right)x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^4 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x^2 + 6*(b^3*d
^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2
*c*d^2)*x^2)*log(b*x^2 + a) - 6*(b^3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 +
 a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c
^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 +
(b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*
b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2
*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*
b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [B] time = 15.0777, size = 643, normalized size = 5.1

$$\frac{3b^2d\log\left(x^2+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{2(ad-bc)^4}-\frac{3b^2d\log\left(x^2+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\cdots}{\cdots}\right)}{2(ad-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] 3*b**2*d*log(x**2 + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5
/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3
*d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 +
 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*c)**
4) - 3*b**2*d*log(x**2 + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*
d**5/(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*
c**3*d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d*
*2 - 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*
c)**4) + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**4 + x**2*(3
*a*b*d**2 + 9*b**2*c*d))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a**2*
b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4 + 1
2*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c*d**
4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**2*(8
*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*c**4
*d - 4*b**4*c**5))
```

Giac [A] time = 1.17023, size = 309, normalized size = 2.45

$$\frac{3\,b^3d\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}-\frac{b^5}{2\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)\left(bx^2+a\right)}+\frac{5\,b^2d^3+\cdots}{4\,(bc-ad)^4\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/2*b^3*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/2*b^5/((b^6*
c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x^2 + a)) + 1/4*(5*
b^2*d^3 + 6*(b^4*c*d^2 - a*b^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b
*x^2 + a) - a*d/(b*x^2 + a) + d)^2)
```

3.314 $\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3} \, dx$

Optimal. Leaf size=230

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{b}{2a\left(a+bx^2\right)\left(c+\right.}$$

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.301425, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{b}{2a\left(a+bx^2\right)\left(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2d^2\right)-6bd}{\left(a+bx^2\right)\left(c+dx^2\right)^2}}{8ac(bc-ad)^2}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}$$

Mathematica [A] time = 0.406944, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4} + \frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} - \frac{4b^3x}{a\left(a+bx^2\right)(ad-bc)^3} + \frac{d^2x(11bc}{c^2\left(c+dx^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

Maple [A] time = 0.013, size = 403, normalized size = 1.8

$$\frac{3d^5x^3a^2}{8(ad-bc)^4\left(dx^2+c\right)^2c^2} - \frac{7d^4x^3ab}{4(ad-bc)^4\left(dx^2+c\right)^2c} + \frac{11d^3b^2x^3}{8(ad-bc)^4\left(dx^2+c\right)^2} + \frac{5d^4xa^2}{8(ad-bc)^4\left(dx^2+c\right)^2c} - \frac{9}{4(ad-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(d*x^2+c)^2
/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*b^2*x^3+5/8*d^4/(a*d-b*c)^4/(d*
x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a*b*x+13/8*d^2/(a*d-b*c)^4
/(d*x^2+c)^2*b^2*c*x+3/8*d^4/(a*d-b*c)^4/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a^2-7/4*d^3/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+35/
8*d^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/2*b^3/(a*d-b*c)
^4*x/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4*x/a/(b*x^2+a)*c-7/2*b^3/(a*d-b*c)^4/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+1/2*b^4/(a*d-b*c)^4/a/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.2844, size = 6472, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
```

```
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17894, size = 448, normalized size = 1.95

$$\frac{b^3x}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left(bx^2+a\right)}+\frac{\left(b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4\right)\sqrt{ab}}+\frac{}{8\left(b^4c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.315 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(\cdots}{2c^2\left(c+\cdots\right.}$$

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c + d*x^2)^2)
 + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)) + Log[x]/(a^2*c^3)
 - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^4) - (d^2*(6*b^2*c
^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rubi [A] time = 0.240651, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(\cdots}{2c^2\left(c+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c + d*x^2)^2)
 + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)) + Log[x]/(a^2*c^3)
 - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^4) - (d^2*(6*b^2*c
^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^3}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^3x}+\frac{b^4}{a(-bc+ad)^3(a+bx)^2}+\frac{b^4(-bc+4ad)}{a^2(-bc+ad)^4(a+bx)}-\frac{d^3}{c(bc-ad)^2(\cdots}\right.\right.$$

$$=\frac{b^3}{2a(bc-ad)^3\left(a+bx^2\right)}+\frac{d^2}{4c(bc-ad)^2\left(c+dx^2\right)^2}+\frac{d^2(3bc-ad)}{2c^2(bc-ad)^3\left(c+dx^2\right)}+\frac{\log(x)}{a^2c^3}\cdots$$

Mathematica [A] time = 0.302499, size = 187, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{2d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^3(bc-ad)^4}+\frac{2b^3(4ad-bc)\log\left(a+bx^2\right)}{a^2(bc-ad)^4}+\frac{4\log(x)}{a^2c^3}-\frac{2b^3}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{2d}{c^2\left(c+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-2*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^
2)^2) + (2*d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (4*Log[x])/
(a^2*c^3) + (2*b^3*(-(b*c) + 4*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^4) - (
2*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^4)
)/4
```

Maple [B] time = 0.023, size = 374, normalized size = 2.

$$-\frac{d^4\ln\left(dx^2+c\right)a^2}{2c^3\left(ad-bc\right)^4}+2\frac{d^3\ln\left(dx^2+c\right)ab}{c^2\left(ad-bc\right)^4}-3\frac{d^2\ln\left(dx^2+c\right)b^2}{c\left(ad-bc\right)^4}+\frac{a^2d^4}{4c\left(ad-bc\right)^4\left(dx^2+c\right)^2}-\frac{abd^3}{2\left(ad-bc\right)^4\left(dx^2+c\right)^2}+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*d^4/c^3/(a*d-b*c)^4*ln(d*x^2+c)*a^2+2*d^3/c^2/(a*d-b*c)^4*ln(d*x^2+c)*
a*b-3*d^2/c/(a*d-b*c)^4*ln(d*x^2+c)*b^2+1/4*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a
^2-1/2*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a*b+1/4*d^2*c/(a*d-b*c)^4/(d*x^2+c)^2*b^
2+1/2*d^4/c^2/(a*d-b*c)^4/(d*x^2+c)*a^2-2*d^3/c/(a*d-b*c)^4/(d*x^2+c)*a*b+3
/2*d^2/(a*d-b*c)^4/(d*x^2+c)*b^2+ln(x)/a^2/c^3+2*b^3/a/(a*d-b*c)^4*ln(b*x^2
+a)*d-1/2*b^4/a^2/(a*d-b*c)^4*ln(b*x^2+a)*c-1/2*b^3/(a*d-b*c)^4/(b*x^2+a)*d
+1/2*b^4/a/(a*d-b*c)^4/(b*x^2+a)*c
```

Maxima [B] time = 1.08827, size = 711, normalized size = 3.7

$$-\frac{\left(b^4c-4ab^3d\right)\log\left(bx^2+a\right)}{2\left(a^2b^4c^4-4a^3b^3c^3d+6a^4b^2c^2d^2-4a^5bcd^3+a^6d^4\right)}-\frac{\left(6b^2c^2d^2-4abcd^3+a^2d^4\right)\log\left(dx^2+c\right)}{2\left(b^4c^7-4ab^3c^6d+6a^2b^2c^5d^2-4a^3bc^4d^3+a^4c^3d^4\right)}+\frac{}{4\left(a^2b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*
a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/2*(6*b^2*c^2*d^2 - 4*a*b*c*d
^3 + a^2*d^4)*log(d*x^2 + c)/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 -
 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) + 1/4*(2*b^3*c^4 + 7*a^2*b*c^2*d^2 - 3*a^3*
c*d^3 + 2*(b^3*c^2*d^2 + 3*a*b^2*c*d^3 - a^2*b*d^4)*x^4 + (4*b^3*c^3*d + 7*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 2*a^3*d^4)*x^2)/(a^2*b^3*c^7 - 3*a^3*b^2*c^
6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 +
3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2
 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*
b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 1/2
```

```
*log(x^2)/(a^2*c^3)
```

Fricas [B] time = 60.7381, size = 2079, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^4*c^6 - 2*a^2*b^3*c^5*d + 7*a^3*b^2*c^4*d^2 - 10*a^4*b*c^3*d^3 +
 3*a^5*c^2*d^4 + 2*(a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3 - 4*a^3*b^2*c^2*d^4 +
 a^4*b*c*d^5)*x^4 + (4*a*b^4*c^5*d + 3*a^2*b^3*c^4*d^2 - 4*a^3*b^2*c^3*d^3
- 5*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2 - 2*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + (b^
5*c^4*d^2 - 4*a*b^4*c^3*d^3)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 4*a^2*b
^3*c^3*d^3)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 8*a^2*b^3*c^4*d^2)*x^2)*log(b*
x^2 + a) - 2*(6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (6*a^2*b^
3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (12*a^2*b^3*c^3*d^3 - 2*a^3*
b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (6*a^2*b^3*c^4*d^2 + 8*a^3*b^2
*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(d*x^2 + c) + 4*(a*b^4*c^
6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (
b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b
*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*
c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b
^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(x)
)/(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^
7*c^5*d^4 + (a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^
5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^6 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 +
 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^4
 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 -
 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18703, size = 635, normalized size = 3.31

$$-\frac{\left(b^5c-4\,ab^4d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4\right)}-\frac{\left(6\,b^2c^2d^3-4\,abcd^4+a^2d^5\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d
 + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) - 1/2*(6*b^2*c^2*d^3 -
4*a*b*c*d^4 + a^2*d^5)*log(abs(d*x^2 + c))/(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5) + 1/2*(b^5*c*x^2 - 4*a*b^
4*d*x^2 + 2*a*b^4*c - 5*a^2*b^3*d)/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*
b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*(b*x^2 + a)) + 1/4*(18*b^2*c^2*d^4*x
^4 - 12*a*b*c*d^5*x^4 + 3*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 32*a*b*c^2*d^4
*x^2 + 8*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 22*a*b*c^3*d^3 + 6*a^2*c^2*d^4)/(
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4)*(d*x^2 + c)^2) + 1/2*log(x^2)/(a^2*c^3)
```

3.316 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=297

$$\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-$$

```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c + d*x^2)^2) + b/(2*a*
(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 + 13*a*b*c*d - 5*a
^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*
c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c
- a*d)^4)
```

Rubi [A] time = 0.496568, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c + d*x^2)^2) + b/(2*a*
(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 + 13*a*b*c*d - 5*a
^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*
c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c
- a*d)^4)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+2ad-7bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(6b^2c^2-8abcd+5a^2d^2\right)-1}{x^2\left(a+bx^2\right)\left(c+d\right.}}{8ac(bc-a}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2+13abcd-5a\right.}{8ac^2(bc-ad)^3x\left(c+d\right.}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

Mathematica [A] time = 0.439648, size = 210, normalized size = 0.71

$$\frac{1}{8}\left(-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^4} + \frac{4b^4x}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{12b^{7/2}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} - \frac{8}{a^2c^3x} + \frac{}{c^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x) + (4*b^4*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (2*d^3*x)/
(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-15*b*c + 7*a*d)*x)/(c^3*(b*c - a
*d)^3*(c + d*x^2)) + (12*b^(7/2)*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^4))/8
```

Maple [A] time = 0.02, size = 428, normalized size = 1.4

$$-\frac{7\,d^6x^3a^2}{8\,c^3\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{11\,d^5x^3ab}{4\,c^2\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{15\,d^4x^3b^2}{8\,c\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{9\,a^2d^5x}{8\,c^2\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{}{4\,c\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -7/8*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a^2+11/4*d^5/c^2/(a*d-b*c)^4/(d*x^
2+c)^2*x^3*a*b-15/8*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2-9/8*d^5/c^2/(a*d-
b*c)^4/(d*x^2+c)^2*a^2*x+13/4*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a*b*x-17/8*d^3/
(a*d-b*c)^4/(d*x^2+c)^2*b^2*x-15/8*d^5/c^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x
*d/(c*d)^(1/2))*a^2+27/4*d^4/c^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a*b-63/8*d^3/c/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/
a^2/c^3/x+1/2*b^4/a/(a*d-b*c)^4*x/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x/(b*
x^2+a)*c+9/2*b^4/a/(a*d-b*c)^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-3/2*b^
5/a^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 23.7778, size = 7524, normalized size = 25.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3
*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^
2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^
4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d
^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*
c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4
*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (
b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5
```

*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((21*
a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3
 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^
2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2
*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^2 - 2*c
*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*
a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d
 - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*
d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2
 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9
- 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x),
-1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b*c^3*d^
3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^
4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c^4*d^2
+ 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^
4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3
- 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^
2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a
^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31
*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3
 + 5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((b^5*c^4*d^2 - 3*a*
b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5
+ (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*
c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2
*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 +
a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d
^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9
- 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4
 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2
- 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d
 + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^
2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)
x^6 + 2(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2
*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*
d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d
^5)*x^2 + 24*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*
c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d
^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*
c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3
*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*
a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d
^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^
5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6
*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*
c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b
^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^
4)*x), -1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b
*c^3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3
*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c
^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*
d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c
^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*
c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b
^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*
d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c
*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*
b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^

```
4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 +
5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^5*c^7*d^2 - 4*a^3*
b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 +
(2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^
4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2
*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3
 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a
^7*c^5*d^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20822, size = 581, normalized size = 1.96

$$-\frac{3\left(b^{5}c-3\,ab^{4}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4}\right)\sqrt{ab}}-\frac{3\left(21\,b^{2}c^{2}d^{3}-18\,abcd^{4}+5\,a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{7}-4\,ab^{3}c^{6}d+6\,a^{2}b^{2}c^{5}d^{2}-4\,a^{3}bc^{4}d^{3}+a^{4}c^{3}d^{4}\right)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^
3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b^2
*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4*a*
b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) -
 1/2*(3*b^4*c^3*x^2 - 6*a*b^3*c^2*d*x^2 + 6*a^2*b^2*c*d^2*x^2 - 2*a^3*b*d^3
*x^2 + 2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3)/((a^2*b^3
*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(b*x^3 + a*x)) - 1/
8*(15*b*c*d^4*x^3 - 7*a*d^5*x^3 + 17*b*c^2*d^3*x - 9*a*c*d^4*x)/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

3.317 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=215

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}$$

```
[Out] -1/(2*a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/(4*c^2*(b*
c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b*c - a*d)^3*(c + d*x
^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (b^4*(2*b*c - 5*a*d)*Log[a + b*
x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Lo
g[c + d*x^2])/(2*c^4*(b*c - a*d)^4)
```

Rubi [A] time = 0.294329, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/(4*c^2*(b*
c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b*c - a*d)^3*(c + d*x
^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (b^4*(2*b*c - 5*a*d)*Log[a + b*
x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Lo
g[c + d*x^2])/(2*c^4*(b*c - a*d)^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^3x^2} + \frac{-2bc-3ad}{a^3c^4x} - \frac{b^5}{a^2(-bc+ad)^3(a+bx)^2} - \frac{b^5(-2bc+5ad)}{a^3(-bc+ad)^4(a+b\ldots}\right.\right.$$
$$= -\frac{1}{2a^2c^3x^2} - \frac{b^4}{2a^2(bc-ad)^3\left(a+bx^2\right)} - \frac{d^3}{4c^2(bc-ad)^2\left(c+dx^2\right)^2} - \frac{d^3(2bc-ad)}{c^3(bc-ad)^3\left(c+d\ldots\right.}$$

Mathematica [A] time = 0.321434, size = 208, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^4} + \frac{2b^4}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{2b^4(2bc-5ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^4} - \frac{4\log(x)(3\ldots}{a^3\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a^2*c^3*x^2) + (2*b^4)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - d^3/(c^2*(
b*c - a*d)^2*(c + d*x^2)^2) + (4*d^3*(-2*b*c + a*d))/(c^3*(b*c - a*d)^3*(c
+ d*x^2)) - (4*(2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (2*b^4*(2*b*c - 5*a*d)*L
og[a + b*x^2])/(a^3*(b*c - a*d)^4) + (2*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^
2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^4))/4
```

Maple [A] time = 0.028, size = 405, normalized size = 1.9

$$\frac{3\,d^5\ln\left(dx^2+c\right)a^2}{2\,c^4\,(ad-bc)^4} - 5\,\frac{d^4\ln\left(dx^2+c\right)ab}{c^3\,(ad-bc)^4} + 5\,\frac{d^3\ln\left(dx^2+c\right)b^2}{c^2\,(ad-bc)^4} - \frac{d^5a^2}{4\,c^2\,(ad-bc)^4\left(dx^2+c\right)^2} + \frac{d^4ab}{2\,c\,(ad-bc)^4\left(dx^2+c\right)^2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/2*d^5/c^4/(a*d-b*c)^4*ln(d*x^2+c)*a^2-5*d^4/c^3/(a*d-b*c)^4*ln(d*x^2+c)*a
*b+5*d^3/c^2/(a*d-b*c)^4*ln(d*x^2+c)*b^2-1/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^
2*a^2+1/2*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a*b-1/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2
*b^2-d^5/c^3/(a*d-b*c)^4/(d*x^2+c)*a^2+3*d^4/c^2/(a*d-b*c)^4/(d*x^2+c)*a*b-
2*d^3/c/(a*d-b*c)^4/(d*x^2+c)*b^2-1/2/a^2/c^3/x^2-3/a^2/c^4*ln(x)*d-2/a^3/c
^3*ln(x)*b-5/2*b^4/a^2/(a*d-b*c)^4*ln(b*x^2+a)*d+b^5/a^3/(a*d-b*c)^4*ln(b*x
^2+a)*c+1/2*b^4/a/(a*d-b*c)^4/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4/(b*x^2+a)
*c
```

Maxima [B] time = 1.18909, size = 879, normalized size = 4.09

$$\frac{\left(2\,b^5c-5\,ab^4d\right)\log\left(bx^2+a\right)}{2\left(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4\right)} + \frac{\left(10\,b^2c^2d^3-10\,abcd^4+3\,a^2d^5\right)\log\left(dx^2+c\right)}{2\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4\right)} - \frac{2\ldots}{4\left((a\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6
*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) + 1/2*(10*b^2*c^2*d^3 - 10*a*b*
c*d^4 + 3*a^2*d^5)*log(d*x^2 + c)/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*
d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/4*(2*a*b^3*c^5 - 6*a^2*b^2*c^4*d +
 6*a^3*b*c^3*d^2 - 2*a^4*c^2*d^3 + 2*(2*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 7*a
^2*b^2*c*d^4 - 3*a^3*b*d^5)*x^6 + (8*b^4*c^4*d - 10*a*b^3*c^3*d^2 + 15*a^2*
b^2*c^2*d^3 + 5*a^3*b*c*d^4 - 6*a^4*d^5)*x^4 + (4*b^4*c^5 - 2*a*b^3*c^4*d -
 6*a^2*b^2*c^3*d^2 + 19*a^3*b*c^2*d^3 - 9*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2
 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^8 + (2*a^2*b^4*
c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5
)*x^6 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3
- 2*a^6*c^4*d^4)*x^4 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a
^6*c^5*d^3)*x^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

Fricas [B] time = 86.5762, size = 2446, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b^4*c^7 - 8*a^3*b^3*c^6*d + 12*a^4*b^2*c^5*d^2 - 8*a^5*b*c^4*d^
3 + 2*a^6*c^3*d^4 + 2*(2*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 10*a^3*b^3*c^3
*d^4 - 10*a^4*b^2*c^2*d^5 + 3*a^5*b*c*d^6)*x^6 + (8*a*b^5*c^6*d - 18*a^2*b^
4*c^5*d^2 + 25*a^3*b^3*c^4*d^3 - 10*a^4*b^2*c^3*d^4 - 11*a^5*b*c^2*d^5 + 6*
a^6*c*d^6)*x^4 + (4*a*b^5*c^7 - 6*a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 + 25*a^
4*b^2*c^4*d^3 - 28*a^5*b*c^3*d^4 + 9*a^6*c^2*d^5)*x^2 - 2*((2*b^6*c^5*d^2 -
 5*a*b^5*c^4*d^3)*x^8 + (4*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3)
*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2)*x^4 + (2*a*b^5*c^7 -
5*a^2*b^4*c^6*d)*x^2)*log(b*x^2 + a) - 2*((10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*
c*d^6 + 3*a^5*b*d^7)*x^8 + (20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^5 - 4*a^5
*b*c*d^6 + 3*a^6*d^7)*x^6 + (10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a
^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (10*a^4*b^2*c^4*d^3 - 10*a^5*b*c^3*d^4 +
3*a^6*c^2*d^5)*x^2)*log(d*x^2 + c) + 4*((2*b^6*c^5*d^2 - 5*a*b^5*c^4*d^3 +
10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (4*b^6*c^6*d - 8
*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^
5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*
c^5*d^2 + 10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a^5*b*c^2*d^5 + 6*a^
6*c*d^6)*x^4 + (2*a*b^5*c^7 - 5*a^2*b^4*c^6*d + 10*a^4*b^2*c^4*d^3 - 10*a^5
*b*c^3*d^4 + 3*a^6*c^2*d^5)*x^2)*log(x))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*
d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^8 + (2*a^3*b
^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^
7*b*c^5*d^5 + a^8*c^4*d^6)*x^6 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^
3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^4 + (a^4
*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6
*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17303, size = 861, normalized size = 4.

$$\frac{\left(2\,b^6c-5\,ab^5d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)}+\frac{\left(10\,b^2c^2d^4-10\,abcd^5+3\,a^2d^6\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*
d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) + 1/2*(10*b^2*c^2*d^4
- 10*a*b*c*d^5 + 3*a^2*d^6)*log(abs(d*x^2 + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^
2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5) + 1/4*(10*a^2*b^3*c^
2*d^3*x^4 - 10*a^3*b^2*c*d^4*x^4 + 3*a^4*b*d^5*x^4 - 4*b^5*c^5*x^2 + 10*a*b
^4*c^4*d*x^2 - 12*a^2*b^3*c^3*d^2*x^2 + 18*a^3*b^2*c^2*d^3*x^2 - 12*a^4*b*c
*d^4*x^2 + 3*a^5*d^5*x^2 - 2*a*b^4*c^5 + 8*a^2*b^3*c^4*d - 12*a^3*b^2*c^3*d
^2 + 8*a^4*b*c^2*d^3 - 2*a^5*c*d^4)/((a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4
*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4)*(b*x^4 + a*x^2)) - 1/4*(30*b^
2*c^2*d^5*x^4 - 30*a*b*c*d^6*x^4 + 9*a^2*d^7*x^4 + 68*b^2*c^3*d^4*x^2 - 72*
a*b*c^2*d^5*x^2 + 22*a^2*c*d^6*x^2 + 39*b^2*c^4*d^3 - 44*a*b*c^3*d^4 + 14*a
^2*c^2*d^5)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3
 + a^4*c^4*d^4)*(d*x^2 + c)^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

3.318 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=377

$$\frac{d(-7a^2d^2+15abcd+4b^2c^2)}{8ac^2x^3(c+dx^2)(bc-ad)^3}-\frac{75a^2bcd^2-35a^3d^3-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-24a^2b^2c^2d^2+75a^3bcd^3-35a^4d^4-24ab^3c^3d}{8a^3c^4x(bc-ad)^3}$$

```
[Out] -(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(24*a^2*c^3*(b
*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^2*b^2*c^2*d^2 + 75*a
^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/(4
*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*
(c + d*x^2)^2) + (d*(4*b^2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c - a
*d)^3*x^3*(c + d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^
2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.68604, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$\frac{d(-7a^2d^2+15abcd+4b^2c^2)}{8ac^2x^3(c+dx^2)(bc-ad)^3}-\frac{75a^2bcd^2-35a^3d^3-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-24a^2b^2c^2d^2+75a^3bcd^3-35a^4d^4-24ab^3c^3d}{8a^3c^4x(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] -(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(24*a^2*c^3*(b
*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^2*b^2*c^2*d^2 + 75*a
^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/(4
*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*
(c + d*x^2)^2) + (d*(4*b^2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c - a
*d)^3*x^3*(c + d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^
2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
```

```
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-5bc+2ad-9bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(10b^2c^2-8abcd+7\ldots\right.}{x^4\left(a+bx\ldots\right.}}{8ac(b\ldots}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2+15abc\ldots\right.}{8ac^2(bc-ad)^3x^3\ldots}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{\ldots}{2a(bc-a\ldots}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+75a^3\ldots}{8a^3c^4(bc-ad)^3x}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+75a^3\ldots}{8a^3c^4(bc-ad)^3x}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+75a^3\ldots}{8a^3c^4(bc-ad)^3x}$$

Mathematica [A] time = 0.450843, size = 230, normalized size = 0.61

$$\frac{1}{24}\left(\frac{3d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{9/2}(bc-ad)^4}-\frac{12b^5x}{a^3\left(a+bx^2\right)(ad-bc)^3}+\frac{12b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)^4}+\frac{72ad \ldots}{a^3c \ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x^3) + (48*b*c + 72*a*d)/(a^3*c^4*x) - (12*b^5*x)/(a^3*(-(b*c)
 + a*d)^3*(a + b*x^2)) + (6*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (3*d
^4*(19*b*c - 11*a*d)*x)/(c^4*(b*c - a*d)^3*(c + d*x^2)) + (12*b^(9/2)*(5*b*
c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d)^4) + (3*d^(7/
2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^
(9/2)*(b*c - a*d)^4))/24
```

Maple [A] time = 0.025, size = 455, normalized size = 1.2

$$\frac{11\,d^7x^3a^2}{8\,c^4\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{15\,d^6x^3ab}{4\,c^3\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{19\,d^5x^3b^2}{8\,c^2\,(ad-bc)^4\left(dx^2+c\right)^2}+\frac{13\,d^6a^2x}{8\,c^3\,(ad-bc)^4\left(dx^2+c\right)^2}-\frac{1 \ldots}{4\,c^2\,(ad- \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 11/8*d^7/c^4/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a^2-15/4*d^6/c^3/(a*d-b*c)^4/(d*x^
2+c)^2*x^3*a*b+19/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+13/8*d^6/c^3/(a
*d-b*c)^4/(d*x^2+c)^2*a^2*x-17/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*a*b*x+21/8
*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*b^2*x+35/8*d^6/c^4/(a*d-b*c)^4/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))*a^2-55/4*d^5/c^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/
(c*d)^(1/2))*a*b+99/8*d^4/c^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2
))*b^2-1/3/a^2/c^3/x^3+3/a^2/c^4/x*d+2/a^3/c^3/x*b-1/2*b^5/a^2/(a*d-b*c)^4*
x/(b*x^2+a)*d+1/2*b^6/a^3/(a*d-b*c)^4*x/(b*x^2+a)*c-11/2*b^5/a^2/(a*d-b*c)^
4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+5/2*b^6/a^3/(a*d-b*c)^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 66.8721, size = 8631, normalized size = 22.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c
^4*d^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3
*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a
*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d
^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d -
184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b
*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*
b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 +
12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d
^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^
2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6
 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*
b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*
a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4
 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*
b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8
*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 +
 (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 -
7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*
a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7
 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4
 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*
c*d^6 + 35*a^5*b*d^7)*x^8 - (120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4
*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 10
5*a^6*d^7)*x^6 - (60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a
^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x
^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4
*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 3*((99*a^3*b^3*c^2*d^5 - 110
*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2
*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*
c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110
*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((5
*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 - 1
1*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5
 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt
(-b/a) - a)/(b*x^2 + a)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3
*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^
4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a
^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a
^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a
^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/
48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c^4*d
^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2
*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a*b^5
*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 -
 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d - 184*
a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2
*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*
c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 24*(
(5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 -
 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x
^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b
```

```
^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a
^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5
 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^
4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9
+ (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*
d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d
- 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*
x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3
 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*
c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c
^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - (120*
b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 -
253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - (60*b^6*c^7 - 52
*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*
d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*
c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c
^2*d^5)*x^2 - 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d -
17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*
a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arct
an(x*sqrt(b/a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7
)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^
6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 +
 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d
^5)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d
^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^
5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7
*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3
*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*
b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*
d^4)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17197, size = 495, normalized size = 1.31

$$\frac{b^5x}{2\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3\right)\left(bx^2+a\right)}+\frac{\left(5\,b^6c-11\,ab^5d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4\right)\sqrt{ab}}+\frac{(9}{8\left(b^4c^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(b*x^2
 + a)) + 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4
```

```
*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) +
1/8*(99*b^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b
^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)
*sqrt(c*d)) + 1/8*(19*b*c*d^5*x^3 - 11*a*d^6*x^3 + 21*b*c^2*d^4*x - 13*a*c*
d^5*x)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x^2 +
c)^2) + 1/3*(6*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^3*c^4*x^3)
```

3.319 $\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$

Optimal. Leaf size=96

$$\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{a^3Ax^{m+1}}{m+1}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m) + (3*a*b*
(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(7 + m))/(7 + m) + (b
^3*B*x^(9 + m))/(9 + m)
```

Rubi [A] time = 0.0651068, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{a^3Ax^{m+1}}{m+1}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m) + (3*a*b*
(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(7 + m))/(7 + m) + (b
^3*B*x^(9 + m))/(9 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx &= \int \left(a^3Ax^m+a^2(3Ab+aB)x^{2+m}+3ab(Ab+aB)x^{4+m}+b^2(Ab+3aB)x^{6+m}+b^3Bx^{8+m}\right)\\ &= \frac{a^3Ax^{1+m}}{1+m}+\frac{a^2(3Ab+aB)x^{3+m}}{3+m}+\frac{3ab(Ab+aB)x^{5+m}}{5+m}+\frac{b^2(Ab+3aB)x^{7+m}}{7+m}+\frac{b^3Bx^{9+m}}{9+m}\end{aligned}$$

Mathematica [A] time = 0.102434, size = 89, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2x^2(aB+3Ab)}{m+3}+\frac{a^3A}{m+1}+\frac{b^2x^6(3aB+Ab)}{m+7}+\frac{3abx^4(aB+Ab)}{m+5}+\frac{b^3Bx^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^2)/(3 + m) + (3*a*b*(A*b
+ a*B)*x^4)/(5 + m) + (b^2*(A*b + 3*a*B)*x^6)/(7 + m) + (b^3*B*x^8)/(9 + m)
)
```

Maple [B] time = 0.007, size = 474, normalized size = 4.9

$$x^{1+m}\left(Bb^3m^4x^8+16\,Bb^3m^3x^8+Ab^3m^4x^6+3\,Bab^2m^4x^6+86\,Bb^3m^2x^8+18\,Ab^3m^3x^6+54\,Bab^2m^3x^6+176\,Bb^3mx^8+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b^3*m^4*x^8+16*B*b^3*m^3*x^8+A*b^3*m^4*x^6+3*B*a*b^2*m^4*x^6+86*
B*b^3*m^2*x^8+18*A*b^3*m^3*x^6+54*B*a*b^2*m^3*x^6+176*B*b^3*m*x^8+3*A*a*b^2
*m^4*x^4+104*A*b^3*m^2*x^6+3*B*a^2*b*m^4*x^4+312*B*a*b^2*m^2*x^6+105*B*b^3*
x^8+60*A*a*b^2*m^3*x^4+222*A*b^3*m*x^6+60*B*a^2*b*m^3*x^4+666*B*a*b^2*m*x^6
+3*A*a^2*b*m^4*x^2+390*A*a*b^2*m^2*x^4+135*A*b^3*x^6+B*a^3*m^4*x^2+390*B*a^
2*b*m^2*x^4+405*B*a*b^2*x^6+66*A*a^2*b*m^3*x^2+900*A*a*b^2*m*x^4+22*B*a^3*m
^3*x^2+900*B*a^2*b*m*x^4+A*a^3*m^4+492*A*a^2*b*m^2*x^2+567*A*a*b^2*x^4+164*
B*a^3*m^2*x^2+567*B*a^2*b*x^4+24*A*a^3*m^3+1374*A*a^2*b*m*x^2+458*B*a^3*m*x
^2+206*A*a^3*m^2+945*A*a^2*b*x^2+315*B*a^3*x^2+744*A*a^3*m+945*A*a^3)/(9+m)
/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.07209, size = 865, normalized size = 9.01

$$\left(\left(Bb^3m^4+16\,Bb^3m^3+86\,Bb^3m^2+176\,Bb^3m+105\,Bb^3\right)x^9+\left(\left(3\,Bab^2+Ab^3\right)m^4+405\,Bab^2+135\,Ab^3+18\left(3\,Bab^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^3*m^4 + 16*B*b^3*m^3 + 86*B*b^3*m^2 + 176*B*b^3*m + 105*B*b^3)*x^9 +
((3*B*a*b^2 + A*b^3)*m^4 + 405*B*a*b^2 + 135*A*b^3 + 18*(3*B*a*b^2 + A*b^3)
*m^3 + 104*(3*B*a*b^2 + A*b^3)*m^2 + 222*(3*B*a*b^2 + A*b^3)*m)*x^7 + 3*((B
*a^2*b + A*a*b^2)*m^4 + 189*B*a^2*b + 189*A*a*b^2 + 20*(B*a^2*b + A*a*b^2)*
m^3 + 130*(B*a^2*b + A*a*b^2)*m^2 + 300*(B*a^2*b + A*a*b^2)*m)*x^5 + ((B*a^
3 + 3*A*a^2*b)*m^4 + 315*B*a^3 + 945*A*a^2*b + 22*(B*a^3 + 3*A*a^2*b)*m^3 +
 164*(B*a^3 + 3*A*a^2*b)*m^2 + 458*(B*a^3 + 3*A*a^2*b)*m)*x^3 + (A*a^3*m^4
+ 24*A*a^3*m^3 + 206*A*a^3*m^2 + 744*A*a^3*m + 945*A*a^3)*x)*x^m/(m^5 + 25*
m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 2.95055, size = 2069, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4*x**4) - A*b
**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B*a*b**2/(2*x**2)
+ B*b**3*log(x), Eq(m, -9)), (-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3*A*
a*b**2/(2*x**2) + A*b**3*log(x) - B*a**3/(4*x**4) - 3*B*a**2*b/(2*x**2) + 3
*B*a*b**2*log(x) + B*b**3*x**2/2, Eq(m, -7)), (-A*a**3/(4*x**4) - 3*A*a**2*
b/(2*x**2) + 3*A*a*b**2*log(x) + A*b**3*x**2/2 - B*a**3/(2*x**2) + 3*B*a**2
*b*log(x) + 3*B*a*b**2*x**2/2 + B*b**3*x**4/4, Eq(m, -5)), (-A*a**3/(2*x**2
) + 3*A*a**2*b*log(x) + 3*A*a*b**2*x**2/2 + A*b**3*x**4/4 + B*a**3*log(x) +
 3*B*a**2*b*x**2/2 + 3*B*a*b**2*x**4/4 + B*b**3*x**6/6, Eq(m, -3)), (A*a**3
*log(x) + 3*A*a**2*b*x**2/2 + 3*A*a*b**2*x**4/4 + A*b**3*x**6/6 + B*a**3*x*
*2/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x**8/8, Eq(m, -1)), (A*
a**3*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24
*A*a**3*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 206*A*a**3*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 744*A*a**3*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 945*A*a**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 3*A*a**2*b*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 66*A*a**2*b*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 492*A*a**2*b*m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 1374*A*a**2*b*m*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 945*A*a**2*b*x**3*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 3*A*a*b**2*m**4*x**5*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*A*a*b**2*m**3*x**5*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*A*a*b**2*m**2*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*A*a*b**
2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 567*A
*a*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + A
*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 18*A*b**3*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 104*A*b**3*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 222*A*b**3*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 135*A*b**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + B*a**3*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 22*B*a**3*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 164*B*a**3*m**2*x**3*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 458*B*a**3*m*x**3*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 315*B*a**3*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3*B*a**2*b*m**4*x**5*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*B*a**2*b*m**3*x**5*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*B*a**2*b*m**2
*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*B*a*
*2*b*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 56
7*B*a**2*b*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 3*B*a*b**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 54*B*a*b**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 312*B*a*b**2*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 666*B*a*b**2*m*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 405*B*a*b**2*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + B*b**3*m**4*x**9*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 16*B*b**3*m**3*x**9*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*B*b**3*m**2*x**9*x**m/(m**5
```

```
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*B*b**3*m*x**9*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*B*b**3*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] time = 1.19561, size = 801, normalized size = 8.34

$$\frac{Bb^3m^4x^9x^m + 16\,Bb^3m^3x^9x^m + 3\,Bab^2m^4x^7x^m + Ab^3m^4x^7x^m + 86\,Bb^3m^2x^9x^m + 54\,Bab^2m^3x^7x^m + 18\,Ab^3m^3x^7x^m + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^3*m^4*x^9*x^m + 16*B*b^3*m^3*x^9*x^m + 3*B*a*b^2*m^4*x^7*x^m + A*b^3*m
^4*x^7*x^m + 86*B*b^3*m^2*x^9*x^m + 54*B*a*b^2*m^3*x^7*x^m + 18*A*b^3*m^3*x
^7*x^m + 176*B*b^3*m*x^9*x^m + 3*B*a^2*b*m^4*x^5*x^m + 3*A*a*b^2*m^4*x^5*x^
m + 312*B*a*b^2*m^2*x^7*x^m + 104*A*b^3*m^2*x^7*x^m + 105*B*b^3*x^9*x^m + 6
0*B*a^2*b*m^3*x^5*x^m + 60*A*a*b^2*m^3*x^5*x^m + 666*B*a*b^2*m*x^7*x^m + 22
2*A*b^3*m*x^7*x^m + B*a^3*m^4*x^3*x^m + 3*A*a^2*b*m^4*x^3*x^m + 390*B*a^2*b
*m^2*x^5*x^m + 390*A*a*b^2*m^2*x^5*x^m + 405*B*a*b^2*x^7*x^m + 135*A*b^3*x^
7*x^m + 22*B*a^3*m^3*x^3*x^m + 66*A*a^2*b*m^3*x^3*x^m + 900*B*a^2*b*m*x^5*x
^m + 900*A*a*b^2*m*x^5*x^m + A*a^3*m^4*x*x^m + 164*B*a^3*m^2*x^3*x^m + 492*
A*a^2*b*m^2*x^3*x^m + 567*B*a^2*b*x^5*x^m + 567*A*a*b^2*x^5*x^m + 24*A*a^3*
m^3*x*x^m + 458*B*a^3*m*x^3*x^m + 1374*A*a^2*b*m*x^3*x^m + 206*A*a^3*m^2*x*
x^m + 315*B*a^3*x^3*x^m + 945*A*a^2*b*x^3*x^m + 744*A*a^3*m*x*x^m + 945*A*a
^3*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.320 $\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) + (b*(A*b +
 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.0421679, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) + (b*(A*b +
 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx &= \int \left(a^2Ax^m+a(2Ab+aB)x^{2+m}+b(Ab+2aB)x^{4+m}+b^2Bx^{6+m}\right) dx\\ &= \frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{3+m}}{3+m}+\frac{b(Ab+2aB)x^{5+m}}{5+m}+\frac{b^2Bx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.0500152, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2A}{m+1}+\frac{bx^4(2aB+Ab)}{m+5}+\frac{ax^2(aB+2Ab)}{m+3}+\frac{b^2Bx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^2)/(3 + m) + (b*(A*b + 2*a*
B)*x^4)/(5 + m) + (b^2*B*x^6)/(7 + m))
```

Maple [B] time = 0.004, size = 262, normalized size = 3.7

$$\frac{x^{1+m}\left(Bb^2m^3x^6+9\,Bb^2m^2x^6+Ab^2m^3x^4+2\,Babm^3x^4+23\,Bb^2mx^6+11\,Ab^2m^2x^4+22\,Babm^2x^4+15\,Bb^2x^6+2\,Aab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b^2*m^3*x^6+9*B*b^2*m^2*x^6+A*b^2*m^3*x^4+2*B*a*b*m^3*x^4+23*B*b
^2*m*x^6+11*A*b^2*m^2*x^4+22*B*a*b*m^2*x^4+15*B*b^2*x^6+2*A*a*b*m^3*x^2+31*
A*b^2*m*x^4+B*a^2*m^3*x^2+62*B*a*b*m*x^4+26*A*a*b*m^2*x^2+21*A*b^2*x^4+13*B
*a^2*m^2*x^2+42*B*a*b*x^4+A*a^2*m^3+94*A*a*b*m*x^2+47*B*a^2*m*x^2+15*A*a^2*
m^2+70*A*a*b*x^2+35*B*a^2*x^2+71*A*a^2*m+105*A*a^2)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.07757, size = 490, normalized size = 6.9

$$\frac{\left(\left(Bb^2m^3+9\,Bb^2m^2+23\,Bb^2m+15\,Bb^2\right)x^7+\left(\left(2\,Bab+Ab^2\right)m^3+42\,Bab+21\,Ab^2+11\,\left(2\,Bab+Ab^2\right)m^2+31\,\left(2\,Ba\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 9*B*b^2*m^2 + 23*B*b^2*m + 15*B*b^2)*x^7 + ((2*B*a*b + A*b^2)
*m^3 + 42*B*a*b + 21*A*b^2 + 11*(2*B*a*b + A*b^2)*m^2 + 31*(2*B*a*b + A*b^2
)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^3 + 35*B*a^2 + 70*A*a*b + 13*(B*a^2 + 2*A*a
*b)*m^2 + 47*(B*a^2 + 2*A*a*b)*m)*x^3 + (A*a^2*m^3 + 15*A*a^2*m^2 + 71*A*a^
2*m + 105*A*a^2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 1.71208, size = 1044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) - B*a**2/(4*
x**4) - B*a*b/x**2 + B*b**2*log(x), Eq(m, -7)), (-A*a**2/(4*x**4) - A*a*b/x
**2 + A*b**2*log(x) - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2, Eq(
```

```
m, -5)), (-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 + B*a**2*log(x)
 + B*a*b*x**2 + B*b**2*x**4/4, Eq(m, -3)), (A*a**2*log(x) + A*a*b*x**2 + A*
b**2*x**4/4 + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6, Eq(m, -1)), (A*
a**2*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a**2*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a**2*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 2*A*a*b*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 1
76*m + 105) + 26*A*a*b*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 94*A*a*b*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*A*
a*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b**2*m**3*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b**2*m**2*x**5*x**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*b**2*m*x**5*x**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 21*A*b**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + B*a**2*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
 105) + 13*B*a**2*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 47*B*a**2*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*B*a**2
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*B*a*b*m**3*x**5*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 22*B*a*b*m**2*x**5*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 62*B*a*b*m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 42*B*a*b*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + B*b**2*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*B*b**2*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*B*
b**2*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*B*b**2*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [B] time = 1.23412, size = 448, normalized size = 6.31

$$\frac{Bb^2m^3x^7x^m + 9\,Bb^2m^2x^7x^m + 2\,Babm^3x^5x^m + Ab^2m^3x^5x^m + 23\,Bb^2mx^7x^m + 22\,Babm^2x^5x^m + 11\,Ab^2m^2x^5x^m + 15\,Bb^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^7*x^m + 9*B*b^2*m^2*x^7*x^m + 2*B*a*b*m^3*x^5*x^m + A*b^2*m^3*
x^5*x^m + 23*B*b^2*m*x^7*x^m + 22*B*a*b*m^2*x^5*x^m + 11*A*b^2*m^2*x^5*x^m
+ 15*B*b^2*x^7*x^m + B*a^2*m^3*x^3*x^m + 2*A*a*b*m^3*x^3*x^m + 62*B*a*b*m*x
^5*x^m + 31*A*b^2*m*x^5*x^m + 13*B*a^2*m^2*x^3*x^m + 26*A*a*b*m^2*x^3*x^m +
 42*B*a*b*x^5*x^m + 21*A*b^2*x^5*x^m + A*a^2*m^3*x*x^m + 47*B*a^2*m*x^3*x^m
 + 94*A*a*b*m*x^3*x^m + 15*A*a^2*m^2*x*x^m + 35*B*a^2*x^3*x^m + 70*A*a*b*x^
3*x^m + 71*A*a^2*m*x*x^m + 105*A*a^2*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

3.321 $\int x^m \left(a + bx^2\right) \left(A + Bx^2\right) dx$

Optimal. Leaf size=45

$$\frac{x^{m+3}(aB + Ab)}{m + 3} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+5}}{m + 5}$$

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*x^(5 + m))
/(5 + m)
```

Rubi [A] time = 0.0214541, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{x^{m+3}(aB + Ab)}{m + 3} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+5}}{m + 5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*x^(5 + m))
/(5 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^2\right) \left(A + Bx^2\right) dx &= \int \left(aAx^m + (Ab + aB)x^{2+m} + bBx^{4+m}\right) dx \\ &= \frac{aAx^{1+m}}{1 + m} + \frac{(Ab + aB)x^{3+m}}{3 + m} + \frac{bBx^{5+m}}{5 + m}\end{aligned}$$

Mathematica [A] time = 0.0300509, size = 42, normalized size = 0.93

$$x^{m+1}\left(\frac{x^2(aB + Ab)}{m + 3} + \frac{aA}{m + 1} + \frac{bBx^4}{m + 5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^2)/(3 + m) + (b*B*x^4)/(5 + m))
```

Maple [B] time = 0.005, size = 110, normalized size = 2.4

$$\frac{x^{1+m}\left(Bbm^2x^4 + 4\,Bbmx^4 + Abm^2x^2 + Bam^2x^2 + 3\,bBx^4 + 6\,Abmx^2 + 6\,Bamx^2 + Aam^2 + 5\,Ax^2b + 5\,Bx^2a + 8\,Aam\right.}{(5 + m)\,(3 + m)\,(1 + m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b*m^2*x^4+4*B*b*m*x^4+A*b*m^2*x^2+B*a*m^2*x^2+3*B*b*x^4+6*A*b*m*
x^2+6*B*a*m*x^2+A*a*m^2+5*A*b*x^2+5*B*a*x^2+8*A*a*m+15*A*a)/(5+m)/(3+m)/(1+
m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0196, size = 211, normalized size = 4.69

$$\frac{\left(\left(Bbm^2 + 4\,Bbm + 3\,Bb\right)x^5 + \left((Ba + Ab)m^2 + 5\,Ba + 5\,Ab + 6\,(Ba + Ab)m\right)x^3 + \left(Aam^2 + 8\,Aam + 15\,Aa\right)x\right)x^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 4*B*b*m + 3*B*b)*x^5 + ((B*a + A*b)*m^2 + 5*B*a + 5*A*b + 6*(B*
a + A*b)*m)*x^3 + (A*a*m^2 + 8*A*a*m + 15*A*a)*x)*x^m/(m^3 + 9*m^2 + 23*m +
 15)
```

Sympy [A] time = 0.876223, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{4x^4} - \frac{Ab}{2x^2} - \frac{Ba}{2x^2} + Bb\log(x) \\ -\frac{Aa}{2x^2} + Ab\log(x) + Ba\log(x) + \frac{Bbx^2}{2} \\ Aa\log(x) + \frac{Abx^2}{2} + \frac{Bax^2}{2} + \frac{Bbx^4}{4} \\ \frac{Aam^2xx^m}{m^3+9m^2+23m+15} + \frac{8Aamxx^m}{m^3+9m^2+23m+15} + \frac{15Aaxx^m}{m^3+9m^2+23m+15} + \frac{Abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{6Abmx^3x^m}{m^3+9m^2+23m+15} + \frac{5Abx^3x^m}{m^3+9m^2+23m+15} + \frac{Bam^2x^3x^m}{m^3+9m^2+23m+15} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a/(4*x**4) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(x), Eq(m,
-5)), (-A*a/(2*x**2) + A*b*log(x) + B*a*log(x) + B*b*x**2/2, Eq(m, -3)), (A
*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**4/4, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 9*m**2 + 23*m + 15) + 8*A*a*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15
) + 15*A*a*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + A*b*m**2*x**3*x**m/(m**3 +
9*m**2 + 23*m + 15) + 6*A*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*A*b
*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*a*m**2*x**3*x**m/(m**3 + 9*m**2
+ 23*m + 15) + 6*B*a*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*B*a*x**3*x
```

```
**m/(m**3 + 9*m**2 + 23*m + 15) + B*b*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m
+ 15) + 4*B*b*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*B*b*x**5*x**m/(m*
*3 + 9*m**2 + 23*m + 15), True))
```

Giac [B] time = 1.13565, size = 193, normalized size = 4.29

$$\frac{Bbm^2x^5x^m + 4\,Bbmx^5x^m + Bam^2x^3x^m + Abm^2x^3x^m + 3\,Bbx^5x^m + 6\,Bamx^3x^m + 6\,Abmx^3x^m + Aam^2xx^m + 5\,Bax^3x^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^5*x^m + 4*B*b*m*x^5*x^m + B*a*m^2*x^3*x^m + A*b*m^2*x^3*x^m + 3*
B*b*x^5*x^m + 6*B*a*m*x^3*x^m + 6*A*b*m*x^3*x^m + A*a*m^2*x*x^m + 5*B*a*x^3
*x^m + 5*A*b*x^3*x^m + 8*A*a*m*x*x^m + 15*A*a*x*x^m)/(m^3 + 9*m^2 + 23*m +
15)
```

3.322 $\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi [A] time = 0.0330978, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {459, 364}

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{Bx^{1+m}}{b(1+m)}-\frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^2}\,dx}{b(1+m)}\\ &= \frac{Bx^{1+m}}{b(1+m)}+\frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}\end{aligned}$$

Mathematica [A] time = 0.059056, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{\left(B x^{2}+A\right) x^{m}}{b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B x^{2}+A\right) x^{m}}{b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B x^{2}+A\right) x^{m}}{b x^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Sympy [C] time = 4.79152, size = 190, normalized size = 2.88

$$\frac{Amxx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Axx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Bmx^{3}x^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + A*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*m*x**3*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5
/2)) + 3*B*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

3.323 $\int \frac{x^m\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(b - b*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rubi [A] time = 0.0439417, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(b - b*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{(Ab-aB)x^{1+m}}{2ab\left(a+bx^2\right)}+\frac{(aB(1+m)+A(b-bm))\int \frac{x^m}{a+bx^2}\,dx}{2ab}\\ &= \frac{(Ab-aB)x^{1+m}}{2ab\left(a+bx^2\right)}+\frac{(aB(1+m)+A(b-bm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0558589, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2
*b*(1 + m))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)\,x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2+A)x^m}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 35.6348, size = 906, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] A*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2
)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + B*(-a
*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**3*x**
m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2
)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*
x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m
/2 + 5/2)) - b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5
/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gam
ma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) -
3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 +
3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{(bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

3.324 $\int \frac{x^m\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^
3*b*(1 + m))
```

Rubi [A] time = 0.0415536, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^
3*b*(1 + m))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{(Ab-aB)x^{1+m}}{4ab\left(a+bx^2\right)^2}+\frac{(-Ab(-3+m)+aB(1+m))\int \frac{x^m}{\left(a+bx^2\right)^2}\,dx}{4ab}\\ &= \frac{(Ab-aB)x^{1+m}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab(3-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{4a^3b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0570247, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^3
*b*(1 + m))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)\,x^m}{(bx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)x^m}{(bx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(Bx^2+A)x^m}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] time = 132.097, size = 3053, normalized size = 32.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] A*(a**2*m**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32
*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x*x**m*lerchphi(b*x**2*exp_
polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 6
4*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*
*2*m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a**2*m*x*x**m*lerc
hphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamm
a(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/
2 + 3/2)) + 8*a**2*m*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64
*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**
2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + 10*a**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2
 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3
/2)) + 2*a*b*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3
/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**3*x**m*lerchphi(b
*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2
+ 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/
2)) - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*
a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*b*
m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b*
*2*x**4*gamma(m/2 + 3/2)) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gam
ma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m
/2 + 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/
2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 +
3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a*b*x**3*x**m*gamma(m/2 + 1/
2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b*
*2*x**4*gamma(m/2 + 3/2)) + b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I
*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*
b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m**2
*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**
2*x**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x*
*2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**
m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**
5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2))) + B*(a**2*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2
*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) + 3*a**2*m**2*x**3*
x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*
a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4
*gamma(m/2 + 5/2)) - 2*a**2*m**2*x**3*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(
m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2
+ 5/2)) - a**2*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)
*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/
2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 3*a**2*x**3*x**m*lerchphi(b*x**2
*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2
```

```
) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) +
 18*a**2*x**3*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x
**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) + 2*a*b*m**3*x**
5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(3
2*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x*
*4*gamma(m/2 + 5/2)) + 6*a*b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x*
*2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 2*a*b*m**2*x**5
*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2
 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 2*a*b*m*x**5*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2
 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5
/2)) - 4*a*b*m*x**5*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a*
*4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 6*a*b*x*
*5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(
32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x
**4*gamma(m/2 + 5/2)) + 6*a*b*x**5*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2
 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5
/2)) + b**2*m**3*x**7*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)
*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/
2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) + 3*b**2*m**2*x**7*x**m*lerchphi(b
*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2
+ 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/
2)) - b**2*m*x**7*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gam
ma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) +
 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 3*b**2*x**7*x**m*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) +
64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{(bx^2 + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

3.325 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=151

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc}{m+9}$$

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(3 + m) + (
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m) + (d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^(9 + m))/(
9 + m) + (b^2*d^3*x^(11 + m))/(11 + m)
```

Rubi [A] time = 0.0885689, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(3 + m) + (
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m) + (d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^(9 + m))/(
9 + m) + (b^2*d^3*x^(11 + m))/(11 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \int \left(a^2c^3x^m+ac^2(2bc+3ad)x^{2+m}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{4+m}+d\left(3b^2c^2+6abcd+\right.\right.$$

$$=\frac{a^2c^3x^{1+m}}{1+m}+\frac{ac^2(2bc+3ad)x^{3+m}}{3+m}+\frac{c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5+m}}{5+m}+\frac{d\left(3b^2c^2+6abcd+\right.}{7+m}$$

Mathematica [A] time = 0.103946, size = 141, normalized size = 0.93

$$x^m\left(\frac{dx^7\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{cx^5\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{a^2c^3x}{m+1}+\frac{ac^2x^3(3ad+2bc)}{m+3}+\frac{bd^2x^9(2ad+3bc)}{m+9}+\frac{b^2d^3x}{m+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] x^m*((a^2*c^3*x)/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^3)/(3 + m) + (c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^5)/(5 + m) + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
```

```
^2)*x^7)/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^9)/(9 + m) + (b^2*d^3*x^11)/(11
 + m))
```

Maple [B] time = 0.01, size = 976, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] x^(1+m)*(b^2*d^3*m^5*x^10+25*b^2*d^3*m^4*x^10+2*a*b*d^3*m^5*x^8+3*b^2*c*d^2
*m^5*x^8+230*b^2*d^3*m^3*x^10+54*a*b*d^3*m^4*x^8+81*b^2*c*d^2*m^4*x^8+950*b
^2*d^3*m^2*x^10+a^2*d^3*m^5*x^6+6*a*b*c*d^2*m^5*x^6+524*a*b*d^3*m^3*x^8+3*b
^2*c^2*d*m^5*x^6+786*b^2*c*d^2*m^3*x^8+1689*b^2*d^3*m*x^10+29*a^2*d^3*m^4*x
^6+174*a*b*c*d^2*m^4*x^6+2244*a*b*d^3*m^2*x^8+87*b^2*c^2*d*m^4*x^6+3366*b^2
*c*d^2*m^2*x^8+945*b^2*d^3*x^10+3*a^2*c*d^2*m^5*x^4+302*a^2*d^3*m^3*x^6+6*a
*b*c^2*d*m^5*x^4+1812*a*b*c*d^2*m^3*x^6+4082*a*b*d^3*m*x^8+b^2*c^3*m^5*x^4+
906*b^2*c^2*d*m^3*x^6+6123*b^2*c*d^2*m*x^8+93*a^2*c*d^2*m^4*x^4+1366*a^2*d^
3*m^2*x^6+186*a*b*c^2*d*m^4*x^4+8196*a*b*c*d^2*m^2*x^6+2310*a*b*d^3*x^8+31*
b^2*c^3*m^4*x^4+4098*b^2*c^2*d*m^2*x^6+3465*b^2*c*d^2*x^8+3*a^2*c^2*d*m^5*x
^2+1050*a^2*c*d^2*m^3*x^4+2577*a^2*d^3*m*x^6+2*a*b*c^3*m^5*x^2+2100*a*b*c^2
*d*m^3*x^4+15462*a*b*c*d^2*m*x^6+350*b^2*c^3*m^3*x^4+7731*b^2*c^2*d*m*x^6+9
9*a^2*c^2*d*m^4*x^2+5190*a^2*c*d^2*m^2*x^4+1485*a^2*d^3*x^6+66*a*b*c^3*m^4*
x^2+10380*a*b*c^2*d*m^2*x^4+8910*a*b*c*d^2*x^6+1730*b^2*c^3*m^2*x^4+4455*b^
2*c^2*d*x^6+a^2*c^3*m^5+1218*a^2*c^2*d*m^3*x^2+10467*a^2*c*d^2*m*x^4+812*a*
b*c^3*m^3*x^2+20934*a*b*c^2*d*m*x^4+3489*b^2*c^3*m*x^4+35*a^2*c^3*m^4+6786*
a^2*c^2*d*m^2*x^2+6237*a^2*c*d^2*x^4+4524*a*b*c^3*m^2*x^2+12474*a*b*c^2*d*x
^4+2079*b^2*c^3*x^4+470*a^2*c^3*m^3+16059*a^2*c^2*d*m*x^2+10706*a*b*c^3*m*x
^2+3010*a^2*c^3*m^2+10395*a^2*c^2*d*x^2+6930*a*b*c^3*x^2+9129*a^2*c^3*m+103
95*a^2*c^3)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.01973, size = 1740, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] ((b^2*d^3*m^5 + 25*b^2*d^3*m^4 + 230*b^2*d^3*m^3 + 950*b^2*d^3*m^2 + 1689*b
^2*d^3*m + 945*b^2*d^3)*x^11 + ((3*b^2*c*d^2 + 2*a*b*d^3)*m^5 + 3465*b^2*c*
```

```
d^2 + 2310*a*b*d^3 + 27*(3*b^2*c*d^2 + 2*a*b*d^3)*m^4 + 262*(3*b^2*c*d^2 +
2*a*b*d^3)*m^3 + 1122*(3*b^2*c*d^2 + 2*a*b*d^3)*m^2 + 2041*(3*b^2*c*d^2 + 2
*a*b*d^3)*m)*x^9 + ((3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^5 + 4455*b^2*c^
2*d + 8910*a*b*c*d^2 + 1485*a^2*d^3 + 29*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d
^3)*m^4 + 302*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^3 + 1366*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*m^2 + 2577*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*
m)*x^7 + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^5 + 2079*b^2*c^3 + 12474*
a*b*c^2*d + 6237*a^2*c*d^2 + 31*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^4 +
 350*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^3 + 1730*(b^2*c^3 + 6*a*b*c^2*
d + 3*a^2*c*d^2)*m^2 + 3489*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m)*x^5 +
((2*a*b*c^3 + 3*a^2*c^2*d)*m^5 + 6930*a*b*c^3 + 10395*a^2*c^2*d + 33*(2*a*b
*c^3 + 3*a^2*c^2*d)*m^4 + 406*(2*a*b*c^3 + 3*a^2*c^2*d)*m^3 + 2262*(2*a*b*c
^3 + 3*a^2*c^2*d)*m^2 + 5353*(2*a*b*c^3 + 3*a^2*c^2*d)*m)*x^3 + (a^2*c^3*m^
5 + 35*a^2*c^3*m^4 + 470*a^2*c^3*m^3 + 3010*a^2*c^3*m^2 + 9129*a^2*c^3*m +
10395*a^2*c^3)*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 1952
4*m + 10395)
```

Sympy [A] time = 5.01584, size = 4345, normalized size = 28.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((-a**2*c**3/(10*x**10) - 3*a**2*c**2*d/(8*x**8) - a**2*c*d**2/(2*
x**6) - a**2*d**3/(4*x**4) - a*b*c**3/(4*x**8) - a*b*c**2*d/x**6 - 3*a*b*c*
d**2/(2*x**4) - a*b*d**3/x**2 - b**2*c**3/(6*x**6) - 3*b**2*c**2*d/(4*x**4)
 - 3*b**2*c*d**2/(2*x**2) + b**2*d**3*log(x), Eq(m, -11)), (-a**2*c**3/(8*x
**8) - a**2*c**2*d/(2*x**6) - 3*a**2*c*d**2/(4*x**4) - a**2*d**3/(2*x**2) -
 a*b*c**3/(3*x**6) - 3*a*b*c**2*d/(2*x**4) - 3*a*b*c*d**2/x**2 + 2*a*b*d**3
*log(x) - b**2*c**3/(4*x**4) - 3*b**2*c**2*d/(2*x**2) + 3*b**2*c*d**2*log(x
) + b**2*d**3*x**2/2, Eq(m, -9)), (-a**2*c**3/(6*x**6) - 3*a**2*c**2*d/(4*x
**4) - 3*a**2*c*d**2/(2*x**2) + a**2*d**3*log(x) - a*b*c**3/(2*x**4) - 3*a*
b*c**2*d/x**2 + 6*a*b*c*d**2*log(x) + a*b*d**3*x**2 - b**2*c**3/(2*x**2) +
3*b**2*c**2*d*log(x) + 3*b**2*c*d**2*x**2/2 + b**2*d**3*x**4/4, Eq(m, -7)),
 (-a**2*c**3/(4*x**4) - 3*a**2*c**2*d/(2*x**2) + 3*a**2*c*d**2*log(x) + a**
2*d**3*x**2/2 - a*b*c**3/x**2 + 6*a*b*c**2*d*log(x) + 3*a*b*c*d**2*x**2 + a
*b*d**3*x**4/2 + b**2*c**3*log(x) + 3*b**2*c**2*d*x**2/2 + 3*b**2*c*d**2*x*
*4/4 + b**2*d**3*x**6/6, Eq(m, -5)), (-a**2*c**3/(2*x**2) + 3*a**2*c**2*d*l
og(x) + 3*a**2*c*d**2*x**2/2 + a**2*d**3*x**4/4 + 2*a*b*c**3*log(x) + 3*a*b
*c**2*d*x**2 + 3*a*b*c*d**2*x**4/2 + a*b*d**3*x**6/3 + b**2*c**3*x**2/2 + 3
*b**2*c**2*d*x**4/4 + b**2*c*d**2*x**6/2 + b**2*d**3*x**8/8, Eq(m, -3)), (a
**2*c**3*log(x) + 3*a**2*c**2*d*x**2/2 + 3*a**2*c*d**2*x**4/4 + a**2*d**3*x
**6/6 + a*b*c**3*x**2 + 3*a*b*c**2*d*x**4/2 + a*b*c*d**2*x**6 + a*b*d**3*x*
*8/4 + b**2*c**3*x**4/4 + b**2*c**2*d*x**6/2 + 3*b**2*c*d**2*x**8/8 + b**2*
d**3*x**10/10, Eq(m, -1)), (a**2*c**3*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**2*c**3*m**4*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*
a**2*c**3*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 3010*a**2*c**3*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*c**3*m*x*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a*
*2*c**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 3*a**2*c**2*d*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 99*a**2*c**2*d*m**4*x**3*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1218*a*
```

*2*c**2*d*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 6786*a**2*c**2*d*m**2*x**3*x**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 16059*a**2*c**2*d*m*x*
*3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 10395*a**2*c**2*d*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 3*a**2*c*d**2*m**5*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 93*a**2*c*d**2*m
4*x5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 1050*a**2*c*d**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 5190*a**2*c*d**2*m**2*x**5*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 104
67*a**2*c*d**2*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
2 + 19524*m + 10395) + 6237*a2*c*d**2*x**5*x**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + a**2*d**3*m**5*x**7*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29
*a**2*d**3*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 302*a**2*d**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1366*a**2*d**3*m**2*x**7
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2577*a**2*d**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 1485*a**2*d**3*x**7*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*c**3*m**5*x**3*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 66*a*b*c**3*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 812*a*b*c**3*m**3*x**3*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4524*a*b*c**3*m**2*x**3
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 10706*a*b*c**3*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 6930*a*b*c**3*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c**2*d*m**5*x**5*x
m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 186*a*b*c**2*d*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 2100*a*b*c**2*d*m**3*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10380*a*b*c**2*d
*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 20934*a*b*c**2*d*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 12474*a*b*c**2*d*x**5*x**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c*d
2*m5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + 174*a*b*c*d**2*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1812*a*b*c*d**2*m**3*x**7*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8
196*a*b*c*d**2*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 15462*a*b*c*d**2*m*x**7*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8910*a*b*c*d**2*x**7
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2*a*b*d**3*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 54*a*b*d**3*m**4*x**9*x**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 524*a*b*d**3*m**3*x**9
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2244*a*b*d**3*m**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 4082*a*b*d**3*m*x**9*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2310*a*b*d**3*x**9*x
m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ b**2*c**3*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
2 + 19524*m + 10395) + 31*b2*c**3*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*c**3*m**3*x**5*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 1730*b**2*c**3*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1

```
2139*m**2 + 19524*m + 10395) + 3489*b**2*c**3*m*x**5*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*c**3*x**5
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 3*b**2*c**2*d*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 87*b**2*c**2*d*m**4*x**7*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 906*b**2*c**2*d*
m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 4098*b**2*c**2*d*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 7731*b**2*c**2*d*m*x**7*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4455*
b**2*c**2*d*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 3*b**2*c*d**2*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 81*b**2*c*d**2*m**4*x**9*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
786*b**2*c*d**2*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 3366*b**2*c*d**2*m**2*x**9*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6123*b**2*c*d**2
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 3465*b**2*c*d**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + b**2*d**3*m**5*x**11*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**2*d**3*m*
*4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 230*b**2*d**3*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 950*b**2*d**3*m**2*x**11*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b*
*2*d**3*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 945*b**2*d**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

Giac [B] time = 1.2044, size = 1609, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (b^2*d^3*m^5*x^11*x^m + 25*b^2*d^3*m^4*x^11*x^m + 3*b^2*c*d^2*m^5*x^9*x^m +
 2*a*b*d^3*m^5*x^9*x^m + 230*b^2*d^3*m^3*x^11*x^m + 81*b^2*c*d^2*m^4*x^9*x^
m + 54*a*b*d^3*m^4*x^9*x^m + 950*b^2*d^3*m^2*x^11*x^m + 3*b^2*c^2*d*m^5*x^7
*x^m + 6*a*b*c*d^2*m^5*x^7*x^m + a^2*d^3*m^5*x^7*x^m + 786*b^2*c*d^2*m^3*x^
9*x^m + 524*a*b*d^3*m^3*x^9*x^m + 1689*b^2*d^3*m*x^11*x^m + 87*b^2*c^2*d*m^
4*x^7*x^m + 174*a*b*c*d^2*m^4*x^7*x^m + 29*a^2*d^3*m^4*x^7*x^m + 3366*b^2*c
*d^2*m^2*x^9*x^m + 2244*a*b*d^3*m^2*x^9*x^m + 945*b^2*d^3*x^11*x^m + b^2*c^
3*m^5*x^5*x^m + 6*a*b*c^2*d*m^5*x^5*x^m + 3*a^2*c*d^2*m^5*x^5*x^m + 906*b^2
*c^2*d*m^3*x^7*x^m + 1812*a*b*c*d^2*m^3*x^7*x^m + 302*a^2*d^3*m^3*x^7*x^m +
 6123*b^2*c*d^2*m*x^9*x^m + 4082*a*b*d^3*m*x^9*x^m + 31*b^2*c^3*m^4*x^5*x^m
 + 186*a*b*c^2*d*m^4*x^5*x^m + 93*a^2*c*d^2*m^4*x^5*x^m + 4098*b^2*c^2*d*m^
2*x^7*x^m + 8196*a*b*c*d^2*m^2*x^7*x^m + 1366*a^2*d^3*m^2*x^7*x^m + 3465*b^
2*c*d^2*x^9*x^m + 2310*a*b*d^3*x^9*x^m + 2*a*b*c^3*m^5*x^3*x^m + 3*a^2*c^2*
d*m^5*x^3*x^m + 350*b^2*c^3*m^3*x^5*x^m + 2100*a*b*c^2*d*m^3*x^5*x^m + 1050
*a^2*c*d^2*m^3*x^5*x^m + 7731*b^2*c^2*d*m*x^7*x^m + 15462*a*b*c*d^2*m*x^7*x
^m + 2577*a^2*d^3*m*x^7*x^m + 66*a*b*c^3*m^4*x^3*x^m + 99*a^2*c^2*d*m^4*x^3
*x^m + 1730*b^2*c^3*m^2*x^5*x^m + 10380*a*b*c^2*d*m^2*x^5*x^m + 5190*a^2*c*
d^2*m^2*x^5*x^m + 4455*b^2*c^2*d*x^7*x^m + 8910*a*b*c*d^2*x^7*x^m + 1485*a^
2*d^3*x^7*x^m + a^2*c^3*m^5*x*x^m + 812*a*b*c^3*m^3*x^3*x^m + 1218*a^2*c^2*
d*m^3*x^3*x^m + 3489*b^2*c^3*m*x^5*x^m + 20934*a*b*c^2*d*m*x^5*x^m + 10467*
```

```
a^2*c*d^2*m*x^5*x^m + 35*a^2*c^3*m^4*x*x^m + 4524*a*b*c^3*m^2*x^3*x^m + 678
6*a^2*c^2*d*m^2*x^3*x^m + 2079*b^2*c^3*x^5*x^m + 12474*a*b*c^2*d*x^5*x^m +
6237*a^2*c*d^2*x^5*x^m + 470*a^2*c^3*m^3*x*x^m + 10706*a*b*c^3*m*x^3*x^m +
16059*a^2*c^2*d*m*x^3*x^m + 3010*a^2*c^3*m^2*x*x^m + 6930*a*b*c^3*x^3*x^m +
 10395*a^2*c^2*d*x^3*x^m + 9129*a^2*c^3*m*x*x^m + 10395*a^2*c^3*x*x^m)/(m^6
 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

3.326 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=109

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 + m) + ((b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d*(b*c + a*d)*x^(7 +
m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rubi [A] time = 0.0622496, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 + m) + ((b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d*(b*c + a*d)*x^(7 +
m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \int \left(a^2c^2x^m+2ac(bc+ad)x^{2+m}+\left(b^2c^2+4abcd+a^2d^2\right)x^{4+m}+2bd(bc+ad)x^{6+m}+b^2d\right.$$

$$=\frac{a^2c^2x^{1+m}}{1+m}+\frac{2ac(bc+ad)x^{3+m}}{3+m}+\frac{\left(b^2c^2+4abcd+a^2d^2\right)x^{5+m}}{5+m}+\frac{2bd(bc+ad)x^{7+m}}{7+m}+\frac{b^2}{9}$$

Mathematica [A] time = 0.0723732, size = 101, normalized size = 0.93

$$x^m\left(\frac{x^5\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x}{m+1}+\frac{2bdx^7(ad+bc)}{m+7}+\frac{2acx^3(ad+bc)}{m+3}+\frac{b^2d^2x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] x^m*((a^2*c^2*x)/(1 + m) + (2*a*c*(b*c + a*d)*x^3)/(3 + m) + ((b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^5)/(5 + m) + (2*b*d*(b*c + a*d)*x^7)/(7 + m) + (b^2*d^
2*x^9)/(9 + m))
```

Maple [B] time = 0.007, size = 569, normalized size = 5.2

$$x^{1+m} \left(b^2 d^2 m^4 x^8 + 16 b^2 d^2 m^3 x^8 + 2 a b d^2 m^4 x^6 + 2 b^2 c d m^4 x^6 + 86 b^2 d^2 m^2 x^8 + 36 a b d^2 m^3 x^6 + 36 b^2 c d m^3 x^6 + 176 b^2 d^2 m\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] x^(1+m)*(b^2*d^2*m^4*x^8+16*b^2*d^2*m^3*x^8+2*a*b*d^2*m^4*x^6+2*b^2*c*d*m^4
*x^6+86*b^2*d^2*m^2*x^8+36*a*b*d^2*m^3*x^6+36*b^2*c*d*m^3*x^6+176*b^2*d^2*m
*x^8+a^2*d^2*m^4*x^4+4*a*b*c*d*m^4*x^4+208*a*b*d^2*m^2*x^6+b^2*c^2*m^4*x^4+
208*b^2*c*d*m^2*x^6+105*b^2*d^2*x^8+20*a^2*d^2*m^3*x^4+80*a*b*c*d*m^3*x^4+4
44*a*b*d^2*m*x^6+20*b^2*c^2*m^3*x^4+444*b^2*c*d*m*x^6+2*a^2*c*d*m^4*x^2+130
*a^2*d^2*m^2*x^4+2*a*b*c^2*m^4*x^2+520*a*b*c*d*m^2*x^4+270*a*b*d^2*x^6+130*
b^2*c^2*m^2*x^4+270*b^2*c*d*x^6+44*a^2*c*d*m^3*x^2+300*a^2*d^2*m*x^4+44*a*b
*c^2*m^3*x^2+1200*a*b*c*d*m*x^4+300*b^2*c^2*m*x^4+a^2*c^2*m^4+328*a^2*c*d*m
^2*x^2+189*a^2*d^2*x^4+328*a*b*c^2*m^2*x^2+756*a*b*c*d*x^4+189*b^2*c^2*x^4+
24*a^2*c^2*m^3+916*a^2*c*d*m*x^2+916*a*b*c^2*m*x^2+206*a^2*c^2*m^2+630*a^2*
c*d*x^2+630*a*b*c^2*x^2+744*a^2*c^2*m+945*a^2*c^2)/(9+m)/(7+m)/(5+m)/(3+m)/
(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.857922, size = 984, normalized size = 9.03

$$\left(\left(b^2 d^2 m^4 + 16 b^2 d^2 m^3 + 86 b^2 d^2 m^2 + 176 b^2 d^2 m + 105 b^2 d^2\right) x^9 + 2 \left(\left(b^2 c d + a b d^2\right) m^4 + 135 b^2 c d + 135 a b d^2 + 18 \left(b^2 c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*m^4 + 16*b^2*d^2*m^3 + 86*b^2*d^2*m^2 + 176*b^2*d^2*m + 105*b^2*d
^2)*x^9 + 2*((b^2*c*d + a*b*d^2)*m^4 + 135*b^2*c*d + 135*a*b*d^2 + 18*(b^2*
c*d + a*b*d^2)*m^3 + 104*(b^2*c*d + a*b*d^2)*m^2 + 222*(b^2*c*d + a*b*d^2)*
m)*x^7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^4 + 189*b^2*c^2 + 756*a*b*c*d +
 189*a^2*d^2 + 20*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^3 + 130*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*m^2 + 300*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m)*x^5 + 2*((a*b
*c^2 + a^2*c*d)*m^4 + 315*a*b*c^2 + 315*a^2*c*d + 22*(a*b*c^2 + a^2*c*d)*m^
3 + 164*(a*b*c^2 + a^2*c*d)*m^2 + 458*(a*b*c^2 + a^2*c*d)*m)*x^3 + (a^2*c^2
*m^4 + 24*a^2*c^2*m^3 + 206*a^2*c^2*m^2 + 744*a^2*c^2*m + 945*a^2*c^2)*x)*x
^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 3.0362, size = 2363, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**2*c**2/(8*x**8) - a**2*c*d/(3*x**6) - a**2*d**2/(4*x**4) - a
*b*c**2/(3*x**6) - a*b*c*d/x**4 - a*b*d**2/x**2 - b**2*c**2/(4*x**4) - b**2
*c*d/x**2 + b**2*d**2*log(x), Eq(m, -9)), (-a**2*c**2/(6*x**6) - a**2*c*d/(
2*x**4) - a**2*d**2/(2*x**2) - a*b*c**2/(2*x**4) - 2*a*b*c*d/x**2 + 2*a*b*d
**2*log(x) - b**2*c**2/(2*x**2) + 2*b**2*c*d*log(x) + b**2*d**2*x**2/2, Eq(
m, -7)), (-a**2*c**2/(4*x**4) - a**2*c*d/x**2 + a**2*d**2*log(x) - a*b*c**2
/x**2 + 4*a*b*c*d*log(x) + a*b*d**2*x**2 + b**2*c**2*log(x) + b**2*c*d*x**2
 + b**2*d**2*x**4/4, Eq(m, -5)), (-a**2*c**2/(2*x**2) + 2*a**2*c*d*log(x) +
 a**2*d**2*x**2/2 + 2*a*b*c**2*log(x) + 2*a*b*c*d*x**2 + a*b*d**2*x**4/2 +
b**2*c**2*x**2/2 + b**2*c*d*x**4/2 + b**2*d**2*x**6/6, Eq(m, -3)), (a**2*c*
*2*log(x) + a**2*c*d*x**2 + a**2*d**2*x**4/4 + a*b*c**2*x**2 + a*b*c*d*x**4
 + a*b*d**2*x**6/3 + b**2*c**2*x**4/4 + b**2*c*d*x**6/3 + b**2*d**2*x**8/8,
 Eq(m, -1)), (a**2*c**2*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**2*c**2*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 206*a**2*c**2*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 744*a**2*c**2*m*x*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 945*a**2*c**2*x*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 2*a**2*c*d*m**4*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a**2*c*d*m**3*x**3*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a**2*c*d*m**2*x**3*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a**2*c*d*m*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a**2*
c*d*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + a**2*
d**2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 20*a**2*d**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 130*a**2*d**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 300*a**2*d**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 189*a**2*d**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 2*a*b*c**2*m**4*x**3*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a*b*c**2*m**3*x**3*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*c**2*m**2*x**3*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*c**2*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*c**2*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b*c*d
*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 80*
a*b*c*d*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 520*a*b*c*d*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 1200*a*b*c*d*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 756*a*b*c*d*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 36*a*b*d**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 208*a*b*d**2*m**2*x**7*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*a*b*d**2*m*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*a*b*d**2*x**7*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*c**2*m**4*x**5*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 20*b**2*c**2*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 130*b
**2*c**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 300*b**2*c**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 189*b**2*c**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
```

```
 1689*m + 945) + 2*b**2*c*d*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 36*b**2*c*d*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 208*b**2*c*d*m**2*x**7*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b**2*c*d*m*x**7*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*b**2*c*d*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*d**2*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**2*d**2*m**3*
x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**2*d
**2*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
176*b**2*d**2*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 105*b**2*d**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945), True))
```

Giac [B] time = 1.19144, size = 949, normalized size = 8.71

$$\frac{b^2d^2m^4x^9x^m + 16\,b^2d^2m^3x^9x^m + 2\,b^2cdm^4x^7x^m + 2\,abd^2m^4x^7x^m + 86\,b^2d^2m^2x^9x^m + 36\,b^2cdm^3x^7x^m + 36\,abd^2m^3x^7x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*m^4*x^9*x^m + 16*b^2*d^2*m^3*x^9*x^m + 2*b^2*c*d*m^4*x^7*x^m + 2*a
*b*d^2*m^4*x^7*x^m + 86*b^2*d^2*m^2*x^9*x^m + 36*b^2*c*d*m^3*x^7*x^m + 36*a
*b*d^2*m^3*x^7*x^m + 176*b^2*d^2*m*x^9*x^m + b^2*c^2*m^4*x^5*x^m + 4*a*b*c*
d*m^4*x^5*x^m + a^2*d^2*m^4*x^5*x^m + 208*b^2*c*d*m^2*x^7*x^m + 208*a*b*d^2
*m^2*x^7*x^m + 105*b^2*d^2*x^9*x^m + 20*b^2*c^2*m^3*x^5*x^m + 80*a*b*c*d*m^
3*x^5*x^m + 20*a^2*d^2*m^3*x^5*x^m + 444*b^2*c*d*m*x^7*x^m + 444*a*b*d^2*m*
x^7*x^m + 2*a*b*c^2*m^4*x^3*x^m + 2*a^2*c*d*m^4*x^3*x^m + 130*b^2*c^2*m^2*x
^5*x^m + 520*a*b*c*d*m^2*x^5*x^m + 130*a^2*d^2*m^2*x^5*x^m + 270*b^2*c*d*x^
7*x^m + 270*a*b*d^2*x^7*x^m + 44*a*b*c^2*m^3*x^3*x^m + 44*a^2*c*d*m^3*x^3*x
^m + 300*b^2*c^2*m*x^5*x^m + 1200*a*b*c*d*m*x^5*x^m + 300*a^2*d^2*m*x^5*x^m
 + a^2*c^2*m^4*x*x^m + 328*a*b*c^2*m^2*x^3*x^m + 328*a^2*c*d*m^2*x^3*x^m +
189*b^2*c^2*x^5*x^m + 756*a*b*c*d*x^5*x^m + 189*a^2*d^2*x^5*x^m + 24*a^2*c^
2*m^3*x*x^m + 916*a*b*c^2*m*x^3*x^m + 916*a^2*c*d*m*x^3*x^m + 206*a^2*c^2*m
^2*x*x^m + 630*a*b*c^2*x^3*x^m + 630*a^2*c*d*x^3*x^m + 744*a^2*c^2*m*x*x^m
+ 945*a^2*c^2*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.327 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=71

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) + (b*(b*c +
 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.0374977, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) + (b*(b*c +
 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx &= \int \left(a^2cx^m+a(2bc+ad)x^{2+m}+b(bc+2ad)x^{4+m}+b^2dx^{6+m}\right) \, dx\\ &= \frac{a^2cx^{1+m}}{1+m}+\frac{a(2bc+ad)x^{3+m}}{3+m}+\frac{b(bc+2ad)x^{5+m}}{5+m}+\frac{b^2dx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.0633677, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2c}{m+1}+\frac{bx^4(2ad+bc)}{m+5}+\frac{ax^2(ad+2bc)}{m+3}+\frac{b^2dx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*c)/(1 + m) + (a*(2*b*c + a*d)*x^2)/(3 + m) + (b*(b*c + 2*a*
d)*x^4)/(5 + m) + (b^2*d*x^6)/(7 + m))
```

Maple [B] time = 0.006, size = 262, normalized size = 3.7

$$\frac{x^{1+m}\left(b^{2}dm^{3}x^{6}+9\,b^{2}dm^{2}x^{6}+2\,abdm^{3}x^{4}+b^{2}cm^{3}x^{4}+23\,b^{2}dmx^{6}+22\,abdm^{2}x^{4}+11\,b^{2}cm^{2}x^{4}+15\,b^{2}dx^{6}+a^{2}dm^{3}x^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] x^(1+m)*(b^2*d*m^3*x^6+9*b^2*d*m^2*x^6+2*a*b*d*m^3*x^4+b^2*c*m^3*x^4+23*b^2
*d*m*x^6+22*a*b*d*m^2*x^4+11*b^2*c*m^2*x^4+15*b^2*d*x^6+a^2*d*m^3*x^2+2*a*b
*c*m^3*x^2+62*a*b*d*m*x^4+31*b^2*c*m*x^4+13*a^2*d*m^2*x^2+26*a*b*c*m^2*x^2+
42*a*b*d*x^4+21*b^2*c*x^4+a^2*c*m^3+47*a^2*d*m*x^2+94*a*b*c*m*x^2+15*a^2*c*
m^2+35*a^2*d*x^2+70*a*b*c*x^2+71*a^2*c*m+105*a^2*c)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.881687, size = 490, normalized size = 6.9

$$\frac{\left(\left(b^{2}dm^{3}+9\,b^{2}dm^{2}+23\,b^{2}dm+15\,b^{2}d\right)x^{7}+\left(\left(b^{2}c+2\,abd\right)m^{3}+21\,b^{2}c+42\,abd+11\,\left(b^{2}c+2\,abd\right)m^{2}+31\,\left(b^{2}c+2\,a\right.\right.\right.}{m^{4}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] ((b^2*d*m^3 + 9*b^2*d*m^2 + 23*b^2*d*m + 15*b^2*d)*x^7 + ((b^2*c + 2*a*b*d)
*m^3 + 21*b^2*c + 42*a*b*d + 11*(b^2*c + 2*a*b*d)*m^2 + 31*(b^2*c + 2*a*b*d
)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 70*a*b*c + 35*a^2*d + 13*(2*a*b*c + a^2
*d)*m^2 + 47*(2*a*b*c + a^2*d)*m)*x^3 + (a^2*c*m^3 + 15*a^2*c*m^2 + 71*a^2*
c*m + 105*a^2*c)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 1.68384, size = 1044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] Piecewise((-a**2*c/(6*x**6) - a**2*d/(4*x**4) - a*b*c/(2*x**4) - a*b*d/x**2
 - b**2*c/(2*x**2) + b**2*d*log(x), Eq(m, -7)), (-a**2*c/(4*x**4) - a**2*d/
(2*x**2) - a*b*c/x**2 + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x**2/2, Eq(
```

```
m, -5)), (-a**2*c/(2*x**2) + a**2*d*log(x) + 2*a*b*c*log(x) + a*b*d*x**2 +
b**2*c*x**2/2 + b**2*d*x**4/4, Eq(m, -3)), (a**2*c*log(x) + a**2*d*x**2/2 +
 a*b*c*x**2 + a*b*d*x**4/2 + b**2*c*x**4/4 + b**2*d*x**6/6, Eq(m, -1)), (a*
*2*c*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a**2*c*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a**2*c*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*c*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + a**2*d*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + 13*a**2*d*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 47*a**2*d*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a
**2*d*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*c*m**3*x**
3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*a*b*c*m**2*x**3*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*a*b*c*m*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 70*a*b*c*x**3*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 2*a*b*d*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 22*a*b*d*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 62*a*b*d*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 42*a*b*d*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**2*c*m**3*x**5*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b**2*c*m**2*x**5*x**m/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105) + 31*b**2*c*m*x**5*x**m/(m**4 + 16*m**3 +
86*m**2 + 176*m + 105) + 21*b**2*c*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b**2*d*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*b**2*d*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b*
*2*d*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*b**2*d*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [B] time = 1.15522, size = 448, normalized size = 6.31

$$\frac{b^2dm^3x^7x^m + 9\,b^2dm^2x^7x^m + b^2cm^3x^5x^m + 2\,abdm^3x^5x^m + 23\,b^2dmx^7x^m + 11\,b^2cm^2x^5x^m + 22\,abdm^2x^5x^m + 15\,b^2dx^7x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*d*m^3*x^7*x^m + 9*b^2*d*m^2*x^7*x^m + b^2*c*m^3*x^5*x^m + 2*a*b*d*m^3*
x^5*x^m + 23*b^2*d*m*x^7*x^m + 11*b^2*c*m^2*x^5*x^m + 22*a*b*d*m^2*x^5*x^m
+ 15*b^2*d*x^7*x^m + 2*a*b*c*m^3*x^3*x^m + a^2*d*m^3*x^3*x^m + 31*b^2*c*m*x
^5*x^m + 62*a*b*d*m*x^5*x^m + 26*a*b*c*m^2*x^3*x^m + 13*a^2*d*m^2*x^3*x^m +
 21*b^2*c*x^5*x^m + 42*a*b*d*x^5*x^m + a^2*c*m^3*x*x^m + 94*a*b*c*m*x^3*x^m
 + 47*a^2*d*m*x^3*x^m + 15*a^2*c*m^2*x*x^m + 70*a*b*c*x^3*x^m + 35*a^2*d*x^
3*x^m + 71*a^2*c*m*x*x^m + 105*a^2*c*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

3.328 $\int \frac{x^m\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=94

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

```
[Out] -((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d*(3 + m))
+ ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(c*d^2*(1 + m))
```

Rubi [A] time = 0.0597319, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 364}

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d*(3 + m))
+ ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(c*d^2*(1 + m))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(-\frac{b(bc-2ad)x^m}{d^2}+\frac{b^2x^{2+m}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^m}{d^2\left(c+dx^2\right)}\right)dx\\
&= -\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)}+\frac{(bc-ad)^2\int \frac{x^m}{c+dx^2}\,dx}{d^2}\\
&= -\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{cd^2(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0971034, size = 118, normalized size = 1.26

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(1,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/
(1 + m) + b*x^2*((2*a*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((d*x^2)/
c)])/(3 + m) + (b*x^2*Hypergeometric2F1[1, (5 + m)/2, (7 + m)/2, -((d*x^2)/
c)])/(5 + m))))/c
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 x^m}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 x^m}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)x^m}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d*x^2 + c), x)
```

Sympy [C] time = 9.36321, size = 299, normalized size = 3.18

$$\frac{a^2 m x x^m \Phi\left(\frac{dx^2 e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{a^2 x x^m \Phi\left(\frac{dx^2 e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{abmx^3 x^m \Phi\left(\frac{dx^2 e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{2c\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2*m*x*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*c*gamma(m/2 + 3/2)) + a**2*x*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + a*b*m*x**3*x**m*l
erchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*c*gamma
(m/2 + 5/2)) + 3*a*b*x**3*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(2*c*gamma(m/2 + 5/2)) + b**2*m*x**5*x**m*lerchphi(d*
x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2
)) + 5*b**2*x**5*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamm
a(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 x^m}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

3.329 $\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=120

$$-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x^{m+1}}{d^2(m+1)}$$

```
[Out] (b^2*x^(1 + m))/(d^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*c*d^2*(c + d*x
^2)) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*d^2*(1 + m))
```

Rubi [A] time = 0.103223, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {463, 459, 364}

$$-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x^{m+1}}{d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^(1 + m))/(d^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*c*d^2*(c + d*x
^2)) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*d^2*(1 + m))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{\int \frac{x^m(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^2)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^{1+m}}{d^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(ad(1-m)+bc(3+m)))\int \frac{x^m}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^{1+m}}{d^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{(bc-ad)(ad(1-m)+bc(3+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(1+m)}$$

Mathematica [A] time = 0.108257, size = 118, normalized size = 0.98

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(2,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(2,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/
(1 + m) + b*x^2*((2*a*Hypergeometric2F1[2, (3 + m)/2, (5 + m)/2, -((d*x^2)/
c)])/(3 + m) + (b*x^2*Hypergeometric2F1[2, (5 + m)/2, (7 + m)/2, -((d*x^2)/
c)])/(5 + m))))/c^2
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2x^m}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2x^m}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)x^m}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\left(a + bx^2\right)^2}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 x^m}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

3.330 $\int \frac{x^m\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)+2abcd\left(1-m^2\right)+b^2c^2\left(m^2+4m+3\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3d^2(m+1)}-\frac{x^{m+1}(bc-ad)(ad(3-m}{8c^2d^2\left(c+d\right.}$$

```
[Out] ((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(a*d*(3 -
m) + b*c*(5 + m))*x^(1 + m))/(8*c^2*d^2*(c + d*x^2)) + ((2*a*b*c*d*(1 - m^2
) + a^2*d^2*(3 - 4*m + m^2) + b^2*c^2*(3 + 4*m + m^2))*x^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^3*d^2*(1 + m))
```

Rubi [A] time = 0.153598, antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {463, 457, 364}

$$\frac{x^{m+1}\left(\frac{(1-m)\left(4a^2d^2-(m+1)(bc-ad)^2\right)}{c^2(m+1)}+4b^2\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8cd^2}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{8c^2d^2\left(c+dx^2\right)}+\frac{x^{m+1}(bc-a}{4cd^2\left(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(a*d*(3 -
m) + b*c*(5 + m))*x^(1 + m))/(8*c^2*d^2*(c + d*x^2)) + ((4*b^2 + ((1 - m)*(
4*a^2*d^2 - (b*c - a*d)^2*(1 + m)))/(c^2*(1 + m)))*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c*d^2)
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{1+m}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^m\left(-4a^2d^2+(bc-ad)^2(1+m)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{1+m}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2\left(c+dx^2\right)} + -\frac{\left(-4b^2c^2d(1+m)-d(-1+m)\left(-4a^2c\right.\right.}{8c^2c}$$

$$= \frac{(bc-ad)^2x^{1+m}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2\left(c+dx^2\right)} + \frac{\left(4b^2c^2(1+m)+(1-m)\left(4a^2d^2-(bc\right.\right.}{8c^3c}$$

Mathematica [A] time = 0.107786, size = 118, normalized size = 0.69

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(3,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(3,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/
(1 + m) + b*x^2*((2*a*Hypergeometric2F1[3, (3 + m)/2, (5 + m)/2, -((d*x^2)/
c)])/(3 + m) + (b*x^2*Hypergeometric2F1[3, (5 + m)/2, (7 + m)/2, -((d*x^2)/
c)])/(5 + m))))/c^3
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2x^m}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2x^m}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)x^{m}}{d^{3}x^{6}+3\,cd^{2}x^{4}+3\,c^{2}dx^{2}+c^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x
^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}\left(a+bx^{2}\right)^{2}}{\left(c+dx^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{2}x^{m}}{\left(dx^{2}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

3.331 $\int \frac{x^m\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=133

$$\frac{dx^{m+1}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{x^{m+1}(bc-ad)^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^3x^{m+5}}{b(m+5)}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (d^2*(3*b*c
 - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(5 + m)) + ((b*c - a*
d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*b^3*(1 + m))
```

Rubi [A] time = 0.0860909, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 364}

$$\frac{dx^{m+1}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{x^{m+1}(bc-ad)^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^3x^{m+5}}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (d^2*(3*b*c
 - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(5 + m)) + ((b*c - a*
d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*b^3*(1 + m))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m\left(c+dx^2\right)^3}{a+bx^2}\,dx=\int\left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^m}{b^3}+\frac{d^2(3bc-ad)x^{2+m}}{b^2}+\frac{d^3x^{4+m}}{b}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3\right.}{b^3\left(a+bx^2\right)}\right.$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{1+m}}{b^3(1+m)}+\frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)}+\frac{d^3x^{5+m}}{b(5+m)}+\frac{(bc-ad)^3\int\frac{x^m}{a+bx^2}\,dx}{b^3}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{1+m}}{b^3(1+m)}+\frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)}+\frac{d^3x^{5+m}}{b(5+m)}+\frac{(bc-ad)^3x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2}\right.}{ab^3(1+m)}$$

Mathematica [C] time = 1.36008, size = 114, normalized size = 0.86

$$\frac{x^{m+1}\left(dx^2\left(3c^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+3}{2}\right)+dx^2\left(3c\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+7}{2}\right)\right)\right)+c^3\Phi\left(-\frac{bx^2}{a},1,\frac{m+1}{2}\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(c^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(3*c^2*
HurwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*(3*c*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/
2]))))/(2*a)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3\right)x^m}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b*x^2 + a), x)
```

Sympy [C] time = 16.9858, size = 411, normalized size = 3.09

$$\frac{c^3mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^3xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3c^2dmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] c**3*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + 3*c**2*d*m*x**3*x
**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*
gamma(m/2 + 5/2)) + 9*c**2*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*c*d**2*m*x**5*x**m
*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gam
ma(m/2 + 7/2)) + 15*c*d**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + d**3*m*x**7*x**m*lerch
phi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2
 + 9/2)) + 7*d**3*x**7*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2
)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

3.332 $\int \frac{x^m(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=94

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

```
[Out] (d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3 + m)) + (
(b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a*b^2*(1 + m))
```

Rubi [A] time = 0.0576673, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 364}

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3 + m)) + (
(b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a*b^2*(1 + m))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \int \left(\frac{d(2bc-ad)x^m}{b^2}+\frac{d^2x^{2+m}}{b}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^m}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2\int \frac{x^m}{a+bx^2}\,dx}{b^2}\\
&= \frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab^2(1+m)}
\end{aligned}$$

Mathematica [C] time = 0.414335, size = 85, normalized size = 0.9

$$\frac{x^{m+1}\left(c^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+1}{2}\right)+dx^2\left(2c\Phi\left(-\frac{bx^2}{a},1,\frac{m+3}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(2*c*Hu
rwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^2)
/a), 1, (5 + m)/2])))/(2*a)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^2 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^2 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^2x^4+2\,cdx^2+c^2)x^m}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b*x^2 + a), x)
```

Sympy [C] time = 9.32418, size = 299, normalized size = 3.18

$$\frac{c^2mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{cdmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] c**2*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c*d*m*x**3*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*a*gamma
(m/2 + 5/2)) + 3*c*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(2*a*gamma(m/2 + 5/2)) + d**2*m*x**5*x**m*lerchphi(b*
x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2
)) + 5*d**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamm
a(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

3.333 $\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

```
[Out] (d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi [A] time = 0.0311237, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {459, 364}

$$\frac{x^{m+1}(bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(c+dx^2\right)}{a+bx^2}\,dx&=\frac{dx^{1+m}}{b(1+m)}-\frac{(-bc(1+m)+ad(1+m))\int \frac{x^m}{a+bx^2}\,dx}{b(1+m)}\\&=\frac{dx^{1+m}}{b(1+m)}+\frac{(bc-ad)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}\end{aligned}$$

Mathematica [A] time = 0.054659, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ad\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{\left(d x^2 + c\right) x^m}{b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^2 + c\right) x^m}{b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d x^2 + c\right) x^m}{b x^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

Sympy [C] time = 4.81527, size = 190, normalized size = 2.88

$$\frac{cmxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{cxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{dmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] c*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + c*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*m*x**3*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5
/2)) + 3*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

3.334 $\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=102

$$\frac{bx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

```
[Out] (b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(
b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

Rubi [A] time = 0.0437234, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {482, 364}

$$\frac{bx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(
b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{b\int \frac{x^m}{a+bx^2}\,dx}{bc-ad}-\frac{d\int \frac{x^m}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{bx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(bc-ad)(1+m)}-\frac{dx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{c(bc-ad)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0535, size = 85, normalized size = 0.83

$$\frac{x^{m+1}\left(ad\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-bc\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(b*c*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [C] time = 12.0885, size = 354, normalized size = 3.47

$$\frac{amx^m\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{x^3\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}-\frac{3ax^m\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{x^3\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*m*x**m*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1, 3/2 - m/2)*gamma(3/2 - m/2
)**2/(x**3*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2
- m/2)*gamma(5/2 - m/2))) - 3*a*x**m*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1
, 3/2 - m/2)*gamma(3/2 - m/2)**2/(x**3*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2
- m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2))) + b*m*x**m*lerchphi(c
*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)*gamma(5/2 - m/2)/
(x*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*g
amma(5/2 - m/2))) - b*x**m*lerchphi(c*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/
2)*gamma(1/2 - m/2)*gamma(5/2 - m/2)/(x*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2
 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.335 $\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=156

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rubi [A] time = 0.189939, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {472, 584, 364}

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{x^m\left(2ad-bc(1-m)-bd(1-m)x^2\right)}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \left(\frac{b(-bc(1-m)+ad(3-m))x^m}{(bc-ad)\left(a+bx^2\right)} + \frac{2ad^2x^m}{(-bc+ad)\left(c+dx^2\right)}\right)dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{d^2\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc(1-m)-ad(3-m)))\int \frac{x^m}{a+bx^2}\,dx}{2a(bc-ad)^2}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{b(ad(3-m)-b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)} + \frac{d^2x^{1+m}}{c\cdots}$$

Mathematica [A] time = 0.0827292, size = 127, normalized size = 0.81

$$\frac{x^{m+1}\left(a^2d^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-abcd\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bc(bc-ad)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/
a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] + b
*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(
a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2dx^6 + (b^2c + 2\,abd)x^4 + a^2c + (2\,abc + a^2d)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)
*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^2 (dx^2 + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.336 $\int \frac{x^m}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=234

$$\frac{bx^{m+1}\left(a^2d^2\left(m^2-8m+15\right)-2abcd\left(m^2-6m+5\right)+b^2c^2\left(m^2-4m+3\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{8a^3(m+1)(bc-ad)^3}+\frac{bx^{m+1}(bc(3-m}{8a^2\left(a+bx^2\right.}$$

```
[Out] (b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(b*c*(3 - m) - a*d*(7 -
m))*x^(1 + m))/(8*a^2*(b*c - a*d)^2*(a + b*x^2)) + (b*(a^2*d^2*(15 - 8*m +
m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3 - 4*m + m^2))*x^(1 + m)*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(8*a^3*(b*c - a*d)^3*(
1 + m)) - (d^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])/(c*(b*c - a*d)^3*(1 + m))
```

Rubi [A] time = 0.373813, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 579, 584, 364}

$$\frac{bx^{m+1}\left(a^2d^2\left(m^2-8m+15\right)-2abcd\left(m^2-6m+5\right)+b^2c^2\left(m^2-4m+3\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{8a^3(m+1)(bc-ad)^3}+\frac{bx^{m+1}(bc(3-m}{8a^2\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```


```
[Out] (b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(b*c*(3 - m) - a*d*(7 -
m))*x^(1 + m))/(8*a^2*(b*c - a*d)^2*(a + b*x^2)) + (b*(a^2*d^2*(15 - 8*m +
m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3 - 4*m + m^2))*x^(1 + m)*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(8*a^3*(b*c - a*d)^3*(
1 + m)) - (d^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])/(c*(b*c - a*d)^3*(1 + m))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} - \frac{\int \frac{x^m(4ad-bc(3-m)-bd(3-m)x^2)}{(a+bx^2)^2(c+dx^2)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\int \frac{x^m(8a^2d^2-abcd(7-8m+m^2)+b^2c^2(3-4m\ldots}{(a+bx^2)(\ldots}}{8a^2(bc\ldots}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\int \left(\frac{b(a^2d^2(15-8m+m^2)-2abcd(5-6m+m^2\ldots}{(bc-ad)(a+bx^2)\ldots}\right.}{8a^2(\ldots}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} - \frac{d^3\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{\left(b\left(a^2d^2\left(15-8\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{b\left(a^2d^2\left(15-8m+m^2\right)-2abcd\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0509, size = 54, normalized size = 0.23

$$\frac{x^{m+1}F_1\left(\frac{m+1}{2};3,1;\frac{m+1}{2}+1;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{a^3c(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*AppellF1[(1 + m)/2, 3, 1, 1 + (1 + m)/2, -((b*x^2)/a), -((d*x^2)
/c)])/(a^3*c*(1 + m))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^3dx^8+(b^3c+3\,ab^2d)x^6+3\,(ab^2c+a^2bd)x^4+a^3c+(3\,a^2bc+a^3d)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*x
^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

3.337 $\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=201

$$-\frac{dx^{m+1}\left(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1)\right)}{2ab^3(m+1)}+\frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)}$$

```
[Out] -(d*(2*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(1 + m))/(2
*a*b^3*(1 + m)) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3 + m))/(2*a*b^2*(3 +
 m)) + ((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/(2*a*b*(a + b*x^2)) + ((b*c -
a*d)^2*(a*d*(5 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^3*(1 + m))
```

Rubi [A] time = 0.224249, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {468, 570, 364}

$$-\frac{dx^{m+1}\left(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1)\right)}{2ab^3(m+1)}+\frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] -(d*(2*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(1 + m))/(2
*a*b^3*(1 + m)) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3 + m))/(2*a*b^2*(3 +
 m)) + ((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/(2*a*b*(a + b*x^2)) + ((b*c -
a*d)^2*(a*d*(5 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^3*(1 + m))
```


Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^m\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x^{1+m}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x^m\left(c+dx^2\right)\left(-c(bc(1-m)+ad(1+m))+d(bc(3+m)-ad(5+m))x^2\right)}{a+bx^2}\,dx}{2ab}\\
&= \frac{(bc-ad)x^{1+m}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)}-\frac{\int\left(\frac{d\left(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m)\right)x^m}{b^2}+\frac{d^2(bc(3+m)-ad(5+m))x^{2+m}}{b}+\frac{(-b^{\cdots}}{}\right.}{2ab}\\
&= -\frac{d\left(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m)\right)x^{1+m}}{2ab^3(1+m)}-\frac{d^2(bc(3+m)-ad(5+m))x^{3+m}}{2ab^2(3+m)}+\frac{(bc\cdots}{}\\
&= -\frac{d\left(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m)\right)x^{1+m}}{2ab^3(1+m)}-\frac{d^2(bc(3+m)-ad(5+m))x^{3+m}}{2ab^2(3+m)}+\frac{(bc\cdots}{}
\end{aligned}
$$

Mathematica [C] time = 4.78303, size = 2524, normalized size = 12.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -(x^(1 + m)*(a*(945 + 744*m + 206*m^2 + 24*m^3 + m^4)*(c^3*(-47 + 52*m + 6*
m^2 + 4*m^3 + m^4) + 3*c^2*d*(1 + m)^4*x^2 + 3*c*d^2*(1 + m)^4*x^4 + d^3*(1
 + m)^4*x^6)*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] - 3*a*(945 + 744*m
 + 206*m^2 + 24*m^3 + m^4)*(c^3*(3 + m)^4 + 3*c^2*d*(65 + 92*m + 54*m^2 + 1
2*m^3 + m^4)*x^2 + 3*c*d^2*(3 + m)^4*x^4 + d^3*(3 + m)^4*x^6)*HurwitzLerchP
hi[-((b*x^2)/a), 1, (3 + m)/2] + 1771875*a*c^3*HurwitzLerchPhi[-((b*x^2)/a)
, 1, (5 + m)/2] + 2812500*a*c^3*m*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/
2] + 1927500*a*c^3*m^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 745500
*a*c^3*m^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 178050*a*c^3*m^4*H
urwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26892*a*c^3*m^5*HurwitzLerchPh
i[-((b*x^2)/a), 1, (5 + m)/2] + 2508*a*c^3*m^6*HurwitzLerchPhi[-((b*x^2)/a)
, 1, (5 + m)/2] + 132*a*c^3*m^7*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2]
 + 3*a*c^3*m^8*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5315625*a*c^2*
d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8437500*a*c^2*d*m*x^2*H
urwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5782500*a*c^2*d*m^2*x^2*Hurwit
zLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2236500*a*c^2*d*m^3*x^2*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (5 + m)/2] + 534150*a*c^2*d*m^4*x^2*HurwitzLerchPhi[-
((b*x^2)/a), 1, (5 + m)/2] + 80676*a*c^2*d*m^5*x^2*HurwitzLerchPhi[-((b*x^2
)/a), 1, (5 + m)/2] + 7524*a*c^2*d*m^6*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1,
 (5 + m)/2] + 396*a*c^2*d*m^7*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/
2] + 9*a*c^2*d*m^8*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 572386
5*a*c*d^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8894988*a*c*d^2
*m*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5978628*a*c*d^2*m^2*x^
4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2276532*a*c*d^2*m^3*x^4*Hur
witzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 538038*a*c*d^2*m^4*x^4*HurwitzLe
rchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 80820*a*c*d^2*m^5*x^4*HurwitzLerchPhi[
-((b*x^2)/a), 1, (5 + m)/2] + 7524*a*c*d^2*m^6*x^4*HurwitzLerchPhi[-((b*x^2
)/a), 1, (5 + m)/2] + 396*a*c*d^2*m^7*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1,
(5 + m)/2] + 9*a*c*d^2*m^8*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2]
+ 1771875*a*d^3*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2812500*a
```

```
*d^3*m*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 1927500*a*d^3*m^2*
x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 745500*a*d^3*m^3*x^6*Hurw
itzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 178050*a*d^3*m^4*x^6*HurwitzLerch
Phi[-((b*x^2)/a), 1, (5 + m)/2] + 26892*a*d^3*m^5*x^6*HurwitzLerchPhi[-((b*
x^2)/a), 1, (5 + m)/2] + 2508*a*d^3*m^6*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1
, (5 + m)/2] + 132*a*d^3*m^7*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2
] + 3*a*d^3*m^8*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] - 2268945*a
*c^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3082884*a*c^3*m*HurwitzL
erchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1793204*a*c^3*m^2*HurwitzLerchPhi[-((
b*x^2)/a), 1, (7 + m)/2] - 585452*a*c^3*m^3*HurwitzLerchPhi[-((b*x^2)/a), 1
, (7 + m)/2] - 117670*a*c^3*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2]
 - 14940*a*c^3*m^5*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1172*a*c^3
*m^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 52*a*c^3*m^7*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (7 + m)/2] - a*c^3*m^8*HurwitzLerchPhi[-((b*x^2)/a),
1, (7 + m)/2] - 6806835*a*c^2*d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m
)/2] - 9248652*a*c^2*d*m*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] -
5379612*a*c^2*d*m^2*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 17563
56*a*c^2*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 353010*a*c
^2*d*m^4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 44820*a*c^2*d*m^
5*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3516*a*c^2*d*m^6*x^2*Hu
rwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 156*a*c^2*d*m^7*x^2*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3*a*c^2*d*m^8*x^2*HurwitzLerchPhi[-((b*x
^2)/a), 1, (7 + m)/2] - 6806835*a*c*d^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1
, (7 + m)/2] - 9248652*a*c*d^2*m*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 +
m)/2] - 5379612*a*c*d^2*m^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2]
 - 1756356*a*c*d^2*m^3*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 35
3010*a*c*d^2*m^4*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 44820*a*
c*d^2*m^5*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3516*a*c*d^2*m^
6*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 156*a*c*d^2*m^7*x^4*Hur
witzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3*a*c*d^2*m^8*x^4*HurwitzLerchPh
i[-((b*x^2)/a), 1, (7 + m)/2] - 2042145*a*d^3*x^6*HurwitzLerchPhi[-((b*x^2)
/a), 1, (7 + m)/2] - 2858964*a*d^3*m*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (
7 + m)/2] - 1708052*a*d^3*m^2*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/
2] - 569804*a*d^3*m^3*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 116
278*a*d^3*m^4*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 14892*a*d^3
*m^5*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1172*a*d^3*m^6*x^6*H
urwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 52*a*d^3*m^7*x^6*HurwitzLerchP
hi[-((b*x^2)/a), 1, (7 + m)/2] - a*d^3*m^8*x^6*HurwitzLerchPhi[-((b*x^2)/a)
, 1, (7 + m)/2] + 1536*b*c^3*x^2*HypergeometricPFQ[{2, 2, 2, 2, 3/2 + m/2},
 {1, 1, 1, 11/2 + m/2}, -((b*x^2)/a)] + 4608*b*c^2*d*x^4*HypergeometricPFQ[
{2, 2, 2, 2, 3/2 + m/2}, {1, 1, 1, 11/2 + m/2}, -((b*x^2)/a)] + 4608*b*c*d^
2*x^6*HypergeometricPFQ[{2, 2, 2, 2, 3/2 + m/2}, {1, 1, 1, 11/2 + m/2}, -((
b*x^2)/a)] + 1536*b*d^3*x^8*HypergeometricPFQ[{2, 2, 2, 2, 3/2 + m/2}, {1,
1, 1, 11/2 + m/2}, -((b*x^2)/a)]))/(192*a^3*(3 + m)*(5 + m)*(7 + m)*(9 + m)
)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3 x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^3 x^m}{(bx^2 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3)x^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b^2*x^4 + 2*a*b*x
^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (c + dx^2)^3}{(a + bx^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**m*(c + d*x**2)**3/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^3 x^m}{(bx^2 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

3.338 $\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=120

$$\frac{x^{m+1}(bc-ad)(ad(m+3)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x^{m+1}}{b^2(m+1)}$$

```
[Out] (d^2*x^(1 + m))/(b^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*a*b^2*(a + b*x
^2)) + ((b*c - a*d)*(a*d*(3 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^2*(1 + m))
```

Rubi [A] time = 0.105725, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {463, 459, 364}

$$\frac{x^{m+1}(bc-ad)(ad(m+3)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x^{m+1}}{b^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```


```
[Out] (d^2*x^(1 + m))/(b^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*a*b^2*(a + b*x
^2)) + ((b*c - a*d)*(a*d*(3 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^2*(1 + m))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2} dx = \frac{(bc-ad)^2 x^{1+m}}{2ab^2 (a+bx^2)} - \frac{\int \frac{x^m(-2b^2c^2+(bc-ad)^2(1+m)-2abd^2x^2)}{a+bx^2} dx}{2ab^2}$$

$$= \frac{d^2x^{1+m}}{b^2(1+m)} + \frac{(bc-ad)^2 x^{1+m}}{2ab^2 (a+bx^2)} - -\frac{(-2a^2bd^2(1+m) - b(1+m)(-2b^2c^2 + (bc-ad)^2(1+m))) \int \frac{}{a}}{2ab^3(1+m)}$$

$$= \frac{d^2x^{1+m}}{b^2(1+m)} + \frac{(bc-ad)^2 x^{1+m}}{2ab^2 (a+bx^2)} + \frac{(bc-ad)(bc(1-m)+ad(3+m))x^{1+m} {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\frac{bx^2}{a}\right)}{2a^2b^2(1+m)}$$

Mathematica [C] time = 1.98636, size = 895, normalized size = 7.46

$$x^{m+1}\left(ad^2x^4\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)m^6 + ac^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)m^6 + 2acdx^2\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)m^6 + 30ad^2x^4\Phi\left(-\frac{bx^2}{a},1,\frac{m+5}{2}\right)m^5\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(9 - 5*m + 3*m^2 + m^3) + 2*
c*d*(1 + m)^3*x^2 + d^2*(1 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a), 1, (1
+ m)/2] - 2*a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(3 + m)^3 + 2*c*d*(31 + 31*m
 + 9*m^2 + m^3)*x^2 + d^2*(3 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a), 1, (
3 + m)/2] + 13125*a*c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 16750
*a*c^2*m*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8775*a*c^2*m^2*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2420*a*c^2*m^3*HurwitzLerchPhi[-((
b*x^2)/a), 1, (5 + m)/2] + 371*a*c^2*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (
5 + m)/2] + 30*a*c^2*m^5*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + a*c^
2*m^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26250*a*c*d*x^2*Hurwitz
LerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 33500*a*c*d*m*x^2*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + 17550*a*c*d*m^2*x^2*HurwitzLerchPhi[-((b*x^2)/a
), 1, (5 + m)/2] + 4840*a*c*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 +
 m)/2] + 742*a*c*d*m^4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 60
*a*c*d*m^5*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2*a*c*d*m^6*x^
2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 10605*a*d^2*x^4*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (5 + m)/2] + 14206*a*d^2*m*x^4*HurwitzLerchPhi[-((b*x
^2)/a), 1, (5 + m)/2] + 7847*a*d^2*m^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1,
 (5 + m)/2] + 2276*a*d^2*m^3*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2
] + 363*a*d^2*m^4*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 30*a*d^
2*m^5*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + a*d^2*m^6*x^4*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] - 128*b*c^2*x^2*HypergeometricPFQ[{2
, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2)/a)] - 256*b*c*d*x^4*Hyperg
eometricPFQ[{2, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2)/a)] - 128*b*
d^2*x^6*HypergeometricPFQ[{2, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2
)/a)]))/(32*a^3*(3 + m)*(5 + m)*(7 + m))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^2 x^m}{(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2 x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)x^m}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**m*(c + d*x**2)**2/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2 x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

3.339 $\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rubi [A] time = 0.042393, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {457, 364}

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x^{1+m}}{2ab\left(a+bx^2\right)}+\frac{(ad(1+m)+b(c-cm))\int \frac{x^m}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x^{1+m}}{2ab\left(a+bx^2\right)}+\frac{(ad(1+m)+b(c-cm))x^{1+m}\,_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0643271, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((bc-ad)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ad\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2
*b*(1 + m))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^2+c\right)x^m}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] time = 37.3548, size = 906, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2
)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + d*(-a
*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**3*x**
m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2
)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*
x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m
/2 + 5/2)) - b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5
/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gam
ma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) -
3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 +
3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

3.340 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=156

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rubi [A] time = 0.160639, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {472, 584, 364}

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{x^m\left(2ad-bc(1-m)-bd(1-m)x^2\right)}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \left(\frac{b(-bc(1-m)+ad(3-m))x^m}{(bc-ad)\left(a+bx^2\right)} + \frac{2ad^2x^m}{(-bc+ad)\left(c+dx^2\right)}\right)dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{d^2\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc(1-m)-ad(3-m)))\int \frac{x^m}{a+bx^2}\,dx}{2a(bc-ad)^2}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{b(ad(3-m)-b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)} + \frac{d^2x^{1+m}}{c\ldots}$$

Mathematica [A] time = 0.0721027, size = 127, normalized size = 0.81

$$\frac{x^{m+1}\left(a^2d^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-abcd\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bc(bc-ad)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/
a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] + b
*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(
a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2dx^6 + \left(b^2c + 2\,abd\right)x^4 + a^2c + \left(2\,abc + a^2d\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)
*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2 + a\right)^2\left(dx^2 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.341 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{b^2x^{m+1}(ad(5-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^3}-\frac{d^2x^{m+1}(ad(1-m)-bc(5-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2(m+1)(bc-ad)^3}+\frac{}{2}$$

```
[Out] (d*(b*c + a*d)*x^(1 + m))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(1 + m))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^2*(a*d*(5 - m) - b*(c - c*m
))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a
^2*(b*c - a*d)^3*(1 + m)) - (d^2*(a*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(b*c - a*d)^3*
(1 + m))
```

Rubi [A] time = 0.388053, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 579, 584, 364}

$$-\frac{b^2x^{m+1}(ad(5-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^3}-\frac{d^2x^{m+1}(ad(1-m)-bc(5-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2(m+1)(bc-ad)^3}+\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c + a*d)*x^(1 + m))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(1 + m))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^2*(a*d*(5 - m) - b*(c - c*m
))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a
^2*(b*c - a*d)^3*(1 + m)) - (d^2*(a*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(b*c - a*d)^3*
(1 + m))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{x^m\left(2ad-bc(1-m)-bd(3-m)x^2\right)}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{x^m\left(2\left(4abcd-b^2c^2(1-m)-a^2d^2(1-m)\right)\right.}{\left(a+bx^2\right)\left(c+dx^2\right.}}{4ac(bc-ad}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \left(\frac{2b^2c(-bc(1-m)+ad(5-m))x^m}{(bc-ad)\left(a+bx^2\right)} + \frac{2a}{}\right.}{4ac(bc-}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(d^2(ad(1-m)-bc(5-m))\right)\int}{2c(bc-ad)^3}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^2(bc(1-m)-ad(5-m))x^{1+}}{2a^2(bc-ad)^3}$$

Mathematica [C] time = 0.0848654, size = 54, normalized size = 0.23

$$\frac{x^{m+1}F_1\left(\frac{m+1}{2};2,2;\frac{m+1}{2}+1;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{a^2c^2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[(1 + m)/2, 2, 2, 1 + (1 + m)/2, -((b*x^2)/a), -((d*x^2)
/c)])/(a^2*c^2*(1 + m))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2d^2x^8+2\left(b^2cd+abd^2\right)x^6+\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(abc^2+a^2cd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.342 $\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=325

$$\frac{d^2x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)-2abcd\left(m^2-8m+7\right)+b^2c^2\left(m^2-12m+35\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3(m+1)(bc-ad)^4}+\frac{dx^{m+1}\left(-a^2d^2(3\right.}{8ac^2\left(\right.}$$

```
[Out] (d*(2*b*c + a*d)*x^(1 + m))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(1 +
 m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 - a^2*d^2*
(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2))
- (b^3*(a*d*(7 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*(b*c - a*d)^4*(1 + m)) + (d^2*(b^2*c^2*(
35 - 12*m + m^2) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^2))*x^(
1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^3*(b*
c - a*d)^4*(1 + m))
```

Rubi [A] time = 0.613702, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 579, 584, 364}

$$\frac{d^2x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)-2abcd\left(m^2-8m+7\right)+b^2c^2\left(m^2-12m+35\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3(m+1)(bc-ad)^4}+\frac{dx^{m+1}\left(-a^2d^2(3\right.}{8ac^2\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (d*(2*b*c + a*d)*x^(1 + m))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(1 +
 m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 - a^2*d^2*
(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2))
- (b^3*(a*d*(7 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*(b*c - a*d)^4*(1 + m)) + (d^2*(b^2*c^2*(
35 - 12*m + m^2) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^2))*x^(
1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^3*(b*
c - a*d)^4*(1 + m))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
```

```
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{x^m\left(2ad-bc(1-m)-bd(5-m)x^2\right)}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{x^m\left(2\left(8abcd-2b^2c^2(1-m)-a^2d\right.\right.}{\left(a+bx^2\right.}}{8ac(b}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2-a^2d^2(3-m)+\right.}{8ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2-a^2d^2(3-m)+\right.}{8ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2-a^2d^2(3-m)+\right.}{8ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2-a^2d^2(3-m)+\right.}{8ac^2(bc-ad)^3}$$

Mathematica [C] time = 0.159945, size = 54, normalized size = 0.17

$$\frac{x^{m+1}F_1\left(\frac{m+1}{2};2,3;\frac{m+1}{2}+1;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{a^2c^3(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (x^(1 + m)*AppellF1[(1 + m)/2, 2, 3, 1 + (1 + m)/2, -((b*x^2)/a), -((d*x^2)
/c)])/(a^2*c^3*(1 + m))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2d^3x^{10}+\left(3\,b^2cd^2+2\,abd^3\right)x^8+\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+a^2c^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+\left(2\,abc^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^3*x^10 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^
2)*x^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

3.343 $\int x^{7/2} \left(a + bx^2\right) \left(A + Bx^2\right) dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB + Ab) + \frac{2}{9}aAx^{9/2} + \frac{2}{17}bBx^{17/2}$$

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2))/17
```

Rubi [A] time = 0.0159898, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB + Ab) + \frac{2}{9}aAx^{9/2} + \frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a + bx^2\right) \left(A + Bx^2\right) dx &= \int \left(aAx^{7/2} + (Ab + aB)x^{11/2} + bBx^{15/2}\right) dx\\ &= \frac{2}{9}aAx^{9/2} + \frac{2}{13}(Ab + aB)x^{13/2} + \frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0155964, size = 33, normalized size = 0.85

$$\frac{2x^{9/2}\left(153x^2(aB + Ab) + 221aA + 117bBx^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(221*a*A + 153*(A*b + a*B)*x^2 + 117*b*B*x^4))/1989
```

Maple [A] time = 0.002, size = 32, normalized size = 0.8

$$\frac{234\,bBx^4 + 306\,Ax^2b + 306\,Bx^2a + 442\,Aa}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1989*x^(9/2)*(117*B*b*x^4+153*A*b*x^2+153*B*a*x^2+221*A*a)
```

Maxima [A] time = 1.02808, size = 36, normalized size = 0.92

$$\frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{13}\,(Ba + Ab)x^{\frac{13}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/9*A*a*x^(9/2)
```

Fricas [A] time = 0.845224, size = 89, normalized size = 2.28

$$\frac{2}{1989}\left(117\,Bbx^{8} + 153\,(Ba + Ab)x^{6} + 221\,Aax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*b*x^8 + 153*(B*a + A*b)*x^6 + 221*A*a*x^4)*sqrt(x)
```

Sympy [A] time = 11.125, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Bax^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(17/2
)/17
```

Giac [A] time = 1.138, size = 39, normalized size = 1.

$$\frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{13}\,Bax^{\frac{13}{2}} + \frac{2}{13}\,Abx^{\frac{13}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/9*A*a*x^(9/2)
```

3.344 $\int x^{5/2} \left(a+bx^2\right) \left(A+Bx^2\right) dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2))/15
```

Rubi [A] time = 0.0159631, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)\left(A+Bx^2\right)dx&=\int\left(aAx^{5/2}+(Ab+aB)x^{9/2}+bBx^{13/2}\right)dx\\&=\frac{2}{7}aAx^{7/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0145277, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(105x^2(aB+Ab)+165aA+77bBx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(7/2)*(165*a*A + 105*(A*b + a*B)*x^2 + 77*b*B*x^4))/1155
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{154\,bBx^4+210\,Ax^2b+210\,Bx^2a+330\,Aa}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1155*x^(7/2)*(77*B*b*x^4+105*A*b*x^2+105*B*a*x^2+165*A*a)
```

Maxima [A] time = 1.04101, size = 36, normalized size = 0.92

$$\frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{11}\,(Ba + Ab)x^{\frac{11}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/7*A*a*x^(7/2)
```

Fricas [A] time = 0.852479, size = 88, normalized size = 2.26

$$\frac{2}{1155}\left(77\,Bbx^7 + 105\,(Ba + Ab)x^5 + 165\,Aax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b*x^7 + 105*(B*a + A*b)*x^5 + 165*A*a*x^3)*sqrt(x)
```

Sympy [A] time = 5.66925, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(15/2
)/15
```

Giac [A] time = 1.13482, size = 39, normalized size = 1.

$$\frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{11}\,Bax^{\frac{11}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/7*A*a*x^(7/2)
```

3.345 $\int x^{3/2} \left(a + bx^2\right) \left(A + Bx^2\right) dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB + Ab) + \frac{2}{5}aAx^{5/2} + \frac{2}{13}bBx^{13/2}$$

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/13
```

Rubi [A] time = 0.015995, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB + Ab) + \frac{2}{5}aAx^{5/2} + \frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(a + bx^2\right) \left(A + Bx^2\right) dx &= \int \left(aAx^{3/2} + (Ab + aB)x^{7/2} + bBx^{11/2}\right) dx\\ &= \frac{2}{5}aAx^{5/2} + \frac{2}{9}(Ab + aB)x^{9/2} + \frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0133352, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2} \left(65x^2(aB + Ab) + 117aA + 45bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(117*a*A + 65*(A*b + a*B)*x^2 + 45*b*B*x^4))/585
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{90\,bBx^4 + 130\,Ax^2b + 130\,Bx^2a + 234\,Aa}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/585*x^(5/2)*(45*B*b*x^4+65*A*b*x^2+65*B*a*x^2+117*A*a)
```

Maxima [A] time = 1.05933, size = 36, normalized size = 0.92

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{9}\,(Ba + Ab)x^{\frac{9}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/5*A*a*x^(5/2)
```

Fricas [A] time = 0.881758, size = 85, normalized size = 2.18

$$\frac{2}{585}\left(45\,Bbx^{6} + 65\,(Ba + Ab)x^{4} + 117\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b*x^6 + 65*(B*a + A*b)*x^4 + 117*A*a*x^2)*sqrt(x)
```

Sympy [A] time = 2.56887, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(13/2)/13
```

Giac [A] time = 1.21152, size = 39, normalized size = 1.

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{9}\,Bax^{\frac{9}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/5*A*a*x^(5/2)
```

3.346 $\int \sqrt{x}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/11
```

Rubi [A] time = 0.0153497, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]

[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{5/2}+bBx^{9/2}\right)dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0146051, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}\left(33x^2(aB+Ab)+77aA+21bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]

[Out] (2*x^(3/2)*(77*a*A + 33*(A*b + a*B)*x^2 + 21*b*B*x^4))/231
```

Maple [A] time = 0.002, size = 32, normalized size = 0.8

$$\frac{42\,bBx^4+66\,Ax^2b+66\,Bx^2a+154\,Aa}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*B*b*x^4+33*A*b*x^2+33*B*a*x^2+77*A*a)
```

Maxima [A] time = 1.02638, size = 36, normalized size = 0.92

$$\frac{2}{11} Bbx^{\frac{11}{2}} + \frac{2}{7} (Ba + Ab)x^{\frac{7}{2}} + \frac{2}{3} Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*(B*a + A*b)*x^(7/2) + 2/3*A*a*x^(3/2)
```

Fricas [A] time = 0.889685, size = 81, normalized size = 2.08

$$\frac{2}{231} \left(21 \, Bbx^{5} + 33 \, (Ba + Ab)x^{3} + 77 \, Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b*x^5 + 33*(B*a + A*b)*x^3 + 77*A*a*x)*sqrt(x)
```

Sympy [A] time = 1.67964, size = 37, normalized size = 0.95

$$\frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{11}{2}}}{11} + \frac{2x^{\frac{7}{2}} \, (Ab + Ba)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + 2*x**(7/2)*(A*b + B*a)/7
```

Giac [A] time = 1.1176, size = 39, normalized size = 1.

$$\frac{2}{11} Bbx^{\frac{11}{2}} + \frac{2}{7} Bax^{\frac{7}{2}} + \frac{2}{7} Abx^{\frac{7}{2}} + \frac{2}{3} Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/3*A*a*x^(3/2)
```

3.347 $\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}} dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rubi [A] time = 0.0153744, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{3/2}+bBx^{7/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{9}bBx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0139022, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x}\left(9x^2(aB+Ab)+45aA+5bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a*A + 9*(A*b + a*B)*x^2 + 5*b*B*x^4))/45
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$\frac{10\,bBx^4+18\,Ax^2b+18\,Bx^2a+90\,Aa}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*B*b*x^4+9*A*b*x^2+9*B*a*x^2+45*A*a)
```

Maxima [A] time = 1.00321, size = 36, normalized size = 0.97

$$\frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,(Ba + Ab)x^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*(B*a + A*b)*x^(5/2) + 2*A*a*sqrt(x)
```

Fricas [A] time = 0.828137, size = 74, normalized size = 2.

$$\frac{2}{45}\left(5\,Bbx^4 + 9\,(Ba + Ab)x^2 + 45\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b*x^4 + 9*(B*a + A*b)*x^2 + 45*A*a)*sqrt(x)
```

Sympy [A] time = 0.76436, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(9/2)/9
```

Giac [A] time = 1.12298, size = 39, normalized size = 1.05

$$\frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2*A*a*sqrt(x)
```

3.348 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rubi [A] time = 0.015213, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)\sqrt{x}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0104049, size = 35, normalized size = 0.95

$$\frac{2\left(-21aA+7aBx^2+7Abx^2+3bBx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-21*a*A + 7*A*b*x^2 + 7*a*B*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$-\frac{-6\,bBx^4-14\,Ax^2b-14\,Bx^2a+42\,Aa}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/21*(-3*B*b*x^4-7*A*b*x^2-7*B*a*x^2+21*A*a)/x^(1/2)
```

Maxima [A] time = 1.00286, size = 36, normalized size = 0.97

$$\frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{3}\,(Ba + Ab)x^{\frac{3}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*(B*a + A*b)*x^(3/2) - 2*A*a/sqrt(x)
```

Fricas [A] time = 0.852547, size = 74, normalized size = 2.

$$\frac{2\left(3\,Bbx^4 + 7\,(Ba + Ab)x^2 - 21\,Aa\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*B*b*x^4 + 7*(B*a + A*b)*x^2 - 21*A*a)/sqrt(x)
```

Sympy [A] time = 0.983058, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(7/2)/7
```

Giac [A] time = 1.1786, size = 39, normalized size = 1.05

$$\frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{3}\,Bax^{\frac{3}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2*A*a/sqrt(x)
```

3.349 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rubi [A] time = 0.016267, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{3/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{5}bBx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.012476, size = 36, normalized size = 0.97

$$\frac{2\left(3bx^2\left(5A+Bx^2\right)-5a\left(A-3Bx^2\right)\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-5*a*(A - 3*B*x^2) + 3*b*x^2*(5*A + B*x^2)))/(15*x^(3/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$-\frac{-6\,bBx^4-30\,Ax^2b-30\,Bx^2a+10\,Aa}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/15*(-3*B*b*x^4-15*A*b*x^2-15*B*a*x^2+5*A*a)/x^(3/2)
```

Maxima [A] time = 1.03156, size = 36, normalized size = 0.97

$$\frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,(Ba + Ab)\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*(B*a + A*b)*sqrt(x) - 2/3*A*a/x^(3/2)
```

Fricas [A] time = 0.663719, size = 74, normalized size = 2.

$$\frac{2\left(3\,Bbx^4 + 15\,(Ba + Ab)x^2 - 5\,Aa\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*b*x^4 + 15*(B*a + A*b)*x^2 - 5*A*a)/x^(3/2)
```

Sympy [A] time = 1.25066, size = 42, normalized size = 1.14

$$-\frac{2Aa}{3x^{\frac{3}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(5/2)/5
```

Giac [A] time = 1.16129, size = 39, normalized size = 1.05

$$\frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/3*A*a/x^(3/2)
```

3.350 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/3
```

Rubi [A] time = 0.0156172, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{x^{3/2}}+bB\sqrt{x}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}-\frac{2(Ab+aB)}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0126628, size = 36, normalized size = 0.97

$$\frac{10bx^2(Bx^2-3A)-6a(A+5Bx^2)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (10*b*x^2*(-3*A + B*x^2) - 6*a*(A + 5*B*x^2))/(15*x^(5/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$-\frac{-10\,bBx^4+30\,Ax^2b+30\,Bx^2a+6\,Aa}{15}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/15*(-5*B*b*x^4+15*A*b*x^2+15*B*a*x^2+3*A*a)/x^(5/2)
```

Maxima [A] time = 1.03911, size = 39, normalized size = 1.05

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\left(5\,(Ba+Ab)x^2+Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*(B*a + A*b)*x^2 + A*a)/x^(5/2)
```

Fricas [A] time = 0.858949, size = 74, normalized size = 2.

$$\frac{2\left(5\,Bbx^4-15\,(Ba+Ab)x^2-3\,Aa\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*B*b*x^4 - 15*(B*a + A*b)*x^2 - 3*A*a)/x^(5/2)
```

Sympy [A] time = 1.86614, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{2Ab}{\sqrt{x}} - \frac{2Ba}{\sqrt{x}} + \frac{2Bbx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*x**(3/2)/3
```

Giac [A] time = 1.13884, size = 42, normalized size = 1.14

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\left(5\,Bax^2+5\,Abx^2+Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*B*a*x^2 + 5*A*b*x^2 + A*a)/x^(5/2)
```

3.351 $\int x^{7/2} \left(a + bx^2\right)^2 \left(A + Bx^2\right) dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{13}ax^{13/2}(aB + 2Ab) + \frac{2}{21}b^2Bx^{21/2}$$

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rubi [A] time = 0.0299327, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{13}ax^{13/2}(aB + 2Ab) + \frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a + bx^2\right)^2 \left(A + Bx^2\right) dx &= \int \left(a^2Ax^{7/2} + a(2Ab + aB)x^{11/2} + b(Ab + 2aB)x^{15/2} + b^2Bx^{19/2}\right) dx\\ &= \frac{2}{9}a^2Ax^{9/2} + \frac{2}{13}a(2Ab + aB)x^{13/2} + \frac{2}{17}b(Ab + 2aB)x^{17/2} + \frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0310451, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2A + 819bx^4(2aB + Ab) + 1071ax^2(aB + 2Ab) + 663b^2Bx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*A + 1071*a*(2*A*b + a*B)*x^2 + 819*b*(A*b + 2*a*B)*x^4
 + 663*b^2*B*x^6))/13923
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{1326\,Bb^2x^6+1638\,Ab^2x^4+3276\,Bx^4ab+4284\,aAbx^2+2142\,Bx^2a^2+3094\,a^2A}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/13923*x^(9/2)*(663*B*b^2*x^6+819*A*b^2*x^4+1638*B*a*b*x^4+2142*A*a*b*x^2+
1071*B*a^2*x^2+1547*A*a^2)
```

Maxima [A] time = 1.05195, size = 69, normalized size = 1.1

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{17}\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/9*A*a^2*x^(9/2) +
 2/13*(B*a^2 + 2*A*a*b)*x^(13/2)
```

Fricas [A] time = 0.797779, size = 146, normalized size = 2.32

$$\frac{2}{13923}\left(663\,Bb^2x^{10}+819\left(2\,Bab+Ab^2\right)x^8+1547\,Aa^2x^4+1071\left(Ba^2+2\,Aab\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*b^2*x^10 + 819*(2*B*a*b + A*b^2)*x^8 + 1547*A*a^2*x^4 + 1071
*(B*a^2 + 2*A*a*b)*x^6)*sqrt(x)
```

Sympy [A] time = 20.9895, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(21/2)/21
```

Giac [A] time = 1.12682, size = 72, normalized size = 1.14

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/9*A*a^2*x^(9/2)
```

3.352 $\int x^{5/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rubi [A] time = 0.0299246, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)dx &= \int \left(a^2Ax^{5/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{17/2}\right)dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0288283, size = 63, normalized size = 1.

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{2310\,Bb^2x^6+2926\,Ab^2x^4+5852\,Bx^4ab+7980\,aAbx^2+3990\,Bx^2a^2+6270\,a^2A}{21945}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*B*b^2*x^6+1463*A*b^2*x^4+2926*B*a*b*x^4+3990*A*a*b*x^
2+1995*B*a^2*x^2+3135*A*a^2)
```

Maxima [A] time = 1.05328, size = 69, normalized size = 1.1

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{2}{15}\left(2\,Bab + Ab^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}} + \frac{2}{11}\left(Ba^2 + 2\,Aab\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/7*A*a^2*x^(7/2) +
 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Fricas [A] time = 0.815691, size = 147, normalized size = 2.33

$$\frac{2}{21945}\left(1155\,Bb^2x^9 + 1463\left(2\,Bab + Ab^2\right)x^7 + 3135\,Aa^2x^3 + 1995\left(Ba^2 + 2\,Aab\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^2*x^9 + 1463*(2*B*a*b + A*b^2)*x^7 + 3135*A*a^2*x^3 + 199
5*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A] time = 11.4721, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7} + \frac{4Aabx^{\frac{11}{2}}}{11} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(15/2)/15 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(19/2)/19
```

Giac [A] time = 1.15334, size = 72, normalized size = 1.14

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{15}\,Babx^{\frac{15}{2}} + \frac{2}{15}\,Ab^2x^{\frac{15}{2}} + \frac{2}{11}\,Ba^2x^{\frac{11}{2}} + \frac{4}{11}\,Aabx^{\frac{11}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/7*A*a^2*x^(7/2)
```

3.353 $\int x^{3/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rubi [A] time = 0.0301834, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)dx &= \int \left(a^2Ax^{3/2}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{11/2}+b^2Bx^{15/2}\right)dx\\ &= \frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0280528, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2A+765bx^4(2aB+Ab)+1105ax^2(aB+2Ab)+585b^2Bx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*A + 1105*a*(2*A*b + a*B)*x^2 + 765*b*(A*b + 2*a*B)*x^4
 + 585*b^2*B*x^6))/9945
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{1170\,Bb^2x^6+1530\,Ab^2x^4+3060\,Bx^4ab+4420\,aAbx^2+2210\,Bx^2a^2+3978\,a^2A}{9945}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/9945*x^(5/2)*(585*B*b^2*x^6+765*A*b^2*x^4+1530*B*a*b*x^4+2210*A*a*b*x^2+1
105*B*a^2*x^2+1989*A*a^2)
```

Maxima [A] time = 1.03582, size = 69, normalized size = 1.1

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/5*A*a^2*x^(5/2) +
 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Fricas [A] time = 0.740549, size = 143, normalized size = 2.27

$$\frac{2}{9945}\left(585\,Bb^2x^8+765\left(2\,Bab+Ab^2\right)x^6+1989\,Aa^2x^2+1105\left(Ba^2+2\,Aab\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*b^2*x^8 + 765*(2*B*a*b + A*b^2)*x^6 + 1989*A*a^2*x^2 + 1105*(
B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A] time = 5.92429, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(17/2)/17
```

Giac [A] time = 1.14374, size = 72, normalized size = 1.14

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{9}\,Ba^2x^{\frac{9}{2}}+\frac{4}{9}\,Aabx^{\frac{9}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/5*A*a^2*x^(5/2)
```

3.354 $\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rubi [A] time = 0.0293406, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx &= \int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{5/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{13/2}\right)dx\\ &= \frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0287531, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2A+105bx^4(2aB+Ab)+165ax^2(aB+2Ab)+77b^2Bx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*A + 165*a*(2*A*b + a*B)*x^2 + 105*b*(A*b + 2*a*B)*x^4 +
 77*b^2*B*x^6))/1155
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$\frac{154 Bb^{2}x^{6}+210 Ab^{2}x^{4}+420 Bx^{4}ab+660 aAbx^{2}+330 Bx^{2}a^{2}+770 a^{2}A}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*B*b^2*x^6+105*A*b^2*x^4+210*B*a*b*x^4+330*A*a*b*x^2+165*
B*a^2*x^2+385*A*a^2)
```

Maxima [A] time = 1.04412, size = 69, normalized size = 1.1

$$\frac{2}{15}Bb^{2}x^{\frac{15}{2}}+\frac{2}{11}\left(2 Bab+Ab^{2}\right)x^{\frac{11}{2}}+\frac{2}{3}Aa^{2}x^{\frac{3}{2}}+\frac{2}{7}\left(Ba^{2}+2 Aab\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/3*A*a^2*x^(3/2) +
 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Fricas [A] time = 0.912158, size = 136, normalized size = 2.16

$$\frac{2}{1155}\left(77 Bb^{2}x^{7}+105\left(2 Bab+Ab^{2}\right)x^{5}+385 Aa^{2}x+165\left(Ba^{2}+2 Aab\right)x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^2*x^7 + 105*(2*B*a*b + A*b^2)*x^5 + 385*A*a^2*x + 165*(B*a^2
 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A] time = 2.46814, size = 66, normalized size = 1.05

$$\frac{2Aa^{2}x^{\frac{3}{2}}}{3}+\frac{2Bb^{2}x^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ab^{2}+2Bab\right)}{11}+\frac{2x^{\frac{7}{2}}\left(2Aab+Ba^{2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*x**(11/2)*(A*b**2 + 2*B*a*b
)/11 + 2*x**(7/2)*(2*A*a*b + B*a**2)/7
```

Giac [A] time = 1.15207, size = 72, normalized size = 1.14

$$\frac{2}{15}Bb^{2}x^{\frac{15}{2}}+\frac{4}{11}Babx^{\frac{11}{2}}+\frac{2}{11}Ab^{2}x^{\frac{11}{2}}+\frac{2}{7}Ba^{2}x^{\frac{7}{2}}+\frac{4}{7}Aabx^{\frac{7}{2}}+\frac{2}{3}Aa^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2/3*A*a^2*x^(3/2)
```

3.355 $\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(9/2
))/9 + (2*b^2*B*x^(13/2))/13
```

Rubi [A] time = 0.0322777, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(9/2
))/9 + (2*b^2*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2A}{\sqrt{x}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{7/2}+b^2Bx^{11/2}\right)dx\\ &= 2a^2A\sqrt{x}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0279692, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2A+65bx^4(2aB+Ab)+117ax^2(aB+2Ab)+45b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*A + 117*a*(2*A*b + a*B)*x^2 + 65*b*(A*b + 2*a*B)*x^4 +
45*b^2*B*x^6))/585
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{90\,Bb^2x^6+130\,Ab^2x^4+260\,Bx^4ab+468\,aAbx^2+234\,Bx^2a^2+1170\,a^2A}{585}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*B*b^2*x^6+65*A*b^2*x^4+130*B*a*b*x^4+234*A*a*b*x^2+117*B*
a^2*x^2+585*A*a^2)
```

Maxima [A] time = 1.08106, size = 69, normalized size = 1.13

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{9}\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+2\,Aa^2\sqrt{x}+\frac{2}{5}\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2*A*a^2*sqrt(x) + 2/5
*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Fricas [A] time = 0.903859, size = 131, normalized size = 2.15

$$\frac{2}{585}\left(45\,Bb^2x^6+65\left(2\,Bab+Ab^2\right)x^4+585\,Aa^2+117\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^2*x^6 + 65*(2*B*a*b + A*b^2)*x^4 + 585*A*a^2 + 117*(B*a^2 + 2
*A*a*b)*x^2)*sqrt(x)
```

Sympy [A] time = 2.12217, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*x**(
5/2)/5 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(13/2)/13
```

Giac [A] time = 1.16364, size = 72, normalized size = 1.18

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{5}\,Ba^2x^{\frac{5}{2}}+\frac{4}{5}\,Aabx^{\frac{5}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/5*B*a^2*x^(
5/2) + 4/5*A*a*b*x^(5/2) + 2*A*a^2*sqrt(x)
```

3.356 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rubi [A] time = 0.030532, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{5/2}+b^2Bx^{9/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{11}b^2Bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0203783, size = 60, normalized size = 0.98

$$\frac{-154a^2\left(3A-Bx^2\right)+44abx^2\left(7A+3Bx^2\right)+6b^2x^4\left(11A+7Bx^2\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-154*a^2*(3*A - B*x^2) + 44*a*b*x^2*(7*A + 3*B*x^2) + 6*b^2*x^4*(11*A + 7*
B*x^2))/(231*Sqrt[x])
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{-42 Bb^2x^6-66 Ab^2x^4-132 Bx^4ab-308 aAbx^2-154 Bx^2a^2+462 a^2A}{231}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/231*(-21*B*b^2*x^6-33*A*b^2*x^4-66*B*a*b*x^4-154*A*a*b*x^2-77*B*a^2*x^2+
231*A*a^2)/x^(1/2)
```

Maxima [A] time = 1.04592, size = 69, normalized size = 1.13

$$\frac{2}{11} Bb^2x^{\frac{11}{2}}+\frac{2}{7}\left(2 Bab+Ab^2\right)x^{\frac{7}{2}}-\frac{2 Aa^2}{\sqrt{x}}+\frac{2}{3}\left(Ba^2+2 Aab\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) - 2*A*a^2/sqrt(x) + 2/3
*(B*a^2 + 2*A*a*b)*x^(3/2)
```

Fricas [A] time = 0.886639, size = 130, normalized size = 2.13

$$\frac{2\left(21 Bb^2x^6+33\left(2 Bab+Ab^2\right)x^4-231 Aa^2+77\left(Ba^2+2 Aab\right)x^2\right)}{231\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b^2*x^6 + 33*(2*B*a*b + A*b^2)*x^4 - 231*A*a^2 + 77*(B*a^2 + 2*
A*a*b)*x^2)/sqrt(x)
```

Sympy [A] time = 2.5104, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{7}{2}}}{7}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*x**
(3/2)/3 + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(11/2)/11
```

Giac [A] time = 1.16097, size = 72, normalized size = 1.18

$$\frac{2}{11}Bb^2x^{\frac{11}{2}} + \frac{4}{7}Babx^{\frac{7}{2}} + \frac{2}{7}Ab^2x^{\frac{7}{2}} + \frac{2}{3}Ba^2x^{\frac{3}{2}} + \frac{4}{3}Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2*A*a^2/sqrt(x)
```

3.357 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rubi [A] time = 0.0288711, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{3/2}+b^2Bx^{7/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{9}b^2Bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0178349, size = 57, normalized size = 0.93

$$\frac{-30a^2(A-3Bx^2)+36abx^2(5A+Bx^2)+2b^2x^4(9A+5Bx^2)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-30*a^2*(A - 3*B*x^2) + 36*a*b*x^2*(5*A + B*x^2) + 2*b^2*x^4*(9*A + 5*B*x^
2))/(45*x^(3/2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$-\frac{-10\,Bb^2x^6-18\,Ab^2x^4-36\,Bx^4ab-180\,aAbx^2-90\,Bx^2a^2+30\,a^2A}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/45*(-5*B*b^2*x^6-9*A*b^2*x^4-18*B*a*b*x^4-90*A*a*b*x^2-45*B*a^2*x^2+15*A
*a^2)/x^(3/2)
```

Maxima [A] time = 1.07084, size = 69, normalized size = 1.13

$$\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{2}{5}\left(2\,Bab+Ab^2\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) - 2/3*A*a^2/x^(3/2) + 2*(
B*a^2 + 2*A*a*b)*sqrt(x)
```

Fricas [A] time = 0.724228, size = 124, normalized size = 2.03

$$\frac{2\left(5\,Bb^2x^6+9\left(2\,Bab+Ab^2\right)x^4-15\,Aa^2+45\left(Ba^2+2\,Aab\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b^2*x^6 + 9*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 45*(B*a^2 + 2*A*a*
b)*x^2)/x^(3/2)
```

Sympy [A] time = 3.01178, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{5}{2}}}{5}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(9/2)/9
```

Giac [A] time = 1.13106, size = 72, normalized size = 1.18

$$\frac{2}{9}Bb^2x^{\frac{9}{2}} + \frac{4}{5}Babx^{\frac{5}{2}} + \frac{2}{5}Ab^2x^{\frac{5}{2}} + 2Ba^2\sqrt{x} + 4Aab\sqrt{x} - \frac{2Aa^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2*B*a^2*sqrt(x)
 + 4*A*a*b*sqrt(x) - 2/3*A*a^2/x^(3/2)
```

3.358 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b + 2*a*B)*x
^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rubi [A] time = 0.0287255, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b + 2*a*B)*x
^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{x^{3/2}}+b(Ab+2aB)\sqrt{x}+b^2Bx^{5/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}-\frac{2a(2Ab+aB)}{\sqrt{x}}+\frac{2}{3}b(Ab+2aB)x^{3/2}+\frac{2}{7}b^2Bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0155888, size = 57, normalized size = 0.93

$$\frac{-42a^2\left(A+5Bx^2\right)+140abx^2\left(Bx^2-3A\right)+10b^2x^4\left(7A+3Bx^2\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*A + B*x^2) + 10*b^2*x^4*(7*A + 3*B*x^2) - 42*a^2*(A + 5*B*
x^2))/(105*x^(5/2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$-\frac{-30\,B b^{2} x^{6}-70\,A b^{2} x^{4}-140\,B x^{4} a b+420\,a A b x^{2}+210\,B x^{2} a^{2}+42\,a^{2} A}{105}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/105*(-15*B*b^2*x^6-35*A*b^2*x^4-70*B*a*b*x^4+210*A*a*b*x^2+105*B*a^2*x^2
+21*A*a^2)/x^(5/2)
```

Maxima [A] time = 1.01632, size = 72, normalized size = 1.18

$$\frac{2}{7}\,B b^{2} x^{\frac{7}{2}}+\frac{2}{3}\left(2\,B a b+A b^{2}\right)x^{\frac{3}{2}}-\frac{2\left(A a^{2}+5\left(B a^{2}+2\,A a b\right)x^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) - 2/5*(A*a^2 + 5*(B*a^2 +
 2*A*a*b)*x^2)/x^(5/2)
```

Fricas [A] time = 0.796934, size = 130, normalized size = 2.13

$$\frac{2\left(15\,B b^{2} x^{6}+35\left(2\,B a b+A b^{2}\right)x^{4}-21\,A a^{2}-105\left(B a^{2}+2\,A a b\right)x^{2}\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 - 21*A*a^2 - 105*(B*a^2 + 2*
A*a*b)*x^2)/x^(5/2)
```

Sympy [A] time = 4.14279, size = 76, normalized size = 1.25

$$-\frac{2Aa^{2}}{5x^{\frac{5}{2}}}-\frac{4Aab}{\sqrt{x}}+\frac{2Ab^{2}x^{\frac{3}{2}}}{3}-\frac{2Ba^{2}}{\sqrt{x}}+\frac{4Babx^{\frac{3}{2}}}{3}+\frac{2Bb^{2}x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*x**(3/2)/3 - 2*B*a**2/s
qrt(x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(7/2)/7
```

Giac [A] time = 1.12807, size = 74, normalized size = 1.21

$$\frac{2}{7}\,Bb^{2}x^{\frac{7}{2}}+\frac{4}{3}\,Babx^{\frac{3}{2}}+\frac{2}{3}\,Ab^{2}x^{\frac{3}{2}}-\frac{2\left(5\,Ba^{2}x^{2}+10\,Aabx^{2}+Aa^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) - 2/5*(5*B*a^2*x^
2 + 10*A*a*b*x^2 + A*a^2)/x^(5/2)
```

3.359 $\int x^{7/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$

Optimal. Leaf size=85

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

Rubi [A] time = 0.0420046, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx &= \int \left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x^{19/2}+b^3Bx^{23/2}\right. \\ &= \frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{25}b\end{aligned}$$

Mathematica [A] time = 0.0416952, size = 85, normalized size = 1.

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

Maple [A] time = 0.004, size = 80, normalized size = 0.9

$$\frac{27846\,Bb^3x^8+33150\,x^6Ab^3+99450\,x^6Bab^2+122850\,x^4Aab^2+122850\,x^4Ba^2b+160650\,x^2Aa^2b+53550\,x^2Ba^3+773}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/348075*x^(9/2)*(13923*B*b^3*x^8+16575*A*b^3*x^6+49725*B*a*b^2*x^6+61425*A
*a*b^2*x^4+61425*B*a^2*b*x^4+80325*A*a^2*b*x^2+26775*B*a^3*x^2+38675*A*a^3)
```

Maxima [A] time = 1.0274, size = 99, normalized size = 1.16

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,Bab^2 + Ab^3\right)x^{\frac{21}{2}} + \frac{6}{17}\left(Ba^2b + Aab^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}} + \frac{2}{13}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

Fricas [A] time = 0.76582, size = 205, normalized size = 2.41

$$\frac{2}{348075}\left(13923\,Bb^3x^{12} + 16575\left(3\,Bab^2 + Ab^3\right)x^{10} + 61425\left(Ba^2b + Aab^2\right)x^8 + 38675\,Aa^3x^4 + 26775\left(Ba^3 + 3\,Aa^2b\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*b^3*x^12 + 16575*(3*B*a*b^2 + A*b^3)*x^10 + 61425*(B*a^2*
b + A*a*b^2)*x^8 + 38675*A*a^3*x^4 + 26775*(B*a^3 + 3*A*a^2*b)*x^6)*sqrt(x)
```

Sympy [A] time = 34.0296, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9} + \frac{6Aa^2bx^{\frac{13}{2}}}{13} + \frac{6Aab^2x^{\frac{17}{2}}}{17} + \frac{2Ab^3x^{\frac{21}{2}}}{21} + \frac{2Ba^3x^{\frac{13}{2}}}{13} + \frac{6Ba^2bx^{\frac{17}{2}}}{17} + \frac{2Bab^2x^{\frac{21}{2}}}{7} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(17/2)/17 + 2*
B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(25/2)/25
```

Giac [A] time = 1.14904, size = 104, normalized size = 1.22

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{7}\,Bab^2x^{\frac{21}{2}} + \frac{2}{21}\,Ab^3x^{\frac{21}{2}} + \frac{6}{17}\,Ba^2bx^{\frac{17}{2}} + \frac{6}{17}\,Aab^2x^{\frac{17}{2}} + \frac{2}{13}\,Ba^3x^{\frac{13}{2}} + \frac{6}{13}\,Aa^2bx^{\frac{13}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 6/17*B*a
^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b*
x^(13/2) + 2/9*A*a^3*x^(9/2)
```

3.360 $\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx$

Optimal. Leaf size=85

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

Rubi [A] time = 0.040575, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx &= \int \left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)x^{17/2}+b^3Bx^{21}\right.\\ &= \frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{2}\end{aligned}$$

Mathematica [A] time = 0.0381048, size = 85, normalized size = 1.

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{14630\,Bb^3x^8+17710\,x^6Ab^3+53130\,x^6Bab^2+67298\,x^4Aab^2+67298\,x^4Ba^2b+91770\,x^2Aa^2b+30590\,x^2Ba^3+4807}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/168245*x^(7/2)*(7315*B*b^3*x^8+8855*A*b^3*x^6+26565*B*a*b^2*x^6+33649*A*a
*b^2*x^4+33649*B*a^2*b*x^4+45885*A*a^2*b*x^2+15295*B*a^3*x^2+24035*A*a^3)
```

Maxima [A] time = 1.05957, size = 99, normalized size = 1.16

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{2}{19}\left(3\,Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{2}{5}\left(Ba^2b + Aab^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}} + \frac{2}{11}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Fricas [A] time = 0.870319, size = 201, normalized size = 2.36

$$\frac{2}{168245}\left(7315\,Bb^3x^{11} + 8855\left(3\,Bab^2 + Ab^3\right)x^9 + 33649\left(Ba^2b + Aab^2\right)x^7 + 24035\,Aa^3x^3 + 15295\left(Ba^3 + 3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*b^3*x^11 + 8855*(3*B*a*b^2 + A*b^3)*x^9 + 33649*(B*a^2*b +
 A*a*b^2)*x^7 + 24035*A*a^3*x^3 + 15295*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

Sympy [A] time = 20.6412, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7} + \frac{6Aa^2bx^{\frac{11}{2}}}{11} + \frac{2Aab^2x^{\frac{15}{2}}}{5} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{11}{2}}}{11} + \frac{2Ba^2bx^{\frac{15}{2}}}{5} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(11/2)/11 + 2*A*a*b**2*x**(15/2)/5 + 2*
A*b**3*x**(19/2)/19 + 2*B*a**3*x**(11/2)/11 + 2*B*a**2*b*x**(15/2)/5 + 6*B*
a*b**2*x**(19/2)/19 + 2*B*b**3*x**(23/2)/23
```

Giac [A] time = 1.15107, size = 104, normalized size = 1.22

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{2}{5}\,Ba^2bx^{\frac{15}{2}} + \frac{2}{5}\,Aab^2x^{\frac{15}{2}} + \frac{2}{11}\,Ba^3x^{\frac{11}{2}} + \frac{6}{11}\,Aa^2bx^{\frac{11}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 2/5*B*a
^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b*x
^(11/2) + 2/7*A*a^3*x^(7/2)
```

3.361 $\int x^{3/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

Rubi [A] time = 0.0403044, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx=\int\left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{15/2}+b^3Bx^{19/2}\right)$$
$$=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{21}b^3$$

Mathematica [A] time = 0.0360561, size = 85, normalized size = 1.

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{6630\,Bb^3x^8+8190\,x^6Ab^3+24570\,x^6Bab^2+32130\,x^4Aab^2+32130\,x^4Ba^2b+46410\,x^2Aa^2b+15470\,x^2Ba^3+27846\,Aa^3}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/69615*x^(5/2)*(3315*B*b^3*x^8+4095*A*b^3*x^6+12285*B*a*b^2*x^6+16065*A*a*
b^2*x^4+16065*B*a^2*b*x^4+23205*A*a^2*b*x^2+7735*B*a^3*x^2+13923*A*a^3)
```

Maxima [A] time = 1.1162, size = 99, normalized size = 1.16

$$\frac{2}{21}Bb^3x^{\frac{21}{2}} + \frac{2}{17}\left(3Bab^2 + Ab^3\right)x^{\frac{17}{2}} + \frac{6}{13}\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + \frac{2}{5}Aa^3x^{\frac{5}{2}} + \frac{2}{9}\left(Ba^3 + 3Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Fricas [A] time = 0.82957, size = 198, normalized size = 2.33

$$\frac{2}{69615}\left(3315Bb^3x^{10} + 4095\left(3Bab^2 + Ab^3\right)x^8 + 16065\left(Ba^2b + Aab^2\right)x^6 + 13923Aa^3x^2 + 7735\left(Ba^3 + 3Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*B*b^3*x^10 + 4095*(3*B*a*b^2 + A*b^3)*x^8 + 16065*(B*a^2*b +
A*a*b^2)*x^6 + 13923*A*a^3*x^2 + 7735*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] time = 11.3913, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5} + \frac{2Aa^2bx^{\frac{9}{2}}}{3} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{17}{2}}}{17} + \frac{2Ba^3x^{\frac{9}{2}}}{9} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{17}{2}}}{17} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(13/2)/13 + 2*A
*b**3*x**(17/2)/17 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*
b**2*x**(17/2)/17 + 2*B*b**3*x**(21/2)/21
```

Giac [A] time = 1.13626, size = 104, normalized size = 1.22

$$\frac{2}{21}Bb^3x^{\frac{21}{2}} + \frac{6}{17}Bab^2x^{\frac{17}{2}} + \frac{2}{17}Ab^3x^{\frac{17}{2}} + \frac{6}{13}Ba^2bx^{\frac{13}{2}} + \frac{6}{13}Aab^2x^{\frac{13}{2}} + \frac{2}{9}Ba^3x^{\frac{9}{2}} + \frac{2}{3}Aa^2bx^{\frac{9}{2}} + \frac{2}{5}Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^
(9/2) + 2/5*A*a^3*x^(5/2)
```

3.362 $\int \sqrt{x}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{3}a^3Ax^{3/2}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(19/2))/19
```

Rubi [A] time = 0.0412365, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{3}a^3Ax^{3/2}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{13/2}+b^3Bx^{17/2}\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{19}\end{aligned}$$

Mathematica [A] time = 0.03822, size = 71, normalized size = 0.84

$$\frac{2x^{3/2}\left(3135a^2x^2(aB+3Ab)+7315a^3A+1463b^2x^6(3aB+Ab)+5985abx^4(aB+Ab)+1155b^3Bx^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(7315*a^3*A + 3135*a^2*(3*A*b + a*B)*x^2 + 5985*a*b*(A*b + a*B)*
x^4 + 1463*b^2*(A*b + 3*a*B)*x^6 + 1155*b^3*B*x^8))/21945
```

Maple [A] time = 0.004, size = 80, normalized size = 0.9

$$\frac{2310\,b^3Bx^8+2926\,x^6b^3A+8778\,x^6ab^2B+11970\,x^4ab^2A+11970\,x^4a^2bB+18810\,x^2Aa^2b+6270\,x^2Ba^3+14630\,a^3A}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*B*b^3*x^8+1463*A*b^3*x^6+4389*B*a*b^2*x^6+5985*A*a*b^
2*x^4+5985*B*a^2*b*x^4+9405*A*a^2*b*x^2+3135*B*a^3*x^2+7315*A*a^3)
```

Maxima [A] time = 1.08603, size = 99, normalized size = 1.16

$$\frac{2}{19}Bb^3x^{\frac{19}{2}} + \frac{2}{15}\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{6}{11}\left(Ba^2b + Aab^2\right)x^{\frac{11}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2}{7}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Fricas [A] time = 0.854758, size = 192, normalized size = 2.26

$$\frac{2}{21945}\left(1155\,Bb^3x^9 + 1463\left(3\,Bab^2 + Ab^3\right)x^7 + 5985\left(Ba^2b + Aab^2\right)x^5 + 7315\,Aa^3x + 3135\left(Ba^3 + 3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^3*x^9 + 1463*(3*B*a*b^2 + A*b^3)*x^7 + 5985*(B*a^2*b + A*
a*b^2)*x^5 + 7315*A*a^3*x + 3135*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] time = 3.65904, size = 95, normalized size = 1.12

$$\frac{2Aa^3x^{\frac{3}{2}}}{3} + \frac{2Bb^3x^{\frac{19}{2}}}{19} + \frac{2x^{\frac{15}{2}}\left(Ab^3 + 3Bab^2\right)}{15} + \frac{2x^{\frac{11}{2}}\left(3Aab^2 + 3Ba^2b\right)}{11} + \frac{2x^{\frac{7}{2}}\left(3Aa^2b + Ba^3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*x**(15/2)*(A*b**3 + 3*B*a*b
**2)/15 + 2*x**(11/2)*(3*A*a*b**2 + 3*B*a**2*b)/11 + 2*x**(7/2)*(3*A*a**2*b
 + B*a**3)/7
```

Giac [A] time = 1.13797, size = 104, normalized size = 1.22

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 6/11*B*a
^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^(
7/2) + 2/3*A*a^3*x^(3/2)
```

3.363 $\int \frac{(a+bx^2)^3 (A+Bx^2)}{\sqrt{x}} \, dx$

Optimal. Leaf size=83

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b + a*B)*x^(9
/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(17/2))/17
```

Rubi [A] time = 0.0403782, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b + a*B)*x^(9
/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3 (A+Bx^2)}{\sqrt{x}} \, dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{11/2}+b^3Bx^{15/2}\right) dx\\ &= 2a^3A\sqrt{x}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{17}b^3Bx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0406482, size = 71, normalized size = 0.86

$$\frac{2\sqrt{x}\left(663a^2x^2(aB+3Ab)+3315a^3A+255b^2x^6(3aB+Ab)+1105abx^4(aB+Ab)+195b^3Bx^8\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3315*a^3*A + 663*a^2*(3*A*b + a*B)*x^2 + 1105*a*b*(A*b + a*B)*x
^4 + 255*b^2*(A*b + 3*a*B)*x^6 + 195*b^3*B*x^8))/3315
```

Maple [A] time = 0.005, size = 80, normalized size = 1.

$$\frac{390\,b^{3}Bx^{8}+510\,x^{6}b^{3}A+1530\,x^{6}ab^{2}B+2210\,x^{4}ab^{2}A+2210\,x^{4}a^{2}bB+3978\,x^{2}Aa^{2}b+1326\,x^{2}Ba^{3}+6630\,a^{3}A}{3315}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/3315*x^(1/2)*(195*B*b^3*x^8+255*A*b^3*x^6+765*B*a*b^2*x^6+1105*A*a*b^2*x^
4+1105*B*a^2*b*x^4+1989*A*a^2*b*x^2+663*B*a^3*x^2+3315*A*a^3)
```

Maxima [A] time = 1.47263, size = 99, normalized size = 1.19

$$\frac{2}{17}\,Bb^{3}x^{\frac{17}{2}}+\frac{2}{13}\left(3\,Bab^{2}+Ab^{3}\right)x^{\frac{13}{2}}+\frac{2}{3}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{9}{2}}+2\,Aa^{3}\sqrt{x}+\frac{2}{5}\left(Ba^{3}+3\,Aa^{2}b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Fricas [A] time = 0.700195, size = 184, normalized size = 2.22

$$\frac{2}{3315}\left(195\,Bb^{3}x^{8}+255\left(3\,Bab^{2}+Ab^{3}\right)x^{6}+1105\left(Ba^{2}b+Aab^{2}\right)x^{4}+3315\,Aa^{3}+663\left(Ba^{3}+3\,Aa^{2}b\right)x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*B*b^3*x^8 + 255*(3*B*a*b^2 + A*b^3)*x^6 + 1105*(B*a^2*b + A*a*b
^2)*x^4 + 3315*A*a^3 + 663*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

Sympy [A] time = 4.95304, size = 112, normalized size = 1.35

$$2Aa^{3}\sqrt{x}+\frac{6Aa^{2}bx^{\frac{5}{2}}}{5}+\frac{2Aab^{2}x^{\frac{9}{2}}}{3}+\frac{2Ab^{3}x^{\frac{13}{2}}}{13}+\frac{2Ba^{3}x^{\frac{5}{2}}}{5}+\frac{2Ba^{2}bx^{\frac{9}{2}}}{3}+\frac{6Bab^{2}x^{\frac{13}{2}}}{13}+\frac{2Bb^{3}x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 2*A*a*b**2*x**(9/2)/3 + 2*A*b**3
*x**(13/2)/13 + 2*B*a**3*x**(5/2)/5 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(17/2)/17
```

Giac [A] time = 1.11773, size = 104, normalized size = 1.25

$$\frac{2}{17}Bb^3x^{\frac{17}{2}}+\frac{6}{13}Bab^2x^{\frac{13}{2}}+\frac{2}{13}Ab^3x^{\frac{13}{2}}+\frac{2}{3}Ba^2bx^{\frac{9}{2}}+\frac{2}{3}Aab^2x^{\frac{9}{2}}+\frac{2}{5}Ba^3x^{\frac{5}{2}}+\frac{6}{5}Aa^2bx^{\frac{5}{2}}+2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 2/3*B*a
^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2
) + 2*A*a^3*sqrt(x)
```

3.364 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(15/2))/15
```

Rubi [A] time = 0.0437627, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{9/2}+b^3Bx^{13/2}\right)dx\\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{11}b^2(Ab+3aB)x^{11/2}+\frac{2}{15}b^3Bx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0216523, size = 81, normalized size = 0.98

$$\frac{330a^2bx^2(7A+3Bx^2)-770a^3(3A-Bx^2)+90ab^2x^4(11A+7Bx^2)+14b^3x^6(15A+11Bx^2)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-770*a^3*(3*A - B*x^2) + 330*a^2*b*x^2*(7*A + 3*B*x^2) + 90*a*b^2*x^4*(11*
A + 7*B*x^2) + 14*b^3*x^6*(15*A + 11*B*x^2))/(1155*Sqrt[x])
```

Maple [A] time = 0.005, size = 80, normalized size = 1.

$$-\frac{-154\,b^3Bx^8-210\,x^6b^3A-630\,x^6ab^2B-990\,x^4ab^2A-990\,x^4a^2bB-2310\,x^2Aa^2b-770\,x^2Ba^3+2310\,a^3A}{1155}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/1155*(-77*B*b^3*x^8-105*A*b^3*x^6-315*B*a*b^2*x^6-495*A*a*b^2*x^4-495*B*
a^2*b*x^4-1155*A*a^2*b*x^2-385*B*a^3*x^2+1155*A*a^3)/x^(1/2)
```

Maxima [A] time = 1.35472, size = 99, normalized size = 1.19

$$\frac{2}{15}\,Bb^3x^{\frac{15}{2}}+\frac{2}{11}\left(3\,Bab^2+Ab^3\right)x^{\frac{11}{2}}+\frac{6}{7}\left(Ba^2b+Aab^2\right)x^{\frac{7}{2}}-\frac{2\,Aa^3}{\sqrt{x}}+\frac{2}{3}\left(Ba^3+3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2*A*a^3/sqrt(x) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Fricas [A] time = 0.779691, size = 181, normalized size = 2.18

$$\frac{2\left(77\,Bb^3x^8+105\left(3\,Bab^2+Ab^3\right)x^6+495\left(Ba^2b+Aab^2\right)x^4-1155\,Aa^3+385\left(Ba^3+3\,Aa^2b\right)x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^3*x^8 + 105*(3*B*a*b^2 + A*b^3)*x^6 + 495*(B*a^2*b + A*a*b^2
)*x^4 - 1155*A*a^3 + 385*(B*a^3 + 3*A*a^2*b)*x^2)/sqrt(x)
```

Sympy [A] time = 5.60799, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{\sqrt{x}}+2Aa^2bx^{\frac{3}{2}}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{2Ba^3x^{\frac{3}{2}}}{3}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+\frac{6Bab^2x^{\frac{11}{2}}}{11}+\frac{2Bb^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b**3*
x**(11/2)/11 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x**
(11/2)/11 + 2*B*b**3*x**(15/2)/15
```

Giac [A] time = 1.33175, size = 104, normalized size = 1.25

$$\frac{2}{15}Bb^3x^{\frac{15}{2}} + \frac{6}{11}Bab^2x^{\frac{11}{2}} + \frac{2}{11}Ab^3x^{\frac{11}{2}} + \frac{6}{7}Ba^2bx^{\frac{7}{2}} + \frac{6}{7}Aab^2x^{\frac{7}{2}} + \frac{2}{3}Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2)
- 2*A*a^3/sqrt(x)
```

3.365 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=83

$$2a^2\sqrt{x}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(13/2))/13
```

Rubi [A] time = 0.0432517, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2\sqrt{x}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{3/2}+b^2(Ab+3aB)x^{7/2}+b^3Bx^{11/2}\right)dx\\ &= -\frac{2a^3A}{3x^{3/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{5}ab(Ab+aB)x^{5/2}+\frac{2}{9}b^2(Ab+3aB)x^{9/2}+\frac{2}{13}b^3Bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0261891, size = 78, normalized size = 0.94

$$\frac{702a^2bx^2(5A+Bx^2)-390a^3(A-3Bx^2)+78ab^2x^4(9A+5Bx^2)+10b^3x^6(13A+9Bx^2)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-390*a^3*(A - 3*B*x^2) + 702*a^2*b*x^2*(5*A + B*x^2) + 78*a*b^2*x^4*(9*A +
 5*B*x^2) + 10*b^3*x^6*(13*A + 9*B*x^2))/(585*x^(3/2))
```

Maple [A] time = 0.006, size = 80, normalized size = 1.

$$-\frac{-90\,b^{3}B x^{8}-130\,x^{6}b^{3}A-390\,x^{6}a b^{2}B-702\,x^{4}a b^{2}A-702\,x^{4}a^{2}b B-3510\,A a^{2}b x^{2}-1170\,B a^{3}x^{2}+390\,a^{3}A}{585}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/585*(-45*B*b^3*x^8-65*A*b^3*x^6-195*B*a*b^2*x^6-351*A*a*b^2*x^4-351*B*a^
2*b*x^4-1755*A*a^2*b*x^2-585*B*a^3*x^2+195*A*a^3)/x^(3/2)
```

Maxima [A] time = 1.032, size = 99, normalized size = 1.19

$$\frac{2}{13}\,B b^{3}x^{\frac{13}{2}}+\frac{2}{9}\left(3\,B a b^{2}+A b^{3}\right)x^{\frac{9}{2}}+\frac{6}{5}\left(B a^{2}b+A a b^{2}\right)x^{\frac{5}{2}}-\frac{2\,A a^{3}}{3\,x^{\frac{3}{2}}}+2\left(B a^{3}+3\,A a^{2}b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2) + 6/5*(B*a^2*b + A*a*
b^2)*x^(5/2) - 2/3*A*a^3/x^(3/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Fricas [A] time = 0.864668, size = 177, normalized size = 2.13

$$\frac{2\left(45\,B b^{3}x^{8}+65\left(3\,B a b^{2}+A b^{3}\right)x^{6}+351\left(B a^{2}b+A a b^{2}\right)x^{4}-195\,A a^{3}+585\left(B a^{3}+3\,A a^{2}b\right)x^{2}\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^3*x^8 + 65*(3*B*a*b^2 + A*b^3)*x^6 + 351*(B*a^2*b + A*a*b^2)*
x^4 - 195*A*a^3 + 585*(B*a^3 + 3*A*a^2*b)*x^2)/x^(3/2)
```

Sympy [A] time = 6.68447, size = 110, normalized size = 1.33

$$-\frac{2Aa^{3}}{3x^{\frac{3}{2}}}+6Aa^{2}b\sqrt{x}+\frac{6Aab^{2}x^{\frac{5}{2}}}{5}+\frac{2Ab^{3}x^{\frac{9}{2}}}{9}+2Ba^{3}\sqrt{x}+\frac{6Ba^{2}bx^{\frac{5}{2}}}{5}+\frac{2Bab^{2}x^{\frac{9}{2}}}{3}+\frac{2Bb^{3}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(5/2)/5 + 2*A*b
**3*x**(9/2)/9 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(5/2)/5 + 2*B*a*b**2*x**(
9/2)/3 + 2*B*b**3*x**(13/2)/13
```

Giac [A] time = 1.13398, size = 104, normalized size = 1.25

$$\frac{2}{13}Bb^3x^{\frac{13}{2}} + \frac{2}{3}Bab^2x^{\frac{9}{2}} + \frac{2}{9}Ab^3x^{\frac{9}{2}} + \frac{6}{5}Ba^2bx^{\frac{5}{2}} + \frac{6}{5}Aab^2x^{\frac{5}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6/5*B*a^2*b
*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2/3*
A*a^3/x^(3/2)
```

3.366 $\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}} dx$

Optimal. Leaf size=81

$$-\frac{2a^2(aB+3Ab)}{\sqrt{x}}-\frac{2a^3A}{5x^{5/2}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A*b + a*B)*
x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(11/2))/11
```

Rubi [A] time = 0.0427194, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2(aB+3Ab)}{\sqrt{x}}-\frac{2a^3A}{5x^{5/2}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A*b + a*B)*
x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}dx &= \int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{x^{3/2}}+3ab(Ab+aB)\sqrt{x}+b^2(Ab+3aB)x^{5/2}+b^3Bx^{9/2}\right)dx\\ &= -\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(3Ab+aB)}{\sqrt{x}}+2ab(Ab+aB)x^{3/2}+\frac{2}{7}b^2(Ab+3aB)x^{7/2}+\frac{2}{11}b^3Bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0216963, size = 78, normalized size = 0.96

$$\frac{2\left(385a^2bx^2\left(Bx^2-3A\right)-77a^3\left(A+5Bx^2\right)+55ab^2x^4\left(7A+3Bx^2\right)+5b^3x^6\left(11A+7Bx^2\right)\right)}{385x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(385*a^2*b*x^2*(-3*A + B*x^2) + 55*a*b^2*x^4*(7*A + 3*B*x^2) - 77*a^3*(A
 + 5*B*x^2) + 5*b^3*x^6*(11*A + 7*B*x^2)))/(385*x^(5/2))
```

Maple [A] time = 0.006, size = 80, normalized size = 1.

$$-\frac{-70 b^{3} B x^{8}-110 x^{6} b^{3} A-330 x^{6} a b^{2} B-770 A a b^{2} x^{4}-770 B a^{2} b x^{4}+2310 A a^{2} b x^{2}+770 B a^{3} x^{2}+154 a^{3} A}{385} x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/385*(-35*B*b^3*x^8-55*A*b^3*x^6-165*B*a*b^2*x^6-385*A*a*b^2*x^4-385*B*a^
2*b*x^4+1155*A*a^2*b*x^2+385*B*a^3*x^2+77*A*a^3)/x^(5/2)
```

Maxima [A] time = 1.04165, size = 101, normalized size = 1.25

$$\frac{2}{11} B b^{3} x^{\frac{11}{2}}+\frac{2}{7}\left(3 B a b^{2}+A b^{3}\right) x^{\frac{7}{2}}+2\left(B a^{2} b+A a b^{2}\right) x^{\frac{3}{2}}-\frac{2\left(A a^{3}+5\left(B a^{3}+3 A a^{2} b\right) x^{2}\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 2*(B*a^2*b + A*a*b^
2)*x^(3/2) - 2/5*(A*a^3 + 5*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

Fricas [A] time = 0.76869, size = 176, normalized size = 2.17

$$\frac{2\left(35 B b^{3} x^{8}+55\left(3 B a b^{2}+A b^{3}\right) x^{6}+385\left(B a^{2} b+A a b^{2}\right) x^{4}-77 A a^{3}-385\left(B a^{3}+3 A a^{2} b\right) x^{2}\right)}{385 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*B*b^3*x^8 + 55*(3*B*a*b^2 + A*b^3)*x^6 + 385*(B*a^2*b + A*a*b^2)*
x^4 - 77*A*a^3 - 385*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

Sympy [A] time = 8.91241, size = 107, normalized size = 1.32

$$-\frac{2Aa^{3}}{5x^{\frac{5}{2}}}-\frac{6Aa^{2}b}{\sqrt{x}}+2Aab^{2}x^{\frac{3}{2}}+\frac{2Ab^{3}x^{\frac{7}{2}}}{7}-\frac{2Ba^{3}}{\sqrt{x}}+2Ba^{2}bx^{\frac{3}{2}}+\frac{6Bab^{2}x^{\frac{7}{2}}}{7}+\frac{2Bb^{3}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) - 6*A*a**2*b/sqrt(x) + 2*A*a*b**2*x**(3/2) + 2*A*b**
3*x**(7/2)/7 - 2*B*a**3/sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a*b**2*x**(7/2)
/7 + 2*B*b**3*x**(11/2)/11
```

Giac [A] time = 1.12703, size = 107, normalized size = 1.32

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{7}\,Bab^2x^{\frac{7}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+2\,Ba^2bx^{\frac{3}{2}}+2\,Aab^2x^{\frac{3}{2}}-\frac{2\left(5\,Ba^3x^2+15\,Aa^2bx^2+Aa^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 2*B*a^2*b*x
^(3/2) + 2*A*a*b^2*x^(3/2) - 2/5*(5*B*a^3*x^2 + 15*A*a^2*b*x^2 + A*a^3)/x^(
5/2)
```

3.367 $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=276

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab-aB)\tan^{-1}}{\sqrt{2}b^{13/4}}$$

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2) + (2*B*x^(
9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(
5/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.258758, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab-aB)\tan^{-1}}{\sqrt{2}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2) + (2*B*x^(
9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(
5/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(13/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{a+bx^2}\,dx = \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^2}\,dx}{9b}$$
$$= \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{x^{3/2}}{a+bx^2}\,dx}{b^2}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^3}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(2a^2(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^3}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^3} + \cdots$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\cdots\right.}{2b^{7/2}}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}} + \cdots$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}} + \frac{a^{5/4}(Ab-a\cdots}{\cdots}$$

Mathematica [A] time = 0.276573, size = 227, normalized size = 0.82

$$\frac{\frac{45\sqrt{2}a^{5/4}(aB-Ab)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)\right)}{\sqrt[4]{b}} + \frac{90\sqrt{2}a^{5/4}(aB-Ab)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{\sqrt[4]{b}} + 72bx^{5/2}(\cdots}{180b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (360*a*(-(A*b) + a*B)*Sqrt[x] + 72*b*(A*b - a*B)*x^(5/2) + 40*b^2*B*x^(9/2)
 + (90*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]))/b^(1/4) + (45*S
qrt[2]*a^(5/4)*(-(A*b) + a*B)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
))/b^(1/4))/(180*b^3)
```

Maple [A] time = 0.009, size = 330, normalized size = 1.2

$$\frac{2B}{9b}x^{\frac{9}{2}} + \frac{2A}{5b}x^{\frac{5}{2}} - \frac{2Ba}{5b^2}x^{\frac{5}{2}} - 2\frac{aA\sqrt{x}}{b^2} + 2\frac{a^2B\sqrt{x}}{b^3} + \frac{a\sqrt{2}A}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{a\sqrt{2}A}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/5/b*A*x^(5/2)-2/5/b^2*B*x^(5/2)*a-2/b^2*a*A*x^(1/2)+2/b^3
*a^2*B*x^(1/2)+1/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)+1)+1/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)-1)+1/4*a/b^2*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*
2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2
*a^2/b^3*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/
2*a^2/b^3*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-1
/4*a^2/b^3*(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.932851, size = 1477, normalized size = 5.35

$$180\, b^{3} \left(-\frac{B^{4} a^{9} - 4\, A B^{3} a^{8} b + 6\, A^{2} B^{2} a^{7} b^{2} - 4\, A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{b^{6} \sqrt{-\frac{B^{4} a^{9} - 4\, A B^{3} a^{8} b + 6\, A^{2} B^{2} a^{7} b^{2} - 4\, A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}} + \left(B^{2} a^{4} - 2\, A B a^{3} b + A^{2} a^{2} b \right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*
b^3 + A^4*a^5*b^4)/b^13)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*a^9 - 4*A*B^3*a^
8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13) + (B^2*a^4 -
 2*A*B*a^3*b + A^2*a^2*b^2)*x)*b^10*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*
a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(3/4) + (B*a^2*b^10 - A*a*b^
11)*sqrt(x)*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^
3 + A^4*a^5*b^4)/b^13)^(3/4))/(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2
- 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)) + 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A
^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(b^3*(-(B^4*
a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^
13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) - 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b +
6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(-b^3*(-(
B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4
)/b^13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) + 4*(5*B*b^2*x^4 + 45*B*a^2 - 45*A
*a*b - 9*(B*a*b - A*b^2)*x^2)*sqrt(x))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14532, size = 402, normalized size = 1.46

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*((a*b^3)^
(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)
^(1/4)*A*a*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 1/4*sq
rt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/b^4 + 2/45*(5*B*b^8*x^(9/2) - 9*B*a*b^7*x^(5/2) +
9*A*b^8*x^(5/2) + 45*B*a^2*b^6*sqrt(x) - 45*A*a*b^7*sqrt(x))/b^9
```

3.368 $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2} dx$

Optimal. Leaf size=257

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\tan}{\sqrt{2}b}$$

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*(A*b - a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3
/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(
11/4)) - (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.205399, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\tan}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*(A*b - a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3
/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(
11/4)) - (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{a+bx^2}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{7b}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} + \frac{(a(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(a(Ab-aB))\,\text{Subst}\left(\right.}{b^{5}}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3} - \frac{(a(Ab-aB))\,\text{Su}}{}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{a^{3/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}} + \frac{a^{3/4}(Ab-aB)\log}{}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} + \frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{11/4}} - \frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}b^{11/4}}$$

Mathematica [A] time = 0.135887, size = 110, normalized size = 0.43

$$\frac{2b^{3/4}x^{3/2}\left(-7aB+7Ab+3bBx^2\right)-21(-a)^{3/4}(aB-Ab)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+21(-a)^{3/4}(aB-Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{21b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*b^(3/4)*x^(3/2)*(7*A*b - 7*a*B + 3*b*B*x^2) - 21*(-a)^(3/4)*(-(A*b) + a*
B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + 21*(-a)^(3/4)*(-(A*b) + a*B)*ArcT
anh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(21*b^(11/4))
```

Maple [A] time = 0.009, size = 308, normalized size = 1.2

$$\frac{2B}{7b}x^{\frac{7}{2}}+\frac{2A}{3b}x^{\frac{3}{2}}-\frac{2Ba}{3b^2}x^{\frac{3}{2}}-\frac{a\sqrt{2}A}{2b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a\sqrt{2}A}{4b^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/3/b*x^(3/2)*A-2/3/b^2*x^(3/2)*B*a-1/2*a/b^2/(1/b*a)^(1/4)
*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-1/4*a/b^2/(1/b*a)^(1/4)*
2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*B*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/4*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*B*ln((
```

```
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(
1/2)+(1/b*a)^(1/2)))+1/2*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/
b*a)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.98935, size = 1823, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b
^3 + A^4*a^3*b^4)/b^11)^(1/4)*arctan((sqrt((B^6*a^10 - 6*A*B^5*a^9*b + 15*A
^2*B^4*a^8*b^2 - 20*A^3*B^3*a^7*b^3 + 15*A^4*B^2*a^6*b^4 - 6*A^5*B*a^5*b^5
+ A^6*a^4*b^6)*x - (B^4*a^7*b^5 - 4*A*B^3*a^6*b^6 + 6*A^2*B^2*a^5*b^7 - 4*A
^3*B*a^4*b^8 + A^4*a^3*b^9)*sqrt(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*
b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11))*b^3*(-(B^4*a^7 - 4*A*B^3*a^6*b
+ 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4) + (B^3*a^5
*b^3 - 3*A*B^2*a^4*b^4 + 3*A^2*B*a^3*b^5 - A^3*a^2*b^6)*sqrt(x)*(-(B^4*a^7
- 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^
(1/4))/(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4
*a^3*b^4)) - 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*
B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6
*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 -
3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)*sqrt(x)) + 21*b^2*(-(B^4*a^7
 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)
^(1/4)*log(-b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^
4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b
^2 - A^3*a^2*b^3)*sqrt(x)) + 4*(3*B*b*x^3 - 7*(B*a - A*b)*x)*sqrt(x))/b^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.1624, size = 356, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} Ba - \left(ab^3\right)^{\frac{3}{4}} Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^5} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} Ba - \left(ab^3\right)^{\frac{3}{4}} Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^5} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 + 1/2*sqrt(2)*((a*b^3)^(3/4)
*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^5 + 2/21*(3*B*b^6*x^(7/2) - 7*B*a*b^5*x^(3/2) + 7*A*b^6*x^(3/2))/
b^7
```

3.369 $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=255

$$\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{\sqrt[4]{a}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{a}(A}{}$$

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(1/4)*(
A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4))
 + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(2*Sqrt[2]*b^(9/4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.202483, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{\sqrt[4]{a}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{a}(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(1/4)*(
A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4))
 + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(2*Sqrt[2]*b^(9/4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{a+bx^2}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{5b}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{b^2}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}(Ab-aB)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2} - \frac{\left(\sqrt{a}(Ab-aB)\right)\mathrm{Subst}}{b^2}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}(Ab-aB)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{5/2}} - \frac{\left(\sqrt{a}(Ab-aB)\right)\mathrm{Su}}{}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{a}\right.}{2\sqrt{}}$$
$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}b^{9/4}}$$

Mathematica [A] time = 0.205342, size = 208, normalized size = 0.82

$$\frac{40\sqrt{x}(Ab-aB)+\frac{5\sqrt{2}\sqrt[4]{a}(Ab-aB)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)\right)}{\sqrt[4]{b}}+\frac{10\sqrt{2}\sqrt[4]{a}(Ab-aB)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{b}}}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (40*(A*b - a*B)*Sqrt[x] + 8*b*B*x^(5/2) + (10*Sqrt[2]*a^(1/4)*(A*b - a*B)*(
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]))/b^(1/4) + (5*Sqrt[2]*a^(1/4)*(A*b - a*B)*(Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/b^(1/4))/(20*b^2)
```

Maple [A] time = 0.008, size = 299, normalized size = 1.2

$$\frac{2B}{5b}x^{\frac{5}{2}}+2\frac{A\sqrt{x}}{b}-2\frac{Ba\sqrt{x}}{b^2}-\frac{\sqrt{2}A}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}A}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{\sqrt{2}A}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/b*A*x^(1/2)-2/b^2*B*a*x^(1/2)-1/2/b*(1/b*a)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b*(1/b*a)^(1/4)*2^(1/2)*A*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-1/4/b*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(
1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2
```

```
)+(1/b*a)^(1/2)))+1/2/b^2*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1
/4)*x^(1/2)+1)*a+1/2/b^2*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)-1)*a+1/4/b^2*(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0013, size = 1365, normalized size = 5.35

$$-\frac{20\,b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}}+\left(B^2a^2-2\,ABab+A^2b^2\right)xb^7}}{B^4a^5-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*
b^3 + A^4*a*b^4)/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*a^5 - 4*A*B^3*a^4*b
 + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9) + (B^2*a^2 - 2*A*B
*a*b + A^2*b^2)*x)*b^7*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A
^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3/4) + (B*a*b^7 - A*b^8)*sqrt(x)*(-(B^4*a^5
 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3
/4))/(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a
*b^4)) + 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2
*b^3 + A^4*a*b^4)/b^9)^(1/4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2
*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) -
 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A
^4*a*b^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^
2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) - 4*(B*b
*x^2 - 5*B*a + 5*A*b)*sqrt(x))/b^2
```

Sympy [A] time = 34.8597, size = 382, normalized size = 1.5

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{\sqrt[4]{-1}A\sqrt[4]{a}b\left(\frac{1}{b}\right)^{\frac{9}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2}-\frac{\sqrt[4]{-1}A\sqrt[4]{a}b\left(\frac{1}{b}\right)^{\frac{9}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2}+\sqrt[4]{-1}A\sqrt[4]{a}b\left(\frac{1}{b}\right)^{\frac{9}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)+\frac{2A\sqrt{x}}{b}-\frac{\sqrt[4]{-1}Ba}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
b, Eq(a, 0)), ((-1)**(1/4)*A*a**(1/4)*b*(1/b)**(9/4)*log(-(-1)**(1/4)*a**(1
/4)*(1/b)**(1/4) + sqrt(x))/2 - (-1)**(1/4)*A*a**(1/4)*b*(1/b)**(9/4)*log((
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 + (-1)**(1/4)*A*a**(1/4)*b*(1
/b)**(9/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4))) + 2*A*sqrt(x)/
b - (-1)**(1/4)*B*a**(5/4)*(1/b)**(9/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1
/4) + sqrt(x))/2 + (-1)**(1/4)*B*a**(5/4)*(1/b)**(9/4)*log((-1)**(1/4)*a**(
1/4)*(1/b)**(1/4) + sqrt(x))/2 - (-1)**(1/4)*B*a**(5/4)*(1/b)**(9/4)*atan((
-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4))) - 2*B*a*sqrt(x)/b**2 + 2*B*x**(
5/2)/(5*b), True))
```

Giac [A] time = 1.17794, size = 355, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} Ba - \left(ab^3\right)^{\frac{1}{4}} Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} Ba - \left(ab^3\right)^{\frac{1}{4}} Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*((a*b^3)^(1/4)
*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*((a*b^3
)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^3 + 2/5*(B*b^4*x^(5/2) - 5*B*a*b^3*sqrt(x) + 5*A*b^4*sqrt(x))/b^5
```

3.370 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) -
 ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rubi [A] time = 0.178335, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) -
 ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{2Bx^{3/2}}{3b}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{3b}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{3b}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^{3/2}}+\frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^{3/2}}\\
&= \frac{2Bx^{3/2}}{3b}+\frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^2}+\frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{\right.}}{2b^2}\\
&= \frac{2Bx^{3/2}}{3b}+\frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(Ab-aB)\log\left(\sqrt{a}-\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0718496, size = 95, normalized size = 0.4

$$\frac{3(Ab-aB)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+(3aB-3Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2\sqrt[4]{-a}b^{3/4}Bx^{3/2}}{3\sqrt[4]{-a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(-a)^(1/4)*b^(3/4)*B*x^(3/2) + 3*(A*b - a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-
a)^(1/4)] + (-3*A*b + 3*a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(3*(-a)
^(1/4)*b^(7/4))
```

Maple [A] time = 0.008, size = 280, normalized size = 1.2

$$\frac{2B}{3b}x^{\frac{3}{2}}+\frac{\sqrt{2}A}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}A}{4b}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+1/2/b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*
x^(1/2)-1)+1/4/b/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2
)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/b/(1/
b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b^2/(1/b*a
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*a-1/4/b^2/(1/b*a)
^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b
*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a-1/2/b^2/(1/b*a)^(1/4)*2^(1/2)*B
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.94803, size = 1698, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b
 + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a
*b^5 + A^6*b^6)*x - (B^4*a^5*b^3 - 4*A*B^3*a^4*b^4 + 6*A^2*B^2*a^3*b^5 - 4*
A^3*B*a^2*b^6 + A^4*a*b^7)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b
^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7)))*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*
A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4) + (B^3*a^3*b^2 -
3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3
*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4))/(B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) + 3*b*(-(B^
4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7
))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B
*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2
 - A^3*b^3)*sqrt(x)) - 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(-a*b^5*(-(B^4*a^4 - 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^
3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.17424, size = 339, normalized size = 1.43

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 1/2*
sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(3/4
)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

3.371 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{3/4}b^{5/4}}$$

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((A*b - a*B)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A
*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(3/4)*b^(5/4))
```

Rubi [A] time = 0.176065, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```



```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((A*b - a*B)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A
*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(3/4)*b^(5/4))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}b} + \frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}b^{3/2}} + \frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right.}{2\sqrt{a}b^{3/2}}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}} +$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}} + \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\right.}{2\sqrt{2}a^{3/}}$$

Mathematica [A] time = 0.128736, size = 166, normalized size = 0.71

$$\frac{(aB-Ab)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.\right.}{2\sqrt{2}a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b + ((-(A*b) + a*B)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(3/4)*b^(5/4))
```

Maple [A] time = 0.007, size = 277, normalized size = 1.2

$$2\,\frac{B\sqrt{x}}{b}+\frac{\sqrt{2}A}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}A}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}\ldots}{2\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+1/2*(1/b*a)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)-1)+1/4*(1/b*a)^(1/4)/a*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*(1/b*a)
^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b*(1/b*a)^(1
/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-1/4/b*(1/b*a)^(1/4)*2
^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4
)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/b*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1
/2)/(1/b*a)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.924506, size = 1347, normalized size = 5.73

$$4\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^2\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}}+\left(B^2a^2-2\,ABab+A^2b^2\right)xa^2b^4\left(-\frac{B\ldots}{\ldots}\right.}}{B^4a^4-4\,AB\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^5))^(1/4)*arctan((sqrt(a^2*b^2*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5)) + (B^2*a^2 - 2*
A*B*a*b + A^2*b^2)*x)*a^2*b^4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(3/4) + (B*a^3*b^4 - A*a^2*b^5)*sqr
t(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)) + b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b
+ 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*
b)*sqrt(x)) - b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*
b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2
*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt
(x)) + 4*B*sqrt(x))/b
```

Sympy [A] time = 10.1895, size = 371, normalized size = 1.58

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} \\ -\frac{\sqrt[4]{-1}A\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{47}{4}}}+\frac{\sqrt[4]{-1}A\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{47}{4}}}-\frac{\sqrt[4]{-1}A\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}b^{12}\left(\frac{1}{b}\right)^{\frac{47}{4}}}+\frac{\sqrt[4]{-1}B\sqrt[4]{a}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{13}\left(\frac{1}{b}\right)^{\frac{47}{4}}}-\frac{\sqrt[4]{-1}B\sqrt[4]{a}\log\left(\sqrt[4]{-1}\right.}{2b^{13}\left(\frac{1}{b}\right)}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*sqrt(x) + 2*B*x**(5/2)/5)/a, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x
))/b, Eq(a, 0)), (-(-1)**(1/4)*A*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + s
qrt(x))/(2*a**(3/4)*b**12*(1/b)**(47/4)) + (-1)**(1/4)*A*log((-1)**(1/4)*a*
*(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)*b**12*(1/b)**(47/4)) - (-1)**(1/
4)*A*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(3/4)*b**12*(1/b
)**(47/4)) + (-1)**(1/4)*B*a**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
+ sqrt(x))/(2*b**13*(1/b)**(47/4)) - (-1)**(1/4)*B*a**(1/4)*log((-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**13*(1/b)**(47/4)) + (-1)**(1/4)*B*a
**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**13*(1/b)**(47
/4)) + 2*B*sqrt(x)/b, True))
```

Giac [A] time = 1.17921, size = 339, normalized size = 1.44

$$\frac{2B\sqrt{x}}{b}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{2ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/2*sq
rt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*((a*b^3)^(1/4)*
B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
```

3.372 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{5/4}b^{3/4}}$$

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) +
((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rubi [A] time = 0.180586, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```


```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) +
((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(5/4)*b^(3/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a\sqrt{b}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a\sqrt{b}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2ab} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right.}{2ab}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}} -$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\right.}{2\sqrt{2}a^{5}}$$

Mathematica [A] time = 0.0855865, size = 74, normalized size = 0.31

$$\frac{\frac{(aB-Ab)\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}b^{3/4}}-\frac{2A}{\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((-2*A)/Sqrt[x] + ((-(A*b) + a*B)*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + A
rcTanh[(a*b^(1/4)*Sqrt[x])/(-a)^(5/4)]))/((-a)^(1/4)*b^(3/4)))/a
```

Maple [A] time = 0.009, size = 277, normalized size = 1.2

$$-\frac{\sqrt{2}A}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{\sqrt{2}A}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{\sqrt{2}A}{4a}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/
a/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-1/4/a/(1/
b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/b/(1/b*a)^(1/4)*2^(1/2)*B
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2/b/(1/b*a)^(1/4)*2^(1/2)*B*arct
an(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/4/b/(1/b*a)^(1/4)*2^(1/2)*B*ln((x-(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2)))-2*A/a/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.913593, size = 1735, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)*x - (B^4*a^7*b - 4*A*B^3*a^6*b^2 + 6*A^2*B^2*a^5*b^3 - 4*A^3*B*a^4*b^4
 + A^4*a^3*b^5)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3)))*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a
^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4) + (B^3*a^4*b - 3*A*B^2*a
^3*b^2 + 3*A^2*B*a^2*b^3 - A^3*a*b^4)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b +
6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4))/(B^4*a^4 - 4
*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) - a*x*(-(B^4*a
^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3)
)^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*
b^2 - A^3*b^3)*sqrt(x)) + a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-(B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4)
- (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 4*A*sqrt(x
))/(a*x)
```

Sympy [A] time = 18.2556, size = 374, normalized size = 1.59

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}} - \frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} - \frac{2B}{\sqrt{x}}}{b} \\ -\frac{2A}{a\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} A b^{16} \left(\frac{1}{b}\right)^{\frac{63}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} A b^{16} \left(\frac{1}{b}\right)^{\frac{63}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} A b^{11} \left(\frac{1}{b}\right)^{\frac{43}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} B b^{15} \left(\frac{1}{b}\right.}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/sqrt(x) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), ((-2*A/(5*x**(5/2)) - 2*B/sqrt(
x))/b, Eq(a, 0)), (-2*A/(a*sqrt(x)) + (-1)**(3/4)*A*b**16*(1/b)**(63/4)*log
(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)) - (-1)**(3/4)*A
*b**16*(1/b)**(63/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a*
*(5/4)) - (-1)**(3/4)*A*b**11*(1/b)**(43/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1
/4)*(1/b)**(1/4)))/a**(5/4) - (-1)**(3/4)*B*b**15*(1/b)**(63/4)*log(-(-1)**
(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)) + (-1)**(3/4)*B*b**15*(
1/b)**(63/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4))
+ (-1)**(3/4)*B*b**10*(1/b)**(43/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b
)**(1/4)))/a**(1/4), True))
```

Giac [A] time = 1.16346, size = 339, normalized size = 1.44

$$-\frac{2\,A}{a\sqrt{x}}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1
/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)
^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a^2*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

3.373 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((A*b - a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4))
- ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rubi [A] time = 0.176935, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((A*b - a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4))
- ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)}dx = -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}dx}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^{3/2}}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x, \sqrt{x}\right)}{2a^{3/2}\sqrt{b}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x\right.}{2a^{3/2}\sqrt{b}}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{(Ab-aB)\log\left(\right.}{2}$$

Mathematica [A] time = 0.135059, size = 168, normalized size = 0.71

$$\frac{(Ab-aB)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}}{}\right.\right.}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

Maple [A] time = 0.009, size = 280, normalized size = 1.2

$$-\frac{2A}{3a}x^{-\frac{3}{2}} - \frac{\sqrt{2}Ab}{2a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) - \frac{\sqrt{2}Ab}{4a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -2/3*A/a/x^(3/2)-1/2/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)-1)*b-1/4/a^2*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b-
1/2/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b+1
/2/a*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/4/a*
(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/a*(1/b*a)^(1/4)*2^(1/2
```

```
)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.98895, size = 1347, normalized size = 5.68

$$-\frac{12\,ax^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^4\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}}+\left(B^2a^2-2\,ABab+A^2b^2\right)xa^5b\left(-\frac{B^4}{}\right.}}{B^4a^4-4\,AB^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*arctan((sqrt(a^4*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b)) + (B^2*a^2 - 2*A*
B*a*b + A^2*b^2)*x)*a^5*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(3/4) + (B*a^6*b - A*a^5*b^2)*sqrt(x)*(-(
B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7
*b))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
A^4*b^4)) + 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(
x)) - 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2
*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x)) +
 4*A*sqrt(x))/(a*x^2)
```

Sympy [A] time = 54.609, size = 364, normalized size = 1.54

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{a} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{\sqrt[4]{-1}A\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}A\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}A\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}B\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b\left(\frac{1}{b}\right)^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}B\log\left(\sqrt[4]{}\right.}{2a^{\frac{3}{4}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/(3*x**(3/2)))/b, Eq(a, 0)), (-2*A/(3*a*x**(3/2)) + (-1)**(1/4)*A*log(-(-1
)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)*(1/b)**(3/4)) - (-1)*
*(1/4)*A*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)*(1/b)
**(3/4)) + (-1)**(1/4)*A*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/
(a**(7/4)*(1/b)**(3/4)) - (-1)**(1/4)*B*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1
/4) + sqrt(x))/(2*a**(3/4)*b*(1/b)**(3/4)) + (-1)**(1/4)*B*log((-1)**(1/4)*
a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)*b*(1/b)**(3/4)) - (-1)**(1/4)*
B*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(3/4)*b*(1/b)**(3/4
)), True))
```

Giac [A] time = 1.16036, size = 339, normalized size = 1.43

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/2*sqrt(2)*((a*b^3)^(
1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*
sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1
/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 1/4*sqr
t(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^2*b) - 2/3*A/(a*x^(3/2))
```

3.374 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)} dx$

Optimal. Leaf size=255

$$\frac{2(Ab-aB)}{a^2\sqrt{x}}+\frac{\sqrt[4]{b}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-}{}$$

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(A*b - a*B)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4
)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/
4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rubi [A] time = 0.205779, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2(Ab-aB)}{a^2\sqrt{x}}+\frac{\sqrt[4]{b}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```


```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(A*b - a*B)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4
)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/
4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)}\,dx = -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(b(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(2b(Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\left(\sqrt{b}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{\left(\sqrt{b}(Ab-aB)\right)\text{Subst}\left(\int\right.}{a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}\right.}{2a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}} - \frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{a}+\right.}{2\sqrt{2}a^{9/4}}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} +$$

Mathematica [C] time = 0.014353, size = 46, normalized size = 0.18

$$-\frac{2\left(5x^2(aB-Ab)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)+aA\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*A + 5*(-(A*b) + a*B)*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a
)]))/(5*a^2*x^(5/2))
```

Maple [A] time = 0.011, size = 299, normalized size = 1.2

$$-\frac{2\,A}{5\,a}x^{-\frac{5}{2}}+2\,\frac{Ab}{a^2\sqrt{x}}-2\,\frac{B}{a\sqrt{x}}+\frac{\sqrt{2}Ab}{2\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}Ab}{2\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}Ab}{4\,a^2}\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a),x)
```

```
[Out] -2/5*A/a/x^(5/2)+2/a^2/x^(1/2)*A*b-2/a/x^(1/2)*B+1/2/a^2/(1/b*a)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b+1/2/a^2/(1/b*a)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b+1/4/a^2/(1/b*a)^(1/4)*2^(1/2
)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(
1/2)*2^(1/2)+(1/b*a)^(1/2)))*b-1/2/a/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/a/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b
```

```
*a)^(1/4)*x^(1/2)-1)-1/4/a/(1/b*a)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.02237, size = 1793, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^
3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*arctan((sqrt((B^6*a^6*b^2 - 6*A*B^5*a^5*b^3
 + 15*A^2*B^4*a^4*b^4 - 20*A^3*B^3*a^3*b^5 + 15*A^4*B^2*a^2*b^6 - 6*A^5*B*a
*b^7 + A^6*b^8)*x - (B^4*a^9*b - 4*A*B^3*a^8*b^2 + 6*A^2*B^2*a^7*b^3 - 4*A^
3*B*a^6*b^4 + A^4*a^5*b^5)*sqrt(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9))*a^2*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2
+ 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4) + (B^3*a^5*b - 3*
A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3 - A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4*b - 4*A*B
^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4))/(B^4*
a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)) - 5
*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4
 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2
 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) + 5*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3
*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-a^7*(-(
B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/
a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x
)) + 4*(5*(B*a - A*b)*x^2 + A*a)*sqrt(x))/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.17123, size = 362, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b^{2}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b^{2}} + \sqrt{2}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) - 1/2*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^
3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) -
1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 2/5*(5*B*a*x^2 - 5*A*b*x^2 + A*a)/(a
^2*x^(5/2))
```

3.375 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=310

$$-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{\sqrt{x}(5Ab-9aB)}{2b^3}+\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{\cdots}\right.}{8\sqrt{2}b^{1\cdots}}$$

```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10*a*b^2) +
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5*A*b - 9*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(
13/4)) + (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.243083, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{\sqrt{x}(5Ab-9aB)}{2b^3}+\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{\cdots}\right.}{8\sqrt{2}b^{1\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10*a*b^2) +
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5*A*b - 9*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(
13/4)) + (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} + \frac{\left(-\frac{5Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} + \frac{(5Ab-9aB)\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b^2}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} - \frac{(a(5Ab-9aB))\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^3}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} - \frac{(a(5Ab-9aB))\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \ldots\right.}{2b^3}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} - \frac{\left(\sqrt{a}(5Ab-9aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,d\ldots\right.}{4b^3}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} - \frac{\left(\sqrt{a}(5Ab-9aB)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\ldots}\right.}{8b^{7/2}}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} + \frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\ldots}\right.}{8\sqrt{2}b^{13/4}}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)} + \frac{\sqrt[4]{a}(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{13/4}} - \ldots$$

Mathematica [A] time = 0.429392, size = 385, normalized size = 1.24

$$-\frac{40a^2\sqrt[4]{b}B\sqrt{x}}{a+bx^2} - 45\sqrt{2}a^{5/4}B\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + 45\sqrt{2}a^{5/4}B\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + \frac{40aAb^{5/4}\sqrt{x}}{a+bx^2} - \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (160*A*b^(5/4)*Sqrt[x] - 320*a*b^(1/4)*B*Sqrt[x] + 32*b^(5/4)*B*x^(5/2) + (
40*a*A*b^(5/4)*Sqrt[x])/(a + b*x^2) - (40*a^2*b^(1/4)*B*Sqrt[x])/(a + b*x^2
) - 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)] + 25*Sqrt[2]*a^(1/4)*A*b*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 45*Sqrt[2]*a^(5/4)*B*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 25*Sqrt[2]*a^(1/4)*A*b*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*a^(5/4)*B*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(80*b^(13/4))
```

Maple [A] time = 0.014, size = 339, normalized size = 1.1

$$\frac{2B}{5b^2}x^{\frac{5}{2}} + 2\frac{A\sqrt{x}}{b^2} - 4\frac{Ba\sqrt{x}}{b^3} + \frac{Aa}{2b^2\left(bx^2+a\right)}\sqrt{x} - \frac{a^2B}{2b^3\left(bx^2+a\right)}\sqrt{x} - \frac{5\sqrt{2}A}{8b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) - \frac{5\sqrt{2}A}{8b^2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/5/b^2*B*x^(5/2)+2/b^2*A*x^(1/2)-4/b^3*B*a*x^(1/2)+1/2*a/b^2*x^(1/2)/(b*x^
2+a)*A-1/2*a^2/b^3*x^(1/2)/(b*x^2+a)*B-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*A*arct
an(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2)))+9/8*a/b^3*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1
/4)*x^(1/2)+1)+9/8*a/b^3*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)-1)+9/16*a/b^3*(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2
)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.00914, size = 1775, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2
*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*arctan((sqrt
(b^6*sqrt(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500
*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13) + (81*B^2*a^2 - 90*A*B*a*b + 25*A^2*b
^2)*x)*b^10*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4
500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4) + (9*B*a*b^10 - 5*A*b^11)*sq
rt(x)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^
3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4))/(6561*B^4*a^5 - 14580*A*B^3*a^4*b
 + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)) + 5*(b^4*x^
2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 45
00*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(b^3*(-(6561*B^4*a^5 - 145
80*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4
)/b^13)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) - 5*(b^4*x^2 + a*b^3)*(-(6561*B^4*
a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*
A^4*a*b^4)/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150
*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a
 - 5*A*b)*sqrt(x)) - 4*(4*B*b^2*x^4 - 45*B*a^2 + 25*A*a*b - 4*(9*B*a*b - 5*
A*b^2)*x^2)*sqrt(x))/(b^4*x^2 + a*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16991, size = 402, normalized size = 1.3

$$\frac{\sqrt{2}\left(9\left(ab^3\right)^{\frac{1}{4}}Ba-5\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(9\left(ab^3\right)^{\frac{1}{4}}Ba-5\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{}\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/8*sqrt(2)*(9*(a*b^3)
^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^
3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/16*s
qrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/b^4 - 1/2*(B*a^2*sqrt(x) - A*a*b*sqrt(x))/((b*x^2
 + a)*b^3) + 2/5*(B*b^8*x^(5/2) - 10*B*a*b^7*sqrt(x) + 5*A*b^8*sqrt(x))/b^1
0
```

3.376 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{x^{3/2}(3Ab-7aB)}{6ab^2}+\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}-\frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}-\frac{(3A}{}$$

```
[Out] -((3*A*b - 7*a*B)*x^(3/2))/(6*a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*
x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/
4)) - ((3*A*b - 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rubi [A] time = 0.21859, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}(3Ab-7aB)}{6ab^2}+\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}-\frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}-\frac{(3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((3*A*b - 7*a*B)*x^(3/2))/(6*a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*
x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/
4)) - ((3*A*b - 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{\left(-\frac{3Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} - \frac{(3Ab-7aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} + \frac{(3Ab-7aB)\,\mathrm{S}}{}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^3} + \frac{(3Ab-7}{}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB}{}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} - \frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} + \frac{(3Ab-7aB)\tan^{-1}\left(1-\right.}{4\sqrt{2}\sqrt[4]{a}b^{11/4}}$$

Mathematica [C] time = 0.192071, size = 136, normalized size = 0.47

$$\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3ab^2} + \frac{3(Ab-2aB)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+(6aB-3Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2\sqrt[4]{-a}b^{3/4}Bx^{3/2}}{3\sqrt[4]{-a}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*(-a)^(1/4)*b^(3/4)*B*x^(3/2) + 3*(A*b - 2*a*B)*ArcTan[(b^(1/4)*Sqrt[x])/
(-a)^(1/4)] + (-3*A*b + 6*a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(3*(-
a)^(1/4)*b^(11/4)) + (2*(-(A*b) + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/
4, -((b*x^2)/a)])/(3*a*b^2)
```

Maple [A] time = 0.013, size = 317, normalized size = 1.1

$$\frac{2B}{3b^2}x^{\frac{3}{2}} - \frac{A}{2b\left(bx^2+a\right)}x^{\frac{3}{2}} + \frac{Ba}{2b^2\left(bx^2+a\right)}x^{\frac{3}{2}} - \frac{7\sqrt{2}Ba}{16b^3}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{7\sqrt{}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/2/b*x^(3/2)/(b*x^2+a)*A+1/2/b^2*x^(3/2)/(b*x^2+a)*B*a-7
/16/b^3/(1/b*a)^(1/4)*2^(1/2)*B*a*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*
```

```
a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-7/8/b^3/(1/b*a)^
(1/4)*2^(1/2)*B*a*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-7/8/b^3/(1/b*a)^(
1/4)*2^(1/2)*B*a*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/16/b^2/(1/b*a)^(
1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*A*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.999223, size = 2141, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B
^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*arctan((sqrt((11
7649*B^6*a^6 - 302526*A*B^5*a^5*b + 324135*A^2*B^4*a^4*b^2 - 185220*A^3*B^3
*a^3*b^3 + 59535*A^4*B^2*a^2*b^4 - 10206*A^5*B*a*b^5 + 729*A^6*b^6)*x - (24
01*B^4*a^5*b^5 - 4116*A*B^3*a^4*b^6 + 2646*A^2*B^2*a^3*b^7 - 756*A^3*B*a^2*
b^8 + 81*A^4*a*b^9)*sqrt(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a
^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11)))*b^3*(-(2401*B^4*a^4 - 411
6*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^1
1))^(1/4) + (343*B^3*a^3*b^3 - 441*A*B^2*a^2*b^4 + 189*A^2*B*a*b^5 - 27*A^3
*b^6)*sqrt(x)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 7
56*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4))/(2401*B^4*a^4 - 4116*A*B^3*a^
3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)) - 3*(b^3*x^2 +
a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*
B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(2401*B^4*a^4 - 4116*A*B^
3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3
/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*sqrt(x
)) + 3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*
a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-a*b^8*(-(2401*
B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^
4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 -
 27*A^3*b^3)*sqrt(x)) - 4*(4*B*b*x^3 + (7*B*a - 3*A*b)*x)*sqrt(x))/(b^3*x^2
 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1643, size = 382, normalized size = 1.32

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2}+\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\,(bx^2+a)b^2}-\frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5}-\frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\mathrm{a}}{8\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 + 1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*b^2) - 1/8
*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) - 1/8*sqrt(2)*(7*(a*b^3
)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*
(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
 - 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sq
rt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
```

3.377 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{3/4}b^{9/4}}$$

```
[Out] -((A*b - 5*a*B)*Sqrt[x])/(2*a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x^
2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b
 - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(3/4)*b^(9/4))
```

Rubi [A] time = 0.208611, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 5*a*B)*Sqrt[x])/(2*a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x^
2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b
 - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(3/4)*b^(9/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{\left(-\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}b^2} + \frac{(Ab-5aB)\,\text{Su}}{}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}b^{5/2}} + \frac{(Ab-5a}{}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}} + \frac{(Ab-5aB)}{}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(Ab-5aB)\tan^{-1}\left(1+\right.}{4\sqrt{2}a^{3/4}b^{9/4}}$$

Mathematica [A] time = 0.388784, size = 353, normalized size = 1.24

$$\frac{\frac{2\sqrt{2}(5aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}} - \frac{\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} + \frac{\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} -}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (32*b^(1/4)*B*Sqrt[x] - (8*A*b^(5/4)*Sqrt[x])/(a + b*x^2) + (8*a*b^(1/4)*B*
Sqrt[x])/(a + b*x^2) + (2*Sqrt[2]*(-(A*b) + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*(A*b - 5*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*A*b*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + 5*Sqrt[2]*a^(1/4)*B*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (Sqrt[2]*A*b*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) - 5*Sqrt[2]*a^(
1/4)*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*b^(9
/4))
```

Maple [A] time = 0.012, size = 323, normalized size = 1.1

$$2\,\frac{B\sqrt{x}}{b^2} - \frac{A}{2\,b\left(bx^2+a\right)}\sqrt{x} + \frac{Ba}{2\,b^2\left(bx^2+a\right)}\sqrt{x} + \frac{\sqrt{2}A}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{\sqrt{2}A}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2*B/b^2*x^(1/2)-1/2/b*x^(1/2)/(b*x^2+a)*A+1/2/b^2*x^(1/2)/(b*x^2+a)*B*a+1/8
/b*(1/b*a)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/8/b*
(1/b*a)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/16/b*(1
/b*a)^(1/4)/a*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-5/8/b^2*(1/b*a)^(1/4)*2^(1
/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)
*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*B
*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2
)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.888488, size = 1594, normalized size = 5.61

$$4\left(b^3x^2+ab^2\right)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^4\sqrt{-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}}+(25\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2
*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*arctan((sqrt(a^2*b^4*sqrt
(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^9)) + (25*B^2*a^2 - 10*A*B*a*b + A^2*b^2)*x)*a^2*b^7*(-(625*
B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)
/(a^3*b^9))^(3/4) + (5*B*a^3*b^7 - A*a^2*b^8)*sqrt(x)*(-(625*B^4*a^4 - 500*
A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(3
/4))/(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3
+ A^4*b^4)) + (b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*
B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(a*b^2*(-(625*B
^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/
(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) - (b^3*x^2 + a*b^2)*(-(625*B^4*a^
4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*
b^9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b
^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) +
4*(4*B*b*x^2 + 5*B*a - A*b)*sqrt(x))/(b^3*x^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15135, size = 382, normalized size = 1.35

$$\frac{2\,B\sqrt{x}}{b^2} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)\right.}{2\left(\frac{a}{b}\right.}\right.}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/
8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/16*sqrt(2)*(5*(a*b^
3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a*b^3) + 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/2*(B*a*sqrt(x)
- A*b*sqrt(x))/((b*x^2 + a)*b^2)
```

3.378 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=261

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^{5/4}b^{7/4}}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
 + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(5/4)*b^(7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rubi [A] time = 0.181585, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^{5/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
 + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(5/4)*b^(7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{2ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4ab^{3/2}} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x,\right.}{4ab^{3/2}}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8ab^2} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\right.}{8ab^2}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}} - \frac{(Ab+3aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}}{8\sqrt{2}a^{5/4}b^{7/4}}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}} + \frac{(Ab+3aB}{}$$

Mathematica [C] time = 0.11935, size = 95, normalized size = 0.36

$$\frac{2x^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2b} + \frac{B\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b^(1/4)*Sqrt[x])/(-a)
^(5/4)]))/((-a)^(1/4)*b^(7/4)) + (2*(A*b - a*B)*x^(3/2)*Hypergeometric2F1[3
/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2*b)
```

Maple [A] time = 0.012, size = 305, normalized size = 1.2

$$\frac{Ab-Ba}{2\,ab\left(bx^2+a\right)}x^{\frac{3}{2}} + \frac{\sqrt{2}A}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}A}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}A}{16\,ab}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)+1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/16/a/b/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/
b*a)^(1/2)))+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
```

```
/2)-1)+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)
+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.997139, size = 1968, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*a - A*b)*x^(3/2) + 4*(a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B
^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*
arctan((sqrt((729*B^6*a^6 + 1458*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 + 540*A
^3*B^3*a^3*b^3 + 135*A^4*B^2*a^2*b^4 + 18*A^5*B*a*b^5 + A^6*b^6)*x - (81*B^
4*a^7*b^3 + 108*A*B^3*a^6*b^4 + 54*A^2*B^2*a^5*b^5 + 12*A^3*B*a^4*b^6 + A^4
*a^3*b^7)*sqrt(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)/(a^5*b^7)))*a*b^2*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4) - (27*B^3*a^4*
b^2 + 27*A*B^2*a^3*b^3 + 9*A^2*B*a^2*b^4 + A^3*a*b^5)*sqrt(x)*(-(81*B^4*a^4
 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^
7))^(1/4))/(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*
b^3 + A^4*b^4)) - (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(a^4*b^5*(-
(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b
^3)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2
*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-(8
1*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4
)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3
)*sqrt(x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21028, size = 369, normalized size = 1.41

$$-\frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{2\left(bx^2 + a\right)ab} + \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4} + \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*a*b) + 1/8*sqrt(2)*(3*(a*b^3)
^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2
*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 1/8*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^
3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^2*b^4) - 1/16*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*lo
g(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/16*sqrt(2)*(3*
(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^4)
```

3.379 $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{7/4}b^{5/4}}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4))
 - ((3*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.180924, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4))
 - ((3*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{2ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}b} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,\right.}{4a^{3/2}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}b^{3/2}} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}\right.}{8a^{3/2}b^{3/2}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} - \frac{(3Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} - \frac{(3Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}} - (3Ab+$$

Mathematica [A] time = 0.244844, size = 203, normalized size = 0.78

$$\frac{\frac{(aB+3Ab)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)\right)}{a^{7/4}\sqrt[4]{b}} - 32B\sqrt{}}{48b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (-32*B*Sqrt[x] + ((3*A*b + a*B)*(8*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a +
 b*x^2)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])))/(a^(7/4)*b^(1/4)))/(48*b*(a + b*x^2))
```

Maple [A] time = 0.012, size = 305, normalized size = 1.2

$$\frac{Ab-Ba}{2\,ab\left(bx^2+a\right)}\sqrt{x} + \frac{3\sqrt{2}A}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{3\sqrt{2}A}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x^2+a)+3/8/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
```

```
(1/b*a)^(1/2)))+3/8/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)+1)+1/8/a/b*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)-1)+1/16/a/b*(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1
/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/8/a/b
*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0147, size = 1559, normalized size = 5.97

$$4\left(ab^2x^2+a^2b\right)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^4b^2\sqrt{-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}}+\left(B^2a^2\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*arctan((sqrt(a^4*b^2*sqrt
(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^5)) + (B^2*a^2 + 6*A*B*a*b + 9*A^2*b^2)*x)*a^5*b^4*(-(B^4*a^4
+ 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*
b^5))^(3/4) - (B*a^6*b^4 + 3*A*a^5*b^5)*sqrt(x)*(-(B^4*a^4 + 12*A*B^3*a^3*b
 + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(3/4))/(B^
4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)
) + (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 12*A*
B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(
1/4) + (B*a + 3*A*b)*sqrt(x)) - (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a
^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*
log(-a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b
^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - 4*(B*a - A*b)*
sqrt(x))/(a*b^2*x^2 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21231, size = 369, normalized size = 1.41

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/8*sqrt(2)*((a*b^
3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3
*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b
^2) - 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a*b)
```

3.380 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4\sqrt{2}a^{9/4}b^{3/4}}$$

```
[Out] -(5*A*b - a*B)/(2*a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x^2))
+ ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*
a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b -
 a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(9/4)*b^(3/4))
```

Rubi [A] time = 0.211744, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4\sqrt{2}a^{9/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```


```
[Out] -(5*A*b - a*B)/(2*a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x^2))
+ ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*
a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b -
 a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(9/4)*b^(3/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} + \frac{\left(\frac{5Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{(5Ab-aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{(5Ab-aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} + \frac{(5Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^2\sqrt{b}} - \frac{(5Ab-aB)\,\text{Subst}\left(\right.}{4a}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{(5Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^2b} - \frac{(5Ab-aB)\,\text{Sul}}{}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{(5Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}} + \frac{(5Ab-aB)\log\left(\right.}{8}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)} + \frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{3/4}} - \frac{(5Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{4\sqrt{2}a^{9/4}b^{3/4}}$$

Mathematica [C] time = 0.212999, size = 117, normalized size = 0.4

$$\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)+3A\left((-a)^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-(-a)^{3/4}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-\frac{2a}{\sqrt{x}}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (3*A*((-2*a)/Sqrt[x] + (-a)^(3/4)*b^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/
4)] - (-a)^(3/4)*b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)]) + 2*(-(A*b)
 + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^3)
```

Maple [A] time = 0.013, size = 323, normalized size = 1.1

$$-2\,\frac{A}{a^2\sqrt{x}} - \frac{Ab}{2\,a^2\left(bx^2+a\right)}x^{\frac{3}{2}} + \frac{B}{2\,a\left(bx^2+a\right)}x^{\frac{3}{2}} - \frac{5\sqrt{2}A}{16\,a^2}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -2*A/a^2/x^(1/2)-1/2/a^2*x^(3/2)/(b*x^2+a)*A*b+1/2/a*x^(3/2)/(b*x^2+a)*B-5/
16/a^2/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-5/8/a^2/(1/b*a)^(1/
```

```
4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8/a^2/(1/b*a)^(1/4)*
2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+1/16/a/b/(1/b*a)^(1/4)*2^
(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(
1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/8/a/b/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.07271, size = 2026, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*arctan((sqrt((B^6*a^6 -
 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 - 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2
*a^2*b^4 - 18750*A^5*B*a*b^5 + 15625*A^6*b^6)*x - (B^4*a^9*b - 20*A*B^3*a^8
*b^2 + 150*A^2*B^2*a^7*b^3 - 500*A^3*B*a^6*b^4 + 625*A^4*a^5*b^5)*sqrt(-(B^
4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^
4)/(a^9*b^3)))*a^2*b*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 50
0*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4) + (B^3*a^5*b - 15*A*B^2*a^4*b
^2 + 75*A^2*B*a^3*b^3 - 125*A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4 - 20*A*B^3*a^3*
b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4))/
(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4
*b^4)) - (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*
b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(a^7*b^2*(-(B^4*a^
4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(
a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)
*sqrt(x)) + (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a
^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-(B^
4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^
4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*
b^3)*sqrt(x)) + 4*((B*a - 5*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^3 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21587, size = 375, normalized size = 1.3

$$\frac{Bax^2 - 5\,Abx^2 - 4\,Aa}{2\left(bx^{\frac{5}{2}} + a\sqrt{x}\right)a^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\mathrm{arcta}}{8\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*x^2 - 5*A*b*x^2 - 4*A*a)/((b*x^(5/2) + a*sqrt(x))*a^2) + 1/8*sqrt(
2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*
B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b^3) - 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^
(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/1
6*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

3.381 $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{(7Ab}{}$$

```
[Out] -(7*A*b - 3*a*B)/(6*a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x^2)
) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
- ((7*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi [A] time = 0.213974, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{(7Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(7*A*b - 3*a*B)/(6*a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x^2)
) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
- ((7*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}+\frac{\left(\frac{7Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}-\frac{(7Ab-3aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}-\frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}-\frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{5/2}}-\frac{(7Ab-3aB)\,\text{S}}{}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}-\frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{5/2}\sqrt{b}}-\frac{(7Ab-3}{}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}+\frac{(7Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3a}{}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}+\frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\tan^{-1}\left(1\right.}{4\sqrt{2}a^{11/4}\sqrt[4]{}}$$

Mathematica [A] time = 0.401288, size = 355, normalized size = 1.23

$$-\frac{24a^{3/4}Ab\sqrt{x}}{a+bx^2}-\frac{32a^{3/4}A}{x^{3/2}}+\frac{24a^{7/4}B\sqrt{x}}{a+bx^2}+21\sqrt{2}Ab^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-21\sqrt{2}Ab^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(3/4)*A)/x^(3/2) - (24*a^(3/4)*A*b*Sqrt[x])/(a + b*x^2) + (24*a^(7/
4)*B*Sqrt[x])/(a + b*x^2) + (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + 21*Sqrt[2]*A*b^(3/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (9*Sqrt[2]*a*B*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) - 21*Sqrt[2]*A
*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (9*Sq
rt[2]*a*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/
4))/(48*a^(11/4))
```

Maple [A] time = 0.014, size = 317, normalized size = 1.1

$$-\frac{2A}{3a^2}x^{-\frac{3}{2}}-\frac{Ab}{2a^2\left(bx^2+a\right)}\sqrt{x}+\frac{B}{2a\left(bx^2+a\right)}\sqrt{x}-\frac{7\sqrt{2}Ab}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{7\sqrt{2}Ab}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/3*A/a^2/x^(3/2)-1/2/a^2*x^(1/2)/(b*x^2+a)*A*b+1/2/a*x^(1/2)/(b*x^2+a)*B-
7/8/a^3*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b-7
/8/a^3*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b-7/
16/a^3*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b+3/8/a^2*(1/b*a)^(
1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/8/a^2*(1/b*a)^(1/4
)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/16/a^2*(1/b*a)^(1/4)*
2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.89974, size = 1700, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*
B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*arctan((sqrt
(a^6*sqrt(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b)) + (9*B^2*a^2 - 42*A*B*a*b + 49*A^2*b^2)*x
)*a^8*b*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B
*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(3/4) + (3*B*a^9*b - 7*A*a^8*b^2)*sqrt(x)*
(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 +
 2401*A^4*b^4)/(a^11*b))^(3/4))/(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b*x^4 + a^3*x^2)*(-(
81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b))^(1/4)*log(a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*
A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a
 - 7*A*b)*sqrt(x)) - 3*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*
b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)
*log(-a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) - 4*((3
*B*a - 7*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18449, size = 382, normalized size = 1.32

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}b}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{\right.}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{8a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/8*sqrt(2)*(3*(a*
b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a -
 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3
*b) - 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a^2) - 2/3*A/(a^2*x^(3/2))
```

3.382 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.}{8\sqrt{2}a^{13/4}}$$

```
[Out] -(9*A*b - 5*a*B)/(10*a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt[x]) + (A*
b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*b
- 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)
) + (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.236951, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.}{8\sqrt{2}a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -(9*A*b - 5*a*B)/(10*a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt[x]) + (A*
b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*b
- 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)
) + (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}+\frac{\left(\frac{9Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}-\frac{(9Ab-5aB)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}+\frac{(b(9Ab-5aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}+\frac{(b(9Ab-5aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}-\frac{\left(\sqrt{b}(9Ab-5aB)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}+\frac{(9Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}+\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}-\frac{\sqrt[4]{b}(9Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(9Ab}{}$$

Mathematica [C] time = 0.417775, size = 151, normalized size = 0.49

$$\frac{2bx^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4}+\frac{4Ab-2aB}{a^3\sqrt{x}}-\frac{2A}{5a^2x^{5/2}}+\frac{\sqrt[4]{b}(aB-2Ab)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{13/4}}+\frac{\sqrt[4]{b}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{-}}\right.}{(-a)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*A)/(5*a^2*x^(5/2)) + (4*A*b - 2*a*B)/(a^3*Sqrt[x]) + (b^(1/4)*(-2*A*b +
 a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(13/4) + (b^(1/4)*(2*A*b -
 a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(13/4) + (2*b*(A*b - a*B)
*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^4)
```

Maple [A] time = 0.016, size = 339, normalized size = 1.1

$$-\frac{2A}{5a^2}x^{-\frac{5}{2}}+4\frac{Ab}{a^3\sqrt{x}}-2\frac{B}{a^2\sqrt{x}}+\frac{Ab^2}{2a^3\left(bx^2+a\right)}x^{\frac{3}{2}}-\frac{Bb}{2a^2\left(bx^2+a\right)}x^{\frac{3}{2}}+\frac{9b\sqrt{2}A}{16a^3}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5*A/a^2/x^(5/2)+4/a^3/x^(1/2)*A*b-2/a^2/x^(1/2)*B+1/2/a^3*b^2*x^(3/2)/(b
*x^2+a)*A-1/2/a^2*b*x^(3/2)/(b*x^2+a)*B+9/16/a^3*b/(1/b*a)^(1/4)*2^(1/2)*A*
ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2)))+9/8/a^3*b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(
1/b*a)^(1/4)*x^(1/2)+1)+9/8/a^3*b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1
/b*a)^(1/4)*x^(1/2)-1)-5/16/a^2/(1/b*a)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2)))-5/8/a^2/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+
1)-5/8/a^2/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.03055, size = 2310, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 121
50*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*arctan((
sqrt((15625*B^6*a^6*b^2 - 168750*A*B^5*a^5*b^3 + 759375*A^2*B^4*a^4*b^4 - 1
822500*A^3*B^3*a^3*b^5 + 2460375*A^4*B^2*a^2*b^6 - 1771470*A^5*B*a*b^7 + 53
1441*A^6*b^8)*x - (625*B^4*a^11*b - 4500*A*B^3*a^10*b^2 + 12150*A^2*B^2*a^9
*b^3 - 14580*A^3*B*a^8*b^4 + 6561*A^4*a^7*b^5)*sqrt(-(625*B^4*a^4*b - 4500*
A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a
^13))*a^3*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 1
4580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4) + (125*B^3*a^6*b - 675*A*B^2*a
^5*b^2 + 1215*A^2*B*a^4*b^3 - 729*A^3*a^3*b^4)*sqrt(x)*(-(625*B^4*a^4*b - 4
500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^
5)/a^13)^(1/4))/(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3
 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)) - 5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*
a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 65
61*A^4*b^5)/a^13)^(1/4)*log(a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12
150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*
B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) +
5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B
^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-a^10*(-(625
*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4
 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^
2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) + 4*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2
+ 4*(5*B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17309, size = 409, normalized size = 1.32

$$-\frac{Babx^{\frac{3}{2}} - Ab^2x^{\frac{3}{2}}}{2\left(bx^2 + a\right)a^3} - \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba - 9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b^2} - \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba - 9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\text{arctar}}{8a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b*x^(3/2) - A*b^2*x^(3/2))/((b*x^2 + a)*a^3) - 1/8*sqrt(2)*(5*(a*
b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*B*a -
 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/
4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*s
qrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/5*(5*B*a*x^2 - 10*A*b*x^2 + A*a)/(a
^3*x^(5/2))
```

3.383 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=316

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\tan^{-1}(}{32\sqrt{2}a^{3/4}b}$$

```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*a*b*(a + b
*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2)) - (5*(A*b - 9*a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/
4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(3/4)*b^(13/4)) - (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*
B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*
a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.239701, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\tan^{-1}(}{32\sqrt{2}a^{3/4}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```


```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*a*b*(a + b
*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2)) - (5*(A*b - 9*a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/
4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(3/4)*b^(13/4)) - (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*
B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*
a^(3/4)*b^(13/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx=\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{\left(-\frac{Ab}{2}+\frac{9aB}{2}\right)\int\frac{x^{7/2}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{(5(Ab-9aB))\int\frac{x^{3/2}}{a+bx^2}\,dx}{32ab^2}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\int\frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\mathrm{Subst}\left(\int\frac{1}{a+bx^4}\,dx,x,\right.}{16b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\mathrm{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,\right.}{32\sqrt{a}b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{64\sqrt{a}b^{7/2}}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{5(Ab-9aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{64\sqrt{2}a^{3/4}b^{13/4}}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{5(Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+$$

Mathematica [A] time = 0.466303, size = 402, normalized size = 1.27

$$\frac{10\sqrt{2}(9aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{10\sqrt{2}(Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{5\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{5\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (256*b^(1/4)*B*Sqrt[x] + (32*a*A*b^(5/4)*Sqrt[x])/(a + b*x^2)^2 - (32*a^2*b
^(1/4)*B*Sqrt[x])/(a + b*x^2)^2 - (72*A*b^(5/4)*Sqrt[x])/(a + b*x^2) + (136
*a*b^(1/4)*B*Sqrt[x])/(a + b*x^2) + (10*Sqrt[2]*(-(A*b) + 9*a*B)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (10*Sqrt[2]*(A*b - 9*a*B)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (5*Sqrt[2]*A*b*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + 45*Sqrt[2]
*a^(1/4)*B*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (5*
Sqrt[2]*A*b*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(
3/4) - 45*Sqrt[2]*a^(1/4)*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(128*b^(13/4))
```

Maple [A] time = 0.015, size = 363, normalized size = 1.2

$$2\frac{B\sqrt{x}}{b^3} - \frac{9A}{16b\left(bx^2+a\right)^2}x^{\frac{5}{2}} + \frac{17Ba}{16b^2\left(bx^2+a\right)^2}x^{\frac{5}{2}} - \frac{5Aa}{16b^2\left(bx^2+a\right)^2}\sqrt{x} + \frac{13a^2B}{16b^3\left(bx^2+a\right)^2}\sqrt{x} + \frac{5\sqrt{2}A}{64b^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*B/b^3*x^(1/2)-9/16/b/(b*x^2+a)^2*x^(5/2)*A+17/16/b^2/(b*x^2+a)^2*x^(5/2)*
B*a-5/16/b^2/(b*x^2+a)^2*A*x^(1/2)*a+13/16/b^3/(b*x^2+a)^2*B*x^(1/2)*a^2+5/
64/b^2*(1/b*a)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+5/
64/b^2*(1/b*a)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+5/
128/b^2*(1/b*a)^(1/4)/a*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*
a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-45/64/b^3*(1/b*a
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-45/64/b^3*(1/b*a)
^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-45/128/b^3*(1/b*a)
^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b
*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.914373, size = 1787, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3
*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*arct
an((sqrt(a^2*b^6*sqrt(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b
^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13)) + (81*B^2*a^2 - 18*A*B*a*b + A^2
*b^2)*x)*a^2*b^10*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
- 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4) + (9*B*a^3*b^10 - A*a^2*b^11)
*sqrt(x)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*
B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4))/(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 48
6*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)) + 5*(b^5*x^4 + 2*a*b^4*x^2 +
 a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3
*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(5*a*b^3*(-(6561*B^4*a^4 - 2916*A*
B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/
4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561
*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^
4)/(a^3*b^13))^(1/4)*log(-5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*
```

```
A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a -
A*b)*sqrt(x)) + 4*(32*B*b^2*x^4 + 45*B*a^2 - 5*A*a*b + 9*(9*B*a*b - A*b^2)*
x^2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1913, size = 410, normalized size = 1.3

$$\frac{2\,B\sqrt{x}}{b^3} - \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4} - \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{\right.}\right.}{}\right.}{64\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^3 - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5
/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5/128*sqrt(2)*(9*(a
*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/(a*b^4) + 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) + 1/16*(17*B*a*
b*x^(5/2) - 9*A*b^2*x^(5/2) + 13*B*a^2*sqrt(x) - 5*A*a*b*sqrt(x))/((b*x^2 +
 a)^2*b^3)
```

3.384 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=293

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{5/4}b^{11/4}}$$

```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3/2))/(16*a
*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(5/4)*b^(11/4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.215887, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{5/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```


```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3/2))/(16*a
*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(5/4)*b^(11/4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}+\frac{\left(\frac{Ab}{2}+\frac{7aB}{2}\right)\int\frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}+\frac{(3(Ab+7aB))\int\frac{\sqrt{x}}{a+bx^2}\,dx}{32ab^2}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}+\frac{(3(Ab+7aB))\,\text{Subst}\left(\int\frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab^2}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}-\frac{(3(Ab+7aB))\,\text{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32ab^{5/2}}+\frac{(3(Ab+7aB))}{}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}+\frac{(3(Ab+7aB))\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64ab^3}+\frac{(3(Ab+7}{}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}+\frac{3(Ab+7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(Ab+7aB)}{}$$

$$=\frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)}-\frac{3(Ab+7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}}+\frac{3(Ab+7aB)\tan^{-1}\left(1+\right.}{32\sqrt{2}a^{5/4}b^{11/4}}$$

Mathematica [C] time = 0.213494, size = 137, normalized size = 0.47

$$\frac{2b^{3/4}x^{3/2}(Ab-2aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)+2b^{3/4}x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)+3(-a)^{7/4}B\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}}{(-a)}\right.\right.}{3a^2b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*(-a)^(7/4)*B*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b^(1/4)*
Sqrt[x])/(-a)^(5/4)]) + 2*b^(3/4)*(A*b - 2*a*B)*x^(3/2)*Hypergeometric2F1[3
/4, 2, 7/4, -((b*x^2)/a)] + 2*b^(3/4)*(-(A*b) + a*B)*x^(3/2)*Hypergeometric
2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^2*b^(11/4))
```

Maple [A] time = 0.015, size = 325, normalized size = 1.1

$$2\,\frac{1}{\left(bx^2+a\right)^2}\left(1/32\,\frac{(3\,Ab-11\,Ba)\,x^{7/2}}{ab}-1/32\,\frac{(Ab+7\,Ba)\,x^{3/2}}{b^2}\right)+\frac{3\,\sqrt{2}A}{64\,b^2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\,\sqrt{2}A}{64\,b^2a}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(3*A*b-11*B*a)/a/b*x^(7/2)-1/32*(A*b+7*B*a)/b^2*x^(3/2))/(b*x^2+a)^
2+3/64/b^2/a/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1
```

```
)+3/64/b^2/a/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1
)+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(
1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+21/64/b^3/(1
/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+21/64/b^3/(1/
b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+21/128/b^3/(1/
b*a)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 0.985894, size = 2226, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/64*(12*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)
*arctan((sqrt((117649*B^6*a^6 + 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
+ 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 + 42*A^5*B*a*b^5 + A^6*b^6)*x
- (2401*B^4*a^7*b^5 + 1372*A*B^3*a^6*b^6 + 294*A^2*B^2*a^5*b^7 + 28*A^3*B*a
^4*b^8 + A^4*a^3*b^9)*sqrt(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*
a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)))*a*b^3*(-(2401*B^4*a^4 + 13
72*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)
)^(1/4) - (343*B^3*a^4*b^3 + 147*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 + A^3*a*b
^6)*sqrt(x)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A
^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4))/(2401*B^4*a^4 + 1372*A*B^3*a^3*b +
 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)) - 3*(a*b^4*x^4 + 2*a^2*b^
3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(27*a^4*b^8*(-(2401*B^4*a^4
 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*
b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3
)*sqrt(x)) + 3*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372
*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^
(1/4)*log(-27*a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*
b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A
*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + 4*((11*B*a*b - 3*A*b^2)*x
^3 + (7*B*a^2 + A*a*b)*x)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1938, size = 396, normalized size = 1.35

$$-\frac{11 \, Babx^{\frac{7}{2}} - 3 \, Ab^{2}x^{\frac{7}{2}} + 7 \, Ba^{2}x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{16 \, {\left(bx^{2} + a\right)}^{2} ab^{2}} + \frac{3 \, \sqrt{2} {\left(7 \, \left(ab^{3}\right)^{\frac{3}{4}} Ba + \left(ab^{3}\right)^{\frac{3}{4}} Ab\right)} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 \, a^{2} b^{5}} + \frac{3 \, \sqrt{2} {\left(7 \, \left(ab^{3}\right)^{\frac{3}{4}}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*a*b*x^(7/2) - 3*A*b^2*x^(7/2) + 7*B*a^2*x^(3/2) + A*a*b*x^(3/2)
)/((b*x^2 + a)^2*a*b^2) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
2*b^5) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 3/128*s
qrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^2*b^5) + 3/128*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*
b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5)
```

3.385 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=298

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\tan^{-1}}{32\sqrt{2}a^{7/4}}$$

```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sqrt[x])/(16
*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.211935, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\tan^{-1}}{32\sqrt{2}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sqrt[x])/(16
*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}dx = \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} + \frac{\left(\frac{3Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} + \frac{(3Ab+5aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}dx}{32ab^2}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} + \frac{(3Ab+5aB)\,\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x, \sqrt{x}\right)}{16ab^2}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} + \frac{(3Ab+5aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{32a^{3/2}b^2} + \frac{(3Ab+5aB)}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} + \frac{(3Ab+5aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x, \sqrt{x}\right)}{64a^{3/2}b^{5/2}} + \frac{(3Ab+}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} - \frac{(3Ab+5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}} + \frac{(3Ab+5}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)} - \frac{(3Ab+5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{9/4}} + \frac{(3Ab+5aB)\tan^{-1}\left(\right.}{32\sqrt{2}a^{7/4}b}$$

Mathematica [A] time = 0.442023, size = 389, normalized size = 1.31

$$\frac{\frac{8Ab^{5/4}\sqrt{x}}{a^2+abx^2} - \frac{2\sqrt{2}(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\sqrt{2}(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}} - \frac{3\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}} + \frac{3\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{a^{7/4}}}{128b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-32*A*b^(5/4)*Sqrt[x])/(a + b*x^2)^2 + (32*a*b^(1/4)*B*Sqrt[x])/(a + b*x^
2)^2 - (72*b^(1/4)*B*Sqrt[x])/(a + b*x^2) + (8*A*b^(5/4)*Sqrt[x])/(a^2 + a*
b*x^2) - (2*Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/a^(7/4) + (2*Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*A*b*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) - (5*Sqrt[2]*B*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (3*Sqrt[2]*A*b*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (5*Sqrt[2]*B*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(128*b^(9/4))
```

Maple [A] time = 0.014, size = 334, normalized size = 1.1

$$2\frac{1}{\left(bx^2+a\right)^2}\left(1/32\frac{(Ab-9Ba)x^{5/2}}{ab} - 1/32\frac{(3Ab+5Ba)\sqrt{x}}{b^2}\right) + \frac{3\sqrt{2}A}{64ba^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{3\sqrt{2}A}{64ba^2}\sqrt[4]{\frac{a}{b}}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(A*b-9*B*a)/a/b*x^(5/2)-1/32*(3*A*b+5*B*a)/b^2*x^(1/2))/(b*x^2+a)^2
+3/64/b/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
+3/64/b/a^2*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+5/64/b^2/a*(1
/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+5/64/b^2/a*(1
/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+5/128/b^2/a*(
1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0764, size = 1817, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a
^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4
)*arctan((sqrt(a^4*b^4*sqrt(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2
*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9)) + (25*B^2*a^2 + 30*A*B*
a*b + 9*A^2*b^2)*x)*a^5*b^7*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^
2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4) - (5*B*a^6*b^7 +
 3*A*a^5*b^8)*sqrt(x)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*
b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4))/(625*B^4*a^4 + 1500*A
*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)) + (a*b^4
*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^
2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(a^2*b^2*
(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3
+ 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - (a*b^4*x^4 + 2*
a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2
*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(625*B
^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - 4*(5*B*a^2 + 3*A*a*b +
(9*B*a*b - A*b^2)*x^2)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16297, size = 402, normalized size = 1.35

$$\frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^3}+\frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/64*sqrt(2)*(5
*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/128*sqrt(2)*(5*(a*b^3)^(1/4
)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^3) - 1/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1/16*(9*B*a*b*x
^(5/2) - A*b^2*x^(5/2) + 5*B*a^2*sqrt(x) + 3*A*a*b*sqrt(x))/((b*x^2 + a)^2*
a*b^2)
```

3.386 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=298

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{9/4}b^{7/4}}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^(3/2))/(16
*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(9/4)*b^(7/4)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.216198, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^(3/2))/(16
*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(9/4)*b^(7/4)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{\left(\frac{5Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} - \frac{(5Ab+3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2b^{3/2}} + \frac{(5Ab+3aB)\,\text{Su}}{}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b^2} + \frac{(5Ab+3a}{}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}} - \frac{(5Ab+3aB)}{}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} - \frac{(5Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{7/4}} + \frac{(5Ab+3aB)\tan^{-1}\left(1+\right.}{32\sqrt{2}a^{9/4}b^{7/4}}$$

Mathematica [C] time = 0.0527267, size = 62, normalized size = 0.21

$$\frac{2x^{3/2}\left((Ab-aB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)+aB\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(a*B*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)] + (A*b - a*B)*
Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)]))/(3*a^3*b)
```

Maple [A] time = 0.014, size = 335, normalized size = 1.1

$$2\frac{1}{\left(bx^2+a\right)^2}\left(1/32\frac{(5Ab+3Ba)x^{7/2}}{a^2}+1/32\frac{(9Ab-Ba)x^{3/2}}{ab}\right)+\frac{5\sqrt{2}A}{64a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\sqrt{2}A}{64a^2b}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(5*A*b+3*B*a)/a^2*x^(7/2)+1/32*(9*A*b-B*a)/a/b*x^(3/2))/(b*x^2+a)^2
+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

```
+5/128/a^2/b/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/64/a/b^2/(1
/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/64/a/b^2/(1
/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/128/a/b^2/(
1/b*a)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0065, size = 2276, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^
3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/
4)*arctan((sqrt((729*B^6*a^6 + 7290*A*B^5*a^5*b + 30375*A^2*B^4*a^4*b^2 + 6
7500*A^3*B^3*a^3*b^3 + 84375*A^4*B^2*a^2*b^4 + 56250*A^5*B*a*b^5 + 15625*A^
6*b^6)*x - (81*B^4*a^9*b^3 + 540*A*B^3*a^8*b^4 + 1350*A^2*B^2*a^7*b^5 + 150
0*A^3*B*a^6*b^6 + 625*A^4*a^5*b^7)*sqrt(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7)))*a^2*b^2*(-
(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 6
25*A^4*b^4)/(a^9*b^7))^(1/4) - (27*B^3*a^5*b^2 + 135*A*B^2*a^4*b^3 + 225*A^
2*B*a^3*b^4 + 125*A^3*a^2*b^5)*sqrt(x)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4))/(81*
B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A
^4*b^4)) - (a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*
a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(
1/4)*log(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1
500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a
^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x)) + (a^2*b^3*x^4 + 2*a^3*b^2*x
^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A
^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(81*B^4*a^4 + 540
*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*
b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3
)*sqrt(x)) - 4*((3*B*a*b + 5*A*b^2)*x^3 - (B*a^2 - 9*A*a*b)*x)*sqrt(x))/(a^
2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17936, size = 402, normalized size = 1.35

$$\frac{3\,Babx^{\frac{7}{2}}+5\,Ab^{2}x^{\frac{7}{2}}-Ba^{2}x^{\frac{3}{2}}+9\,Aabx^{\frac{3}{2}}}{16\,\left(bx^{2}+a\right)^{2}a^{2}b}+\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba+5\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{4}}+\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*a*b*x^(7/2) + 5*A*b^2*x^(7/2) - B*a^2*x^(3/2) + 9*A*a*b*x^(3/2))/
((b*x^2 + a)^2*a^2*b) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
3*b^4) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) - 1/128
*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/128*sqrt(2)*(3*(a*b^3)^(3/4)*B*a +
 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^4)
```

3.387 $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\tan^{-1}}{32\sqrt{2}a^{11/4}}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt[x])/(16*a
^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.212456, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\tan^{-1}}{32\sqrt{2}a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt[x])/(16*a
^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{\left(\frac{7Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} + \frac{(3(7Ab+aB))\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} + \frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} + \frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{5/2}b} + \frac{(3(7Ab+a}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} + \frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{5/2}b^{3/2}} + \frac{(3(7Ab}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} - \frac{3(7Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}} + \frac{3(7Ab+a}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)} - \frac{3(7Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}b^{5/4}} + \frac{3(7Ab+aB)\tan^{-1}\left(1\right.}{32\sqrt{2}a^{11/4}b^{5}}$$

Mathematica [A] time = 0.279884, size = 230, normalized size = 0.78

$$\frac{\frac{(aB+7Ab)\left(7\left(a+bx^2\right)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)\right)}{a^{11/4}\sqrt[4]{b}}}{896b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] (-256*B*Sqrt[x] + ((7*A*b + a*B)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 7*(a + b*x^2
)*(8*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a + b*x^2)*(2*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))))/(a^(11/4)*b^(1/4)))/(8
96*b*(a + b*x^2)^2)
```

Maple [A] time = 0.015, size = 325, normalized size = 1.1

$$2\frac{1}{\left(bx^2+a\right)^2}\left(1/32\frac{(7Ab+Ba)x^{5/2}}{a^2}+1/32\frac{(11Ab-3Ba)\sqrt{x}}{ab}\right)+\frac{21\sqrt{2}A}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21\sqrt{2}A}{64a^3}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*A*b+B*a)/a^2*x^(5/2)+1/32*(11*A*b-3*B*a)/a/b*x^(1/2))/(b*x^2+a)^
2+21/64/a^3*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
+21/64/a^3*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+
21/128/a^3*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/64/a^2/b*(1/b
*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/64/a^2/b*(1/b
*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/128/a^2/b*(1/
b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.950624, size = 1782, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b
+ 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*
arctan((sqrt(a^6*b^2*sqrt(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2
+ 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)) + (B^2*a^2 + 14*A*B*a*b + 49
*A^2*b^2)*x)*a^8*b^4*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 13
72*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4) - (B*a^9*b^4 + 7*A*a^8*b^5
)*sqrt(x)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*
b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4))/(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*
B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b^3*x^4 + 2*a^3*b^
2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*log(3*a^3*b*(-(B^4*a^4 + 28*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)
)^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b
)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b^5))^(1/4)*log(-3*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 29
4*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*
(B*a + 7*A*b)*sqrt(x)) - 4*(3*B*a^2 - 11*A*a*b - (B*a*b + 7*A*b^2)*x^2)*sqr
t(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16115, size = 396, normalized size = 1.35

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/64*sqrt(2)*((a*
b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a
+ 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^2) - 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 1/16*(B*a*b*x^(5/2) + 7
*A*b^2*x^(5/2) - 3*B*a^2*sqrt(x) + 11*A*a*b*sqrt(x))/((b*x^2 + a)^2*a^2*b)
```

3.388 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{13/4}b^{3/4}}$$

```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x
^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) + (5*(9*A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4))
- (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[
2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1
3/4)*b^(3/4))
```

Rubi [A] time = 0.233806, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\right.}{32\sqrt{2}a^{13/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```


```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x
^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) + (5*(9*A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4))
- (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[
2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1
3/4)*b^(3/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{\left(\frac{9Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \cdots\right.}{16a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, \cdots\right.}{32a^3\sqrt{b}}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2} \cdots\right.}{64a^3b}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{5(9Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} \cdots\right.}{64\sqrt{2}a^{13/4}b^{3/4}}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}} - \cdots$$

Mathematica [C] time = 0.194449, size = 147, normalized size = 0.46

$$\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4} - \frac{2Abx^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4} - \frac{2A}{a^3\sqrt{x}} + \frac{A\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{13/4}} + \frac{aA\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*A)/(a^3*Sqrt[x]) + (A*b^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a
)^(13/4) + (a*A*b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4)
- (2*A*b*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^4) + (2
*(-(A*b) + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^
4)
```

Maple [A] time = 0.019, size = 363, normalized size = 1.1

$$-2\,\frac{A}{a^3\sqrt{x}} - \frac{13\,Ab^2}{16\,a^3\left(bx^2+a\right)^2}x^{\frac{7}{2}} + \frac{5\,Bb}{16\,a^2\left(bx^2+a\right)^2}x^{\frac{7}{2}} - \frac{17\,Ab}{16\,a^2\left(bx^2+a\right)^2}x^{\frac{3}{2}} + \frac{9\,B}{16\,a\left(bx^2+a\right)^2}x^{\frac{3}{2}} - \frac{45\,\sqrt{2}A}{128\,a^3}\ln\left(\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -2*A/a^3/x^(1/2)-13/16/a^3/(b*x^2+a)^2*x^(7/2)*A*b^2+5/16/a^2/(b*x^2+a)^2*x
^(7/2)*B*b-17/16/a^2/(b*x^2+a)^2*A*x^(3/2)*b+9/16/a/(b*x^2+a)^2*B*x^(3/2)-4
5/128/a^3/(1/b*a)^(1/4)*2^(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*
a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-45/64/a^3/(1/b*a
)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-45/64/a^3/(1/b*a)
^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+5/128/a^2/b/(1/b*a
)^(1/4)*2^(1/2)*B*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)
*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+5/64/a^2/b/(1/b*a)^(1/4)*2^(1/2)
*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.06905, size = 2244, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b +
486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*ar
ctan((sqrt((B^6*a^6 - 54*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 - 14580*A^3*B^3
*a^3*b^3 + 98415*A^4*B^2*a^2*b^4 - 354294*A^5*B*a*b^5 + 531441*A^6*b^6)*x -
 (B^4*a^11*b - 36*A*B^3*a^10*b^2 + 486*A^2*B^2*a^9*b^3 - 2916*A^3*B*a^8*b^4
 + 6561*A^4*a^7*b^5)*sqrt(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
- 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3)))*a^3*b*(-(B^4*a^4 - 36*A*B^3
*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))
^(1/4) + (B^3*a^6*b - 27*A*B^2*a^5*b^2 + 243*A^2*B*a^4*b^3 - 729*A^3*a^3*b^
4)*sqrt(x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a
*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4))/(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2
*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)) - 5*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*
B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(125*a^10*b^2*(-(B^4*a^4 - 36*
A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*
b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3
)*sqrt(x)) + 5*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^
3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1
/4)*log(-125*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2
916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2
*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) + 4*(5*(B*a*b - 9*A*b^2)*x
^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^
3 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22196, size = 405, normalized size = 1.26

$$-\frac{2A}{a^3\sqrt{x}} + \frac{5Babx^{\frac{7}{2}} - 13Ab^2x^{\frac{7}{2}} + 9Ba^2x^{\frac{3}{2}} - 17Aabx^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^3} + \frac{5\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^4b^3} + \frac{5\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/16*(5*B*a*b*x^(7/2) - 13*A*b^2*x^(7/2) + 9*B*a^2*x^(
3/2) - 17*A*a*b*x^(3/2))/((b*x^2 + a)^2*a^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*
B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(
3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^4*b^3) - 5/128*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) + 5/128*sqrt(2)*((a*b
^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^4*b^3)
```

3.389 $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=322

$$\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\right.}{64\sqrt{2}a^{15/4}}$$

```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(3/2)*(a +
b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x^2)) + (7*(11*A*b -
3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*
b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*
(11*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi [A] time = 0.236954, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\right.}{64\sqrt{2}a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```



```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(3/2)*(a +
b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x^2)) + (7*(11*A*b -
3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*
b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*
(11*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{\left(\frac{11Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{(7(11Ab-3aB))\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\,\text{Subst}\left(\int \frac{1}{a+b\ldots}\right.}{16a^3}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\,\text{Subst}\left(\int \frac{\sqrt{a}-\ldots}{a+\ldots}\right.}{32a^{7/2}}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\,\text{Subst}\left(\int \frac{\ldots}{\frac{\sqrt{a}}{\sqrt{b}}-\ldots}\right.}{64a^{7/2}\sqrt{b}}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{7(11Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\ldots}\right.}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{7(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}{32\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.431408, size = 400, normalized size = 1.24

$$\frac{-\frac{96a^{7/4}Ab\sqrt{x}}{\left(a+bx^2\right)^2} - \frac{360a^{3/4}Ab\sqrt{x}}{a+bx^2} - \frac{256a^{3/4}A}{x^{3/2}} + \frac{96a^{11/4}B\sqrt{x}}{\left(a+bx^2\right)^2} + \frac{168a^{7/4}B\sqrt{x}}{a+bx^2} + 231\sqrt{2}Ab^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 231\sqrt{2}A\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-256*a^(3/4)*A)/x^(3/2) - (96*a^(7/4)*A*b*Sqrt[x])/(a + b*x^2)^2 + (96*a^
(11/4)*B*Sqrt[x])/(a + b*x^2)^2 - (360*a^(3/4)*A*b*Sqrt[x])/(a + b*x^2) + (
168*a^(7/4)*B*Sqrt[x])/(a + b*x^2) + (42*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (42*Sqrt[2]*(11*A*b - 3*a*B
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + 231*Sqrt[2]*A*b^
(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (63*Sqrt
[2]*a*B*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)
 - 231*Sqrt[2]*A*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x] + (63*Sqrt[2]*a*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/b^(1/4))/(384*a^(15/4))
```

Maple [A] time = 0.017, size = 357, normalized size = 1.1

$$-\frac{2A}{3a^3}x^{-\frac{3}{2}}-\frac{15Ab^2}{16a^3\left(bx^2+a\right)^2}x^{\frac{5}{2}}+\frac{7Bb}{16a^2\left(bx^2+a\right)^2}x^{\frac{5}{2}}-\frac{19Ab}{16a^2\left(bx^2+a\right)^2}\sqrt{x}+\frac{11B}{16a\left(bx^2+a\right)^2}\sqrt{x}-\frac{77\sqrt{2}Ab}{64a^4}\sqrt[4]{\frac{a}{b}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/3*A/a^3/x^(3/2)-15/16/a^3/(b*x^2+a)^2*x^(5/2)*A*b^2+7/16/a^2/(b*x^2+a)^2
*x^(5/2)*B*b-19/16/a^2/(b*x^2+a)^2*A*x^(1/2)*b+11/16/a/(b*x^2+a)^2*B*x^(1/2
)-77/64/a^4*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
*b-77/64/a^4*(1/b*a)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1
)*b-77/128/a^4*(1/b*a)^(1/4)*2^(1/2)*A*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b+21/64/a^3
*(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+21/64/a^3*
(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+21/128/a^3*
(1/b*a)^(1/4)*2^(1/2)*B*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.11157, size = 1901, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B^3
*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)
)^(1/4)*arctan((sqrt(a^8*sqrt(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^
2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)) + (9*B^2*a^2 - 66*
A*B*a*b + 121*A^2*b^2)*x)*a^11*b*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^
2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4) + (3*B*a
^12*b - 11*A*a^11*b^2)*sqrt(x)*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*
B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4))/(81*B^4*a
^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^
4*b^4)) + 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B
^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*
b))^(1/4)*log(7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2
 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*
sqrt(x)) - 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*
B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15
```

```
*b))^(1/4)*log(-7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b
^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b
)*sqrt(x)) - 4*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*
a*b)*x^2)*sqrt(x))/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18111, size = 410, normalized size = 1.27

$$\frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^4b}+\frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)\right.}{2\left(\frac{a}{b}\right.}\right.}{64a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/64*sqrt(2)*(3*
(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*
B*a - 11*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^4*b) - 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/3*A/(a^3*x^(3/2))
 + 1/16*(7*B*a*b*x^(5/2) - 15*A*b^2*x^(5/2) + 11*B*a^2*sqrt(x) - 19*A*a*b*s
qrt(x))/((b*x^2 + a)^2*a^3)
```

3.390 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13$$

```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(16*a^4*Sqr
t[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13*A*b - 5*a*B)/(16*a^
2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) +
(9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4))
```

Rubi [A] time = 0.26158, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```


```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(16*a^4*Sqr
t[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13*A*b - 5*a*B)/(16*a^
2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) +
(9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{\left(\frac{13Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{(9(13Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{(9(13Ab-5aB))\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{(9b(13Ab-5a}{32a}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{(9b(13Ab-5a}{}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{\left(9\sqrt{b}(13Ab-5\right.}{}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{(9(13Ab-5aB}{}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{9\sqrt[4]{b}(13Ab-5a}{}$$

$$=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9\sqrt[4]{b}(13Ab-5a}{3}$$

Mathematica [C] time = 0.473284, size = 189, normalized size = 0.55

$$-\frac{2bx^{3/2}(aB-2Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5}+\frac{2bx^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5}+\frac{6Ab-2aB}{a^4\sqrt{x}}-\frac{2A}{5a^3x^{5/2}}+\frac{\sqrt[4]{b}(3Ab-aB)\tan}{(-a)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*A)/(5*a^3*x^(5/2)) + (6*A*b - 2*a*B)/(a^4*Sqrt[x]) + (b^(1/4)*(3*A*b -
a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4) + (b^(1/4)*(-3*A*b +
 a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4) - (2*b*(-2*A*b + a
*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^5) + (2*b*(A
*b - a*B)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^5)
```

Maple [A] time = 0.02, size = 381, normalized size = 1.1

$$-\frac{2A}{5a^3}x^{-\frac{5}{2}}+6\frac{Ab}{a^4\sqrt{x}}-2\frac{B}{a^3\sqrt{x}}+\frac{21b^3A}{16a^4\left(bx^2+a\right)^2}x^{\frac{7}{2}}-\frac{13b^2B}{16a^3\left(bx^2+a\right)^2}x^{\frac{7}{2}}+\frac{25Ab^2}{16a^3\left(bx^2+a\right)^2}x^{\frac{3}{2}}-\frac{17Bb}{16a^2\left(bx^2+a\right)^2}x^{\frac{3}{2}}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/5*A/a^3/x^(5/2)+6/a^4/x^(1/2)*A*b-2/a^3/x^(1/2)*B+21/16/a^4*b^3/(b*x^2+a
)^2*x^(7/2)*A-13/16/a^3*b^2/(b*x^2+a)^2*x^(7/2)*B+25/16/a^3*b^2/(b*x^2+a)^2
*A*x^(3/2)-17/16/a^2*b/(b*x^2+a)^2*B*x^(3/2)+117/128/a^4*b/(1/b*a)^(1/4)*2^
(1/2)*A*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+117/64/a^4*b/(1/b*a)^(1/4)*2^(1/2)*A*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+117/64/a^4*b/(1/b*a)^(1/4)*2^(1/2)*A*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-45/128/a^3/(1/b*a)^(1/4)*2^(1/2)*B*ln(
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))-45/64/a^3/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b
*a)^(1/4)*x^(1/2)+1)-45/64/a^3/(1/b*a)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(1/b*
a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.883611, size = 2507, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A
*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a
^17)^(1/4)*arctan((sqrt((15625*B^6*a^6*b^2 - 243750*A*B^5*a^5*b^3 + 1584375
*A^2*B^4*a^4*b^4 - 5492500*A^3*B^3*a^3*b^5 + 10710375*A^4*B^2*a^2*b^6 - 111
38790*A^5*B*a*b^7 + 4826809*A^6*b^8)*x - (625*B^4*a^13*b - 6500*A*B^3*a^12*
b^2 + 25350*A^2*B^2*a^11*b^3 - 43940*A^3*B*a^10*b^4 + 28561*A^4*a^9*b^5)*sq
rt(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3
*B*a*b^4 + 28561*A^4*b^5)/a^17))*a^4*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4) +
(125*B^3*a^7*b - 975*A*B^2*a^6*b^2 + 2535*A^2*B*a^5*b^3 - 2197*A^3*a^4*b^4)
*sqrt(x)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43
940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4))/(625*B^4*a^4*b - 6500*A*B^3*a
^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)) - 45*(
a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*lo
g(729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 -
43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A
*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 45*(a^4*b^2*x^7
+ 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*
B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*log(-729*a^13*
```

```
(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B
*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A*B^2*a^2*b^
2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 4*(45*(5*B*a*b^2 - 13*A*b^3
)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 32*A*a^3 + 32*(5*B*a^3 - 13*A*a^2
*b)*x^2)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21917, size = 440, normalized size = 1.28

$$-\frac{9\,\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{5}b^{2}}-\frac{9\,\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{64\,a^{5}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(
2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) - 9/64*sqrt(2)*
(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 9/128*sqrt(2)*(5*(a*b^3)^(
3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(a^5*b^2) - 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*
b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) - 1/16*(13*B
*a*b^2*x^(7/2) - 21*A*b^3*x^(7/2) + 17*B*a^2*b*x^(3/2) - 25*A*a*b^2*x^(3/2)
)/((b*x^2 + a)^2*a^4) - 2/5*(5*B*a*x^2 - 15*A*b*x^2 + A*a)/(a^4*x^(5/2))
```

3.391 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right) dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c + 2*a*d)*
x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rubi [A] time = 0.0327075, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c + 2*a*d)*
x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)dx &= \int \left(a^2cx^{7/2}+a(2bc+ad)x^{11/2}+b(bc+2ad)x^{15/2}+b^2dx^{19/2}\right)dx\\ &= \frac{2}{9}a^2cx^{9/2}+\frac{2}{13}a(2bc+ad)x^{13/2}+\frac{2}{17}b(bc+2ad)x^{17/2}+\frac{2}{21}b^2dx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0324698, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2c+819bx^4(2ad+bc)+1071ax^2(ad+2bc)+663b^2dx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*c + 1071*a*(2*b*c + a*d)*x^2 + 819*b*(b*c + 2*a*d)*x^4
 + 663*b^2*d*x^6))/13923
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{1326\,b^2dx^6+3276\,x^4abd+1638\,b^2cx^4+2142\,x^2a^2d+4284\,abcx^2+3094\,a^2c}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^2*d*x^6+1638*a*b*d*x^4+819*b^2*c*x^4+1071*a^2*d*x^2+
2142*a*b*c*x^2+1547*a^2*c)
```

Maxima [A] time = 1.04874, size = 69, normalized size = 1.1

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\left(b^2c+2\,abd\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}+\frac{2}{13}\left(2\,abc+a^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*(b^2*c + 2*a*b*d)*x^(17/2) + 2/9*a^2*c*x^(9/2) +
 2/13*(2*a*b*c + a^2*d)*x^(13/2)
```

Fricas [A] time = 0.755708, size = 146, normalized size = 2.32

$$\frac{2}{13923}\left(663\,b^2dx^{10}+819\left(b^2c+2\,abd\right)x^8+1547\,a^2cx^4+1071\left(2\,abc+a^2d\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^2*d*x^10 + 819*(b^2*c + 2*a*b*d)*x^8 + 1547*a^2*c*x^4 + 1071
*(2*a*b*c + a^2*d)*x^6)*sqrt(x)
```

Sympy [A] time = 20.147, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{9}{2}}}{9}+\frac{2a^2dx^{\frac{13}{2}}}{13}+\frac{4abcx^{\frac{13}{2}}}{13}+\frac{4abdx^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{17}{2}}}{17}+\frac{2b^2dx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(9/2)/9 + 2*a**2*d*x**(13/2)/13 + 4*a*b*c*x**(13/2)/13 + 4*a*b*
d*x**(17/2)/17 + 2*b**2*c*x**(17/2)/17 + 2*b**2*d*x**(21/2)/21
```

Giac [A] time = 1.15688, size = 72, normalized size = 1.14

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\,b^2cx^{\frac{17}{2}}+\frac{4}{17}\,abdx^{\frac{17}{2}}+\frac{4}{13}\,abcx^{\frac{13}{2}}+\frac{2}{13}\,a^2dx^{\frac{13}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*b^2*c*x^(17/2) + 4/17*a*b*d*x^(17/2) + 4/13*a*b*
c*x^(13/2) + 2/13*a^2*d*x^(13/2) + 2/9*a^2*c*x^(9/2)
```

3.392 $\int x^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rubi [A] time = 0.0292791, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2cx^{5/2}+a(2bc+ad)x^{9/2}+b(bc+2ad)x^{13/2}+b^2dx^{17/2}\right)\,dx\\ &= \frac{2}{7}a^2cx^{7/2}+\frac{2}{11}a(2bc+ad)x^{11/2}+\frac{2}{15}b(bc+2ad)x^{15/2}+\frac{2}{19}b^2dx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0282473, size = 63, normalized size = 1.

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{2310\,b^2dx^6+5852\,x^4abd+2926\,b^2cx^4+3990\,x^2a^2d+7980\,abcx^2+6270\,a^2c}{21945}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*b^2*d*x^6+2926*a*b*d*x^4+1463*b^2*c*x^4+1995*a^2*d*x^
2+3990*a*b*c*x^2+3135*a^2*c)
```

Maxima [A] time = 1.07416, size = 69, normalized size = 1.1

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}} + \frac{2}{15}\left(b^2c + 2\,abd\right)x^{\frac{15}{2}} + \frac{2}{7}\,a^2cx^{\frac{7}{2}} + \frac{2}{11}\left(2\,abc + a^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*(b^2*c + 2*a*b*d)*x^(15/2) + 2/7*a^2*c*x^(7/2) +
 2/11*(2*a*b*c + a^2*d)*x^(11/2)
```

Fricas [A] time = 0.819483, size = 147, normalized size = 2.33

$$\frac{2}{21945}\left(1155\,b^2dx^9 + 1463\left(b^2c + 2\,abd\right)x^7 + 3135\,a^2cx^3 + 1995\left(2\,abc + a^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d*x^9 + 1463*(b^2*c + 2*a*b*d)*x^7 + 3135*a^2*c*x^3 + 199
5*(2*a*b*c + a^2*d)*x^5)*sqrt(x)
```

Sympy [A] time = 11.211, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{7}{2}}}{7} + \frac{2a^2dx^{\frac{11}{2}}}{11} + \frac{4abcx^{\frac{11}{2}}}{11} + \frac{4abdx^{\frac{15}{2}}}{15} + \frac{2b^2cx^{\frac{15}{2}}}{15} + \frac{2b^2dx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(7/2)/7 + 2*a**2*d*x**(11/2)/11 + 4*a*b*c*x**(11/2)/11 + 4*a*b*
d*x**(15/2)/15 + 2*b**2*c*x**(15/2)/15 + 2*b**2*d*x**(19/2)/19
```

Giac [A] time = 1.14771, size = 72, normalized size = 1.14

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}} + \frac{2}{15}\,b^2cx^{\frac{15}{2}} + \frac{4}{15}\,abdx^{\frac{15}{2}} + \frac{4}{11}\,abcx^{\frac{11}{2}} + \frac{2}{11}\,a^2dx^{\frac{11}{2}} + \frac{2}{7}\,a^2cx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*b^2*c*x^(15/2) + 4/15*a*b*d*x^(15/2) + 4/11*a*b*
c*x^(11/2) + 2/11*a^2*d*x^(11/2) + 2/7*a^2*c*x^(7/2)
```

3.393 $\int x^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c + 2*a*d)*x^
(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rubi [A] time = 0.0298885, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c + 2*a*d)*x^
(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx &= \int \left(a^2cx^{3/2}+a(2bc+ad)x^{7/2}+b(bc+2ad)x^{11/2}+b^2dx^{15/2}\right) \, dx\\ &= \frac{2}{5}a^2cx^{5/2}+\frac{2}{9}a(2bc+ad)x^{9/2}+\frac{2}{13}b(bc+2ad)x^{13/2}+\frac{2}{17}b^2dx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0282503, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2c+765bx^4(2ad+bc)+1105ax^2(ad+2bc)+585b^2dx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*c + 1105*a*(2*b*c + a*d)*x^2 + 765*b*(b*c + 2*a*d)*x^4
 + 585*b^2*d*x^6))/9945
```

Maple [A] time = 0.004, size = 56, normalized size = 0.9

$$\frac{1170\,b^{2}d x^{6}+3060\,x^{4}abd+1530\,b^{2}c x^{4}+2210\,x^{2}a^{2}d+4420\,abc x^{2}+3978\,a^{2}c}{9945}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/9945*x^(5/2)*(585*b^2*d*x^6+1530*a*b*d*x^4+765*b^2*c*x^4+1105*a^2*d*x^2+2
210*a*b*c*x^2+1989*a^2*c)
```

Maxima [A] time = 1.04525, size = 69, normalized size = 1.1

$$\frac{2}{17}\,b^{2}d x^{\frac{17}{2}}+\frac{2}{13}\left(b^{2}c+2\,abd\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^{2}c x^{\frac{5}{2}}+\frac{2}{9}\left(2\,abc+a^{2}d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*(b^2*c + 2*a*b*d)*x^(13/2) + 2/5*a^2*c*x^(5/2) +
 2/9*(2*a*b*c + a^2*d)*x^(9/2)
```

Fricas [A] time = 0.744456, size = 143, normalized size = 2.27

$$\frac{2}{9945}\left(585\,b^{2}d x^{8}+765\left(b^{2}c+2\,abd\right)x^{6}+1989\,a^{2}c x^{2}+1105\left(2\,abc+a^{2}d\right)x^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d*x^8 + 765*(b^2*c + 2*a*b*d)*x^6 + 1989*a^2*c*x^2 + 1105*(
2*a*b*c + a^2*d)*x^4)*sqrt(x)
```

Sympy [A] time = 5.93053, size = 80, normalized size = 1.27

$$\frac{2a^{2}cx^{\frac{5}{2}}}{5}+\frac{2a^{2}dx^{\frac{9}{2}}}{9}+\frac{4abcx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{13}{2}}}{13}+\frac{2b^{2}cx^{\frac{13}{2}}}{13}+\frac{2b^{2}dx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(5/2)/5 + 2*a**2*d*x**(9/2)/9 + 4*a*b*c*x**(9/2)/9 + 4*a*b*d*x*
*(13/2)/13 + 2*b**2*c*x**(13/2)/13 + 2*b**2*d*x**(17/2)/17
```

Giac [A] time = 1.15493, size = 72, normalized size = 1.14

$$\frac{2}{17}\,b^{2}d x^{\frac{17}{2}}+\frac{2}{13}\,b^{2}c x^{\frac{13}{2}}+\frac{4}{13}\,abd x^{\frac{13}{2}}+\frac{4}{9}\,abc x^{\frac{9}{2}}+\frac{2}{9}\,a^{2}d x^{\frac{9}{2}}+\frac{2}{5}\,a^{2}c x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*b^2*c*x^(13/2) + 4/13*a*b*d*x^(13/2) + 4/9*a*b*c
*x^(9/2) + 2/9*a^2*d*x^(9/2) + 2/5*a^2*c*x^(5/2)
```

3.394 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c + 2*a*d)*x^
(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rubi [A] time = 0.0285362, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c + 2*a*d)*x^
(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2c\sqrt{x}+a(2bc+ad)x^{5/2}+b(bc+2ad)x^{9/2}+b^2dx^{13/2}\right)\,dx\\ &= \frac{2}{3}a^2cx^{3/2}+\frac{2}{7}a(2bc+ad)x^{7/2}+\frac{2}{11}b(bc+2ad)x^{11/2}+\frac{2}{15}b^2dx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0295099, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2c+105bx^4(2ad+bc)+165ax^2(ad+2bc)+77b^2dx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*c + 165*a*(2*b*c + a*d)*x^2 + 105*b*(b*c + 2*a*d)*x^4 +
 77*b^2*d*x^6))/1155
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{154\,b^2dx^6 + 420\,x^4abd + 210\,b^2cx^4 + 330\,x^2a^2d + 660\,abcx^2 + 770\,a^2c}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^2*d*x^6+210*a*b*d*x^4+105*b^2*c*x^4+165*a^2*d*x^2+330*
a*b*c*x^2+385*a^2*c)
```

Maxima [A] time = 1.0643, size = 69, normalized size = 1.1

$$\frac{2}{15}\,b^2dx^{\frac{15}{2}} + \frac{2}{11}\left(b^2c + 2\,abd\right)x^{\frac{11}{2}} + \frac{2}{3}\,a^2cx^{\frac{3}{2}} + \frac{2}{7}\left(2\,abc + a^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*(b^2*c + 2*a*b*d)*x^(11/2) + 2/3*a^2*c*x^(3/2) +
 2/7*(2*a*b*c + a^2*d)*x^(7/2)
```

Fricas [A] time = 0.707551, size = 136, normalized size = 2.16

$$\frac{2}{1155}\left(77\,b^2dx^7 + 105\left(b^2c + 2\,abd\right)x^5 + 385\,a^2cx + 165\left(2\,abc + a^2d\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d*x^7 + 105*(b^2*c + 2*a*b*d)*x^5 + 385*a^2*c*x + 165*(2*a*b
*c + a^2*d)*x^3)*sqrt(x)
```

Sympy [A] time = 2.49226, size = 66, normalized size = 1.05

$$\frac{2a^2cx^{\frac{3}{2}}}{3} + \frac{2b^2dx^{\frac{15}{2}}}{15} + \frac{2x^{\frac{11}{2}}\left(2abd + b^2c\right)}{11} + \frac{2x^{\frac{7}{2}}\left(a^2d + 2abc\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

```
[Out] 2*a**2*c*x**(3/2)/3 + 2*b**2*d*x**(15/2)/15 + 2*x**(11/2)*(2*a*b*d + b**2*c
)/11 + 2*x**(7/2)*(a**2*d + 2*a*b*c)/7
```

Giac [A] time = 1.17171, size = 72, normalized size = 1.14

$$\frac{2}{15}\,b^2dx^{\frac{15}{2}} + \frac{2}{11}\,b^2cx^{\frac{11}{2}} + \frac{4}{11}\,abdx^{\frac{11}{2}} + \frac{4}{7}\,abcx^{\frac{7}{2}} + \frac{2}{7}\,a^2dx^{\frac{7}{2}} + \frac{2}{3}\,a^2cx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*b^2*c*x^(11/2) + 4/11*a*b*d*x^(11/2) + 4/7*a*b*c
*x^(7/2) + 2/7*a^2*d*x^(7/2) + 2/3*a^2*c*x^(3/2)
```

3.395 $\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a*d)*x^(9/2
))/9 + (2*b^2*d*x^(13/2))/13
```

Rubi [A] time = 0.0290469, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a*d)*x^(9/2
))/9 + (2*b^2*d*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c}{\sqrt{x}}+a(2bc+ad)x^{3/2}+b(bc+2ad)x^{7/2}+b^2dx^{11/2}\right)dx\\ &= 2a^2c\sqrt{x}+\frac{2}{5}a(2bc+ad)x^{5/2}+\frac{2}{9}b(bc+2ad)x^{9/2}+\frac{2}{13}b^2dx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0309814, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2c+65bx^4(2ad+bc)+117ax^2(ad+2bc)+45b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*c + 117*a*(2*b*c + a*d)*x^2 + 65*b*(b*c + 2*a*d)*x^4 +
45*b^2*d*x^6))/585
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{90\, b^2 d x^6 + 260\, x^4 abd + 130\, b^2 c x^4 + 234\, x^2 a^2 d + 468\, abc x^2 + 1170\, a^2 c}{585}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*b^2*d*x^6+130*a*b*d*x^4+65*b^2*c*x^4+117*a^2*d*x^2+234*a*
b*c*x^2+585*a^2*c)
```

Maxima [A] time = 1.1155, size = 69, normalized size = 1.13

$$\frac{2}{13}\, b^2 d x^{\frac{13}{2}} + \frac{2}{9}\left(b^2 c + 2\, abd\right)x^{\frac{9}{2}} + 2\, a^2 c\sqrt{x} + \frac{2}{5}\left(2\, abc + a^2 d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*(b^2*c + 2*a*b*d)*x^(9/2) + 2*a^2*c*sqrt(x) + 2/5
*(2*a*b*c + a^2*d)*x^(5/2)
```

Fricas [A] time = 0.908064, size = 131, normalized size = 2.15

$$\frac{2}{585}\left(45\, b^2 d x^6 + 65\left(b^2 c + 2\, abd\right)x^4 + 585\, a^2 c + 117\left(2\, abc + a^2 d\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 65*(b^2*c + 2*a*b*d)*x^4 + 585*a^2*c + 117*(2*a*b*c +
 a^2*d)*x^2)*sqrt(x)
```

Sympy [A] time = 2.14039, size = 78, normalized size = 1.28

$$2a^2 c\sqrt{x} + \frac{2a^2 d x^{\frac{5}{2}}}{5} + \frac{4abc x^{\frac{5}{2}}}{5} + \frac{4abd x^{\frac{9}{2}}}{9} + \frac{2b^2 c x^{\frac{9}{2}}}{9} + \frac{2b^2 d x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*a**2*c*sqrt(x) + 2*a**2*d*x**(5/2)/5 + 4*a*b*c*x**(5/2)/5 + 4*a*b*d*x**(9
/2)/9 + 2*b**2*c*x**(9/2)/9 + 2*b**2*d*x**(13/2)/13
```

Giac [A] time = 1.16651, size = 72, normalized size = 1.18

$$\frac{2}{13}\, b^2 d x^{\frac{13}{2}} + \frac{2}{9}\, b^2 c x^{\frac{9}{2}} + \frac{4}{9}\, abd x^{\frac{9}{2}} + \frac{4}{5}\, abc x^{\frac{5}{2}} + \frac{2}{5}\, a^2 d x^{\frac{5}{2}} + 2\, a^2 c\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*b^2*c*x^(9/2) + 4/9*a*b*d*x^(9/2) + 4/5*a*b*c*x^(
5/2) + 2/5*a^2*d*x^(5/2) + 2*a^2*c*sqrt(x)
```

3.396 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c + 2*a*d)*x^(
7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rubi [A] time = 0.030651, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c + 2*a*d)*x^(
7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^2c}{x^{3/2}}+a(2bc+ad)\sqrt{x}+b(bc+2ad)x^{5/2}+b^2dx^{9/2}\right)dx\\ &= -\frac{2a^2c}{\sqrt{x}}+\frac{2}{3}a(2bc+ad)x^{3/2}+\frac{2}{7}b(bc+2ad)x^{7/2}+\frac{2}{11}b^2dx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0212652, size = 60, normalized size = 0.98

$$\frac{-154a^2\left(3c-dx^2\right)+44abx^2\left(7c+3dx^2\right)+6b^2x^4\left(11c+7dx^2\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-154*a^2*(3*c - d*x^2) + 44*a*b*x^2*(7*c + 3*d*x^2) + 6*b^2*x^4*(11*c + 7*
d*x^2))/(231*Sqrt[x])
```

Maple [A] time = 0.003, size = 56, normalized size = 0.9

$$-\frac{-42\,b^2dx^6-132\,x^4abd-66\,b^2cx^4-154\,x^2a^2d-308\,abcx^2+462\,a^2c}{231}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x)
```

```
[Out] -2/231*(-21*b^2*d*x^6-66*a*b*d*x^4-33*b^2*c*x^4-77*a^2*d*x^2-154*a*b*c*x^2+
231*a^2*c)/x^(1/2)
```

Maxima [A] time = 1.07836, size = 69, normalized size = 1.13

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}}+\frac{2}{7}\left(b^2c+2\,abd\right)x^{\frac{7}{2}}-\frac{2\,a^2c}{\sqrt{x}}+\frac{2}{3}\left(2\,abc+a^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*(b^2*c + 2*a*b*d)*x^(7/2) - 2*a^2*c/sqrt(x) + 2/3
*(2*a*b*c + a^2*d)*x^(3/2)
```

Fricas [A] time = 0.628379, size = 130, normalized size = 2.13

$$\frac{2\left(21\,b^2dx^6+33\left(b^2c+2\,abd\right)x^4-231\,a^2c+77\left(2\,abc+a^2d\right)x^2\right)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*d*x^6 + 33*(b^2*c + 2*a*b*d)*x^4 - 231*a^2*c + 77*(2*a*b*c +
a^2*d)*x^2)/sqrt(x)
```

Sympy [A] time = 2.46512, size = 78, normalized size = 1.28

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2b^2dx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

```
[Out] -2*a**2*c/sqrt(x) + 2*a**2*d*x**(3/2)/3 + 4*a*b*c*x**(3/2)/3 + 4*a*b*d*x**(
7/2)/7 + 2*b**2*c*x**(7/2)/7 + 2*b**2*d*x**(11/2)/11
```

Giac [A] time = 1.17261, size = 72, normalized size = 1.18

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}}+\frac{2}{7}\,b^2cx^{\frac{7}{2}}+\frac{4}{7}\,abdx^{\frac{7}{2}}+\frac{4}{3}\,abcx^{\frac{3}{2}}+\frac{2}{3}\,a^2dx^{\frac{3}{2}}-\frac{2\,a^2c}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*b^2*c*x^(7/2) + 4/7*a*b*d*x^(7/2) + 4/3*a*b*c*x^(
3/2) + 2/3*a^2*d*x^(3/2) - 2*a^2*c/sqrt(x)
```

3.397 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c + 2*a*d)*x^(
5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rubi [A] time = 0.0288909, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c + 2*a*d)*x^(
5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx &= \int \left(\frac{a^2c}{x^{5/2}}+\frac{a(2bc+ad)}{\sqrt{x}}+b(bc+2ad)x^{3/2}+b^2dx^{7/2}\right)dx\\ &= -\frac{2a^2c}{3x^{3/2}}+2a(2bc+ad)\sqrt{x}+\frac{2}{5}b(bc+2ad)x^{5/2}+\frac{2}{9}b^2dx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0200688, size = 57, normalized size = 0.93

$$\frac{-30a^2\left(c-3dx^2\right)+36abx^2\left(5c+dx^2\right)+2b^2x^4\left(9c+5dx^2\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-30*a^2*(c - 3*d*x^2) + 36*a*b*x^2*(5*c + d*x^2) + 2*b^2*x^4*(9*c + 5*d*x^
2))/(45*x^(3/2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$-\frac{-10\,b^{2}d x^{6}-36\,x^{4}abd-18\,b^{2}c x^{4}-90\,x^{2}a^{2}d-180\,abc x^{2}+30\,a^{2}c}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x)
```

```
[Out] -2/45*(-5*b^2*d*x^6-18*a*b*d*x^4-9*b^2*c*x^4-45*a^2*d*x^2-90*a*b*c*x^2+15*a
^2*c)/x^(3/2)
```

Maxima [A] time = 1.0184, size = 69, normalized size = 1.13

$$\frac{2}{9}\,b^{2}d x^{\frac{9}{2}}+\frac{2}{5}\left(b^{2}c+2\,abd\right)x^{\frac{5}{2}}-\frac{2\,a^{2}c}{3\,x^{\frac{3}{2}}}+2\left(2\,abc+a^{2}d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*(b^2*c + 2*a*b*d)*x^(5/2) - 2/3*a^2*c/x^(3/2) + 2*(
2*a*b*c + a^2*d)*sqrt(x)
```

Fricas [A] time = 0.870297, size = 124, normalized size = 2.03

$$\frac{2\left(5\,b^{2}d x^{6}+9\left(b^{2}c+2\,abd\right)x^{4}-15\,a^{2}c+45\left(2\,abc+a^{2}d\right)x^{2}\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*d*x^6 + 9*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 45*(2*a*b*c + a^2*
d)*x^2)/x^(3/2)
```

Sympy [A] time = 2.97023, size = 76, normalized size = 1.25

$$-\frac{2a^{2}c}{3x^{\frac{3}{2}}}+2a^{2}d\sqrt{x}+4abc\sqrt{x}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^{2}cx^{\frac{5}{2}}}{5}+\frac{2b^{2}dx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

```
[Out] -2*a**2*c/(3*x**(3/2)) + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 4*a*b*d*x**(5
/2)/5 + 2*b**2*c*x**(5/2)/5 + 2*b**2*d*x**(9/2)/9
```

Giac [A] time = 1.17291, size = 72, normalized size = 1.18

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}} + \frac{2}{5}\,b^2cx^{\frac{5}{2}} + \frac{4}{5}\,abdx^{\frac{5}{2}} + 4\,abc\sqrt{x} + 2\,a^2d\sqrt{x} - \frac{2\,a^2c}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*b^2*c*x^(5/2) + 4/5*a*b*d*x^(5/2) + 4*a*b*c*sqrt(x)
 + 2*a^2*d*sqrt(x) - 2/3*a^2*c/x^(3/2)
```

3.398 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c + 2*a*d)*x
^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rubi [A] time = 0.0294659, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c + 2*a*d)*x
^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2c}{x^{7/2}}+\frac{a(2bc+ad)}{x^{3/2}}+b(bc+2ad)\sqrt{x}+b^2dx^{5/2}\right)dx\\ &= -\frac{2a^2c}{5x^{5/2}}-\frac{2a(2bc+ad)}{\sqrt{x}}+\frac{2}{3}b(bc+2ad)x^{3/2}+\frac{2}{7}b^2dx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0163813, size = 57, normalized size = 0.93

$$\frac{-42a^2(c+5dx^2)+140abx^2(dx^2-3c)+10b^2x^4(7c+3dx^2)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*c + d*x^2) + 10*b^2*x^4*(7*c + 3*d*x^2) - 42*a^2*(c + 5*d*
x^2))/(105*x^(5/2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$-\frac{-30\,b^2dx^6-140\,x^4abd-70\,b^2cx^4+210\,x^2a^2d+420\,abcx^2+42\,a^2c}{105}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x)
```

```
[Out] -2/105*(-15*b^2*d*x^6-70*a*b*d*x^4-35*b^2*c*x^4+105*a^2*d*x^2+210*a*b*c*x^2
+21*a^2*c)/x^(5/2)
```

Maxima [A] time = 1.11314, size = 72, normalized size = 1.18

$$\frac{2}{7}\,b^2dx^{\frac{7}{2}}+\frac{2}{3}\left(b^2c+2\,abd\right)x^{\frac{3}{2}}-\frac{2\left(a^2c+5\left(2\,abc+a^2d\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*(b^2*c + 2*a*b*d)*x^(3/2) - 2/5*(a^2*c + 5*(2*a*b*c
 + a^2*d)*x^2)/x^(5/2)
```

Fricas [A] time = 0.861375, size = 130, normalized size = 2.13

$$\frac{2\left(15\,b^2dx^6+35\left(b^2c+2\,abd\right)x^4-21\,a^2c-105\left(2\,abc+a^2d\right)x^2\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d*x^6 + 35*(b^2*c + 2*a*b*d)*x^4 - 21*a^2*c - 105*(2*a*b*c +
a^2*d)*x^2)/x^(5/2)
```

Sympy [A] time = 4.1883, size = 76, normalized size = 1.25

$$-\frac{2a^2c}{5x^{\frac{5}{2}}}-\frac{2a^2d}{\sqrt{x}}-\frac{4abc}{\sqrt{x}}+\frac{4abdx^{\frac{3}{2}}}{3}+\frac{2b^2cx^{\frac{3}{2}}}{3}+\frac{2b^2dx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

```
[Out] -2*a**2*c/(5*x**(5/2)) - 2*a**2*d/sqrt(x) - 4*a*b*c/sqrt(x) + 4*a*b*d*x**(3
/2)/3 + 2*b**2*c*x**(3/2)/3 + 2*b**2*d*x**(7/2)/7
```

Giac [A] time = 1.13856, size = 74, normalized size = 1.21

$$\frac{2}{7}\,b^{2}dx^{\frac{7}{2}} + \frac{2}{3}\,b^{2}cx^{\frac{3}{2}} + \frac{4}{3}\,abdx^{\frac{3}{2}} - \frac{2\left(10\,abcx^{2} + 5\,a^{2}dx^{2} + a^{2}c\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*b^2*c*x^(3/2) + 4/3*a*b*d*x^(3/2) - 2/5*(10*a*b*c*x
^2 + 5*a^2*d*x^2 + a^2*c)/x^(5/2)
```

3.399 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

Rubi [A] time = 0.0494179, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\int\left(a^2c^2x^{7/2}+2ac(bc+ad)x^{11/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+2bd(bc+ad)x^{19/2}+\right.$$
$$=\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{13}ac(bc+ad)x^{13/2}+\frac{2}{17}\left(b^2c^2+4abcd+a^2d^2\right)x^{17/2}+\frac{4}{21}bd(bc+ad)x^2$$

Mathematica [A] time = 0.0349686, size = 97, normalized size = 1.

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

Maple [A] time = 0.004, size = 97, normalized size = 1.

$$\frac{27846\,b^2d^2x^8+66300\,x^6abd^2+66300\,x^6b^2cd+40950\,x^4a^2d^2+163800\,x^4abcd+40950\,x^4b^2c^2+107100\,x^2a^2cd+10710}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/348075*x^(9/2)*(13923*b^2*d^2*x^8+33150*a*b*d^2*x^6+33150*b^2*c*d*x^6+204
75*a^2*d^2*x^4+81900*a*b*c*d*x^4+20475*b^2*c^2*x^4+53550*a^2*c*d*x^2+53550*
a*b*c^2*x^2+38675*a^2*c^2)
```

Maxima [A] time = 1.08637, size = 115, normalized size = 1.19

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\left(b^2cd+abd^2\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}+\frac{4}{13}\left(abc^2+a^2cd\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*(b^2*c*d + a*b*d^2)*x^(21/2) + 2/17*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(17/2) + 2/9*a^2*c^2*x^(9/2) + 4/13*(a*b*c^2 + a^2*
c*d)*x^(13/2)
```

Fricas [A] time = 0.758657, size = 227, normalized size = 2.34

$$\frac{2}{348075}\left(13923\,b^2d^2x^{12}+33150\left(b^2cd+abd^2\right)x^{10}+20475\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+38675\,a^2c^2x^4+53550\left(abc^2+a^2c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^2*x^12 + 33150*(b^2*c*d + a*b*d^2)*x^10 + 20475*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 38675*a^2*c^2*x^4 + 53550*(a*b*c^2 + a^2*c
*d)*x^6)*sqrt(x)
```

Sympy [A] time = 34.5392, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{9}{2}}}{9}+\frac{4a^2cdx^{\frac{13}{2}}}{13}+\frac{2a^2d^2x^{\frac{17}{2}}}{17}+\frac{4abc^2x^{\frac{13}{2}}}{13}+\frac{8abcdx^{\frac{17}{2}}}{17}+\frac{4abd^2x^{\frac{21}{2}}}{21}+\frac{2b^2c^2x^{\frac{17}{2}}}{17}+\frac{4b^2cdx^{\frac{21}{2}}}{21}+\frac{2b^2d^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(9/2)/9 + 4*a**2*c*d*x**(13/2)/13 + 2*a**2*d**2*x**(17/2)/17
 + 4*a*b*c**2*x**(13/2)/13 + 8*a*b*c*d*x**(17/2)/17 + 4*a*b*d**2*x**(21/2)/
21 + 2*b**2*c**2*x**(17/2)/17 + 4*b**2*c*d*x**(21/2)/21 + 2*b**2*d**2*x**(2
```

```
5/2)/25
```

Giac [A] time = 1.14502, size = 127, normalized size = 1.31

$$\frac{2}{25}b^2d^2x^{\frac{25}{2}} + \frac{4}{21}b^2cdx^{\frac{21}{2}} + \frac{4}{21}abd^2x^{\frac{21}{2}} + \frac{2}{17}b^2c^2x^{\frac{17}{2}} + \frac{8}{17}abcdx^{\frac{17}{2}} + \frac{2}{17}a^2d^2x^{\frac{17}{2}} + \frac{4}{13}abc^2x^{\frac{13}{2}} + \frac{4}{13}a^2cdx^{\frac{13}{2}} + \frac{2}{9}a^2c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*b^2*c*d*x^(21/2) + 4/21*a*b*d^2*x^(21/2) + 2/1
7*b^2*c^2*x^(17/2) + 8/17*a*b*c*d*x^(17/2) + 2/17*a^2*d^2*x^(17/2) + 4/13*a
*b*c^2*x^(13/2) + 4/13*a^2*c*d*x^(13/2) + 2/9*a^2*c^2*x^(9/2)
```

3.400 $\int x^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

Rubi [A] time = 0.0459392, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^{5/2}+2ac(bc+ad)x^{9/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+2bd(bc+ad)x^{17/2}+b^2d^2x^{21/2}\right)dx\\ &= \frac{2}{7}a^2c^2x^{7/2}+\frac{4}{11}ac(bc+ad)x^{11/2}+\frac{2}{15}\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+\frac{4}{19}bd(bc+ad)x^{19/2}+\frac{2}{23}b^2d^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.0330486, size = 97, normalized size = 1.

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$\frac{43890 b^2d^2x^8 + 106260 x^6abd^2 + 106260 x^6b^2cd + 67298 x^4a^2d^2 + 269192 x^4abcd + 67298 x^4b^2c^2 + 183540 x^2a^2cd + }{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*b^2*d^2*x^8+53130*a*b*d^2*x^6+53130*b^2*c*d*x^6+336
49*a^2*d^2*x^4+134596*a*b*c*d*x^4+33649*b^2*c^2*x^4+91770*a^2*c*d*x^2+91770
*a*b*c^2*x^2+72105*a^2*c^2)
```

Maxima [A] time = 1.04838, size = 115, normalized size = 1.19

$$\frac{2}{23} b^2d^2x^{\frac{23}{2}} + \frac{4}{19} \left(b^2cd + abd^2\right)x^{\frac{19}{2}} + \frac{2}{15} \left(b^2c^2 + 4 abcd + a^2d^2\right)x^{\frac{15}{2}} + \frac{2}{7} a^2c^2x^{\frac{7}{2}} + \frac{4}{11} \left(abc^2 + a^2cd\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*(b^2*c*d + a*b*d^2)*x^(19/2) + 2/15*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(15/2) + 2/7*a^2*c^2*x^(7/2) + 4/11*(a*b*c^2 + a^2*
c*d)*x^(11/2)
```

Fricas [A] time = 0.74764, size = 225, normalized size = 2.32

$$\frac{2}{504735} \left(21945 b^2d^2x^{11} + 53130 \left(b^2cd + abd^2\right)x^9 + 33649 \left(b^2c^2 + 4 abcd + a^2d^2\right)x^7 + 72105 a^2c^2x^3 + 91770 \left(abc^2 + a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^2*x^11 + 53130*(b^2*c*d + a*b*d^2)*x^9 + 33649*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^7 + 72105*a^2*c^2*x^3 + 91770*(a*b*c^2 + a^2*c*
d)*x^5)*sqrt(x)
```

Sympy [A] time = 20.5765, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{7}{2}}}{7} + \frac{4a^2cdx^{\frac{11}{2}}}{11} + \frac{2a^2d^2x^{\frac{15}{2}}}{15} + \frac{4abc^2x^{\frac{11}{2}}}{11} + \frac{8abcdx^{\frac{15}{2}}}{15} + \frac{4abd^2x^{\frac{19}{2}}}{19} + \frac{2b^2c^2x^{\frac{15}{2}}}{15} + \frac{4b^2cdx^{\frac{19}{2}}}{19} + \frac{2b^2d^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(7/2)/7 + 4*a**2*c*d*x**(11/2)/11 + 2*a**2*d**2*x**(15/2)/15
 + 4*a*b*c**2*x**(11/2)/11 + 8*a*b*c*d*x**(15/2)/15 + 4*a*b*d**2*x**(19/2)/
19 + 2*b**2*c**2*x**(15/2)/15 + 4*b**2*c*d*x**(19/2)/19 + 2*b**2*d**2*x**(2
```

```
3/2)/23
```

Giac [A] time = 1.14413, size = 127, normalized size = 1.31

$$\frac{2}{23}\,b^{2}d^{2}x^{\frac{23}{2}}+\frac{4}{19}\,b^{2}cdx^{\frac{19}{2}}+\frac{4}{19}\,abd^{2}x^{\frac{19}{2}}+\frac{2}{15}\,b^{2}c^{2}x^{\frac{15}{2}}+\frac{8}{15}\,abcdx^{\frac{15}{2}}+\frac{2}{15}\,a^{2}d^{2}x^{\frac{15}{2}}+\frac{4}{11}\,abc^{2}x^{\frac{11}{2}}+\frac{4}{11}\,a^{2}cdx^{\frac{11}{2}}+\frac{2}{7}\,a^{2}c^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*b^2*c*d*x^(19/2) + 4/19*a*b*d^2*x^(19/2) + 2/1
5*b^2*c^2*x^(15/2) + 8/15*a*b*c*d*x^(15/2) + 2/15*a^2*d^2*x^(15/2) + 4/11*a
*b*c^2*x^(11/2) + 4/11*a^2*c*d*x^(11/2) + 2/7*a^2*c^2*x^(7/2)
```

3.401 $\int x^{3/2} \left(a + bx^2\right)^2 \left(c + dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{13}x^{13/2}\left(a^2d^2 + 4abcd + b^2c^2\right) + \frac{2}{5}a^2c^2x^{5/2} + \frac{4}{17}bdx^{17/2}(ad + bc) + \frac{4}{9}acx^{9/2}(ad + bc) + \frac{2}{21}b^2d^2x^{21/2}$$

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

Rubi [A] time = 0.0497726, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{13}x^{13/2}\left(a^2d^2 + 4abcd + b^2c^2\right) + \frac{2}{5}a^2c^2x^{5/2} + \frac{4}{17}bdx^{17/2}(ad + bc) + \frac{4}{9}acx^{9/2}(ad + bc) + \frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(a + bx^2\right)^2\left(c + dx^2\right)^2 \, dx = \int \left(a^2c^2x^{3/2} + 2ac(bc + ad)x^{7/2} + \left(b^2c^2 + 4abcd + a^2d^2\right)x^{11/2} + 2bd(bc + ad)x^{15/2} +$$
$$= \frac{2}{5}a^2c^2x^{5/2} + \frac{4}{9}ac(bc + ad)x^{9/2} + \frac{2}{13}\left(b^2c^2 + 4abcd + a^2d^2\right)x^{13/2} + \frac{4}{17}bd(bc + ad)x^{17/}$$

Mathematica [A] time = 0.0316799, size = 97, normalized size = 1.

$$\frac{2}{13}x^{13/2}\left(a^2d^2 + 4abcd + b^2c^2\right) + \frac{2}{5}a^2c^2x^{5/2} + \frac{4}{17}bdx^{17/2}(ad + bc) + \frac{4}{9}acx^{9/2}(ad + bc) + \frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

Maple [A] time = 0.004, size = 97, normalized size = 1.

$$\frac{6630\,b^2d^2x^8+16380\,x^6abd^2+16380\,x^6b^2cd+10710\,x^4a^2d^2+42840\,x^4abcd+10710\,x^4b^2c^2+30940\,x^2a^2cd+30940\,ac^2}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*b^2*d^2*x^8+8190*a*b*d^2*x^6+8190*b^2*c*d*x^6+5355*a^
2*d^2*x^4+21420*a*b*c*d*x^4+5355*b^2*c^2*x^4+15470*a^2*c*d*x^2+15470*a*b*c^
2*x^2+13923*a^2*c^2)
```

Maxima [A] time = 1.10538, size = 115, normalized size = 1.19

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\left(b^2cd+abd^2\right)x^{\frac{17}{2}}+\frac{2}{13}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}+\frac{4}{9}\left(abc^2+a^2cd\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*(b^2*c*d + a*b*d^2)*x^(17/2) + 2/13*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(13/2) + 2/5*a^2*c^2*x^(5/2) + 4/9*(a*b*c^2 + a^2*c
*d)*x^(9/2)
```

Fricas [A] time = 0.884202, size = 220, normalized size = 2.27

$$\frac{2}{69615}\left(3315\,b^2d^2x^{10}+8190\left(b^2cd+abd^2\right)x^8+5355\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+13923\,a^2c^2x^2+15470\left(abc^2+a^2cd\right)x^4\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^2*x^10 + 8190*(b^2*c*d + a*b*d^2)*x^8 + 5355*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^6 + 13923*a^2*c^2*x^2 + 15470*(a*b*c^2 + a^2*c*d)*x
^4)*sqrt(x)
```

Sympy [A] time = 11.3792, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{5}{2}}}{5}+\frac{4a^2cdx^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{13}{2}}}{13}+\frac{4abc^2x^{\frac{9}{2}}}{9}+\frac{8abcdx^{\frac{13}{2}}}{13}+\frac{4abd^2x^{\frac{17}{2}}}{17}+\frac{2b^2c^2x^{\frac{13}{2}}}{13}+\frac{4b^2cdx^{\frac{17}{2}}}{17}+\frac{2b^2d^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(5/2)/5 + 4*a**2*c*d*x**(9/2)/9 + 2*a**2*d**2*x**(13/2)/13 +
 4*a*b*c**2*x**(9/2)/9 + 8*a*b*c*d*x**(13/2)/13 + 4*a*b*d**2*x**(17/2)/17 +
 2*b**2*c**2*x**(13/2)/13 + 4*b**2*c*d*x**(17/2)/17 + 2*b**2*d**2*x**(21/2)
```

```
/21
```

Giac [A] time = 1.59996, size = 127, normalized size = 1.31

$$\frac{2}{21}b^2d^2x^{\frac{21}{2}} + \frac{4}{17}b^2cdx^{\frac{17}{2}} + \frac{4}{17}abd^2x^{\frac{17}{2}} + \frac{2}{13}b^2c^2x^{\frac{13}{2}} + \frac{8}{13}abcdx^{\frac{13}{2}} + \frac{2}{13}a^2d^2x^{\frac{13}{2}} + \frac{4}{9}abc^2x^{\frac{9}{2}} + \frac{4}{9}a^2cdx^{\frac{9}{2}} + \frac{2}{5}a^2c^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*b^2*c*d*x^(17/2) + 4/17*a*b*d^2*x^(17/2) + 2/1
3*b^2*c^2*x^(13/2) + 8/13*a*b*c*d*x^(13/2) + 2/13*a^2*d^2*x^(13/2) + 4/9*a*
b*c^2*x^(9/2) + 4/9*a^2*c*d*x^(9/2) + 2/5*a^2*c^2*x^(5/2)
```

3.402 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15/2))/15 + (2*b^2*d^2
*x^(19/2))/19
```

Rubi [A] time = 0.0465173, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15/2))/15 + (2*b^2*d^2
*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2\sqrt{x}+2ac(bc+ad)x^{5/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+2bd(bc+ad)x^{13/2}+b^2d^2x\right.\\ &= \frac{2}{3}a^2c^2x^{3/2}+\frac{4}{7}ac(bc+ad)x^{7/2}+\frac{2}{11}\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+\frac{4}{15}bd(bc+ad)x^{15/2}+\end{aligned}$$

Mathematica [A] time = 0.0342166, size = 83, normalized size = 0.86

$$\frac{2x^{3/2}\left(1995x^4\left(a^2d^2+4abcd+b^2c^2\right)+7315a^2c^2+2926bdx^6(ad+bc)+6270acx^2(ad+bc)+1155b^2d^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2*c^2 + 6270*a*c*(b*c + a*d)*x^2 + 1995*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 + 2926*b*d*(b*c + a*d)*x^6 + 1155*b^2*d^2*x^8))/21945
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$\frac{2310\,b^{2}d^{2}x^{8}+5852\,x^{6}abd^{2}+5852\,x^{6}b^{2}cd+3990\,x^{4}a^{2}d^{2}+15960\,x^{4}abcd+3990\,x^{4}b^{2}c^{2}+12540\,x^{2}a^{2}cd+12540\,ac^{2}b\ldots}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*b^2*d^2*x^8+2926*a*b*d^2*x^6+2926*b^2*c*d*x^6+1995*a^
2*d^2*x^4+7980*a*b*c*d*x^4+1995*b^2*c^2*x^4+6270*a^2*c*d*x^2+6270*a*b*c^2*x
^2+7315*a^2*c^2)
```

Maxima [A] time = 1.07824, size = 115, normalized size = 1.19

$$\frac{2}{19}\,b^{2}d^{2}x^{\frac{19}{2}}+\frac{4}{15}\,\left(b^{2}cd+abd^{2}\right)x^{\frac{15}{2}}+\frac{2}{11}\,\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^{2}c^{2}x^{\frac{3}{2}}+\frac{4}{7}\,\left(abc^{2}+a^{2}cd\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*(b^2*c*d + a*b*d^2)*x^(15/2) + 2/11*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(11/2) + 2/3*a^2*c^2*x^(3/2) + 4/7*(a*b*c^2 + a^2*c
*d)*x^(7/2)
```

Fricas [A] time = 0.736918, size = 213, normalized size = 2.2

$$\frac{2}{21945}\,\left(1155\,b^{2}d^{2}x^{9}+2926\,\left(b^{2}cd+abd^{2}\right)x^{7}+1995\,\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{5}+7315\,a^{2}c^{2}x+6270\,\left(abc^{2}+a^{2}cd\right)x^{3}\right)\sqrt{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^2*x^9 + 2926*(b^2*c*d + a*b*d^2)*x^7 + 1995*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^5 + 7315*a^2*c^2*x + 6270*(a*b*c^2 + a^2*c*d)*x^3)*s
qrt(x)
```

Sympy [A] time = 3.73765, size = 110, normalized size = 1.13

$$\frac{2a^{2}c^{2}x^{\frac{3}{2}}}{3}+\frac{2b^{2}d^{2}x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{15}{2}}\left(2abd^{2}+2b^{2}cd\right)}{15}+\frac{2x^{\frac{11}{2}}\left(a^{2}d^{2}+4abcd+b^{2}c^{2}\right)}{11}+\frac{2x^{\frac{7}{2}}\left(2a^{2}cd+2abc^{2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

```
[Out] 2*a**2*c**2*x**(3/2)/3 + 2*b**2*d**2*x**(19/2)/19 + 2*x**(15/2)*(2*a*b*d**2
 + 2*b**2*c*d)/15 + 2*x**(11/2)*(a**2*d**2 + 4*a*b*c*d + b**2*c**2)/11 + 2*
x**(7/2)*(2*a**2*c*d + 2*a*b*c**2)/7
```

Giac [A] time = 1.2185, size = 127, normalized size = 1.31

$$\frac{2}{19}b^2d^2x^{\frac{19}{2}} + \frac{4}{15}b^2cdx^{\frac{15}{2}} + \frac{4}{15}abd^2x^{\frac{15}{2}} + \frac{2}{11}b^2c^2x^{\frac{11}{2}} + \frac{8}{11}abcdx^{\frac{11}{2}} + \frac{2}{11}a^2d^2x^{\frac{11}{2}} + \frac{4}{7}abc^2x^{\frac{7}{2}} + \frac{4}{7}a^2cdx^{\frac{7}{2}} + \frac{2}{3}a^2c^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*b^2*c*d*x^(15/2) + 4/15*a*b*d^2*x^(15/2) + 2/1
1*b^2*c^2*x^(11/2) + 8/11*a*b*c*d*x^(11/2) + 2/11*a^2*d^2*x^(11/2) + 4/7*a*
b*c^2*x^(7/2) + 4/7*a^2*c*d*x^(7/2) + 2/3*a^2*c^2*x^(3/2)
```

3.403 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

Rubi [A] time = 0.0459258, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c^2}{\sqrt{x}}+2ac(bc+ad)x^{3/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+2bd(bc+ad)x^{11/2}+b^2d^2x^{15/}\right.\\ &= 2a^2c^2\sqrt{x}+\frac{4}{5}ac(bc+ad)x^{5/2}+\frac{2}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+\frac{4}{13}bd(bc+ad)x^{13/2}+\frac{2}{17}b\end{aligned}$$

Mathematica [A] time = 0.0313195, size = 95, normalized size = 1.

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

Maple [A] time = 0.005, size = 97, normalized size = 1.

$$\frac{1170 b^2d^2x^8 + 3060 x^6abd^2 + 3060 x^6b^2cd + 2210 x^4a^2d^2 + 8840 x^4abcd + 2210 x^4b^2c^2 + 7956 x^2a^2cd + 7956 ac^2bx^2 + 19}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*b^2*d^2*x^8+1530*a*b*d^2*x^6+1530*b^2*c*d*x^6+1105*a^2*
d^2*x^4+4420*a*b*c*d*x^4+1105*b^2*c^2*x^4+3978*a^2*c*d*x^2+3978*a*b*c^2*x^2
+9945*a^2*c^2)
```

Maxima [A] time = 1.02062, size = 115, normalized size = 1.21

$$\frac{2}{17} b^2d^2x^{\frac{17}{2}} + \frac{4}{13} \left(b^2cd + abd^2\right)x^{\frac{13}{2}} + \frac{2}{9} \left(b^2c^2 + 4 abcd + a^2d^2\right)x^{\frac{9}{2}} + 2 a^2c^2\sqrt{x} + \frac{4}{5} \left(abc^2 + a^2cd\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*(b^2*c*d + a*b*d^2)*x^(13/2) + 2/9*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(9/2) + 2*a^2*c^2*sqrt(x) + 4/5*(a*b*c^2 + a^2*c*d)*
x^(5/2)
```

Fricas [A] time = 0.70194, size = 208, normalized size = 2.19

$$\frac{2}{9945} \left(585 b^2d^2x^8 + 1530 \left(b^2cd + abd^2\right)x^6 + 1105 \left(b^2c^2 + 4 abcd + a^2d^2\right)x^4 + 9945 a^2c^2 + 3978 \left(abc^2 + a^2cd\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^2*x^8 + 1530*(b^2*c*d + a*b*d^2)*x^6 + 1105*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^4 + 9945*a^2*c^2 + 3978*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(
x)
```

Sympy [A] time = 4.98539, size = 134, normalized size = 1.41

$$2a^2c^2\sqrt{x} + \frac{4a^2cdx^{\frac{5}{2}}}{5} + \frac{2a^2d^2x^{\frac{9}{2}}}{9} + \frac{4abc^2x^{\frac{5}{2}}}{5} + \frac{8abcdx^{\frac{9}{2}}}{9} + \frac{4abd^2x^{\frac{13}{2}}}{13} + \frac{2b^2c^2x^{\frac{9}{2}}}{9} + \frac{4b^2cdx^{\frac{13}{2}}}{13} + \frac{2b^2d^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*a**2*c**2*sqrt(x) + 4*a**2*c*d*x**(5/2)/5 + 2*a**2*d**2*x**(9/2)/9 + 4*a*
b*c**2*x**(5/2)/5 + 8*a*b*c*d*x**(9/2)/9 + 4*a*b*d**2*x**(13/2)/13 + 2*b**2
```

```
*c**2*x**(9/2)/9 + 4*b**2*c*d*x**(13/2)/13 + 2*b**2*d**2*x**(17/2)/17
```

Giac [A] time = 1.17831, size = 127, normalized size = 1.34

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}} + \frac{4}{13}\,b^2cdx^{\frac{13}{2}} + \frac{4}{13}\,abd^2x^{\frac{13}{2}} + \frac{2}{9}\,b^2c^2x^{\frac{9}{2}} + \frac{8}{9}\,abcdx^{\frac{9}{2}} + \frac{2}{9}\,a^2d^2x^{\frac{9}{2}} + \frac{4}{5}\,abc^2x^{\frac{5}{2}} + \frac{4}{5}\,a^2cdx^{\frac{5}{2}} + 2\,a^2c^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*b^2*c*d*x^(13/2) + 4/13*a*b*d^2*x^(13/2) + 2/9
*b^2*c^2*x^(9/2) + 8/9*a*b*c*d*x^(9/2) + 2/9*a^2*d^2*x^(9/2) + 4/5*a*b*c^2*
x^(5/2) + 4/5*a^2*c*d*x^(5/2) + 2*a^2*c^2*sqrt(x)
```

3.404 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2))/11 + (2*b^2*d^2*x^
(15/2))/15
```

Rubi [A] time = 0.0475267, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2))/11 + (2*b^2*d^2*x^
(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{3/2}}+2ac(bc+ad)\sqrt{x}+\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+2bd(bc+ad)x^{9/2}+b^2d^2x^{13/2}\right)d\\ &= -\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{3}ac(bc+ad)x^{3/2}+\frac{2}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+\frac{4}{11}bd(bc+ad)x^{11/2}+\frac{2}{15}b^2d^2\end{aligned}$$

Mathematica [A] time = 0.0316485, size = 83, normalized size = 0.87

$$\frac{2\left(165x^4\left(a^2d^2+4abcd+b^2c^2\right)-1155a^2c^2+210bdx^6(ad+bc)+770acx^2(ad+bc)+77b^2d^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2*c^2 + 770*a*c*(b*c + a*d)*x^2 + 165*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 210*b*d*(b*c + a*d)*x^6 + 77*b^2*d^2*x^8))/(1155*Sqrt[x])
```

Maple [A] time = 0.004, size = 97, normalized size = 1.

$$-\frac{-154 b^2 d^2 x^8 - 420 x^6 abd^2 - 420 x^6 b^2 cd - 330 x^4 a^2 d^2 - 1320 x^4 abcd - 330 x^4 b^2 c^2 - 1540 x^2 a^2 cd - 1540 ac^2 bx^2 + 231}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^2*x^8-210*a*b*d^2*x^6-210*b^2*c*d*x^6-165*a^2*d^2*x^4-66
0*a*b*c*d*x^4-165*b^2*c^2*x^4-770*a^2*c*d*x^2-770*a*b*c^2*x^2+1155*a^2*c^2)
/x^(1/2)
```

Maxima [A] time = 1.0307, size = 115, normalized size = 1.21

$$\frac{2}{15} b^2 d^2 x^{\frac{15}{2}} + \frac{4}{11} \left(b^2 cd + abd^2\right) x^{\frac{11}{2}} + \frac{2}{7} \left(b^2 c^2 + 4 abcd + a^2 d^2\right) x^{\frac{7}{2}} - \frac{2 a^2 c^2}{\sqrt{x}} + \frac{4}{3} \left(abc^2 + a^2 cd\right) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*(b^2*c*d + a*b*d^2)*x^(11/2) + 2/7*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(7/2) - 2*a^2*c^2/sqrt(x) + 4/3*(a*b*c^2 + a^2*c*d)*
x^(3/2)
```

Fricas [A] time = 0.721341, size = 203, normalized size = 2.14

$$\frac{2 \left(77 b^2 d^2 x^8 + 210 \left(b^2 cd + abd^2\right) x^6 + 165 \left(b^2 c^2 + 4 abcd + a^2 d^2\right) x^4 - 1155 a^2 c^2 + 770 \left(abc^2 + a^2 cd\right) x^2\right)}{1155 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^8 + 210*(b^2*c*d + a*b*d^2)*x^6 + 165*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 - 1155*a^2*c^2 + 770*(a*b*c^2 + a^2*c*d)*x^2)/sqrt(x)
```

Sympy [A] time = 5.54516, size = 134, normalized size = 1.41

$$-\frac{2a^2c^2}{\sqrt{x}} + \frac{4a^2cdx^{\frac{3}{2}}}{3} + \frac{2a^2d^2x^{\frac{7}{2}}}{7} + \frac{4abc^2x^{\frac{3}{2}}}{3} + \frac{8abcdx^{\frac{7}{2}}}{7} + \frac{4abd^2x^{\frac{11}{2}}}{11} + \frac{2b^2c^2x^{\frac{7}{2}}}{7} + \frac{4b^2cdx^{\frac{11}{2}}}{11} + \frac{2b^2d^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

```
[Out] -2*a**2*c**2/sqrt(x) + 4*a**2*c*d*x**(3/2)/3 + 2*a**2*d**2*x**(7/2)/7 + 4*a
*b*c**2*x**(3/2)/3 + 8*a*b*c*d*x**(7/2)/7 + 4*a*b*d**2*x**(11/2)/11 + 2*b**
```

```
2*c**2*x**(7/2)/7 + 4*b**2*c*d*x**(11/2)/11 + 2*b**2*d**2*x**(15/2)/15
```

Giac [A] time = 1.62242, size = 127, normalized size = 1.34

$$\frac{2}{15}\,b^2d^2x^{\frac{15}{2}}+\frac{4}{11}\,b^2cdx^{\frac{11}{2}}+\frac{4}{11}\,abd^2x^{\frac{11}{2}}+\frac{2}{7}\,b^2c^2x^{\frac{7}{2}}+\frac{8}{7}\,abcdx^{\frac{7}{2}}+\frac{2}{7}\,a^2d^2x^{\frac{7}{2}}+\frac{4}{3}\,abc^2x^{\frac{3}{2}}+\frac{4}{3}\,a^2cdx^{\frac{3}{2}}-\frac{2\,a^2c^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*b^2*c*d*x^(11/2) + 4/11*a*b*d^2*x^(11/2) + 2/7
*b^2*c^2*x^(7/2) + 8/7*a*b*c*d*x^(7/2) + 2/7*a^2*d^2*x^(7/2) + 4/3*a*b*c^2*
x^(3/2) + 4/3*a^2*c*d*x^(3/2) - 2*a^2*c^2/sqrt(x)
```

3.405 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}} dx$

Optimal. Leaf size=95

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))/9 + (2*b^2*d^2*x^(1
3/2))/13
```

Rubi [A] time = 0.0466474, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))/9 + (2*b^2*d^2*x^(1
3/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}} dx &= \int \left(\frac{a^2c^2}{x^{5/2}}+\frac{2ac(bc+ad)}{\sqrt{x}}+\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+2bd(bc+ad)x^{7/2}+b^2d^2x^{11/2}\right) d\\ &= -\frac{2a^2c^2}{3x^{3/2}}+4ac(bc+ad)\sqrt{x}+\frac{2}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+\frac{4}{9}bd(bc+ad)x^{9/2}+\frac{2}{13}b^2d^2\end{aligned}$$

Mathematica [A] time = 0.0318285, size = 83, normalized size = 0.87

$$\frac{2\left(117x^4\left(a^2d^2+4abcd+b^2c^2\right)-195a^2c^2+130bdx^6(ad+bc)+1170acx^2(ad+bc)+45b^2d^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (2*(-195*a^2*c^2 + 1170*a*c*(b*c + a*d)*x^2 + 117*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 130*b*d*(b*c + a*d)*x^6 + 45*b^2*d^2*x^8))/(585*x^(3/2))
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$-\frac{-90\,b^{2}d^{2}x^{8}-260\,x^{6}abd^{2}-260\,x^{6}b^{2}cd-234\,x^{4}a^{2}d^{2}-936\,x^{4}abcd-234\,x^{4}b^{2}c^{2}-2340\,x^{2}a^{2}cd-2340\,ac^{2}bx^{2}+390\,a^{2}c^{2}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*b^2*d^2*x^8-130*a*b*d^2*x^6-130*b^2*c*d*x^6-117*a^2*d^2*x^4-468
*a*b*c*d*x^4-117*b^2*c^2*x^4-1170*a^2*c*d*x^2-1170*a*b*c^2*x^2+195*a^2*c^2)
/x^(3/2)
```

Maxima [A] time = 1.02923, size = 115, normalized size = 1.21

$$\frac{2}{13}\,b^{2}d^{2}x^{\frac{13}{2}}+\frac{4}{9}\left(b^{2}cd+abd^{2}\right)x^{\frac{9}{2}}+\frac{2}{5}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{\frac{5}{2}}-\frac{2\,a^{2}c^{2}}{3\,x^{\frac{3}{2}}}+4\left(abc^{2}+a^{2}cd\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*(b^2*c*d + a*b*d^2)*x^(9/2) + 2/5*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(5/2) - 2/3*a^2*c^2/x^(3/2) + 4*(a*b*c^2 + a^2*c*d)*sq
rt(x)
```

Fricas [A] time = 0.802958, size = 201, normalized size = 2.12

$$\frac{2\left(45\,b^{2}d^{2}x^{8}+130\left(b^{2}cd+abd^{2}\right)x^{6}+117\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{4}-195\,a^{2}c^{2}+1170\left(abc^{2}+a^{2}cd\right)x^{2}\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d^2*x^8 + 130*(b^2*c*d + a*b*d^2)*x^6 + 117*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^4 - 195*a^2*c^2 + 1170*(a*b*c^2 + a^2*c*d)*x^2)/x^(3/2)
```

Sympy [A] time = 6.6326, size = 133, normalized size = 1.4

$$-\frac{2a^{2}c^{2}}{3x^{\frac{3}{2}}}+4a^{2}cd\sqrt{x}+\frac{2a^{2}d^{2}x^{\frac{5}{2}}}{5}+4abc^{2}\sqrt{x}+\frac{8abcdx^{\frac{5}{2}}}{5}+\frac{4abd^{2}x^{\frac{9}{2}}}{9}+\frac{2b^{2}c^{2}x^{\frac{5}{2}}}{5}+\frac{4b^{2}cdx^{\frac{9}{2}}}{9}+\frac{2b^{2}d^{2}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

```
[Out] -2*a**2*c**2/(3*x**(3/2)) + 4*a**2*c*d*sqrt(x) + 2*a**2*d**2*x**(5/2)/5 + 4
*a*b*c**2*sqrt(x) + 8*a*b*c*d*x**(5/2)/5 + 4*a*b*d**2*x**(9/2)/9 + 2*b**2*c
```

```
**2*x**(5/2)/5 + 4*b**2*c*d*x**(9/2)/9 + 2*b**2*d**2*x**(13/2)/13
```

Giac [A] time = 1.15176, size = 127, normalized size = 1.34

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,b^2cdx^{\frac{9}{2}}+\frac{4}{9}\,abd^2x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^2x^{\frac{5}{2}}+\frac{8}{5}\,abcdx^{\frac{5}{2}}+\frac{2}{5}\,a^2d^2x^{\frac{5}{2}}+4\,abc^2\sqrt{x}+4\,a^2cd\sqrt{x}-\frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*b^2*c*d*x^(9/2) + 4/9*a*b*d^2*x^(9/2) + 2/5*b^2
*c^2*x^(5/2) + 8/5*a*b*c*d*x^(5/2) + 2/5*a^2*d^2*x^(5/2) + 4*a*b*c^2*sqrt(x
) + 4*a^2*c*d*sqrt(x) - 2/3*a^2*c^2/x^(3/2)
```

3.406 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}} dx$

Optimal. Leaf size=95

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2))/7 + (2*b^2*d^2*x^
(11/2))/11
```

Rubi [A] time = 0.0471844, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2))/7 + (2*b^2*d^2*x^
(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{7/2}}+\frac{2ac(bc+ad)}{x^{3/2}}+\left(b^2c^2+4abcd+a^2d^2\right)\sqrt{x}+2bd(bc+ad)x^{5/2}+b^2d^2x^{9/2}\right)dx\\ &= -\frac{2a^2c^2}{5x^{5/2}}-\frac{4ac(bc+ad)}{\sqrt{x}}+\frac{2}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+\frac{4}{7}bd(bc+ad)x^{7/2}+\frac{2}{11}b^2d^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0373862, size = 83, normalized size = 0.87

$$\frac{2\left(385x^4\left(a^2d^2+4abcd+b^2c^2\right)-231a^2c^2+330bdx^6(ad+bc)-2310acx^2(ad+bc)+105b^2d^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^2 - 2310*a*c*(b*c + a*d)*x^2 + 385*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 330*b*d*(b*c + a*d)*x^6 + 105*b^2*d^2*x^8))/(1155*x^(5/2))
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$-\frac{-210 b^2 d^2 x^8 - 660 x^6 abd^2 - 660 x^6 b^2 cd - 770 x^4 a^2 d^2 - 3080 x^4 abcd - 770 x^4 b^2 c^2 + 4620 x^2 a^2 cd + 4620 ac^2 bx^2 + 46}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*b^2*d^2*x^8-330*a*b*d^2*x^6-330*b^2*c*d*x^6-385*a^2*d^2*x^4-1
540*a*b*c*d*x^4-385*b^2*c^2*x^4+2310*a^2*c*d*x^2+2310*a*b*c^2*x^2+231*a^2*c
^2)/x^(5/2)
```

Maxima [A] time = 1.06232, size = 117, normalized size = 1.23

$$\frac{2}{11} b^2 d^2 x^{\frac{11}{2}} + \frac{4}{7} \left(b^2 cd + abd^2\right) x^{\frac{7}{2}} + \frac{2}{3} \left(b^2 c^2 + 4 abcd + a^2 d^2\right) x^{\frac{3}{2}} - \frac{2 \left(a^2 c^2 + 10 \left(abc^2 + a^2 cd\right) x^2\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*(b^2*c*d + a*b*d^2)*x^(7/2) + 2/3*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(3/2) - 2/5*(a^2*c^2 + 10*(a*b*c^2 + a^2*c*d)*x^2)/x^(
5/2)
```

Fricas [A] time = 0.75379, size = 204, normalized size = 2.15

$$\frac{2 \left(105 b^2 d^2 x^8 + 330 \left(b^2 cd + abd^2\right) x^6 + 385 \left(b^2 c^2 + 4 abcd + a^2 d^2\right) x^4 - 231 a^2 c^2 - 2310 \left(abc^2 + a^2 cd\right) x^2\right)}{1155 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^2*d^2*x^8 + 330*(b^2*c*d + a*b*d^2)*x^6 + 385*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 - 231*a^2*c^2 - 2310*(a*b*c^2 + a^2*c*d)*x^2)/x^(5/2)
```

Sympy [A] time = 8.94829, size = 133, normalized size = 1.4

$$-\frac{2a^2c^2}{5x^{\frac{5}{2}}} - \frac{4a^2cd}{\sqrt{x}} + \frac{2a^2d^2x^{\frac{3}{2}}}{3} - \frac{4abc^2}{\sqrt{x}} + \frac{8abcdx^{\frac{3}{2}}}{3} + \frac{4abd^2x^{\frac{7}{2}}}{7} + \frac{2b^2c^2x^{\frac{3}{2}}}{3} + \frac{4b^2cdx^{\frac{7}{2}}}{7} + \frac{2b^2d^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

```
[Out] -2*a**2*c**2/(5*x**(5/2)) - 4*a**2*c*d/sqrt(x) + 2*a**2*d**2*x**(3/2)/3 - 4
*a*b*c**2/sqrt(x) + 8*a*b*c*d*x**(3/2)/3 + 4*a*b*d**2*x**(7/2)/7 + 2*b**2*c
```

```
**2*x**(3/2)/3 + 4*b**2*c*d*x**(7/2)/7 + 2*b**2*d**2*x**(11/2)/11
```

Giac [A] time = 1.16628, size = 130, normalized size = 1.37

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}} + \frac{4}{7}\,b^2cdx^{\frac{7}{2}} + \frac{4}{7}\,abd^2x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^2x^{\frac{3}{2}} + \frac{8}{3}\,abcdx^{\frac{3}{2}} + \frac{2}{3}\,a^2d^2x^{\frac{3}{2}} - \frac{2\left(10\,abc^2x^2 + 10\,a^2cdx^2 + a^2c^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*b^2*c*d*x^(7/2) + 4/7*a*b*d^2*x^(7/2) + 2/3*b^2
*c^2*x^(3/2) + 8/3*a*b*c*d*x^(3/2) + 2/3*a^2*d^2*x^(3/2) - 2/5*(10*a*b*c^2*
x^2 + 10*a^2*c*d*x^2 + a^2*c^2)/x^(5/2)
```

3.407 $\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/}$$

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
(29/2))/29
```

Rubi [A] time = 0.064872, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{7/2}+ac^2(2bc+3ad)x^{11/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+d\left(3b^2c^2+6al\right.\right.\\ &= \frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2(2bc+3ad)x^{13/2}+\frac{2}{17}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{17/2}+\frac{2}{21}d\left(3b^2c^2\right.\end{aligned}$$

Mathematica [A] time = 0.0426096, size = 139, normalized size = 1.

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
```

```
(29/2))/29
```

Maple [A] time = 0.006, size = 138, normalized size = 1.

$$\frac{696150\,b^{2}d^{3}x^{10}+1615068\,x^{8}abd^{3}+2422602\,x^{8}b^{2}cd^{2}+961350\,x^{6}a^{2}d^{3}+5768100\,x^{6}abcd^{2}+2884050\,x^{6}b^{2}c^{2}d+3562650}{10094175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/10094175*x^(9/2)*(348075*b^2*d^3*x^10+807534*a*b*d^3*x^8+1211301*b^2*c*d^
2*x^8+480675*a^2*d^3*x^6+2884050*a*b*c*d^2*x^6+1442025*b^2*c^2*d*x^6+178132
5*a^2*c*d^2*x^4+3562650*a*b*c^2*d*x^4+593775*b^2*c^3*x^4+2329425*a^2*c^2*d*
x^2+1552950*a*b*c^3*x^2+1121575*a^2*c^3)
```

Maxima [A] time = 1.03665, size = 171, normalized size = 1.23

$$\frac{2}{29}\,b^{2}d^{3}x^{\frac{29}{2}}+\frac{2}{25}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{\frac{25}{2}}+\frac{2}{21}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{21}{2}}+\frac{2}{9}\,a^{2}c^{3}x^{\frac{9}{2}}+\frac{2}{17}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{\frac{1}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 2/25*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(25/2) + 2/21*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(21/2) + 2/9*a^2*c^3*x^(9/2) + 2/17*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(17/2) + 2/13*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(13/2)
```

Fricas [A] time = 0.841121, size = 332, normalized size = 2.39

$$\frac{2}{10094175}\left(348075\,b^{2}d^{3}x^{14}+403767\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{12}+480675\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{10}+1121575\,a^{2}c^{3}x^{4}+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/10094175*(348075*b^2*d^3*x^14 + 403767*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 4
80675*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1121575*a^2*c^3*x^4 + 59
3775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^8 + 776475*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^6)*sqrt(x)
```

Sympy [A] time = 55.1669, size = 192, normalized size = 1.38

$$\frac{2a^{2}c^{3}x^{\frac{9}{2}}}{9}+\frac{6a^{2}c^{2}dx^{\frac{13}{2}}}{13}+\frac{6a^{2}cd^{2}x^{\frac{17}{2}}}{17}+\frac{2a^{2}d^{3}x^{\frac{21}{2}}}{21}+\frac{4abc^{3}x^{\frac{13}{2}}}{13}+\frac{12abc^{2}dx^{\frac{17}{2}}}{17}+\frac{4abcd^{2}x^{\frac{21}{2}}}{7}+\frac{4abd^{3}x^{\frac{25}{2}}}{25}+\frac{2b^{2}c^{3}x^{\frac{17}{2}}}{17}+\frac{2b^{2}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(9/2)/9 + 6*a**2*c**2*d*x**(13/2)/13 + 6*a**2*c*d**2*x**(17/
2)/17 + 2*a**2*d**3*x**(21/2)/21 + 4*a*b*c**3*x**(13/2)/13 + 12*a*b*c**2*d*
x**(17/2)/17 + 4*a*b*c*d**2*x**(21/2)/7 + 4*a*b*d**3*x**(25/2)/25 + 2*b**2*
c**3*x**(17/2)/17 + 2*b**2*c**2*d*x**(21/2)/7 + 6*b**2*c*d**2*x**(25/2)/25
+ 2*b**2*d**3*x**(29/2)/29
```

Giac [A] time = 1.17091, size = 182, normalized size = 1.31

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{6}{25}\,b^2cd^2x^{\frac{25}{2}} + \frac{4}{25}\,abd^3x^{\frac{25}{2}} + \frac{2}{7}\,b^2c^2dx^{\frac{21}{2}} + \frac{4}{7}\,abcd^2x^{\frac{21}{2}} + \frac{2}{21}\,a^2d^3x^{\frac{21}{2}} + \frac{2}{17}\,b^2c^3x^{\frac{17}{2}} + \frac{12}{17}\,abc^2dx^{\frac{17}{2}} + \frac{6}{17}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 6/25*b^2*c*d^2*x^(25/2) + 4/25*a*b*d^3*x^(25/2) + 2
/7*b^2*c^2*d*x^(21/2) + 4/7*a*b*c*d^2*x^(21/2) + 2/21*a^2*d^3*x^(21/2) + 2/
17*b^2*c^3*x^(17/2) + 12/17*a*b*c^2*d*x^(17/2) + 6/17*a^2*c*d^2*x^(17/2) +
4/13*a*b*c^3*x^(13/2) + 6/13*a^2*c^2*d*x^(13/2) + 2/9*a^2*c^3*x^(9/2)
```

3.408 $\int x^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2$$

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
(27/2))/27
```

Rubi [A] time = 0.0647248, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{5/2}+ac^2(2bc+3ad)x^{9/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+d\left(3b^2c^2+6abcd\right.\right.\\ &= \frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2(2bc+3ad)x^{11/2}+\frac{2}{15}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+\frac{2}{19}d\left(3b^2c^2+\right.\end{aligned}$$

Mathematica [A] time = 0.0400455, size = 139, normalized size = 1.

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
```

```
(27/2))/27
```

Maple [A] time = 0.005, size = 138, normalized size = 1.

$$\frac{336490\,b^2d^3x^{10} + 790020\,x^8abd^3 + 1185030\,x^8b^2cd^2 + 478170\,x^6a^2d^3 + 2869020\,x^6abcd^2 + 1434510\,x^6b^2c^2d + 18170}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/4542615*x^(7/2)*(168245*b^2*d^3*x^10+395010*a*b*d^3*x^8+592515*b^2*c*d^2*
x^8+239085*a^2*d^3*x^6+1434510*a*b*c*d^2*x^6+717255*b^2*c^2*d*x^6+908523*a^
2*c*d^2*x^4+1817046*a*b*c^2*d*x^4+302841*b^2*c^3*x^4+1238895*a^2*c^2*d*x^2+
825930*a*b*c^3*x^2+648945*a^2*c^3)
```

Maxima [A] time = 1.03671, size = 171, normalized size = 1.23

$$\frac{2}{27}\,b^2d^3x^{\frac{27}{2}} + \frac{2}{23}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{23}{2}} + \frac{2}{19}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{19}{2}} + \frac{2}{7}\,a^2c^3x^{\frac{7}{2}} + \frac{2}{15}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 2/23*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(23/2) + 2/19*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(19/2) + 2/7*a^2*c^3*x^(7/2) + 2/15*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(15/2) + 2/11*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(11/2)
```

Fricas [A] time = 0.769891, size = 328, normalized size = 2.36

$$\frac{2}{4542615}\left(168245\,b^2d^3x^{13} + 197505\left(3\,b^2cd^2 + 2\,abd^3\right)x^{11} + 239085\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^9 + 648945\,a^2c^3x^3 + 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*b^2*d^3*x^13 + 197505*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 23
9085*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 648945*a^2*c^3*x^3 + 30284
1*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + 412965*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^5)*sqrt(x)
```

Sympy [A] time = 35.0576, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{7}{2}}}{7} + \frac{6a^2c^2dx^{\frac{11}{2}}}{11} + \frac{2a^2cd^2x^{\frac{15}{2}}}{5} + \frac{2a^2d^3x^{\frac{19}{2}}}{19} + \frac{4abc^3x^{\frac{11}{2}}}{11} + \frac{4abc^2dx^{\frac{15}{2}}}{5} + \frac{12abcd^2x^{\frac{19}{2}}}{19} + \frac{4abd^3x^{\frac{23}{2}}}{23} + \frac{2b^2c^3x^{\frac{15}{2}}}{15} + 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(7/2)/7 + 6*a**2*c**2*d*x**(11/2)/11 + 2*a**2*c*d**2*x**(15/
2)/5 + 2*a**2*d**3*x**(19/2)/19 + 4*a*b*c**3*x**(11/2)/11 + 4*a*b*c**2*d*x*
*(15/2)/5 + 12*a*b*c*d**2*x**(19/2)/19 + 4*a*b*d**3*x**(23/2)/23 + 2*b**2*c
**3*x**(15/2)/15 + 6*b**2*c**2*d*x**(19/2)/19 + 6*b**2*c*d**2*x**(23/2)/23
+ 2*b**2*d**3*x**(27/2)/27
```

Giac [A] time = 1.18821, size = 182, normalized size = 1.31

$$\frac{2}{27}b^2d^3x^{\frac{27}{2}}+\frac{6}{23}b^2cd^2x^{\frac{23}{2}}+\frac{4}{23}abd^3x^{\frac{23}{2}}+\frac{6}{19}b^2c^2dx^{\frac{19}{2}}+\frac{12}{19}abcd^2x^{\frac{19}{2}}+\frac{2}{19}a^2d^3x^{\frac{19}{2}}+\frac{2}{15}b^2c^3x^{\frac{15}{2}}+\frac{4}{5}abc^2dx^{\frac{15}{2}}+\frac{2}{5}a^2cc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 6/23*b^2*c*d^2*x^(23/2) + 4/23*a*b*d^3*x^(23/2) + 6
/19*b^2*c^2*d*x^(19/2) + 12/19*a*b*c*d^2*x^(19/2) + 2/19*a^2*d^3*x^(19/2) +
 2/15*b^2*c^3*x^(15/2) + 4/5*a*b*c^2*d*x^(15/2) + 2/5*a^2*c*d^2*x^(15/2) +
4/11*a*b*c^3*x^(11/2) + 6/11*a^2*c^2*d*x^(11/2) + 2/7*a^2*c^3*x^(7/2)
```

3.409 $\int x^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}($$

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
5/2))/25
```

Rubi [A] time = 0.0632243, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}($$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
5/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{3/2}+ac^2(2bc+3ad)x^{7/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+d\left(3b^2c^2+6abc\right.\right.\\ &= \frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2(2bc+3ad)x^{9/2}+\frac{2}{13}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+\frac{2}{17}d\left(3b^2c^2\right.\end{aligned}$$

Mathematica [A] time = 0.037461, size = 139, normalized size = 1.

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}($$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
```

5/2))/25

Maple [A] time = 0.006, size = 138, normalized size = 1.

$$\frac{27846\,b^2d^3x^{10} + 66300\,x^8abd^3 + 99450\,x^8b^2cd^2 + 40950\,x^6a^2d^3 + 245700\,x^6abcd^2 + 122850\,x^6b^2c^2d + 160650\,x^4a^2cd^2 +}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/348075*x^(5/2)*(13923*b^2*d^3*x^10+33150*a*b*d^3*x^8+49725*b^2*c*d^2*x^8+
20475*a^2*d^3*x^6+122850*a*b*c*d^2*x^6+61425*b^2*c^2*d*x^6+80325*a^2*c*d^2*
x^4+160650*a*b*c^2*d*x^4+26775*b^2*c^3*x^4+116025*a^2*c^2*d*x^2+77350*a*b*c
^3*x^2+69615*a^2*c^3)
```

Maxima [A] time = 1.05972, size = 171, normalized size = 1.23

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{17}{2}} + \frac{2}{5}\,a^2c^3x^{\frac{5}{2}} + \frac{2}{13}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{1}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(21/2) + 2/17*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(17/2) + 2/5*a^2*c^3*x^(5/2) + 2/13*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(13/2) + 2/9*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(9/2)
```

Fricas [A] time = 0.76166, size = 319, normalized size = 2.29

$$\frac{2}{348075}\left(13923\,b^2d^3x^{12} + 16575\left(3\,b^2cd^2 + 2\,abd^3\right)x^{10} + 20475\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^8 + 69615\,a^2c^3x^2 + 26775\left(b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^3*x^12 + 16575*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 20475
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 69615*a^2*c^3*x^2 + 26775*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 38675*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
)*sqrt(x)
```

Sympy [A] time = 21.2537, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{5}{2}}}{5} + \frac{2a^2c^2dx^{\frac{9}{2}}}{3} + \frac{6a^2cd^2x^{\frac{13}{2}}}{13} + \frac{2a^2d^3x^{\frac{17}{2}}}{17} + \frac{4abc^3x^{\frac{9}{2}}}{9} + \frac{12abc^2dx^{\frac{13}{2}}}{13} + \frac{12abcd^2x^{\frac{17}{2}}}{17} + \frac{4abd^3x^{\frac{21}{2}}}{21} + \frac{2b^2c^3x^{\frac{13}{2}}}{13} + \frac{6b^2c^2}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(5/2)/5 + 2*a**2*c**2*d*x**(9/2)/3 + 6*a**2*c*d**2*x**(13/2)
/13 + 2*a**2*d**3*x**(17/2)/17 + 4*a*b*c**3*x**(9/2)/9 + 12*a*b*c**2*d*x**(
13/2)/13 + 12*a*b*c*d**2*x**(17/2)/17 + 4*a*b*d**3*x**(21/2)/21 + 2*b**2*c*
*3*x**(13/2)/13 + 6*b**2*c**2*d*x**(17/2)/17 + 2*b**2*c*d**2*x**(21/2)/7 +
2*b**2*d**3*x**(25/2)/25
```

Giac [A] time = 1.17432, size = 182, normalized size = 1.31

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}} + \frac{2}{7}\,b^2cd^2x^{\frac{21}{2}} + \frac{4}{21}\,abd^3x^{\frac{21}{2}} + \frac{6}{17}\,b^2c^2dx^{\frac{17}{2}} + \frac{12}{17}\,abcd^2x^{\frac{17}{2}} + \frac{2}{17}\,a^2d^3x^{\frac{17}{2}} + \frac{2}{13}\,b^2c^3x^{\frac{13}{2}} + \frac{12}{13}\,abc^2dx^{\frac{13}{2}} + \frac{6}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/7*b^2*c*d^2*x^(21/2) + 4/21*a*b*d^3*x^(21/2) + 6/
17*b^2*c^2*d*x^(17/2) + 12/17*a*b*c*d^2*x^(17/2) + 2/17*a^2*d^3*x^(17/2) +
2/13*b^2*c^3*x^(13/2) + 12/13*a*b*c^2*d*x^(13/2) + 6/13*a^2*c*d^2*x^(13/2)
+ 4/9*a*b*c^3*x^(9/2) + 2/3*a^2*c^2*d*x^(9/2) + 2/5*a^2*c^3*x^(5/2)
```

3.410 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad$$

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
3/2))/23
```

Rubi [A] time = 0.0635255, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
3/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3\sqrt{x}+ac^2(2bc+3ad)x^{5/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+d\left(3b^2c^2+6abcd+a\right.\right.\\ &= \frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2(2bc+3ad)x^{7/2}+\frac{2}{11}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+\frac{2}{15}d\left(3b^2c^2+6a\right.\end{aligned}$$

Mathematica [A] time = 0.0375953, size = 139, normalized size = 1.

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
```

```
3/2))/23
```

Maple [A] time = 0.007, size = 138, normalized size = 1.

$$\frac{43890\,b^2d^3x^{10}+106260\,x^8abd^3+159390\,x^8b^2cd^2+67298\,x^6a^2d^3+403788\,x^6abcd^2+201894\,x^6b^2c^2d+275310\,x^4a^2}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x)
```

```
[Out] 2/504735*x^(3/2)*(21945*b^2*d^3*x^10+53130*a*b*d^3*x^8+79695*b^2*c*d^2*x^8+
33649*a^2*d^3*x^6+201894*a*b*c*d^2*x^6+100947*b^2*c^2*d*x^6+137655*a^2*c*d^
2*x^4+275310*a*b*c^2*d*x^4+45885*b^2*c^3*x^4+216315*a^2*c^2*d*x^2+144210*a*
b*c^3*x^2+168245*a^2*c^3)
```

Maxima [A] time = 1.04112, size = 171, normalized size = 1.23

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{15}{2}}+\frac{2}{3}\,a^2c^3x^{\frac{3}{2}}+\frac{2}{11}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 2/19*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(19/2) + 2/15*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(15/2) + 2/3*a^2*c^3*x^(3/2) + 2/11*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(11/2) + 2/7*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(7/2)
```

Fricas [A] time = 0.80791, size = 316, normalized size = 2.27

$$\frac{2}{504735}\left(21945\,b^2d^3x^{11}+26565\left(3\,b^2cd^2+2\,abd^3\right)x^9+33649\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+168245\,a^2c^3x+45885\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^3*x^11 + 26565*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 33649*
(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + 168245*a^2*c^3*x + 45885*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 72105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3)*
sqrt(x)
```

Sympy [A] time = 5.20731, size = 155, normalized size = 1.12

$$\frac{2a^2c^3x^{\frac{3}{2}}}{3}+\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}\left(2abd^3+3b^2cd^2\right)}{19}+\frac{2x^{\frac{15}{2}}\left(a^2d^3+6abcd^2+3b^2c^2d\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3a^2cd^2+6abc^2d+b^2c^3\right)}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

```
[Out] 2*a**2*c**3*x**(3/2)/3 + 2*b**2*d**3*x**(23/2)/23 + 2*x**(19/2)*(2*a*b*d**3
 + 3*b**2*c*d**2)/19 + 2*x**(15/2)*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*
d)/15 + 2*x**(11/2)*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3)/11 + 2*x**(7
/2)*(3*a**2*c**2*d + 2*a*b*c**3)/7
```

Giac [A] time = 1.17829, size = 182, normalized size = 1.31

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}} + \frac{6}{19}\,b^2cd^2x^{\frac{19}{2}} + \frac{4}{19}\,abd^3x^{\frac{19}{2}} + \frac{2}{5}\,b^2c^2dx^{\frac{15}{2}} + \frac{4}{5}\,abcd^2x^{\frac{15}{2}} + \frac{2}{15}\,a^2d^3x^{\frac{15}{2}} + \frac{2}{11}\,b^2c^3x^{\frac{11}{2}} + \frac{12}{11}\,abc^2dx^{\frac{11}{2}} + \frac{6}{11}\,a^2cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 6/19*b^2*c*d^2*x^(19/2) + 4/19*a*b*d^3*x^(19/2) + 2
/5*b^2*c^2*d*x^(15/2) + 4/5*a*b*c*d^2*x^(15/2) + 2/15*a^2*d^3*x^(15/2) + 2/
11*b^2*c^3*x^(11/2) + 12/11*a*b*c^2*d*x^(11/2) + 6/11*a^2*c*d^2*x^(11/2) +
4/7*a*b*c^3*x^(7/2) + 6/7*a^2*c^2*d*x^(7/2) + 2/3*a^2*c^3*x^(3/2)
```

3.411 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad$$

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

Rubi [A] time = 0.0647974, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c^3}{\sqrt{x}}+ac^2(2bc+3ad)x^{3/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+d\left(3b^2c^2+6abcd+a^2d\right.\right.\\ &= 2a^2c^3\sqrt{x}+\frac{2}{5}ac^2(2bc+3ad)x^{5/2}+\frac{2}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+\frac{2}{13}d\left(3b^2c^2+6abcd\right.\end{aligned}$$

Mathematica [A] time = 0.0388549, size = 137, normalized size = 1.

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
```

```
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

Maple [A] time = 0.007, size = 138, normalized size = 1.

$$\frac{6630\,b^{2}d^{3}x^{10}+16380\,x^{8}abd^{3}+24570\,x^{8}b^{2}cd^{2}+10710\,x^{6}a^{2}d^{3}+64260\,x^{6}abcd^{2}+32130\,x^{6}b^{2}c^{2}d+46410\,x^{4}a^{2}cd^{2}+9282}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2/69615*x^(1/2)*(3315*b^2*d^3*x^10+8190*a*b*d^3*x^8+12285*b^2*c*d^2*x^8+535
5*a^2*d^3*x^6+32130*a*b*c*d^2*x^6+16065*b^2*c^2*d*x^6+23205*a^2*c*d^2*x^4+4
6410*a*b*c^2*d*x^4+7735*b^2*c^3*x^4+41769*a^2*c^2*d*x^2+27846*a*b*c^3*x^2+6
9615*a^2*c^3)
```

Maxima [A] time = 1.089, size = 171, normalized size = 1.25

$$\frac{2}{21}\,b^{2}d^{3}x^{\frac{21}{2}}+\frac{2}{17}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{\frac{17}{2}}+\frac{2}{13}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{13}{2}}+2\,a^{2}c^{3}\sqrt{x}+\frac{2}{9}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 2/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(17/2) + 2/13*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(13/2) + 2*a^2*c^3*sqrt(x) + 2/9*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(9/2) + 2/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
5/2)
```

Fricas [A] time = 0.821759, size = 305, normalized size = 2.23

$$\frac{2}{69615}\left(3315\,b^{2}d^{3}x^{10}+4095\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{8}+5355\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{6}+69615\,a^{2}c^{3}+7735\left(b^{2}c^{3}+6\,abc\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^3*x^10 + 4095*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 5355*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 69615*a^2*c^3 + 7735*(b^2*c^3 + 6*a
*b*c^2*d + 3*a^2*c*d^2)*x^4 + 13923*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt(x)
```

Sympy [A] time = 10.4738, size = 190, normalized size = 1.39

$$2a^{2}c^{3}\sqrt{x}+\frac{6a^{2}c^{2}dx^{\frac{5}{2}}}{5}+\frac{2a^{2}cd^{2}x^{\frac{9}{2}}}{3}+\frac{2a^{2}d^{3}x^{\frac{13}{2}}}{13}+\frac{4abc^{3}x^{\frac{5}{2}}}{5}+\frac{4abc^{2}dx^{\frac{9}{2}}}{3}+\frac{12abcd^{2}x^{\frac{13}{2}}}{13}+\frac{4abd^{3}x^{\frac{17}{2}}}{17}+\frac{2b^{2}c^{3}x^{\frac{9}{2}}}{9}+\frac{6b^{2}c^{2}dx}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] 2*a**2*c**3*sqrt(x) + 6*a**2*c**2*d*x**(5/2)/5 + 2*a**2*c*d**2*x**(9/2)/3 +
 2*a**2*d**3*x**(13/2)/13 + 4*a*b*c**3*x**(5/2)/5 + 4*a*b*c**2*d*x**(9/2)/3
 + 12*a*b*c*d**2*x**(13/2)/13 + 4*a*b*d**3*x**(17/2)/17 + 2*b**2*c**3*x**(9
/2)/9 + 6*b**2*c**2*d*x**(13/2)/13 + 6*b**2*c*d**2*x**(17/2)/17 + 2*b**2*d*
*3*x**(21/2)/21
```

Giac [A] time = 1.69149, size = 182, normalized size = 1.33

$$\frac{2}{21}b^2d^3x^{\frac{21}{2}} + \frac{6}{17}b^2cd^2x^{\frac{17}{2}} + \frac{4}{17}abd^3x^{\frac{17}{2}} + \frac{6}{13}b^2c^2dx^{\frac{13}{2}} + \frac{12}{13}abcd^2x^{\frac{13}{2}} + \frac{2}{13}a^2d^3x^{\frac{13}{2}} + \frac{2}{9}b^2c^3x^{\frac{9}{2}} + \frac{4}{3}abc^2dx^{\frac{9}{2}} + \frac{2}{3}a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 6/17*b^2*c*d^2*x^(17/2) + 4/17*a*b*d^3*x^(17/2) + 6
/13*b^2*c^2*d*x^(13/2) + 12/13*a*b*c*d^2*x^(13/2) + 2/13*a^2*d^3*x^(13/2) +
 2/9*b^2*c^3*x^(9/2) + 4/3*a*b*c^2*d*x^(9/2) + 2/3*a^2*c*d^2*x^(9/2) + 4/5*
a*b*c^3*x^(5/2) + 6/5*a^2*c^2*d*x^(5/2) + 2*a^2*c^3*sqrt(x)
```

3.412 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc$$

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

Rubi [A] time = 0.0638812, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^2c^3}{x^{3/2}}+ac^2(2bc+3ad)\sqrt{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5/2}+d\left(3b^2c^2+6abcd+a^2d^2\right)x\right.\\ &= -\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2(2bc+3ad)x^{3/2}+\frac{2}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+\frac{2}{11}d\left(3b^2c^2+6abcd+a\right.\end{aligned}$$

Mathematica [A] time = 0.0513395, size = 137, normalized size = 1.

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

Maple [A] time = 0.007, size = 138, normalized size = 1.

$$-\frac{-2310\,b^2d^3x^{10}-5852\,x^8abd^3-8778\,x^8b^2cd^2-3990\,x^6a^2d^3-23940\,x^6abcd^2-11970\,x^6b^2c^2d-18810\,x^4a^2cd^2-37}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x)
```

```
[Out] -2/21945*(-1155*b^2*d^3*x^10-2926*a*b*d^3*x^8-4389*b^2*c*d^2*x^8-1995*a^2*d
^3*x^6-11970*a*b*c*d^2*x^6-5985*b^2*c^2*d*x^6-9405*a^2*c*d^2*x^4-18810*a*b*
c^2*d*x^4-3135*b^2*c^3*x^4-21945*a^2*c^2*d*x^2-14630*a*b*c^3*x^2+21945*a^2*
c^3)/x^(1/2)
```

Maxima [A] time = 1.06686, size = 171, normalized size = 1.25

$$\frac{2}{19}\,b^2d^3x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{11}{2}}-\frac{2\,a^2c^3}{\sqrt{x}}+\frac{2}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/15*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(15/2) + 2/11*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(11/2) - 2*a^2*c^3/sqrt(x) + 2/7*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(7/2) + 2/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
3/2)
```

Fricas [A] time = 0.860847, size = 304, normalized size = 2.22

$$\frac{2\left(1155\,b^2d^3x^{10}+1463\left(3\,b^2cd^2+2\,abd^3\right)x^8+1995\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-21945\,a^2c^3+3135\left(b^2c^3+6\,abc^2d\right.}{21945\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^3*x^10 + 1463*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1995*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 21945*a^2*c^3 + 3135*(b^2*c^3 + 6*a
*b*c^2*d + 3*a^2*c*d^2)*x^4 + 7315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/sqrt(x)
```

Sympy [A] time = 11.564, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{\sqrt{x}}+2a^2c^2dx^{\frac{3}{2}}+\frac{6a^2cd^2x^{\frac{7}{2}}}{7}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{4abc^3x^{\frac{3}{2}}}{3}+\frac{12abc^2dx^{\frac{7}{2}}}{7}+\frac{12abcd^2x^{\frac{11}{2}}}{11}+\frac{4abd^3x^{\frac{15}{2}}}{15}+\frac{2b^2c^3x^{\frac{7}{2}}}{7}+\frac{6b^2c^2}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

```
[Out] -2*a**2*c**3/sqrt(x) + 2*a**2*c**2*d*x**(3/2) + 6*a**2*c*d**2*x**(7/2)/7 +
2*a**2*d**3*x**(11/2)/11 + 4*a*b*c**3*x**(3/2)/3 + 12*a*b*c**2*d*x**(7/2)/7
 + 12*a*b*c*d**2*x**(11/2)/11 + 4*a*b*d**3*x**(15/2)/15 + 2*b**2*c**3*x**(7
/2)/7 + 6*b**2*c**2*d*x**(11/2)/11 + 2*b**2*c*d**2*x**(15/2)/5 + 2*b**2*d**
3*x**(19/2)/19
```

Giac [A] time = 1.15516, size = 182, normalized size = 1.33

$$\frac{2}{19}b^2d^3x^{\frac{19}{2}} + \frac{2}{5}b^2cd^2x^{\frac{15}{2}} + \frac{4}{15}abd^3x^{\frac{15}{2}} + \frac{6}{11}b^2c^2dx^{\frac{11}{2}} + \frac{12}{11}abcd^2x^{\frac{11}{2}} + \frac{2}{11}a^2d^3x^{\frac{11}{2}} + \frac{2}{7}b^2c^3x^{\frac{7}{2}} + \frac{12}{7}abc^2dx^{\frac{7}{2}} + \frac{6}{7}a^2cd^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/5*b^2*c*d^2*x^(15/2) + 4/15*a*b*d^3*x^(15/2) + 6/
11*b^2*c^2*d*x^(11/2) + 12/11*a*b*c*d^2*x^(11/2) + 2/11*a^2*d^3*x^(11/2) +
2/7*b^2*c^3*x^(7/2) + 12/7*a*b*c^2*d*x^(7/2) + 6/7*a^2*c*d^2*x^(7/2) + 4/3*
a*b*c^3*x^(3/2) + 2*a^2*c^2*d*x^(3/2) - 2*a^2*c^3/sqrt(x)
```

3.413 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3b$$

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

Rubi [A] time = 0.0644083, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3b$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{5/2}}\,dx=\int \left(\frac{a^2c^3}{x^{5/2}}+\frac{ac^2(2bc+3ad)}{\sqrt{x}}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+d\left(3b^2c^2+6abcd+a^2d^2\right)x$$

$$=-\frac{2a^2c^3}{3x^{3/2}}+2ac^2(2bc+3ad)\sqrt{x}+\frac{2}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5/2}+\frac{2}{9}d\left(3b^2c^2+6abcd+\right.$$

Mathematica [A] time = 0.0523587, size = 137, normalized size = 1.

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3b$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
```

```
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

Maple [A] time = 0.007, size = 138, normalized size = 1.

$$-\frac{-1170\,b^2d^3x^{10} - 3060\,x^8abd^3 - 4590\,x^8b^2cd^2 - 2210\,x^6a^2d^3 - 13260\,x^6abcd^2 - 6630\,x^6b^2c^2d - 11934\,x^4a^2cd^2 - 23868\,x \cdots}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x)
```

```
[Out] -2/9945*(-585*b^2*d^3*x^10-1530*a*b*d^3*x^8-2295*b^2*c*d^2*x^8-1105*a^2*d^3
*x^6-6630*a*b*c*d^2*x^6-3315*b^2*c^2*d*x^6-5967*a^2*c*d^2*x^4-11934*a*b*c^2
*d*x^4-1989*b^2*c^3*x^4-29835*a^2*c^2*d*x^2-19890*a*b*c^3*x^2+3315*a^2*c^3)
/x^(3/2)
```

Maxima [A] time = 1.08122, size = 171, normalized size = 1.25

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}} + \frac{2}{13}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{13}{2}} + \frac{2}{9}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{9}{2}} - \frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}} + \frac{2}{5}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{5}{2}} + 2\,(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 2/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(13/2) + 2/9*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(9/2) - 2/3*a^2*c^3/x^(3/2) + 2/5*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(5/2) + 2*(2*a*b*c^3 + 3*a^2*c^2*d)*sqrt(
x)
```

Fricas [A] time = 0.905874, size = 298, normalized size = 2.18

$$\frac{2\left(585\,b^2d^3x^{10} + 765\left(3\,b^2cd^2 + 2\,abd^3\right)x^8 + 1105\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^6 - 3315\,a^2c^3 + 1989\left(b^2c^3 + 6\,abc^2d + 3\,a^2 \cdots\right.\right.}{9945\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^3*x^10 + 765*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1105*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 3315*a^2*c^3 + 1989*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^4 + 9945*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(3/2)
```

Sympy [A] time = 12.5521, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{3x^{\frac{3}{2}}} + 6a^2c^2d\sqrt{x} + \frac{6a^2cd^2x^{\frac{5}{2}}}{5} + \frac{2a^2d^3x^{\frac{9}{2}}}{9} + 4abc^3\sqrt{x} + \frac{12abc^2dx^{\frac{5}{2}}}{5} + \frac{4abcd^2x^{\frac{9}{2}}}{3} + \frac{4abd^3x^{\frac{13}{2}}}{13} + \frac{2b^2c^3x^{\frac{5}{2}}}{5} + \frac{2b^2c^2dx^{\frac{9}{2}}}{3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)
```

```
[Out] -2*a**2*c**3/(3*x**(3/2)) + 6*a**2*c**2*d*sqrt(x) + 6*a**2*c*d**2*x**(5/2)/
5 + 2*a**2*d**3*x**(9/2)/9 + 4*a*b*c**3*sqrt(x) + 12*a*b*c**2*d*x**(5/2)/5
+ 4*a*b*c*d**2*x**(9/2)/3 + 4*a*b*d**3*x**(13/2)/13 + 2*b**2*c**3*x**(5/2)/
5 + 2*b**2*c**2*d*x**(9/2)/3 + 6*b**2*c*d**2*x**(13/2)/13 + 2*b**2*d**3*x**
(17/2)/17
```

Giac [A] time = 1.18935, size = 182, normalized size = 1.33

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}}+\frac{6}{13}\,b^2cd^2x^{\frac{13}{2}}+\frac{4}{13}\,abd^3x^{\frac{13}{2}}+\frac{2}{3}\,b^2c^2dx^{\frac{9}{2}}+\frac{4}{3}\,abcd^2x^{\frac{9}{2}}+\frac{2}{9}\,a^2d^3x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^3x^{\frac{5}{2}}+\frac{12}{5}\,abc^2dx^{\frac{5}{2}}+\frac{6}{5}\,a^2cd^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 6/13*b^2*c*d^2*x^(13/2) + 4/13*a*b*d^3*x^(13/2) + 2
/3*b^2*c^2*d*x^(9/2) + 4/3*a*b*c*d^2*x^(9/2) + 2/9*a^2*d^3*x^(9/2) + 2/5*b^
2*c^3*x^(5/2) + 12/5*a*b*c^2*d*x^(5/2) + 6/5*a^2*c*d^2*x^(5/2) + 4*a*b*c^3*
sqrt(x) + 6*a^2*c^2*d*sqrt(x) - 2/3*a^2*c^3/x^(3/2)
```

3.414 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}$$

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(11/2))/11 + (2*b^2*d^3*x^(15/2
))/15
```

Rubi [A] time = 0.0627778, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(11/2))/11 + (2*b^2*d^3*x^(15/2
))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^2c^3}{x^{7/2}}+\frac{ac^2(2bc+3ad)}{x^{3/2}}+c\left(b^2c^2+6abcd+3a^2d^2\right)\sqrt{x}+d\left(3b^2c^2+6abcd+a^2d^2\right)x^{5/2}\right.\\ &= -\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(2bc+3ad)}{\sqrt{x}}+\frac{2}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+\frac{2}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)\end{aligned}$$

Mathematica [A] time = 0.0481395, size = 121, normalized size = 0.88

$$\frac{2\left(165dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+385cx^4\left(3a^2d^2+6abcd+b^2c^2\right)-231a^2c^3-1155ac^2x^2(3ad+2bc)+105bd^2x^8(2ad+\right.}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^3 - 1155*a*c^2*(2*b*c + 3*a*d)*x^2 + 385*c*(b^2*c^2 + 6*a*b*
c*d + 3*a^2*d^2)*x^4 + 165*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 105*b*
d^2*(3*b*c + 2*a*d)*x^8 + 77*b^2*d^3*x^10))/(1155*x^(5/2))
```

Maple [A] time = 0.006, size = 138, normalized size = 1.

$$-\frac{-154\,b^{2}d^{3}x^{10}-420\,x^{8}abd^{3}-630\,x^{8}b^{2}cd^{2}-330\,x^{6}a^{2}d^{3}-1980\,x^{6}abcd^{2}-990\,x^{6}b^{2}c^{2}d-2310\,x^{4}a^{2}cd^{2}-4620\,x^{4}abc^{2}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^3*x^10-210*a*b*d^3*x^8-315*b^2*c*d^2*x^8-165*a^2*d^3*x^6
-990*a*b*c*d^2*x^6-495*b^2*c^2*d*x^6-1155*a^2*c*d^2*x^4-2310*a*b*c^2*d*x^4-
385*b^2*c^3*x^4+3465*a^2*c^2*d*x^2+2310*a*b*c^3*x^2+231*a^2*c^3)/x^(5/2)
```

Maxima [A] time = 1.06531, size = 174, normalized size = 1.27

$$\frac{2}{15}\,b^{2}d^{3}x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{\frac{11}{2}}+\frac{2}{7}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{7}{2}}+\frac{2}{3}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{\frac{3}{2}}-\frac{2\left(a^{2}c^{3}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 2/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(11/2) + 2/7*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(7/2) + 2/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^(3/2) - 2/5*(a^2*c^3 + 5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

Fricas [A] time = 0.806183, size = 293, normalized size = 2.14

$$\frac{2\left(77\,b^{2}d^{3}x^{10}+105\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{8}+165\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{6}-231\,a^{2}c^{3}+385\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd\right.\right.}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^3*x^10 + 105*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 165*(3*b^2*c^
2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 231*a^2*c^3 + 385*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^4 - 1155*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

Sympy [A] time = 16.2194, size = 185, normalized size = 1.35

$$-\frac{2a^{2}c^{3}}{5x^{\frac{5}{2}}}-\frac{6a^{2}c^{2}d}{\sqrt{x}}+2a^{2}cd^{2}x^{\frac{3}{2}}+\frac{2a^{2}d^{3}x^{\frac{7}{2}}}{7}-\frac{4abc^{3}}{\sqrt{x}}+4abc^{2}dx^{\frac{3}{2}}+\frac{12abcd^{2}x^{\frac{7}{2}}}{7}+\frac{4abd^{3}x^{\frac{11}{2}}}{11}+\frac{2b^{2}c^{3}x^{\frac{3}{2}}}{3}+\frac{6b^{2}c^{2}dx^{\frac{7}{2}}}{7}+\frac{6b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

```
[Out] -2*a**2*c**3/(5*x**(5/2)) - 6*a**2*c**2*d/sqrt(x) + 2*a**2*c*d**2*x**(3/2)
+ 2*a**2*d**3*x**(7/2)/7 - 4*a*b*c**3/sqrt(x) + 4*a*b*c**2*d*x**(3/2) + 12*
a*b*c*d**2*x**(7/2)/7 + 4*a*b*d**3*x**(11/2)/11 + 2*b**2*c**3*x**(3/2)/3 +
6*b**2*c**2*d*x**(7/2)/7 + 6*b**2*c*d**2*x**(11/2)/11 + 2*b**2*d**3*x**(15/
2)/15
```

Giac [A] time = 1.16136, size = 185, normalized size = 1.35

$$\frac{2}{15}\,b^2d^3x^{\frac{15}{2}} + \frac{6}{11}\,b^2cd^2x^{\frac{11}{2}} + \frac{4}{11}\,abd^3x^{\frac{11}{2}} + \frac{6}{7}\,b^2c^2dx^{\frac{7}{2}} + \frac{12}{7}\,abcd^2x^{\frac{7}{2}} + \frac{2}{7}\,a^2d^3x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^3x^{\frac{3}{2}} + 4\,abc^2dx^{\frac{3}{2}} + 2\,a^2cd^2x^{\frac{3}{2}} - \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 6/11*b^2*c*d^2*x^(11/2) + 4/11*a*b*d^3*x^(11/2) + 6
/7*b^2*c^2*d*x^(7/2) + 12/7*a*b*c*d^2*x^(7/2) + 2/7*a^2*d^3*x^(7/2) + 2/3*b
^2*c^3*x^(3/2) + 4*a*b*c^2*d*x^(3/2) + 2*a^2*c*d^2*x^(3/2) - 2/5*(10*a*b*c^
3*x^2 + 15*a^2*c^2*d*x^2 + a^2*c^3)/x^(5/2)
```

3.415 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=311

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\tan^{-}}{\sqrt{2}d^{1}}$$

```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d^3) - (2*b
*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*d) - (c^(5/4)*(b*c -
 a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) +
 (c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqr
t[2]*d^(17/4)) - (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)
)
```

Rubi [A] time = 0.312552, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\tan^{-}}{\sqrt{2}d^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```


```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d^3) - (2*b
*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*d) - (c^(5/4)*(b*c -
 a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) +
 (c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqr
t[2]*d^(17/4)) - (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)
)
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{7/2}}{d^2}+\frac{b^2x^{11/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{7/2}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(bc-ad)^2\int \frac{x^{7/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{\left(c(bc-ad)^2\right)\int \frac{x^{3/2}}{c+dx^2}\,dx}{d^3}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^2(bc-ad)^2\right)\int \frac{1}{\sqrt{x}(c+dx^2}}{d^4}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(2c^2(bc-ad)^2\right)\mathrm{Subst}\left(\int\right.}{d^4}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^{3/2}(bc-ad)^2\right)\mathrm{Subst}\left(\int\right.}{d^4}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^{3/2}(bc-ad)^2\right)\mathrm{Subst}\left(\int\right.}{2d}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{c}-\right.}{2\sqrt{2}d^{1}}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\right.}{\sqrt{2}d^{17/4}}$$

Mathematica [A] time = 0.134275, size = 299, normalized size = 0.96

$$-585\sqrt{2}c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+585\sqrt{2}c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-1170$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-4680*c*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 936*d^(5/4)*(b*c - a*d)^2*x^(5/2)
- 520*b*d^(9/4)*(b*c - 2*a*d)*x^(9/2) + 360*b^2*d^(13/4)*x^(13/2) - 1170*Sq
rt[2]*c^(5/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] +
 1170*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)] - 585*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x] + 585*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2340*d^(17/4))
```

Maple [B] time = 0.015, size = 545, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/13*b^2*x^(13/2)/d+4/9/d*x^(9/2)*a*b-2/9/d^2*x^(9/2)*b^2*c+2/5/d*x^(5/2)*a
^2-4/5/d^2*x^(5/2)*a*b*c+2/5/d^3*x^(5/2)*b^2*c^2-2/d^2*a^2*c*x^(1/2)+4/d^3*
a*b*c^2*x^(1/2)-2/d^4*b^2*c^3*x^(1/2)+1/2*c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-c^2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2*c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*a^2-c^2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*b^2+1/4*c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-1/2*
c^2/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/4*c^3/d^4*(c/d)^(1/4)*2^
(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.11715, size = 2813, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(b^8
*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4
*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 +
a^8*c^5*d^8)/d^17) + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b
*c^3*d^3 + a^4*c^2*d^4)*x)*d^13*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c
^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 2
8*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4) - (b^2*c^3*d
^13 - 2*a*b*c^2*d^14 + a^2*c*d^15)*sqrt(x)*(-(b^8*c^13 - 8*a*b^7*c^12*d + 2
8*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*
c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4))/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8)) + 585*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*
d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^
6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(d^4*(-(b^8*c
^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b
^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^
8*c^5*d^8)/d^17)^(1/4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) - 585
*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
```

```
*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(-d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d +
 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^
3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)
 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) + 4*(45*b^2*d^3*x^6 - 585*b
^2*c^3 + 1170*a*b*c^2*d - 585*a^2*c*d^2 - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^4 +
117*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40537, size = 589, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}} abc^2d + \left(cd^3\right)^{\frac{1}{4}} a^2cd^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}} abc^2d + (cd}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1
/4)*a^2*c*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(
1/4))/d^5 + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d
+ (c*d^3)^(1/4)*a^2*c*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/d^5 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/
4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/d^5 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*
b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/d^5 + 2/585*(45*b^2*d^12*x^(13/2) - 65*b^2*c*d^11*x^(9/2) + 130*a
*b*d^12*x^(9/2) + 117*b^2*c^2*d^10*x^(5/2) - 234*a*b*c*d^11*x^(5/2) + 117*a
^2*d^12*x^(5/2) - 585*b^2*c^3*d^9*sqrt(x) + 1170*a*b*c^2*d^10*sqrt(x) - 585
*a^2*c*d^11*sqrt(x))/d^13
```

3.416 $\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=290

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\right.}{\sqrt{2}d^{15/4}}$$

```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(7*d^2) + (
2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]
*d^(15/4)) + (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

Rubi [A] time = 0.254077, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\right.}{\sqrt{2}d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```


```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(7*d^2) + (
2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]
*d^(15/4)) + (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{5/2}}{d^2}+\frac{b^2x^{9/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{5/2}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{(bc-ad)^2\int \frac{x^{5/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(c(bc-ad)^2\right)\int \frac{\sqrt{x}}{c+dx^2}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(2c(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{\left(c(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^{7/2}}-\frac{(c(\cdots}{\cdots}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(c(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2d^4}-\cdots$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\cdots$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-a\cdots}{\cdots}$$

Mathematica [A] time = 0.104447, size = 276, normalized size = 0.95

$$\frac{-231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+462\sqrt{2}c^{3\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (616*d^(3/4)*(b*c - a*d)^2*x^(3/2) - 264*b*d^(7/4)*(b*c - 2*a*d)*x^(7/2) +
168*b^2*d^(11/4)*x^(11/2) + 462*Sqrt[2]*c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 462*Sqrt[2]*c^(3/4)*(b*c - a*d)^2*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 231*Sqrt[2]*c^(3/4)*(b*c - a*d)^2
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + 231*Sqrt[2]*c
^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(924*d^(15/4))
```

Maple [B] time = 0.01, size = 504, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/11*b^2*x^(11/2)/d+4/7/d*x^(7/2)*a*b-2/7/d^2*x^(7/2)*b^2*c+2/3/d*x^(3/2)*a
^2-4/3/d^2*x^(3/2)*c*a*b+2/3/d^3*x^(3/2)*b^2*c^2-1/2*c/d^2/(c/d)^(1/4)*2^(1
```

```
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+c^2/d^3/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+c^2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4*c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
))*a^2+1/2*c^2/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-1/4*c^3/d^4/(c/
d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.18633, size = 3549, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/462*(924*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*arctan((sqrt((b^12*c^16 - 12*a*
b^11*c^15*d + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^1
2*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 +
 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a^11
*b*c^5*d^11 + a^12*c^4*d^12)*x - (b^8*c^11*d^7 - 8*a*b^7*c^10*d^8 + 28*a^2*
b^6*c^9*d^9 - 56*a^3*b^5*c^8*d^10 + 70*a^4*b^4*c^7*d^11 - 56*a^5*b^3*c^6*d^
12 + 28*a^6*b^2*c^5*d^13 - 8*a^7*b*c^4*d^14 + a^8*c^3*d^15)*sqrt(-(b^8*c^11
 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^
7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3
*d^8)/d^15))*d^4*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3
*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6
 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4) - (b^6*c^8*d^4 - 6*a*b^5*c^7*
d^5 + 15*a^2*b^4*c^6*d^6 - 20*a^3*b^3*c^5*d^7 + 15*a^4*b^2*c^4*d^8 - 6*a^5*
b*c^3*d^9 + a^6*c^2*d^10)*sqrt(x)*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4))/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
^8)) - 231*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(d^11*(-(b^8*c^11 - 8*a*b^7*
c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*
a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)
```

```
^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 231*d^3*(
-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*
a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7
 + a^8*c^3*d^8)/d^15)^(1/4)*log(-d^11*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3/4) + (b^6*c
^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c
^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 4*(21*b^2*d^2*x^5 - 33*(
b^2*c*d - 2*a*b*d^2)*x^3 + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)*sqrt(x))/d
^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20272, size = 520, normalized size = 1.79

$$-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^6} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a\right.}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^6 - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^6 + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*
c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^6 - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^6 +
 2/231*(21*b^2*d^10*x^(11/2) - 33*b^2*c*d^9*x^(7/2) + 66*a*b*d^10*x^(7/2) +
 77*b^2*c^2*d^8*x^(3/2) - 154*a*b*c*d^9*x^(3/2) + 77*a^2*d^10*x^(3/2))/d^11
```

3.417 $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=288

$$-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2\sqrt{x}(bc-ad)^2}{d^3}+\frac{\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{\cdots}\right.}{2\sqrt{2}d^{13/4}}$$

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^2) + (2*b^
2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13
/4)) - (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rubi [A] time = 0.239919, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2\sqrt{x}(bc-ad)^2}{d^3}+\frac{\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{\cdots}\right.}{2\sqrt{2}d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^2) + (2*b^
2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13
/4)) - (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{3/2}}{d^2}+\frac{b^2x^{7/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{3/2}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{(bc-ad)^2\int \frac{x^{3/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(c(bc-ad)^2\right)\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(2c(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^3}-$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right.}{2d^{7/2}}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-a}{}$$

Mathematica [A] time = 0.101572, size = 276, normalized size = 0.96

$$-72bd^{5/4}x^{5/2}(bc-2ad)+360\sqrt[4]{d}\sqrt{x}(bc-ad)^2+45\sqrt{2}\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-45\sqrt{2}\sqrt[4]{c}(bc-a$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (360*d^(1/4)*(b*c - a*d)^2*Sqrt[x] - 72*b*d^(5/4)*(b*c - 2*a*d)*x^(5/2) + 4
0*b^2*d^(9/4)*x^(9/2) + 90*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)] - 90*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(
b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(1
80*d^(13/4))
```

Maple [B] time = 0.01, size = 495, normalized size = 1.7

$$\frac{2\,b^2}{9\,d}x^{\frac{9}{2}}+\frac{4\,ab}{5\,d}x^{\frac{5}{2}}-\frac{2\,b^2c}{5\,d^2}x^{\frac{5}{2}}+2\,\frac{a^2\sqrt{x}}{d}-4\,\frac{abc\sqrt{x}}{d^2}+2\,\frac{b^2c^2\sqrt{x}}{d^3}-\frac{\sqrt{2}a^2}{2\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}abc}{d^2}\sqrt[4]{\frac{c}{d}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/9*b^2*x^(9/2)/d+4/5/d*x^(5/2)*a*b-2/5/d^2*x^(5/2)*b^2*c+2/d*a^2*x^(1/2)-4
/d^2*c*a*b*x^(1/2)+2/d^3*b^2*c^2*x^(1/2)-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*a*b*c-1/2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)*b^2*c^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*a^2+1/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a*b*c-1/2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1)*b^2*c^2-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/2/d^2*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b*c-1/4/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.12437, size = 2633, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/90*(180*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5
*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8
*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*arctan((sqrt(d^6*sqrt(-(b^8*c^9 - 8
*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d
^13) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)*x)*d^10*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^
6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^
7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4) - (b^2*c^2*d^10 - 2*a*b*c*d^11 + a^2*d
^12)*sqrt(x)*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c
^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a
^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4))/(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^
6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 +
28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)) + 45*d^3*(-(b^8*c^9 - 8*
a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
- 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^
13)^(1/4)*log(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*
b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6
- 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(x)) - 45*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^
3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^
6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(-d^3*(-(b^8*c^9 - 8*a*b^7*
c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a
```

```
^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1
/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(5*b^2*d^2*x^4 + 45*b^2*
c^2 - 90*a*b*c*d + 45*a^2*d^2 - 9*(b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(x))/d^3
```

Sympy [A] time = 126.589, size = 661, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9), Eq(
c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2
)/13)/c, Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
)/d, Eq(c, 0)), ((-1)**(1/4)*a**2*c**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)*
*(1/4) + sqrt(x))/(2*d**3*(1/d)**(7/4)) - (-1)**(1/4)*a**2*c**(1/4)*log((-1
)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**3*(1/d)**(7/4)) + (-1)**(1/
4)*a**2*c**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**3*(1
/d)**(7/4)) + 2*a**2*sqrt(x)/d - (-1)**(1/4)*a*b*c**(5/4)*log(-(-1)**(1/4)*
c**(1/4)*(1/d)**(1/4) + sqrt(x))/(d**4*(1/d)**(7/4)) + (-1)**(1/4)*a*b*c**(
5/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(d**4*(1/d)**(7/4)) -
 2*(-1)**(1/4)*a*b*c**(5/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)
))/(d**4*(1/d)**(7/4)) - 4*a*b*c*sqrt(x)/d**2 + 4*a*b*x**(5/2)/(5*d) + (-1)
**(1/4)*b**2*c**(9/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*
d**5*(1/d)**(7/4)) - (-1)**(1/4)*b**2*c**(9/4)*log((-1)**(1/4)*c**(1/4)*(1/
d)**(1/4) + sqrt(x))/(2*d**5*(1/d)**(7/4)) + (-1)**(1/4)*b**2*c**(9/4)*atan
((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**5*(1/d)**(7/4)) + 2*b**2*
c**2*sqrt(x)/d**3 - 2*b**2*c*x**(5/2)/(5*d**2) + 2*b**2*x**(9/2)/(9*d), Tru
e))
```

Giac [A] time = 1.16225, size = 520, normalized size = 1.81

$$-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^4}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)\right.}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^4 - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^4 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^4 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^4 +
 2/45*(5*b^2*d^8*x^(9/2) - 9*b^2*c*d^7*x^(5/2) + 18*a*b*d^8*x^(5/2) + 45*b^
2*c^2*d^6*sqrt(x) - 90*a*b*c*d^7*sqrt(x) + 45*a^2*d^8*sqrt(x))/d^9
```

3.418 $\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=268

$$-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)}{}$$

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11
/4))
```

Rubi [A] time = 0.233037, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11
/4))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)\sqrt{x}}{d^2}+\frac{b^2x^{5/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{x}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\int\frac{\sqrt{x}}{c+dx^2}\,dx}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int\frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^{5/2}}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{\sqrt{c}+}{c+}\right.}{d^{5/2}}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2d^3}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{c}+\right.}{2\sqrt{2}\sqrt[4]{c}}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+$$

Mathematica [A] time = 0.112958, size = 249, normalized size = 0.93

$$\frac{-56b\sqrt[4]{c}d^{3/4}x^{3/2}(bc-2ad)+21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{84\sqrt[4]{c}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-56*b*c^(1/4)*d^(3/4)*(b*c - 2*a*d)*x^(3/2) + 24*b^2*c^(1/4)*d^(7/4)*x^(7/
2) - 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
 + 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] +
 21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x] - 21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(84*c^(1/4)*d^(11/4))
```

Maple [B] time = 0.009, size = 461, normalized size = 1.7

$$\frac{2b^2}{7d}x^{\frac{7}{2}}+\frac{4ab}{3d}x^{\frac{3}{2}}-\frac{2b^2c}{3d^2}x^{\frac{3}{2}}+\frac{\sqrt{2}a^2}{2d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{\sqrt{2}abc}{d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}b^2c^2}{2d^3}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x)
```

```
[Out] 2/7*b^2*x^(7/2)/d+4/3*b/d*x^(3/2)*a-2/3*b^2/d^2*x^(3/2)*c+1/2/d/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*c+1/2/d^3/(c/d)^(1/4)*2^(1/2)*a
```

```
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2*c^2+1/2/d/(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-1/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*c+1/2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*b^2*c^2+1/4/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*
a^2-1/2/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b*c+1/4/d^3/(c/d)^(1/4)*
2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2)))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.1626, size = 3380, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/42*(84*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)*x - (b^8*c^9*d^5 - 8*a*b^7*c^8*d^6 + 28*a^2*b^6*c^7*d^7 - 56*
a^3*b^5*c^6*d^8 + 70*a^4*b^4*c^5*d^9 - 56*a^5*b^3*c^4*d^10 + 28*a^6*b^2*c^3
*d^11 - 8*a^7*b*c^2*d^12 + a^8*c*d^13)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*
a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*
d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11)))*d^3*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c*d^11))^(1/4) - (b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20
*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8 + a^6*d^9)*sqrt(x)*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c*d^11))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*
d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(c*d^8
*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)
) + 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
```

```
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(-c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 4*(3*b^2*d*x^3 - 7*(b^2*c
 - 2*a*b*d)*x)*sqrt(x))/d^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)

[Out] Exception raised: TypeError
```

Giac [A] time = 1.16849, size = 487, normalized size = 1.82

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2d\right.}{2\,cd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="giac")

[Out] 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^5) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^5) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/
4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^5) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*
b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^5) + 2/21*(3*b^2*d^6*x^(7/2) - 7*b^2*c*d^5*x^(3/2) + 14*a*b*d^6*
x^(3/2))/d^7
```

3.419 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)}\,dx$

Optimal. Leaf size=266

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}c^{3/4}d^{9/4}}$$

```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c - a*d)^2*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) + (
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/
4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rubi [A] time = 0.211948, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```


```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c - a*d)^2*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) + (
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/
4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)}dx = \int \left(-\frac{b(bc-2ad)}{d^2\sqrt{x}} + \frac{b^2x^{3/2}}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2\sqrt{x}(c+dx^2)}\right)dx$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}(c+dx^2)}dx}{d^2}$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}dx, x, \sqrt{x}\right)}{d^2}$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}dx, x, \sqrt{x}\right)}{\sqrt{c}d^2} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{\sqrt{c}}{c}\right.}{\sqrt{c}d^2}$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx, x, \sqrt{x}\right)}{2\sqrt{c}d^{5/2}} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\right.}{2}$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2\log\left(\sqrt{c}+\right.}{2\sqrt{2}c}$$
$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}$$

Mathematica [A] time = 0.110403, size = 249, normalized size = 0.94

$$\frac{-40bc^{3/4}\sqrt[4]{d}\sqrt{x}(bc-2ad) - 5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right) + 5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{20c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*Sqrt[x] + 8*b^2*c^(3/4)*d^(5/4)*x^(5/2
) - 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
+ 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] -
5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x] + 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(20*c^(3/4)*d^(9/4))
```

Maple [B] time = 0.009, size = 452, normalized size = 1.7

$$\frac{2b^2}{5d}x^{\frac{5}{2}} + 4\frac{ab\sqrt{x}}{d} - 2\frac{b^2\sqrt{x}c}{d^2} + \frac{\sqrt{2}a^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{\sqrt{2}ab}{d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{c\sqrt{2}b^2}{2d^2}\sqrt[4]{\frac{c}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2)/d+4*b/d*a*x^(1/2)-2*b^2/d^2*x^(1/2)*c+1/2*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d*(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(
```

```
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*a^2-1/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b+1/2/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-1/2/d*(c/d)^
(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.15252, size = 2581, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan((sqrt(c^2*d^4*sqrt(-(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c
^3*d^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*x)*c^2*d^7*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4) - (b^2*c^4*d^7 - 2*a*b*c^3*d^8
 + a^2*c^2*d^9)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 5*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*sqrt(x)) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(b^2*d*x^2
 - 5*b^2*c + 10*a*b*d)*sqrt(x))/d^2
```

Sympy [A] time = 30.7488, size = 612, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5
/2)/5)/d, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/
9)/c, Eq(d, 0)), (-(-1)**(1/4)*a**2*d**10*(1/d)**(41/4)*log(-(-1)**(1/4)*c*
*(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(3/4)) + (-1)**(1/4)*a**2*d**10*(1/d)*
*(41/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(3/4)) - (-1
)**(1/4)*a**2*d**10*(1/d)**(41/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)*
*(1/4)))/c**(3/4) + (-1)**(1/4)*a*b*c**(1/4)*d**9*(1/d)**(41/4)*log(-(-1)**
(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x)) - (-1)**(1/4)*a*b*c**(1/4)*d**9*(1/d
)**(41/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x)) + 2*(-1)**(1/4)*
a*b*c**(1/4)*d**9*(1/d)**(41/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(
1/4))) + 4*a*b*sqrt(x)/d - (-1)**(1/4)*b**2*c**(5/4)*d**8*(1/d)**(41/4)*log
(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/2 + (-1)**(1/4)*b**2*c**(5/4
)*d**8*(1/d)**(41/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/2 - (
-1)**(1/4)*b**2*c**(5/4)*d**8*(1/d)**(41/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1
/4)*(1/d)**(1/4))) - 2*b**2*c*sqrt(x)/d**2 + 2*b**2*x**(5/2)/(5*d), True))
```

Giac [A] time = 1.15362, size = 486, normalized size = 1.83

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2 \left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{3}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2 \left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}}\right.}{2\,cd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^3) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^3) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^3) + 2/5*(b^2*d^4*x^(5/2) - 5*b^2*c*d^3*sqrt(x) + 10*a*b*d^4*sqr
t(x))/d^5
```

3.420 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\right.}{\sqrt{2}c^{5/4}d^{7/4}}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^
2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4))
- ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*c^(5/4)*d^(7/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```

Rubi [A] time = 0.266351, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\right.}{\sqrt{2}c^{5/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```


```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^
2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4))
- ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*c^(5/4)*d^(7/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = -\frac{2a^2}{c\sqrt{x}}+\frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-ad)+\frac{1}{2}b^2cx^2\right)}{c+dx^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{cd^{3/2}}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{\right.}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2cd^2}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\right.}{2cd^2}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{2\sqrt{2}c^{5/4}d^{7/4}}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.122271, size = 261, normalized size = 1.

$$\frac{-24a^2\sqrt[4]{c}d^{7/4}-3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+}{12c^{5/4}d^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (-24*a^2*c^(1/4)*d^(7/4) + 8*b^2*c^(5/4)*d^(3/4)*x^2 + 6*Sqrt[2]*(b*c - a*d
)^2*Sqrt[x]*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 6*Sqrt[2]*(b*c
- a*d)^2*Sqrt[x]*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 3*Sqrt[2]*
(b*c - a*d)^2*Sqrt[x]*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x] + 3*Sqrt[2]*(b*c - a*d)^2*Sqrt[x]*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(12*c^(5/4)*d^(7/4)*Sqrt[x])
```

Maple [B] time = 0.011, size = 439, normalized size = 1.7

$$\frac{2b^2}{3d}x^{\frac{3}{2}}-\frac{\sqrt{2}a^2}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}ab}{d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{c\sqrt{2}b^2}{2d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x)
```

```
[Out] 2/3*b^2*x^(3/2)/d-1/2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)*a^2+1/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*
```

```
b-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1
/2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/d/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2*c/d^2/(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4/c/(c/d)^(1/4)*2^
(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2)))*a^2+1/2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-1
/4*c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2*a^2/c/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.23888, size = 3394, normalized size = 13.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="fricas")

[Out] 1/6*(12*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11
*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)*x - (b^8*c^11*d^3 - 8*a*b^7*c^10*d^4 + 28*a^2*b^6*c^9*d^5 -
56*a^3*b^5*c^8*d^6 + 70*a^4*b^4*c^7*d^7 - 56*a^5*b^3*c^6*d^8 + 28*a^6*b^2*c
^5*d^9 - 8*a^7*b*c^4*d^10 + a^8*c^3*d^11)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7)))*c*d^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^5*d^7))^(1/4) - (b^6*c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d
^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)
*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(
1/4)*log(c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 +
 a^6*d^6)*sqrt(x)) + 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
```

```
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*log(-c^4*d^5*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 4*(
b^2*c*x^2 - 3*a^2*d)*sqrt(x))/(c*d*x)
```

Sympy [A] time = 28.8324, size = 597, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3),
Eq(c, 0) & Eq(d, 0)), ((-2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2
)/7)/c, Eq(d, 0)), ((-2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)
/3)/d, Eq(c, 0)), (-2*a**2/(c*sqrt(x)) + (-1)**(3/4)*a**2*log(-(-1)**(1/4)*
c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*d**14*(1/d)**(57/4)) - (-1)**(
3/4)*a**2*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*d**1
4*(1/d)**(57/4)) - (-1)**(3/4)*a**2*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d
)**(1/4)))/(c**(5/4)*d**14*(1/d)**(57/4)) - (-1)**(3/4)*a*b*log(-(-1)**(1/4
)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(c**(1/4)*d**15*(1/d)**(57/4)) + (-1)**(
3/4)*a*b*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(c**(1/4)*d**15*(
1/d)**(57/4)) + 2*(-1)**(3/4)*a*b*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)*
*(1/4)))/(c**(1/4)*d**15*(1/d)**(57/4)) + (-1)**(3/4)*b**2*c**(3/4)*log(-(-
1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**16*(1/d)**(57/4)) - (-1)**
(3/4)*b**2*c**(3/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**
16*(1/d)**(57/4)) - (-1)**(3/4)*b**2*c**(3/4)*atan((-1)**(3/4)*sqrt(x)/(c**
(1/4)*(1/d)**(1/4)))/(d**16*(1/d)**(57/4)) + 2*b**2*x**(3/2)/(3*d), True))
```

Giac [A] time = 1.21228, size = 464, normalized size = 1.78

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d}-\frac{2\,a^2}{c\sqrt{x}}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^4}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2
- 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/2*sqrt(2)*((c*d^3)^
(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) + 1/4*sq
rt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) - 1/4*sqrt(
2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4)
```

3.421 $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)} dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\tan^{-1}}{\sqrt{2}c^{7/4}}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) +
((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rubi [A] time = 0.264903, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\tan^{-1}}{\sqrt{2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) +
((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = -\frac{2a^2}{3cx^{3/2}} + \frac{2\int \frac{\frac{3}{2}a(2bc-ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}(c+dx^2)}\,dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}(c+dx^2)}\,dx}{cd}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{cd}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}d} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx,\right.}{c^{3/2}d}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}d^{3/2}} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{d}}}\right.}{2c^{3/2}d^{3/2}}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\right.}{2\sqrt{2}c^{7/4}d^{5/4}}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} + \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc-a}{}$$

Mathematica [A] time = 0.116511, size = 261, normalized size = 1.

$$\frac{-8a^2c^{3/4}d^{5/4} + 3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right) - 3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{12c^{7/4}d^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-8*a^2*c^(3/4)*d^(5/4) + 24*b^2*c^(7/4)*d^(1/4)*x^2 + 6*Sqrt[2]*(b*c - a*d
)^2*x^(3/2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 6*Sqrt[2]*(b*c
- a*d)^2*x^(3/2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 3*Sqrt[2]*
(b*c - a*d)^2*x^(3/2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x] - 3*Sqrt[2]*(b*c - a*d)^2*x^(3/2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(12*c^(7/4)*d^(5/4)*x^(3/2))
```

Maple [B] time = 0.011, size = 439, normalized size = 1.7

$$2\frac{b^2\sqrt{x}}{d} - \frac{d\sqrt{2}a^2}{2c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{\sqrt{2}ab}{c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{\sqrt{2}b^2}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x)
```

```
[Out] 2*b^2*x^(1/2)/d-1/2/c^2*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*a^2+1/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
```

```
a*b-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2
/c^2*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/c*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2/d*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4/c^2*d*(c/d)^(1/4)*
2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b
-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.09562, size = 2591, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="fricas")

[Out] -1/6*(12*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*arctan((sqrt(c^4*d^2*sqrt(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^7*d^5)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*x)*c^5*d^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4) - (b^2*c^7*d^4 - 2*a*b*c^6
*d^5 + a^2*c^5*d^6)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4))/(b^8*c^8 - 8*a*b^7*c
^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^
5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 3*c*d*x^2*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(x)) - 3*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(-c^
2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) -
4*(3*b^2*c*x^2 - a^2*d)*sqrt(x))/(c*d*x^2)
```

Sympy [A] time = 52.9118, size = 581, normalized size = 2.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x))/d, Eq(c, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x*
*(5/2)/5)/c, Eq(d, 0)), (-2*a**2/(3*c*x**(3/2)) + (-1)**(1/4)*a**2*log(-(-1
)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(7/4)*(1/d)**(3/4)) - (-1)*
*(1/4)*a**2*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(7/4)*(1
/d)**(3/4)) + (-1)**(1/4)*a**2*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1
/4)))/(c**(7/4)*(1/d)**(3/4)) - (-1)**(1/4)*a*b*log(-(-1)**(1/4)*c**(1/4)*(
1/d)**(1/4) + sqrt(x))/(c**(3/4)*d*(1/d)**(3/4)) + (-1)**(1/4)*a*b*log((-1)
**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(c**(3/4)*d*(1/d)**(3/4)) - 2*(-1)
**(1/4)*a*b*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(c**(3/4)*d*(
1/d)**(3/4)) + (-1)**(1/4)*b**2*c**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(
1/4) + sqrt(x))/(2*d**2*(1/d)**(3/4)) - (-1)**(1/4)*b**2*c**(1/4)*log((-1)*
*(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**2*(1/d)**(3/4)) + (-1)**(1/4)
*b**2*c**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**2*(1/d
)**(3/4)) + 2*b**2*sqrt(x)/d, True))
```

Giac [A] time = 1.16316, size = 464, normalized size = 1.78

$$\frac{2\,b^2\sqrt{x}}{d} - \frac{2\,a^2}{3\,cx^{\frac{3}{2}}} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}} abcd + \left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
- 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/2*sqrt(2)*((c*d^3)^
(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/4*sq
rt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2) + 1/4*sqrt(
2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2)
```

3.422 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1\right.}{\sqrt{2}c^{9/4}d^{3/}}$$

```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*c - a*d)^2
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) +
 ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(
9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```

Rubi [A] time = 0.280172, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 453, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1\right.}{\sqrt{2}c^{9/4}d^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```


```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*c - a*d)^2
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) +
 ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(
9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = -\frac{2a^2}{5cx^{5/2}} + \frac{2\int \frac{\frac{5}{2}a(2bc-ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2\sqrt{d}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,d\right.}{c^2\sqrt{d}}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2d} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}+}\right.}{2c^2d}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\right.}{2\sqrt{2}c^{9/4}d^{3/4}}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}} + \frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}} + \frac{(bc-}{}$$

Mathematica [A] time = 0.113732, size = 254, normalized size = 0.95

$$\frac{-\frac{8a^2c^{5/4}}{x^{5/2}} + \frac{5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}} - \frac{5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}} - \frac{10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{3/4}} + \frac{10\sqrt{2}(bc-ad)^2}{}}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a^2*c^(5/4))/x^(5/2) + (40*a*c^(1/4)*(-2*b*c + a*d))/Sqrt[x] - (10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) +
 (10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d
^(3/4) + (5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/d^(3/4) - (5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(20*c^(9/4))
```

Maple [B] time = 0.012, size = 452, normalized size = 1.7

$$\frac{d\sqrt{2}a^2}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - \frac{\sqrt{2}ab}{c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{\sqrt{2}b^2}{2d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{d\sqrt{2}a^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x)
```

```
[Out] 1/2/c^2*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2/d/(c/d)^
```

```
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2/c^2*d/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-1/c/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/2/d/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/4/c^2*d/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))*a^2-1/2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/4/d/(c/d)^(1/
4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/5*a^2/c/x^(5/2)+2*a^2/c^2/x^(1/2)*d-4*a/c
/x^(1/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.19988, size = 3421, normalized size = 12.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)*x - (b^8*c^13*d - 8*a*b^7*c^12*d^2 + 28*a^2*b^6*c^11*d^3
 - 56*a^3*b^5*c^10*d^4 + 70*a^4*b^4*c^9*d^5 - 56*a^5*b^3*c^8*d^6 + 28*a^6*b
^2*c^7*d^7 - 8*a^7*b*c^6*d^8 + a^8*c^5*d^9)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3)))*c^2*d*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^9*d^3))^(1/4) - (b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d
^3 - 20*a^3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^
7)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 5*c^2*x^3*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3
))^(1/4)*log(c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a
^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d
```

```
^5 + a^6*d^6)*sqrt(x)) + 5*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*log(-c^7*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x))
 + 4*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1824, size = 477, normalized size = 1.79

$$-\frac{2\left(10\,abcx^2-5\,a^2dx^2+a^2c\right)}{5\,c^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -2/5*(10*a*b*c*x^2 - 5*a^2*d*x^2 + a^2*c)/(c^2*x^(5/2)) + 1/2*sqrt(2)*((c*d
^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan
(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/2
*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a
^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/
(c^3*d^3) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^
3*d^3) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*
d^3)
```

3.423 $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)} dx$

Optimal. Leaf size=269

$$-\frac{2a^2}{7cx^{7/2}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}-$$

```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1
/4))
```

Rubi [A] time = 0.275532, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 453, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2}{7cx^{7/2}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```


```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1
/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = -\frac{2a^2}{7cx^{7/2}} + \frac{2\int \frac{\frac{7}{2}a(2bc-ad)+\frac{7}{2}b^2cx^2}{x^{5/2}(c+dx^2)}\,dx}{7c}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}(c+dx^2)}\,dx}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{d}}{c+dx^4}\right.}{c^{5/2}}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}\sqrt{d}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int -\right.}{2c^{5/2}}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\right.}{2\sqrt{2}c^{11/4}}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}} - \frac{(b}{}$$

Mathematica [A] time = 0.115048, size = 254, normalized size = 0.94

$$\frac{-\frac{24a^2c^{7/4}}{x^{7/2}} + \frac{56ac^{3/4}(ad-2bc)}{x^{3/2}} - \frac{21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}} + \frac{21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}} - \frac{42\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\right.}{\sqrt[4]{d}}}{84c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] ((-24*a^2*c^(7/4))/x^(7/2) + (56*a*c^(3/4)*(-2*b*c + a*d))/x^(3/2) - (42*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(1/4)
+ (42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
d^(1/4) - (21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/d^(1/4) + (21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(1/4))/(84*c^(11/4))
```

Maple [B] time = 0.013, size = 461, normalized size = 1.7

$$\frac{\sqrt{2}a^2d^2}{2c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) - \frac{\sqrt{2}abd}{c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{2}b^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x)
```

```
[Out] 1/2/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2*d^2-1
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*d+1/2/c*
```

```
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/4/c^3*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2*d^2-1/2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))*a*b*d+1/4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/2/c^3*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2-1/c^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d+1/2/c*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2/7*a^2/c/x^(7/2)+2/3*a^2
/c^2/x^(3/2)*d-4/3*a/c/x^(3/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.07121, size = 2587, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*arctan((sqrt(c^6*sqrt(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*x)*c^8*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4) - (b^2*c^10*d - 2*a*b*c^9*d^2 + a^2
*c^8*d^3)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 21*c^2*x^4*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(
c^11*d))^(1/4)*log(c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sqrt(x)) - 21*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(-c^3*(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(3*a^2*c + 7*(2
```

```
*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17359, size = 478, normalized size = 1.78

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^3d} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}\right.}{2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^3*d) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c^3*d) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^3*d) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c^3*d) - 2/21*(14*a*b*c*x^2 - 7*a^2*d*x^2 + 3*a^2*c)/(c^2*x^(7/2))
```

3.424 $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=288

$$\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}-\frac{\sqrt[4]{b}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{}\right.}{2\sqrt{2}a^{13/4}}$$

```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2*(b*c - a*
d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^
2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(13/4)) + (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.316244, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}-\frac{\sqrt[4]{b}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{}\right.}{2\sqrt{2}a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2*(b*c - a*
d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^
2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(13/4)) + (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}dx = -\frac{2c^2}{9ax^{9/2}} + \frac{2\int \frac{-\frac{9}{2}c(bc-2ad)+\frac{9}{2}ad^2x^2}{x^{7/2}(a+bx^2)}dx}{9a}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} + \frac{(bc-ad)^2\int \frac{1}{x^{3/2}(a+bx^2)}dx}{a^2}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(b(bc-ad)^2)\int \frac{\sqrt{x}}{a+bx^2}dx}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(2b(bc-ad)^2)\operatorname{Subst}\left(\int \frac{x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{(\sqrt{b}(bc-ad)^2)\operatorname{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^3} - \frac{(\sqrt{b}(bc-}{}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x, \sqrt{x}\right)}{2a^3} - \frac{(bc-ad)^2}{}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}} + \frac{\sqrt[4]{b}(bc-}{}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}} - \frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}}{\sqrt{2}a^{13/}}$$

Mathematica [C] time = 0.195968, size = 101, normalized size = 0.35

$$-\frac{2\left(a\left(a^2\left(5c^2+18cdx^2+45d^2x^4\right)-9abcx^2\left(c+10dx^2\right)+45b^2c^2x^4\right)+15bx^6(bc-ad)^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{45a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 10*d*x^2) + a^2*(5*c^2 + 18*c*d*x
^2 + 45*d^2*x^4)) + 15*b*(b*c - a*d)^2*x^6*Hypergeometric2F1[3/4, 1, 7/4, -
((b*x^2)/a)]))/(45*a^4*x^(9/2))
```

Maple [B] time = 0.013, size = 495, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x)
```

```
[Out] -2/9*c^2/a/x^(9/2)-2/a/x^(1/2)*d^2+4/a^2/x^(1/2)*c*b*d-2/a^3/x^(1/2)*b^2*c^
2-4/5*c/a/x^(5/2)*d+2/5*c^2/a^2/x^(5/2)*b-1/2/a/(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^2+1/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b*c*d-1/2/a^3/(1/b*a)^(1/4)*2^(1/2)*arctan(
```

```
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b^2*c^2-1/2/a/(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^2+1/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b*c*d-1/2/a^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b^2*c^2-1/4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(
1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2
)+(1/b*a)^(1/2)))*d^2+1/2/a^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(
1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
)*b*c*d-1/4/a^3/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(
1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.11924, size = 3498, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*
b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6
- 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*arctan((sqrt((b^14*c^12 - 12*a*b
^13*c^11*d + 66*a^2*b^12*c^10*d^2 - 220*a^3*b^11*c^9*d^3 + 495*a^4*b^10*c^8
*d^4 - 792*a^5*b^9*c^7*d^5 + 924*a^6*b^8*c^6*d^6 - 792*a^7*b^7*c^5*d^7 + 49
5*a^8*b^6*c^4*d^8 - 220*a^9*b^5*c^3*d^9 + 66*a^10*b^4*c^2*d^10 - 12*a^11*b^
3*c*d^11 + a^12*b^2*d^12)*x - (a^7*b^9*c^8 - 8*a^8*b^8*c^7*d + 28*a^9*b^7*c
^6*d^2 - 56*a^10*b^6*c^5*d^3 + 70*a^11*b^5*c^4*d^4 - 56*a^12*b^4*c^3*d^5 +
28*a^13*b^3*c^2*d^6 - 8*a^14*b^2*c*d^7 + a^15*b*d^8)*sqrt(-(b^9*c^8 - 8*a*b
^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 5
6*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)
)*a^3*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3
+ 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*
c*d^7 + a^8*b*d^8)/a^13)^(1/4) - (a^3*b^7*c^6 - 6*a^4*b^6*c^5*d + 15*a^5*b^
5*c^4*d^2 - 20*a^6*b^4*c^3*d^3 + 15*a^7*b^3*c^2*d^4 - 6*a^8*b^2*c*d^5 + a^9
*b*d^6)*sqrt(x)*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^
6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 -
8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2
*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5
 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)) - 45*a^3*x^5*(-(b^9*c
^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c
^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*
d^8)/a^13)^(1/4)*log(a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 -
 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*
c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5
*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b
^2*c*d^5 + a^6*b*d^6)*sqrt(x)) + 45*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28
```

```
*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3
*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(-a
^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 +
70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*
d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^
2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*
sqrt(x)) - 4*(45*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 5*a^2*c^2 - 9*(a*b*c
^2 - 2*a^2*c*d)*x^2)*sqrt(x))/(a^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.68607, size = 527, normalized size = 1.83

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^4b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a\right.}{2\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4
))/(a^4*b^2) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d
 + (a*b^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3
)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^4*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(
3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a^4*b^2) - 2/45*(45*b^2*c^2*x^4 - 90*a*b*c*d*x^4 + 45*a^2*d^2
*x^4 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 5*a^2*c^2)/(a^3*x^(9/2))
```

3.425 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=375

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{\sqrt{x}(13bc-5ad)(bc-ad)}{2d^4}+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{8\sqrt{2}d^{17/4}}$$

```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d)*(b*c - a
*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b*c - a*d)^2*x^(9/2))
/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c
- 5*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*d^(17/4)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*
b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rubi [A] time = 0.437007, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{\sqrt{x}(13bc-5ad)(bc-ad)}{2d^4}+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{8\sqrt{2}d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d)*(b*c - a
*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b*c - a*d)^2*x^(9/2))
/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c
- 5*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*d^(17/4)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*
b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx=\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{\int \frac{x^{7/2}\left(\frac{1}{2}(3bc-5ad)(3bc-ad)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$=\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{((13bc-5ad)(bc-ad))\int \frac{x^{7/2}}{c+dx^2}\,dx}{4cd^2}$$

$$=-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}+\frac{((13bc-5ad)(bc-ad))\int \frac{x^{3/2}}{c+dx^2}\,dx}{4d^3}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{(c(13bc-}{}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{(c(13bc-}{}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{(\sqrt{c}(13bc}{}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}-\frac{(\sqrt{c}(13bc}{}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}+\frac{\sqrt[4]{c}(13bc-}{}$$

$$=\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}+\frac{\sqrt[4]{c}(13bc-}{}$$

Mathematica [A] time = 0.345805, size = 372, normalized size = 0.99

$$\frac{1440\sqrt[4]{d}\sqrt{x}\left(a^2d^2-4abcd+3b^2c^2\right)+45\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-45\sqrt{2}\sqrt[4]{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (1440*d^(1/4)*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[x] - 576*b*d^(5/4)*(b*
c - a*d)*x^(5/2) + 160*b^2*d^(9/4)*x^(9/2) + (360*c*d^(1/4)*(b*c - a*d)^2*S
qrt[x])/(c + d*x^2) + 90*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 90*Sqrt[2]*c^(1/4)*(13*
b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)] + 45*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(13*b^
2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(720*d^(17/4))
```

Maple [A] time = 0.017, size = 563, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/9*b^2*x^(9/2)/d^2+4/5/d^2*x^(5/2)*a*b-4/5/d^3*x^(5/2)*b^2*c+2/d^2*a^2*x^(
1/2)-8/d^3*c*a*b*x^(1/2)+6/d^4*b^2*c^2*x^(1/2)+1/2*c/d^2*x^(1/2)/(d*x^2+c)*
a^2-c^2/d^3*x^(1/2)/(d*x^2+c)*a*b+1/2*c^3/d^4*x^(1/2)/(d*x^2+c)*b^2-5/8/d^2
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+9/4*c/d^3*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-13/8*c^2/d^4*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-5/8/d^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+9/4*c/d^3*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-13/8*c^2/d^4*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-5/16/d^2*(c/d)^(1/4)*2^
(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2)))*a^2+9/8*c/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a
*b-13/16*c^2/d^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.20173, size = 3289, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 37247
6*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 18684
0*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*
d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d
 + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4
 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 62
5*a^8*c*d^8)/d^17) + (169*b^4*c^4 - 468*a*b^3*c^3*d + 454*a^2*b^2*c^2*d^2 -
 180*a^3*b*c*d^3 + 25*a^4*d^4)*x)*d^13*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*
d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^
4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 6
25*a^8*c*d^8)/d^17)^(3/4) - (13*b^2*c^2*d^13 - 18*a*b*c*d^14 + 5*a^2*d^15)*
sqrt(x)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 48
5784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55
```

```
100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(3/4))/(285
61*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c
^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^
3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)) + 45*(d^5*x^2 + c*d^4)*(-(2856
1*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^
6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3
*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(d^4*(-(28561*b^8
*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3
 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6
- 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*sqrt(x)) - 45*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*
b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^
4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2
*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(-d^4*(-(28561*b^8*c^9 - 158184*a*b^7*
c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^
5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7
 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(
x)) - 4*(20*b^2*d^3*x^6 + 585*b^2*c^3 - 810*a*b*c^2*d + 225*a^2*c*d^2 - 4*(
13*b^2*c*d^2 - 18*a*b*d^3)*x^4 + 36*(13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^
3)*x^2)*sqrt(x))/(d^5*x^2 + c*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17975, size = 594, normalized size = 1.58

$$-\frac{\sqrt{2}\left(13\left(cd^3\right)^{\frac{1}{4}}b^2c^2-18\left(cd^3\right)^{\frac{1}{4}}abcd+5\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,d^5}-\frac{\sqrt{2}\left(13\left(cd^3\right)^{\frac{1}{4}}b^2c^2-18\left(cd^3\right)^{\frac{1}{4}}ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/d^5 - 1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b
*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/d^5 - 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(
c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/d^5 + 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*
d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/d^5 + 1/2*(b^2*c^3*sqrt(x) - 2*a*b*c^2*d*sqrt(x) + a^2
*c*d^2*sqrt(x))/((d*x^2 + c)*d^4) + 2/45*(5*b^2*d^16*x^(9/2) - 18*b^2*c*d^1
5*x^(5/2) + 18*a*b*d^16*x^(5/2) + 135*b^2*c^2*d^14*sqrt(x) - 180*a*b*c*d^15
*sqrt(x) + 45*a^2*d^16*sqrt(x))/d^18
```

3.426 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=346

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3}+\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}}-\frac{(11bc-3ad)(bc-}{}$$

```
[Out] -((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(6*c*d^3) + (2*b^2*x^(7/2))/(7*d^2)
 + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11*b*c - 3*a*d)*(b*c -
 a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(
15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)
*d^(15/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rubi [A] time = 0.317422, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3}+\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}}-\frac{(11bc-3ad)(bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(6*c*d^3) + (2*b^2*x^(7/2))/(7*d^2)
 + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11*b*c - 3*a*d)*(b*c -
 a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(
15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)
*d^(15/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}\left(-4a^2d^2+7(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{((11bc-3ad)(bc-ad))\int \frac{x^{5/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4d^3}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{c+dx^4}\right.}{2d^3}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{((11bc-3ad)(bc-ad))\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}}{c+dx^4}\right.}{4d^{7/2}}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}-\sqrt{2}}\right.}{8d^4}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\right.}{8\sqrt{2}\sqrt[4]{c}d^{15/4}}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{}}{\sqrt[4]{c}}\right.}{4\sqrt{2}\sqrt[4]{c}d^{15/4}}$$

Mathematica [A] time = 0.182954, size = 337, normalized size = 0.97

$$\frac{\frac{21\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}} - \frac{21\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}} - \frac{42\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\tan}{\sqrt[4]{c}}}{336d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-448*b*d^(3/4)*(b*c - a*d)*x^(3/2) + 96*b^2*d^(7/4)*x^(7/2) - (168*d^(3/4)
*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (42*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d
+ 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(1/4) + (42*S
qrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/c^(1/4) + (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4) - (21
*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4))/(336*d^(15/4))
```

Maple [A] time = 0.017, size = 523, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/7*b^2*x^(7/2)/d^2+4/3*b/d^2*x^(3/2)*a-4/3*b^2/d^3*x^(3/2)*c-1/2/d*x^(3/2)
/(d*x^2+c)*a^2+1/d^2*x^(3/2)/(d*x^2+c)*c*a*b-1/2/d^3*x^(3/2)/(d*x^2+c)*b^2*
c^2-7/8/d^3/(c/d)^(1/4)*2^(1/2)*c*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-7/4/d^3/(c/d)^(1/4)*
2^(1/2)*c*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-7/4/d^3/(c/d)^(1/4)*2^(
1/2)*c*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+11/16/d^4/(c/d)^(1/4)*2^(1
/2)*b^2*c^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2)))+11/8/d^4/(c/d)^(1/4)*2^(1/2)*b^2*c^2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)+11/8/d^4/(c/d)^(1/4)*2^(1/2)*b^2*c^2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16/d^2/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+3/8/d^2/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3
/8/d^2/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.2089, size = 4166, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*
a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a
^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*
d^15))^(1/4)*arctan((sqrt((1771561*b^12*c^12 - 13528284*a*b^11*c^11*d + 459
43458*a^2*b^10*c^10*d^2 - 91492940*a^3*b^9*c^9*d^3 + 118659255*a^4*b^8*c^8*
d^4 - 105323064*a^5*b^7*c^7*d^5 + 65490076*a^6*b^6*c^6*d^6 - 28724472*a^7*b
^5*c^5*d^7 + 8825895*a^8*b^4*c^4*d^8 - 1855980*a^9*b^3*c^3*d^9 + 254178*a^1
0*b^2*c^2*d^10 - 20412*a^11*b*c*d^11 + 729*a^12*d^12)*x - (14641*b^8*c^9*d^
7 - 74536*a*b^7*c^8*d^8 + 158268*a^2*b^6*c^7*d^9 - 181720*a^3*b^5*c^6*d^10
+ 122566*a^4*b^4*c^5*d^11 - 49560*a^5*b^3*c^4*d^12 + 11772*a^6*b^2*c^3*d^13
 - 1512*a^7*b*c^2*d^14 + 81*a^8*c*d^15)*sqrt(-(14641*b^8*c^8 - 74536*a*b^7*
c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^
4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 +
81*a^8*d^8)/(c*d^15)))*d^4*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^
2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5
*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^
15))^(1/4) - (1331*b^6*c^6*d^4 - 5082*a*b^5*c^5*d^5 + 7557*a^2*b^4*c^4*d^6
- 5516*a^3*b^3*c^3*d^7 + 2061*a^4*b^2*c^2*d^8 - 378*a^5*b*c*d^9 + 27*a^6*d^
10)*sqrt(x)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 -
 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 +
```

```
11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4))/(14
641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c
^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2
*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)) - 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*
c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 +
 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1
512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(c*d^11*(-(14641*b^8*c^8 -
 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 1225
66*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a
^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d
 + 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378
*a^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) + 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8
 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 12
2566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512
*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(-c*d^11*(-(14641*b^8*c^8 - 7
4536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566
*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7
*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d +
 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a
^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) - 4*(12*b^2*d^2*x^5 - 4*(11*b^2*c*d - 14*
a*b*d^2)*x^3 - 7*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(d^4*x^2
 + c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21053, size = 558, normalized size = 1.61

$$-\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\left(dx^2+c\right)d^3} + \frac{\sqrt{2}\left(11\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 14\left(cd^3\right)^{\frac{3}{4}}abcd + 3\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^6} + \frac{\sqrt{2}(1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*d
^3) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*
(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(c*d^6) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^
(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^6) - 1/16*sqrt(2)*(11*(c*d^3)^(3/4)*b
^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 1/16*sqrt(2)*(11*(c*d^3)^(3/4)
*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
```

```
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 2/21*(3*b^2*d^12*x^(7/2) -
14*b^2*c*d^11*x^(3/2) + 14*a*b*d^12*x^(3/2))/d^14
```

3.427 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=346

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}-\frac{(bc-ad)(9}{}$$

```
[Out] -((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(2*c*d^3) + (2*b^2*x^(5/2))/(5*d^2) +
((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(9*b*c - a*d)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(13/4))
 + ((b*c - a*d)*(9*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*d^(13/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4)) +
 ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```

Rubi [A] time = 0.328888, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}-\frac{(bc-ad)(9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] -((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(2*c*d^3) + (2*b^2*x^(5/2))/(5*d^2) +
((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(9*b*c - a*d)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(13/4))
 + ((b*c - a*d)*(9*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*d^(13/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4)) +
 ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}\left(-4a^2d^2+5(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}dx}{2cd^2}$$

$$= \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(9bc-ad))\int \frac{x^{3/2}}{c+dx^2}dx}{4cd^2}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}dx}{4d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}\left(\int \frac{1}{c+dx^4}dx, x, \right.}{2d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}dx\right.}{4\sqrt{c}d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\right.}{8\sqrt{c}d^{7/2}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(9bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{8\sqrt{2}c^{3/4}d^{13/4}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}d^{13/4}} +$$

Mathematica [A] time = 0.177896, size = 333, normalized size = 0.96

$$\frac{-\frac{5\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}+\frac{5\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}-\frac{10\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\tan^{-1}\left(1-\right.}{c^{3/4}}}{80d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-320*b*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^2*d^(5/4)*x^(5/2) - (40*d^(1/4)*
(b*c - a*d)^2*Sqrt[x])/(c + d*x^2) - (10*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d +
a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) + (10*Sqrt[
2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/c^(3/4) - (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4) + (5*Sqrt[2]*(9*
b^2*c^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/c^(3/4))/(80*d^(13/4))
```

Maple [A] time = 0.014, size = 523, normalized size = 1.5

$$\frac{2b^2}{5d^2}x^{\frac{5}{2}} + 4\frac{ab\sqrt{x}}{d^2} - 4\frac{b^2\sqrt{x}c}{d^3} - \frac{a^2}{2d\left(dx^2+c\right)}\sqrt{x} + \frac{abc}{d^2\left(dx^2+c\right)}\sqrt{x} - \frac{b^2c^2}{2d^3\left(dx^2+c\right)}\sqrt{x} + \frac{\sqrt{2}a^2}{8cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/5*b^2*x^(5/2)/d^2+4*b/d^2*a*x^(1/2)-4*b^2/d^3*x^(1/2)*c-1/2/d*x^(1/2)/(d*
x^2+c)*a^2+1/d^2*x^(1/2)/(d*x^2+c)*c*a*b-1/2/d^3*x^(1/2)/(d*x^2+c)*b^2*c^2+
1/8/d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-5/4/d
^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+9/8/d^3*(c
/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/8/d*(c/d)^(
1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-5/4/d^2*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+9/8/d^3*(c/d)^(1/4)*c*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/16/d*(c/d)^(1/4)/c*2^(1/
2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))*a^2-5/8/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+9/
16/d^3*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.08931, size = 3036, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(20*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*
b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*
c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)
*arctan((sqrt(c^2*d^6*sqrt(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b
^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c
^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13)) + (81*
b^4*c^4 - 180*a*b^3*c^3*d + 118*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + a^4*d^4)
*x)*c^2*d^10*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 -
45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*
a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(3/4) - (9*b^2*c^4*
d^10 - 10*a*b*c^3*d^11 + a^2*c^2*d^12)*sqrt(x)*(-(6561*b^8*c^8 - 29160*a*b^
7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4
*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^
8)/(c^3*d^13))^(3/4))/(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6
*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5
 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)) + 5*(d^4*x^2 + c*d^3)*(
-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*
c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^
```

```
6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*log(c*d^3*(-(6561*b^8*c^8 -
 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*
a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d
^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x
)) - 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^
6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^
3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*l
og(-c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 457
20*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6
*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 1
0*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(4*b^2*d^2*x^4 - 45*b^2*c^2 + 50*a*b*c*d
- 5*a^2*d^2 - 4*(9*b^2*c*d - 10*a*b*d^2)*x^2)*sqrt(x))/(d^4*x^2 + c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21174, size = 551, normalized size = 1.59

$$\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^{4}}+\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd\right.\right.}{8\,cd^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1
/4))/(c*d^4) + 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(c*d^4) + 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(
c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c
*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^
2*d^2*sqrt(x))/((d*x^2 + c)*d^3) + 2/5*(b^2*d^8*x^(5/2) - 10*b^2*c*d^7*sqrt
(x) + 10*a*b*d^8*sqrt(x))/d^10
```

3.428 $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=310

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-a}{}$$

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d*x^2)) + (
(b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4
*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(5/4)*d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```

Rubi [A] time = 0.281799, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d*x^2)) + (
(b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4
*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(5/4)*d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-4a^2d^2+3(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{4cd^{5/2}} - \frac{((bc-ad)(7bc}{}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{8cd^3} - \frac{((bc-ad)}{}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(7bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}} + \frac{(bc-ad)(7bc}{}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{(bc-ad)(7bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}d^{11/4}} - \frac{(bc-ad)(7bc+ad)\tan}{4\sqrt{2}c^{5/4}d^{1}}$$

Mathematica [A] time = 0.178779, size = 319, normalized size = 1.03

$$-\frac{\frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}} + \frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}} + \frac{6\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\tan^{-1}}{c^{5/4}}}{48d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (32*b^2*d^(3/4)*x^(3/2) + (24*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c*(c + d*x^2)
) + (6*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/c^(5/4) - (6*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (3*Sqrt[2]*(7*b^2
*c^2 - 6*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(48*d^(11/4))
```

Maple [B] time = 0.014, size = 499, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 2/3*b^2*x^(3/2)/d^2+1/2/c*x^(3/2)/(d*x^2+c)*a^2-1/d*x^(3/2)/(d*x^2+c)*a*b+1
/2/d^2*c*x^(3/2)/(d*x^2+c)*b^2+1/8/d/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
```

```
c/d)^(1/4)*x^(1/2)-1)*a^2+3/4/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*a*b-7/8/d^3*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*b^2+1/16/d/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+3/8/d^2
/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-7/16/d^3*c/(c/d)^(1/4)*2^(1/2)*ln
((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))*b^2+1/8/d/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)*a^2+3/4/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)*a*b-7/8/d^3*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.14783, size = 3846, normalized size = 12.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a
^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*
c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)
*arctan((sqrt((117649*b^12*c^12 - 605052*a*b^11*c^11*d + 1195698*a^2*b^10*c
^10*d^2 - 1049580*a^3*b^9*c^9*d^3 + 247695*a^4*b^8*c^8*d^4 + 184968*a^5*b^7
*c^7*d^5 - 73604*a^6*b^6*c^6*d^6 - 26424*a^7*b^5*c^5*d^7 + 5055*a^8*b^4*c^4
*d^8 + 3060*a^9*b^3*c^3*d^9 + 498*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^
12*d^12)*x - (2401*b^8*c^11*d^5 - 8232*a*b^7*c^10*d^6 + 9212*a^2*b^6*c^9*d^
7 - 2520*a^3*b^5*c^8*d^8 - 1434*a^4*b^4*c^7*d^9 + 360*a^5*b^3*c^6*d^10 + 18
8*a^6*b^2*c^5*d^11 + 24*a^7*b*c^4*d^12 + a^8*c^3*d^13)*sqrt(-(2401*b^8*c^8
- 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4
*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 +
 a^8*d^8)/(c^5*d^11)))*c*d^3*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4) +
(343*b^6*c^7*d^3 - 882*a*b^5*c^6*d^4 + 609*a^2*b^4*c^5*d^5 + 36*a^3*b^3*c^4
*d^6 - 87*a^4*b^2*c^3*d^7 - 18*a^5*b*c^2*d^8 - a^6*c*d^9)*sqrt(x)*(-(2401*b
^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1
434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*
c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4))/(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212
*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^
3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)) - 3*(c*d^3*x^2
 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 252
0*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^
2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)*log(c^4*d^8*(-(2401
```

```
*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 -
 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*
b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6 - 882*a*b^5*c^5*d + 609
*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5
 - a^6*d^6)*sqrt(x)) + 3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7
*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4
 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c
^5*d^11))^(1/4)*log(-c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) -
(343*b^6*c^6 - 882*a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 -
 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x)) - 4*(4*b^2*c*d*x^3
 + (7*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(c*d^3*x^2 + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20098, size = 524, normalized size = 1.69

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2}+\frac{b^2c^2x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^2d^2x^{\frac{3}{2}}}{2\left(dx^2+c\right)cd^2}-\frac{\sqrt{2}\left(7\left(cd^3\right)^{\frac{3}{4}}b^2c^2-6\left(cd^3\right)^{\frac{3}{4}}abcd-\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^2 + 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^
(3/2))/((d*x^2 + c)*c*d^2) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^
3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b
^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) + 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5) - 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5)
```

3.429 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2} \, dx$

Optimal. Leaf size=312

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad)}{}$$

```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2)) + ((b*c
 - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rubi [A] time = 0.34237, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```


```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2)) + ((b*c
 - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2}\,dx = \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{\int \frac{\frac{1}{2}(bc-3ad)(bc+ad)-2b^2cdx^2}{\sqrt{x}(c+dx^2)}\,dx}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\int \frac{1}{\sqrt{x}(c+dx^2)}\,dx}{4cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}d^2} - \frac{((bc-ad)(5bc+}{}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{3/2}d^{5/2}} - \frac{((bc-ad)(5}{}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(5bc+}{}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(5bc+3ad)\tan^{-1}}{4\sqrt{2}c^{7/4}d^{9/4}}$$

Mathematica [A] time = 0.17088, size = 318, normalized size = 1.02

$$\frac{\frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} - \frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} + \frac{2\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\tan^{-1}\left(1-\right.}{c^{7/4}}}{16d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (32*b^2*d^(1/4)*Sqrt[x] + (8*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c*(c + d*x^2))
 + (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^
2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/c^(7/4) - (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(16*d^(9/4
))
```

Maple [B] time = 0.016, size = 496, normalized size = 1.6

$$2\,\frac{b^2\sqrt{x}}{d^2} + \frac{a^2}{2\,c\,(dx^2+c)}\sqrt{x} - \frac{ab}{d\,(dx^2+c)}\sqrt{x} + \frac{b^2c}{2\,d^2\,(dx^2+c)}\sqrt{x} + \frac{3\,\sqrt{2}a^2}{8\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{2}ab}{4\,cd}\sqrt[4]{\frac{c}{d}}\,\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2*b^2*x^(1/2)/d^2+1/2/c*x^(1/2)/(d*x^2+c)*a^2-1/d*x^(1/2)/(d*x^2+c)*a*b+1/2
/d^2*c*x^(1/2)/(d*x^2+c)*b^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)-1)*a^2+1/4/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b-5/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/8/d/c*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-5/16/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))*b^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)*a^2+1/4/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a
*b-5/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.05354, size = 2944, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^
6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^
5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*ar
ctan((sqrt(c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*
d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 32
4*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*b^4*c^4
- 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*
d^7*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*
c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 +
 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4) + (5*b^2*c^7*d^7 - 2*a*b*c^
6*d^8 - 3*a^2*c^5*d^9)*sqrt(x)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))
/(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*
b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^
4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^
8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2
*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3
*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)
```

```
 - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^3*x^2 + c^2*d^2)*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3
 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^
7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 1000
*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4
*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8
*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(
4*b^2*c*d*x^2 + 5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x))/(c*d^3*x^2 + c^2*
d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18832, size = 524, normalized size = 1.68

$$\frac{2\,b^2\sqrt{x}}{d^2}-\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{1}{4}}abcd-3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^3}-\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{1}{4}}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^2 - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*
a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
+ 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
- 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c
*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*lo
g(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/16*sqrt(2)*(5*
(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*
log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/2*(b^2*c^2*
sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c*d^2)
```

3.430 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2} \, dx$

Optimal. Leaf size=333

$$-\frac{x^{3/2}(5a^2d^2-2abcd+b^2c^2)}{2c^2d(c+dx^2)}-\frac{2a^2}{c\sqrt{x}(c+dx^2)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5a}{}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^(3/
2))/(2*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*d)*(3*
b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9
/4)*d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*
b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8
*Sqrt[2]*c^(9/4)*d^(7/4))
```

Rubi [A] time = 0.327528, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}(5a^2d^2-2abcd+b^2c^2)}{2c^2d(c+dx^2)}-\frac{2a^2}{c\sqrt{x}(c+dx^2)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```


```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^(3/
2))/(2*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*d)*(3*
b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9
/4)*d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*
b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8
*Sqrt[2]*c^(9/4)*d^(7/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{c\sqrt{x}(c+dx^2)} + \frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-5ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \cdots\right.}{2c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} - \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx, \cdots\right.}{4c^2d^{3/2}}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2 \cdots}\right.}{8c^2d^2}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\cdots\right.}{8\sqrt{2}c^{9/4}d^{7/4}}$$

$$= -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} - \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}d^{7/4}} + \cdots$$

Mathematica [A] time = 0.188094, size = 317, normalized size = 0.95

$$\frac{\frac{\sqrt{2}(-5a^2d^2+2abcd+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}} + \frac{\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}} + \frac{2\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\tan^{-1}\left(1-\cdots\right.}{d^{7/4}}\cdots}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(1/4))/Sqrt[x] - (8*c^(1/4)*(b*c - a*d)^2*x^(3/2))/(d*(c + d*x^
2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a
^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (Sqrt[2]*(
3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4))/(16*c
^(9/4))
```

Maple [A] time = 0.016, size = 495, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(3/2)/(d*x^2+c)*a^2+1/c*x^(3/2)/(d*x^2+c)*a*b-1/2/d*x^(3/2)/(d
*x^2+c)*b^2-5/8/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*a^2+1/4/c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*
b+3/8/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-5/8
/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/4/c/d/
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/8/d^2/(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-5/16/c^2/(c/d)^(1/
4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/8/c/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)*a*b+3/16/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2*a^2/c^2/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.21648, size = 3861, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*
arctan((sqrt((729*b^12*c^12 + 2916*a*b^11*c^11*d - 2430*a^2*b^10*c^10*d^2 -
 19980*a^3*b^9*c^9*d^3 + 135*a^4*b^8*c^8*d^4 + 59976*a^5*b^7*c^7*d^5 + 6364
*a^6*b^6*c^6*d^6 - 99960*a^7*b^5*c^5*d^7 + 375*a^8*b^4*c^4*d^8 + 92500*a^9*
b^3*c^3*d^9 - 18750*a^10*b^2*c^2*d^10 - 37500*a^11*b*c*d^11 + 15625*a^12*d^
12)*x - (81*b^8*c^13*d^3 + 216*a*b^7*c^12*d^4 - 324*a^2*b^6*c^11*d^5 - 984*
a^3*b^5*c^10*d^6 + 646*a^4*b^4*c^9*d^7 + 1640*a^5*b^3*c^8*d^8 - 900*a^6*b^2
*c^7*d^9 - 1000*a^7*b*c^6*d^10 + 625*a^8*c^5*d^11)*sqrt(-(81*b^8*c^8 + 216*
a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d
^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^
8*d^8)/(c^9*d^7)))*c^2*d^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^
6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 -
900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4) + (2
7*b^6*c^8*d^2 + 54*a*b^5*c^7*d^3 - 99*a^2*b^4*c^6*d^4 - 172*a^3*b^3*c^5*d^5
 + 165*a^4*b^2*c^4*d^6 + 150*a^5*b*c^3*d^7 - 125*a^6*c^2*d^8)*sqrt(x)*(-(81
*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 64
6*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4))/(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^
2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 98
4*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^
```

```
2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(c^7*d^5*(-
(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d -
99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*
c*d^5 - 125*a^6*d^6)*sqrt(x)) + (c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(c^9*d^7))^(1/4)*log(-c^7*d^5*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))
^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^
3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x)) - 4*(
4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(x))/(c^2*d^2*x^3 +
c^3*d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23198, size = 525, normalized size = 1.58

$$-\frac{b^2c^2x^2 - 2\,abcdx^2 + 5\,a^2d^2x^2 + 4\,a^2cd}{2\left(dx^{\frac{5}{2}} + c\sqrt{x}\right)c^2d} + \frac{\sqrt{2}\left(3\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 2\left(cd^3\right)^{\frac{3}{4}}abcd - 5\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right.}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{8\,c^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 5*a^2*d^2*x^2 + 4*a^2*c*d)/((d*x^(5/2)
+ c*sqrt(x))*c^2*d) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4
)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^
2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) - 1/16*sqrt(2)*(3*
(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*
log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4) + 1/16*sqrt(2)*(
3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4)
```

3.431 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad}{}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) - ((3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*S
qrt[x])/(6*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*
(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
11/4)*d^(5/4)) - ((b*c - a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*
Sqrt[2]*c^(11/4)*d^(5/4))
```

Rubi [A] time = 0.343347, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```


```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) - ((3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*S
qrt[x])/(6*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*
(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
11/4)*d^(5/4)) - ((b*c - a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*
Sqrt[2]*c^(11/4)*d^(5/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{3cx^{3/2}(c+dx^2)}+\frac{2\int \frac{\frac{1}{2}a(6bc-7ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}(c+dx^2)^2}dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\int \frac{1}{\sqrt{x}(c+dx^2)}dx}{4c^2d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\,\text{Subst}\left(\int \frac{1}{c+dx^4}dx,x,\sqrt{x}\right.}{2c^2d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}dx,x,\right.}{4c^{5/2}d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}d\right.}{8c^{5/2}d^{3/2}}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}-\frac{(bc-ad)(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{8\sqrt{2}c^{11/4}d^{5/4}}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}-\frac{(bc-ad)(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc}{}$$

Mathematica [A] time = 0.184479, size = 315, normalized size = 0.95

$$\frac{-\frac{3\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{3\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}-\frac{6\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\tan^{-1}\left(1-\right.}{d^{5/4}}}{48c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(3/4))/x^(3/2) - (24*c^(3/4)*(b*c - a*d)^2*Sqrt[x])/(d*(c + d*x
^2)) - (6*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (6*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (3*Sqrt[2]*(b^
2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/d^(5/4) + (3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4))/(48*c^(11/
4))
```

Maple [A] time = 0.017, size = 498, normalized size = 1.5

$$-\frac{a^2d}{2c^2(dx^2+c)}\sqrt{x}+\frac{ab}{c(dx^2+c)}\sqrt{x}-\frac{b^2}{2d(dx^2+c)}\sqrt{x}-\frac{7d\sqrt{2}a^2}{8c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\sqrt{2}ab}{4c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(1/2)/(d*x^2+c)*a^2+1/c*x^(1/2)/(d*x^2+c)*a*b-1/2/d*x^(1/2)/(d
*x^2+c)*b^2-7/8/c^3*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)*a^2+3/4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a*b+1/8/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-7
/8/c^3*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/4/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/8/c/d*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-7/16/c^3*d*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*a*b+1/16/c/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c^2/x
^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.967523, size = 2965, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b
^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*
d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(
1/4)*arctan((sqrt(c^6*d^2*sqrt(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6
*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 +
9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)) + (b^4*
c^4 + 12*a*b^3*c^3*d + 22*a^2*b^2*c^2*d^2 - 84*a^3*b*c*d^3 + 49*a^4*d^4)*x)
*c^8*d^4*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^
5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6
- 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3/4) + (b^2*c^10*d^4 + 6*a*
b*c^9*d^5 - 7*a^2*c^8*d^6)*sqrt(x)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^
6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d
^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3
/4))/(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3
- 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232
*a^7*b*c*d^7 + 2401*a^8*d^8)) + 3*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24
*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4
*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 240
1*a^8*d^8)/(c^11*d^5))^(1/4)*log(c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^
2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c
```

```
^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)
)^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) - 3*(c^2*d^2*x^4 + c^3
*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5
*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 -
 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)*log(-c^3*d*(-(b^8*c^8 +
 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*
c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 +
2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)
) + 4*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)*sqrt(x))/(c^2*d
^2*x^4 + c^3*d*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20827, size = 518, normalized size = 1.56

$$-\frac{2\,a^2}{3\,c^2x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd-7\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^2*x^(3/2)) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/
4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/2*(b^2*c^2*sqr
t(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c^2*d)
```

3.432 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2} dx$

Optimal. Leaf size=363

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\left(c+dx^2\right)}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-$$

```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5/2)*(c + d
*x^2)) - (5*b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)/(10*c^2*d*Sqrt[x]*(c + d*x^2)
) - ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a
*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(8*Sqrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*d^(3/4))
```

Rubi [A] time = 0.377162, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\left(c+dx^2\right)}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```


```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5/2)*(c + d
*x^2)) - (5*b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)/(10*c^2*d*Sqrt[x]*(c + d*x^2)
) - ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a
*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(8*Sqrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*d^(3/4))
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{5cx^{5/2}(c+dx^2)} + \frac{2\int \frac{\frac{1}{2}a(10bc-9ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)^2}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} - \frac{((bc-9ad)(bc-ad))\int \frac{1}{x^{3/2}(c+dx^2)}\,dx}{4c^2d}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} + \frac{((bc-9ad)(bc-ad))\int \frac{\sqrt{}}{c+}}{4c^3}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} + \frac{((bc-9ad)(bc-ad))\,\mathrm{Subs}}{2c^3}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} - \frac{((bc-9ad)(bc-ad))\,\mathrm{Subs}}{4c^3\sqrt{}}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} + \frac{((bc-9ad)(bc-ad))\,\mathrm{Subs}}{8}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} + \frac{(bc-9ad)(bc-ad)\log(\sqrt{}}{8\sqrt{2}c^{13}}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} - \frac{(bc-9ad)(bc-ad)\tan^{-1}}{4\sqrt{2}c^{13/4}d^{3/4}}$$

Mathematica [A] time = 0.198704, size = 333, normalized size = 0.92

$$\frac{\frac{5\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}} - \frac{5\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}} - \frac{10\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\tan^{-1}(1}{d^{3/4}}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(5/4))/x^(5/2) + (320*a*c^(1/4)*(-(b*c) + a*d))/Sqrt[x] + (40*c
^(1/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (10*Sqrt[2]*(b^2*c^2 - 10*a*b*c
*d + 9*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (1
0*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4) - (5*Sqrt
[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(80*c^(13/4))
```

Maple [A] time = 0.02, size = 524, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/2/c^3*x^(3/2)/(d*x^2+c)*a^2*d^2-1/c^2*x^(3/2)/(d*x^2+c)*a*b*d+1/2/c*x^(3/
2)/(d*x^2+c)*b^2+9/16/c^3*d/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+9/8/c^
3*d/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8/c^3*d
/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-5/8/c^2/(c/d
)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-5/4/c^2/(c/d)^(1/4)*2^(1/2)*a*b*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/4/c^2/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/c/d/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)+1/8/c/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8
/c/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5*a^2/
c^2/x^(5/2)+4*a^2/c^3/x^(1/2)*d-4*a/c^2/x^(1/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.20001, size = 3997, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c
^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d
^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^
(1/4)*arctan((sqrt((b^12*c^12 - 60*a*b^11*c^11*d + 1554*a^2*b^10*c^10*d^2 -
 22700*a^3*b^9*c^9*d^3 + 205215*a^4*b^8*c^8*d^4 - 1188600*a^5*b^7*c^7*d^5 +
 4443580*a^6*b^6*c^6*d^6 - 10697400*a^7*b^5*c^5*d^7 + 16622415*a^8*b^4*c^4*
d^8 - 16548300*a^9*b^3*c^3*d^9 + 10195794*a^10*b^2*c^2*d^10 - 3542940*a^11*
b*c*d^11 + 531441*a^12*d^12)*x - (b^8*c^15*d - 40*a*b^7*c^14*d^2 + 636*a^2*
b^6*c^13*d^3 - 5080*a^3*b^5*c^12*d^4 + 21286*a^4*b^4*c^11*d^5 - 45720*a^5*b
^3*c^10*d^6 + 51516*a^6*b^2*c^9*d^7 - 29160*a^7*b*c^8*d^8 + 6561*a^8*c^7*d^
9)*sqrt(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5
*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^
6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3)))*c^3*d*(-(b^8*c^8 - 40*a*
b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*
d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6
561*a^8*d^8)/(c^13*d^3))^(1/4) - (b^6*c^9*d - 30*a*b^5*c^8*d^2 + 327*a^2*b^
4*c^7*d^3 - 1540*a^3*b^3*c^6*d^4 + 2943*a^4*b^2*c^5*d^5 - 2430*a^5*b*c^4*d^
6 + 729*a^6*c^3*d^7)*sqrt(x)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*
```

```
d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5
+ 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1/
4))/(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3
+ 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 2
9160*a^7*b*c*d^7 + 6561*a^8*d^8)) - 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40
*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c
^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7
+ 6561*a^8*d^8)/(c^13*d^3))^(1/4)*log(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d
+ 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 4572
0*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^
8)/(c^13*d^3))^(3/4) + (b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 15
40*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)
*sqrt(x)) + 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b
^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c
^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^
3))^(1/4)*log(-c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 -
 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 515
16*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(3/4) +
(b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 29
43*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 4*(5*(b^2*c
^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2 - 9*a^2*c*d)*x
^2)*sqrt(x))/(c^3*d*x^5 + c^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21053, size = 541, normalized size = 1.49

$$\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\left(dx^2+c\right)c^3} - \frac{2\left(10\,abcx^2 - 10\,a^2dx^2 + a^2c\right)}{5\,c^3x^{\frac{5}{2}}} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 10\left(cd^3\right)^{\frac{3}{4}}abcd + 9\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\text{ar}}{8\,c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*c^
3) - 2/5*(10*a*b*c*x^2 - 10*a^2*d*x^2 + a^2*c)/(c^3*x^(5/2)) + 1/8*sqrt(2)*
((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2
)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^
3) + 1/8*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d
^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c
/d)^(1/4))/(c^4*d^3) - 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/
4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(c^4*d^3) + 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3
```

```
/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c^4*d^3)
```

3.433 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=440

$$\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{3/4}d^{17/4}}$$

```
[Out] -((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(16*c*d^4) + ((117*b^2*c^
2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b*c - a*d)^2*x^(9/2))
/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b*c - a*d)*x^(9/2))/(16*c^2*d^2
*(c + d*x^2)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 -
 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d
^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(17/4))
```

Rubi [A] time = 0.380913, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{3/4}d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] -((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(16*c*d^4) + ((117*b^2*c^
2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b*c - a*d)^2*x^(9/2))
/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b*c - a*d)*x^(9/2))/(16*c^2*d^2
*(c + d*x^2)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 -
 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d
^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(17/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{7/2}\left(\frac{1}{2}\left(-8a^2d^2+9(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\int \frac{x^{7/2}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(117b^2c^2-9\right.}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc}{}$$

Mathematica [A] time = 0.36706, size = 383, normalized size = 0.87

$$\frac{-\frac{40\sqrt[4]{d}\sqrt{x}\left(9a^2d^2-34abcd+25b^2c^2\right)}{c+dx^2} - \frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}} + \frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{c^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-1280*b*d^(1/4)*(3*b*c - 2*a*d)*Sqrt[x] + 256*b^2*d^(5/4)*x^(5/2) + (160*c
*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (40*d^(1/4)*(25*b^2*c^2 - 3
4*a*b*c*d + 9*a^2*d^2)*Sqrt[x])/(c + d*x^2) - (10*Sqrt[2]*(117*b^2*c^2 - 90
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4
) + (10*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*
```

```
d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) - (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d +
 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(
3/4) + (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4))/(640*d^(17/4))
```

Maple [A] time = 0.019, size = 590, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/5*b^2/d^3*x^(5/2)+4*b/d^3*a*x^(1/2)-6*b^2/d^4*x^(1/2)*c-9/16/d/(d*x^2+c)^
2*x^(5/2)*a^2+17/8/d^2/(d*x^2+c)^2*x^(5/2)*a*b*c-25/16/d^3/(d*x^2+c)^2*x^(5
/2)*b^2*c^2-5/16/d^2/(d*x^2+c)^2*x^(1/2)*a^2*c+13/8/d^3/(d*x^2+c)^2*x^(1/2)
*a*b*c^2-21/16/d^4/(d*x^2+c)^2*x^(1/2)*b^2*c^3+5/64/d^2*(c/d)^(1/4)/c*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-45/32/d^3*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+117/64/d^4*(c/d)^(1/4)*c*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/d^2*(c/d)^(1/4)/c*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/32/d^3*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+117/64/d^4*(c/d)^(1/4)*c*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/d^2*(c/d)^(1/4)/c*2^(1/2)*ln((x+
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2)))*a^2-45/64/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+117/128/d
^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.965665, size = 3676, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(20*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*arctan((sqrt(c^2*d^8*
sqrt(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^
```

```
2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^
3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3
*d^17)) + (13689*b^4*c^4 - 21060*a*b^3*c^3*d + 9270*a^2*b^2*c^2*d^2 - 900*a
^3*b*c*d^3 + 25*a^4*d^4)*x)*c^2*d^13*(-(187388721*b^8*c^8 - 576580680*a*b^7
*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*
a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 4500
0*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4) - (117*b^2*c^4*d^13 - 90*a*b
*c^3*d^14 + 5*a^2*c^2*d^15)*sqrt(x)*(-(187388721*b^8*c^8 - 576580680*a*b^7*
c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000
*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4))/(187388721*b^8*c^8 - 5765806
80*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 12
4525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^
6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)) + 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 -
 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c
^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^
17))^(1/4)*log(c*d^4*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 6973176
60*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4
- 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 +
625*a^8*d^8)/(c^3*d^17))^(1/4) + (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqr
t(x)) - 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*log(-c*d^4*(-(1873887
21*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*
a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 12
73500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)
+ (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 4*(32*b^2*d^3*x^6 - 585
*b^2*c^3 + 450*a*b*c^2*d - 25*a^2*c*d^2 - 32*(13*b^2*c*d^2 - 10*a*b*d^3)*x^
4 - 9*(117*b^2*c^2*d - 90*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(x))/(d^6*x^4 + 2
*c*d^5*x^2 + c^2*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19657, size = 609, normalized size = 1.38

$$\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}abcd+5\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^5}+\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(c*d^5) + 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1
/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/128*sqrt(2)*(117*(c*d^3)^(1/4)*b
^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/128*sqrt(2)*(117*(c*d^3)^(1/
4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/16*(25*b^2*c^2*d*x^(5/2
) - 34*a*b*c*d^2*x^(5/2) + 9*a^2*d^3*x^(5/2) + 21*b^2*c^3*sqrt(x) - 26*a*b*
c^2*d*sqrt(x) + 5*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*d^4) + 2/5*(b^2*d^12*x^
(5/2) - 15*b^2*c*d^11*sqrt(x) + 10*a*b*d^12*sqrt(x))/d^15
```

3.434 $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=401

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{64\sqrt{2}c^{5/4}d^{15/4}}$$

```
[Out] -((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(48*c*d^2) + ((b*c - a*d)^
2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b*c + a*d)*x^(7/2))/(
16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b
^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(15/4))
```

Rubi [A] time = 0.343386, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{64\sqrt{2}c^{5/4}d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(48*c*d^2) + ((b*c - a*d)^
2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b*c + a*d)*x^(7/2))/(
16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b
^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(15/4))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}\left(-8a^2d^2+7(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)\int \frac{x^{5/2}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(77b^2c^2-42a\right.}{}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(77b^2c^2-42a\right.}{}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(77b^2c^2-42a\right.}{}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(77b^2c^2-42a\right.}{}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(77b^2c^2-42a\right.}{}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(77b^2c^2-42a\right.}{}$$

Mathematica [A] time = 0.235513, size = 363, normalized size = 0.91

$$\frac{\frac{24d^{3/4}x^{3/2}\left(3a^2d^2-22abcd+19b^2c^2\right)}{c\left(c+dx^2\right)} - \frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}} + \frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{c^{5/4}}}{38}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(3/4)*x^(3/2) - (96*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2
 + (24*d^(3/4)*(19*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*x^(3/2))/(c*(c + d*x^2
)) + (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (3*Sqrt[
2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3
*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/
4))/(384*d^(15/4))
```

Maple [A] time = 0.018, size = 562, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/3*b^2/d^3*x^(3/2)+3/16/(d*x^2+c)^2/c*x^(7/2)*a^2-11/8/d/(d*x^2+c)^2*x^(7/
2)*a*b+19/16/d^2/(d*x^2+c)^2*c*x^(7/2)*b^2-1/16/d/(d*x^2+c)^2*x^(3/2)*a^2-7
/8/d^2/(d*x^2+c)^2*x^(3/2)*c*a*b+15/16/d^3/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64
/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+21/32/
d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-77/64/d^4
*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/d^2/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/d^3/(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64/d^4*c/(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/d^2/c/(c/d
)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+21/64/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))*a*b-77/128/d^4*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.18075, size = 4574, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 7669
7544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 45
36*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*arctan((sqrt((208422380089*b
^12*c^12 - 682109607564*a*b^11*c^11*d + 881427350034*a^2*b^10*c^10*d^2 - 54
3593843100*a^3*b^9*c^9*d^3 + 136525986135*a^4*b^8*c^8*d^4 + 8334677736*a^5*
b^7*c^7*d^5 - 7849956996*a^6*b^6*c^6*d^6 - 324727704*a^7*b^5*c^5*d^7 + 2072
41335*a^8*b^4*c^4*d^8 + 32148900*a^9*b^3*c^3*d^9 + 2030994*a^10*b^2*c^2*d^1
0 + 61236*a^11*b*c*d^11 + 729*a^12*d^12)*x - (35153041*b^8*c^11*d^7 - 76697
544*a*b^7*c^10*d^8 + 57274140*a^2*b^6*c^9*d^9 - 13854456*a^3*b^5*c^8*d^10 -
 1457946*a^4*b^4*c^7*d^11 + 539784*a^5*b^3*c^6*d^12 + 86940*a^6*b^2*c^5*d^1
```

```
3 + 4536*a^7*b*c^4*d^14 + 81*a^8*c^3*d^15)*sqrt(-(35153041*b^8*c^8 - 766975
44*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457
946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15)))*c*d^4*(-(35153041*b^8*c^8 - 7669754
4*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14579
46*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4) + (456533*b^6*c^7*d^4 - 747054*
a*b^5*c^6*d^5 + 354123*a^2*b^4*c^5*d^6 - 15876*a^3*b^3*c^4*d^7 - 13797*a^4*
b^2*c^3*d^8 - 1134*a^5*b*c^2*d^9 - 27*a^6*c*d^10)*sqrt(x)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4))/(35153041*b^8*c^8
- 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^
3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^
6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)) - 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^
3)*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 -
13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5
 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)
*log(c^4*d^11*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6
*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*
b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d
^15))^(3/4) - (456533*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2
 - 15876*a^3*b^3*c^3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^
6*d^6)*sqrt(x)) + 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(-c^4*d^11*(-(35
153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456
*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940
*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (4565
33*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^
3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 4
*(32*b^2*c*d^2*x^5 + (121*b^2*c^2*d - 66*a*b*c*d^2 + 9*a^2*d^3)*x^3 + (77*b
^2*c^3 - 42*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2
 + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22289, size = 576, normalized size = 1.44

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3} + \frac{19\,b^2c^2dx^{\frac{7}{2}} - 22\,abcd^2x^{\frac{7}{2}} + 3\,a^2d^3x^{\frac{7}{2}} + 15\,b^2c^3x^{\frac{3}{2}} - 14\,abc^2dx^{\frac{3}{2}} - a^2cd^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2cd^3} - \frac{\sqrt{2}\Big(77\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 42\,\left(cd^3\right)^{\frac{3}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*(19*b^2*c^2*d*x^(7/2) - 22*a*b*c*d^2*x^(7/2) + 3
*a^2*d^3*x^(7/2) + 15*b^2*c^3*x^(3/2) - 14*a*b*c^2*d*x^(3/2) - a^2*c*d^2*x^
(3/2))/((d*x^2 + c)^2*c*d^3) - 1/64*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*
(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) - 1/64*sqrt(2)*(77*(c*d^3
)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) + 1
/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^
3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^6
) - 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*
(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c
^2*d^6)
```

3.435 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=402

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{64\sqrt{2}c^{7/4}d^{13/4}}$$

```
[Out] -((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(16*c*d^2) + ((b*c - a*d)^
2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b*c + 3*a*d)*x^(5/2))
/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45
*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(13/4))
```

Rubi [A] time = 0.329627, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{64\sqrt{2}c^{7/4}d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] -((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(16*c*d^2) + ((b*c - a*d)^
2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b*c + 3*a*d)*x^(5/2))
/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45
*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(13/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}\left(-8a^2d^2+5(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\int \frac{x^{3/2}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2-10a\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.372445, size = 361, normalized size = 0.9

$$\frac{\frac{8\sqrt[4]{d}\sqrt{x}\left(a^2d^2-18abcd+17b^2c^2\right)}{c\left(c+dx^2\right)} + \frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} - \frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} \ldots}{128d^{1\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(1/4)*Sqrt[x] - (32*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2
 + (8*d^(1/4)*(17*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*Sqrt[x])/(c*(c + d*x^2))
+ (2*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^
2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(4
5*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/c^(7/4) - (Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(12
8*d^(13/4))
```

Maple [A] time = 0.017, size = 568, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2*b^2/d^3*x^(1/2)+1/16/(d*x^2+c)^2/c*x^(5/2)*a^2-9/8/d/(d*x^2+c)^2*x^(5/2)*
a*b+17/16/d^2/(d*x^2+c)^2*c*x^(5/2)*b^2-3/16/d/(d*x^2+c)^2*x^(1/2)*a^2-5/8/
d^2/(d*x^2+c)^2*x^(1/2)*c*a*b+13/16/d^3/(d*x^2+c)^2*x^(1/2)*b^2*c^2+3/64/d/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/d^2/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64/d^3*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/d/c^2*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+5/32/d^2/c*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-45/64/d^3*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/d/c^2*(c/d)^(1/
4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+5/64/d^2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))*a*b-45/128/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.964721, size = 3345, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*
a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b
^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7
 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*arctan((sqrt(c^4*d^6*sqrt(-(4100625*b^8*c^
8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 -
 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13)) + (2025*b^4*c^4 - 900*a*b^3*c^3*d -
170*a^2*b^2*c^2*d^2 + 60*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*d^10*(-(4100625*b^
8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d
^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 +
1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(3/4) + (45*b^2*c^7*d^10 - 10*a*
```

```
b*c^6*d^11 - 3*a^2*c^5*d^12)*sqrt(x)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7
*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8
*d^8)/(c^7*d^13))^(3/4))/(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^
2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*
b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 3
6600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)
/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(-c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(3
2*b^2*c*d^2*x^4 + 45*b^2*c^3 - 10*a*b*c^2*d - 3*a^2*c*d^2 + (81*b^2*c^2*d -
 18*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20879, size = 575, normalized size = 1.43

$$\frac{2\,b^2\sqrt{x}}{d^3} - \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4} - \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^3 - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/
4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2
*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/128*sqrt(2
)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^
2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1/128*s
qrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/
```

```
4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1
/16*(17*b^2*c^2*d*x^(5/2) - 18*a*b*c*d^2*x^(5/2) + a^2*d^3*x^(5/2) + 13*b^2
*c^3*sqrt(x) - 10*a*b*c^2*d*sqrt(x) - 3*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c
*d^3)
```

3.436 $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3} dx$

Optimal. Leaf size=364

$$\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}}$$

```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(11*b*c + 5*
a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(1
1/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(6
4*Sqrt[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*d^
(11/4))
```

Rubi [A] time = 0.292597, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(11*b*c + 5*
a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(1
1/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(6
4*Sqrt[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*d^
(11/4))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-8a^2d^2+3(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\int \frac{\sqrt{x}}{c+dx^2}dx}{32c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}dx, x,\right.}{16c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}dx,\right.}{32c^2d^{5/2}}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x\right.}{64c^2d^3}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{\right.}}{64\sqrt{2}c^{9/4}d^{11/4}}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}} -$$

Mathematica [A] time = 0.203085, size = 339, normalized size = 0.93

$$\frac{-\frac{8\sqrt[4]{c}d^{3/4}x^{3/2}\left(-5a^2d^2-6abcd+11b^2c^2\right)}{c+dx^2}+\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}\left(5a^2d^2+6abcd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((32*c^(5/4)*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 - (8*c^(1/4)*d^(3
/4)*(11*b^2*c^2 - 6*a*b*c*d - 5*a^2*d^2)*x^(3/2))/(c + d*x^2) - 2*Sqrt[2]*(
21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)] + 2*Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(21*b^2
*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(128*c^(9/4)*d^(11/4))
```

Maple [A] time = 0.017, size = 514, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 2*(1/32*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)/c^2/d*x^(7/2)+1/32*(9*a^2*d^2-2*a*
b*c*d-7*b^2*c^2)/c/d^2*x^(3/2))/(d*x^2+c)^2+5/64/d/c^2/(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/32/d^2/c/(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/d/c^2/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/32/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/d/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))*a^2+3/64/d^2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+21/128/d^3/(c/d
)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.29552, size = 4366, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264
*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4
*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c
*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*arctan((sqrt((85766121*b^12*c^12 + 14
7027636*a*b^11*c^11*d + 227542770*a^2*b^10*c^10*d^2 + 215040420*a^3*b^9*c^9
*d^3 + 181522215*a^4*b^8*c^8*d^4 + 112905576*a^5*b^7*c^7*d^5 + 63002556*a^6
*b^6*c^6*d^6 + 26882280*a^7*b^5*c^5*d^7 + 10290375*a^8*b^4*c^4*d^8 + 290250
0*a^9*b^3*c^3*d^9 + 731250*a^10*b^2*c^2*d^10 + 112500*a^11*b*c*d^11 + 15625
*a^12*d^12)*x - (194481*b^8*c^13*d^5 + 222264*a*b^7*c^12*d^6 + 280476*a^2*b
^6*c^11*d^7 + 176904*a^3*b^5*c^10*d^8 + 112806*a^4*b^4*c^9*d^9 + 42120*a^5*
b^3*c^8*d^10 + 15900*a^6*b^2*c^7*d^11 + 3000*a^7*b*c^6*d^12 + 625*a^8*c^5*d
^13)*sqrt(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 +
176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 1
5900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11)))*c^2*d^3
*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a
^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6
*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4) - (9261*b^
6*c^8*d^3 + 7938*a*b^5*c^7*d^4 + 8883*a^2*b^4*c^6*d^5 + 3996*a^3*b^3*c^5*d^
6 + 2115*a^4*b^2*c^4*d^7 + 450*a^5*b*c^3*d^8 + 125*a^6*c^2*d^9)*sqrt(x)*(-(
194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b
^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2
*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4))/(194481*b^8*c
^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 +
```

```
112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3
000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(
-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3
*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b
^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*log(c^7*d^8*
(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^
3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*
b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4) + (9261*b^6
*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 211
5*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x)) + (c^2*d^4*x^4
+ 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*
a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a
^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c
^9*d^11))^(1/4)*log(-c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 28047
6*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120
*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/
(c^9*d^11))^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2
 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*
d^6)*sqrt(x)) + 4*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (7*b^2*c^
3 + 2*a*b*c^2*d - 9*a^2*c*d^2)*x)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c
^4*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29434, size = 562, normalized size = 1.54

$$-\frac{11\,b^2c^2dx^{\frac{7}{2}} - 6\,abcd^2x^{\frac{7}{2}} - 5\,a^2d^3x^{\frac{7}{2}} + 7\,b^2c^3x^{\frac{3}{2}} + 2\,abc^2dx^{\frac{3}{2}} - 9\,a^2cd^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2c^2d^2} + \frac{\sqrt{2}\left(21\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 6\,\left(cd^3\right)^{\frac{3}{4}}abcd + 5\,\left(c\right.\right.}{64\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*b^2*c^2*d*x^(7/2) - 6*a*b*c*d^2*x^(7/2) - 5*a^2*d^3*x^(7/2) + 7*b
^2*c^3*x^(3/2) + 2*a*b*c^2*d*x^(3/2) - 9*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^2*
c^2*d^2) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d
 + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(c^3*d^5) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(
c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) - 1/128*sqrt(2)*(21*(c*d^
3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5) + 1/128*sqrt(2)*(21*(
c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5)
```

3.437 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

Optimal. Leaf size=364

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}$$

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c + 7*a
*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9
/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d +
21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*d^
(9/4))
```

Rubi [A] time = 0.297039, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c + 7*a
*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9
/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d +
21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*d^
(9/4))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx = \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{\int \frac{\frac{1}{2}(-8a^2d^2+(bc-ad)^2)-4b^2cdx^2}{\sqrt{x}(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\sqrt{x}(c+dx^2)}\,dx}{32c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x,\sqrt{x}\right)}{16c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,x,\sqrt{\right.}{32c^{5/2}d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx,\right.}{64c^{5/2}d^{5/2}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}c^{11/4}d^{9/4}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b}{}$$

Mathematica [A] time = 0.195867, size = 339, normalized size = 0.93

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}\sqrt{x}(-7a^2d^2-2abcd+9b^2c^2)}{c+dx^2} - \sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}(21a^2d^2+6abcd+5b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (8*c^(3/4)*d^(1
/4)*(9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(c + d*x^2) - 2*Sqrt[2]*(5
*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)] + 2*Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(128*c^(11/4)*d^(9/4))
```

Maple [A] time = 0.018, size = 514, normalized size = 1.4

$$2\frac{1}{(dx^2+c)^2}\left(1/32\frac{(7a^2d^2+2cabd-9b^2c^2)x^{5/2}}{c^2d}+1/32\frac{(11a^2d^2-6cabd-5b^2c^2)\sqrt{x}}{d^2c}\right)+\frac{21\sqrt{2}a^2}{64c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^(5/2)+1/32*(11*a^2*d^2-6*a*
b*c*d-5*b^2*c^2)/d^2/c*x^(1/2))/(d*x^2+c)^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/32/c^2/d*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+5/64/c/d^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/32/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+5/64/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
))*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+5/128/c/d^2*(c/d
)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.33032, size = 3345, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*arctan((sqrt(c^6*d^4*sqrt(-(625*b^8*c^
8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 1128
06*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 2222
64*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x)*c^8*d^7*(-(625*b
^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 +
 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4) - (5*b^2*c^10*d^7 +
 6*a*b*c^9*d^8 + 21*a^2*c^8*d^9)*sqrt(x)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d
+ 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 +
176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1944
81*a^8*d^8)/(c^11*d^9))^(3/4))/(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*
b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b
^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8))
 + (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*
d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
+ 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 19
```

```
4481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7
*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*sqr
t(x)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b
^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d
^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*sqrt(x)) - 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2 + (9*b^2*c^2*d - 2*
a*b*c*d^2 - 7*a^2*d^3)*x^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20145, size = 562, normalized size = 1.54

$$\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}} b^2c^2 + 6\,\left(cd^3\right)^{\frac{1}{4}} abcd + 21\,\left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3} + \frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}} b^2c^2 + 6\,\left(cd^3\right)^{\frac{1}{4}} abcd + 21}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)
^(1/4))/(c^3*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)
*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2
*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(1/4)
*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/16*(9*b^2*c^2*d*x^(5/2)
 - 2*a*b*c*d^2*x^(5/2) - 7*a^2*d^3*x^(5/2) + 5*b^2*c^3*sqrt(x) + 6*a*b*c^2*
d*sqrt(x) - 11*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^2*d^2)
```

3.438 $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=399

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{(5ad(\ldots}{}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) - ((b^2*c^2 - 2*a*b*c*d + 9*a^2*d^2)*x^(
3/2))/(4*c^2*d*(c + d*x^2)^2) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*x^(3/2
))/(16*c^3*d*(c + d*x^2)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^
2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*d^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*d^(7/4))
```

Rubi [A] time = 0.39452, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{(5ad(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) - ((b^2*c^2 - 2*a*b*c*d + 9*a^2*d^2)*x^(
3/2))/(4*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (5*a*(2*b*c - 9*a*d))/c^2)*x^
(3/2))/(16*c*(c + d*x^2)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^
2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*d^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*d^(7/4))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx = -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} + \frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-9ad)+\frac{1}{2}b^2cx^2\right)}{\left(c+dx^2\right)^3}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{1}{8}\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)\int \frac{\sqrt{x}}{\left(c+dx^2\right)^2}\,dx$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)\cdots}{32c\cdots}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)\cdots}{\cdots}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} - \frac{\left(3b^2c^2+10abcd-\cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} + \frac{\left(3b^2c^2+10abcd-\cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} + \frac{\left(3b^2c^2+10abcd-\cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2} - \frac{\left(b^2c^2-2abcd+9a^2d^2\right)x^{3/2}}{4c^2d\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)} - \frac{\left(3b^2c^2+10abcd-\cdots\right.}{32\cdots}$$

Mathematica [A] time = 0.40807, size = 364, normalized size = 0.91

$$\frac{\frac{8\sqrt[4]{c}x^{3/2}\left(-13a^2d^2+10abcd+3b^2c^2\right)}{d\left(c+dx^2\right)}+\frac{\sqrt{2}\left(-45a^2d^2+10abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{\sqrt{2}\left(45a^2d^2-10abcd-3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\cdots\right)}{d^{7/4}}\cdots}{128c^{13/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(1/4))/Sqrt[x] - (32*c^(5/4)*(b*c - a*d)^2*x^(3/2))/(d*(c + d*
x^2)^2) + (8*c^(1/4)*(3*b^2*c^2 + 10*a*b*c*d - 13*a^2*d^2)*x^(3/2))/(d*(c +
 d*x^2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 10*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 10*a*b*c
*d - 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (
Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 10*a*b*c*d
+ 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d
^(7/4))/(128*c^(13/4))
```

Maple [A] time = 0.022, size = 568, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] -13/16/c^3/(d*x^2+c)^2*x^(7/2)*a^2*d^2+5/8/c^2/(d*x^2+c)^2*x^(7/2)*a*b*d+3/
16/c/(d*x^2+c)^2*x^(7/2)*b^2-17/16/c^2/(d*x^2+c)^2*d*x^(3/2)*a^2+9/8/c/(d*x
^2+c)^2*x^(3/2)*a*b-1/16/(d*x^2+c)^2/d*x^(3/2)*b^2-45/64/c^3/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-45/64/c^3/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-45/128/c^3/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))*a^2+5/64/c^2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+3/128/c/d
^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2*a^2/c^3/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.51384, size = 4365, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*
c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4
 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
4100625*a^8*d^8)/(c^13*d^7))^(1/4)*arctan((sqrt((729*b^12*c^12 + 14580*a*b^
11*c^11*d + 55890*a^2*b^10*c^10*d^2 - 553500*a^3*b^9*c^9*d^3 - 3479625*a^4*
b^8*c^8*d^4 + 10305000*a^5*b^7*c^7*d^5 + 75317500*a^6*b^6*c^6*d^6 - 1545750
00*a^7*b^5*c^5*d^7 - 782915625*a^8*b^4*c^4*d^8 + 1868062500*a^9*b^3*c^3*d^9
 + 2829431250*a^10*b^2*c^2*d^10 - 11071687500*a^11*b*c*d^11 + 8303765625*a^
12*d^12)*x - (81*b^8*c^15*d^3 + 1080*a*b^7*c^14*d^4 + 540*a^2*b^6*c^13*d^5
- 36600*a^3*b^5*c^12*d^6 - 42650*a^4*b^4*c^11*d^7 + 549000*a^5*b^3*c^10*d^8
 + 121500*a^6*b^2*c^9*d^9 - 3645000*a^7*b*c^8*d^10 + 4100625*a^8*c^7*d^11)*
```

```
sqrt(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*
c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c
^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7)))*c^3*d^2*(-(81*
b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 -
42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3
645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4) + (27*b^6*c^9*d^2 +
 270*a*b^5*c^8*d^3 - 315*a^2*b^4*c^7*d^4 - 7100*a^3*b^3*c^6*d^5 + 4725*a^4*
b^2*c^5*d^6 + 60750*a^5*b*c^4*d^7 - 91125*a^6*c^3*d^8)*sqrt(x)*(-(81*b^8*c^
8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*
a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000
*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4))/(81*b^8*c^8 + 1080*a*b^7
*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
 4100625*a^8*d^8)) - (c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*log(c^10*d^5*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (27*b^6*c^6 + 270*a*b^5*c
^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 +
60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) + (c^3*d^3*x^5 + 2*c^4*d^2*x^3
+ c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a
^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^
6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*lo
g(-c^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*
a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a
^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) -
 (27*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3
 + 4725*a^4*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) - 4*(
32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2*c^3 - 1
8*a*b*c^2*d + 81*a^2*c*d^2)*x^2)*sqrt(x))/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^
5*d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20441, size = 576, normalized size = 1.44

$$-\frac{2\,a^2}{c^3\sqrt{x}}+\frac{3\,b^2c^2dx^{\frac{7}{2}}+10\,abcd^2x^{\frac{7}{2}}-13\,a^2d^3x^{\frac{7}{2}}-b^2c^3x^{\frac{3}{2}}+18\,abc^2dx^{\frac{3}{2}}-17\,a^2cd^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2c^3d}+\frac{\sqrt{2}\Big(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+10\,\left(cd^3\right)^{\frac{3}{4}}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2/(c^3*sqrt(x)) + 1/16*(3*b^2*c^2*d*x^(7/2) + 10*a*b*c*d^2*x^(7/2) - 1
3*a^2*d^3*x^(7/2) - b^2*c^3*x^(3/2) + 18*a*b*c^2*d*x^(3/2) - 17*a^2*c*d^2*x
^(3/2))/((d*x^2 + c)^2*c^3*d) + 1/64*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*
(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(
2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) + 1/64*sqrt(2)*(3*(c*d^3
)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) -
1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d
^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^
4) + 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45
*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
c^4*d^4)
```

3.439 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\sqrt{x}\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}+\frac{(7}{}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) - ((3*b^2*c^2 - 6*a*b*c*d + 11*a^2*d^2
)*Sqrt[x])/(12*c^2*d*(c + d*x^2)^2) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))
*Sqrt[x])/(48*c^3*d*(c + d*x^2)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Ar
cTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4))
+ ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11
*a*d))*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*d^(5/4)
)
```

Rubi [A] time = 0.405564, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}+\frac{(7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) - ((3*b^2*c^2 - 6*a*b*c*d + 11*a^2*d^2
)*Sqrt[x])/(12*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (7*a*(6*b*c - 11*a*d))/
c^2)*Sqrt[x])/(48*c*(c + d*x^2)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Ar
cTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4))
+ ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11
*a*d))*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*d^(5/4)
)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx = -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}+\frac{2\int \frac{\frac{1}{2}a(6bc-11ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\left(c+dx^2\right)^3}\,dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{1}{24}\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\int \frac{1}{\sqrt{x}\left(c+\right.}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-1}{c^2}\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-1}{c^2}\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-1}{c^2}\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}+\frac{\left(3b^2c^2+42ab\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}-\frac{\left(3b^2c^2+42ab\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)}-\frac{\left(3b^2c^2+42ab\right.}{}$$

Mathematica [A] time = 0.259722, size = 365, normalized size = 0.91

$$\frac{\frac{24c^{3/4}\sqrt{x}\left(-15a^2d^2+14abcd+b^2c^2\right)}{d\left(c+dx^2\right)}+\frac{3\sqrt{2}\left(77a^2d^2-42abcd-3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{3\sqrt{2}\left(-77a^2d^2+42abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{d^{5/4}}}{384c^{15/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(3/4))/x^(3/2) - (96*c^(7/4)*(b*c - a*d)^2*Sqrt[x])/(d*(c + d*
x^2)^2) + (24*c^(3/4)*(b^2*c^2 + 14*a*b*c*d - 15*a^2*d^2)*Sqrt[x])/(d*(c +
d*x^2)) + (6*Sqrt[2]*(-3*b^2*c^2 - 42*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (6*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c*
d - 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (3
*Sqrt[2]*(-3*b^2*c^2 - 42*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4) + (3*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c
*d - 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
```

```
)/d^(5/4))/(384*c^(15/4))
```

Maple [A] time = 0.022, size = 562, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] -15/16/c^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+7/8/c^2/(d*x^2+c)^2*x^(5/2)*a*b*d+1/
16/c/(d*x^2+c)^2*x^(5/2)*b^2-19/16/c^2/(d*x^2+c)^2*d*x^(1/2)*a^2+11/8/c/(d*
x^2+c)^2*x^(1/2)*a*b-3/16/(d*x^2+c)^2/d*x^(1/2)*b^2-77/64/c^4*d*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+21/32/c^3*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/64/c^2/d*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64/c^4*d*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/c^3*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128/c^4*d*(c/d)^(1/4)*2^(1/2)*ln
((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))*a^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+3/128
/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c^3/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.36903, size = 3515, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a
*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b
^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544
*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*arctan((sqrt(c^8*d^2*sqr
t(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*
c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6
*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5)) + (9*b^
4*c^4 + 252*a*b^3*c^3*d + 1302*a^2*b^2*c^2*d^2 - 6468*a^3*b*c*d^3 + 5929*a^
4*d^4)*x)*c^11*d^4*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2
```

```
 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*
d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(
c^15*d^5))^(3/4) + (3*b^2*c^13*d^4 + 42*a*b*c^12*d^5 - 77*a^2*c^11*d^6)*sqr
t(x)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*
b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140
*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(3/
4))/(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5
*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^
6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)) + 3*(c^3*d^3*x^6
+ 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b
^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^
5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*
a^8*d^8)/(c^15*d^5))^(1/4)*log(c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 869
40*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 138
54456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 3
5153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*
sqrt(x)) - 3*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536
*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4
*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 766975
44*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(-c^4*d*(-(81*b^8*c
^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6
 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2
+ 42*a*b*c*d - 77*a^2*d^2)*sqrt(x)) + 4*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a
*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b^2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2
)*sqrt(x))/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22447, size = 575, normalized size = 1.43

$$-\frac{2\,a^2}{3\,c^3x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\,\left(cd^3\right)^{\frac{1}{4}}abcd-77\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^4d^2}+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^3*x^(3/2)) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)
^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)
*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/128*s
```

```
qrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/
4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2) - 1/
128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3
)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2
) + 1/16*(b^2*c^2*d*x^(5/2) + 14*a*b*c*d^2*x^(5/2) - 15*a^2*d^3*x^(5/2) - 3
*b^2*c^3*sqrt(x) + 22*a*b*c^2*d*sqrt(x) - 19*a^2*c*d^2*sqrt(x))/((d*x^2 + c
)^2*c^3*d)
```

3.440 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=439

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\,(c+dx^2)^2}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)^2}+\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2}{}$$

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2)/(5*c*x^(
5/2)*(c + d*x^2)^2) - (5*b^2*c^2 - 10*a*b*c*d + 13*a^2*d^2)/(20*c^2*d*Sqrt[
x]*(c + d*x^2)^2) - (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(80*c^3*d*Sqrt[x]
*(c + d*x^2)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*
a*d*(10*b*c - 13*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(
3/4)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(3/4))
```

Rubi [A] time = 0.450207, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {462, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\,(c+dx^2)^2}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)^2}+\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2)/(5*c*x^(
5/2)*(c + d*x^2)^2) - (5*b^2*c^2 - 10*a*b*c*d + 13*a^2*d^2)/(20*c^2*d*Sqrt[
x]*(c + d*x^2)^2) - ((5*b^2)/d - (9*a*(10*b*c - 13*a*d))/c^2)/(80*c*Sqrt[x]
*(c + d*x^2)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*
a*d*(10*b*c - 13*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(
3/4)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(3/4))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} + \frac{2\int \frac{\frac{1}{2}a(10bc-13ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)^3}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} + \frac{1}{40}\left(-\frac{5b^2}{d} + \frac{9a(10bc-13ad)}{c^2}\right)\int \frac{1}{x^{3/2}(c+d\cdots}$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} - \frac{\left(\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}\right)\int \frac{\cdots}{x^{3}\cdots}}{32c}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} + \frac{(5b^2c\cdots}{\cdots}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} + \frac{(5b^2c\cdots}{\cdots}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} - \frac{(5b^2c\cdots}{\cdots}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} + \frac{(5b^2c\cdots}{\cdots}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} + \frac{(5b^2c\cdots}{\cdots}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}(c+dx^2)} - \frac{(5b^2c\cdots}{\cdots}$$

Mathematica [A] time = 0.380915, size = 382, normalized size = 0.87

$$\frac{40\sqrt[4]{c}x^{3/2}(21a^2d^2-26abcd+5b^2c^2)}{c+dx^2} + \frac{5\sqrt{2}(117a^2d^2-90abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}} - \frac{5\sqrt{2}(117a^2d^2-90abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\cdots\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(5/4))/x^(5/2) + (1280*a*c^(1/4)*(-2*b*c + 3*a*d))/Sqrt[x] + (
160*c^(5/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (40*c^(1/4)*(5*b^2*c^2 -
 26*a*b*c*d + 21*a^2*d^2)*x^(3/2))/(c + d*x^2) - (10*Sqrt[2]*(5*b^2*c^2 - 9
0*a*b*c*d + 117*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(
3/4) + (10*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d
+ 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
d^(3/4) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(640*c^(17/4))
```

Maple [A] time = 0.023, size = 590, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x)
```

```
[Out] 21/16/c^4/(d*x^2+c)^2*x^(7/2)*a^2*d^3-13/8/c^3/(d*x^2+c)^2*x^(7/2)*a*b*d^2+
5/16/c^2/(d*x^2+c)^2*x^(7/2)*b^2*d+25/16/c^3/(d*x^2+c)^2*x^(3/2)*a^2*d^2-17
/8/c^2/(d*x^2+c)^2*x^(3/2)*a*b*d+9/16/c/(d*x^2+c)^2*x^(3/2)*b^2+117/128/c^4
*d/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+117/64/c^4*d/(c/d)^(1/4)*2^(1/2
)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+117/64/c^4*d/(c/d)^(1/4)*2^(1/2
)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-45/64/c^3/(c/d)^(1/4)*2^(1/2)*a
*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))-45/32/c^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)-45/32/c^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)+5/128/c^2/d/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+5/64/c^2
/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/64/c^2/d
/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5*a^2/c^3/
x^(5/2)+6*a^2/c^4/x^(1/2)*d-4*a/c^3/x^(1/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63098, size = 4849, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(20*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b
^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57
6580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*arctan((sqrt((15
625*b^12*c^12 - 1687500*a*b^11*c^11*d + 78131250*a^2*b^10*c^10*d^2 - 201993
7500*a^3*b^9*c^9*d^3 + 31839834375*a^4*b^8*c^8*d^4 - 314326575000*a^5*b^7*c
^7*d^5 + 1936382557500*a^6*b^6*c^6*d^6 - 7355241855000*a^7*b^5*c^5*d^7 + 17
434219710375*a^8*b^4*c^4*d^8 - 25881265273500*a^9*b^3*c^3*d^9 + 23425464012
210*a^10*b^2*c^2*d^10 - 11839219392780*a^11*b*c*d^11 + 2565164201769*a^12*d
^12)*x - (625*b^8*c^17*d - 45000*a*b^7*c^16*d^2 + 1273500*a^2*b^6*c^15*d^3
- 17739000*a^3*b^5*c^14*d^4 + 124525350*a^4*b^4*c^13*d^5 - 415092600*a^5*b^
3*c^12*d^6 + 697317660*a^6*b^2*c^11*d^7 - 576580680*a^7*b*c^10*d^8 + 187388
721*a^8*c^9*d^9)*sqrt(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c
^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a
^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388
721*a^8*d^8)/(c^17*d^3)))*c^4*d*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4) - (125*b^6*c^10*d - 6750*a*b^5*c^
9*d^2 + 130275*a^2*b^4*c^8*d^3 - 1044900*a^3*b^3*c^7*d^4 + 3048435*a^4*b^2*
c^6*d^5 - 3696030*a^5*b*c^5*d^6 + 1601613*a^6*c^4*d^7)*sqrt(x)*(-(625*b^8*c
^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3
 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^
2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4))/(
625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^
5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 6973176
60*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)) - 5*(c^4*d
^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(c^13*d^2*(-(625*b^8*c^8 - 450
00*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 12452
5350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^
6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4) + (125*b^6
*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3
+ 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*sqrt(x))
 + 5*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7
*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4
*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57658068
0*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(-c^13*d^2*(-(625*b
^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5
*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^
6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4
) + (125*b^6*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*
b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d
^6)*sqrt(x)) - 4*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2
*c^3 + 9*(5*b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 -
13*a^2*c^2*d)*x^2)*sqrt(x))/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24481, size = 599, normalized size = 1.36

$$\frac{5\,b^2c^2dx^{\frac{7}{2}} - 26\,abcd^2x^{\frac{7}{2}} + 21\,a^2d^3x^{\frac{7}{2}} + 9\,b^2c^3x^{\frac{3}{2}} - 34\,abc^2dx^{\frac{3}{2}} + 25\,a^2cd^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2c^4} - \frac{2\left(10\,abcx^2 - 15\,a^2dx^2 + a^2c\right)}{5\,c^4x^{\frac{5}{2}}} + \frac{\sqrt{2}\Big(5\,\left(cd^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b^2*c^2*d*x^(7/2) - 26*a*b*c*d^2*x^(7/2) + 21*a^2*d^3*x^(7/2) + 9*b
^2*c^3*x^(3/2) - 34*a*b*c^2*d*x^(3/2) + 25*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^
2*c^4) - 2/5*(10*a*b*c*x^2 - 15*a^2*d*x^2 + a^2*c)/(c^4*x^(5/2)) + 1/64*sqr
t(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^5*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*
c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2
 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b
^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3)
```

3.441 $\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=328

$$\frac{2dx^{7/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\cdots\right.}{2\sqrt{2}b^{19/4}}$$

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b^2) + (2*d^3*x^(15/2
))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) +
(a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.293491, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2dx^{7/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\cdots\right.}{2\sqrt{2}b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b^2) + (2*d^3*x^(15/2
))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) +
(a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{b^3} + \frac{d^2(3bc-ad)x^{9/2}}{b^2} + \frac{d^3x^{13/2}}{b} + \frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-\right.}{b^3\left(a+bx^2\right)}\right.$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} + \frac{(bc-ad)^3\int\frac{x^{5/2}}{a+bx^2}dx}{b^3}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} - \frac{\left(a(bc-a\right.}{}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} - \frac{\left(2a(bc-\right.}{}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} + \frac{\left(a(bc-a\right.}{}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} - \frac{\left(a(bc-a\right.}{}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} - \frac{a^{3/4}(bc-}{}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} + \frac{a^{3/4}(bc-}{}$$

Mathematica [C] time = 0.385138, size = 132, normalized size = 0.4

$$\frac{2x^{3/2}\left(165a^2bd^2\left(7c+dx^2\right)-385a^3d^3-15ab^2d\left(77c^2+33cdx^2+7d^2x^4\right)-385(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)+b^3\left(495c\right.\right.}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(-385*a^3*d^3 + 165*a^2*b*d^2*(7*c + d*x^2) - 15*a*b^2*d*(77*c^2
 + 33*c*d*x^2 + 7*d^2*x^4) + b^3*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 +
 77*d^3*x^6) - 385*(b*c - a*d)^3*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a
)]))/(1155*b^4)
```

Maple [B] time = 0.01, size = 721, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/15*d^3*x^(15/2)/b-2/11/b^2*x^(11/2)*a*d^3+6/11/b*x^(11/2)*c*d^2+2/7/b^3*x
^(7/2)*a^2*d^3-6/7/b^2*x^(7/2)*a*c*d^2+6/7/b*x^(7/2)*c^2*d-2/3/b^4*x^(3/2)*
a^3*d^3+2/b^3*x^(3/2)*a^2*c*d^2-2/b^2*x^(3/2)*a*c^2*d+2/3/b*x^(3/2)*c^3+1/2
*a^4/b^5/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-
3/2*a^3/b^4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c
```

```
*d^2+3/2*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)
+1)*c^2*d-1/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)+1)*c^3+1/2*a^4/b^5/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)-1)*d^3-3/2*a^3/b^4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)-1)*c*d^2+3/2*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x^(1/2)-1)*c^2*d-1/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b
*a)^(1/4)*x^(1/2)-1)*c^3+1/4*a^4/b^5/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1
/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2)))*d^3-3/4*a^3/b^4/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*
2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d
^2+3/4*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d-1/4*a/b
^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07219, size = 5611, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2310*(4620*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*
d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924
*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b
^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/
4)*arctan((sqrt((a^4*b^18*c^18 - 18*a^5*b^17*c^17*d + 153*a^6*b^16*c^16*d^2
 - 816*a^7*b^15*c^15*d^3 + 3060*a^8*b^14*c^14*d^4 - 8568*a^9*b^13*c^13*d^5
+ 18564*a^10*b^12*c^12*d^6 - 31824*a^11*b^11*c^11*d^7 + 43758*a^12*b^10*c^1
0*d^8 - 48620*a^13*b^9*c^9*d^9 + 43758*a^14*b^8*c^8*d^10 - 31824*a^15*b^7*c
^7*d^11 + 18564*a^16*b^6*c^6*d^12 - 8568*a^17*b^5*c^5*d^13 + 3060*a^18*b^4*
c^4*d^14 - 816*a^19*b^3*c^3*d^15 + 153*a^20*b^2*c^2*d^16 - 18*a^21*b*c*d^17
 + a^22*d^18)*x - (a^3*b^21*c^12 - 12*a^4*b^20*c^11*d + 66*a^5*b^19*c^10*d^
2 - 220*a^6*b^18*c^9*d^3 + 495*a^7*b^17*c^8*d^4 - 792*a^8*b^16*c^7*d^5 + 92
4*a^9*b^15*c^6*d^6 - 792*a^10*b^14*c^5*d^7 + 495*a^11*b^13*c^4*d^8 - 220*a^
12*b^12*c^3*d^9 + 66*a^13*b^11*c^2*d^10 - 12*a^14*b^10*c*d^11 + a^15*b^9*d^
12)*sqrt(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*
a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c
^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9
 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19))*b^5*(-(a^3*b
^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3
+ 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^1
0*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c
^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4) + (a^2*b^14*c^9 - 9*a^3
```

```
*b^13*c^8*d + 36*a^4*b^12*c^7*d^2 - 84*a^5*b^11*c^6*d^3 + 126*a^6*b^10*c^5*
d^4 - 126*a^7*b^9*c^4*d^5 + 84*a^8*b^8*c^3*d^6 - 36*a^9*b^7*c^2*d^7 + 9*a^1
0*b^6*c*d^8 - a^11*b^5*d^9)*sqrt(x)*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d +
 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8
*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^
4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^
15*d^12)/b^19)^(1/4))/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^1
0*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 9
24*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12
*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)) - 1155
*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6
*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*
d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 +
66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(b^14*(
-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c
^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 -
792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^1
3*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 -
 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^
5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a
^10*b*c*d^8 - a^11*d^9)*sqrt(x)) + 1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*
c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 -
 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^1
1*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d
^11 + a^15*d^12)/b^19)^(1/4)*log(-b^14*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*
d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*
a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4
*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 +
 a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d
^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^
8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt(x)) -
4*(77*b^3*d^3*x^7 + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 165*(3*b^3*c^2*d -
3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*x)*sqrt(x))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19339, size = 717, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
```

```
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^7 - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^7 + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^7 - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*
(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 + 2/1155*(77*b^
14*d^3*x^(15/2) + 315*b^14*c*d^2*x^(11/2) - 105*a*b^13*d^3*x^(11/2) + 495*b
^14*c^2*d*x^(7/2) - 495*a*b^13*c*d^2*x^(7/2) + 165*a^2*b^12*d^3*x^(7/2) + 3
85*b^14*c^3*x^(3/2) - 1155*a*b^13*c^2*d*x^(3/2) + 1155*a^2*b^12*c*d^2*x^(3/
2) - 385*a^3*b^11*d^3*x^(3/2))/b^15
```

3.442 $\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=326

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2\sqrt{x}(bc-ad)^3}{b^4}+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}$$

```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(5
/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) + (2*d^3*x^(13/2))/(13
*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/
4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.276661, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2\sqrt{x}(bc-ad)^3}{b^4}+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```


```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(5
/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) + (2*d^3*x^(13/2))/(13
*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/
4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(17/4))
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{b^3}+\frac{d^2(3bc-ad)x^{7/2}}{b^2}+\frac{d^3x^{11/2}}{b}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-\right.}{b^3\left(a+bx^2\right)}\right.$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{(bc-ad)^3\int\frac{x^{3/2}}{a+bx^2}dx}{b^3}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(a(bc-ad\right.}{}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(2a(bc-a\right.}{}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(\sqrt{a}(bc-a\right.}{}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(\sqrt{a}(bc-a\right.}{}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}(bc-a}{}$$

$$=\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}(bc-a}{}$$

Mathematica [C] time = 0.376934, size = 133, normalized size = 0.41

$$\frac{2\sqrt{x}\left(117a^2bd^2\left(15c+dx^2\right)-585a^3d^3-13ab^2d\left(135c^2+27cdx^2+5d^2x^4\right)-585(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)+3b^3\left(11\right.\right.}{585b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-585*a^3*d^3 + 117*a^2*b*d^2*(15*c + d*x^2) - 13*a*b^2*d*(135*c
^2 + 27*c*d*x^2 + 5*d^2*x^4) + 3*b^3*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^
4 + 15*d^3*x^6) - 585*(b*c - a*d)^3*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2
)/a)]))/(585*b^4)
```

Maple [B] time = 0.01, size = 712, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/13*d^3*x^(13/2)/b-2/9/b^2*x^(9/2)*a*d^3+2/3/b*x^(9/2)*c*d^2+2/5/b^3*x^(5/
2)*a^2*d^3-6/5/b^2*x^(5/2)*a*c*d^2+6/5/b*x^(5/2)*c^2*d-2/b^4*a^3*d^3*x^(1/2
)+6/b^3*a^2*c*d^2*x^(1/2)-6/b^2*a*c^2*d*x^(1/2)+2/b*c^3*x^(1/2)+1/4/b^4*(1/
b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1
/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a^3*d^3-3/4/b^3*(1/b*a)^(1/4)*2
```

```
^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*
x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a^2*c*d^2+3/4/b^2*(1/b*a)^(1/4)*2^(1/2)*ln(
(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))*a*c^2*d-1/4/b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2)))*c^3+1/2/b^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)+1)*a^3*d^3-3/2/b^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)*a^2*c*d^2+3/2/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x^(1/2)+1)*a*c^2*d-1/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x^(1/2)+1)*c^3+1/2/b^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x^(1/2)-1)*a^3*d^3-3/2/b^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x^(1/2)-1)*a^2*c*d^2+3/2/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x^(1/2)-1)*a*c^2*d-1/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x^(1/2)-1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77466, size = 4181, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2
 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^
7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^
3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*ar
ctan((sqrt(b^8*sqrt(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d
^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*
a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*
c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*b^13*(-(a*b^12*c^12 - 12*a^2*b^11*c^
11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 7
92*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^
4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11
+ a^13*d^12)/b^17)^(3/4) + (b^16*c^3 - 3*a*b^15*c^2*d + 3*a^2*b^14*c*d^2 -
a^3*b^13*d^3)*sqrt(x)*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^1
0*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 9
24*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b
^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(3/
4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*
c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 -
 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11
*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)) + 585*b^4*(-(a*b^12*c^12 - 1
```

```
2*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^
8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7
 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a
^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(b^4*(-(a*b^12*c^12 - 12*a^2*b^11*
c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 -
 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*
b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^1
1 + a^13*d^12)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(x)) - 585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^
(1/4)*log(-b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*
b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*
d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4) - (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 4*(45*b^3*d^3*
x^6 + 585*b^3*c^3 - 1755*a*b^2*c^2*d + 1755*a^2*b*c*d^2 - 585*a^3*d^3 + 65*
(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 117*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^
3)*x^2)*sqrt(x))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20111, size = 717, normalized size = 2.2

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^5}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c
^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*
(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 2/585*(45*b^1
2*d^3*x^(13/2) + 195*b^12*c*d^2*x^(9/2) - 65*a*b^11*d^3*x^(9/2) + 351*b^12*
c^2*d*x^(5/2) - 351*a*b^11*c*d^2*x^(5/2) + 117*a^2*b^10*d^3*x^(5/2) + 585*b
```

```
^12*c^3*sqrt(x) - 1755*a*b^11*c^2*d*sqrt(x) + 1755*a^2*b^10*c*d^2*sqrt(x) -
 585*a^3*b^9*d^3*sqrt(x))/b^13
```

3.443 $\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=306

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\right.}{2\sqrt{}}$$

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*(3*b*c - a
*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c - a*d)^3*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/
4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rubi [A] time = 0.245434, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```


```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*(3*b*c - a
*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c - a*d)^3*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/
4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```


Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{d^2(3bc-ad)x^{5/2}}{b^2} + \frac{d^3x^{9/2}}{b} + \frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a\right.}{b^3\left(a+bx^2\right)}\right.$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\int \frac{\sqrt{x}}{a+bx^2}dx}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\right.}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} - \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\right.}{b^{7/2}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}\right.}{2b^4}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} - \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

Mathematica [C] time = 0.359545, size = 97, normalized size = 0.32

$$\frac{2x^{3/2}\left(ad\left(77a^2d^2-33abd\left(7c+dx^2\right)+3b^2\left(77c^2+33cdx^2+7d^2x^4\right)\right)+77(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{231ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(a*d*(77*a^2*d^2 - 33*a*b*d*(7*c + d*x^2) + 3*b^2*(77*c^2 + 33*c
*d*x^2 + 7*d^2*x^4)) + 77*(b*c - a*d)^3*Hypergeometric2F1[3/4, 1, 7/4, -((b
*x^2)/a)]))/(231*a*b^3)
```

Maple [B] time = 0.01, size = 659, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/11*d^3*x^(11/2)/b-2/7*d^3/b^2*x^(7/2)*a+6/7*d^2/b*x^(7/2)*c+2/3*d^3/b^3*x
^(3/2)*a^2-2*d^2/b^2*x^(3/2)*c*a+2*d/b*x^(3/2)*c^2-1/2/b^4/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*a^3*d^3+3/2/b^3/(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*a^2*c*d^2-3/2/b^2/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*a*c^2*d+1/2/b/(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3-1/2/b^4/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*a^3*d^3+3/2/b^3/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*a^2*c*d^2-3/2/b^2/
```

```
(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*a*c^2*d+1/2/b
/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3-1/4/b^4/
(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x
+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a^3*d^3+3/4/b^3/(1/b*a)^(1/4
)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*a^2*c*d^2-3/4/b^2/(1/b*a)^(1/4)*2^(1/2)*
ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2)))*a*c^2*d+1/4/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^
(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*
a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6619, size = 5393, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/462*(924*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*arcta
n((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^1
5*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^
12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9
*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^
12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^1
5*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x -
(a*b^19*c^12 - 12*a^2*b^18*c^11*d + 66*a^3*b^17*c^10*d^2 - 220*a^4*b^16*c^9
*d^3 + 495*a^5*b^15*c^8*d^4 - 792*a^6*b^14*c^7*d^5 + 924*a^7*b^13*c^6*d^6 -
 792*a^8*b^12*c^5*d^7 + 495*a^9*b^11*c^4*d^8 - 220*a^10*b^10*c^3*d^9 + 66*a
^11*b^9*c^2*d^10 - 12*a^12*b^8*c*d^11 + a^13*b^7*d^12)*sqrt(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a*b^15)))*b^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a*b^15))^(1/4) + (b^13*c^9 - 9*a*b^12*c^8*d + 36*a^2*b^11*c^7*d^2 - 84*a
^3*b^10*c^6*d^3 + 126*a^4*b^9*c^5*d^4 - 126*a^5*b^8*c^4*d^5 + 84*a^6*b^7*c^
3*d^6 - 36*a^7*b^6*c^2*d^7 + 9*a^8*b^5*c*d^8 - a^9*b^4*d^9)*sqrt(x)*(-(b^12
*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
```

```
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4))/(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)) - 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4
)*log(a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(3/4) - (b^9*
c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5
*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 +
9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a*b^15))^(1/4)*log(-a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^
6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6
- 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 4*(21*b^2*d^3*x^
5 + 33*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^
3)*x)*sqrt(x))/b^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.19675, size = 662, normalized size = 2.16

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} b^3c^3 - 3\left(ab^3\right)^{\frac{3}{4}} ab^2c^2d + 3\left(ab^3\right)^{\frac{3}{4}} a^2bcd^2 - \left(ab^3\right)^{\frac{3}{4}} a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^6} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} b^3c^3 - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^6) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^
3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)
```

```
^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^6) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^
2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b
^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3
)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6)
+ 2/231*(21*b^10*d^3*x^(11/2) + 99*b^10*c*d^2*x^(7/2) - 33*a*b^9*d^3*x^(7/2
) + 231*b^10*c^2*d*x^(3/2) - 231*a*b^9*c*d^2*x^(3/2) + 77*a^2*b^8*d^3*x^(3/
2))/b^11
```

3.444 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)}\,dx$

Optimal. Leaf size=304

$$\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\ldots\right.}{2\sqrt{2}a^{3/4}b^{13/4}}$$

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b*c - a*d)*
x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) -
((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.249237, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\ldots\right.}{2\sqrt{2}a^{3/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b*c - a*d)*
x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) -
((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3\sqrt{x}} + \frac{d^2(3bc-ad)x^{3/2}}{b^2} + \frac{d^3x^{7/2}}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\sqrt{x}\left(a+bx^2\right)}\right)$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx\right.}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx\right.}{\sqrt{a}b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\right.}{2\sqrt{a}b^{7/2}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}a^{3/4}b^{13/4}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{13/4}}$$

Mathematica [C] time = 0.343753, size = 96, normalized size = 0.32

$$\frac{2\sqrt{x}\left(ad\left(45a^2d^2-9abd\left(15c+dx^2\right)+b^2\left(135c^2+27cdx^2+5d^2x^4\right)\right)+45(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{45ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[x]*(a*d*(45*a^2*d^2 - 9*a*b*d*(15*c + d*x^2) + b^2*(135*c^2 + 27*c*
d*x^2 + 5*d^2*x^4)) + 45*(b*c - a*d)^3*Hypergeometric2F1[1/4, 1, 5/4, -((b*
x^2)/a)]))/(45*a*b^3)
```

Maple [B] time = 0.01, size = 650, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/9*d^3*x^(9/2)/b-2/5*d^3/b^2*x^(5/2)*a+6/5*d^2/b*x^(5/2)*c+2*d^3/b^3*a^2*x
^(1/2)-6*d^2/b^2*c*a*x^(1/2)+6*d/b*c^2*x^(1/2)-1/2/b^3*(1/b*a)^(1/4)*a^2*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+3/2/b^2*(1/b*a)^(1/4)*a*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-3/2/b*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+1/2*(1/b*a)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3-1/4/b^3*(1/b*a)^(1/4)*a^2*
2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+3/4/b^2*(1/b*a)^(1/4)*a*2^(1/2)*ln((x+
```

```
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/
2)+(1/b*a)^(1/2)))*c*d^2-3/4/b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)
))*c^2*d+1/4*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3-1/2/b^3*(
1/b*a)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+3/2/b^
2*(1/b*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-3/2
/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+1/2*
(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.38563, size = 4100, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="fricas")

[Out] -1/90*(180*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arc
tan((sqrt(a^2*b^6*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) +
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^2*b^10*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^3*b^13))^(3/4) + (a^2*b^13*c^3 - 3*a^3*b^12*c^2*d + 3*a^
4*b^11*c*d^2 - a^5*b^10*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(a^3*b^13))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 45*b^3*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
```

```
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(x)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*
b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x))
 - 4*(5*b^2*d^3*x^4 + 135*b^2*c^2*d - 135*a*b*c*d^2 + 45*a^2*d^3 + 9*(3*b^2
*c*d^2 - a*b*d^3)*x^2)*sqrt(x))/b^3
```

Sympy [A] time = 106.066, size = 889, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)
/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2*c**3*sqrt(x) + 6*c**2*d*
x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13/2)/13)/a, Eq(b, 0)), ((-2*
c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2
)/9)/b, Eq(a, 0)), ((-1)**(1/4)*a**(9/4)*d**3*log(-(-1)**(1/4)*a**(1/4)*(1/
b)**(1/4) + sqrt(x))/(2*b**7*(1/b)**(15/4)) - (-1)**(1/4)*a**(9/4)*d**3*log
((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**7*(1/b)**(15/4)) + (-1)
**(1/4)*a**(9/4)*d**3*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b*
*7*(1/b)**(15/4)) - 3*(-1)**(1/4)*a**(5/4)*c*d**2*log(-(-1)**(1/4)*a**(1/4)
*(1/b)**(1/4) + sqrt(x))/(2*b**6*(1/b)**(15/4)) + 3*(-1)**(1/4)*a**(5/4)*c*
d**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**6*(1/b)**(15/4)
) - 3*(-1)**(1/4)*a**(5/4)*c*d**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)*
*(1/4)))/(b**6*(1/b)**(15/4)) + 3*(-1)**(1/4)*a**(1/4)*c**2*d*log(-(-1)**(1
/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**5*(1/b)**(15/4)) - 3*(-1)**(1/4)
*a**(1/4)*c**2*d*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**5*(
1/b)**(15/4)) + 3*(-1)**(1/4)*a**(1/4)*c**2*d*atan((-1)**(3/4)*sqrt(x)/(a**
(1/4)*(1/b)**(1/4)))/(b**5*(1/b)**(15/4)) + 2*a**2*d**3*sqrt(x)/b**3 - 6*a*
c*d**2*sqrt(x)/b**2 - 2*a*d**3*x**(5/2)/(5*b**2) + 6*c**2*d*sqrt(x)/b + 6*c
*d**2*x**(5/2)/(5*b) + 2*d**3*x**(9/2)/(9*b) - (-1)**(1/4)*c**3*log(-(-1)**
(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)*b**4*(1/b)**(15/4)) + (-
1)**(1/4)*c**3*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)
*b**4*(1/b)**(15/4)) - (-1)**(1/4)*c**3*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*
(1/b)**(1/4)))/(a**(3/4)*b**4*(1/b)**(15/4)), True))
```

Giac [B] time = 1.21652, size = 662, normalized size = 2.18

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^
3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)
^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
+ 2/45*(5*b^8*d^3*x^(9/2) + 27*b^8*c*d^2*x^(5/2) - 9*a*b^7*d^3*x^(5/2) + 13
5*b^8*c^2*d*sqrt(x) - 135*a*b^7*c*d^2*sqrt(x) + 45*a^2*b^6*d^3*sqrt(x))/b^9
```

3.445 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=284

$$-\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{5/4}b^{11/4}}$$

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2*d^3*x^(7/
2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) +
((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.290217, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{5/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2*d^3*x^(7/
2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) +
((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^2\left(a+bx^4\right)}\,dx,x,\sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{c^3}{ax^2}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^6}{b}+\frac{(-bc+ad)^3x^2}{ab^2\left(a+bx^4\right)}\right)dx,x,\sqrt{x}\right)$$

$$= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{ab^2}$$

$$= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{ab^{5/2}}-\frac{(bc-ad)^3\,\text{Su}}{}$$

$$= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2ab^3}-\frac{(bc-ad}{}$$

$$= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3}{}$$

$$= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1+\right.}{\sqrt{2}a^{5/4}b^{11/4}}$$

Mathematica [C] time = 0.334505, size = 89, normalized size = 0.31

$$-\frac{2\left(a\left(7a^2d^3x^2-3abd^2x^2\left(7c+dx^2\right)+21b^2c^3\right)+7x^2(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{21a^2b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(21*b^2*c^3 + 7*a^2*d^3*x^2 - 3*a*b*d^2*x^2*(7*c + d*x^2)) + 7*(b*c
- a*d)^3*x^2*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]))/(21*a^2*b^2*Sqr
t[x])
```

Maple [B] time = 0.012, size = 622, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2/7*d^3*x^(7/2)/b-2/3*d^3/b^2*x^(3/2)*a+2*d^2/b*x^(3/2)*c+1/2*a^2/b^3/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-3/2*a/b^2/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2+3/2/b/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d-1/2/a/(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+1/4*a^2/b^3/(1/
b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1
/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3-3/4*a/b^2/(1/b*a)^(1/4)*2^(
1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^
```

```
(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2+3/4/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*
a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2)))*c^2*d-1/4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^
3+1/2*a^2/b^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
*d^3-3/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1
)*c*d^2+3/2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)
*c^2*d-1/2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*
c^3-2*c^3/a/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.56615, size = 5403, normalized size = 19.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="fricas")

[Out] -1/42*(84*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*
arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^
3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a
^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 4862
0*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 185
64*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 81
6*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)
*x - (a^3*b^17*c^12 - 12*a^4*b^16*c^11*d + 66*a^5*b^15*c^10*d^2 - 220*a^6*b
^14*c^9*d^3 + 495*a^7*b^13*c^8*d^4 - 792*a^8*b^12*c^7*d^5 + 924*a^9*b^11*c^
6*d^6 - 792*a^10*b^10*c^5*d^7 + 495*a^11*b^9*c^4*d^8 - 220*a^12*b^8*c^3*d^9
 + 66*a^13*b^7*c^2*d^10 - 12*a^14*b^6*c*d^11 + a^15*b^5*d^12)*sqrt(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11)))*a*b^3*(-(b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(a^5*b^11))^(1/4) + (a*b^12*c^9 - 9*a^2*b^11*c^8*d + 36*a^3*b^
10*c^7*d^2 - 84*a^4*b^9*c^6*d^3 + 126*a^5*b^8*c^5*d^4 - 126*a^6*b^7*c^4*d^5
 + 84*a^7*b^6*c^3*d^6 - 36*a^8*b^5*c^2*d^7 + 9*a^9*b^4*c*d^8 - a^10*b^3*d^9
)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
```

```
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4))/(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)) - 21*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a^5*b^11))^(1/4)*log(a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 21*a*b^2*x*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(-a^4*b^8*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8
*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^
5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a
^9*d^9)*sqrt(x)) - 4*(3*a*b*d^3*x^4 - 21*b^2*c^3 + 7*(3*a*b*c*d^2 - a^2*d^3
)*x^2)*sqrt(x))/(a*b^2*x)
```

Sympy [A] time = 103.445, size = 847, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(5*x**(5/2)) - 6*c**2*d/sqrt(x) + 2*c*d**2*x**(3/2)
 + 2*d**3*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/sqrt(x) + 2*c**2*d*x
**(3/2) + 6*c*d**2*x**(7/2)/7 + 2*d**3*x**(11/2)/11)/a, Eq(b, 0)), ((-2*c**
3/(5*x**(5/2)) - 6*c**2*d/sqrt(x) + 2*c*d**2*x**(3/2) + 2*d**3*x**(7/2)/7)/
b, Eq(a, 0)), (-(-1)**(3/4)*a**(7/4)*d**3*log(-(-1)**(1/4)*a**(1/4)*(1/b)**
(1/4) + sqrt(x))/(2*b**5*(1/b)**(9/4)) + (-1)**(3/4)*a**(7/4)*d**3*log((-1)
**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**5*(1/b)**(9/4)) + (-1)**(3/4
)*a**(7/4)*d**3*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**5*(1/
b)**(9/4)) + 3*(-1)**(3/4)*a**(3/4)*c*d**2*log(-(-1)**(1/4)*a**(1/4)*(1/b)*
*(1/4) + sqrt(x))/(2*b**4*(1/b)**(9/4)) - 3*(-1)**(3/4)*a**(3/4)*c*d**2*log
((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**4*(1/b)**(9/4)) - 3*(-1
)**(3/4)*a**(3/4)*c*d**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/
(b**4*(1/b)**(9/4)) - 2*a*d**3*x**(3/2)/(3*b**2) + 2*c*d**2*x**(3/2)/b + 2*
d**3*x**(7/2)/(7*b) - 2*c**3/(a*sqrt(x)) - 3*(-1)**(3/4)*c**2*d*log(-(-1)**
(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b**3*(1/b)**(9/4)) + 3*(
-1)**(3/4)*c**2*d*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1
/4)*b**3*(1/b)**(9/4)) + 3*(-1)**(3/4)*c**2*d*atan((-1)**(3/4)*sqrt(x)/(a**
(1/4)*(1/b)**(1/4)))/(a**(1/4)*b**3*(1/b)**(9/4)) + (-1)**(3/4)*c**3*log(-(
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*b**2*(1/b)**(9/4))
- (-1)**(3/4)*c**3*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(
5/4)*b**2*(1/b)**(9/4)) - (-1)**(3/4)*c**3*atan((-1)**(3/4)*sqrt(x)/(a**(1/
```

```
4)*(1/b)**(1/4)))/(a**(5/4)*b**2*(1/b)**(9/4)), True))
```

Giac [B] time = 1.24331, size = 624, normalized size = 2.2

$$-\frac{2c^3}{a\sqrt{x}} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^5} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*c^3/(a*sqrt(x)) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a
*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 1/2*sq
rt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4
)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) - 1
/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)
^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(a^2*b^5) + 2/21*(3*b^6*d^3*x^(7/2) + 21*b^6*c*d^2*x^(3/
2) - 7*a*b^5*d^3*x^(3/2))/b^7
```

3.446 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)} dx$

Optimal. Leaf size=284

$$\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{7/4}b^{9/4}}$$

```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d^3*x^(5/2)
)/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c
 - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.26692, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{7/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d^3*x^(5/2)
)/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c
 - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(7/4)*b^(9/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^4(a+bx^4)}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax^4}+\frac{d^3x^4}{b}+\frac{(-bc+ad)^3}{ab^2(a+bx^4)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^2}$$
$$= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}b^2}-\frac{(bc-ad)^3\,\text{S}}{}$$
$$= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}b^{5/2}}-\frac{(bc-a}{}$$
$$= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}+\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad}{}$$
$$= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1\right.}{\sqrt{2}a^{7/4}b^{9/4}}$$

Mathematica [C] time = 0.356801, size = 89, normalized size = 0.31

$$-\frac{2\left(a\left(15a^2d^3x^2-3abd^2x^2\left(15c+dx^2\right)+5b^2c^3\right)+15x^2(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{15a^2b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(5*b^2*c^3 + 15*a^2*d^3*x^2 - 3*a*b*d^2*x^2*(15*c + d*x^2)) + 15*(b*
c - a*d)^3*x^2*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]))/(15*a^2*b^2*x
^(3/2))
```

Maple [B] time = 0.013, size = 616, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b-2*d^3/b^2*a*x^(1/2)+6*d^2/b*x^(1/2)*c-2/3*c^3/a/x^(3/2)+1
/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-
3/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2+3
/2/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d-1/
2/a^2*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3+1
/2*a/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-
3/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2+3
/2/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d-1/
```

```
2/a^2*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+1
/4*a/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3-3/4/b*(1/b*a)^(
1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^
(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2+3/4/a*(1/b*a)^(1/4)*2^(1/2)*ln(
(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))*c^2*d-1/4/a^2*b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(
1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a
)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.37986, size = 4078, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(60*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*
arctan((sqrt(a^4*b^4*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10
*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 92
4*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3
*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))
+ (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^5*b^7*(-(b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(a^7*b^9))^(3/4) + (a^5*b^10*c^3 - 3*a^6*b^9*c^2*d + 3*a^7
*b^8*c*d^2 - a^8*b^7*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a
^7*b^9))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 15*a*b^2*x^2*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
```

```
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 15*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(x)) + 4*(3*a*b*d^3*x^4 - 5*b^2*c^3 + 15*(3*a*b*c*d^2 - a^2*d^3)*x^2)*s
qrt(x))/(a*b^2*x^2)
```

Sympy [A] time = 157.665, size = 842, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x)
 + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) + 6*c**
2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/a, Eq(b, 0)), ((-2*c
**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(5/2)/5
)/b, Eq(a, 0)), (-(-1)**(1/4)*a**(5/4)*b**30*d**3*(1/b)**(129/4)*log(-(-1)*
*(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 + (-1)**(1/4)*a**(5/4)*b**30*d**3
*(1/b)**(129/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 - (-1)**
(1/4)*a**(5/4)*d**3*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**
(1/4)))/b**2 + 3*(-1)**(1/4)*a**(1/4)*b**31*c*d**2*(1/b)**(129/4)*log(-(-1)
**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 - 3*(-1)**(1/4)*a**(1/4)*b**31*c
*d**2*(1/b)**(129/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/2 + 3
*(-1)**(1/4)*a**(1/4)*c*d**2*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4
)*(1/b)**(1/4)))/b - 2*a*d**3*sqrt(x)/b**2 + 6*c*d**2*sqrt(x)/b + 2*d**3*x*
*(5/2)/(5*b) - 2*c**3/(3*a*x**(3/2)) - 3*(-1)**(1/4)*b**32*c**2*d*(1/b)**(1
29/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + 3*(-
1)**(1/4)*b**32*c**2*d*(1/b)**(129/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(2*a**(3/4)) - 3*(-1)**(1/4)*c**2*d*(1/b)**(1/4)*atan((-1)**(3/
4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4) + (-1)**(1/4)*b**33*c**3*(1/b)
**(129/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) -
(-1)**(1/4)*b**33*c**3*(1/b)**(129/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(2*a**(7/4)) + (-1)**(1/4)*b*c**3*(1/b)**(1/4)*atan((-1)**(3/4)
*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(7/4), True))
```

Giac [B] time = 1.44698, size = 622, normalized size = 2.19

$$-\frac{2\,c^3}{3\,ax^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)
*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/2*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3
*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^
(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) +
 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^2*b^3) + 2/5*(b^4*d^3*x^(5/2) + 15*b^4*c*d^2*sqrt(x
) - 5*a*b^3*d^3*sqrt(x))/b^5
```

3.447 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$

Optimal. Leaf size=283

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{9/4}b^{7/4}}$$

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2*d^3*x^(3/
2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.275281, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2*d^3*x^(3/
2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^6(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{c^3}{ax^6} + \frac{c^2(-bc+3ad)}{a^2x^2} + \frac{d^3x^2}{b} - \frac{(-bc+ad)^3x^2}{a^2b(a+bx^4)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b^{3/2}} + \frac{(bc-ad)^3\,\text{Sub}}{}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^2b^2} + \frac{(bc-ad)^3}{}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} - \frac{(bc-ad)^3\,\text{lo}}{}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{(bc-ad)^3\tan^{-1}\left(1 + \right.}{\sqrt{2}a^{9/4}b^{7/4}}$$

Mathematica [C] time = 0.352557, size = 88, normalized size = 0.31

$$-\frac{2\left(a\left(-5a^2d^3x^4 + 3abc^2\left(c+15dx^2\right) - 15b^2c^3x^2\right) - 5x^4(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{15a^3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-15*b^2*c^3*x^2 - 5*a^2*d^3*x^4 + 3*a*b*c^2*(c + 15*d*x^2)) - 5*(b*
c - a*d)^3*x^4*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]))/(15*a^3*b*x^(
5/2))
```

Maple [B] time = 0.015, size = 616, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/3*d^3*x^(3/2)/b-2/5*c^3/a/x^(5/2)-6*c^2/a/x^(1/2)*d+2*c^3/a^2/x^(1/2)*b-1
/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+
3/2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-3
/2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+1/
2/a^2*b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3-1
/2*a/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+
3/2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-3
/2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+1/
```

```
2/a^2*b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3-1
/4*a/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+3/4/b/(1/b*a)^(
1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^
(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2-3/4/a/(1/b*a)^(1/4)*2^(1/2)*ln(
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))*c^2*d+1/4/a^2*b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(
1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a
)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05288, size = 5407, normalized size = 19.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(60*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*
arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^
3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a
^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 4862
0*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 185
64*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 81
6*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)
*x - (a^5*b^15*c^12 - 12*a^6*b^14*c^11*d + 66*a^7*b^13*c^10*d^2 - 220*a^8*b
^12*c^9*d^3 + 495*a^9*b^11*c^8*d^4 - 792*a^10*b^10*c^7*d^5 + 924*a^11*b^9*c
^6*d^6 - 792*a^12*b^8*c^5*d^7 + 495*a^13*b^7*c^4*d^8 - 220*a^14*b^6*c^3*d^9
 + 66*a^15*b^5*c^2*d^10 - 12*a^16*b^4*c*d^11 + a^17*b^3*d^12)*sqrt(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7)))*a^2*b^2*(-(b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(a^9*b^7))^(1/4) + (a^2*b^11*c^9 - 9*a^3*b^10*c^8*d + 36*a^4*
b^9*c^7*d^2 - 84*a^5*b^8*c^6*d^3 + 126*a^6*b^7*c^5*d^4 - 126*a^7*b^6*c^4*d^
5 + 84*a^8*b^5*c^3*d^6 - 36*a^9*b^4*c^2*d^7 + 9*a^10*b^3*c*d^8 - a^11*b^2*d
^9)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^
3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66
```

```
*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4))/(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)) - 15*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(a^9*b^7))^(1/4)*log(a^7*b^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a
^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3
*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 15*a^2*b*x^
3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*d^3*x^4 - 3*a*b*c^3 + 15*(b^2*c^3 - 3*a*b*c^2*
d)*x^2)*sqrt(x))/(a^2*b*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24746, size = 614, normalized size = 2.17

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b}+\frac{2\left(5\,bc^3x^2-15\,ac^2dx^2-ac^3\right)}{5\,a^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan}{2\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 2/5*(5*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c^3)/(a^2*x^(5/2)
) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a
*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) + 1/2*sqrt(2)*((a*b^3)^(3
/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (
```

```
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3
/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/4*sqrt(2)*((a*b^
3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^
2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^3*b^4)
```

3.448 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx$

Optimal. Leaf size=283

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{11/4}b^{5/4}}$$

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) + (2*d^3*Sqr
t[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.258136, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) + (2*d^3*Sqr
t[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{9/2}\left(a+bx^2\right)}dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^8\left(a+bx^4\right)}dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^8}+\frac{c^2(-bc+3ad)}{a^2x^4}-\frac{(-bc+ad)^3}{a^2b\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x, \sqrt{x}\right)}{a^2b}$$
$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^{5/2}b}+\frac{(bc-ad)^3\,\text{Sub}}{}$$
$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx, x, \sqrt{x}\right)}{2a^{5/2}b^{3/2}}+\frac{(bc-ad)^3}{}$$
$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\,\text{lo}}{}$$
$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}a^{11/4}b^{5/4}}$$

Mathematica [C] time = 0.356971, size = 88, normalized size = 0.31

$$-\frac{2\left(a\left(-21a^2d^3x^4+3abc^2\left(c+7dx^2\right)-7b^2c^3x^2\right)-21x^4(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21a^3bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-7*b^2*c^3*x^2 - 21*a^2*d^3*x^4 + 3*a*b*c^2*(c + 7*d*x^2)) - 21*(b*
c - a*d)^3*x^4*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]))/(21*a^3*b*x^(
7/2))
```

Maple [B] time = 0.013, size = 622, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x)
```

```
[Out] 2*d^3*x^(1/2)/b-2/7*c^3/a/x^(7/2)-2*c^2/a/x^(3/2)*d+2/3*c^3/a^2/x^(3/2)*b-1
/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+3/2/
a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-3/2/a
^2*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+1/
2/a^3*b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3
-1/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+3/
2/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-3/2
/a^2*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+
```

```
1/2/a^3*b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c
^3-1/4/b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+3/4/a*(1/b*a)^(
1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^
(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2-3/4/a^2*b*(1/b*a)^(1/4)*2^(1/2)
*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2
)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d+1/4/a^3*b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74324, size = 4086, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/42*(84*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4
)*arctan((sqrt(a^6*b^2*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5
)) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^8*b^4*(-(b^12*c^12 - 12*a
*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 49
5*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*
c*d^11 + a^12*d^12)/(a^11*b^5))^(3/4) + (a^8*b^7*c^3 - 3*a^9*b^6*c^2*d + 3*
a^10*b^5*c*d^2 - a^11*b^4*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(a^11*b^5))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 21*a^2*b*x^4*
(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(a^3*b*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
```

```
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 21*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(-a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(x)) - 4*(21*a^2*d^3*x^4 - 3*a*b*c^3 + 7*(b^2*c^3 - 3*a*b*c^2*d)*
x^2)*sqrt(x))/(a^2*b*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1795, size = 614, normalized size = 2.17

$$\frac{2\,d^3\sqrt{x}}{b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/2*sqrt(
2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a
^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 1/4*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^3*b^2) + 2/21*(7*b*c^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(
a^2*x^(7/2))
```

3.449 $\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=303

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+$$

```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) - (2*c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a*d)^3*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4))
 - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(2*Sqrt[2]*a^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi [A] time = 0.290693, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) - (2*c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a*d)^3*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4))
 - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(2*Sqrt[2]*a^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{11/2}\left(a+bx^2\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^{10}\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \left(\frac{c^3}{ax^{10}} + \frac{c^2(-bc+3ad)}{a^2x^6} + \frac{c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3x^2} + \frac{(-bc+ad)^3x^2}{a^3\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{\left(2(bc-ad)^3\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^3}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^3\sqrt{b}} -$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{\right.}{2a^3b}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right.}{2\sqrt{2}a^{13/4}b^{3/4}}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}b^{3/4}} - \frac{(bc-a}{}$$

Mathematica [C] time = 0.36461, size = 101, normalized size = 0.33

$$-\frac{2\left(ac\left(a^2\left(5c^2+27cdx^2+135d^2x^4\right)-9abcx^2\left(c+15dx^2\right)+45b^2c^2x^4\right)+15x^6(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{45a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*c*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 15*d*x^2) + a^2*(5*c^2 + 27*c*d
*x^2 + 135*d^2*x^4)) + 15*(b*c - a*d)^3*x^6*Hypergeometric2F1[3/4, 1, 7/4,
-((b*x^2)/a)]))/(45*a^4*x^(9/2))
```

Maple [B] time = 0.015, size = 650, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x)
```

```
[Out] -2/9*c^3/a/x^(9/2)-6*c/a/x^(1/2)*d^2+6*c^2/a^2/x^(1/2)*b*d-2*c^3/a^3/x^(1/2
)*b^2-6/5*c^2/a/x^(5/2)*d+2/5*c^3/a^2/x^(5/2)*b+1/2/b/(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-3/2/a/(1/b*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2+3/2/a^2*b/(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d-1/2/a^3*b^2/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3+1/2/b/(1/b*a)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-3/2/a/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2+3/2/a^2*b/(1/b*a)^(1/4)*2^(1/
```

```
2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d-1/2/a^3*b^2/(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+1/4/b/(1/b*a)^(1/4)*2^(
1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3-3/4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)
^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2)))*c*d^2+3/4/a^2*b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/
2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*
c^2*d-1/4/a^3*b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)
+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01762, size = 5445, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/90*(180*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)
*arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a
^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*
a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 486
20*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18
564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 8
16*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18
)*x - (a^7*b^13*c^12 - 12*a^8*b^12*c^11*d + 66*a^9*b^11*c^10*d^2 - 220*a^10
*b^10*c^9*d^3 + 495*a^11*b^9*c^8*d^4 - 792*a^12*b^8*c^7*d^5 + 924*a^13*b^7*
c^6*d^6 - 792*a^14*b^6*c^5*d^7 + 495*a^15*b^5*c^4*d^8 - 220*a^16*b^4*c^3*d^
9 + 66*a^17*b^3*c^2*d^10 - 12*a^18*b^2*c*d^11 + a^19*b*d^12)*sqrt(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3)))*a^3*b*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
 a^12*d^12)/(a^13*b^3))^(1/4) + (a^3*b^10*c^9 - 9*a^4*b^9*c^8*d + 36*a^5*b^
8*c^7*d^2 - 84*a^6*b^7*c^6*d^3 + 126*a^7*b^6*c^5*d^4 - 126*a^8*b^5*c^4*d^5
+ 84*a^9*b^4*c^3*d^6 - 36*a^10*b^3*c^2*d^7 + 9*a^11*b^2*c*d^8 - a^12*b*d^9)
*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b
^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^
6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^
```

```
10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) - 45*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(a^13*b^3))^(1/4)*log(a^10*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 45*a^3*x^5*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)*log(-a^10*b^2*(-(b^
12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*c^3 + 45*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)
*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d)*x^2)*sqrt(x))/(a^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22414, size = 652, normalized size = 2.15

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) - 1/2*sqrt(2)*((a*b^3)^(3/4)
*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b
^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
```

```
/b)^(1/4))/(a^4*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)
*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 1/4*sqrt(2)*((a*b^3)^
(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
 (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a^4*b^3) - 2/45*(45*b^2*c^3*x^4 - 135*a*b*c^2*d*x^4 + 135*a^2*c*d^2*x^4 - 9
*a*b*c^3*x^2 + 27*a^2*c^2*d*x^2 + 5*a^2*c^3)/(a^3*x^(9/2))
```

3.450 $\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx$

Optimal. Leaf size=305

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{3/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a
*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi [A] time = 0.267611, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{3/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a
*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^{12}(a+bx^4)}dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{c^3}{ax^{12}} + \frac{c^2(-bc+3ad)}{a^2x^8} + \frac{c(b^2c^2-3abcd+3a^2d^2)}{a^3x^4} + \frac{(-bc+ad)^3}{a^3(a+bx^4)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x, \sqrt{x}\right)}{a^3}$$
$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx, x, \sqrt{x}\right)}{a^{7/2}}$$
$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}dx, x,\right.}{2a^{7/2}\sqrt{b}}$$
$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} + \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\right.}}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}$$
$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} + \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{(bc-}{}$$

Mathematica [C] time = 0.358459, size = 102, normalized size = 0.33

$$-\frac{2\left(ac\left(3a^2\left(7c^2+33cdx^2+77d^2x^4\right)-33abcx^2\left(c+7dx^2\right)+77b^2c^2x^4\right)+231x^6(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{231a^4x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*c*(77*b^2*c^2*x^4 - 33*a*b*c*x^2*(c + 7*d*x^2) + 3*a^2*(7*c^2 + 33*c
*d*x^2 + 77*d^2*x^4)) + 231*(b*c - a*d)^3*x^6*Hypergeometric2F1[1/4, 1, 5/4
, -((b*x^2)/a)]))/(231*a^4*x^(11/2))
```

Maple [B] time = 0.015, size = 659, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x)
```

```
[Out] -2/11*c^3/a/x^(11/2)-2*c/a/x^(3/2)*d^2+2*c^2/a^2/x^(3/2)*b*d-2/3*c^3/a^3/x^
(3/2)*b^2-6/7*c^2/a/x^(7/2)*d+2/7*c^3/a^2/x^(7/2)*b+1/2/a*(1/b*a)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-3/2/a^2*(1/b*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b*c*d^2+3/2/a^3*(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b^2*c^2*d-1/2/a^4*(1/b*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b^3*c^3+1/2/a*(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-3/2/a^2*(1/b*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b*c*d^2+3/2/a^3*(1/b*a)
```

```
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b^2*c^2*d-1/2/a^4*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b^3*c^3+1/4/a*(
1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3-3/4/a^2*(1/b*a)^(1/4)*2^(
1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b*c*d^2+3/4/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(
1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2
)+(1/b*a)^(1/2)))*b^2*c^2*d-1/4/a^4*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2)))*b^3*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.3615, size = 4089, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/462*(924*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*a
rctan((sqrt(a^8*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b)) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^15*b))^(3/4) + (a^11*b^4*c^3 - 3*a^12*b^3*c^2*d + 3*a^13*b^2*c
*d^2 - a^14*b*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))
^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 231*a^3*x^6*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*log(a^4*(-(b^12*c^12 - 12*a*b^11*c^
```

```
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
 a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(x)) - 231*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*
b))^(1/4)*log(-a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 4*(21*a^2*c
^3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 - 3*a^2*c^2
*d)*x^2)*sqrt(x))/(a^3*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16721, size = 652, normalized size = 2.14

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^4b}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a^4*b) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b
^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^
3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b)
 - 2/231*(77*b^2*c^3*x^4 - 231*a*b*c^2*d*x^4 + 231*a^2*c*d^2*x^4 - 33*a*b*c
^3*x^2 + 99*a^2*c^2*d*x^2 + 21*a^2*c^3)/(a^3*x^(11/2))
```

3.451 $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

Optimal. Leaf size=325

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{5/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc$$

```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b*c - a*d)^3)/(a^4*Sq
rt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^
(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(2*Sqrt[2]*a^(17/4))
```

Rubi [A] time = 0.305925, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{5/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b*c - a*d)^3)/(a^4*Sq
rt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^
(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(2*Sqrt[2]*a^(17/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{15/2}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^{14}\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{c^3}{ax^{14}} + \frac{c^2(-bc+3ad)}{a^2x^{10}} + \frac{c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3x^6} + \frac{(-bc+ad)^3}{a^4x^2} - \frac{b(-bc+ad)^3}{a^4\left(a+bx^4\right.}\right.\right.$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{\left(2b(bc-ad)^3\right)\text{Subst}}{a}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} - \frac{\left(\sqrt{b}(bc-ad)^3\right)\text{Subs}}{}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{(bc-ad)^3\,\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{\sqrt[4]{b}(bc-ad)^3\log\left(\sqrt{a}\right.}{2\sqrt{}}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} - \frac{\sqrt[4]{b}(bc-ad)^3\tan^{-1}\left(\right.}{\sqrt{2}a^{17/4}}$$

Mathematica [C] time = 0.393541, size = 148, normalized size = 0.46

$$-\frac{2\left(a\left(-13a^2bcx^2\left(5c^2+27cdx^2+135d^2x^4\right)+3a^3\left(65c^2dx^2+15c^3+117cd^2x^4+195d^3x^6\right)+117ab^2c^2x^4\left(c+15dx^2\right)-\right.\right.}{585a^5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-585*b^3*c^3*x^6 + 117*a*b^2*c^2*x^4*(c + 15*d*x^2) - 13*a^2*b*c*x^
2*(5*c^2 + 27*c*d*x^2 + 135*d^2*x^4) + 3*a^3*(15*c^3 + 65*c^2*d*x^2 + 117*c
*d^2*x^4 + 195*d^3*x^6)) - 195*b*(b*c - a*d)^3*x^8*Hypergeometric2F1[3/4, 1
, 7/4, -((b*x^2)/a)]))/(585*a^5*x^(13/2))
```

Maple [B] time = 0.015, size = 712, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x)
```

```
[Out] -2/13*c^3/a/x^(13/2)-2/a/x^(1/2)*d^3+6/a^2/x^(1/2)*c*d^2*b-6/a^3/x^(1/2)*c^
2*d*b^2+2/a^4/x^(1/2)*c^3*b^3-6/5*c/a/x^(5/2)*d^2+6/5*c^2/a^2/x^(5/2)*b*d-2
/5*c^3/a^3/x^(5/2)*b^2-2/3*c^2/a/x^(9/2)*d+2/9*c^3/a^2/x^(9/2)*b-1/2/a/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+3/2/a^2/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b*c*d^2-3/2/a^3/(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b^2*c^2*d+1/2/a
^4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b^3*c^3-1/
```

```
2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+3/2/a
^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b*c*d^2-3/
2/a^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b^2*c^2
*d+1/2/a^4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*b^
3*c^3-1/4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*
a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+3/4/a^2/(1/b
*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b*c*d^2-3/4/a^3/(1/b*a)^(1/4)*2^
(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b^2*c^2*d+1/4/a^4/(1/b*a)^(1/4)*2^(1/2)*ln((
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(
1/2)+(1/b*a)^(1/2)))*b^3*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57927, size = 5557, normalized size = 17.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2
 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a
^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^
3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4
)*arctan((sqrt((b^20*c^18 - 18*a*b^19*c^17*d + 153*a^2*b^18*c^16*d^2 - 816*
a^3*b^17*c^15*d^3 + 3060*a^4*b^16*c^14*d^4 - 8568*a^5*b^15*c^13*d^5 + 18564
*a^6*b^14*c^12*d^6 - 31824*a^7*b^13*c^11*d^7 + 43758*a^8*b^12*c^10*d^8 - 48
620*a^9*b^11*c^9*d^9 + 43758*a^10*b^10*c^8*d^10 - 31824*a^11*b^9*c^7*d^11 +
 18564*a^12*b^8*c^6*d^12 - 8568*a^13*b^7*c^5*d^13 + 3060*a^14*b^6*c^4*d^14
- 816*a^15*b^5*c^3*d^15 + 153*a^16*b^4*c^2*d^16 - 18*a^17*b^3*c*d^17 + a^18
*b^2*d^18)*x - (a^9*b^13*c^12 - 12*a^10*b^12*c^11*d + 66*a^11*b^11*c^10*d^2
 - 220*a^12*b^10*c^9*d^3 + 495*a^13*b^9*c^8*d^4 - 792*a^14*b^8*c^7*d^5 + 92
4*a^15*b^7*c^6*d^6 - 792*a^16*b^6*c^5*d^7 + 495*a^17*b^5*c^4*d^8 - 220*a^18
*b^4*c^3*d^9 + 66*a^19*b^3*c^2*d^10 - 12*a^20*b^2*c*d^11 + a^21*b*d^12)*sqr
t(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*
d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792
*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*
c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17))*a^4*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4) + (a^4*b^10*c^9 - 9*a^5*b^9*c^8*d + 36
*a^6*b^8*c^7*d^2 - 84*a^7*b^7*c^6*d^3 + 126*a^8*b^6*c^5*d^4 - 126*a^9*b^5*c
^4*d^5 + 84*a^10*b^4*c^3*d^6 - 36*a^11*b^3*c^2*d^7 + 9*a^12*b^2*c*d^8 - a^1
```

```
3*b*d^9)*sqrt(x)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 2
20*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b
^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^
9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4))/(
b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3
+ 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7
*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*
d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)) - 585*a^4*x^7*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(a^13*(-(b^13*c^12 - 12*a*b^12*c^11
*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 79
2*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5
*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11
+ a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2
 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*
b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt(x)) +
585*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^
3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^
6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 6
6*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(-a^
13*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^
9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 7
92*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^
3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*
a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4
 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^
2*c*d^8 - a^9*b*d^9)*sqrt(x)) + 4*(585*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*(a*b^2*c^3 - 3*a^2*b*c^2*d + 3*a^3*c
*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)*sqrt(x))/(a^4*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23084, size = 724, normalized size = 2.23

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} b^3c^3 - 3\left(ab^3\right)^{\frac{3}{4}} ab^2c^2d + 3\left(ab^3\right)^{\frac{3}{4}} a^2bcd^2 - \left(ab^3\right)^{\frac{3}{4}} a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^5b^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}} b^3c^3 - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 1/2*sqrt(2)*((a*b^3)^(3/4)*
b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^
3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^5*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*
a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 1/4*sqrt(2)*((a*b^3)^(
3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
^5*b^2) + 2/585*(585*b^3*c^3*x^6 - 1755*a*b^2*c^2*d*x^6 + 1755*a^2*b*c*d^2*
x^6 - 585*a^3*d^3*x^6 - 117*a*b^2*c^3*x^4 + 351*a^2*b*c^2*d*x^4 - 351*a^3*c
*d^2*x^4 + 65*a^2*b*c^3*x^2 - 195*a^3*c^2*d*x^2 - 45*a^3*c^3)/(a^4*x^(13/2)
)
```

3.452 $\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=409

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}+\frac{\sqrt{x}(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log}{8\sqrt{2}}$$

```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2 - 39*a*b*
c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17*a*d)*x^(9/2))/(18*b
^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)
) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d
)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (
a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b
*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.46727, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 467, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}+\frac{\sqrt{x}(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2 - 39*a*b*
c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17*a*d)*x^(9/2))/(18*b
^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)
) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d
)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (
a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b
*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*b^(21/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(c+dx^4\right)^2\left(5c+17dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \left(\frac{(5bc-17ad)(bc-ad)^2}{b^4} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^4}{b^3} + \frac{d^2(39bc-17ad)x^8}{b^2} + \frac{17d^3}{b}\right.\right.}{2b}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3}{26}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3}{26}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3}{26}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3}{26}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3}{26}$$

Mathematica [C] time = 2.57421, size = 419, normalized size = 1.02

$$\frac{35a^2\left(-9945a\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^2bx^2\left(250563c^2dx^2+83521c^3+255555cd^2x^4+83521d^3x^6\right)+a^3\left(194481c^2dx^2+6\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (35*a^2*(-32768*b^4*x^8*(c + d*x^2)^3 + 8704*a*b^3*x^6*(1609*c^3 + 3423*c^2
*d*x^2 + 3267*c*d^2*x^4 + 1069*d^3*x^6) + 9945*a^4*(64827*c^3 + 194481*c^2*
d*x^2 + 194481*c*d^2*x^4 + 62651*d^3*x^6) + 3978*a^3*b*x^2*(176389*c^3 + 52
9167*c^2*d*x^2 + 541647*c*d^2*x^4 + 177477*d^3*x^6) + 221*a^2*b^2*x^4*(8576
91*c^3 + 2417553*c^2*d*x^2 + 2528145*c*d^2*x^4 + 846811*d^3*x^6) - 9945*a*(
b^3*x^6*(2827*c^3 + 6561*c^2*d*x^2 + 6561*c*d^2*x^4 + 2187*d^3*x^6) + a*b^2
*x^4*(28561*c^3 + 82227*c^2*d*x^2 + 85683*c*d^2*x^4 + 28561*d^3*x^6) + a^3*
(64827*c^3 + 194481*c^2*d*x^2 + 194481*c*d^2*x^4 + 62651*d^3*x^6) + a^2*b*x
^2*(83521*c^3 + 250563*c^2*d*x^2 + 255555*c*d^2*x^4 + 83521*d^3*x^6))*Hyper
geometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]) - 98304*b^6*x^12*(c + d*x^2)^3*Hyp
ergeometricPFQ[{2, 2, 2, 2, 13/4}, {1, 1, 1, 29/4}, -((b*x^2)/a)])/(8910720
0*a^3*b^5*x^(11/2))
```

Maple [B] time = 0.018, size = 804, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] -3/2*a^2/b^3*x^(1/2)/(b*x^2+a)*c^2*d+17/8*a^3/b^5*(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+17/8*a^3/b^5*(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+17/16*a^3/b^5*(1/b*a)^(1/4)*2^
(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+3/2*a^3/b^4*x^(1/2)/(b*x^2+a)*c*d^2-39/8
*a^2/b^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^
2+27/8*a/b^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*
c^2*d-39/16*a^2/b^4*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/
2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2+27
/16*a/b^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d-39/8*a^2/b^4
*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2+27/8*a
/b^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d-12
/5/b^3*x^(5/2)*a*c*d^2+18/b^4*a^2*c*d^2*x^(1/2)+2/b^2*c^3*x^(1/2)+2/13*d^3*
x^(13/2)/b^2-12/b^3*a*c^2*d*x^(1/2)-1/2*a^4/b^5*x^(1/2)/(b*x^2+a)*d^3+1/2*a
/b^2*x^(1/2)/(b*x^2+a)*c^3-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x^(1/2)+1)*c^3-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b
*a)^(1/4)*x^(1/2)-1)*c^3-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2)))*c^3+6/5/b^4*x^(5/2)*a^2*d^3+6/5/b^2*x^(5/2)*c^2*d-8/b^5*a^3*d^3*x^(1/
2)-4/9/b^3*x^(9/2)*a*d^3+2/3/b^2*x^(9/2)*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.42914, size = 5115, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4680*(2340*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d +
 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^
4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c
^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11
*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*arctan(
(sqrt(b^10*sqrt(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10
*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*
b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 1263145
5*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21) + (25*b^6*c^6 - 270*a*b^5*c^5
```

```
*d + 1119*a^2*b^4*c^4*d^2 - 2276*a^3*b^3*c^3*d^3 + 2439*a^4*b^2*c^2*d^4 - 1
326*a^5*b*c*d^5 + 289*a^6*d^6)*x)*b^16*(-(625*a*b^12*c^12 - 13500*a^2*b^11*
c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^
8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a
^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168
018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4)
 + (5*b^19*c^3 - 27*a*b^18*c^2*d + 39*a^2*b^17*c*d^2 - 17*a^3*b^16*d^3)*sqr
t(x)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2
- 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^
5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*
c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^1
2*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4))/(625*a*b^12*c^12 - 13500*a^2*b^1
1*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*
b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976
*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 31
68018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)) + 585*(b
^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^1
0*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6
*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 126314
55*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*log(b^5*(-(625*a*b^12*
c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^
9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^
6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a
^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*
a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*
a^3*d^3)*sqrt(x)) - 585*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^
11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5
*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 1422597
6*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3
168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1
/4)*log(-b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c
^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^
7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*
a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 76
6428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c
^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*sqrt(x)) + 4*(180*b^4*d^3*x^8 + 2925*a*
b^3*c^3 - 15795*a^2*b^2*c^2*d + 22815*a^3*b*c*d^2 - 9945*a^4*d^3 + 20*(39*b
^4*c*d^2 - 17*a*b^3*d^3)*x^6 + 52*(27*b^4*c^2*d - 39*a*b^3*c*d^2 + 17*a^2*b
^2*d^3)*x^4 + 468*(5*b^4*c^3 - 27*a*b^3*c^2*d + 39*a^2*b^2*c*d^2 - 17*a^3*b
*d^3)*x^2)*sqrt(x))/(b^6*x^2 + a*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22219, size = 810, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(
a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^6 - 1/8*sqrt(2)*(5*(a*b^3)^(1
/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 -
 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/b^6 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)
^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^
3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/16*sqrt(2)*(5*(
a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*
b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/b^6 + 1/2*(a*b^3*c^3*sqrt(x) - 3*a^2*b^2*c^2*d*sqrt(x) + 3*a^3*b
*c*d^2*sqrt(x) - a^4*d^3*sqrt(x))/((b*x^2 + a)*b^5) + 2/585*(45*b^24*d^3*x^
(13/2) + 195*b^24*c*d^2*x^(9/2) - 130*a*b^23*d^3*x^(9/2) + 351*b^24*c^2*d*x
^(5/2) - 702*a*b^23*c*d^2*x^(5/2) + 351*a^2*b^22*d^3*x^(5/2) + 585*b^24*c^3
*sqrt(x) - 3510*a*b^23*c^2*d*sqrt(x) + 5265*a^2*b^22*c*d^2*sqrt(x) - 2340*a
^3*b^21*d^3*sqrt(x))/b^26
```

3.453 $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=374

$$\frac{dx^{3/2}(5a^2d^2-11abcd+7b^2c^2)}{2b^4}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(b}{}$$

```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2*(11*b*c -
 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2) - (x^(3/2)*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(b*c -
 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```

Rubi [A] time = 0.414985, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 467, 570, 297, 1162, 617, 204, 1165, 628}

$$\frac{dx^{3/2}(5a^2d^2-11abcd+7b^2c^2)}{2b^4}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2*(11*b*c -
 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2) - (x^(3/2)*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(b*c -
 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(c+dx^4\right)^2\left(3c+15dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \left(\frac{3d\left(7b^2c^2-11abcd+5a^2d^2\right)x^2}{b^3} + \frac{3d^2(11bc-5ad)x^6}{b^2} + \frac{15d^3x^{10}}{b} + \frac{3\left(b^3c^3-7ab^2c^2d+\right.}{b^3\left(a\right.}\right.\right.}{2b}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{(3(bc-}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} - \frac{(3(bc-}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{(3(bc-}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{3(bc-}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} - \frac{3(bc-}{}$$

Mathematica [C] time = 2.03165, size = 377, normalized size = 1.01

$$\frac{385\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(9a^2bx^2\left(50625c^2dx^2+16875c^3+52033cd^2x^4+16875d^3x^6\right)+a^3\left(390963c^2dx^2+130321c^3+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (-32768*b^3*x^6*(c + d*x^2)^3 - 45*a*b^2*x^4*(122993*c^3 + 299987*c^2*d*x^2
 + 322515*c*d^2*x^4 + 109553*d^3*x^6) - 330*a^2*b*x^2*(112027*c^3 + 336081*
c^2*d*x^2 + 350865*c*d^2*x^4 + 114907*d^3*x^6) - 385*a^3*(130321*c^3 + 3909
63*c^2*d*x^2 + 390963*c*d^2*x^4 + 124561*d^3*x^6) + 385*(b^3*x^6*(3553*c^3
+ 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6) + 3*a*b^2*x^4*(14641*c^3
+ 41235*c^2*d*x^2 + 43923*c*d^2*x^4 + 14641*d^3*x^6) + 9*a^2*b*x^2*(16875*c
^3 + 50625*c^2*d*x^2 + 52033*c*d^2*x^4 + 16875*d^3*x^6) + a^3*(130321*c^3 +
 390963*c^2*d*x^2 + 390963*c*d^2*x^4 + 124561*d^3*x^6))*Hypergeometric2F1[3
/4, 1, 7/4, -((b*x^2)/a)])/(887040*a*b^4*x^(9/2)) - (128*b*x^(11/2)*(c + d*
x^2)^3*HypergeometricPFQ[{2, 2, 2, 2, 11/4}, {1, 1, 1, 27/4}, -((b*x^2)/a)]
)/(72105*a^3)
```

Maple [B] time = 0.017, size = 748, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/11*d^3*x^(11/2)/b^2-4/7*d^3/b^3*x^(7/2)*a+6/7*d^2/b^2*x^(7/2)*c+2*d^3/b^4
*x^(3/2)*a^2-4*d^2/b^3*x^(3/2)*a*c+2*d/b^2*x^(3/2)*c^2+1/2/b^4*x^(3/2)/(b*x
^2+a)*a^3*d^3-3/2/b^3*x^(3/2)/(b*x^2+a)*a^2*c*d^2+3/2/b^2*x^(3/2)/(b*x^2+a)
*a*c^2*d-1/2/b*x^(3/2)/(b*x^2+a)*c^3-15/16/b^5/(1/b*a)^(1/4)*2^(1/2)*a^3*d^
3*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/
2)*2^(1/2)+(1/b*a)^(1/2)))-15/8/b^5/(1/b*a)^(1/4)*2^(1/2)*a^3*d^3*arctan(2^
(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-15/8/b^5/(1/b*a)^(1/4)*2^(1/2)*a^3*d^3*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+33/16/b^4/(1/b*a)^(1/4)*2^(1/2)*a^2*c*d^
2*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/
2)*2^(1/2)+(1/b*a)^(1/2)))+33/8/b^4/(1/b*a)^(1/4)*2^(1/2)*a^2*c*d^2*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+33/8/b^4/(1/b*a)^(1/4)*2^(1/2)*a^2*c*d^2*a
rctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-21/16/b^3/(1/b*a)^(1/4)*2^(1/2)*a*c^
2*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(
1/2)*2^(1/2)+(1/b*a)^(1/2)))-21/8/b^3/(1/b*a)^(1/4)*2^(1/2)*a*c^2*d*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-21/8/b^3/(1/b*a)^(1/4)*2^(1/2)*a*c^2*d*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*c^3*ln(
(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(1/b
*a)^(1/4)*x^(1/2)+1)+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(1/b*
a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65904, size = 6029, normalized size = 16.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/616*(924*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10
*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^
7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d
^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 +
 625*a^12*d^12)/(a*b^19))^(1/4)*arctan((sqrt((b^18*c^18 - 42*a*b^17*c^17*d
+ 801*a^2*b^16*c^16*d^2 - 9200*a^3*b^15*c^15*d^3 + 71220*a^4*b^14*c^14*d^4
- 394392*a^5*b^13*c^13*d^5 + 1619684*a^6*b^12*c^12*d^6 - 5050512*a^7*b^11*c
^11*d^7 + 12147630*a^8*b^10*c^10*d^8 - 22765820*a^9*b^9*c^9*d^9 + 33419166*
a^10*b^8*c^8*d^10 - 38446992*a^11*b^7*c^7*d^11 + 34503236*a^12*b^6*c^6*d^12
 - 23888280*a^13*b^5*c^5*d^13 + 12508500*a^14*b^4*c^4*d^14 - 4790000*a^15*b
^3*c^3*d^15 + 1265625*a^16*b^2*c^2*d^16 - 206250*a^17*b*c*d^17 + 15625*a^18
```

```
*d^18)*x - (a*b^21*c^12 - 28*a^2*b^20*c^11*d + 338*a^3*b^19*c^10*d^2 - 2316
*a^4*b^18*c^9*d^3 + 10015*a^5*b^17*c^8*d^4 - 28856*a^6*b^16*c^7*d^5 + 57148
*a^7*b^15*c^6*d^6 - 78968*a^8*b^14*c^5*d^7 + 76111*a^9*b^13*c^4*d^8 - 50220
*a^10*b^12*c^3*d^9 + 21650*a^11*b^11*c^2*d^10 - 5500*a^12*b^10*c*d^11 + 625
*a^13*b^9*d^12)*sqrt(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2
 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 5
7148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 5022
0*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12
*d^12)/(a*b^19)))*b^5*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d
^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 +
 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50
220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^
12*d^12)/(a*b^19))^(1/4) + (b^14*c^9 - 21*a*b^13*c^8*d + 180*a^2*b^12*c^7*d
^2 - 820*a^3*b^11*c^6*d^3 + 2190*a^4*b^10*c^5*d^4 - 3606*a^5*b^9*c^4*d^5 +
3716*a^6*b^8*c^3*d^6 - 2340*a^7*b^7*c^2*d^7 + 825*a^8*b^6*c*d^8 - 125*a^9*b
^5*d^9)*sqrt(x)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2
316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148
*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^
9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^1
2)/(a*b^19))^(1/4))/(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 -
 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 571
48*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*
a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d
^12)) - 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^1
0*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c
^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*
d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11
+ 625*a^12*d^12)/(a*b^19))^(1/4)*log(27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^1
1*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4
- 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 7
6111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 55
00*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4) - 27*(b^9*c^9 - 21*a*b^8*
c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 -
3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^
8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28
*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b
^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*
c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^
2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4)*log(-27*a*b^14
*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9
*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^
6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 +
 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3
/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*
d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 -
2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 4*(28*b^3*
d^3*x^7 + 12*(11*b^3*c*d^2 - 5*a*b^2*d^3)*x^5 + 44*(7*b^3*c^2*d - 11*a*b^2*
c*d^2 + 5*a^2*b*d^3)*x^3 - 77*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5
*a^3*d^3)*x)*sqrt(x))/(b^5*x^2 + a*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27006, size = 745, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3
*d^3*x^(3/2))/((b*x^2 + a)*b^4) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a
*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^
3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
*b^7) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d +
11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) - 3/16*sqrt(2)*((a*
b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c
*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a*b^7) + 3/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2
*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^7) + 2/77*(7*b^20*d^3*x^(11/2)
 + 33*b^20*c*d^2*x^(7/2) - 22*a*b^19*d^3*x^(7/2) + 77*b^20*c^2*d*x^(3/2) -
154*a*b^19*c*d^2*x^(3/2) + 77*a^2*b^18*d^3*x^(3/2))/b^22
```

3.454 $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{90b^4}-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad}{}$$

```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) + (d*(113*b
*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqrt[x]*(c + d*x^2)^2)/
(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - ((b*c - 13*a*d)*(b*c
 - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)
*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(17/4))
```

Rubi [A] time = 0.52928, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 467, 528, 388, 211, 1165, 628, 1162, 617, 204}

$$\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{90b^4}-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) + (d*(113*b
*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqrt[x]*(c + d*x^2)^2)/
(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - ((b*c - 13*a*d)*(b*c
 - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)
*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(17/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)^2\left(c+13dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(c(9bc-13ad)+d(113bc-117ad)x^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{18b^2}$$

$$= \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{c\left(45b^2c^2-178ab\ldots\right.}{}\right.}{}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \ldots$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \ldots$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \ldots$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \ldots$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \ldots$$

Mathematica [C] time = 2.11163, size = 377, normalized size = 0.98

$$585\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(3a^2bx^2\left(85683c^2dx^2+28561c^3+89139cd^2x^4+28561d^3x^6\right)+a^3\left(250563c^2dx^2+83521c^3+2\ldots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (-98304*b^3*x^6*(c + d*x^2)^3 - 585*a^3*(83521*c^3 + 250563*c^2*d*x^2 + 250
563*c*d^2*x^4 + 78529*d^3*x^6) - 234*a^2*b*x^2*(172447*c^3 + 517341*c^2*d*x
^2 + 543261*c*d^2*x^4 + 174943*d^3*x^6) - 13*a*b^2*x^4*(532193*c^3 + 133737
9*c^2*d*x^2 + 1503267*c*d^2*x^4 + 507233*d^3*x^6) + 585*(b^3*x^6*(1009*c^3
+ 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + 9*a*b^2*x^4*(2187*c^3 +
5921*c^2*d*x^2 + 6561*c*d^2*x^4 + 2187*d^3*x^6) + 3*a^2*b*x^2*(28561*c^3 +
85683*c^2*d*x^2 + 89139*c*d^2*x^4 + 28561*d^3*x^6) + a^3*(83521*c^3 + 25056
3*c^2*d*x^2 + 250563*c*d^2*x^4 + 78529*d^3*x^6))*Hypergeometric2F1[1/4, 1,
5/4, -((b*x^2)/a)])/(449280*a*b^4*x^(11/2)) - (128*b*x^(9/2)*(c + d*x^2)^3*
```

```
HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 25/4}, -((b*x^2)/a)])/(41769
*a^3)
```

Maple [B] time = 0.016, size = 748, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/9*d^3/b^2*x^(9/2)-4/5*d^3/b^3*x^(5/2)*a+6/5*d^2/b^2*x^(5/2)*c+6*d^3/b^4*a
^2*x^(1/2)-12*d^2/b^3*c*a*x^(1/2)+6*d/b^2*c^2*x^(1/2)+1/2/b^4*x^(1/2)/(b*x^
2+a)*a^3*d^3-3/2/b^3*x^(1/2)/(b*x^2+a)*a^2*c*d^2+3/2/b^2*x^(1/2)/(b*x^2+a)*
a*c^2*d-1/2/b*x^(1/2)/(b*x^2+a)*c^3-13/8/b^4*(1/b*a)^(1/4)*a^2*2^(1/2)*arct
an(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+27/8/b^3*(1/b*a)^(1/4)*a*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-15/8/b^2*(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+1/8/b*(1/b*a)^(1/4)/a*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3-13/8/b^4*(1/b*a)^(1/4)*a^2*2^(
1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+27/8/b^3*(1/b*a)^(1/4)*a*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-15/8/b^2*(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+1/8/b*(1/b*a)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3-13/16/b^4*(1/b*a)^(1/
4)*a^2*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+27/16/b^3*(1/b*a)^(1/4)*a*2^(1/
2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2-15/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2)))*c^2*d+1/16/b*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)
))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39942, size = 4809, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^
10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b
```

```
^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^
8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276
*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4)*arctan((sqrt(a^2*b^8*sq
rt(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*
c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6
*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*
b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*
d^12)/(a^3*b^17)) + (b^6*c^6 - 30*a*b^5*c^5*d + 279*a^2*b^4*c^4*d^2 - 836*a
^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 702*a^5*b*c*d^5 + 169*a^6*d^6)*x)*a
^2*b^13*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^
3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a
^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 185364
4*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561
*a^12*d^12)/(a^3*b^17))^(3/4) + (a^2*b^16*c^3 - 15*a^3*b^15*c^2*d + 27*a^4*
b^14*c*d^2 - 13*a^5*b^13*d^3)*sqrt(x)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 145
8*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 5350
32*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 25
20207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10
- 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(3/4))/(b^12*c^12 - 6
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a
^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824
*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 8710
26*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)) + 45*(b^5*x
^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1841
2*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 13657
56*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 18
53644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 2
8561*a^12*d^12)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d
 + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4
- 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^
7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2
*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^
3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 45*(b^5*x^2 +
a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3
*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^
6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644
*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*
a^12*d^12)/(a^3*b^17))^(1/4)*log(-a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 53
5032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 +
2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^1
0 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^3 -
15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 4*(20*b^3*d^3*x^6
- 45*b^3*c^3 + 675*a*b^2*c^2*d - 1215*a^2*b*c*d^2 + 585*a^3*d^3 + 4*(27*b^3
*c*d^2 - 13*a*b^2*d^3)*x^4 + 36*(15*b^3*c^2*d - 27*a*b^2*c*d^2 + 13*a^2*b*d
^3)*x^2)*sqrt(x))/(b^5*x^2 + a*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19802, size = 745, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b
^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 -
13*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3
)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d
^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/16*sqrt(2)
*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a
^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a*b^5) - 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^
2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*b^4) + 2/45*(5*b^16*d^3*x
^(9/2) + 27*b^16*c*d^2*x^(5/2) - 18*a*b^15*d^3*x^(5/2) + 135*b^16*c^2*d*sqr
t(x) - 270*a*b^15*c*d^2*sqrt(x) + 135*a^2*b^14*d^3*sqrt(x))/b^18
```

3.455 $\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=376

$$-\frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{6ab^3}+\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\,\mathrm{lo}}{8\sqrt{}}$$

```
[Out] -(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(6*a*b^3) - (d^2*(7*b*c
- 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c + d*x^2)^2)/(2*a*b*
(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/
4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*c + 1
1*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*b^(15/4))
```

Rubi [A] time = 0.425869, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$-\frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{6ab^3}+\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\,\mathrm{lo}}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] -(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(6*a*b^3) - (d^2*(7*b*c
- 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c + d*x^2)^2)/(2*a*b*
(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/
4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*c + 1
1*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*b^(15/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
```

```
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^2\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^2\left(c+dx^4\right)\left(-c(bc+3ad)+d(7bc-11ad)x^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{\mathrm{Subst}\left(\int \left(\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^2}{b^2} + \frac{d^2(7bc-11ad)x^6}{b} - \frac{\left(b^3c^3+9ab^2c^2d-21a^2bc\right.}{b^2\left(a+bx^4\right.}\right.\right.}{2ab}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{\left((bc-ad\right.}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{\left((bc-ad\right.}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{\left((bc-ad\right.}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{(bc-ad)}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{(bc-ad)^2}{}$$

Mathematica [C] time = 2.1093, size = 355, normalized size = 0.94

$$\frac{95a\left(a\left(77a^2\left(50625c^2dx^2+16875c^3+50625cd^2x^4+15467d^3x^6\right)+22abx^2\left(77793c^2dx^2+25931c^3+87201cd^2x^4+2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (95*a*(a*(77*a^2*(16875*c^3 + 50625*c^2*d*x^2 + 50625*c*d^2*x^4 + 15467*d^3
*x^6) + b^2*x^4*(56099*c^3 + 79593*c^2*d*x^2 + 79593*c*d^2*x^4 + 26531*d^3*
x^6) + 22*a*b*x^2*(25931*c^3 + 77793*c^2*d*x^2 + 87201*c*d^2*x^4 + 28043*d^
3*x^6)) - 77*(b^3*x^6*(-101*c^3 + 81*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6)
 + a*b^2*x^4*(2401*c^3 + 6051*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6) +
a^2*b*x^2*(14641*c^3 + 43923*c^2*d*x^2 + 46611*c*d^2*x^4 + 14641*d^3*x^6) +
 a^3*(16875*c^3 + 50625*c^2*d*x^2 + 50625*c*d^2*x^4 + 15467*d^3*x^6))*Hyper
geometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]) - 32768*b^4*x^8*(c + d*x^2)^3*Hype
rgeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1, 1, 23/4}, -((b*x^2)/a)])/(5617920*a
^3*b^3*x^(9/2))
```

Maple [B] time = 0.019, size = 706, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/7*d^3/b^2*x^(7/2)-4/3*d^3/b^3*x^(3/2)*a+2*d^2/b^2*x^(3/2)*c-1/2/b^3*a^2*x
^(3/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(3/2)/(b*x^2+a)*c*d^2-3/2/b*x^(3/2)/(b*x^2
+a)*c^2*d+1/2/a*x^(3/2)/(b*x^2+a)*c^3+11/8/b^4*a^2/(1/b*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-21/8/b^3*a/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2+9/8/b^2/(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+1/8/b/a/(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3+11/8/b^4*a^2/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-21/8/b^3*a/(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2+9/8/b^2/(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+1/8/b/a/(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+11/16/b^4*a^2/(1/b*a)
^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3-21/16/b^3*a/(1/b*a)^(1/4)*2^(1/
2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2+9/16/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b
*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(
1/b*a)^(1/2)))*c^2*d+1/16/b/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(
1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64902, size = 6134, normalized size = 16.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*
b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*
c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*
c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b
*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*arctan((sqrt((b^18*c^18 + 54*a
*b^17*c^17*d + 1089*a^2*b^16*c^16*d^2 + 8976*a^3*b^15*c^15*d^3 + 5940*a^4*b
^14*c^14*d^4 - 279576*a^5*b^13*c^13*d^5 - 338844*a^6*b^12*c^12*d^6 + 600177
6*a^7*b^11*c^11*d^7 - 6412626*a^8*b^10*c^10*d^8 - 62165180*a^9*b^9*c^9*d^9
+ 294333534*a^10*b^8*c^8*d^10 - 671362704*a^11*b^7*c^7*d^11 + 974580036*a^1
2*b^6*c^6*d^12 - 971334936*a^13*b^5*c^5*d^13 + 678512340*a^14*b^4*c^4*d^14
- 328575984*a^15*b^3*c^3*d^15 + 105546969*a^16*b^2*c^2*d^16 - 20292426*a^17
*b*c*d^17 + 1771561*a^18*d^18)*x - (a^3*b^19*c^12 + 36*a^4*b^18*c^11*d + 40
```

```
2*a^5*b^17*c^10*d^2 + 692*a^6*b^16*c^9*d^3 - 10017*a^7*b^15*c^8*d^4 - 5688*
a^8*b^14*c^7*d^5 + 160188*a^9*b^13*c^6*d^6 - 486648*a^10*b^12*c^5*d^7 + 746
703*a^11*b^11*c^4*d^8 - 676588*a^12*b^10*c^3*d^9 + 368082*a^13*b^9*c^2*d^10
 - 111804*a^14*b^8*c*d^11 + 14641*a^15*b^7*d^12)*sqrt(-(b^12*c^12 + 36*a*b^
11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8
*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d
^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*
d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15)))*a*b^4*(-(b^12*c
^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 1001
7*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*
a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*
a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/
4) - (a*b^13*c^9 + 27*a^2*b^12*c^8*d + 180*a^3*b^11*c^7*d^2 - 372*a^4*b^10*
c^6*d^3 - 3186*a^5*b^9*c^5*d^4 + 13194*a^6*b^8*c^4*d^5 - 21372*a^7*b^7*c^3*
d^6 + 17820*a^8*b^6*c^2*d^7 - 7623*a^9*b^5*c*d^8 + 1331*a^10*b^4*d^9)*sqrt(
x)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^
9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d
^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d
^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^
5*b^15))^(1/4))/(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692
*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^
9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^1
2*d^12)) - 21*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a
^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b
^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b
^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^1
1*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(a^4*b^11*(-(b^12*c^12 +
 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4
*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b
^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*
b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) +
(b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 318
6*a^4*b^5*c^5*d^4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a
^7*b^2*c^2*d^7 - 7623*a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) + 21*(a*b^4*x^2
+ a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^
3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b
^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d
^12)/(a^5*b^15))^(1/4)*log(-a^4*b^11*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*
a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*
b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*
b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^
11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) + (b^9*c^9 + 27*a*b^8*c^8*
d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 1319
4*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 - 7623*a^
8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) - 4*(12*a*b^2*d^3*x^5 + 4*(21*a*b^2*c*d^
2 - 11*a^2*b*d^3)*x^3 + 7*(3*b^3*c^3 - 9*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 11*
a^3*d^3)*x)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22588, size = 697, normalized size = 1.85

$$\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\left(bx^2 + a\right)ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 + 9\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 21\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 11\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)}{8\,a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(
a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*
a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^2*b^6) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*
d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) - 1/16*sqrt(
2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*
a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^6) + 1/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(
3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) + 2/21*(3*b^12
*d^3*x^(7/2) + 21*b^12*c*d^2*x^(3/2) - 14*a*b^11*d^3*x^(3/2))/b^14
```

3.456 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2}\,dx$

Optimal. Leaf size=340

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\ldots$$

```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + ((b*c - a*d
)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(
b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
+ (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```

Rubi [A] time = 0.388311, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```


```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + ((b*c - a*d
)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(
b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
+ (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2}dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{(a+bx^4)^2}dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3(a+bx^4)^2}\right)dx, x, \sqrt{x}\right)$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{2\,\text{Subst}\left(\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{(a+bx^4)^2}dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}dx, x,\right.}{2ab^3}$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx\right.}{4a^{3/2}b^3}$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\right.}{8a^{3/2}b^{7/2}}$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{8\sqrt{2}a^{7/4}b^{13/4}}$$
$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+$$

Mathematica [C] time = 2.01997, size = 358, normalized size = 1.05

$$\frac{a\left(45a^2\left(85683c^2dx^2+28561c^3+85683cd^2x^4+25105d^3x^6\right)+18abx^2\left(104781c^2dx^2+34927c^3+119181cd^2x^4+3665\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (a*(45*a^2*(28561*c^3 + 85683*c^2*d*x^2 + 85683*c*d^2*x^4 + 25105*d^3*x^6)
+ b^2*x^4*(50033*c^3 + 98259*c^2*d*x^2 + 98259*c*d^2*x^4 + 32753*d^3*x^6) +
 18*a*b*x^2*(34927*c^3 + 104781*c^2*d*x^2 + 119181*c*d^2*x^4 + 36655*d^3*x^
6)) - 45*(b^3*x^6*(-1151*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 3*a*b
^2*x^4*(625*c^3 + 1491*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + 9*a^2*b*
x^2*(2187*c^3 + 6561*c^2*d*x^2 + 7201*c*d^2*x^4 + 2187*d^3*x^6) + a^3*(2856
1*c^3 + 85683*c^2*d*x^2 + 85683*c*d^2*x^4 + 25105*d^3*x^6))*Hypergeometric2
F1[1/4, 1, 5/4, -((b*x^2)/a)])/(34560*a^2*b^3*x^(11/2)) - (128*b*x^(5/2)*(c
 + d*x^2)^3*HypergeometricPFQ[{5/4, 2, 2, 2, 2}, {1, 1, 1, 21/4}, -((b*x^2)
/a)])/(9945*a^3)
```

Maple [B] time = 0.016, size = 697, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b^2-4*d^3/b^3*a*x^(1/2)+6*d^2/b^2*x^(1/2)*c-1/2/b^3*a^2*x^(
1/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(1/2)/(b*x^2+a)*c*d^2-3/2/b*x^(1/2)/(b*x^2+a
)*c^2*d+1/2/a*x^(1/2)/(b*x^2+a)*c^3+9/8/b^3*a*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3-15/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2+3/8/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+3/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+9/16/b^3*a*(1/b*a)^(1/4)*2^(1/2)*ln(
(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))*d^3-15/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^
(1/2)))*c*d^2+3/16/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d
+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3+9/8/b^3*a*(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3-15/8/b^2*(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2+3/8/b/a*(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+3/8/a^2*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.37465, size = 4316, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(60*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
- 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a
^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)
/(a^7*b^13))^(1/4)*arctan((sqrt(a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d
- 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5
*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4
*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 +
81*a^12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 -
4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x)*a^5
*b^10*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^
9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 3
28*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^1
0*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4) - (a^5
*b^13*c^3 + a^6*b^12*c^2*d - 5*a^7*b^11*c*d^2 + 3*a^8*b^10*d^3)*sqrt(x)*(-(
```

```
b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 1
27*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^
5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2
*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4))/(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)) + 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a
*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*
d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 103
9*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^1
1*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d -
 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) - 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^1
2 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b
^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^
7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 -
540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*
c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^
4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5
*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10
 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2
*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) + 4*(4*a*b^2*d^3*x^4 + 5*b^3*c
^3 - 15*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 45*a^3*d^3 + 12*(5*a*b^2*c*d^2 - 3*a
^2*b*d^3)*x^2)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2229, size = 690, normalized size = 2.03

$$\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{4}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^
(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 3/8*sqrt(2)*((a*b^3)^(1/4)*
```

```
b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^
3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a
*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) - 3/16*sqrt(2)*((a*b^3)
^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 +
3*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(a^2*b^4) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sq
rt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a*b^3) + 2/5*(b^8*d^3*x^(5/2) + 15*b^
8*c*d^2*sqrt(x) - 10*a*b^7*d^3*sqrt(x))/b^10
```

3.457 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2} dx$

Optimal. Leaf size=368

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+ \cdots$$

```
[Out] -(c^2*(5*b*c - a*d))/(2*a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(3/2))/(6*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a + b*x^2)) + ((b*c - a
*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(
5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(11/4))
```

Rubi [A] time = 0.431288, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+ \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(5*b*c - a*d))/(2*a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(3/2))/(6*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a + b*x^2)) + ((b*c - a
*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(
5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(11/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(5bc-ad)+d(3bc-7ad)x^4)}{x^2(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-5bc+ad)}{ax^2} + \frac{d^2(3bc-7ad)x^2}{b} + \frac{(-bc+ad)^2(5bc+7ad)x^2}{ab(a+bx^4)}\right)dx, x, \sqrt{x}\right)}{2ab}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} - \frac{((bc-ad)^2(5bc+7ad))\,\text{Subst}\left(\int\right.}{2a^2b^2}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} + \frac{((bc-ad)^2(5bc+7ad))\,\text{Subst}\left(\int\right.}{4a^2b^{5/2}}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} - \frac{((bc-ad)^2(5bc+7ad))\,\text{Subst}\left(\int\right.}{8a^2b^3}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} - \frac{(bc-ad)^2(5bc+7ad)\log\left(\sqrt{a}-\right.}{8\sqrt{2}a^{9/4}b^{11/4}}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)} + \frac{(bc-ad)^2(5bc+7ad)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{9/4}b^{11/4}}$$

Mathematica [C] time = 1.99514, size = 355, normalized size = 0.96

$$-\frac{32768b^3x^6(c+dx^2)^3\,\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},2,2,2,2\right\},\left\{1,1,1,\frac{19}{4}\right\},-\frac{bx^2}{a}\right)+55\left(a\left(7a^2\left(43923c^2dx^2+14641c^3\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -(55*(a*(-21*b^2*x^4*(-1919*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 18
*a*b*x^2*(361*c^3 + 1083*c^2*d*x^2 + 2427*c*d^2*x^4 + 809*d^3*x^6) + 7*a^2*
(14641*c^3 + 43923*c^2*d*x^2 + 43923*c*d^2*x^4 + 11953*d^3*x^6)) - 7*(b^3*x
^6*(-1919*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 9*a*b^2*x^4*(27*c^3
+ 209*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + 3*a^2*b*x^2*(2401*c^3 + 7203
*c^2*d*x^2 + 8355*c*d^2*x^4 + 2401*d^3*x^6) + a^3*(14641*c^3 + 43923*c^2*d*
x^2 + 43923*c*d^2*x^4 + 11953*d^3*x^6))*Hypergeometric2F1[3/4, 1, 7/4, -((b
*x^2)/a)]) + 32768*b^3*x^6*(c + d*x^2)^3*HypergeometricPFQ[{3/4, 2, 2, 2, 2
}, {1, 1, 1, 19/4}, -((b*x^2)/a)])/(887040*a^3*b^2*x^(9/2))
```

Maple [B] time = 0.02, size = 682, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*d^3*x^(3/2)/b^2-2*c^3/a^2/x^(1/2)+1/2/b^2*a*x^(3/2)/(b*x^2+a)*d^3-3/2/b
*x^(3/2)/(b*x^2+a)*c*d^2+3/2/a*x^(3/2)/(b*x^2+a)*c^2*d-1/2*b/a^2*x^(3/2)/(b
*x^2+a)*c^3-7/16/b^3*a/(1/b*a)^(1/4)*2^(1/2)*d^3*ln((x-(1/b*a)^(1/4)*x^(1/2
)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-7
/8/b^3*a/(1/b*a)^(1/4)*2^(1/2)*d^3*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-
7/8/b^3*a/(1/b*a)^(1/4)*2^(1/2)*d^3*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
+3/16/b/a/(1/b*a)^(1/4)*2^(1/2)*c^2*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(
1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+3/8/b/a/(1/b
*a)^(1/4)*2^(1/2)*c^2*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+3/8/b/a/(1/
b*a)^(1/4)*2^(1/2)*c^2*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-5/16/a^2/(
1/b*a)^(1/4)*2^(1/2)*c^3*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-5/8/a^2/(1/b*a)^(1/4)*2^(
1/2)*c^3*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8/a^2/(1/b*a)^(1/4)*2^(1
/2)*c^3*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+9/16/b^2/(1/b*a)^(1/4)*2^(1
/2)*c*d^2*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/
4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+9/8/b^2/(1/b*a)^(1/4)*2^(1/2)*c*d^2*arct
an(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+9/8/b^2/(1/b*a)^(1/4)*2^(1/2)*c*d^2*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64249, size = 6132, normalized size = 16.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d -
3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 287
28*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*
a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^
11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*arctan((sqrt((15625*b^18*c^
18 - 56250*a*b^17*c^17*d - 84375*a^2*b^16*c^16*d^2 + 570000*a^3*b^15*c^15*d
^3 - 211500*a^4*b^14*c^14*d^4 - 2174040*a^5*b^13*c^13*d^5 + 2720004*a^6*b^1
2*c^12*d^6 + 3321072*a^7*b^11*c^11*d^7 - 8368866*a^8*b^10*c^10*d^8 + 640420
*a^9*b^9*c^9*d^9 + 11255310*a^10*b^8*c^8*d^10 - 8509968*a^11*b^7*c^7*d^11 -
 4831644*a^12*b^6*c^6*d^12 + 9537192*a^13*b^5*c^5*d^13 - 3095820*a^14*b^4*c
^4*d^14 - 2551920*a^15*b^3*c^3*d^15 + 2614689*a^16*b^2*c^2*d^16 - 907578*a^
17*b*c*d^17 + 117649*a^18*d^18)*x - (625*a^5*b^17*c^12 - 1500*a^6*b^16*c^11
*d - 3150*a^7*b^15*c^10*d^2 + 11060*a^8*b^14*c^9*d^3 + 1071*a^9*b^13*c^8*d^
4 - 28728*a^10*b^12*c^7*d^5 + 19068*a^11*b^11*c^6*d^6 + 27144*a^12*b^10*c^5
*d^7 - 37665*a^13*b^9*c^4*d^8 + 2324*a^14*b^8*c^3*d^9 + 19698*a^15*b^7*c^2*
```

```
d^10 - 12348*a^16*b^6*c*d^11 + 2401*a^17*b^5*d^12)*sqrt(-(625*b^12*c^12 - 1
500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a
^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*
b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2
*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11)))*a^2*b^3*(-(6
25*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*
c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*
d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9
+ 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11
))^(1/4) - (125*a^2*b^12*c^9 - 225*a^3*b^11*c^8*d - 540*a^4*b^10*c^7*d^2 +
1308*a^5*b^9*c^6*d^3 + 342*a^6*b^8*c^5*d^4 - 2430*a^7*b^7*c^4*d^5 + 1140*a^
8*b^6*c^3*d^6 + 1260*a^9*b^5*c^2*d^7 - 1323*a^10*b^4*c*d^8 + 343*a^11*b^3*d
^9)*sqrt(x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2
+ 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19
068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*
a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12
*d^12)/(a^9*b^11))^(1/4))/(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^
10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*
c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4
*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11
 + 2401*a^12*d^12)) - 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a
*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^
8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c
^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*
d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(a^7*b^8*
(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*
b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*
c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*
d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*
b^11))^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*
a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3
*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x))
+ 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*
a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b
^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*
c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(-a^7*b^8*(-(625*b^12*c^12 -
1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*
a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7
*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^
2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4) + (125
*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 3
42*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3*d^6 + 1260*a^7
*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*d^3*x^
4 - 12*a*b^2*c^3 - (15*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 7*a^3*d^3)
*x^2)*sqrt(x))/(a^2*b^3*x^3 + a^3*b^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26914, size = 680, normalized size = 1.85

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b^2}-\frac{5\,b^3c^3x^2-3\,ab^2c^2dx^2+3\,a^2bcd^2x^2-a^3d^3x^2+4\,ab^2c^3}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2b^2}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d-9\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2\right.}{8\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(5*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3*a^2*b*c*d^
2*x^2 - a^3*d^3*x^2 + 4*a*b^2*c^3)/((b*x^(5/2) + a*sqrt(x))*a^2*b^2) - 1/8*
sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^
(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^5) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4
)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^5) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)
^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^5) - 1/16*sqrt(2)*
(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^
2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^3*b^5)
```

3.458 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=367

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}+\frac{(bc}{}$$

```
[Out] -(c^2*(7*b*c - 3*a*d))/(6*a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqrt[x])/(2*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a + b*x^2)) + ((b*c - a
*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2*(
7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(9/4))
```

Rubi [A] time = 0.410948, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}+\frac{(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(7*b*c - 3*a*d))/(6*a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqrt[x])/(2*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a + b*x^2)) + ((b*c - a
*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2*(
7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(9/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(7bc-3ad)+d(bc-5ad)x^4)}{x^4(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{d^2(bc-5ad)}{b} + \frac{c^2(-7bc+3ad)}{ax^4} + \frac{(-bc+ad)^2(7bc+5ad)}{ab(a+bx^4)}\right)dx, x, \sqrt{x}\right)}{2ab}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{((bc-ad)^2(7bc+5ad))\,\text{Subst}(\int}{2a^2b^2}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{((bc-ad)^2(7bc+5ad))\,\text{Subst}(\int}{4a^{5/2}b^2}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{((bc-ad)^2(7bc+5ad))\,\text{Subst}(\int}{8a^{5/2}b^{5/2}}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} + \frac{(bc-ad)^2(7bc+5ad)\log(\sqrt{a}-}{8\sqrt{2}a^{11/4}b^{9/4}}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} + \frac{(bc-ad)^2(7bc+5ad)\tan^{-1}(1-}{4\sqrt{2}a^{11/4}b^{9/4}}$$

Mathematica [C] time = 2.1343, size = 350, normalized size = 0.95

$$-\frac{32768b^3x^6(c+dx^2)^3\,\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},2,2,2,2\right\},\left\{1,1,1,\frac{17}{4}\right\},-\frac{bx^2}{a}\right) - 585\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^2bx^2(187\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(39*a*(-5*b^2*x^4*(-869*c^3 + 81*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) +
6*a*b*x^2*(469*c^3 + 1407*c^2*d*x^2 + 2367*c*d^2*x^4 + 789*d^3*x^6) + 15*a^
2*(2187*c^3 + 6561*c^2*d*x^2 + 6561*c*d^2*x^4 + 1547*d^3*x^6)) - 585*(a*b^2
*x^4*(c^3 + 1155*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + b^3*x^6*(-869*c^3 + 8
1*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + a^2*b*x^2*(625*c^3 + 1875*c^2*d*
x^2 + 2259*c*d^2*x^4 + 625*d^3*x^6) + a^3*(2187*c^3 + 6561*c^2*d*x^2 + 6561
*c*d^2*x^4 + 1547*d^3*x^6))*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2)/a)] +
32768*b^3*x^6*(c + d*x^2)^3*HypergeometricPFQ[{1/4, 2, 2, 2, 2}, {1, 1, 1,
17/4}, -((b*x^2)/a)])/(149760*a^3*b^2*x^(11/2))
```

Maple [B] time = 0.019, size = 682, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*d^3/b^2*x^(1/2)-2/3*c^3/a^2/x^(3/2)+1/2*a/b^2*x^(1/2)/(b*x^2+a)*d^3-3/2/b
*x^(1/2)/(b*x^2+a)*c*d^2+3/2/a*x^(1/2)/(b*x^2+a)*c^2*d-1/2/a^2*b*x^(1/2)/(b
*x^2+a)*c^3-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)+1)*d^3+3/8/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/
2)+1)*c*d^2+9/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)+1)*c^2*d-7/8/a^3*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)*c^3-5/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)-1)*d^3+3/8/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(
1/2)-1)*c*d^2+9/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)-1)*c^2*d-7/8/a^3*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)
*x^(1/2)-1)*c^3-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*
2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3
+3/16/a/b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c*d^2+9/16/a^2*(1/
b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1
/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d-7/16/a^3*b*(1/b*a)^(1/4)*
2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.37307, size = 4639, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11
*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4
 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 +
1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 150
0*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*arctan((sqrt(a^6*b^4*sqr
t(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a
^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*
b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*
c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^1
1*b^9)) + (49*b^6*c^6 - 126*a*b^5*c^5*d + 39*a^2*b^4*c^4*d^2 + 124*a^3*b^3*
c^3*d^3 - 81*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 25*a^6*d^6)*x)*a^8*b^7*(-(2
401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^
9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c
^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d
^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9
```

```
))^(3/4) - (7*a^8*b^10*c^3 - 9*a^9*b^9*c^2*d - 3*a^10*b^8*c*d^2 + 5*a^11*b^
7*d^3)*sqrt(x)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^1
0*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^
5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 +
11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a
^12*d^12)/(a^11*b^9))^(3/4))/(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*
a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^
4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*
d^11 + 625*a^12*d^12)) + 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 -
12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 3766
5*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a
^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b
^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(a^3
*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 23
24*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*
a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*
b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/
(a^11*b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)
*sqrt(x)) - 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*
c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8
*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^
7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 -
 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(-a^3*b^2*(-(2401
*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c
^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*
d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9
- 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^
(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)) -
4*(12*a^2*b*d^3*x^4 - 4*a*b^2*c^3 - (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 15*a^3*d^3)*x^2)*sqrt(x))/(a^2*b^3*x^4 + a^3*b^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18381, size = 676, normalized size = 1.84

$$\frac{2\,d^3\sqrt{x}}{b^2} - \frac{2\,c^3}{3\,a^2x^{\frac{3}{2}}} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 9\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 5\,\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 2/3*c^3/(a^2*x^(3/2)) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^
3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(a^3*b^3) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)
*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1
/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b
^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 -
9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)
*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) - 1/2
*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3
*sqrt(x))/((b*x^2 + a)*a^2*b^2)
```

3.459 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=376

$$\frac{c\left(2a^2d^2-15abcd+9b^2c^2\right)}{2a^3b\sqrt{x}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\right.}{8\sqrt{2}a^{13/4}}$$

```
[Out] -(c^2*(9*b*c - 5*a*d))/(10*a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*
a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(5/2)*(a
 + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/
4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*(3*b*
c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(13/4)*b^(7/4))
```

Rubi [A] time = 0.429371, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$\frac{c\left(2a^2d^2-15abcd+9b^2c^2\right)}{2a^3b\sqrt{x}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\right.}{8\sqrt{2}a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(9*b*c - 5*a*d))/(10*a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*
a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(5/2)*(a
 + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/
4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*(3*b*
c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(13/4)*b^(7/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
```

```
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{7/2}\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^6\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(-c(9bc-5ad)-d(bc+3ad)x^4\right)}{x^6\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-9bc+5ad)}{ax^6} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{a^2x^2} - \frac{3(-bc+ad)^2(3bc+ad)x^2}{a^2\left(a+bx^4\right)}\right)\,dx, x, \right.}{2ab}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(3(bc-ad)^2(3bc+ad\right)}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\left(3(bc-ad)^2(3bc+ad\right)}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(3(bc-ad)^2(3bc+ad\right)}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{3(bc-ad)^2(3bc+ad)}{8}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{3(bc-ad)^2(3bc+ad)}{4\sqrt{2}a^{13/}}$$

Mathematica [C] time = 1.89733, size = 353, normalized size = 0.94

$$-\frac{-491520ab^2x^4\left(c+dx^2\right)^3 \text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},2,2,2,2\right\},\left\{1,1,1,\frac{15}{4}\right\},-\frac{bx^2}{a}\right) + 385\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(9a^2b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -(-77*a*(6*a*b*x^2*(-1423*c^3 - 13485*c^2*d*x^2 + 915*c*d^2*x^4 + 305*d^3*x
^6) - 15*b^2*x^4*(-2831*c^3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6
) + 5*a^2*(2401*c^3 + 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 1249*d^3*x^6)) + 38
5*(3*a*b^2*x^4*(c^3 + 1923*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 9*a^2*b*x^2
*(27*c^3 + 81*c^2*d*x^2 - 47*c*d^2*x^4 + 27*d^3*x^6) + b^3*x^6*(-2831*c^3 +
 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + a^3*(2401*c^3 + 7203*c^2*
d*x^2 + 7203*c*d^2*x^4 + 1249*d^3*x^6))*Hypergeometric2F1[3/4, 1, 7/4, -((b
*x^2)/a)] - 491520*a*b^2*x^4*(c + d*x^2)^3*HypergeometricPFQ[{-1/4, 2, 2, 2
, 2}, {1, 1, 1, 15/4}, -((b*x^2)/a)])/(887040*a^4*b*x^(9/2))
```

Maple [B] time = 0.02, size = 697, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5*c^3/a^2/x^(5/2)-6*c^2/a^2/x^(1/2)*d+4*c^3/a^3/x^(1/2)*b-1/2/b*x^(3/2)/
(b*x^2+a)*d^3+3/2/a*x^(3/2)/(b*x^2+a)*c*d^2-3/2/a^2*b*x^(3/2)/(b*x^2+a)*c^2
*d+1/2/a^3*b^2*x^(3/2)/(b*x^2+a)*c^3+3/8/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+3/8/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-15/8/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+9/8/a^3*b/(1/b*a)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(
1/2)+(1/b*a)^(1/2)))*d^3+3/16/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2)))*c*d^2-15/16/a^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(
1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^2*d+
9/16/a^3*b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a
)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3+3/8/b^2/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+3/8/a/b/(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-15/8/a^2/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2*d+9/8/a^3*b/(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65603, size = 5704, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(60*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a
^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^
4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*
d^12)/(a^13*b^7))^(1/4)*arctan((sqrt((729*b^18*c^18 - 7290*a*b^17*c^17*d +
31833*a^2*b^16*c^16*d^2 - 78192*a^3*b^15*c^15*d^3 + 113940*a^4*b^14*c^14*d^
4 - 88920*a^5*b^13*c^13*d^5 + 10180*a^6*b^12*c^12*d^6 + 46320*a^7*b^11*c^11
*d^7 - 35970*a^8*b^10*c^10*d^8 - 220*a^9*b^9*c^9*d^9 + 12078*a^10*b^8*c^8*d
^10 - 3600*a^11*b^7*c^7*d^11 - 1884*a^12*b^6*c^6*d^12 + 936*a^13*b^5*c^5*d^
13 + 180*a^14*b^4*c^4*d^14 - 112*a^15*b^3*c^3*d^15 - 15*a^16*b^2*c^2*d^16 +
 6*a^17*b*c*d^17 + a^18*d^18)*x - (81*a^7*b^15*c^12 - 540*a^8*b^14*c^11*d +
 1458*a^9*b^13*c^10*d^2 - 1932*a^10*b^12*c^9*d^3 + 1039*a^11*b^11*c^8*d^4 +
 328*a^12*b^10*c^7*d^5 - 644*a^13*b^9*c^6*d^6 + 136*a^14*b^8*c^5*d^7 + 127*
a^15*b^7*c^4*d^8 - 44*a^16*b^6*c^3*d^9 - 14*a^17*b^5*c^2*d^10 + 4*a^18*b^4*
c*d^11 + a^19*b^3*d^12)*sqrt(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*
```

```
b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c
^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 -
44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a
^13*b^7)))*a^3*b^2*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10
*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 -
644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^
3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))
^(1/4) - (27*a^3*b^11*c^9 - 135*a^4*b^10*c^8*d + 252*a^5*b^9*c^7*d^2 - 188*
a^6*b^8*c^6*d^3 - 6*a^7*b^7*c^5*d^4 + 78*a^8*b^6*c^4*d^5 - 20*a^9*b^5*c^3*d
^6 - 12*a^10*b^4*c^2*d^7 + 3*a^11*b^3*c*d^8 + a^12*b^2*d^9)*sqrt(x)*(-(81*b
^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^
3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*
a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^
2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4))/(81*b^12*c^12 - 54
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*
b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d
^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^
11*b*c*d^11 + a^12*d^12)) - 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 -
540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^
4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5
*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*
a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(27*a^10*b^5*(-(81*b^12*c^1
2 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 103
9*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5
*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10
+ 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b
^8*c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 +
78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^
8 + a^9*d^9)*sqrt(x)) + 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*
a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^
8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7
 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11
*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(-27*a^10*b^5*(-(81*b^12*c^12 -
 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a
^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^
5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4
*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b^8*
c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*
a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 +
 a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(x))/(a
^3*b^2*x^5 + a^4*b*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25349, size = 682, normalized size = 1.81

$$\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\left(bx^2 + a\right)a^3b} + \frac{2\left(10\,bc^3x^2 - 15\,ac^2dx^2 - ac^3\right)}{5\,a^3x^{\frac{5}{2}}} + \frac{3\,\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 5\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a^3*b) + 2/5*(10*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c
^3)/(a^3*x^(5/2)) + 3/8*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*
a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) + 3/8*sqr
t(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)
*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) - 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3
*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^4) + 3
/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3
)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^4*b^4)
```

3.460 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2} dx$

Optimal. Leaf size=376

$$\frac{c(6a^2d^2-21abcd+11b^2c^2)}{6a^3bx^{3/2}}-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\right.}{8\sqrt{2}a^{15/4}}$$

```
[Out] -(c^2*(11*b*c - 7*a*d))/(14*a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*a*b*c*d +
6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(7/2)*
(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/
4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c
+ a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 0.415759, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{c(6a^2d^2-21abcd+11b^2c^2)}{6a^3bx^{3/2}}-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\right.}{8\sqrt{2}a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(11*b*c - 7*a*d))/(14*a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*a*b*c*d +
6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(7/2)*
(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/
4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c
+ a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(15/4)*b^(5/4))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
```

```
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = 2\,\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^8(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(11bc-7ad)-d(3bc+ad)x^4)}{x^8(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-11bc+7ad)}{ax^8} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{a^2x^4} - \frac{(-bc+ad)^2(11bc+ad)}{a^2(a+bx^4)}\right) dx, x\right.}{2ab}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11bc+a}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11bc+a}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11bc+a}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)^2(11bc+ad}{8}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)^2(11bc+ad}{4\sqrt{2}a}$$

Mathematica [C] time = 1.799, size = 353, normalized size = 0.94

$$-\frac{-229376ab^2x^4(c+dx^2)^3\,\text{HypergeometricPFQ}\left(\left\{-\frac{3}{4},2,2,2,2\right\},\left\{1,1,1,\frac{13}{4}\right\},-\frac{bx^2}{a}\right) + 315\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)(3a^2b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] -(-15*a*(21*a^2*(625*c^3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 241*d^3*x^6) +
 6*a*b*x^2*(-1195*c^3 - 6657*c^2*d*x^2 + 2751*c*d^2*x^4 + 917*d^3*x^6) - 7*
b^2*x^4*(-1823*c^3 + 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6)) + 315
*(3*a^2*b*x^2*(c^3 + 3*c^2*d*x^2 - 1149*c*d^2*x^4 + d^3*x^6) + 9*a*b^2*x^4*
(27*c^3 + 977*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + a^3*(625*c^3 + 1875*
c^2*d*x^2 + 1875*c*d^2*x^4 + 241*d^3*x^6) + b^3*x^6*(-1823*c^3 + 7203*c^2*d
*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6))*Hypergeometric2F1[1/4, 1, 5/4, -((b*
x^2)/a)] - 229376*a*b^2*x^4*(c + d*x^2)^3*HypergeometricPFQ[{-3/4, 2, 2, 2,
 2}, {1, 1, 1, 13/4}, -((b*x^2)/a)])/(241920*a^4*b*x^(11/2))
```

Maple [B] time = 0.021, size = 706, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/7*c^3/a^2/x^(7/2)-2*c^2/a^2/x^(3/2)*d+4/3*c^3/a^3/x^(3/2)*b-1/2/b*x^(1/2
)/(b*x^2+a)*d^3+3/2/a*x^(1/2)/(b*x^2+a)*c*d^2-3/2/a^2*b*x^(1/2)/(b*x^2+a)*c
^2*d+1/2/a^3*b^2*x^(1/2)/(b*x^2+a)*c^3+1/8/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d^3+9/8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c*d^2-21/8/a^3*b*(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^2*d+11/8/a^4*b^2*(1/b*a)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c^3+1/16/a/b*(1/b*a)^(1/4)*2^(1/
2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))*d^3+9/16/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/
b*a)^(1/2)))*c*d^2-21/16/a^3*b*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^
(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)
))*c^2*d+11/16/a^4*b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c^3+1
/8/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d^3+9/
8/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c*d^2-2
1/8/a^3*b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c^2
*d+11/8/a^4*b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+
1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4073, size = 4648, normalized size = 12.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11
*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8
*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d
^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 +
36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*arctan((sqrt(a^8*b^2*sqrt(-
(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588
*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188
*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b
^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^
5)) + (121*b^6*c^6 - 462*a*b^5*c^5*d + 639*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^
3*d^3 + 39*a^4*b^2*c^2*d^4 + 18*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b^4*(-(14641
*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b
^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3
```

```
*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(3
/4) - (11*a^11*b^7*c^3 - 21*a^12*b^6*c^2*d + 9*a^13*b^5*c*d^2 + a^14*b^4*d^
3)*sqrt(x)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10
*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7
*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^
8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d
^12)/(a^15*b^5))^(3/4))/(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^
2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*
a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8
*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^
11 + a^12*d^12)) + 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804
*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703
*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a
^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2
*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(a^4*b*(-(14
641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^
3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*
c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))
^(1/4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) -
 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 3
68082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 -
 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 1
0017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(-a^4*b*(-(14641*b^12*c^12 - 1
11804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 7
46703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5
688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^1
0*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4) + (11*b^3*
c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) - 4*(12*a^2*b*c^3
- 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3)*x^4 - 4*(11*a
*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)*sqrt(x))/(a^3*b^2*x^6 + a^4*b*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19982, size = 687, normalized size = 1.83

$$\frac{\sqrt{2}\left(11\left(ab^3\right)^{\frac{1}{4}}b^3c^3-21\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+9\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b^2}+\frac{\sqrt{2}\left(11\left(ab^3\right)^{\frac{1}{4}}b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a
*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/8*sqrt(2)*(11*(a*b^3)
^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2
 + (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(
a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*
d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*sqrt
(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1
/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^4*b^2) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) +
3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a^3*b) + 2/21*(14*b*c
^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(a^3*x^(7/2))
```

3.461 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(\right.}{\sqrt{2}b^{7/4}(}$$

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (c^(7/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (
a^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*b^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c -
a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*d^(7/4)*(b*c - a*d))
```

Rubi [A] time = 0.557387, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 479, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(\right.}{\sqrt{2}b^{7/4}(}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (c^(7/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (
a^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*b^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c -
a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*d^(7/4)*(b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x^{3/2}}{3bd} - \frac{2\,\text{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{3bd}$$
$$= \frac{2x^{3/2}}{3bd} - \frac{2\,\text{Subst}\left(\int \left(\frac{3a^2dx^2}{(-bc+ad)(a+bx^4)} + \frac{3bc^2x^2}{(bc-ad)(c+dx^4)}\right)dx, x, \sqrt{x}\right)}{3bd}$$
$$= \frac{2x^{3/2}}{3bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}$$
$$= \frac{2x^{3/2}}{3bd} - \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}(bc-ad)} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}(bc-ad)} + \frac{c^2\,\text{Subst}\left(\int\right.}{d^{3/2}}$$
$$= \frac{2x^{3/2}}{3bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^2(bc-ad)} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^2(bc-ad)} + \frac{a^{7/}}{}$$
$$= \frac{2x^{3/2}}{3bd} + \frac{a^{7/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)} - \frac{a^{7/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)} - \frac{c^{7/4}}{}$$
$$= \frac{2x^{3/2}}{3bd} - \frac{a^{7/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)} + \frac{a^{7/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)} + \frac{c^{7/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)} -$$

Mathematica [A] time = 0.245442, size = 411, normalized size = 0.86

$$\frac{\frac{3\sqrt{2}a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}} - \frac{3\sqrt{2}a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}} - \frac{6\sqrt{2}a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{7/4}} + \frac{6\sqrt{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{7/4}} - \frac{8ax^{3/2}}{b}}{12bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*x^(3/2))/b + (8*c*x^(3/2))/d - (6*Sqrt[2]*a^(7/4)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sqrt[2]*a^(7/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sqrt[2]*c^(7/4)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) - (6*Sqrt[2]*c^(7/4)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (3*Sqrt[2]*a^(7/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (3*Sqrt[2]*a^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (3*Sq
rt[2]*c^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d
^(7/4) + (3*Sqrt[2]*c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/d^(7/4))/(12*b*c - 12*a*d)
```

Maple [A] time = 0.012, size = 351, normalized size = 0.7

$$\frac{2}{3\,bd}x^{\frac{3}{2}} + \frac{c^2\sqrt{2}}{(4\,ad-4\,bc)\,d^2}\ln\left(\left(x - \sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x + \sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{c^2\sqrt{2}}{(2\,ad-2\,bc)\,d^2}\arctan\left(\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2/3*x^(3/2)/b/d+1/4*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+
1/2*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)+1/2*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)-1/4*a^2/(a*d-b*c)/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2
)))-1/2*a^2/(a*d-b*c)/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)+1)-1/2*a^2/(a*d-b*c)/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.17737, size = 2858, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*
d^3 + a^4*b^7*d^4))^(1/4)*b*d*arctan(-(sqrt(a^10*x - (a^7*b^5*c^2 - 2*a^8*b
^4*c*d + a^9*b^3*d^2)*sqrt(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)))*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6
*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*(b^3*c - a*b^2*d)
- (a^5*b^3*c - a^6*b^2*d)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*sqrt(x))/a^7) - 12*(-c^7/(b^4*c
^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^
(1/4)*b*d*arctan(-(sqrt(c^10*x - (b^2*c^9*d^3 - 2*a*b*c^8*d^4 + a^2*c^7*d^5
)*sqrt(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11)))*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9
 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*(b*c*d^2 - a*d^3) - (b*c^6*d^2 - a*c^5
*d^3)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11))^(1/4)*sqrt(x))/c^7) + 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d
+ 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqr
t(x) + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^1
1*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)
)^(3/4)) - 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b
^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) - (b^8*c^3 - 3*a*b^7*c^2
*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^
2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) - 3*(-c^7/(b^4*c^4*d
^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4
```

```
)*b*d*log(c^5*sqrt(x) + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^
3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c
*d^10 + a^4*d^11))^(3/4)) + 3*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*
b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*b*d*log(c^5*sqrt(x) - (b^3*
c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4
*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) + 4
*x^(3/2))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")

[Out] Timed out
```

3.462 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}b^{5/4}(bc}$$

```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^
(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*
d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rubi [A] time = 0.489076, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 479, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}b^{5/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^
(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*
d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*d^(5/4)*(b*c - a*d))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{bd} - \frac{2\,\text{Subst}\left(\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{bd}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{c^{3/2}\,\text{Subst}\left(\int\right.}{d(}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)} - \frac{a^{5/4}}{}$$

$$= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\log}{}$$

$$= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}}{}$$

Mathematica [A] time = 0.188199, size = 409, normalized size = 0.86

$$\frac{-\frac{\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{5/4}} + \frac{\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{5/4}} - \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{5/4}} - \frac{8a\sqrt{x}}{b} + \cdots}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*Sqrt[x])/b + (8*c*Sqrt[x])/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqrt[2]*
c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4)
 - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/d^(5/4))/(4*b*c - 4*a*d)
```

Maple [A] time = 0.011, size = 339, normalized size = 0.7

$$2\,\frac{\sqrt{x}}{bd} + \frac{c\sqrt{2}}{4\,(ad-bc)\,d}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) + \frac{c\sqrt{2}}{2\,(ad-bc)\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2*x^(1/2)/b/d+1/4/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/2/d*
c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2/d
*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4/
b*a/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/
b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/b*a/(a*d-b
*c)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b*a/(
a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32207, size = 2757, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(-((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c
*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*
a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(a^2*x + (b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))) - (a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(x))/a^5) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*
b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arcta
n(-((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^
4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/
4)*sqrt(c^2*x + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))) - (b^3
*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*s
qrt(x))/c^5) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b
^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) + (-a^5/(b^9*c^4 - 4*a*b^8
*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c -
 a*b*d)) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c
*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3
*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b
*d)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c
*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3
*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2))
 - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8
 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6
 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4
```

```
*sqrt(x))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")

[Out] Timed out
```

3.463 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}}{\sqrt{2}b^{3/}}$$

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b
*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d
)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(
b*c - a*d))
```

Rubi [A] time = 0.358855, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 481, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}}{\sqrt{2}b^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b
*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d
)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(
b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}(bc-ad)} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}(bc-ad)} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx,\right.}{\sqrt{d}(bc-ad)}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)} - \frac{a^{3/4}\,\text{Subst}\left(\right.}{2}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)} + \frac{c^{3/4}\log\left(\sqrt{\right.}}{2}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\tan}{\sqrt{2}}$$

Mathematica [A] time = 0.112679, size = 142, normalized size = 0.31

$$\frac{\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{3/4}} - \frac{(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{3/4}} - \frac{(-c)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{-c}}\right)}{d^{3/4}} + \frac{(-c)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{-c}}\right)}{d^{3/4}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (((-a)^(3/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(3/4) - ((-c)^(3/4)*Ar
cTan[(d^(1/4)*Sqrt[x])/(-c)^(1/4)])/d^(3/4) - ((-a)^(3/4)*ArcTanh[(b^(1/4)*
Sqrt[x])/(-a)^(1/4)])/b^(3/4) + ((-c)^(3/4)*ArcTanh[(d^(1/4)*Sqrt[x])/(-c)^
(1/4)])/d^(3/4))/(b*c - a*d)
```

Maple [A] time = 0.012, size = 328, normalized size = 0.7

$$-\frac{c\sqrt{2}}{(4\,ad-4\,bc)\,d}\ln\left(\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - \frac{c\sqrt{2}}{(2\,ad-2\,bc)\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-1/2*c/(a*d-b*c)/d/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2*c/(a*d-b*c)/d/
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/4*a/(a*d-b*c)/b
/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*a/(a*d-b*c)/b/(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*a/(a*d-b*c)/b/(1/b
```

```
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66828, size = 2741, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4))^(1/4)*arctan(-(sqrt(a^4*x - (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5
*b*d^2)*sqrt(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*
c*d^3 + a^4*b^3*d^4)))*(b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4) - (a^2*b^2*c - a^3*b
*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 +
a^4*b^3*d^4))^(1/4)*sqrt(x))/a^3) + 2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
+ 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(-(sqrt(c^4*x -
 (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3
*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)))*(b*c*d - a*d^2)*(
-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a
^4*d^7))^(1/4) - (b*c^3*d - a*c^2*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt(x))/c^3) - 1/2*
(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*
b^3*d^4))^(1/4)*log(a^2*sqrt(x) + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3
*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*
a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) - (
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4
*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) +
 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d
^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a
^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*
d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c
^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x
) - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^
4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/
4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.464 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\,\mathrm{l}}{}$$

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b
*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d
)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(
b*c - a*d))
```

Rubi [A] time = 0.362682, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b
*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d
)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(
b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$
$$= -\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\right.}{bc-ad}$$
$$= -\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}(bc-ad)} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt[4]{a}\,\text{Subst}\left(\right.}{2}$$
$$= \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\right.}{2\sqrt{2}\sqrt[4]{d}}$$
$$= \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\tan^{-1}\left(1\right.}{\sqrt{2}\sqrt[4]{d}(b}$$

Mathematica [A] time = 0.11925, size = 364, normalized size = 0.79

$$\frac{\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - \sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + 2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) - 2\sqrt[4]{a}\sqrt[4]{d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*a^(1/4
)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*b^(1/4)*c^(1/4)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + b^(1/4)*c^(1/4)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*b^(1/4)*d^(1/4)*(
b*c - a*d))
```

Maple [A] time = 0.011, size = 304, normalized size = 0.7

$$-\frac{\sqrt{2}}{4\,ad-4\,bc}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) - \frac{\sqrt{2}}{2\,ad-2\,bc}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-1/2/(a*d-b*c)*(c/d)^(1/
```

```
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2/(a*d-b*c)*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/4/(a*d-b*c)*(1/b*a)^(1/4)*2
^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*
x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.35808, size = 2542, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4))^(1/4)*arctan(-((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)) + x)*(-a/(b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4) - (b^4*
c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(x)*(-a/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a)
- 2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
a^4*d^5))^(1/4)*arctan(-((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)) + x)*(-c/(b^4*c^4*d -
4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4) - (b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)*(-c/(b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c
) - 1/2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-a/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4
)*log(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)
*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4
*d^5))^(1/4) + sqrt(x)) - 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*
c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d -
 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sq
rt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.465 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+$$

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*
(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)) - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d))
```

Rubi [A] time = 0.357569, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*
(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)) - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d))
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
```

```
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{c}-}{c+}\right.}{bc-}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}}{}}\right.}{2(bc-}$$

$$= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{d}\log\left(\sqrt{c}-\right.}{2\sqrt{2}}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)} + \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)} + \frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)} - \frac{\sqrt[4]{d}\tan}{\sqrt{2}}$$

Mathematica [A] time = 0.13332, size = 364, normalized size = 0.79

$$\frac{\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - \sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) + 2\sqrt[4]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*b^(1/
4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*a^(1/4)*d^(1/4
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + a^(1/4)*d^(1/4)*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(b*c - a*d))
```

Maple [A] time = 0.011, size = 304, normalized size = 0.7

$$\frac{\sqrt{2}}{4\,ad-4\,bc}\ln\left(\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{\sqrt{2}}{2\,ad-2\,bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/2/(a*d-b*c)/(c/d)^(1/4
```

```
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2/(a*d-b*c)/(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4/(a*d-b*c)/(1/b*a)^(1/4)*2^
(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/(a*d-b*c)/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/(a*d-b*c)/(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.38829, size = 2596, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*arctan(-(sqrt(b^2*x - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)
*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4)))*(b*c - a*d)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^
2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b^2*c - a*b*d)*sqrt(x)*(-b/(a*b^4*c^
4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4))/
b) - 2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 +
 a^4*c*d^4))^(1/4)*arctan(-(sqrt(d^2*x - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c
*d^3)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^
3 + a^4*c*d^4)))*(b*c - a*d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d
^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c*d - a*d^2)*sqrt(x)*(-d/(b^4
*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1
/4))/d) + 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*
b*c*d^3 + a^5*d^4))^(1/4)*log((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
- a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c
*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d
+ 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(a*b^3*c^3 - 3*a
^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d +
6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-d/
(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)
)^(1/4)*log((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^
4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(
3/4) + d*sqrt(x)) + 1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(-(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*
a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4) + d*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.466 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}a^{3/4}(bc}$$

```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*
(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c -
a*d)) + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4
)*(b*c - a*d))
```

Rubi [A] time = 0.347413, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\sqrt{2}\right.}{\sqrt{2}a^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*
(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c -
a*d)) + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4
)*(b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx,x,\sqrt{x}\right)$$
$$= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{bc-ad} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x,\sqrt{x}\right)}{bc-ad}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{a}(bc-ad)} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{a}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx\right.}{\sqrt{c}(bc-ad)}$$
$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{a}(bc-ad)} - \frac{b^{3/4}\,\text{Subs}}{}$$
$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\log\left(\right.}{2}$$
$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\,\text{t}}{}$$

Mathematica [A] time = 0.123238, size = 364, normalized size = 0.79

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-2a^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*b^(3/
4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*a^(3/4)*d^(3/4
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + a^(3/4)*d^(3/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - a^(3/4)*d^(3/4)*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a^(3/4)*c^(3/4)*
(b*c - a*d))
```

Maple [A] time = 0.012, size = 328, normalized size = 0.7

$$\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{d\sqrt{2}}{(2\,ad-2\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x)
```

```
[Out] 1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/2*d/(a*d-b*c)*(c/d
```

```
)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2*d/(a*d-b*c)*(c/
d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4*b/(a*d-b*c)*(1
/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x
-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b/(a*d-b*c)*(1/b*a)^(1/4
)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)*(1/b*a)
^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92799, size = 2714, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
 + a^7*d^4))^(1/4)*arctan(-((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*sqrt(b^2*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))
)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(3/4) - (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*
b*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c
*d^3 + a^7*d^4))^(3/4)*sqrt(x))/b^3) + 2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(-((b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(d^2*x + (b^2*c^4 - 2*
a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5
*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*
a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4) - (b^3*c^5*d - 3*a*
b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
 + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt(x))/d^3)
+ 1/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*
d^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/
2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(1/4)*log(b*sqrt(x) - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^
4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/2*(-
d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^
3*d^4))^(1/4)*log(d*sqrt(x) + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*
d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/2*(-d^3/
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4))^(1/4)*log(d*sqrt(x) - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.467 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}}{\sqrt{2}a^{5/4}}$$

```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(
5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d
)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rubi [A] time = 0.535349, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 480, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}}{\sqrt{2}a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(
5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d
)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{ac\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{ac}$$
$$= -\frac{2}{ac\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{b^2cx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{ad^2x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{ac}$$
$$= -\frac{2}{ac\sqrt{x}} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c(bc-ad)}$$
$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} - \frac{d^{3/2}}{}$$
$$= -\frac{2}{ac\sqrt{x}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)} -$$
$$= -\frac{2}{ac\sqrt{x}} - \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}$$
$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)} - \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)} - \frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{\sqrt{2}c^{5/4}(bc-ad)}$$

Mathematica [A] time = 0.232899, size = 409, normalized size = 0.86

$$\frac{\frac{\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}} - \frac{\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}} - \frac{2\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}} + \frac{8b}{a\sqrt{x}} - \cdots}{4ad-4\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*Sqrt[x]) - (8*d)/(c*Sqrt[x]) - (2*Sqrt[2]*b^(5/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (2*Sqrt[2]*d^(5/4)*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (Sqrt[2]*b^(5/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (Sqrt[2]*b^(5/4)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (Sqrt[
2]*d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5
/4) + (Sqrt[2]*d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/c^(5/4))/(-4*b*c + 4*a*d)
```

Maple [A] time = 0.015, size = 339, normalized size = 0.7

$$-\frac{d\sqrt{2}}{4c(ad-bc)}\ln\left(\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - \frac{d\sqrt{2}}{2c(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-1/2*d/c/(a*d-b*c)/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2*d/c/(a*d-b*c)/
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/4*b/a/(a*d-b*c)
/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b/a/(a*d-b*c)/(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b/a/(a*d-b*c)/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-2/a/c/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79828, size = 2808, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(-(sqrt(b^8*x - (a^3*b^7*c^2 - 2*a^4*b^
6*c*d + a^5*b^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c
^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4)))*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d +
6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d) - (a*b^
5*c - a^2*b^4*d)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*sqrt(x))/b^5) - 4*(-d^5/(b^4*c^9 - 4*a*b^3
*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*ar
ctan(-(sqrt(d^8*x - (b^2*c^5*d^5 - 2*a*b*c^4*d^6 + a^2*c^3*d^7)*sqrt(-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
)))*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d) - (b*c^2*d^4 - a*c*d^5)*(-d^5/(b^4*c^9
- 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)
*sqrt(x))/d^5) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) + (a^4*b^3*c^3 - 3*a
^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c
^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*
a*c*x*log(b^4*sqrt(x) - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^
7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^
2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*sqrt(x) + (b^3*c^7
- 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c
```

```
^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*a*c*x*log(d^4*sqrt(x) - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*
b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4*sqrt(x))/(a*c*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 3.8384, size = 2300, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*I*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 -
 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28
*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*
c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log
(8^(3/4)*a + 4*I*2^(1/4)*(-a^3*b)^(1/4)*sqrt(x)) + 1/2*I*2^(1/4)*(1/2)^(1/4
)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*
b^5*c^5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^
8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7
*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(8^(3/4)*a - 4*I*2^(1/4)*
(-a^3*b)^(1/4)*sqrt(x)) - 1/2*I*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*
a^6*b^3*c^3*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^1
3 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b
^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 +
 a^13*c^5*d^8))^(1/4)*log(8^(3/4)*c + 4*I*2^(1/4)*(-c^3*d)^(1/4)*sqrt(x)) +
 1/2*I*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b
^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28
*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*
c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log
(8^(3/4)*c - 4*I*2^(1/4)*(-c^3*d)^(1/4)*sqrt(x)) - 1/2*2^(1/4)*(1/2)^(1/4)*
(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^
5*c^5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*
b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d
^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(abs(8^(3/4)*a + 4*2^(1/4)*
(-a^3*b)^(1/4)*sqrt(x))) + 1/2*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8
*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(a^5*b^8*c^13
 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^
4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 +
a^13*c^5*d^8))^(1/4)*log(abs(8^(3/4)*a - 4*2^(1/4)*(-a^3*b)^(1/4)*sqrt(x)))
 - 1/2*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b
^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28
```

```
*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*
c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log
(abs(8^(3/4)*c + 4*2^(1/4)*(-c^3*d)^(1/4)*sqrt(x))) + 1/2*2^(1/4)*(1/2)^(1/
4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d
^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a
^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^
7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(abs(8^(3/4)*c - 4*2^(1/
4)*(-c^3*d)^(1/4)*sqrt(x))) - 2/(a*c*sqrt(x))
```

3.468 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\right.}{\sqrt{2}a^{7/4}(}$$

```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b
^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a
*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(2*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rubi [A] time = 0.481307, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\right.}{\sqrt{2}a^{7/4}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b
^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a
*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(2*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{3acx^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{3ac}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c(bc-ad)}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)} - \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)} + \frac{d^2\,\text{Su}\ldots}{}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}(bc-ad)} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \ldots\right)}{2a^{3/2}(bc-ad)}$$

$$= -\frac{2}{3acx^{3/2}} + \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}$$

$$= -\frac{2}{3acx^{3/2}} + \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)} - \frac{d^{7/4}\tan^{-1}\left(1-\ldots\right)}{\sqrt{2}c^{7/4}(bc-ad)}$$

Mathematica [A] time = 0.235657, size = 411, normalized size = 0.86

$$-\frac{\frac{3\sqrt{2}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}} + \frac{3\sqrt{2}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}} - \frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}} + \frac{8b}{ax^{3/2}}\ldots}{12ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(3/2)) - (8*d)/(c*x^(3/2)) - (6*Sqrt[2]*b^(7/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqrt[2]*b^(7/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3
*Sqrt[2]*d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/c^(7/4) - (3*Sqrt[2]*d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/c^(7/4))/(-12*b*c + 12*a*d)
```

Maple [A] time = 0.013, size = 351, normalized size = 0.7

$$-\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) - \frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-1/2/c^2*d^2/(a*
d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2/c^2*d^
2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/3/a
/c/x^(3/2)+1/4/a^2*b^2/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*
x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/
2)))+1/2/a^2*b^2/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)+1)+1/2/a^2*b^2/(a*d-b*c)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.8914, size = 2877, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b
*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*arctan(-((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(b^4*x + (a^4*b^2*c^2 - 2*a
^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^
2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))) - (a^5*b^5*c^3 - 3*a^6*b^4*c^2*d + 3*a
^7*b^3*c*d^2 - a^8*b^2*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^
2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(x))/b^7) - 12*(-d^7/(b^4
*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4)
)^(1/4)*a*c*x^2*arctan(-((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt(d^4*x + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2
)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^
3 + a^4*c^7*d^4))) - (b^3*c^8*d^2 - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3
*c^5*d^5)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^
8*d^3 + a^4*c^7*d^4))^(3/4)*sqrt(x))/d^7) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^
3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log
(b^2*sqrt(x) + (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4
*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a
*c*x^2*log(b^2*sqrt(x) - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c
^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^
4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4
))^(1/4)*a*c*x^2*log(d^2*sqrt(x) + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2
*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3
*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a
^4*c^7*d^4))^(1/4)*a*c*x^2*log(d^2*sqrt(x) - (-d^7/(b^4*c^11 - 4*a*b^3*c^10
```

```
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c
^2*d)) - 4*sqrt(x))/(a*c*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

Giac [C] time = 5.29166, size = 2246, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")

[Out] -1/2*I*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d
^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d
+ 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^
12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(
1/4)*log(2*I*b*sqrt(x) + 2*(-a*b^3)^(1/4)) + 1/2*I*2^(1/4)*(1/2)^(1/4)*(-(b
^11*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^
7*c^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10
*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c
^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(-2*I*b*sqrt(x) + 2*(-a
*b^3)^(1/4)) - 1/2*I*2^(1/4)*(1/2)^(1/4)*(-(a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3
*d^8 + 6*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^11*d^11)/(a^7*b^8*c^15 - 8*a
^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^
11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^
15*c^7*d^8))^(1/4)*log(2*I*d*sqrt(x) + 2*(-c*d^3)^(1/4)) + 1/2*I*2^(1/4)*(1
/2)^(1/4)*(-(a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6*a^9*b^2*c^2*d^9 - 4*a^
10*b*c*d^10 + a^11*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13
*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 +
 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(-2*I*d*s
qrt(x) + 2*(-c*d^3)^(1/4)) + 1/2*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^11 - 4*a*b^1
0*c^10*d + 6*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*d^4)/(a^7*b^
8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70
*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*
c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(abs(2*b*sqrt(x) + 2*(-a*b^3)^(1/4))) - 1
/2*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d^2 -
 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28
*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b
^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)
*log(abs(-2*b*sqrt(x) + 2*(-a*b^3)^(1/4))) + 1/2*2^(1/4)*(1/2)^(1/4)*(-(a^7
*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^
11*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b
^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9
*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(abs(2*d*sqrt(x) + 2*(-c*
d^3)^(1/4))) - 1/2*2^(1/4)*(1/2)^(1/4)*(-(a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d
```

```
^8 + 6*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^11*d^11)/(a^7*b^8*c^15 - 8*a^8
*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11
*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15
*c^7*d^8))^(1/4)*log(abs(-2*d*sqrt(x) + 2*(-c*d^3)^(1/4))) - 2/3/(a*c*x^(3/
2))
```

3.469 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=498

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}a^{9/4}(\cdots}$$

```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c -
a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^
(9/4)*(b*c - a*d)) - (d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*
(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rubi [A] time = 0.687037, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}a^{9/4}(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c -
a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^
(9/4)*(b*c - a*d)) - (d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*
(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2\,\mathrm{Subst}\left(\int \frac{-5(bc+ad)-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{5ac}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{5b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{5a^2d^3x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} - \frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, \ldots\right.}{c^2(bc-ad)}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} + \frac{b^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,d\ldots\right.}{a^2(bc-ad)}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2(bc-ad)} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\ldots}{\sqrt[4]{b}}}\ldots\right.}{2a^2(bc-\ldots}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)} - \frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\ldots\right.}{2\sqrt{2}a^{9/4}(bc\ldots}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)} + \frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)} + \frac{d^{9/4}\ldots}{\ldots}$$

Mathematica [A] time = 0.27421, size = 437, normalized size = 0.88

$$\frac{-\frac{40b^2}{a^2\sqrt{x}} - \frac{5\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}} + \frac{5\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}} + \frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}} - \frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+\ldots\right.}{a^{9/4}}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(5/2)) - (8*d)/(c*x^(5/2)) - (40*b^2)/(a^2*Sqrt[x]) + (40*d^2)/
(c^2*Sqrt[x]) + (10*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[2]*d^(9/4)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(9/4) - (5*Sqrt[2
```

```
]*d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(9/
4))/(-20*b*c + 20*a*d)
```

Maple [A] time = 0.015, size = 375, normalized size = 0.8

$$\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\ln\left(\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/2*d^2/c^2/(a*d
-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2*d^2/c^2
/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/
c/x^(5/2)+2/a/c^2/x^(1/2)*d+2/a^2/c/x^(1/2)*b-1/4*b^2/a^2/(a*d-b*c)/(1/b*a)
^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b^2/a^2/(a*d-b*c)/(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b^2/a^2/(a*d-b*c)/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 25.6548, size = 3024, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(b^14*x - (a^5*b^11*c
^2 - 2*a^6*b^10*c*d + a^7*b^9*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*
d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))*(-b^9/(a^9*b^4*c^4 -
4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(
a^2*b*c - a^3*d) - (a^2*b^8*c - a^3*b^7*d)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*
c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*sqrt(x))/b^9
) - 20*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(d^14*x - (b^2*c^7*d^9
- 2*a*b*c^6*d^10 + a^2*c^5*d^11)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a
^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))*(-d^9/(b^4*c^13 - 4*a*b
```

```
^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*
c^3 - a*c^2*d) - (b*c^3*d^7 - a*c^2*d^8)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d +
 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*sqrt(x))/d^9)
+ 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) + (a^7*b^3*c^3 - 3*a^8*
b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
 + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b
^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)
)^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^
9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^
2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^
12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*
x^3*log(d^7*sqrt(x) + (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7
*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*
d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2
*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*log(d^7*sqrt
(x) - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4
*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^
4))^(3/4)) + 4*(5*(b*c + a*d)*x^2 - a*c)*sqrt(x))/(a^2*c^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

Giac [C] time = 2.85419, size = 1754, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")

[Out] -1/2*I*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11
*d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^
16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 -
 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9
*d^8))^(1/4)*log(8^(3/4)*a + 4*I*2^(1/4)*(-a^3*b)^(1/4)*sqrt(x)) + 1/2*I*2^
(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11*d^2 - 4*
a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28
*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*
b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1
/4)*log(8^(3/4)*a - 4*I*2^(1/4)*(-a^3*b)^(1/4)*sqrt(x)) - 1/2*I*2^(1/4)*(1/
2)^(1/4)*(-(a^9*b^4*c^4*d^9 - 4*a^10*b^3*c^3*d^10 + 6*a^11*b^2*c^2*d^11 - 4
*a^12*b*c*d^12 + a^13*d^13)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6
*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*
d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(8
^(3/4)*c + 4*I*2^(1/4)*(-c^3*d)^(1/4)*sqrt(x)) + 1/2*I*2^(1/4)*(1/2)^(1/4)*
(-(a^9*b^4*c^4*d^9 - 4*a^10*b^3*c^3*d^10 + 6*a^11*b^2*c^2*d^11 - 4*a^12*b*c
```

```
*d^12 + a^13*d^13)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2
 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*
a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(8^(3/4)*c
- 4*I*2^(1/4)*(-c^3*d)^(1/4)*sqrt(x)) - 1/2*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^1
3 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11*d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c
^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*
b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^
11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(abs(8^(3/4)*a + 4*2^(
1/4)*(-a^3*b)^(1/4)*sqrt(x))) + 1/2*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*
b^12*c^12*d + 6*a^2*b^11*c^11*d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/
(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14
*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 -
 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(abs(8^(3/4)*a - 4*2^(1/4)*(-a
^3*b)^(1/4)*sqrt(x))) + 2/5*(5*b*c*x^2 + 5*a*d*x^2 - a*c)/(a^2*c^2*x^(5/2))
```

3.470 $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$\frac{a^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(\right.}{\sqrt{2}b^{5/4}(}$$

```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*d*(b*c - a
*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d
)^2) - (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*
b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(
5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.84591, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 582, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(\right.}{\sqrt{2}b^{5/4}(}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*d*(b*c - a
*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d
)^2) - (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*
b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(
5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^4(5ac+(5bc-4ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{ac(5bc-4ad)+(5b^2c^2-4abcd-4a^2d^2)x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \cdot\right.}{2bd^2(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)^2} - \frac{(c^2(5bc-9\cdots}{\cdots}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)^2} - \frac{a^{5/2}\,\text{Subst}\cdots}{\cdots}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)^2} - \frac{a^{5/2}\,\text{Su}\cdots}{\cdots}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2} - \frac{a^{9/4}\log\cdots}{\cdots}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2} - \frac{a^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}b^{5/4}(bc-ad\cdots}$$

Mathematica [A] time = 0.375928, size = 563, normalized size = 0.99

$$4\sqrt{2}a^{9/4}d^{9/4}(c+dx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 4\sqrt{2}a^{9/4}d^{9/4}(c+dx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + 8\sqrt{2}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*b^(5/4)*c^2*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^(1/4)*d^(1/4)*(b*c - a*d)
^2*Sqrt[x]*(c + d*x^2) + 8*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*A
```

```
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*Sqrt[2]*b^(5/4)*c^(5/4)*(5
*b*c - 9*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2
*Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt[2]*a^(9/4)*d^(9/4)
*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + S
qrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d
)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
16*b^(5/4)*d^(9/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.016, size = 582, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 2/b/d^2*x^(1/2)-1/2*c^2/d/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*c^3/d^2/(a*d-
b*c)^2*x^(1/2)/(d*x^2+c)*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-5/8*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-5/8*c^2/d^2/(a*d-b*c)^2*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+9/16*c/d/(a*d-b*c)^2
*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a-5/16*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4
)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))*b-1/4/b*a^2/(a*d-b*c)^2*(1/b*a)^(1/4)*2^(1/2)*ln
((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2
^(1/2)+(1/b*a)^(1/2)))-1/2/b*a^2/(a*d-b*c)^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/b*a^2/(a*d-b*c)^2*(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 161.814, size = 7129, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9
 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^
4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b
^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^
15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)*arctan(((b^6*c^6*d^7 - 6*a*b^5*c^5*d
^8 + 15*a^2*b^4*c^4*d^9 - 20*a^3*b^3*c^3*d^10 + 15*a^4*b^2*c^2*d^11 - 6*a^5
*b*c*d^12 + a^6*d^13)*sqrt((25*b^2*c^4 - 90*a*b*c^3*d + 81*a^2*c^2*d^2)*x +
 (b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d
^8)*sqrt(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a
^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b
^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^
14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17)))*(-(625*b^4*c^9 - 45
00*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5
*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^
5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 -
8*a^7*b*c*d^16 + a^8*d^17))^(3/4) + (5*b^7*c^8*d^7 - 39*a*b^6*c^7*d^8 + 129
*a^2*b^5*c^6*d^9 - 235*a^3*b^4*c^5*d^10 + 255*a^4*b^3*c^4*d^11 - 165*a^5*b^
2*c^3*d^12 + 59*a^6*b*c^2*d^13 - 9*a^7*c*d^14)*sqrt(x)*(-(625*b^4*c^9 - 450
0*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*
d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5
*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8
*a^7*b*c*d^16 + a^8*d^17))^(3/4))/(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a
^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)) - 16*(-a^9/(b^13*
c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b
^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^
8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*arc
tan(((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^3*d^3 +
15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b
^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4
- 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))
^(3/4)*sqrt(a^4*x + (b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^
3*c*d^3 + a^4*b^2*d^4)*sqrt(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c
^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28
*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))) - (a^2*b^10*c^6 - 6*a^3
*b^9*c^5*d + 15*a^4*b^8*c^4*d^2 - 20*a^5*b^7*c^3*d^3 + 15*a^6*b^6*c^2*d^4 -
 6*a^7*b^5*c*d^5 + a^8*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(3/4)*sqrt(x))/a^9)
 - 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c
^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a
^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 -
a*b*d^4)*x^2)*log(a^2*sqrt(x) + (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)) + 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*
c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 2
8*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*
b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(b^13*c^8 - 8*
a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d
^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^
8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) + (b^2*c^2*d^2 - a*b*c*d^3 +
 (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b
^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7
*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13
 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^
(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b
*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c
^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 +
```

```
 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) - (b^2*c^2*d^2 -
a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d +
12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^
9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b
^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 +
a^8*d^17))^(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) - (b^2*c^2*d^2 - 2*a*b*c*
d^3 + a^2*d^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 -
14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 2
8*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3
*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) + 4*(5
*b*c^2 - 4*a*c*d + 4*(b*c*d - a*d^2)*x^2)*sqrt(x))/(b^2*c^2*d^2 - a*b*c*d^3
 + (b^2*c*d^3 - a*b*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55658, size = 969, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - (
a*b^3)^(1/4)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/
2*(a*b^3)^(1/4)*a^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*(a*b^3)^(1/4)
*a^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2
*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*
(c*d^3)^(1/4)*a*c*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(
c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) -
 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(
2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) - 1/8*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1
/4)*a*c*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^
2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/8*(5*(c*d^3)^(1/4)*b*c^2
 - 9*(c*d^3)^(1/4)*a*c*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/2*c^2*sq
rt(x)/((b*c*d^2 - a*d^3)*(d*x^2 + c)) + 2*sqrt(x)/(b*d^2)
```

3.471 $\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=536

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}b^{3/4}(\cdots}$$

```
[Out] -(c*x^(3/2))/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (a^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) -
(c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3
/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(
7/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.592208, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 470, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}b^{3/4}(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(c*x^(3/2))/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (a^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) -
(c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3
/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(
7/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3ac+(3bc-4ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{4a^2dx^2}{(-bc+ad)\left(a+bx^4\right)} + \frac{c(3bc-7ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(3bc-7ad))\,\text{Subst}\left(\int \frac{x}{c+}\right.}{2d(bc-ad)^2}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}(bc-ad)^2} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \right.}{\sqrt{b}(bc-ad)^2}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\right.}{2b(bc-ad)^2}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2} - \frac{a^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}b^{3/4}(bc-a}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2} + \frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2} - \frac{c^{3/4}(3bc}{}$$

Mathematica [A] time = 0.32764, size = 527, normalized size = 0.98

$$\frac{4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 8\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(3/4)*c*d^(3/4)*(b*c - a*d)*x^(3/2) - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(c +
d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*a^(7/4)*d^
(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]
*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)] + 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(c +
 d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt
[2]*a^(7/4)*d^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*b^(3/4)*c^(
3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(16*b^(3/4)*d^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.017, size = 566, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(3/2)/(d*x^2+
c)*b-7/8*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)*a+3/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)*b-7/8*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*a+3/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*l
n((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2)))*a+3/16*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))*b+1/4*a^2/(a*d-b*c)^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
+1/2*a^2/(a*d-b*c)^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)+1/2*a^2/(a*d-b*c)^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 142.469, size = 7397, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*x^(3/2) - 4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4
*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a
^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^
5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^1
3 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*arctan(((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*sqrt((729*b^6*c^10 - 10206*a*b^5*c^9*d + 59535*a^2*b^4*c^8*d^2 - 1
85220*a^3*b^3*c^7*d^3 + 324135*a^4*b^2*c^6*d^4 - 302526*a^5*b*c^5*d^5 + 117
649*a^6*c^4*d^6)*x - (81*b^8*c^11*d^3 - 1080*a*b^7*c^10*d^4 + 6156*a^2*b^6*
c^9*d^5 - 19560*a^3*b^5*c^8*d^6 + 37846*a^4*b^4*c^7*d^7 - 45640*a^5*b^3*c^6
*d^8 + 33516*a^6*b^2*c^5*d^9 - 13720*a^7*b*c^4*d^10 + 2401*a^8*c^3*d^11)*sq
rt(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d
^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9
- 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*
```

b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15)))*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4) + (27*b^5*c^7*d^2 - 243*a*b^4*c^6*d^3 + 846*a^2*b^3*c^5*d^4 - 1414*a^3*b^2*c^4*d^5 + 1127*a^4*b*c^3*d^6 - 343*a^5*c^2*d^7)*sqrt(x)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4))/(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)) + 16*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*arctan((sqrt(a^10*x - (a^7*b^5*c^4 - 4*a^8*b^4*c^3*d + 6*a^9*b^3*c^2*d^2 - 4*a^10*b^2*c*d^3 + a^11*b*d^4)*sqrt(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8)))*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - (a^5*b^3*c^2 - 2*a^6*b^2*c*d + a^7*b*d^2)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*sqrt(x))/a^7) - 4*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) + (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + 4*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) - (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*log((b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^3*b^3*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(3/4) - (27*b^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3*d^2 - 343*a^3*c^2*d^3)*sqrt(x)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*log(-(b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^3*b^3*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(3/4) - (27*b^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3

```
*d^2 - 343*a^3*c^2*d^3)*sqrt(x)))/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55287, size = 919, normalized size = 1.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + (a*b
^3)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) - 1/2*(a*
b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*
c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/2*(a*b^3)^(3/4)*a*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*
a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/4*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/4*(3*(c*d^
3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sq
rt(2)*a^2*d^6) - 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*log(sqrt(2
)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b
*c*d^5 + sqrt(2)*a^2*d^6) + 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2
*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) - 1/2*c*x^(3/2)/((b*c*d - a*d^2)*(d*x
^2 + c))
```

3.472 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=532

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}}{\sqrt{2}\sqrt[4]{b}}$$

```
[Out] -(c*Sqrt[x])/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) + (a^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) -
(c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*
(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(
b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.529085, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 470, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}}{\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(c*Sqrt[x])/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) + (a^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) -
(c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*
(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(
b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{ac+(bc-4ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(bc-5ad))\,\text{Subst}\left(\int \frac{1}{c+dx}\right.}{2d(bc-ad)^2}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,\right.}{(bc-ad)^2}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}(bc-ad)^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}\right.}{2\sqrt{b}(bc-}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\sqrt[4]{b}(bc-a}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2} - \frac{\sqrt[4]{c}(bc-}{}$$

Mathematica [A] time = 0.319452, size = 523, normalized size = 0.98

$$\frac{-4\sqrt{2}a^{5/4}d^{5/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}a^{5/4}d^{5/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(1/4)*c*d^(1/4)*(b*c - a*d)*Sqrt[x] - 8*Sqrt[2]*a^(5/4)*d^(5/4)*(c +
d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*a^(5/4)*d^
(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]
*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)] + 2*Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(5/4)*d^(5/4)*(c + d*x
^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4*Sqrt[2]*
a^(5/4)*d^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x] - Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*b^(1/4)*c^(1/4)*(
b*c - 5*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(16*b^(1/4)*d^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.015, size = 533, normalized size = 1.

$$\frac{ac}{2\left(ad-bc\right)^2\left(dx^2+c\right)}\sqrt{x} - \frac{bc^2}{2\left(ad-bc\right)^2 d\left(dx^2+c\right)}\sqrt{x} - \frac{5\sqrt{2}a}{8\left(ad-bc\right)^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{c\sqrt{2}b}{8\left(ad-bc\right)^2 d}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(1/2)/(d*x^2+
c)*b-5/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)*a+1/8*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*b-5/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a+1/8*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*b-5/16/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a+1/
16*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/4/(a*d-b*c)^2*a
*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/(a*d-b*c)^2*a*(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2/(a*d-b*c)^2*a*(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 31.9312, size = 6624, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c
^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^
5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*
c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*
d^13))^(1/4)*arctan(((b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c^4*d^6 -
20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10)*sqrt((b
^2*c^2 - 10*a*b*c*d + 25*a^2*d^2)*x + (b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^
2*b^2*c^2*d^4 - 4*a^3*b*c*d^5 + a^4*d^6)*sqrt(-(b^4*c^5 - 20*a*b^3*c^4*d +
150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a
*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9
 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13)))
*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 62
5*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b
^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11
 - 8*a^7*b*c*d^12 + a^8*d^13))^(3/4) + (b^7*c^7*d^4 - 11*a*b^6*c^6*d^5 + 45
*a^2*b^5*c^5*d^6 - 95*a^3*b^4*c^4*d^7 + 115*a^4*b^3*c^3*d^8 - 81*a^5*b^2*c^
2*d^9 + 31*a^6*b*c*d^10 - 5*a^7*d^11)*sqrt(x)*(-(b^4*c^5 - 20*a*b^3*c^4*d +
 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*
```

```
a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^
9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))
^(3/4))/(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3
 + 625*a^4*c*d^4)) - 16*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2
 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^
3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d
^2 - a*d^3)*x^2)*arctan(((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a^5/
(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4
*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
a^8*b*d^8))^(3/4)*sqrt(a^2*x + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7
*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 2
8*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))) - (a*b^7*c^6 - 6*a^2*b^6
*c^5*d + 15*a^3*b^5*c^4*d^2 - 20*a^4*b^4*c^3*d^3 + 15*a^5*b^3*c^2*d^4 - 6*a
^6*b^2*c*d^5 + a^7*b*d^6)*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d
^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*
b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(3/4)*sqrt(x))/a^5) + (b*c^2*d
- a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^
2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d
^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*
b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)*log(
-(b*c - 5*a*d)*sqrt(x) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 2
0*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b
^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70
*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^
12 + a^8*d^13))^(1/4)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4
*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c
*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*
d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^
7*b*c*d^12 + a^8*d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) - (b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500
*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6
*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 +
28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)) - 4*(-a^5/(b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) + (-a^5
/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^
4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
 a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*(-a^5/(b^9*c^8 - 8*
a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4
- 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(
1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) - (-a^5/(b^9
*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5
*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*
b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*c*sqrt(x))/(b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50881, size = 903, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + (a*b^3
)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + 1/2*(a*b^3)
^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2
- 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*
c*d + sqrt(2)*a^2*b*d^2) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) + 1/4*((c*d^3)^(1/4)*b*c
 - 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x
))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^
4) + 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d
)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(
2)*a^2*d^4) - 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d
^3 + sqrt(2)*a^2*d^4) - 1/2*c*sqrt(x)/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.473 $\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=528

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-$$

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((b*c +
 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d
^(3/4)*(b*c - a*d)^2) + ((b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a
*d)^2) - ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.591102, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 471, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((b*c +
 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d
^(3/4)*(b*c - a*d)^2) + ((b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a
*d)^2) - ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a-bx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{4abx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{(bc+3ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right) dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x,\right.}{2(bc-ad)^2}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\left(a\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(a\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,d\right.}{(bc-ad)^2}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx,\right.}{2(bc-ad)^2}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2} + \frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}(bc-a}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} + \frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{(bc-}{}$$

Mathematica [A] time = 0.304355, size = 522, normalized size = 0.99

$$\frac{-4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(1/4)*d^(3/4)*(b*c - a*d)*x^(3/2) + 8*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*
d^(3/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[
2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(
3/4)*(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
 + 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(b*c
 + 3*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(16*c^(1/4)*d^(3/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.015, size = 528, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*b*
c+3/16/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+3/8/(a*d-b*c)^2/(
c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8/(a*d-b*c)^2/
(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/(a*d-b*c)^
2/d/(c/d)^(1/4)*2^(1/2)*b*c*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/8/(a*d-b*c)^2/d/(c/d)^(1/4)*
2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8/(a*d-b*c)^2/d/(c/d)^(
1/4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4*a/(a*d-b*c)^2/(1
/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(
1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*a/(a*d-b*c)^2/(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*a/(a*d-b*c)^2/(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 51.4167, size = 6947, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*
c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56
*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/
4)*arctan(((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt((b^6*c^6 + 18*a*b^5*c^5
*d + 135*a^2*b^4*c^4*d^2 + 540*a^3*b^3*c^3*d^3 + 1215*a^4*b^2*c^2*d^4 + 145
8*a^5*b*c*d^5 + 729*a^6*d^6)*x - (b^8*c^9*d + 8*a*b^7*c^8*d^2 + 12*a^2*b^6*
c^7*d^3 - 40*a^3*b^5*c^6*d^4 - 74*a^4*b^4*c^5*d^5 + 120*a^5*b^3*c^4*d^6 + 1
08*a^6*b^2*c^3*d^7 - 216*a^7*b*c^2*d^8 + 81*a^8*c*d^9)*sqrt(-(b^4*c^4 + 12*
a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d
^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4
*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8
*c*d^11)))*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d
^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a
^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d
^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4) - (b^5*c^5*d + 7*a*b^4*c^4*d^2 +
 10*a^2*b^3*c^3*d^3 - 18*a^3*b^2*c^2*d^4 - 27*a^4*b*c*d^5 + 27*a^5*d^6)*sqr
t(x)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 8
```

```
1*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5
*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8
*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4))/(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^
2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)) - 16*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan((sqrt(a^4*b^2*x - (a^3*b^5*c^4 - 4*
a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*sqrt(-a^3*
b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)))*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4) - (a^2*b^3
*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*sqrt(x))/(a^3*b)) +
 4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sq
rt(x) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - 4*
(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
+ 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*
d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sqrt(
x) - (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log((b^6*c
^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4
*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 5
4*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^
8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a
^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4)
 + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) + (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log(-(b^6*
c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^
4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c
^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*
a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4
) + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) - 4*x^
(3/2))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.5804, size = 922, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c
^2*d^4 + sqrt(2)*a^2*c*d^5) + 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - 1/8*((c*d^3)^(
3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) + 1
/8*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*
a^2*c*d^5) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*
d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2)
+ 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/2*(a*b^3)^(3/4)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2*sqr
t(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*x^(3/2)/((d*x^2 + c)*(b*c - a*d
))
```

3.474 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt[4]{a}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-$$

```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(1/4)*b^(3/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((3*b*c
 + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d
^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a
*d)^2) + ((3*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.475323, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 471, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(1/4)*b^(3/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((3*b*c
 + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d
^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a
*d)^2) + ((3*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{a-3bx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{(2ab)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx,\right.}{2(bc-ad)^2}$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}b\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(\sqrt{a}b\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\right.}{(bc-ad)^2}$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} - \frac{\left(\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \right.}{2(bc-}$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2} - \frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}(bc-a}$$
$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{(3bc}{}$$

Mathematica [A] time = 0.307704, size = 522, normalized size = 0.99

$$\frac{4\sqrt{2}\sqrt[4]{a}b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}\sqrt[4]{a}b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(3/4)*d^(1/4)*(b*c - a*d)*Sqrt[x] + 8*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*
d^(1/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[
2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(
1/4)*(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
 - 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(3*b*c + a*d)*(c + d*x^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(3*b
*c + a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(16*c^(3/4)*d^(1/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.014, size = 528, normalized size = 1.

$$-\frac{ad}{2\left(ad-bc\right)^2\left(dx^2+c\right)}\sqrt{x}+\frac{bc}{2\left(ad-bc\right)^2\left(dx^2+c\right)}\sqrt{x}+\frac{\sqrt{2}ad}{8\left(ad-bc\right)^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\sqrt{2}b}{8\left(ad-bc\right)^2}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*b*
c+1/8/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*a*d+3/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)+1)*b+1/8/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*a*d+3/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*b+1/16/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*d+3
/16/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b-1/4*b/(a*d-b*c)^2*(1
/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(
1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b/(a*d-b*c)^2*(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)^2*(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 18.2557, size = 6483, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*
d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^
10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*
a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4
)*arctan(((b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^
5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7)*sqrt((9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 -
4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 2
8*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^
6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9)))*(-(81*b^4*c^4
 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^
11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*
b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a
^8*c^3*d^9))^(3/4) - (3*b^7*c^9*d - 17*a*b^6*c^8*d^2 + 39*a^2*b^5*c^7*d^3 -
 45*a^3*b^4*c^6*d^4 + 25*a^4*b^3*c^5*d^5 - 3*a^5*b^2*c^4*d^6 - 3*a^6*b*c^3*
d^7 + a^7*c^2*d^8)*sqrt(x)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^
```

```
6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 +
28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(3/4))/(81*b^4*c^4 + 1
08*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)) - 16*(-a*b
^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan(((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*sqrt(b^2*x + (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b^3/(b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8))) - (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
+ 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a*b^3/(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*s
qrt(x))/(a*b^3)) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 10
8*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d
- 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c
^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^
3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c
^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a
^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^
4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8
*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7
*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) - (b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
+ 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9
*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^
6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - 4*(-a*b^3/(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^
(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)) + 4*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*
d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b
^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 -
a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8))^(1/4)) + 4*sqrt(x))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47224, size = 884, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2
*d^2 + sqrt(2)*a^2*c*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/8*(3*(c*d^3)^(1/
4)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) - 1/8*(3*
(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^
3) - (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - (a*b^3)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(
sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/2*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(
2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2
*d^2) + 1/2*sqrt(x)/((d*x^2 + c)*(b*c - a*d))
```

3.475 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(\right.}{\sqrt{2}\sqrt[4]{a}(b}$$

```
[Out] -(d*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) + (b^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) +
(d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*c^(5/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*
(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(
b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.597818, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 472, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(\right.}{\sqrt{2}\sqrt[4]{a}(b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(d*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) + (b^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) +
(d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*c^(5/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*
(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(
b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(4bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \left(\frac{4b^2cx^2}{(bc-ad)\left(a+bx^4\right)} + \frac{d(-5bc+ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right) dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{(d(5bc-ad))\,\text{Subst}\left(\int \frac{x^2}{c+dx}\right.}{2c(bc-ad)^2}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x\right.}{(bc-ad)^2}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{2(bc-ad)^2}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2} - \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\sqrt[4]{a}(bc-a}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2} + \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2} + \frac{\sqrt[4]{d}(5bc}{}$$

Mathematica [A] time = 0.317304, size = 523, normalized size = 0.98

$$\frac{4\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 4\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 8\sqrt{2}b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(1/4)*c^(1/4)*d*(-(b*c) + a*d)*x^(3/2) - 8*Sqrt[2]*b^(5/4)*c^(5/4)*(c
+ d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*b^(5/4)*
c^(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[
2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)] + 2*Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*b^(5/4)*c^(5/4)*(c +
 d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt
[2]*b^(5/4)*c^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] + Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*a^(1/4)*d^(1
/4)*(5*b*c - a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(16*a^(1/4)*c^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.016, size = 533, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x^(3/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(3/2)/(d*x^2+
c)*b+1/8*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)*a-5/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*b+1/8*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*a-5/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*b+1/16*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*
a-5/16/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/4*b/(a*d-b*c)^2
/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b/(a*d-b*c)^2/(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b/(a*d-b*c)^2/(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 69.9565, size = 7121, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*d*x^(3/2) + 4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^
4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^
5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6
*d^7 + a^8*c^5*d^8))^(1/4)*arctan(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt
((15625*b^6*c^6*d^2 - 18750*a*b^5*c^5*d^3 + 9375*a^2*b^4*c^4*d^4 - 2500*a^3
*b^3*c^3*d^5 + 375*a^4*b^2*c^2*d^6 - 30*a^5*b*c*d^7 + a^6*d^8)*x - (625*b^8
*c^11*d - 3000*a*b^7*c^10*d^2 + 5900*a^2*b^6*c^9*d^3 - 6120*a^3*b^5*c^8*d^4
 + 3606*a^4*b^4*c^7*d^5 - 1224*a^5*b^3*c^6*d^6 + 236*a^6*b^2*c^5*d^7 - 24*a
^7*b*c^4*d^8 + a^8*c^3*d^9)*sqrt(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*
a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28
*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c
^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)))*(-(625*b^4*c
^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8))^(1/4) + (125*b^5*c^6*d - 325*a*b^4*c^5*d^2 + 290*a^2*b^3*
c^4*d^3 - 106*a^3*b^2*c^3*d^4 + 17*a^4*b*c^2*d^5 - a^5*c*d^6)*sqrt(x)*(-(62
```

```
5*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^
4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8))^(1/4))/(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^
2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)) + 16*(-b^5/(a*b^8*c^8 - 8*a^2*b^
7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56
*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*
c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan((sqrt(b^8*x - (a*b^9*c^4 -
4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*sqrt(-
b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3
+ 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*
d^7 + a^9*d^8)))*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 -
56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c
^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2) -
(b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d +
28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c
^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*sqrt(x))/b^5)
 - 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c
^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a
^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*lo
g(b^4*sqrt(x) + (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*
b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^
8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*
c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8
))^(3/4)) + 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*
a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*
d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*log(b^4*sqrt(x) - (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2
- 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/
(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70
*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7
+ a^9*d^8))^(3/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^4
*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5
)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*
d^7 + a^8*c^5*d^8))^(1/4)*log((b^6*c^10 - 6*a*b^5*c^9*d + 15*a^2*b^4*c^8*d^
2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6
)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d
^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5
*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 -
8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d - 75*a*b^2*c^2*d^2 +
 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*
b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(1/4)*log(-(b^6*c^10 - 6*a*b^5*c^9*d +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5
*d^5 + a^6*c^4*d^6)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*
d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^1
1*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*
a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d -
75*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)))/(b*c^3 - a*c^2*d + (
b*c^2*d - a*c*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55253, size = 946, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*
c^3*d^3 + sqrt(2)*a^2*c^2*d^4) - 1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqr
t(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + 1/8*(5*(c
*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d
^4) - 1/8*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + s
qrt(2)*a^2*c^2*d^4) + (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqr
t(2)*a^3*b*d^2) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2
)*a^3*b*d^2) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) + 1/
2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*
a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) - 1/2*d*x^(3/2)/((b*
c^2 - a*c*d)*(d*x^2 + c))
```

3.476 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=536

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}}{\sqrt{2}a^{3/4}}$$

```
[Out] -(d*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) +
(d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(
7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.532762, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}}{\sqrt{2}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(d*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) +
(d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(
7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{(d(7bc-3ad))\,\text{Subst}\left(\int\right.}{2c(bc-ad}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}(bc-ad)^2} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,\right.}{\sqrt{a}(bc-ad)^2}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}(bc-ad)^2} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}\right.}{2\sqrt{a}(bc}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}a^{3/4}(bc}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{d^{3/4}(}{}$$

Mathematica [A] time = 0.32826, size = 526, normalized size = 0.98

$$\frac{8a^{3/4}c^{3/4}d\sqrt{x}(ad-bc)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(3ad}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*Sqrt[x] - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c
+ d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*
c^(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[
2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*
(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4*
Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x] + Sqrt[2]*a^(3/4)*d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*a^(3/4)
*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(16*a^(3/4)*c^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.013, size = 566, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x^(1/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(1/2)/(d*x^2+
c)*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a-7/16*d
/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+3/8*d^2/(a*d-b*c)^2/c
^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-7/8*d/(a*d-b
*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+3/8*d^2
/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
a-7/8*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*b+1/4*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
+1/2*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)+1/2*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 100.731, size = 6876, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^4*c^4*d^3 - 41
16*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^
8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^
4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7
 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9*
d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*d
^6)*sqrt((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
a^7*b*c^8*d^7 + a^8*c^7*d^8)))*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2
646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^1
4*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*
a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/
4) + (7*b^7*c^12*d - 45*a*b^6*c^11*d^2 + 123*a^2*b^5*c^10*d^3 - 185*a^3*b^4
*c^9*d^4 + 165*a^4*b^3*c^8*d^5 - 87*a^5*b^2*c^7*d^6 + 25*a^6*b*c^6*d^7 - 3*
a^7*c^5*d^8)*sqrt(x)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a
```

```
^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^
10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4))/(2401*
b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
 81*a^4*d^7)) + 16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^
2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b
^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d
- a*c*d^2)*x^2)*arctan(((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2
 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7
/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 +
 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*
d^7 + a^11*d^8))^(3/4)*sqrt(b^4*x + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*
b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c
^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^
8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))) - (a^2*b^
8*c^6 - 6*a^3*b^7*c^5*d + 15*a^4*b^6*c^4*d^2 - 20*a^5*b^5*c^3*d^3 + 15*a^6*
b^4*c^2*d^4 - 6*a^7*b^3*c*d^5 + a^8*b^2*d^6)*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7
*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*
a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(3/4)*sq
rt(x))/b^7) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 -
 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*
c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a
*c*d^2)*x^2)*log(b^2*sqrt(x) + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^
5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^
5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*
c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28
*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*
c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*
d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b
^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*
c^2 - 2*a^2*b*c*d + a^3*d^2)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)
*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*
b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56
*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*
c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sq
rt(x) + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^1
5 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4
*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^
8*c^7*d^8))^(1/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 8
1*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^
12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8
*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sqrt(x) - (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7
*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))
^(1/4)) - 4*d*sqrt(x))/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47362, size = 909, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + (a*b^3
)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b^3)
^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^
2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/2*(a*b^3)^(1/4)*b*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*
b*c*d + sqrt(2)*a^3*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*
arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/4*(7*(c*d^3)^(1/4)
*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sq
rt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^
2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqr
t(2)*a^2*c^2*d^2) + 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(-sq
rt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b
*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/2*d*sqrt(x)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.477 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=570

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}}{\sqrt{2}a^{5/4}}$$

```
[Out] -(4*b*c - 5*a*d)/(2*a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*Sqrt[x]
*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2
) + (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (b^(9/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5/4
)*(9*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.750091, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}}{\sqrt{2}a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(4*b*c - 5*a*d)/(2*a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*Sqrt[x]
*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2
) + (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (b^(9/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5/4
)*(9*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{4bc-5ad-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^2\left(4b^2c^2+4abcd-5a^2d^2+bd(4bc-5\right.}{\left(a+bx^4\right)\left(c+dx^4\right)}\right.}{2ac^2(bc-ad)}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\mathrm{Subst}\left(\int\left(\frac{4b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{ad^2(-9bc+5}{(-bc+ad)\left(c+\right.}\right.\right.}{2ac^2(bc-ad)}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)^2} + \cdots$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{b^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)^2} - \cdots$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2} - \frac{b^{9/4}\tan^{-1}}{\sqrt{2}a^{5/4}}$$

Mathematica [A] time = 0.673274, size = 540, normalized size = 0.95

$$\frac{1}{16}\left(-\frac{4\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2} + \frac{4\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2} + \frac{8\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{a^{5/4}(bc-ad)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*Sqrt[x]) + (8*d^2*x^(3/2))/(c^2*(b*c - a*d)*(c + d*x^2)) + (8*S
qrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(b*c
 - a*d)^2) - (8*Sqrt[2]*b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(a^(5/4)*(b*c - a*d)^2) + (2*Sqrt[2]*d^(5/4)*(-9*b*c + 5*a*d)*ArcTan[1
- (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^2) + (2*Sqrt[2]*
d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(
9/4)*(b*c - a*d)^2) - (4*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^2) + (4*Sqrt[2]*b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d
)^2) + (Sqrt[2]*d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(-9*b*c
 + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9
/4)*(b*c - a*d)^2))/16
```

Maple [A] time = 0.018, size = 582, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x^(3/2)/
(d*x^2+c)*b-5/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
-5/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)-5/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)+9/16*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+9/8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+9/8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)-2/a/c^2/x^(1/2)-1/4*b^2/a/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2)*l
n((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*
2^(1/2)+(1/b*a)^(1/2)))-1/2*b^2/a/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b^2/a/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 141.711, size = 7727, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] 1/8*(4*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^
4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8
+ 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^
5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^
6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*arctan(((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt((531441*b^6*c^6*d^8 - 1771470*a*b^5*c^5*d^9 + 2460375*a^
2*b^4*c^4*d^10 - 1822500*a^3*b^3*c^3*d^11 + 759375*a^4*b^2*c^2*d^12 - 16875
0*a^5*b*c*d^13 + 15625*a^6*d^14)*x - (6561*b^8*c^13*d^5 - 40824*a*b^7*c^12*
d^6 + 109836*a^2*b^6*c^11*d^7 - 166824*a^3*b^5*c^10*d^8 + 156406*a^4*b^4*c^
9*d^9 - 92680*a^5*b^3*c^8*d^10 + 33900*a^6*b^2*c^7*d^11 - 7000*a^7*b*c^6*d^
12 + 625*a^8*c^5*d^13)*sqrt(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1215
0*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^1
6*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*
a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8)))*
(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^
3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 -
 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b
^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4) + (729*b^5*c^7*d^4 - 2
673*a*b^4*c^6*d^5 + 3834*a^2*b^3*c^5*d^6 - 2690*a^3*b^2*c^4*d^7 + 925*a^4*b
*c^3*d^8 - 125*a^5*c^2*d^9)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d
^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*
a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*
d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^
9*d^8))^(1/4))/(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*
d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)) + 16*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*
c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a
^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(
(a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*arctan((sqrt(b^14*
x - (a^3*b^13*c^4 - 4*a^4*b^12*c^3*d + 6*a^5*b^11*c^2*d^2 - 4*a^6*b^10*c*d^
3 + a^7*b^9*d^4)*sqrt(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*
d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^
11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8)))*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^
7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56
*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)
*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - (a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7
*d^2)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^
5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6
- 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*sqrt(x))/b^9) - 4*(-b^9/(a^5*b^8*c^8 -
8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*
d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8
))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*
sqrt(x) + (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*
c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8
 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c
^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*
d^8))^(3/4)) + 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2
- 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b
^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x
^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) - (a^4*b^6*c^6 - 6*a^5*b^5*c^
5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*
b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d
^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^1
1*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) - ((a*b*c^3*d - a^2*c^2*
d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d
^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*
a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*
d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^
```

```
9*d^8))^(1/4)*log((b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*a^3
*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6561
*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d
^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3
*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11
*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c^3*d^4 - 1215*a*b
^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x)) + ((a*b*c^3*d - a^2*c^
2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3
*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 -
8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^1
3*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*
c^9*d^8))^(1/4)*log(-(b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*
a^3*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6
561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*
c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*
a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c
^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c^3*d^4 - 1215*
a*b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x)) - 4*(4*b*c^2 - 4*a*
c*d + (4*b*c*d - 5*a*d^2)*x^2)*sqrt(x))/((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a
*b*c^4 - a^2*c^3*d)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)

[Out] Timed out
```

Giac [A] time = 1.56095, size = 979, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")

[Out] 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c
^4*d^2 + sqrt(2)*a^2*c^3*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*
a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - 1/8*(9*(c*
d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^
3) + 1/8*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sq
rt(2)*a^2*c^3*d^3) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
+ 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt
(2)*a^4*d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a
^4*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(a*b^
```

```
3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2
*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(4*b*c*d*x^2 - 5*a*d^2*
x^2 + 4*b*c^2 - 4*a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^(5/2) + c*sqrt(x)))
```

3.478 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=570

$$\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(\right.}{\sqrt{2}a^{7/4}(b}$$

```
[Out] -(4*b*c - 7*a*d)/(6*a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*d)*x^(3/2)
*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*
d)^2) + (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (
b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
 + (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.823219, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(\right.}{\sqrt{2}a^{7/4}(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 7*a*d)/(6*a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*d)*x^(3/2)
*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*
d)^2) + (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (
b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
 + (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-7ad-7bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{3\left(4b^2c^2+4abcd-7a^2d^2\right)+3bd(4bc-7a}{\left(a+bx^4\right)\left(c+dx^4\right)}\right.}{6ac^2(bc-ad)}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)^2} + \frac{\left(d^2\right.}{}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)^2} - \frac{b^3\,\text{S}}{}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2} -$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{b^{11/4}\tan^{-}}{\sqrt{2}a^{7}}$$

Mathematica [A] time = 0.660252, size = 542, normalized size = 0.95

$$\frac{1}{48}\left(\frac{12\sqrt{2}b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^2} - \frac{12\sqrt{2}b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^2} + \frac{24\sqrt{2}b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{a^{7/4}(bc-ad)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*x^(3/2)) + (24*d^2*Sqrt[x])/(c^2*(b*c - a*d)*(c + d*x^2)) + (24
*Sqrt[2]*b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(
```

```
b*c - a*d)^2) - (24*Sqrt[2]*b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(a^(7/4)*(b*c - a*d)^2) + (6*Sqrt[2]*d^(7/4)*(-11*b*c + 7*a*d)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^2) + (6*S
qrt[2]*d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(c^(11/4)*(b*c - a*d)^2) + (12*Sqrt[2]*b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^2) - (12*Sqrt[2]*
b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/
4)*(b*c - a*d)^2) + (3*Sqrt[2]*d^(7/4)*(-11*b*c + 7*a*d)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^2) + (3*Sqr
t[2]*d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^2))/48
```

Maple [A] time = 0.018, size = 588, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x^(1/2)/
(d*x^2+c)*b-7/8*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)*a+11/8*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-7/8*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+11/8*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^3/c^3/(a*d-b*c)^2*
(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a+11/16*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4
)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))*b-2/3/a/c^2/x^(3/2)-1/4/a^2*b^3/(a*d-b*c)^2*(1/b
*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/
b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2/a^2*b^3/(a*d-b*c)^2*(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2/a^2*b^3/(a*d-b*c)
^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55399, size = 969, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - (
a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/
2*(a*b^3)^(1/4)*b^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)^(1/4)
*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2
 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7
*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2)
+ 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)
*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)
^(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2
*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) - 1/8*(11*(c*d^3)^(1/4)*b
*c*d - 7*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/2*d^
2*sqrt(x)/((b*c^3 - a*c^2*d)*(d*x^2 + c)) - 2/3/(a*c^2*x^(3/2))
```

3.479 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=618

$$\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}(bc-ad)}+\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-}}{\sqrt{2}a^{9/}}$$

```
[Out] -(4*b*c - 9*a*d)/(10*a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*a*b*c*d -
9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*x^(5/2)*(c
+ d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2
) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (b^(1
3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (
d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.962011, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}(bc-ad)}+\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-}}{\sqrt{2}a^{9/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] -(4*b*c - 9*a*d)/(10*a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*a*b*c*d -
9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*x^(5/2)*(c
+ d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2
) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (b^(1
3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (
d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-9ad-9bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{5\left(4b^2c^2+4abcd-9a^2d^2\right)+5bd(4\ldots}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\ldots\right)}{10ac^2(bc-ad)}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\left(2b^4\right)\text{S}\ldots}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{b^{7/2}\,\text{Sub}\ldots}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{b^3\,\text{Subs}\ldots}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{b^{13/4}\log\ldots}{\ldots}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{b^{13/4}\tan\ldots}{\sqrt{2}a\ldots}$$

Mathematica [A] time = 0.804845, size = 563, normalized size = 0.91

$$\frac{1}{80}\left(\frac{20\sqrt{2}b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^2} - \frac{20\sqrt{2}b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^2} - \frac{40\sqrt{2}b^{13/4}\tan^{-1}\left(1-\ldots\right.}{a^{9/4}(bc-ad)^2}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*x^(5/2)) + (160*(b*c + 2*a*d))/(a^2*c^3*Sqrt[x]) - (40*d^3*x^(3
/2))/(c^3*(b*c - a*d)*(c + d*x^2)) - (40*Sqrt[2]*b^(13/4)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^2) + (40*Sqrt[2]*b^(13/4
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^2) +
(10*Sqrt[2]*d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(c^(13/4)*(b*c - a*d)^2) + (10*Sqrt[2]*d^(9/4)*(-13*b*c + 9*a*d)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^2) + (2
0*Sqrt[2]*b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(a^(9/4)*(b*c - a*d)^2) - (20*Sqrt[2]*b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^2) + (5*Sqrt[2]*d^
(9/4)*(-13*b*c + 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(c^(13/4)*(b*c - a*d)^2) + (5*Sqrt[2]*d^(9/4)*(13*b*c - 9*a*d)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a
*d)^2))/80
```

Maple [A] time = 0.023, size = 612, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/c^3/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)^2*x^(3/2)
/(d*x^2+c)*b+9/16*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)+9/8*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)+9/8*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)-1)-13/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^2/x^(5/2)+4/a/c^3/x^(1/2)*d+2/a^2/c^2
/x^(1/2)*b+1/4*b^3/a^2/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4
)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(
1/2)))+1/2*b^3/a^2/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x^(1/2)+1)+1/2*b^3/a^2/(a*d-b*c)^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60038, size = 965, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x^(3/2)/((b*c^4 - a*c^3*d)*(d*x^2 + c)) + (a*b^3)^(3/4)*b*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*
c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) + (a*b^3)^(3/4)*b*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c^2
 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) - 1/2*(a*b^3)^(3/4)*b*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*
b*c*d + sqrt(2)*a^5*d^2) + 1/2*(a*b^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(
1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*
a^5*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^6 - 2*sqr
t(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^
3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 1/8*(
13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*
d^2) - 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqr
t(2)*a^2*c^4*d^2) + 2/5*(5*b*c*x^2 + 10*a*d*x^2 - a*c)/(a^2*c^3*x^(5/2))
```

3.480 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=631

$$-\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\ldots\right.}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}$$

```
[Out] -(c*Sqrt[x])/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqrt[x])/(16*
d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 -
30*a*b*c*d - 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*
Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(5
/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)
^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + (
(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.81407, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\ldots\right.}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*Sqrt[x])/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqrt[x])/(16*
d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 -
30*a*b*c*d - 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*
Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(5
/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)
^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + (
(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{ac+(bc-8ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{ac(3bc+5ad)+3bc(bc-9ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \right.}{16cd(bc-ad)^2}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2a^2b\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} + \frac{(3}{}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a^{3/2}b\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} +$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x,\right.}{2(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{5/4}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right.}{2\sqrt{2}(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} + \frac{a^{5/4}b^{3/4}}{}$$

Mathematica [A] time = 0.56896, size = 640, normalized size = 1.01

$$\frac{-32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 8*c^(3/4)*d^(1/4)*(b*c - 9*a*d
)*(b*c - a*d)*Sqrt[x]*(c + d*x^2) - 64*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5
/4)*(c + d*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 64*Sqrt[2
]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*
x^2)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(3*b^2*c^2
 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)] - 32*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 32*Sqrt[2]*a^(5/4)
*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c +
d*x^2)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[
2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(128*c^(3/4)*d^(5/4)*(b*c - a*d)^3*(
c + d*x^2)^2)
```

Maple [A] time = 0.018, size = 839, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -9/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+5/8/(a*d-b*c)^3/(d*x^2+c)^2*x
^(5/2)*c*a*b*d-1/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2*c^2-5/16/(a*d-b*c)^
3/(d*x^2+c)^2*c*d*x^(1/2)*a^2+1/8/(a*d-b*c)^3/(d*x^2+c)^2*c^2*x^(1/2)*a*b+3
/16/(a*d-b*c)^3/(d*x^2+c)^2*c^3/d*x^(1/2)*b^2+5/64/(a*d-b*c)^3*d*(c/d)^(1/4
)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+15/32/(a*d-b*c)^3*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-3/64/(a*d-b*c)^3
/d*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/(a*
d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+
15/32/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
*a*b-3/64/(a*d-b*c)^3/d*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*b^2+5/128/(a*d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2
+15/64/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-3/128/(a*d-b*c)
^3/d*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-1/4*b/(a*d-b*c)^3*a*(1/b*a)
^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b/(a*d-b*c)^3*a*(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)^3*a*(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72693, size = 1274, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 - s
qrt(2)*a^3*d^3) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt
(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3
*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2
*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(3*(c*d^3)^(1/4)*b^2*c
^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)
*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sq
rt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) + 1/32*(
3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(
2)*a^3*c*d^5) + 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d -
5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - s
qrt(2)*a^3*c*d^5) - 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*
d - 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^
4 - sqrt(2)*a^3*c*d^5) + 1/16*(b*c*d*x^(5/2) - 9*a*d^2*x^(5/2) - 3*b*c^2*sq
rt(x) - 5*a*c*d*sqrt(x))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(d*x^2 + c)^2
)
```

3.481 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=628

$$\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\cdots\right.}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$

```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))/(16*c*(b*
c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*
b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[
2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3/4)*(
b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(3/4)*b^(5/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^
2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.76127, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\cdots\right.}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```


```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))/(16*c*(b*
c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*
b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[
2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3/4)*(
b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(3/4)*b^(5/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^
2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a-5bx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a(9bc-ad)-b(5bc+3ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx,\right.}{16c(bc-ad)^2}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\left(\frac{32ab^2cx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{\left(5b^2c^2+30abcd\right.}{(bc-ad)(c\right.}\right.}{16c(bc-ad)^2}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(2ab^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} + \frac{(5}{}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(ab^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} -$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^3}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{3/4}b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right.}{2\sqrt{2}(bc-ad)^3}$$

$$= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} - \frac{a^{3/4}b^{5/4}}{}$$

Mathematica [A] time = 0.787059, size = 544, normalized size = 0.87

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}d^{3/4}} - \frac{\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}d^{3/4}} - \frac{2\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan}{c^{5/4}d^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (8*(b*c - a*d)*(5*b*c + 3*a*d)*
x^(3/2))/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*d^(3/4)) + (2*Sqrt[
2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(5/4)*d^(3/4)) - 32*Sqrt[2]*a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 32*Sqrt[2]*a^(3/4)*b^(5/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (Sqrt[2]*(5*b^2*c
^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(c^(5/4)*d^(3/4)) - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*
d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*d
^(3/4)))/(128*(b*c - a*d)^3)
```

Maple [A] time = 0.019, size = 839, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^(7/2)*a^2+1/8/(a*d-b*c)^3/(d*x^2+c)^2*
d^2*x^(7/2)*a*b-5/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(7/2)*b^2-1/16/(a*d-b*c)
^3/(d*x^2+c)^2*x^(3/2)*a^2*d^2+5/8/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*c*a*b*d-
9/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64/(a*d-b*c)^3/c*d/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-15/32/(a*d-b*c)^3/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-5/64/(a*d-b*c)^3*
c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/(a*d
-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-1
5/32/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
a*b-5/64/(a*d-b*c)^3*c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)-1)*b^2+3/128/(a*d-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-
15/64/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-5/128/(a*d-b*c)^
3*c/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/4*b*a/(a*d-b*c)^3/(1/b*a)^
(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)
^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b*a/(a*d-b*c)^3/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b*a/(a*d-b*c)^3/(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86114, size = 1300, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*
a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/
(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 -
sqrt(2)*a^3*c^2*d^6) + 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b
*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*
sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - 1/64*(5*(c*d^3)^(3/4)*b^2*c^
2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)
*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^
4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) + 1/64*(5*(c*d^3)^(3/4)*
b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2
*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - (a*b^3)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b
^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^
3) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d
^2 - sqrt(2)*a^3*b*d^3) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*
b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt
(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/16*(5*b*c*d*x^(7/2) + 3*a*d^2*x^
(7/2) + 9*b*c^2*x^(3/2) - a*c*d*x^(3/2))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*(d*x^2 + c)^2)
```

3.482 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=627

$$-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}$$

```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(16*c*(b*c
- a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/4)*(
b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) -
 ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*
b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.710189, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```


```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(16*c*(b*c
- a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/4)*(
b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) -
 ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*
b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{a-7bx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{a(11bc-3ad)-3b(7bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{16c(bc-ad)^2}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(2ab^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} + \frac{\left(21b \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(\sqrt{a}b^2\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} - \cdots$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(\sqrt{a}b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x} \cdots\right.}{2(bc-ad)^3} \cdots$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\sqrt[4]{a}b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} - \cdots$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\sqrt[4]{a}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} - \frac{\sqrt[4]{a}b^{7/4}\tan \cdots}{\sqrt{2}\cdots}$$

Mathematica [A] time = 0.80391, size = 543, normalized size = 0.87

$$-\frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}\sqrt[4]{d}} + \frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}\sqrt[4]{d}} - \frac{2\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\text{ta}\cdots}{c^{7/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c + a*d)*Sq
rt[x])/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(
```

```
1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(7/4)*d^(1/4)) + (2*Sqrt[2
]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(7/4)*d^(1/4)) + 32*Sqrt[2]*a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 32*Sqrt[2]*a^(1/4)*b^(7/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (Sqrt[2]*(21*b^2*
c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(c^(7/4)*d^(1/4)) + (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)
*d^(1/4)))/(128*(b*c - a*d)^3)
```

Maple [A] time = 0.018, size = 848, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/16/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^(5/2)*a^2+3/8/(a*d-b*c)^3/(d*x^2+c)^2*
d^2*x^(5/2)*a*b-7/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(5/2)*b^2+7/8/(a*d-b*c)^
3/(d*x^2+c)^2*x^(1/2)*c*a*b*d-11/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*b^2*c^2
-3/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a^2*d^2+3/64/(a*d-b*c)^3/c^2*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2-7/32/(a*d-b*c)^3/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d-21/64/(a*
d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64
/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
a^2*d^2-7/32/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a*b*d-21/64/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*b^2+3/128/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))*a^2*d^2-7/64/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b*d-
21/128/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/4*b^2/(a*d-b*
c)^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2
))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b^2/(a*d-b*c)^3*(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b^2/(a*d-b*c
)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.84457, size = 1277, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 -
sqrt(2)*a^3*d^3) - (a*b^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqr
t(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d +
3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*b*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^
2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(21*(c*d^3)^(1/4)*b^2
*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d - 3*sq
rt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/32
*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2
*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt
(2)*a^3*c^2*d^4) + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*
d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 -
 sqrt(2)*a^3*c^2*d^4) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a
*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3
*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/16*(7*b*c*d*x^(5/2) + a*d^2*x^(5/2) + 11*b*
c^2*sqrt(x) - 3*a*c*d*sqrt(x))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(d*x^2
+ c)^2)
```

3.483 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{9/4}(bc-ad)^3}$$

```
[Out] -(d*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d)*x^(3/2))
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(9/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4
)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqr
t[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(9/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b
*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a
*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.817231, antiderivative size = 633, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d)*x^(3/2))
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(9/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4
)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqr
t[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(9/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b
*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a
*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(8bc-5ad-5bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(32b^2c^2-13abcd+5a^2d^2-bd(1\right.}{\left(a+bx^4\right)\left(c+dx^4\right)}\right.}{16c^2(bc-ad)^2}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int\left(\frac{32b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{d\left(45b^2c^2-\right.}{(bc-}\right.\right.}{16c^2(bc-a}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} -$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^3} +$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right.}{2(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right.}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3} + \frac{b^{9/4}\tan}{\sqrt{2}}$$

Mathematica [A] time = 0.980182, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(ad-bc)^3} + \frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\right.}{c^{9/4}(bc-ad)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-13*b*c + 5*a*d)*x^(
3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) - (64*Sqrt[2]*b^
(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d
)^3) + (2*Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^
(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(9/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)^3) + (3
2*Sqrt[2]*b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d +
 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^
(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*
(b*c - a*d)^3))/128
```

Maple [A] time = 0.019, size = 855, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 5/16*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^(7/2)*a^2-9/8*d^3/(a*d-b*c)^3/(d*x^2
+c)^2/c*x^(7/2)*a*b+13/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2+9/16*d^3/
(a*d-b*c)^3/(d*x^2+c)^2/c*x^(3/2)*a^2-13/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3
/2)*a*b+17/16*d/(a*d-b*c)^3/(d*x^2+c)^2*c*x^(3/2)*b^2+5/64*d^2/(a*d-b*c)^3/
c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-9/32*d/(a
*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+4
5/64/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
b^2+5/128*d^2/(a*d-b*c)^3/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-9/64
*d/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+45/128/(a*d-b*c)^
3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+5/64*d^2/(a*d-b*c)^3/c^2/(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-9/32*d/(a*d-b*c)^3/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+45/64/(a*d-b
*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/4*b^2
/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b
*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b^2/(a*d-b*
c)^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b^2/
(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.88241, size = 1307, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4
- sqrt(2)*a^3*c^3*d^5) - 1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*
a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
 - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 +
 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + 1/64*(45*(c*d^3)^(3/4)*b^
2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqr
t(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^
5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) - 1/64*(45*(c*d^3)^(
3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(-sqr
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*
a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + (a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a
^4*d^3) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x
))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^
3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqr
t(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d
 + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/16*(13*b*c*d^2*x^(7/2) - 5*
```

```
a*d^3*x^(7/2) + 17*b*c^2*d*x^(3/2) - 9*a*c*d^2*x^(3/2))/((b^2*c^4 - 2*a*b*c
^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.484 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{11/4}(bc-ad)^3}$$

```
[Out] -(d*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d)*Sqrt[x])
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3
/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 -
 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4
)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b
*c - a*d)^3)
```

Rubi [A] time = 0.831815, antiderivative size = 633, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}c^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d)*Sqrt[x])
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3
/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 -
 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4
)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b
*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-7ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx,x,\sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-45abcd+21a^2d^2-3\cdots}{\left(a+bx^4\right)\left(c+dx\cdots\right.}\cdots\right.}{16c^2(bc-a\cdots}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\cdots\right.}{(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)\cdots}{\sqrt{a}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,\cdots\right.}{2\sqrt{a}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\cdots\right.}{2\sqrt{2}a^{3/4}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^3} + \frac{b^{11\cdots}}{\cdots}$$

Mathematica [A] time = 0.908967, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^3} + \frac{\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\cdots\right.}{c^{11/4}(ad-bc)^3}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-15*b*c + 7*a*d)*Sqr
t[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(11/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) - (64*Sqrt[2]*b
^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a
*d)^3) + (2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) - (2*Sqrt[2
]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(11/4)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(-(b*c) + a*
d)^3) + (32*Sqrt[2]*b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a
*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(c^(11/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d
+ 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
c^(11/4)*(-(b*c) + a*d)^3))/128
```

Maple [A] time = 0.018, size = 882, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 7/16*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^(5/2)*a^2-11/8*d^3/(a*d-b*c)^3/(d*x^
2+c)^2/c*x^(5/2)*a*b+15/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2+11/16*d^
3/(a*d-b*c)^3/(d*x^2+c)^2/c*x^(1/2)*a^2-15/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^
(1/2)*a*b+19/16*d/(a*d-b*c)^3/(d*x^2+c)^2*c*x^(1/2)*b^2+21/128*d^3/(a*d-b*c
)^3/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-33/64*d^2/(a*d-b*c)^3/c^2*
(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+77/128*d/(a*d-b*c)^3/c*(c/d)^(1/4)
*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2)))*b^2+21/64*d^3/(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-33/32*d^2/(a*d-b*c)^3/c^2*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+77/64*d/(a*d-b*c)^3
/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+21/64*d^3/
(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a
^2-33/32*d^2/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*a*b+77/64*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)-1)*b^2-1/4*b^3/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*ln((x+
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/
2)+(1/b*a)^(1/2)))-1/2*b^3/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b^3/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76236, size = 1296, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d
^2 - sqrt(2)*a^4*d^3) + (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^
2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(1/4)*b^2*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)
*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(1/
4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3
 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/3
2*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a
^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)
*a^3*c^3*d^3) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d +
 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2
*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*
d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a
^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) + 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(
```

```
c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)
^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/16*(15*b*c*d^2*x^(5/2) - 7*a*d^3
*x^(5/2) + 19*b*c^2*d*sqrt(x) - 11*a*c*d^2*sqrt(x))/((b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.485 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=681

$$-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}(bc-ad)^2}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-1\right.}{}$$

```
[Out] -(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(16*a*c^3*(b*c - a*d)^2*Sqrt[x]) -
d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c - 9*a*d))/(16*c^2*(b
*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)
*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.00645, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}(bc-ad)^2}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(16*a*c^3*(b*c - a*d)^2*Sqrt[x]) -
d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c - 9*a*d))/(16*c^2*(b
*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)
*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-9ad-9bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-85abcd+45a^2 d}{x^2\left(a+bx^4\right)}\right.}{16c^2(bc}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} - \text{Sub}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} - \text{Sub}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} - \frac{(2b^4}{}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b^{7/2}}{}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} - \frac{b^3\,\text{S}}{}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} - \frac{b^{13/4}}{}$$

$$= -\frac{\frac{32b^2c}{a} - 85bd + \frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b^{13/4}}{}$$

Mathematica [A] time = 1.09256, size = 637, normalized size = 0.94

$$\frac{1}{128}\left(\frac{\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^3} + \frac{\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log}{c^{13/4}(ad-bc)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*Sqrt[x]) + (32*d^2*x^(3/2))/(c^2*(b*c - a*d)*(c + d*x^2)^2) +
(8*d^2*(21*b*c - 13*a*d)*x^(3/2))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (64*Sqr
t[2]*b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*
c) + a*d)^3) + (64*Sqrt[2]*b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(a^(5/4)*(-(b*c) + a*d)^3) - (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*
a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13
```

```
/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2
*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^
3) + (32*Sqrt[2]*b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3) + (32*Sqrt[2]*b^(13/4)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^3) + (Sq
rt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a*d)^3) + (Sqrt[2]
*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(-(b*c) + a*d)^3))/128
```

Maple [A] time = 0.024, size = 900, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -13/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2+17/8*d^4/c^2/(a*d-b*c)^3
/(d*x^2+c)^2*x^(7/2)*a*b-21/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2-17
/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a^2+21/8*d^3/c/(a*d-b*c)^3/(d*x
^2+c)^2*x^(3/2)*a*b-25/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2-45/128*d^
3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-45/64*d^3/c^3/(a
*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-45/
64*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)+65/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)+65/32*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)+65/32*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)-117/128*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2
*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2)))-117/64*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)-117/64*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a/c^3/x^(1/2)+1/4*b^3/a/(a*d-b*c)^3
/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2*b^3/a/(a*d-b*c)^3/(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+1/2*b^3/a/(a*d-b*c
)^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.99924, size = 1332, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*
d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c
^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b^3)^(3/4)*b*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2
)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(3
/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^
3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/
32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 -
sqrt(2)*a^3*c^4*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*
a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 +
3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^
2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^
6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/64*(117*(c*d^3)^
(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(-
sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)
*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/16*(21*
b*c*d^3*x^(7/2) - 13*a*d^4*x^(7/2) + 25*b*c^2*d^2*x^(3/2) - 17*a*c*d^3*x^(3
/2))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/(a*c^3*sqrt(
x))
```

3.486 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=681

$$-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-\right.}{}$$

```
[Out] -(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(48*a*c^3*(b*c - a*d)^2*x^(3/2)) -
 d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c - 11*a*d))/(16*c^2*
(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/
4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2
 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(1
5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
5/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.927946, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```


```
[Out] -(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(48*a*c^3*(b*c - a*d)^2*x^(3/2)) -
 d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c - 11*a*d))/(16*c^2*
(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/
4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2
 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(1
5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
5/4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-11ad-11bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-133abcd+77\ldots}{x^4\left(a+b\ldots\right.}\ldots\right.}{16c^2(\ldots}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} - \ldots$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} - \ldots$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} - \ldots$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} - \ldots$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \ldots$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \ldots$$

Mathematica [A] time = 1.01227, size = 639, normalized size = 0.94

$$\frac{1}{384}\left(\frac{3\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(ad-bc)^3} + \frac{3\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\ldots}{c^{15/4}(bc-a\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*x^(3/2)) + (96*d^2*Sqrt[x])/(c^2*(b*c - a*d)*(c + d*x^2)^2) +
(24*d^2*(23*b*c - 15*a*d)*Sqrt[x])/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (192*S
qrt[2]*b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(-(
b*c) + a*d)^3) + (192*Sqrt[2]*b^(15/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) - (6*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 2
10*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^
(15/4)*(b*c - a*d)^3) + (6*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*
a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(15/4)*(b*c - a*
d)^3) + (96*Sqrt[2]*b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^3) + (96*Sqrt[2]*b^(15/4)*Log[Sqrt[a] +
```

```
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(-(b*c) + a*d)^3) +
(3*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*(-(b*c) + a*d)^3) +
(3*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*(b*c - a*d)^3))/384
```

Maple [A] time = 0.023, size = 906, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -15/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a^2+19/8*d^4/c^2/(a*d-b*c)^3
/(d*x^2+c)^2*x^(5/2)*a*b-23/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2-19
/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a^2+23/8*d^3/c/(a*d-b*c)^3/(d*x
^2+c)^2*x^(1/2)*a*b-27/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*b^2-77/64*d^4
/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a^2+105/32*d^3/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*a*b-165/64*d^2/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64*d^4/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+105/32*d^3/c^3/(a*d-b*c)^3*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-165/64*d^2/c^2/(
a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77
/128*d^4/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+105/64*d^
3/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-165/128*d^2/c^2/
(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3/a/c^3/x^(3/2)+1/4/
a^2*b^4/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/
2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+1/2/a^2*
b^4/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+
1)+1/2/a^2*b^4/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.05851, size = 1343, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*
c*d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(1/4)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b
^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(1/4)*b^3
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*
sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b
^3)^(1/4)*b^3*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^
2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d
^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*
(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^
5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d
^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*
c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/64*(165*(c*d^3)^
(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*l
og(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2
)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) - 1/64*(165*
(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^
2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 -
 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1
/16*(23*b*c*d^3*x^(5/2) - 15*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 19*a*c*
d^3*sqrt(x))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/3/(a
*c^3*x^(3/2))
```

3.487 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=743

$$-\frac{117a^2d^2-189abcd+32b^2c^2}{80ac^3x^{5/2}(bc-ad)^2}+\frac{-189a^2bcd^2+117a^3d^3+32ab^2c^2d+32b^3c^3}{16a^2c^4\sqrt{x}(bc-ad)^2}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\mathrm{lo}}{64\sqrt{2}c^{17/4}(bc}$$

```
[Out] -(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(80*a*c^3*(b*c - a*d)^2*x^(5/2))
+ (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*a^3*d^3)/(16*a^2*c^4
*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) - (d*(2
1*b*c - 13*a*d))/(16*c^2*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3)
 + (b^(17/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4
)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^
3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (b^(17/4)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
9/4)*(b*c - a*d)^3) - (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 -
 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.23311, antiderivative size = 743, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{117a^2d^2-189abcd+32b^2c^2}{80ac^3x^{5/2}(bc-ad)^2}+\frac{-189a^2bcd^2+117a^3d^3+32ab^2c^2d+32b^3c^3}{16a^2c^4\sqrt{x}(bc-ad)^2}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\mathrm{lo}}{64\sqrt{2}c^{17/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(80*a*c^3*(b*c - a*d)^2*x^(5/2))
+ (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*a^3*d^3)/(16*a^2*c^4
*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) - (d*(2
1*b*c - 13*a*d))/(16*c^2*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3)
 + (b^(17/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4
)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^
3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (b^(17/4)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
9/4)*(b*c - a*d)^3) - (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 -
 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
```

```
Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-13ad-13bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-189abcd+11}{x^6(a+b}\right.}{16c^2(}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} - \frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}\left(c+dx^2\right)} - $$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)x^{5/2}}$$

Mathematica [A] time = 1.28456, size = 660, normalized size = 0.89

$$\frac{1}{640}\left(\frac{5\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(ad-bc)^3} + \frac{5\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right.}{c^{17/4}(bc-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*x^(5/2)) + (1280*(b*c + 3*a*d))/(a^2*c^4*Sqrt[x]) - (160*d^3*x
^(3/2))/(c^3*(b*c - a*d)*(c + d*x^2)^2) + (40*d^3*(-29*b*c + 21*a*d)*x^(3/2
```

```
))/(c^4*(b*c - a*d)^2*(c + d*x^2)) + (320*Sqrt[2]*b^(17/4)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(-(b*c) + a*d)^3) - (320*Sqrt[2]*b^
(17/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(-(b*c) + a*
d)^3) + (10*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*d)^3) - (10*Sq
rt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*d)^3) + (160*Sqrt[2]*b^(17/4
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(b*c
 - a*d)^3) + (160*Sqrt[2]*b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(a^(9/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b^
2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(c^(17/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b
^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(c^(17/4)*(b*c - a*d)^3))/640
```

Maple [A] time = 0.026, size = 933, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 21/16*d^6/c^4/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2-25/8*d^5/c^3/(a*d-b*c)^3/
(d*x^2+c)^2*x^(7/2)*a*b+29/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2+2
5/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a^2-29/8*d^4/c^2/(a*d-b*c)^3/(
d*x^2+c)^2*x^(3/2)*a*b+33/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2+117/
128*d^4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+117/64*d^4
/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)+117/64*d^4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)-153/64*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2)))-153/32*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)-153/32*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+221/128*d^2/c^2/(a*d-b*c)^3/(c/d)^(1
/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+221/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(
1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+221/64*d^2/c^2/(a*d-b*c)^3/(
c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^3/x^(5
/2)+6/a/c^4/x^(1/2)*d+2/a^2/c^3/x^(1/2)*b-1/4*b^4/a^2/(a*d-b*c)^3/(1/b*a)^(
1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^
(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-1/2*b^4/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-1/2*b^4/a^2/(a*d-b*c)^3/(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.87921, size = 1350, normalized size = 1.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c
*d^2 - sqrt(2)*a^6*d^3) + (a*b^3)^(3/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^
2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/2*(a*b^3)^(3/4)*b^2*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*s
qrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) + 1/2*(a*b^
3)^(3/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3
*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^
3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d
^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^
2 - sqrt(2)*a^3*c^5*d^3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3
/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d +
 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) + 1/64*(221*(c*d^3)^(3/4)*b
^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^
7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/64*(221*(c*d^3)^(3
/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*
b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/16*(29*b*c*d
```

```
^4*x^(7/2) - 21*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 25*a*c*d^4*x^(3/2))/
((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 2/5*(5*b*c*x^2 + 15
*a*d*x^2 - a*c)/(a^2*c^4*x^(5/2))
```

3.488 $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=624

$$\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{x}(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+\cdots}{\cdots}$$

```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x])/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1
/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4
)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d
)^3) + (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.757192, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{x}(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x])/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1
/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4
)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d
)^3) + (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-3ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{8abc^2-12bc(bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, \cdots\right.}{8bc(bc-ad)^2}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(a(5bc+3ad))\,\text{Subst}\left(\int \frac{1}{a+bx^4} \cdots\right.}{2(bc-ad)^3}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}(5bc+3ad)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\cdots}{a+\cdots}\right.}{4(bc-ad)^3}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}(5bc+3ad)\right)\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{b}}-\cdots}\right.}{8\sqrt{b}(bc-ad)\cdots}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}(5bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\cdots}\right.}{8\sqrt{2}\sqrt[4]{b}(bc-ad)\cdots}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3}$$

Mathematica [A] time = 1.14652, size = 585, normalized size = 0.94

$$\frac{1}{16}\left(\frac{8a\sqrt{x}}{\left(a+bx^2\right)(bc-ad)^2} + \frac{8c\sqrt{x}}{\left(c+dx^2\right)(bc-ad)^2} + \frac{\sqrt{2}\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}(bc-ad)^3} - \frac{\sqrt{2}\sqrt[4]{a}(3ad+5b\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*a*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) + (8*c*Sqrt[x])/((b*c - a*d)^2*(
c + d*x^2)) + (2*Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
```

```
)*Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^3) - (2*Sqrt[2]*a^(1/4)*(5*b*c +
3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^
3) + (2*Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(d^(1/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(d^(1/4)*(b*c - a*d)^3) + (
Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(b^(1/4)*(b*c - a*d)^3) - (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(b^(1/4)*(b*c
- a*d)^3) + (Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(d^(1/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*c^(1/4)
*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(d^(1/4)*(-(b*c) + a*d)^3))/16
```

Maple [A] time = 0.019, size = 740, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a*d-1/2*c^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+
c)*b-5/8/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)*a*d-3/8*c/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*b-5/8/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a*d-3/8*c/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*b-5/16/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*d-
3/16*c/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/2*a^2/(a*d-b*c)
^3*x^(1/2)/(b*x^2+a)*d-1/2*a/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*b*c+3/8*a/(a*d-b
*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d+5/8/(
a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*b*
c+3/8*a/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1
/2)-1)*d+5/8/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)
*x^(1/2)-1)*b*c+3/16*a/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4
)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(
1/2)))*d+5/16/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*b*
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.489 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=609

$$-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad}{}$$

```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(1/4)*(3*b*c + 5*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c -
 a*d)^3) + (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
+ (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(1/4)*(3*b*c + 5*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4
)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.796088, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(1/4)*(3*b*c + 5*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c -
 a*d)^3) + (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
+ (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(1/4)*(3*b*c + 5*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4
)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c-5dx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(12c(bc+ad)-8bcdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,d\right.}{8c(bc-ad)^2}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int\left(\frac{4bc(3bc+5ad)x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{4cd(}{(bc-}\right.\right.}{8c(bc-ad}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(5bc+3ad))\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\right.}{2(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(\sqrt{d}(5bc+3ad)\right)\text{Subst}\left(\int \frac{\sqrt{c}}{c}\right.}{4(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(5bc+3ad)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}}\right.}{8(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{b}(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\right.}{8\sqrt{2}\sqrt[4]{a}(bc-ad}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\sqrt[4]{b}(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3}$$

Mathematica [A] time = 1.10313, size = 583, normalized size = 0.96

$$\frac{1}{16}\left(-\frac{8bx^{3/2}}{\left(a+bx^2\right)(bc-ad)^2} - \frac{8dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2} + \frac{\sqrt{2}\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{b}(5ad}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*x^(3/2))/((b*c - a*d)^2*(a + b*x^2)) - (8*d*x^(3/2))/((b*c - a*d)^2*
(c + d*x^2)) + (2*Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) - (2*Sqrt[2]*b^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c)
+ a*d)^3) + (2*Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(c^(1/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(1/4)*(5*b*c + 3*
a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(1/4)*(b*c - a*d)^3)
 + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*
a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(
-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*
d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(c^(1/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.02, size = 740, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(3/2)/(d*x^2+c
)*b*c+5/16/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*c*ln((x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+5/8/(a*d-b*
c)^3/(c/d)^(1/4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/8/(a*d
-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16*
d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+3/8*d/(a*d-b*c)^3/(c/d
)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8*d/(a*d-b*c)^3/(
c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/2*b/(a*d-b*c)^
3*x^(3/2)/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*c-5/16/(a*d-b
*c)^3/(1/b*a)^(1/4)*2^(1/2)*a*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-5/8/(a*d-b*c)^3/(1
/b*a)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8/(a*d-b*
c)^3/(1/b*a)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-3/16
*b/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+
(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-3/8*b/(a*d-
b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-3/8*
b/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.490 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=601

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\tan^{-1}}{4\sqrt{2}a^{3/4}(bc}$$

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (b^(3/
4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.690909, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\tan^{-1}}{4\sqrt{2}a^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (b^(3/
4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{c-7dx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4c(bc+ad)-24bcdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{}\right.}{8c(bc-ad)^2}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(7bc+ad))\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx,\right.}{2(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(7bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,d\right.}{4\sqrt{c}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{d}(7bc+ad)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}\right.}{8\sqrt{c}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{3/4}(bc+7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{8\sqrt{2}a^{3/4}(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{3/4}(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{4\sqrt{2}a^{3/4}(bc-ad)^3}$$

Mathematica [A] time = 0.997369, size = 575, normalized size = 0.96

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(ad-bc)^3} + \frac{\sqrt{2}b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(bc-ad)^3} + \frac{2\sqrt{2}b^{3/4}(7a}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) - (8*d*Sqrt[x])/((b*c - a*d)^2*
(c + d*x^2)) + (2*Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
```

```
*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) - (2*Sqrt[2]*b^(3/4)*(b*c +
7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*
d)^3) + (2*Sqrt[2]*d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/(c^(3/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(3/4)*(7*b*c + a*d)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*(b*c - a*d)^3) + (Sqrt
[2]*b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(b*c - a*d
)^3) + (Sqrt[2]*d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(7*b*c +
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*(
-(b*c) + a*d)^3))/16
```

Maple [A] time = 0.019, size = 770, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(1/2)/(d*x^2+c
)*b*c+1/8*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)*a+7/8*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*b+1/16*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
*a+7/16*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/8*d^2/(a*d-b
*c)^3*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+7/8*d/(
a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-1/2*
b/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*c
-7/8*b/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/
2)+1)*d-1/8*b^2/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^
(1/4)*x^(1/2)+1)*c-7/8*b/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x^(1/2)-1)*d-1/8*b^2/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c-7/16*b/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/
2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))*d-1/16*b^2/(a*d-b*c)^3*(1/b*a)^(1/4)/a*2^(1/2)*
ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.491 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2} dx$

Optimal. Leaf size=624

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\tan^{-}}{4\sqrt{2}a^{5/4}(bc}$$

```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(3/2))/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^
(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c
- a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b
*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(5/4)*(b*c -
9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^
(5/4)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.858001, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\tan^{-}}{4\sqrt{2}a^{5/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(3/2))/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^
(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c
- a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b
*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(5/4)*(b*c -
9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^
(5/4)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad-5bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-4\left(b^2c^2-8abcd+a^2\right.\right.}{\left(a+bx^4\right)(}\right.}{8ac(bc}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\left(-\frac{4b^2c(bc-9ad)x^2}{(bc-ad)\left(a+bx^4\right)} -\right.\right.}{8ac(bc}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(bc-9ad)\right)\text{Subst}\left(\int \frac{}{a-}\right.}{2a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(b^{3/2}(bc-9ad)\right)\text{Subst}\left(\int\right.}{4a(bc-ad}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{(b(bc-9ad))\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{8a(bc-a}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{5/4}(bc-9ad)\log\left(\sqrt{a}-\sqrt{}\right.}{8\sqrt{2}a^{5/4}(bc-}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{}}{}\right.}{4\sqrt{2}a^{5/4}(bc-ad)^3}$$

Mathematica [A] time = 1.14288, size = 589, normalized size = 0.94

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{5/4}(9ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(ad-bc)^3} + \frac{\sqrt{2}b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(ad-bc)^3} + \frac{2\sqrt{2}b^{5/4}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*b^2*x^(3/2))/(a*(b*c - a*d)^2*(a + b*x^2)) + (8*d^2*x^(3/2))/(c*(b*c -
a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*b^(5/4)*
(-(b*c) + 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-
(b*c) + a*d)^3) + (2*Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(5/4)*(9*b*c
 - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*(b*c - a*d)
^3) + (Sqrt[2]*b^(5/4)*(-(b*c) + 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*b^(5/4)*(b*c
 - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5
/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*
d^(5/4)*(-9*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(c^(5/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.02, size = 778, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x^(3/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x^(3/2)/(d*x^
2+c)*b+1/8*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)*b+1/8*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*b+1/16*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*ln((x
-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2)))*a-9/16*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/2
*b^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3/a*x^(3/2)/(b*x^2+a
)*c+9/8*b/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)+1)*d-1/8*b^2/(a*d-b*c)^3/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x^(1/2)+1)*c+9/8*b/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/b*a)^(1/4)*x^(1/2)-1)*d-1/8*b^2/(a*d-b*c)^3/a/(1/b*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c+9/16*b/(a*d-b*c)^3/(1/b*a)^(1/4)*2^
(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d-1/16*b^2/(a*d-b*c)^3/a/(1/b*a)^(1/4)*2^(1/
2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.492 $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=628

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)}{4\sqrt{2}a^{7/}}$$

```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sqrt[x])/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4
)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2
]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (
b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)
*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.861671, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)}{4\sqrt{2}a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sqrt[x])/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4
)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2
]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (
b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)
*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4\left(3b^2c^2-8abcd+3a^2\right.}{\left(a+bx^4\right)(}\right.}{8ac(bc}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(3bc-11ad)\right)\text{Subst}\left(\int\right.}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(3bc-11ad)\right)\text{Subst}\left(\int\right.}{4a^{3/2}(bc-}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^{3/2}(3bc-11ad)\right)\text{Subst}}{8a^{3/2}(bc}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\right.}{8\sqrt{2}a^{7/4}(bc}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{7/4}(bc-ad)}$$

Mathematica [A] time = 1.20154, size = 593, normalized size = 0.94

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(ad-bc)^3} + \frac{2\sqrt{2}b^{7/}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*b^2*Sqrt[x])/(a*(b*c - a*d)^2*(a + b*x^2)) + (8*d^2*Sqrt[x])/(c*(b*c -
a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 - (Sqrt
```

```
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*b^(7/4)
*(-3*b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*
(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(
11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(7/4)*(b*
c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4
)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(a^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*(-(b*c) + a*d
)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.019, size = 808, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x^(1/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x^(1/2)/(d*x^
2+c)*b+3/8*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)*b+3/8*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/
4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*l
n((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2)))*b+1/2*b^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)
^3/a*x^(1/2)/(b*x^2+a)*c+11/8*b^2/(a*d-b*c)^3/a*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d-3/8*b^3/(a*d-b*c)^3/a^2*(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c+11/8*b^2/(a*d-b*c)^3/a*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d-3/8*b^3/(a*d-
b*c)^3/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c+
11/16*b^2/(a*d-b*c)^3/a*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2
^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d-3/
16*b^3/(a*d-b*c)^3/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.493 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=676

$$-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}(bc-ad)^2}-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{9/4}(bc-ad)^3}$$

```
[Out] -(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[x]) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)*(5*b*c - 13*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b^(
9/4)*(5*b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(1
3*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^
(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9
/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.05594, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}(bc-ad)^2}-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[x]) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)*(5*b*c - 13*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b^(
9/4)*(5*b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(1
3*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^
(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9
/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-9bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4\left(5b^2c^2-8\right.}{x}\right.}{8}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+\right.}$$

Mathematica [A] time = 1.28625, size = 606, normalized size = 0.9

$$\frac{1}{16}\left(-\frac{8b^3x^{3/2}}{a^2\left(a+bx^2\right)(bc-ad)^2} + \frac{\sqrt{2}b^{9/4}(13ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^3} + \frac{\sqrt{2}b^{9/4}(13ad-5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{a^{9/4}(ad-bc)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*Sqrt[x]) - (8*b^3*x^(3/2))/(a^2*(b*c - a*d)^2*(a + b*x^2)) -
(8*d^3*x^(3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(9/4)*(-5*b*
c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(-(b*c)
 + a*d)^3) + (2*Sqrt[2]*b^(9/4)*(-5*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(9/4)*(13*b*c
- 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d
)^3) + (2*Sqrt[2]*d^(9/4)*(-13*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(9/4)*(-5*b*c + 13*a*d
```

```
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(b*c
 - a*d)^3) + (Sqrt[2]*b^(9/4)*(-5*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9
/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(c^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^
3))/16
```

Maple [A] time = 0.025, size = 825, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^4/c^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3*x^(3/2)/
(d*x^2+c)*b-5/16*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
-5/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)-5/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)+13/16*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2)))+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)-2/a^2/c^2/x^(1/2)-1/2*b^3/a/(a*d-b*c)^3*x^(3/2)/(b*x
^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*c-13/16*b^2/a/(a*d-b*c)^3
/(1/b*a)^(1/4)*2^(1/2)*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))
/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-13/8*b^2/a/(a*d-b*c)^3/(1
/b*a)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-13/8*b^2/a/(a
*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+5
/16*b^3/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*ln((x-(1/b*a)^(1/4)*x^(1/2)
*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+5/
8*b^3/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*
x^(1/2)+1)+5/8*b^3/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(
1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.494 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=676

$$-\frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2}+\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{11/4}(bc-ad)^3}$$

```
[Out] -(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^(3/2)) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b
^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(1
1/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqr
t[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^
3) - (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7
*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2
]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^3)
```

Rubi [A] time = 0.966571, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2}+\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] -(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^(3/2)) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b
^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(1
1/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqr
t[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^
3) - (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7
*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2
]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^3)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-11bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4(7b\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)(\ldots}$$

Mathematica [A] time = 1.25323, size = 610, normalized size = 0.9

$$\frac{1}{48}\left(-\frac{24b^3\sqrt{x}}{a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{3\sqrt{2}b^{11/4}(15ad-7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}(ad-bc)^3}+\frac{3\sqrt{2}b^{11/4}(15ad-7bc)\log\left(\sqrt{2}\sqrt[4]{}\right.}{a^{11/4}(bc-a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*x^(3/2)) - (24*b^3*Sqrt[x])/(a^2*(b*c - a*d)^2*(a + b*x^2)) -
 (24*d^3*Sqrt[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (6*Sqrt[2]*b^(11/4)*(-7
*b*c + 15*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*(-(
b*c) + a*d)^3) + (6*Sqrt[2]*b^(11/4)*(-7*b*c + 15*a*d)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*(b*c - a*d)^3) + (6*Sqrt[2]*d^(11/4)*(
15*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b
*c - a*d)^3) + (6*Sqrt[2]*d^(11/4)*(-15*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) + (3*Sqrt[2]*b^(11/4)*(-7
*b*c + 15*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(a^(11/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*b^(11/4)*(-7*b*c + 15*a*d)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^
3) + (3*Sqrt[2]*d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^3) + (3*Sqrt[2]*d^(11/4)*(
15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(c^(11/4)*(-(b*c) + a*d)^3))/48
```

Maple [A] time = 0.025, size = 825, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^4/c^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3*x^(1/2)/
(d*x^2+c)*b-7/8*d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)*a+15/8*d^3/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-7/8*d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+15/8*d^3/c^2/(a*d-b*c)^3*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^4/c^3/(a*d-b*c)^3*
(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a+15/16*d^3/c^2/(a*d-b*c)^3*(c/d)^(1/4
)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))*b-2/3/a^2/c^2/x^(3/2)-1/2*b^3/a/(a*d-b*c)^3*x^(1
/2)/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*c-15/8*b^3/a^2/(a
*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d+7
/8*b^4/a^3/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
^(1/2)+1)*c-15/8*b^3/a^2/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x^(1/2)-1)*d+7/8*b^4/a^3/(a*d-b*c)^3*(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c-15/16*b^3/a^2/(a*d-b*c)^3*(1/b*a)^(1
/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(
1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d+7/16*b^4/a^3/(a*d-b*c)^3*(1/b*a)^(1/
4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1
/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.495 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=731

$$-\frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2}+\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}(bc-ad)^2}+\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^1}{}$$

```
[Out] -(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(10*a^2*c^2*(b*c - a*d)^2*x^(5/2)) + (
(b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*
Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) + b/(2
*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c -
 a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*
d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)
*(b*c - a*d)^3) - (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13/4
)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b
*c - a*d)^3)
```

Rubi [A] time = 1.27615, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2}+\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}(bc-ad)^2}+\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^1}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(10*a^2*c^2*(b*c - a*d)^2*x^(5/2)) + (
(b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*
Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) + b/(2
*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c -
 a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*
d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)
*(b*c - a*d)^3) - (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13/4
)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b
```

```
*c - a*d)^3)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-13bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4(9b^{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)(\cdots}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad}{2ac(bc-ad)^2x^{5/2}}$$

Mathematica [A] time = 1.42183, size = 630, normalized size = 0.86

$$\frac{1}{80}\left(\frac{40b^4x^{3/2}}{a^3\left(a+bx^2\right)(bc-ad)^2} + \frac{5\sqrt{2}b^{13/4}(17ad-9bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}(ad-bc)^3} + \frac{5\sqrt{2}b^{13/4}(17ad-9bc)\log\left(\sqrt{2}\cdots\right.}{a^{13/4}(bc-\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*x^(5/2)) + (320*(b*c + a*d))/(a^3*c^3*Sqrt[x]) + (40*b^4*x^(3
/2))/(a^3*(b*c - a*d)^2*(a + b*x^2)) + (40*d^4*x^(3/2))/(c^3*(b*c - a*d)^2*
(c + d*x^2)) + (10*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)*(b*c - a*d)^3) + (10*Sqrt[2]*b^(13/4)*(
```

```
-9*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)*(
-(b*c) + a*d)^3) + (10*Sqrt[2]*d^(13/4)*(-17*b*c + 9*a*d)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^3) + (10*Sqrt[2]*d^(13/
4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4
)*(b*c - a*d)^3) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(13/4)*(-(b*c) + a*d)^3) + (5*
Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(a^(13/4)*(b*c - a*d)^3) + (5*Sqrt[2]*d^(13/4)*(17*b*c
- 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13
/4)*(b*c - a*d)^3) + (5*Sqrt[2]*d^(13/4)*(17*b*c - 9*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(-(b*c) + a*d)^3))/80
```

Maple [A] time = 0.027, size = 849, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^5/c^3/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a-1/2*d^4/c^2/(a*d-b*c)^3*x^(3/2)
/(d*x^2+c)*b+9/16*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)+9/8*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)+9/8*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)-1)-17/16*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))-17/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)-17/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a^2/c^2/x^(5/2)+4/a^2/c^3/x^(1/2)*d+4/a^3
/c^2/x^(1/2)*b+1/2*b^4/a^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2*b^5/a^3/(a*d
-b*c)^3*x^(3/2)/(b*x^2+a)*c+17/16*b^3/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)
*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2)))+17/8*b^3/a^2/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*
d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+17/8*b^3/a^2/(a*d-b*c)^3/(1/b*a)^
(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-9/16*b^4/a^3/(a*d-b
*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(
1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))-9/8*b^4/a^3/(a*d-b*c
)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-9/8*b^4
/a^3/(a*d-b*c)^3/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/
2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.496 $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\right.}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}$$

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sqrt[x])/(2*
b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17*a*d)*Sqrt[x])/(16*(
b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/
4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*(b*c - a*d)^4) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) +
((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + (a^(1/4)*b^(3/4)*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + ((21*b^2
*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.04155, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\right.}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sqrt[x])/(2*
b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17*a*d)*Sqrt[x])/(16*(
b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/
4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*(b*c - a*d)^4) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) +
((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + (a^(1/4)*b^(3/4)*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + ((21*b^2
*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-7ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{12abc^2-28bc(bc+2ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2} \ldots\right.}{16bc(bc-ad)^2}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} - \ldots$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} - \ldots$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} - \ldots$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} - \ldots$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} + \ldots$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)} + \ldots$$

Mathematica [A] time = 1.33243, size = 604, normalized size = 0.84

$$\frac{-\frac{\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}+\frac{\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}-\frac{2\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\mathrm{ta}}{c^{3/4}\sqrt[4]{d}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((64*a*b*(b*c - a*d)*Sqrt[x])/(a + b*x^2) + (32*c*(b*c - a*d)^2*Sqrt[x])/(c
 + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c + 9*a*d)*Sqrt[x])/(c + d*x^2) + 16*Sqrt
[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] - 16*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*d^(1/4)) + (2*Sqrt[2
]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(3/4)*d^(1/4)) + 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 8*Sqrt[2]*a^(1/
4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x] - (Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)) + (Sqrt[2]*(
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)))/(128*(b*c - a*d)^4)
```

Maple [A] time = 0.023, size = 1066, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -9/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a^2*d^3+1/8/(a*d-b*c)^4/(d*x^2+c)^2*x
^(5/2)*a*b*c*d^2+7/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*b^2*c^2*d-5/16/(a*d-b
*c)^4/(d*x^2+c)^2*x^(1/2)*a^2*c*d^2-3/8/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b
*c^2*d+11/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*b^2*c^3+5/64/(a*d-b*c)^4*(c/d)
^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2+35/32/(a*d-b
*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d+21/64
/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^
2+5/64/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)*a^2*d^2+35/32/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)-1)*a*b*d+21/64/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*a^2*d^2+35/64/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b*
d+21/128/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-1/2*a^2*b/(
a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d+1/2*a*b^2/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*c-7/
8*a*b/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2
)+1)*d-5/8*b^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x^(1/2)+1)*c-7/8*a*b/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x^(1/2)-1)*d-5/8*b^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c-7/16*a*b/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2
)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/
```

```
2)*2^(1/2)+(1/b*a)^(1/2)))*d-5/16*b^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*ln(
(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^
(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.48062, size = 1611, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*
c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4
) - 1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*
b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4
*d^4) + 1/32*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
```

```
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c
^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/32*(21*(c*d^3)^(1
/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^5*
d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c
^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*
d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sq
rt(2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqr
t(2)*a^4*d^4) + 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*c
^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4)
 + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1
/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*
c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3
*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^
3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)
*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/2*a*b*s
qrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)) +
1/16*(7*b*c*d*x^(5/2) + 9*a*d^2*x^(5/2) + 11*b*c^2*sqrt(x) + 5*a*c*d*sqrt(x
))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^2)
```

3.497 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}c^{5/4}(bc-ad)^4}$$

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(16*c*(b*c - a*d)^3*(
c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^
4) + (3*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15
*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c
 - a*d)^4) - (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b
^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(15*b^2*c^2
 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.02006, antiderivative size = 703, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}c^{5/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(16*c*(b*c - a*d)^3*(
c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^
4) + (3*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15
*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c
 - a*d)^4) - (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b
^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(15*b^2*c^2
 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c-9dx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(12c(2bc+ad)-60bcdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2} \cdots\right.}{16c(bc-ad)^2}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} + \text{Sub}\cdots$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} + \text{Sub}\cdots$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{(3b\cdots}{\cdots}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} - \frac{(3b\cdots}{\cdots}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{(3b\cdots}{\cdots}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{3b^{5/\cdots}}{\cdots}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right)} - \frac{3b^{5/\cdots}}{\cdots}$$

Mathematica [A] time = 1.82837, size = 604, normalized size = 0.86

$$\frac{\frac{3\sqrt{2}\sqrt[4]{d}\left(a^2d^2-18abcd-15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{3\sqrt{2}\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{6\sqrt{2}\sqrt[4]{d}\left(-a^2d^2+18abcd+\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*x^(3/2))/(a + b*x^2) - (32*d*(b*c - a*d)^2*x^(3/2))/(
c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(13*b*c + 3*a*d)*x^(3/2))/(c*(c + d*x^2)
) - (48*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/a^(1/4) + (48*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (6*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b
*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (6
*Sqrt[2]*d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4) - (24*Sqrt[2
]*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/a^(1/4) + (3*Sqrt[2]*d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sq
rt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(128*(b*c - a*d)^4)
```

Maple [A] time = 0.023, size = 1067, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(7/2)*a^2+5/8*d^3/(a*d-b*c)^4/(d*x^2+c
)^2*x^(7/2)*a*b-13/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(7/2)*b^2-1/16*d^3/(a
*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+9/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*c
*a*b-17/16*d/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64*d^2/(a*d-b*c)^4/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-27/32*d/(a*d
-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64
/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^
2+3/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)-1)*a^2-27/32*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b-45/64/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*b^2+3/128*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*ln((x
-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2)))*a^2-27/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*
b-45/128/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/2*b^2/(a*
d-b*c)^4*x^(3/2)/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c+9/16
*b/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*a*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2
)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+9/8*b/(a*
d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+
9/8*b/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^
(1/2)-1)+3/16*b^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*c*ln((x-(1/b*a)^(1/4)*x
^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2
```

```
)))+3/8*b^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4
)*x^(1/2)+1)+3/8*b^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/
b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)

[Out] Timed out
```

Giac [B] time = 2.66757, size = 1671, normalized size = 2.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")

[Out] -1/2*b^2*x^(3/2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^
2 + a)) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(
2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sq
rt(2)*a^5*b*d^4) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4
 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*
```

```
c*d^3 + sqrt(2)*a^5*b*d^4) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3
/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3
+ 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6
) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/
4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*
d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/8*((a*b^3)^(3/4)*b
*c + 3*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 -
4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/8*((a*b^3)^(3/4)*b*c + 3*(
a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2
)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(
2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18
*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sq
rt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/
64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2
*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^
2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c
^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 1/16*(13*b*c*d^2*x^(7/2) + 3*a*d^3*x^(7/2)
+ 17*b*c^2*d*x^(3/2) - a*c*d^2*x^(3/2))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

3.498 $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=703

$$\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}c^{7/4}(bc-ad)^4}$$

```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(16*c*(b*c - a*d)^3*(c
 + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4)
+ (d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a
*d)^4) + (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2
+ 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*c^2 + 22*a
*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.00542, antiderivative size = 703, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}c^{7/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(16*c*(b*c - a*d)^3*(c
 + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4)
+ (d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a
*d)^4) + (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2
+ 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*c^2 + 22*a
*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{c-11dx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx,x,\sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{4c(2bc+ad)-84bcdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx,\right.}{16c(bc-ad)^2}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} + \text{Sub}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{\left(b^2\right.}{}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{\left(b^2\right.}{}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} + \frac{\left(b^{3/}\right.}{}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} - \frac{b^{7/4}}{}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)} - \frac{b^{7/4}}{}$$

Mathematica [A] time = 1.77013, size = 603, normalized size = 0.86

$$\frac{\sqrt{2}d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} + \frac{\sqrt{2}d^{3/4}\left(3a^2d^2-22abcd-77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} + \frac{2\sqrt{2}d^{3/4}\left(-3a^2d^2+22abcd+7\right.}{c^{7/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*Sqrt[x])/(a + b*x^2) - (32*d*(b*c - a*d)^2*Sqrt[x])/(
c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(15*b*c + a*d)*Sqrt[x])/(c*(c + d*x^2))
- (16*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/a^(3/4) + (16*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) -
(2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (8*Sqrt
[2]*b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4) + (Sq
rt[2]*d^(3/4)*(-77*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(128*(b*c - a*d)^4)
```

Maple [A] time = 0.023, size = 1094, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(5/2)*a^2+7/8*d^3/(a*d-b*c)^4/(d*x^2+c
)^2*x^(5/2)*a*b-15/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(5/2)*b^2+11/8*d^2/(a
*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*c*a*b-19/16*d/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)
*b^2*c^2-3/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a^2+3/64*d^3/(a*d-b*c)^4/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-11/32*d^2
/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*
b-77/64*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)*b^2+3/64*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*a^2-11/32*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4
)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-11/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*
ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2)))*a*b-77/128*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))*b^2+1/2*b^2/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x^(1/
2)/(b*x^2+a)*c+11/8*b^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x^(1/2)+1)*d+1/8*b^3/(a*d-b*c)^4*(1/b*a)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c+11/8*b^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d+1/8*b^3/(a*d-b*c)^4*(1/b*a)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c+11/16*b^2/(a*d-b*c
)^4*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)
)/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d+1/16*b^3/(a*d-b*c)^4*(
1/b*a)^(1/4)/a*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(
x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.47858, size = 1643, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^
2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a
^5*d^4) + 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4
*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3
+ sqrt(2)*a^5*d^4) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*
c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*
a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) - 1/32*(77
*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*
```

```
a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3
)^(1/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*
b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a
^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)
*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^4*c^
4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c
*d^3 + sqrt(2)*a^5*d^4) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)
*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*
d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/64*(77*(c*d^3)^(1/
4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*
c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c
^2*d^4) - 1/2*b^2*sqrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*(b*x^2 + a)) - 1/16*(15*b*c*d^2*x^(5/2) + a*d^3*x^(5/2) + 19*b*c^2*d*sq
rt(x) - 3*a*c*d^2*sqrt(x))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^
3*c*d^3)*(d*x^2 + c)^2)
```

3.499 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=739

$$\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26a\right.}{}$$

```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(3/2))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 21*a*b*c*d -
 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (b^(9/4)*(b*c -
 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*
(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sq
rt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c
 - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 1
3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)
*(b*c - a*d)^4)
```

Rubi [A] time = 1.15852, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(3/2))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 21*a*b*c*d -
 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (b^(9/4)*(b*c -
 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*
(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sq
rt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c
 - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 1
3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)
```

```
*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad-9bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-4\left(2b^2c^2-16ab\right.\right.}{\left(a+\right.}\right.}{16a}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd-5a^2\right.}{16ac^2(bc-ad)^3\left(c+\right.}$$

Mathematica [A] time = 2.05839, size = 691, normalized size = 0.94

$$\frac{1}{128}\left(\frac{\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^4} - \frac{\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\right.}{c^{9/4}(bc-ad)^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^3*x^(3/2))/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (32*d^2*x^(3/2))/(c*(
b*c - a*d)^2*(c + d*x^2)^2) + (8*d^2*(21*b*c - 5*a*d)*x^(3/2))/(c^2*(b*c -
a*d)^3*(c + d*x^2)) + (16*Sqrt[2]*b^(9/4)*(-(b*c) + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(b*c - a*d)^4) + (16*Sqrt[2]*b^(9/
4)*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(
b*c - a*d)^4) - (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*A
```

```
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^4) + (2*
Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(9/4)*(b*
c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^
(5/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(9/4)*(-(b*c) + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^4) + (Sqr
t[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^4) - (Sqrt[2]*d^(
5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^4))/128
```

Maple [A] time = 0.023, size = 1100, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 5/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(7/2)*a^2-13/8*d^4/(a*d-b*c)^4/(d*x^
2+c)^2/c*x^(7/2)*a*b+21/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*b^2+9/16*d^4
/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(3/2)*a^2-17/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(
3/2)*a*b+25/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(3/2)*b^2+5/64*d^3/(a*d-b*c)
^4/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-13/32*
d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
*a*b+117/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*b^2+5/128*d^3/(a*d-b*c)^4/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
*a^2-13/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+117/1
28*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+5/64*d^3/(a*d-b*c
)^4/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-13/32
*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)*a*b+117/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)*b^2-1/2*b^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a
*x^(3/2)/(b*x^2+a)*c-13/8*b^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d+1/8*b^3/(a*d-b*c)^4/a/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c-13/8*b^2/(a*d-b*c)^4/(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*d+1/8*b^3/(a*d-b*c)^4/a/(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c-13/16*b^2/(a
*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)
^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d+1/16*b^3/(a*d-b*
c)^4/a/(1/b*a)^(1/4)*2^(1/2)*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1
/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.65982, size = 1665, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^(3/2)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^4 - 4
*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3
+ sqrt(2)*a^6*d^4) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^
4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5
*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^
(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^
2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d
^5) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c
^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) - 1/8*((a*b^3)^(3/4
)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*
d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/8*((a*b^3)^(3/4)*b*c - 1
3*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*
sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^2*c^2 -
26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/
```

```
d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*
sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) +
1/64*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c
^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*
b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) + 1/16*(21*b*c*d^3*x^(7/2) - 5*a*d^4*x^(7/
2) + 25*b*c^2*d^2*x^(3/2) - 9*a*c*d^3*x^(3/2))/((b^3*c^5 - 3*a*b^2*c^4*d +
3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.500 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=739

$$\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-\right.}{}$$

```
[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*Sqrt[x])
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 23*a*b*c*d -
 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(11/4)*(b*
c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4
)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2
 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11
/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(1
1/4)*(b*c - 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rubi [A] time = 0.977757, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*Sqrt[x])
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 23*a*b*c*d -
 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(11/4)*(b*
c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4
)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2
 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11
/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(1
1/4)*(b*c - 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
```

```
Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-11bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-4\left(6b^2c^2-16a\cdots\right.}{\cdots}\right.}{16\cdots}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23abcd-\cdots\right.}{16ac^2(bc-ad)^3\left(c\cdots\right.}$$

Mathematica [A] time = 2.0101, size = 692, normalized size = 0.94

$$\frac{1}{128}\left(-\frac{3\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^4}+\frac{3\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{\right.}}{c^{11/4}(bc-ad)^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^3*Sqrt[x])/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (32*d^2*Sqrt[x])/(c*(
b*c - a*d)^2*(c + d*x^2)^2) + (8*d^2*(23*b*c - 7*a*d)*Sqrt[x])/(c^2*(b*c -
a*d)^3*(c + d*x^2)) + (48*Sqrt[2]*b^(11/4)*(-(b*c) + 5*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(b*c - a*d)^4) + (48*Sqrt[2]*b^(11
/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(
b*c - a*d)^4) - (6*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Ar
cTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^4) + (6*
Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(11/4)*(
-(b*c) + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])
/(a^(7/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^4) - (3
*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^4) + (3*Sqrt[
2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^4))/128
```

Maple [A] time = 0.022, size = 1124, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 7/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(5/2)*a^2-15/8*d^4/(a*d-b*c)^4/(d*x^
2+c)^2/c*x^(5/2)*a*b+23/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*b^2+11/16*d^
4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(1/2)*a^2-19/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^
(1/2)*a*b+27/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(1/2)*b^2+21/64*d^4/(a*d-b*
c)^4/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/3
2*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*a*b+165/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*b^2+21/128*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))*a^2-45/64*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)*a*b+165/128*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+21
/64*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)*a^2-45/32*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)*a*b+165/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2*b^3/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d
+1/2*b^4/(a*d-b*c)^4/a*x^(1/2)/(b*x^2+a)*c-15/8*b^3/(a*d-b*c)^4/a*(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d+3/8*b^4/(a*d-b*c)^4/
a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c-15/8*b^
```

```
3/(a*d-b*c)^4/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-
1)*d+3/8*b^4/(a*d-b*c)^4/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x^(1/2)-1)*c-15/16*b^3/(a*d-b*c)^4/a*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*
a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1
/b*a)^(1/2)))*d+3/16*b^4/(a*d-b*c)^4/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x+(1/b*a
)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/
b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.20438, size = 1692, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*sqrt(x)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^
```

```
4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c
*d^3 + sqrt(2)*a^6*d^4) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*
d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*s
qrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d -
30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a
*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*
a^4*c^3*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^
2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a
^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 3/8*((a*b
^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)
*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 3/8*((a*b^3)^
(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^
4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/64*(55*(c*d^3)
^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*lo
g(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)
*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2
)*a^4*c^3*d^4) - 3/64*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*
d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2
- 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 1/16*(23*b*c*d^3*x^(5/2)
 - 7*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 11*a*c*d^3*sqrt(x))/((b^3*c^5 -
 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.501 $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{}\right.}{8\sqrt{2}a^{9/4}(bc-ad)^4}$$

```
[Out] -(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(16*a^2*c^3*(
b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*Sqrt[x]*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d*x
^2)) + (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4)
+ (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (d^(9/4)*(221
*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/
4)*(b*c - a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) + (d^(9/4)*
(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.3996, antiderivative size = 805, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{}\right.}{8\sqrt{2}a^{9/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(16*a^2*c^3*(
b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*Sqrt[x]*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d*x
^2)) + (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4)
+ (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (d^(9/4)*(221
*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/
4)*(b*c - a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) + (d^(9/4)*
```

```
(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-13bdx^4}{x^2(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-4(10b^2c}{}\right.}{}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+25abc\right.}{16ac^2(bc-ad)^3\sqrt{}}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{}{2a(bc-a}$$

Mathematica [A] time = 2.24716, size = 706, normalized size = 0.88

$$\frac{1}{128}\left(-\frac{\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^4} + \frac{\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\text{lo}}{c^{13/4}(bc-ad}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*Sqrt[x]) + (64*b^4*x^(3/2))/(a^2*(-(b*c) + a*d)^3*(a + b*x^2
)) - (32*d^3*x^(3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (8*d^3*(-29*b*c +
```

```
 13*a*d)*x^(3/2))/(c^3*(b*c - a*d)^3*(c + d*x^2)) + (16*Sqrt[2]*b^(13/4)*(5
*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c
 - a*d)^4) + (16*Sqrt[2]*b^(13/4)*(-5*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^4) + (2*Sqrt[2]*d^(9/4)*(221*b
^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(c^(13/4)*(b*c - a*d)^4) - (2*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170*a*b
*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)
*(b*c - a*d)^4) + (8*Sqrt[2]*b^(13/4)*(-5*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^4) + (8*Sqrt[
2]*b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^4) - (Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170
*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(c^(13/4)*(b*c - a*d)^4) + (Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170*a*b*
c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(c^(13/4)*(b*c - a*d)^4))/128
```

Maple [A] time = 0.03, size = 1143, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -13/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a^2+21/8*d^5/c^2/(a*d-b*c)^4
/(d*x^2+c)^2*x^(7/2)*a*b-29/16*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*b^2-17
/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+25/8*d^4/c/(a*d-b*c)^4/(d*x
^2+c)^2*x^(3/2)*a*b-33/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*b^2-45/128*d^
4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-45/64*d^4/c^3/(a
*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-45/
64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)+85/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)+85/32*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)+85/32*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)-221/128*d^2/c/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b
^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))-221/64*d^2/c/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-221/64*d^2/c/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2
)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a^2/c^3/x^(1/2)+1/2*b^4/a/(a*
d-b*c)^4*x^(3/2)/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c+17
/16*b^3/a/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*d*ln((x-(1/b*a)^(1/4)*x^(1/2)*2
^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+17/8
*b^3/a/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(
1/2)+1)+17/8*b^3/a/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*
a)^(1/4)*x^(1/2)-1)-5/16*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*c*ln((x-
(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/
2)+(1/b*a)^(1/2)))-5/8*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2
^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)-5/8*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/
2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.43811, size = 1800, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(
2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt
(2)*a^7*d^4) - 1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^
4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6
*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)
^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d
+ 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4
) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d
^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/8*(5*(a*b^3)^(3/4)*
b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^
2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/8*(5*(a*b^3)^(3/4)*b^2
```

```
*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*
d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) + 1/64*(221*(c*d^3)^(3/4)*b^
2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*
d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d
^4) - 1/64*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d
^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(
2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*
a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/2*(5*b^4*c^3*x^2 - 12*a*b^3*c^2*d*
x^2 + 12*a^2*b^2*c*d^2*x^2 - 4*a^3*b*d^3*x^2 + 4*a*b^3*c^3 - 12*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 - 4*a^4*d^3)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*
c^4*d^2 - a^5*c^3*d^3)*(b*x^(5/2) + a*sqrt(x))) - 1/16*(29*b*c*d^4*x^(7/2)
- 13*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 17*a*c*d^4*x^(3/2))/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

3.502 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{11/4}(bc-ad)^4}$$

```
[Out] -(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(48*a^2*c^3*(
b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(3/2)*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c + d*
x^2)) + (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^
4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/4)
*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*
a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) +
(d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (
d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.32189, antiderivative size = 805, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}a^{11/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] -(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(48*a^2*c^3*(
b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(3/2)*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c + d*
x^2)) + (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^
4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/4)
*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*
a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) +
(d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (
```

```
d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-7bc+4ad-15bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-4(}{}\right.}{}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+2\right.}{16ac^2(bc-a}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

Mathematica [A] time = 2.12416, size = 707, normalized size = 0.88

$$\frac{1}{384}\left(\frac{3\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(bc-ad)^4} - \frac{3\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b\right.}{c^{15/4}(bc}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*x^(3/2)) + (192*b^4*Sqrt[x])/(a^2*(-(b*c) + a*d)^3*(a + b*x^
2)) - (96*d^3*Sqrt[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (24*d^3*(-31*b*c
 + 15*a*d)*Sqrt[x])/(c^3*(b*c - a*d)^3*(c + d*x^2)) + (48*Sqrt[2]*b^(15/4)*
(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*(
b*c - a*d)^4) + (48*Sqrt[2]*b^(15/4)*(-7*b*c + 19*a*d)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*(b*c - a*d)^4) + (6*Sqrt[2]*d^(11/4)*(
```

```
285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)]])/(c^(15/4)*(b*c - a*d)^4) - (6*Sqrt[2]*d^(11/4)*(285*b^2*c^2 - 2
66*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]])/(c^
(15/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^4) + (
24*Sqrt[2]*b^(15/4)*(-7*b*c + 19*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^4) + (3*Sqrt[2]*d^(11/4)*(285*
b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(c^(15/4)*(b*c - a*d)^4) - (3*Sqrt[2]*d^(11/4)*(285*b^
2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(c^(15/4)*(b*c - a*d)^4))/384
```

Maple [A] time = 0.027, size = 1143, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -15/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a^2+23/8*d^5/c^2/(a*d-b*c)^4
/(d*x^2+c)^2*x^(5/2)*a*b-31/16*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*b^2-19
/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a^2+27/8*d^4/c/(a*d-b*c)^4/(d*x
^2+c)^2*x^(1/2)*a*b-35/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*b^2-77/64*d^5
/c^4/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a^2+133/32*d^4/c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*a*b-285/64*d^3/c^2/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64*d^5/c^4/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+133/32*d^4/c^3/(a*d-b*c)^4*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-285/64*d^3/c^2/(
a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77
/128*d^5/c^4/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+133/64*d^
4/c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-285/128*d^3/c^2/
(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3/a^2/c^3/x^(3/2)+1/
2*b^4/a/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(1/2)/(b*
x^2+a)*c+19/8*b^4/a^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b
*a)^(1/4)*x^(1/2)-1)*d-7/8*b^5/a^3/(a*d-b*c)^4*(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)*c+19/16*b^4/a^2/(a*d-b*c)^4*(1/b*a)^(1/4)
*2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4
)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*d-7/16*b^5/a^3/(a*d-b*c)^4*(1/b*a)^(1/4)*
2^(1/2)*ln((x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x-(1/b*a)^(1/4)
*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))*c+19/8*b^4/a^2/(a*d-b*c)^4*(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*d-7/8*b^5/a^3/(a*d-b*c)^4*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.24992, size = 1725, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^4*sqrt(x)/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)
*(b*x^2 + a)) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*
b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a
^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/
4)*a*b^3*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*
d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^
2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8
 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*
d^3 + sqrt(2)*a^4*c^4*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d
^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*
c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c
^4*d^4) - 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4
*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^
7*d^4) + 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4
*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^
7*d^4) - 1/64*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3
```

```
+ 77*(c*d^3)^(1/4)*a^2*d^4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*
sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/64*(285*(c*d^3)^(1/4)*b^2*
c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*log(-sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3
*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*
c^4*d^4) - 1/16*(31*b*c*d^4*x^(5/2) - 15*a*d^5*x^(5/2) + 35*b*c^2*d^3*sqrt(
x) - 19*a*c*d^4*sqrt(x))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*
c^3*d^3)*(d*x^2 + c)^2) - 2/3/(a^2*c^3*x^(3/2))
```

3.503 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=881

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}a^{13/4}(bc-ad)^4}$$

```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/(80*a^2*c^
3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^3*d - 32*a^2*b^2*c^2
*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*c^4*(b*c - a*d)^3*Sqrt[x]) + (
d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c
- a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13
*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*
c - 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/
4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b
^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126
*a*b*c*d + 39*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*
d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*
b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119
*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rubi [A] time = 1.69109, antiderivative size = 881, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}a^{13/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```


```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/(80*a^2*c^
3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^3*d - 32*a^2*b^2*c^2
*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*c^4*(b*c - a*d)^3*Sqrt[x]) + (
d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c
- a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13
*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*
c - 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/
4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b
^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126
*a*b*c*d + 39*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*
```

```
d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*
b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119
*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-17bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-4(18b^2}{}\right.}{}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+29ab\right.}{16ac^2(bc-ad)^3x}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{}{2a(bc-}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\left(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+\right.}{16a^3c^4(bc-ad)^3\sqrt{}}$$

Mathematica [A] time = 2.20188, size = 729, normalized size = 0.83

$$\frac{1}{640}\left(\frac{15\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(bc-ad)^4} - \frac{15\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c\right.}{c^{17/4}(bc-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*x^(5/2)) + (1280*(2*b*c + 3*a*d))/(a^3*c^4*Sqrt[x]) - (320*b
^5*x^(3/2))/(a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (160*d^4*x^(3/2))/(c^3*(b*
c - a*d)^2*(c + d*x^2)^2) + (40*d^4*(37*b*c - 21*a*d)*x^(3/2))/(c^4*(b*c -
a*d)^3*(c + d*x^2)) + (240*Sqrt[2]*b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)*(b*c - a*d)^4) + (240*Sqrt[2]*b^
(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1
3/4)*(b*c - a*d)^4) - (30*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*
a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*
d)^4) + (30*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*d)^4) + (120*S
qrt[2]*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(a^(13/4)*(b*c - a*d)^4) + (120*Sqrt[2]*b^(17/4)*(-3*b*c +
 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(13/
4)*(b*c - a*d)^4) + (15*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(17/4
)*(b*c - a*d)^4) - (15*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(17/4)
*(b*c - a*d)^4))/640
```

Maple [A] time = 0.034, size = 1170, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -21/16*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*d*ln((x-(1/b*a)^(1/4)*x^(1
/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))
-21/8*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(1/b*a)^(1
/4)*x^(1/2)+1)+6/a^2/c^4/x^(1/2)*d+4/a^3/c^3/x^(1/2)*b-2/5/a^2/c^3/x^(5/2)+
21/16*d^7/c^4/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a^2+37/16*d^5/c^2/(a*d-b*c)^4
/(d*x^2+c)^2*x^(7/2)*b^2+25/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+
41/16*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*b^2-1/2*b^5/a^2/(a*d-b*c)^4*x^(
3/2)/(b*x^2+a)*d+1/2*b^6/a^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c+357/128*d^3/c^
2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+357/64*d^3/c^2/(a*d-
b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+357/64
*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)-1)-189/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
-189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)-189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)-21/8*b^4/a^2/(a*d-b*c)^4/(1/b*a)^(1/4)*2^(1/2)
*d*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)+9/16*b^5/a^3/(a*d-b*c)^4/(1/b*a)
^(1/4)*2^(1/2)*c*ln((x-(1/b*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2))/(x+(1/b
*a)^(1/4)*x^(1/2)*2^(1/2)+(1/b*a)^(1/2)))+9/8*b^5/a^3/(a*d-b*c)^4/(1/b*a)^(
1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)+1)+9/8*b^5/a^3/(a*d-b*c
)^4/(1/b*a)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(1/b*a)^(1/4)*x^(1/2)-1)-29/8*d^
6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a*b-33/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c
)^2*x^(3/2)*a*b+117/128*d^5/c^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))+117/64*d^5/c^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)+117/64*d^5/c^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.71873, size = 1740, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^(3/2)/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
(b*x^2 + a)) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^
4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7
*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*
a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2
 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c
^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*
sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 +
 sqrt(2)*a^4*c^5*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/
4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
```

```
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d +
 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4)
 - 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3
*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) +
 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*
d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) -
3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3
)^(3/4)*a^2*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3
*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126
*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sq
rt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 1/
16*(37*b*c*d^5*x^(7/2) - 21*a*d^6*x^(7/2) + 41*b*c^2*d^4*x^(3/2) - 25*a*c*d
^5*x^(3/2))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x
^2 + c)^2) + 2/5*(10*b*c*x^2 + 15*a*d*x^2 - a*c)/(a^3*c^4*x^(5/2))
```

3.504 $\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7*b^4) + (B*(a + b*x^
2)^(9/2))/(9*b^4)
```

Rubi [A] time = 0.0873286, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7*b^4) + (B*(a + b*x^
2)^(9/2))/(9*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)dx=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}(A+Bx)\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(Ab-3aB)(a+bx}{b^3}\right.\right.$$

$$=\frac{a^2(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{B\left(a\right.}{}$$

Mathematica [A] time = 0.055869, size = 75, normalized size = 0.73

$$\frac{\left(a+bx^2\right)^{3/2}\left(24a^2b\left(A+Bx^2\right)-16a^3B-6ab^2x^2\left(6A+5Bx^2\right)+5b^3x^4\left(9A+7Bx^2\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^2) - 6*a*b^2*x^2*(6*A + 5
*B*x^2) + 5*b^3*x^4*(9*A + 7*B*x^2)))/(315*b^4)
```

Maple [A] time = 0.006, size = 77, normalized size = 0.8

$$\frac{35\,Bx^6b^3+45\,Ab^3x^4-30\,Bab^2x^4-36\,Aab^2x^2+24\,Ba^2bx^2+24\,Aa^2b-16\,Ba^3}{315\,b^4}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(3/2)*(35*B*b^3*x^6+45*A*b^3*x^4-30*B*a*b^2*x^4-36*A*a*b^2*
x^2+24*B*a^2*b*x^2+24*A*a^2*b-16*B*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28993, size = 219, normalized size = 2.13

$$\frac{\left(35\,Bb^4x^8+5\left(Bab^3+9\,Ab^4\right)x^6-16\,Ba^4+24\,Aa^3b-3\left(2\,Ba^2b^2-3\,Aab^3\right)x^4+4\left(2\,Ba^3b-3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(B*a*b^3 + 9*A*b^4)*x^6 - 16*B*a^4 + 24*A*a^3*b - 3
*(2*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^2)*sqrt(b*x^
2 + a)/b^4
```

Sympy [A] time = 1.44617, size = 212, normalized size = 2.06

$$\begin{cases} \frac{8Aa^3\sqrt{a+bx^2}}{105b^3} - \frac{4Aa^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{Aax^4\sqrt{a+bx^2}}{35b} + \frac{Ax^6\sqrt{a+bx^2}}{7} - \frac{16Ba^4\sqrt{a+bx^2}}{315b^4} + \frac{8Ba^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2Ba^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{Bax^6\sqrt{a+bx^2}}{63b} + \frac{Bx^8\sqrt{a+bx^2}}{9} \\ \sqrt{a}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*A*a**2*x**2*sqrt(a + b*
x**2)/(105*b**2) + A*a*x**4*sqrt(a + b*x**2)/(35*b) + A*x**6*sqrt(a + b*x**
2)/7 - 16*B*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*B*a**3*x**2*sqrt(a + b*x**
2)/(315*b**3) - 2*B*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + B*a*x**6*sqrt(a
 + b*x**2)/(63*b) + B*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*(A*x**6/
6 + B*x**8/8), True))
```

Giac [A] time = 1.11333, size = 144, normalized size = 1.4

$$\frac{\frac{3\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)A}{b^2}+\frac{\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)B}{b^3}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3
/2)*a^2)*A/b^2 + (35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x
^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B/b^3)/b
```

3.505 $\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=155

$$-\frac{a^2x\sqrt{a+bx^2}(8Ab-5aB)}{128b^3}+\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{ax^3\sqrt{a+bx^2}(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}(8Ab-5aB)}{48b}$$

```
[Out] -(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(128*b^3) + (a*(8*A*b - 5*a*B)*x^3
*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt[a + b*x^2])/(48*b)
+ (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rubi [A] time = 0.07237, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^2x\sqrt{a+bx^2}(8Ab-5aB)}{128b^3}+\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{ax^3\sqrt{a+bx^2}(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}(8Ab-5aB)}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(128*b^3) + (a*(8*A*b - 5*a*B)*x^3
*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt[a + b*x^2])/(48*b)
+ (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} - \frac{(-8Ab+5aB)\int x^4\sqrt{a+bx^2}\,dx}{8b}$$

$$= \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} + \frac{(a(8Ab-5aB))\int\frac{x^4}{\sqrt{a+bx^2}}dx}{48b}$$

$$= \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} - \frac{\left(a^2(8Ab-5a\right.}{64}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5}{}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5}{}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5}{}$$

Mathematica [A] time = 0.26824, size = 130, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(-2a^2b\left(12A+5Bx^2\right)+15a^3B+8ab^2x^2\left(2A+Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)-\frac{3a^{5/2}(5aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*B + 8*a*b^2*x^2*(2*A + B*x^2) + 16*b^3*
x^4*(4*A + 3*B*x^2) - 2*a^2*b*(12*A + 5*B*x^2)) - (3*a^(5/2)*(-8*A*b + 5*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(7/2))
```

Maple [A] time = 0.01, size = 181, normalized size = 1.2

$$\frac{Bx^5}{8b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5Bax^3}{48b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5a^2Bx}{64b^3}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5Ba^3x}{128b^3}\sqrt{bx^2+a} - \frac{5Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}} + \frac{Ax^3}{6b}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*B*x^5*(b*x^2+a)^(3/2)/b-5/48*B/b^2*a*x^3*(b*x^2+a)^(3/2)+5/64*B/b^3*a^2
*x*(b*x^2+a)^(3/2)-5/128*B/b^3*a^3*x*(b*x^2+a)^(1/2)-5/128*B/b^(7/2)*a^4*ln
```

```
(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*A*x^3*(b*x^2+a)^(3/2)/b-1/8*A/b^2*a*x*(b*x^
2+a)^(3/2)+1/16*A/b^2*a^2*x*(b*x^2+a)^(1/2)+1/16*A/b^(5/2)*a^3*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66158, size = 587, normalized size = 3.79

$$\left[-\frac{3\left(5\,Ba^4-8\,Aa^3b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Bb^4x^7+8\left(Bab^3+8\,Ab^4\right)x^5-2\left(5\,Ba^2b^2-8\,Aab^3\right.\right.}{768\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^
2 - 8*A*a*b^3)*x^3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1
/384*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^2 - 8*A*a*b^3)*x^
3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] time = 13.5101, size = 286, normalized size = 1.85

$$-\frac{Aa^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5A\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Abx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x^3}{384b^2\sqrt{1+\frac{b}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - A*a**(3/2)*x**3/(48*b*sqrt(1 +
 b*x**2/a)) + 5*A*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(5/2)) + A*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B
*a**(7/2)*x/(128*b**3*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x**3/(384*b**2*sqr
t(1 + b*x**2/a)) - B*a**(3/2)*x**5/(192*b*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)
*x**7/(48*sqrt(1 + b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
7/2)) + B*b*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.1469, size = 178, normalized size = 1.15

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2+\frac{Bab^5+8\,Ab^6}{b^6}\right)x^2-\frac{5\,Ba^2b^4-8\,Aab^5}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^3-8\,Aa^2b^4\right)}{b^6}\right)\sqrt{bx^2+ax}+\frac{\left(5\,Ba^4-8\,Aa^3b\right)\log}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2 + (B*a*b^5 + 8*A*b^6)/b^6)*x^2 - (5*B*a^2*b^4 - 8*A*a*
b^5)/b^6)*x^2 + 3*(5*B*a^3*b^3 - 8*A*a^2*b^4)/b^6)*sqrt(b*x^2 + a)*x + 1/12
8*(5*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.506 $\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(5/
2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rubi [A] time = 0.061788, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(5/
2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx}(A+Bx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)dx,x,x^2\right)\\
&= -\frac{a(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}
\end{aligned}$$

Mathematica [A] time = 0.0396147, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^2\right)+3b^2x^2\left(7A+5Bx^2\right)\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(8*a^2*B + 3*b^2*x^2*(7*A + 5*B*x^2) - 2*a*b*(7*A + 6*B*
x^2)))/(105*b^3)
```

Maple [A] time = 0.006, size = 53, normalized size = 0.7

$$-\frac{-15\,b^2Bx^4-21\,Ab^2x^2+12\,Babx^2+14\,abA-8\,a^2B}{105\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(-15*B*b^2*x^4-21*A*b^2*x^2+12*B*a*b*x^2+14*A*a*b-8*
B*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83288, size = 166, normalized size = 2.27

$$\frac{\left(15\,Bb^3x^6+3\left(Bab^2+7\,Ab^3\right)x^4+8\,Ba^3-14\,Aa^2b-\left(4\,Ba^2b-7\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 + 3*(B*a*b^2 + 7*A*b^3)*x^4 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 0.705362, size = 162, normalized size = 2.22

$$\begin{cases} -\frac{2Aa^2\sqrt{a+bx^2}}{15b^2}+\frac{Aax^2\sqrt{a+bx^2}}{15b}+\frac{Ax^4\sqrt{a+bx^2}}{5}+\frac{8Ba^3\sqrt{a+bx^2}}{105b^3}-\frac{4Ba^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{Bax^4\sqrt{a+bx^2}}{35b}+\frac{Bx^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a**2*sqrt(a + b*x**2)/(15*b**2) + A*a*x**2*sqrt(a + b*x**2)
/(15*b) + A*x**4*sqrt(a + b*x**2)/5 + 8*B*a**3*sqrt(a + b*x**2)/(105*b**3)
- 4*B*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + B*a*x**4*sqrt(a + b*x**2)/(35
*b) + B*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**6/6),
 True))
```

Giac [A] time = 1.14133, size = 107, normalized size = 1.47

$$\frac{\frac{7\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)A}{b}+\frac{\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)B}{b^2}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A/b + (15*(b*x^2 + a
)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*B/b^2)/b
```

3.507 $\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=122

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3*Sqrt[a +
b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*(2*A*b - a*B)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.0591792, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3*Sqrt[a +
b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*(2*A*b - a*B)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{(-6Ab+3aB)\int x^2\sqrt{a+bx^2}\,dx}{6b}$$

$$= \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} + \frac{(a(2Ab-aB))\int \frac{x^2}{\sqrt{a+bx^2}}dx}{8b}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{\left(a^2(2Ab-aB\right)}{16}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{\left(a^2(2Ab-aB\right)}{}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a^2(2Ab-aB)}{16}$$

Mathematica [A] time = 0.210085, size = 108, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(-3a^2B+2ab\left(3A+Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)+\frac{3a^{3/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-3*a^2*B + 2*a*b*(3*A + B*x^2) + 4*b^2*x^2*(3*
A + 2*B*x^2)) + (3*a^(3/2)*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[1 + (b*x^2)/a]))/(48*b^(5/2))
```

Maple [A] time = 0.007, size = 139, normalized size = 1.1

$$\frac{Bx^3}{6\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Bax}{8\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{a^2Bx}{16\,b^2}\sqrt{bx^2+a} + \frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{Ax}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{aAx}{8\,b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/8*B/b^2*a*x*(b*x^2+a)^(3/2)+1/16*B/b^2*a^2*x*
(b*x^2+a)^(1/2)+1/16*B/b^(5/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*A*x*(b
*x^2+a)^(3/2)/b-1/8*A/b*a*x*(b*x^2+a)^(1/2)-1/8*A/b^(3/2)*a^2*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88832, size = 475, normalized size = 3.89

$$\left[-\frac{3\left(Ba^3 - 2\,Aa^2b\right)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) - 2\left(8\,Bb^3x^5 + 2\left(Bab^2 + 6\,Ab^3\right)x^3 - 3\left(Ba^2b - 2\,Aab^2\right)x\right)\sqrt{bx^2}}{96\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 - 3*(B*a^2*b - 2*A*
a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 -
3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] time = 8.62363, size = 226, normalized size = 1.85

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1 + \frac{bx^2}{a}}} + \frac{3A\sqrt{a}x^3}{8\sqrt{1 + \frac{bx^2}{a}}} - \frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{Abx^5}{4\sqrt{a}\sqrt{1 + \frac{bx^2}{a}}} - \frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1 + \frac{bx^2}{a}}} + \frac{5B\sqrt{a}x^5}{24\sqrt{1 + \frac{bx^2}{a}}} + \frac{Ba^3\operatorname{asi}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) - B*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)
*x**3/(48*b*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a))
+ B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(
1 + b*x**2/a))
```

Giac [A] time = 1.13321, size = 135, normalized size = 1.11

$$\frac{1}{48}\left(2\left(4\,Bx^2 + \frac{Bab^3 + 6\,Ab^4}{b^4}\right)x^2 - \frac{3\left(Ba^2b^2 - 2\,Aab^3\right)}{b^4}\right)\sqrt{bx^2 + a}x - \frac{\left(Ba^3 - 2\,Aa^2b\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2 + (B*a*b^3 + 6*A*b^4)/b^4)*x^2 - 3*(B*a^2*b^2 - 2*A*a*b^3)
/b^4)*sqrt(b*x^2 + a)*x - 1/16*(B*a^3 - 2*A*a^2*b)*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(5/2)
```

3.508 $\int x\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(5*b^2)
```

Rubi [A] time = 0.0362037, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(5*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2}\left(A+Bx^2\right)dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{a+bx}(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0225728, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(-2aB+5Ab+3bBx^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

Maple [A] time = 0.003, size = 31, normalized size = 0.7

$$\frac{3\,bBx^{2}+5\,Ab-2\,Ba}{15\,b^{2}}\left(bx^{2}+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(3/2)*(3*B*b*x^2+5*A*b-2*B*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81207, size = 113, normalized size = 2.46

$$\frac{\left(3\,Bb^{2}x^{4}-2\,Ba^{2}+5\,Aab+\left(Bab+5\,Ab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 +
 a)/b^2
```

Sympy [A] time = 0.329264, size = 110, normalized size = 2.39

$$\begin{cases} \frac{Aa\sqrt{a+bx^2}}{3b}+\frac{Ax^2\sqrt{a+bx^2}}{3}-\frac{2Ba^2\sqrt{a+bx^2}}{15b^2}+\frac{Bax^2\sqrt{a+bx^2}}{15b}+\frac{Bx^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise(((A*a*sqrt(a + b*x**2)/(3*b) + A*x**2*sqrt(a + b*x**2)/3 - 2*B*a**
2*sqrt(a + b*x**2)/(15*b**2) + B*a*x**2*sqrt(a + b*x**2)/(15*b) + B*x**4*sq
```

```
rt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*(A*x**2/2 + B*x**4/4), True))
```

Giac [A] time = 1.11125, size = 63, normalized size = 1.37

$$\frac{5\left(bx^2+a\right)^{\frac{3}{2}}A+\frac{\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)B}{b}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(5*(b*x^2 + a)^(3/2)*A + (3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)
*B/b)/b
```

3.509 $\int \sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=87

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.028411, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} - \frac{(-4Ab+aB)\int \sqrt{a+bx^2}\,dx}{4b}$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4Ab-aB))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4Ab-aB))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b}$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

Mathematica [A] time = 0.147091, size = 85, normalized size = 0.98

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(B\left(a+2bx^2\right)+4Ab\right)-\frac{\sqrt{a}(aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(4*A*b + B*(a + 2*b*x^2)) - (Sqrt[a]*(-4*A*b +
a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(8*b^(3/2))
```

Maple [A] time = 0.006, size = 96, normalized size = 1.1

$$\frac{Bx}{4b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Bax}{8b}\sqrt{bx^2+a} - \frac{a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + \frac{Ax}{2}\sqrt{bx^2+a} + \frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*B/b*a*x*(b*x^2+a)^(1/2)-1/8*B/b^(3/2)*a^2*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78891, size = 370, normalized size = 4.25

$$\left[-\frac{\left(Ba^2-4\,Aab\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(2\,Bb^2x^3+\left(Bab+4\,Ab^2\right)x\right)\sqrt{bx^2+a}}{16\,b^2},\ \frac{\left(Ba^2-4\,Aab\right)\sqrt{-b}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*B*b^2*x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/8*((
B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b^2*x^3
 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 5.16794, size = 144, normalized size = 1.66

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x
**2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.12908, size = 93, normalized size = 1.07

$$\frac{1}{8}\left(2\,Bx^2+\frac{Bab+4\,Ab^2}{b^2}\right)\sqrt{bx^2+a}x+\frac{\left(Ba^2-4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*a^2 - 4*A*
a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.510 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=59

$$A\sqrt{a+bx^2}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0442799, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$A\sqrt{a+bx^2}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx, x, x^2\right)\\ &= \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{1}{2}A\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\ &= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{1}{2}(aA)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{(aA)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} - \sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.0331814, size = 59, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\left(B\left(a+bx^2\right)+3Ab\right)}{3b} - \sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b + B*(a + b*x^2)))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

Maple [A] time = 0.008, size = 57, normalized size = 1.

$$\frac{B}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}} - A\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) + A\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b-A*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(
b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58105, size = 300, normalized size = 5.08

$$\left[\frac{3\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(Bbx^2+Ba+3\,Ab\right)\sqrt{bx^2+a}}{6\,b},\ \frac{3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(Bbx^2+Ba+3\,Ab\right)\sqrt{b}}{3\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b, 1/3*(3*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + (B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 15.2528, size = 76, normalized size = 1.29

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^2}\right)}{2}-\frac{B\left(\begin{cases}-\sqrt{a}x^2 & \text{for } b=0\\ -\frac{2\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**2))/2 -
 B*Piecewise((-sqrt(a)*x**2, Eq(b, 0)), (-2*(a + b*x**2)**(3/2)/(3*b), True
))/2
```

Giac [A] time = 1.12738, size = 81, normalized size = 1.37

$$\frac{Aa\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^2+a}Ab^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] A*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*((b*x^2 + a)^(3/2)*B*b^
2 + 3*sqrt(b*x^2 + a)*A*b^3)/b^3
```

3.511 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{x\sqrt{a+bx^2}(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(a*x) + ((2
*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] time = 0.0333191, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{x\sqrt{a+bx^2}(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(a*x) + ((2
*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^2}dx = -\frac{A\left(a+bx^2\right)^{3/2}}{ax} - \frac{(-2Ab-aB)\int \sqrt{a+bx^2}\,dx}{a}$$
$$= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a} - \frac{A\left(a+bx^2\right)^{3/2}}{ax} - \frac{1}{2}(-2Ab-aB)\int \frac{1}{\sqrt{a+bx^2}}dx$$
$$= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a} - \frac{A\left(a+bx^2\right)^{3/2}}{ax} - \frac{1}{2}(-2Ab-aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a} - \frac{A\left(a+bx^2\right)^{3/2}}{ax} + \frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Mathematica [A] time = 0.148331, size = 71, normalized size = 0.85

$$\frac{1}{2}\sqrt{a+bx^2}\left(\frac{(aB+2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} - \frac{2A}{x} + Bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-2*A)/x + B*x + ((2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/2
```

Maple [A] time = 0.008, size = 93, normalized size = 1.1

$$\frac{Bx}{2}\sqrt{bx^2+a} + \frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{A}{ax}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{Abx}{a}\sqrt{bx^2+a} + A\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] 1/2*x*B*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A*(b*
x^2+a)^(3/2)/a/x+A*b/a*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57556, size = 320, normalized size = 3.81

$$\left[\frac{(Ba+2\,Ab)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(Bbx^2-2\,Ab\right)\sqrt{bx^2+a}}{4\,bx},-\frac{(Ba+2\,Ab)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((B*a + 2*A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a))/(b*x), -1/2*((B*a + 2*A*b)*sqrt(-
b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a)
)/(b*x)]
```

Sympy [A] time = 3.4614, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B*a*a
sinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A] time = 1.13485, size = 113, normalized size = 1.35

$$\frac{1}{2}\sqrt{bx^2+a}Bx+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a) - 1/4*(B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

3.512 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*a*x^2) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rubi [A] time = 0.0629452, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^2}(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*a*x^2) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{(Ab+2aB)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)}{4a}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{1}{4}(Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{(Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2b}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0418051, size = 63, normalized size = 0.75

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\left(2Bx^2-A\right)}{x^2}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-A + 2*B*x^2))/x^2 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/Sqrt[a])/2
```

Maple [A] time = 0.009, size = 106, normalized size = 1.3

$$-B\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)+B\sqrt{bx^2+a}-\frac{A}{2ax^2}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Ab}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{Ab}{2a}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2)-1/2*A*(b
*x^2+a)^(3/2)/a/x^2-1/2*A*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+1
/2*A*b/a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6608, size = 332, normalized size = 3.95

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bax^2-Aa\right)\sqrt{bx^2+a}}{4\,ax^2}, \frac{(2\,Ba+Ab)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,Bax^2-\right.}{2\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*a + A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(2*B*a*x^2 - A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*((2*B*a + A*b)
*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x^2 - A*a)*sqrt(b*x
^2 + a))/(a*x^2)]
```

Sympy [A] time = 19.8911, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a/(sqrt(b)*x*sqrt(a/(b*x
**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A] time = 1.11272, size = 92, normalized size = 1.1

$$\frac{2\,\sqrt{bx^2+a}Bb+\frac{\left(2\,Bab+Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}Ab}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b*x^2 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^2 + a)*A*b/x^2)/b
```

3.513 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt[b]*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0249793, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt[b]*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^4}\,dx = -\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3} + B\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{B\sqrt{a+bx^2}}{x} - \frac{A\left(a+bx^2\right)^{3/2}}{3ax^3} + (bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{B\sqrt{a+bx^2}}{x} - \frac{A\left(a+bx^2\right)^{3/2}}{3ax^3} + (bB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{B\sqrt{a+bx^2}}{x} - \frac{A\left(a+bx^2\right)^{3/2}}{3ax^3} + \sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [A] time = 0.165591, size = 81, normalized size = 1.23

$$\frac{\sqrt{a+bx^2}\left(\frac{3\sqrt{a}\sqrt{b}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}} - \frac{aA+3aBx^2+Abx^2}{x^3}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((a*A + A*b*x^2 + 3*a*B*x^2)/x^3) + (3*Sqrt[a]*Sqrt[b]*B
*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(3*a)
```

Maple [A] time = 0.009, size = 75, normalized size = 1.1

$$-\frac{A}{3\,ax^3}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{B}{ax}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{bBx}{a}\sqrt{bx^2+a} + B\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(3/2)/a/x^3-B/a/x*(b*x^2+a)^(3/2)+B*b/a*x*(b*x^2+a)^(1/2)+
B*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5914, size = 331, normalized size = 5.02

$$\left[\frac{3\,Ba\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left((3\,Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{6\,ax^3},-\frac{3\,Ba\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left((3\,Ba\right.}{3\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3), -1/3*(3*B*a*sqrt(-b)*x
^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^
2 + a))/(a*x^3)]
```

Sympy [A] time = 2.55674, size = 107, normalized size = 1.62

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt(a
)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [B] time = 1.20675, size = 204, normalized size = 3.09

$$-\frac{1}{2}\,B\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)
 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 3*B*a^3*sqrt(b) + A*a^
2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.514 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5} dx$

Optimal. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))/(4*a*x^4)
 + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] time = 0.0688513, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))/(4*a*x^4)
 + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}+\frac{\left(-\frac{Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}-\frac{(b(Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}-\frac{(Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}+\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0702502, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^2\right)\left(2a\left(A+2Bx^2\right)+Abx^2\right)-bx^4\sqrt{\frac{bx^2}{a}+1}(4aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{8ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] (-((a + b*x^2)*(A*b*x^2 + 2*a*(A + 2*B*x^2))) - b*(-(A*b) + 4*a*B)*x^4*Sqrt
[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(8*a*x^4*Sqrt[a + b*x^2])
```

Maple [B] time = 0.01, size = 153, normalized size = 1.7

$$-\frac{A}{4\,ax^4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Ab}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Ab^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{Ab^2}{8\,a^2}\sqrt{bx^2+a}-\frac{B}{2\,ax^2}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bb}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(3/2)/a/x^4+1/8*A*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*A*b^2/a^(3
/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/8*A*b^2/a^2*(b*x^2+a)^(1/2)-1/2
*B/a/x^2*(b*x^2+a)^(3/2)-1/2*B*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)+1/2*B*b/a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60376, size = 394, normalized size = 4.48

$$\left[-\frac{\left(4\,Bab - Ab^2\right)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + 2\left(2\,Aa^2 + \left(4\,Ba^2 + Aab\right)x^2\right)\sqrt{bx^2+a}}{16\,a^2x^4}, \frac{\left(4\,Bab - Ab^2\right)\sqrt{-a}x^4\,\text{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - A*b^2)*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*
x^4), 1/8*((4*B*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4)]
```

Sympy [A] time = 46.1759, size = 144, normalized size = 1.64

$$-\frac{Aa}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^2\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*sqrt(a/(b*
x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + A*b**2*asinh(sqrt(a
)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b*as
inh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

Giac [A] time = 1.11024, size = 162, normalized size = 1.84

$$\frac{\frac{\left(4\,Bab^2 - Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab^2 - 4\,\sqrt{bx^2+a}Ba^2b^2 + \left(bx^2+a\right)^{\frac{3}{2}}Ab^3 + \sqrt{bx^2+a}Aab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (4
*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 + (b*x^2 + a)^(3/2
)*A*b^3 + sqrt(b*x^2 + a)*A*a*b^3)/(a*b^2*x^4))/b
```

3.515 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(5*a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(3/2))/(15*
a^2*x^3)
```

Rubi [A] time = 0.0211285, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{(a+bx^2)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(5*a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(3/2))/(15*
a^2*x^3)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^6}\,dx &= -\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}-\frac{(2Ab-5aB)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{5a}\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{(2Ab-5aB)\left(a+bx^2\right)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.0131374, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{3/2}(3aA+5aBx^2-2Abx^2)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(3*a*A - 2*A*b*x^2 + 5*a*B*x^2))/(15*a^2*x^5)
```

Maple [A] time = 0.004, size = 37, normalized size = 0.7

$$-\frac{-2\,Abx^2+5\,Bax^2+3\,Aa}{15\,x^5a^2}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*A*b*x^2+5*B*a*x^2+3*A*a)/x^5/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54684, size = 126, normalized size = 2.38

$$-\frac{\left(\left(5\,Bab-2\,Ab^2\right)x^4+3\,Aa^2+\left(5\,Ba^2+Aab\right)x^2\right)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*a*b - 2*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^2*x^5)
```

Sympy [B] time = 2.32608, size = 119, normalized size = 2.25

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*sqrt(b)*sqrt(
a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a)
```

Giac [B] time = 1.13378, size = 313, normalized size = 5.91

$$\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{3}{2}}-30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{3}{2}}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{5}{2}}+20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2\cdots}{15\left(\left(\sqrt{b}x-\sqrt{b}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 30*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*B*a*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(5/2) + 2
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) + 10*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*A*a*b^(5/2) - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) +
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2) + 5*B*a^4*b^(3/2) - 2*A*a^
3*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.516 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7} dx$

Optimal. Leaf size=120

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt[a + b*x^2
])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^2*(A*b - 2*a*B)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rubi [A] time = 0.0936198, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt[a + b*x^2
])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^2*(A*b - 2*a*B)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6} + \frac{\left(-\frac{3Ab}{2}+3aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)}{6a}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} - \frac{A\left(a+bx^2\right)^{3/2}}{6ax^6} - \frac{(b(Ab-2aB))\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{16a}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A\left(a+bx^2\right)^{3/2}}{6ax^6} + \frac{\left(b^2(Ab-2aB)\right)\operatorname{Subst}}{32a}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A\left(a+bx^2\right)^{3/2}}{6ax^6} + \frac{(b(Ab-2aB))\operatorname{Subst}}{}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A\left(a+bx^2\right)^{3/2}}{6ax^6} - \frac{b^2(Ab-2aB)\tanh^{-1}}{16a^{5/2}}$$

Mathematica [C] time = 0.0211102, size = 61, normalized size = 0.51

$$-\frac{\left(a+bx^2\right)^{3/2}\left(a^3A+b^2x^6(2aB-Ab)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(a^3*A + b^2*(-(A*b) + 2*a*B)*x^6*Hypergeometric2F1[3/2
, 3, 5/2, 1 + (b*x^2)/a]))/(6*a^4*x^6)
```

Maple [A] time = 0.013, size = 197, normalized size = 1.6

$$-\frac{A}{6\,a x^6}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{Ab}{8\,a^2x^4}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Ab^2}{16\,a^3x^2}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}} + \frac{Ab^3}{16\,a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(3/2)/a/x^6+1/8*A*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16*A*b^2/a^3
/x^2*(b*x^2+a)^(3/2)-1/16*A*b^3/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+1/16*A*b^3/a^3*(b*x^2+a)^(1/2)-1/4*B/a/x^4*(b*x^2+a)^(3/2)+1/8*B*b/a^2/x
^2*(b*x^2+a)^(3/2)+1/8*B*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-
1/8*B*b^2/a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76611, size = 500, normalized size = 4.17

$$\left[-\frac{3\left(2\,Bab^2 - Ab^3\right)\sqrt{a}x^6\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\sqrt{a} + 2\,a}{x^2}\right) + 2\left(3\left(2\,Ba^2b - Aab^2\right)x^4 + 8\,Aa^3 + 2\left(6\,Ba^3 + Aa^2b\right)x^2\right)\sqrt{bx^2 + a}}{96\,a^3x^6}, -\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(2*B*a^2*b - A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
+ A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6), -1/48*(3*(2*B*a*b^2 - A*b^3)*sq
rt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - A*a*b^2)*x^4
+ 8*A*a^3 + 2*(6*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [B] time = 59.6569, size = 226, normalized size = 1.88

$$-\frac{Aa}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5A\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{Ba}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3B\sqrt{}}{8x^3\sqrt{\frac{a}{bx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5*sqrt(a/(b
*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)/(16
*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5
/2)) - B*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*sqrt
```

```
(a/(b*x**2) + 1)) - B*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*asinh(
sqrt(a)/(sqrt(b)*x))/(8*a**(3/2))
```

Giac [A] time = 1.11098, size = 189, normalized size = 1.58

$$-\frac{\frac{3\left(2\,Bab^3-Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{6\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-6\,\sqrt{bx^2+a}Ba^3b^3-3\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+8\left(bx^2+a\right)^{\frac{3}{2}}Aab^4+3\,\sqrt{bx^2+a}Aa^2b^4}{a^2b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(2*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (6*(b*x^2 + a)^(5/2)*B*a*b^3 - 6*sqrt(b*x^2 + a)*B*a^3*b^3 - 3*(b*x^2 +
 a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4 + 3*sqrt(b*x^2 + a)*A*a^2*b^4
)/(a^2*b^3*x^6))/b
```

3.517 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(7*a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(35*
a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rubi [A] time = 0.0336299, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(7*a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(35*
a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^8}\,dx = -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7} - \frac{(4Ab-7aB)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}$$
$$= -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{35a^2x^5} + \frac{(2b(4Ab-7aB))\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{35a^2}$$
$$= -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{35a^2x^5} - \frac{2b(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{105a^3x^3}$$

Mathematica [A] time = 0.0299985, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{3/2}\left(-3a^2\left(5A+7Bx^2\right)+2abx^2\left(6A+7Bx^2\right)-8Ab^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-8*A*b^2*x^4 - 3*a^2*(5*A + 7*B*x^2) + 2*a*b*x^2*(6*A +
 7*B*x^2)))/(105*a^3*x^7)
```

Maple [A] time = 0.005, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-14\,Bx^4ab-12\,aAbx^2+21\,Bx^2a^2+15\,Aa^2}{105\,x^7a^3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*A*b^2*x^4-14*B*a*b*x^4-12*A*a*b*x^2+21*B*a^2*x^2+
15*A*a^2)/x^7/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73242, size = 178, normalized size = 2.12

$$\frac{\left(2\left(7\,Bab^2-4\,Ab^3\right)x^6-\left(7\,Ba^2b-4\,Aab^2\right)x^4-15\,Aa^3-3\left(7\,Ba^3+Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*a*b^2 - 4*A*b^3)*x^6 - (7*B*a^2*b - 4*A*a*b^2)*x^4 - 15*A*a^3
 - 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^7)
```

Sympy [B] time = 3.0506, size = 442, normalized size = 5.26

$$-\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a*b
**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*sqrt(b)*sqrt(a/
(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*
b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

Giac [B] time = 1.14164, size = 389, normalized size = 4.63

$$\frac{4\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Bab^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(5/2) - 175*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*B*a*b^(5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(7/2
) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(5/2) + 140*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a*b^(7/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(5
/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(7/2) + 49*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^4*b^(5/2) - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b
^(7/2) - 7*B*a^5*b^(5/2) + 4*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^7
```

3.518 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=156

$$-\frac{b^2\sqrt{a+bx^2}(5Ab-8aB)}{128a^3x^2}+\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}+\frac{b\sqrt{a+bx^2}(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}(5Ab-8aB)}{48ax^6}-\frac{A}{}$$

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)*Sqrt[a +
b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a^3*x^2)
 - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*A*b - 8*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rubi [A] time = 0.119913, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{b^2\sqrt{a+bx^2}(5Ab-8aB)}{128a^3x^2}+\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}+\frac{b\sqrt{a+bx^2}(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}(5Ab-8aB)}{48ax^6}-\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)*Sqrt[a +
b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a^3*x^2)
 - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*A*b - 8*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} + \frac{\left(-\frac{5Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)}{8a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} - \frac{(b(5Ab-8aB))\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)}{96a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} + \frac{\left(b^2(5Ab-8aB)\right)\text{Subs}}{128}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A\left(a+bx^2\right)^{3/}}{8ax^8}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A\left(a+bx^2\right)^{3/}}{8ax^8}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A\left(a+bx^2\right)^{3/}}{8ax^8}$$

Mathematica [C] time = 0.02195, size = 62, normalized size = 0.4

$$-\frac{\left(a+bx^2\right)^{3/2}\left(3a^4A+b^3x^8(5Ab-8aB)\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{24a^5x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(3*a^4*A + b^3*(5*A*b - 8*a*B)*x^8*Hypergeometric2F1[3/
2, 4, 5/2, 1 + (b*x^2)/a]))/(24*a^5*x^8)
```

Maple [A] time = 0.016, size = 239, normalized size = 1.5

$$-\frac{B}{6\,a x^{6}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{B b}{8\,a^{2} x^{4}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{B b^{2}}{16\,a^{3} x^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{B b^{3}}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{5}{2}}+\frac{B b^{3}}{16\,a^{3}}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/6*B/a/x^6*(b*x^2+a)^(3/2)+1/8*B*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16*B*b^2/a^3
/x^2*(b*x^2+a)^(3/2)-1/16*B*b^3/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+1/16*B*b^3/a^3*(b*x^2+a)^(1/2)-1/8*A*(b*x^2+a)^(3/2)/a/x^8+5/48*A*b/a^2/
x^6*(b*x^2+a)^(3/2)-5/64*A*b^2/a^3/x^4*(b*x^2+a)^(3/2)+5/128*A*b^3/a^4/x^2*
(b*x^2+a)^(3/2)+5/128*A*b^4/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5
/128*A*b^4/a^4*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13965, size = 612, normalized size = 3.92

$$\left[-\frac{3\left(8\,B a b^{3}-5\,A b^{4}\right)\sqrt{a}x^{8}\log\left(-\frac{b x^{2}+2\,\sqrt{b x^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\left(3\left(8\,B a^{2} b^{2}-5\,A a b^{3}\right)x^{6}-48\,A a^{4}-2\left(8\,B a^{3} b-5\,A a^{2} b^{2}\right)x^{4}-\right.}{768\,a^{4} x^{8}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(
8*B*a^3*b - 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*x^8), 1/384*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(8*B*a^3*b -
 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^8)]
```

Sympy [A] time = 98.7911, size = 286, normalized size = 1.83

$$-\frac{Aa}{8\sqrt{b}x^{9}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{7A\sqrt{b}}{48x^{7}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{3}{2}}}{192ax^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{5}{2}}}{384a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{7}{2}}}{128a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5Ab^{4}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{128a^{\frac{7}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] -A*a/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 7*A*sqrt(b)/(48*x**7*sqrt(a/(b
*x**2) + 1)) + A*b**(3/2)/(192*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2)/
(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a**3*x*sqrt(a/(b*x
**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(7/2)) - B*a/(6*sq
rt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*B*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1
)) + B*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(16*a**2*x*sq
rt(a/(b*x**2) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

Giac [A] time = 1.61277, size = 262, normalized size = 1.68

$$\frac{\frac{3\left(8\,Bab^4-5\,Ab^5\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{24\left(bx^2+a\right)^{\frac{7}{2}}Bab^4-88\left(bx^2+a\right)^{\frac{5}{2}}Ba^2b^4+40\left(bx^2+a\right)^{\frac{3}{2}}Ba^3b^4+24\sqrt{bx^2+a}Ba^4b^4-15\left(bx^2+a\right)^{\frac{7}{2}}Ab^5+55\left(bx^2+a\right)^{\frac{5}{2}}Aab^5-73\,(b}{a^3b^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 - 88*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 40*(
b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 15*(b*x^2 + a)^
(7/2)*A*b^5 + 55*(b*x^2 + a)^(5/2)*A*a*b^5 - 73*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^3*b^4*x^8))/b
```

3.519 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2(a+bx^2)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b(a+bx^2)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{(a+bx^2)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(9*a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(21*
a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (8*b^2*(
2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^3)
```

Rubi [A] time = 0.0554634, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8b^2(a+bx^2)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b(a+bx^2)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{(a+bx^2)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(9*a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(21*
a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (8*b^2*(
2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^3)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{10}}\,dx = -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9} - \frac{(6Ab-9aB)\int \frac{\sqrt{a+bx^2}}{x^8}\,dx}{9a}$$
$$= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7} + \frac{(4b(2Ab-3aB))\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{21a^2}$$
$$= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7} - \frac{4b(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{105a^3x^5} - \frac{(8b^2(2Ab-3a}{10}$$
$$= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7} - \frac{4b(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{105a^3x^5} + \frac{8b^2(2Ab-3a}{315}$$

Mathematica [A] time = 0.0382665, size = 81, normalized size = 0.69

$$\frac{\left(a+bx^2\right)^{3/2}\left(6a^2bx^2\left(5A+6Bx^2\right)-5a^3\left(7A+9Bx^2\right)-24ab^2x^4\left(A+Bx^2\right)+16Ab^3x^6\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a^2*b*x^2*(
5*A + 6*B*x^2) - 5*a^3*(7*A + 9*B*x^2)))/(315*a^4*x^9)
```

Maple [A] time = 0.006, size = 83, normalized size = 0.7

$$-\frac{-16\,Ab^3x^6+24\,Bab^2x^6+24\,Aab^2x^4-36\,Ba^2bx^4-30\,Aa^2bx^2+45\,Ba^3x^2+35\,Aa^3}{315\,x^9a^4}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*A*b^3*x^6+24*B*a*b^2*x^6+24*A*a*b^2*x^4-36*B*a^
2*b*x^4-30*A*a^2*b*x^2+45*B*a^3*x^2+35*A*a^3)/x^9/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17845, size = 231, normalized size = 1.97

$$-\frac{\left(8\left(3\,Bab^3-2\,Ab^4\right)x^8-4\left(3\,Ba^2b^2-2\,Aab^3\right)x^6+35\,Aa^4+3\left(3\,Ba^3b-2\,Aa^2b^2\right)x^4+5\left(9\,Ba^4+Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*a*b^3 - 2*A*b^4)*x^8 - 4*(3*B*a^2*b^2 - 2*A*a*b^3)*x^6 + 35*
A*a^4 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 + A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^4*x^9)
```

Sympy [B] time = 4.13412, size = 957, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**6*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**
12*x**14) + 30*A*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x
**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
 40*A*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6
*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*A*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5*b**(9/2)*sqrt(a/(b*
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a
**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3*b**(13/2)*x**4*sqrt(a/(b*x
**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a*b**(17/2)*x**8*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*
b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x*
*8 + 105*a**3*b**6*x**10)
```

Giac [B] time = 1.16652, size = 464, normalized size = 3.97

$$\frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{9}{2}}+63\left(\sqrt{b}x-\sqrt{bx^2+\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(7/2) - 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*B*a*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(
```

```
9/2) + 63*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(7/2) + 378*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b
^(7/2) + 168*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 108*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^4*b^(7/2) - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*
a^3*b^(9/2) - 27*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 18*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(9/2) + 3*B*a^6*b^(7/2) - 2*A*a^5*b^(9/2))
/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.520 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$\frac{b^3\sqrt{a+bx^2}(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-10aB)}{384a^3x^4}-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b\sqrt{a+bx^2}(7Ab-10aB)}{480a^2x^6}$$

```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(384*a^3*x
^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(256*a^4*x^2) - (A*(a + b*x^2)
^(3/2))/(10*a*x^10) - (b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*a^(9/2))
```

Rubi [A] time = 0.146803, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^3\sqrt{a+bx^2}(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-10aB)}{384a^3x^4}-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b\sqrt{a+bx^2}(7Ab-10aB)}{480a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```


```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(384*a^3*x
^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(256*a^4*x^2) - (A*(a + b*x^2)
^(3/2))/(10*a*x^10) - (b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*a^(9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} + \frac{\left(-\frac{7Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx,x,x^2\right)}{10a}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} - \frac{(b(7Ab-10aB))\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx,x,x^2\right)}{160a}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} + \frac{\left(b^2(7Ab-10aB)\right)\text{Su}}{1}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} - \frac{A\left(a+bx\right.}{10ax^1}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} + \frac{b^3(7Ab-}{2}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} + \frac{b^3(7Ab-}{2}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} + \frac{b^3(7Ab-}{2}$$

Mathematica [C] time = 0.0220841, size = 62, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{3/2}\left(3a^5A+b^4x^{10}(10aB-7Ab)\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{30a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(3*a^5*A + b^4*(-7*A*b + 10*a*B)*x^10*Hypergeometric2F1
[3/2, 5, 5/2, 1 + (b*x^2)/a]))/(30*a^6*x^10)
```

Maple [A] time = 0.024, size = 281, normalized size = 1.5

$$-\frac{A}{10\,a x^{10}}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{7\,A b}{80\,a^2 x^8}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{7\,A b^2}{96\,a^3 x^6}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{7\,A b^3}{128\,a^4 x^4}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{7\,A b^4}{256\,a^5 x^2}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{7\,A b^5}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(3/2)/a/x^10+7/80*A*b/a^2/x^8*(b*x^2+a)^(3/2)-7/96*A*b^2/
a^3/x^6*(b*x^2+a)^(3/2)+7/128*A*b^3/a^4/x^4*(b*x^2+a)^(3/2)-7/256*A*b^4/a^5
/x^2*(b*x^2+a)^(3/2)-7/256*A*b^5/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)+7/256*A*b^5/a^5*(b*x^2+a)^(1/2)-1/8*B/a/x^8*(b*x^2+a)^(3/2)+5/48*B*b/a^
2/x^6*(b*x^2+a)^(3/2)-5/64*B*b^2/a^3/x^4*(b*x^2+a)^(3/2)+5/128*B*b^3/a^4/x^
2*(b*x^2+a)^(3/2)+5/128*B*b^4/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
-5/128*B*b^4/a^4*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33929, size = 745, normalized size = 3.94

$$\left[-\frac{15\left(10\,Bab^4-7\,Ab^5\right)\sqrt{a}x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(15\left(10\,Ba^2b^3-7\,Aab^4\right)x^8-10\left(10\,Ba^3b^2-7\,Aa^2b^3\right)x^6+3\right.}{7680\,a^5x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 7*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B
*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4
+ 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^5*x^10), -1/3840*(15*(10
*B*a*b^4 - 7*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(1
0*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*
a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(
b*x^2 + a))/(a^5*x^10)]
```

Sympy [A] time = 114.89, size = 347, normalized size = 1.84

$$-\frac{Aa}{10\sqrt{b}x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{9A\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}}{480ax^7\sqrt{\frac{a}{bx^2}+1}}-\frac{7Ab^{\frac{5}{2}}}{1920a^2x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{7Ab^{\frac{7}{2}}}{768a^3x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{7Ab^{\frac{9}{2}}}{256a^4x\sqrt{\frac{a}{bx^2}+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] -A*a/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 9*A*sqrt(b)/(80*x**9*sqrt(a/
(b*x**2) + 1)) + A*b**(3/2)/(480*a*x**7*sqrt(a/(b*x**2) + 1)) - 7*A*b**(5/2
)/(1920*a**2*x**5*sqrt(a/(b*x**2) + 1)) + 7*A*b**(7/2)/(768*a**3*x**3*sqrt(
a/(b*x**2) + 1)) + 7*A*b**(9/2)/(256*a**4*x*sqrt(a/(b*x**2) + 1)) - 7*A*b**
5*asinh(sqrt(a)/(sqrt(b)*x))/(256*a**(9/2)) - B*a/(8*sqrt(b)*x**9*sqrt(a/(b
*x**2) + 1)) - 7*B*sqrt(b)/(48*x**7*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)/(192
*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*B*b**(5/2)/(384*a**2*x**3*sqrt(a/(b*x**2)
 + 1)) - 5*B*b**(7/2)/(128*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*B*b**4*asinh(sq
rt(a)/(sqrt(b)*x))/(128*a**(7/2))
```

Giac [A] time = 1.15335, size = 311, normalized size = 1.65

$$-\frac{\frac{15\left(10\,Bab^5-7\,Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{150\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-700\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+1280\left(bx^2+a\right)^{\frac{5}{2}}Ba^3b^5-580\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-150\sqrt{bx^2+a}Ba^5b^5-105\left(bx^2+a\right.}{a^4b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 7*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^4) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 - 700*(b*x^2 + a)^(7/2)*B*a^2*b^5
+ 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 - 580*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 105*(b*x^2 + a)^(9/2)*A*b^6 + 490*(b*x^2 + a)^(
7/2)*A*a*b^6 - 896*(b*x^2 + a)^(5/2)*A*a^2*b^6 + 790*(b*x^2 + a)^(3/2)*A*a^
3*b^6 + 105*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^4*b^5*x^10))/b
```

3.521 $\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=103

$$\frac{a^2 \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2} (Ab-3aB)}{9b^4}-\frac{a \left(a+bx^2\right)^{7/2} (2Ab-3aB)}{7b^4}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9*b^4) + (B*(a + b*x^
2)^(11/2))/(11*b^4)
```

Rubi [A] time = 0.0773654, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2 \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2} (Ab-3aB)}{9b^4}-\frac{a \left(a+bx^2\right)^{7/2} (2Ab-3aB)}{7b^4}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9*b^4) + (B*(a + b*x^
2)^(11/2))/(11*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx &= \frac{1}{2} \text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}+\frac{(Ab-3aB}{l}\right.\right.\\
&= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4}+
\end{aligned}$$

Mathematica [A] time = 0.058488, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{5/2}\left(8a^2b\left(11A+15Bx^2\right)-48a^3B-10ab^2x^2\left(22A+21Bx^2\right)+35b^3x^4\left(11A+9Bx^2\right)\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-48*a^3*B + 35*b^3*x^4*(11*A + 9*B*x^2) + 8*a^2*b*(11*A
 + 15*B*x^2) - 10*a*b^2*x^2*(22*A + 21*B*x^2)))/(3465*b^4)
```

Maple [A] time = 0.006, size = 77, normalized size = 0.8

$$\frac{315\,Bx^6b^3+385\,Ab^3x^4-210\,Bab^2x^4-220\,Aab^2x^2+120\,Ba^2bx^2+88\,Aa^2b-48\,Ba^3}{3465\,b^4}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/3465*(b*x^2+a)^(5/2)*(315*B*b^3*x^6+385*A*b^3*x^4-210*B*a*b^2*x^4-220*A*a
*b^2*x^2+120*B*a^2*b*x^2+88*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61489, size = 284, normalized size = 2.76

$$\frac{\left(315\,Bb^5x^{10}+35\left(12\,Bab^4+11\,Ab^5\right)x^8+5\left(3\,Ba^2b^3+110\,Aab^4\right)x^6-48\,Ba^5+88\,Aa^4b-3\left(6\,Ba^3b^2-11\,Aa^2b^3\right)x^4+4\,(6}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*b^5*x^10 + 35*(12*B*a*b^4 + 11*A*b^5)*x^8 + 5*(3*B*a^2*b^3 +
110*A*a*b^4)*x^6 - 48*B*a^5 + 88*A*a^4*b - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x
^4 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 3.99679, size = 260, normalized size = 2.52

$$\begin{cases} \frac{8Aa^4\sqrt{a+bx^2}}{315b^3} - \frac{4Aa^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{Aa^2x^4\sqrt{a+bx^2}}{105b} + \frac{10Aax^6\sqrt{a+bx^2}}{63} + \frac{Abx^8\sqrt{a+bx^2}}{9} - \frac{16Ba^5\sqrt{a+bx^2}}{1155b^4} + \frac{8Ba^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2Ba^3x^4\sqrt{a+bx^2}}{385b^2} + \frac{Ba \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*A*a**3*x**2*sqrt(a + b*
x**2)/(315*b**2) + A*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*A*a*x**6*sqrt(
a + b*x**2)/63 + A*b*x**8*sqrt(a + b*x**2)/9 - 16*B*a**5*sqrt(a + b*x**2)/(
1155*b**4) + 8*B*a**4*x**2*sqrt(a + b*x**2)/(1155*b**3) - 2*B*a**3*x**4*sqr
t(a + b*x**2)/(385*b**2) + B*a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*B*a*x**
8*sqrt(a + b*x**2)/33 + B*b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(3/2)
*(A*x**6/6 + B*x**8/8), True))
```

Giac [B] time = 1.1141, size = 323, normalized size = 3.14

$$\frac{\frac{33\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa}{b^2} + \frac{11\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)Ba}{b^3} + \frac{11\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^
(3/2)*a^2)*A*a/b^2 + 11*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 1
89*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B*a/b^3 + 11*(35*(b*x
^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(
b*x^2 + a)^(3/2)*a^3)*A/b^2 + (315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9
/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 1155*(b*x
^2 + a)^(3/2)*a^4)*B/b^3)/b
```

3.522 $\int x^4 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=188

$$\frac{a^2x^3\sqrt{a+bx^2}(2Ab-aB)}{128b^2}-\frac{3a^3x\sqrt{a+bx^2}(2Ab-aB)}{256b^3}+\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-aB)}{16b}+$$

```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt[a + b*x^2])/(32*b)
+ ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b) + (B*x^5*(a + b*x^2)^(5/2))/
(10*b) + (3*a^4*(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^
(7/2))
```

Rubi [A] time = 0.0963296, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^2x^3\sqrt{a+bx^2}(2Ab-aB)}{128b^2}-\frac{3a^3x\sqrt{a+bx^2}(2Ab-aB)}{256b^3}+\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-aB)}{16b}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```


```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt[a + b*x^2])/(32*b)
+ ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b) + (B*x^5*(a + b*x^2)^(5/2))/
(10*b) + (3*a^4*(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^
(7/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx=\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}-\frac{(-10Ab+5aB)\int x^4\left(a+bx^2\right)^{3/2}dx}{10b}$$
$$=\frac{(2Ab-aB)x^5\left(a+bx^2\right)^{3/2}}{16b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}+\frac{(3a(2Ab-aB))\int x^4\sqrt{a+bx^2}\,dx}{16b}$$
$$=\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)x^5\left(a+bx^2\right)^{3/2}}{16b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}+\frac{\left(a^2(2A\right.}{}$$
$$=\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)x^5\left(a+bx^2\right)^{3/2}}{16b}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}$$

Mathematica [A] time = 0.306261, size = 150, normalized size = 0.8

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(4a^2b^2x^2\left(5A+2Bx^2\right)-10a^3b\left(3A+Bx^2\right)+15a^4B+16ab^3x^4\left(15A+11Bx^2\right)+32b^4x^6\left(5A+4Bx^2\right)\right)\right.}{1280b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^4*B - 10*a^3*b*(3*A + B*x^2) + 4*a^2*b^2*
x^2*(5*A + 2*B*x^2) + 32*b^4*x^6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(15*A + 11*
B*x^2)) - (15*a^(7/2)*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 +
 (b*x^2)/a]))/(1280*b^(7/2))
```

Maple [A] time = 0.01, size = 219, normalized size = 1.2

$$\frac{Bx^5}{10\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bax^3}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^2Bx}{32\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Ba^3x}{128\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,Ba^4x}{256\,b^3}\sqrt{bx^2+a}-\frac{3\,Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/10*B*x^5*(b*x^2+a)^(5/2)/b-1/16*B/b^2*a*x^3*(b*x^2+a)^(5/2)+1/32*B/b^3*a^
2*x*(b*x^2+a)^(5/2)-1/128*B/b^3*a^3*x*(b*x^2+a)^(3/2)-3/256*B/b^3*a^4*x*(b*
x^2+a)^(1/2)-3/256*B/b^(7/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/8*A*x^3*(b
*x^2+a)^(5/2)/b-1/16*A/b^2*a*x*(b*x^2+a)^(5/2)+1/64*A/b^2*a^2*x*(b*x^2+a)^(
3/2)+3/128*A/b^2*a^3*x*(b*x^2+a)^(1/2)+3/128*A/b^(5/2)*a^4*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01368, size = 693, normalized size = 3.69

$$\left[-\frac{15\left(Ba^{5}-2\,Aa^{4}b\right)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\left(128\,Bb^{5}x^{9}+16\left(11\,Bab^{4}+10\,Ab^{5}\right)x^{7}+8\left(Ba^{2}b^{3}+30\,Aa\right.\right.}{2560\,b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*a^5 - 2*A*a^4*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(128*B*b^5*x^9 + 16*(11*B*a*b^4 + 10*A*b^5)*x^7 + 8*(B*a^
2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3)*x^3 + 15*(B*a^4*b -
2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/1280*(15*(B*a^5 - 2*A*a^4*b)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (128*B*b^5*x^9 + 16*(11*B*a*b^4 + 1
0*A*b^5)*x^7 + 8*(B*a^2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3
)*x^3 + 15*(B*a^4*b - 2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] time = 34.0349, size = 345, normalized size = 1.84

$$-\frac{3Aa^{\frac{7}{2}}x}{128b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{\frac{5}{2}}x^{3}}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{13Aa^{\frac{3}{2}}x^{5}}{64\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5A\sqrt{a}bx^{7}}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Aa^{4}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Ab^{2}x^{9}}{8\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{\frac{9}{2}}x}{256b^{3}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - A*a**(5/2)*x**3/(128*b*sqrt
(1 + b*x**2/a)) + 13*A*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*A*sqrt(a)*
b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**
(5/2)) + A*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*B*a**(9/2)*x/(256*b
**3*sqrt(1 + b*x**2/a)) + B*a**(7/2)*x**3/(256*b**2*sqrt(1 + b*x**2/a)) - B
*a**(5/2)*x**5/(640*b*sqrt(1 + b*x**2/a)) + 23*B*a**(3/2)*x**7/(160*sqrt(1
```

```
+ b*x**2/a)) + 19*B*sqrt(a)*b*x**9/(80*sqrt(1 + b*x**2/a)) - 3*B*a**5*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(7/2)) + B*b**2*x**11/(10*sqrt(a)*sqrt(1 + b*x*
*2/a))
```

Giac [A] time = 1.12732, size = 215, normalized size = 1.14

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bbx^2+\frac{11\,Bab^8+10\,Ab^9}{b^8}\right)x^2+\frac{Ba^2b^7+30\,Aab^8}{b^8}\right)x^2-\frac{5\left(Ba^3b^6-2\,Aa^2b^7\right)}{b^8}\right)x^2+\frac{15\left(Ba^4b^5-2\,Aa^3b\right.}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*b*x^2 + (11*B*a*b^8 + 10*A*b^9)/b^8)*x^2 + (B*a^2*b^7
+ 30*A*a*b^8)/b^8)*x^2 - 5*(B*a^3*b^6 - 2*A*a^2*b^7)/b^8)*x^2 + 15*(B*a^4*b
^5 - 2*A*a^3*b^6)/b^8)*sqrt(b*x^2 + a)*x + 3/256*(B*a^5 - 2*A*a^4*b)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.523 $\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B \left(a+bx^2\right)^{9/2}}{9b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(7/
2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rubi [A] time = 0.0592361, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B \left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(7/
2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx &= \frac{1}{2} \text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx) \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{5/2}}{b^2}+\frac{B(a+bx)^{7/2}}{b^2}\right) \, dx, x, x^2\right)\\
&= -\frac{a(Ab-aB) \left(a+bx^2\right)^{5/2}}{5b^3}+\frac{(Ab-2aB) \left(a+bx^2\right)^{7/2}}{7b^3}+\frac{B \left(a+bx^2\right)^{9/2}}{9b^3}
\end{aligned}$$

Mathematica [A] time = 0.0393702, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{5/2} \left(8a^2B-2ab \left(9A+10Bx^2\right)+5b^2x^2 \left(9A+7Bx^2\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2*B + 5*b^2*x^2*(9*A + 7*B*x^2) - 2*a*b*(9*A + 10*B
*x^2)))/(315*b^3)
```

Maple [A] time = 0.005, size = 53, normalized size = 0.7

$$-\frac{-35\,b^2Bx^4-45\,Ab^2x^2+20\,Babx^2+18\,abA-8\,a^2B}{315\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(-35*B*b^2*x^4-45*A*b^2*x^2+20*B*a*b*x^2+18*A*a*b-8*
B*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62572, size = 217, normalized size = 2.97

$$\frac{\left(35\,Bb^4x^8+5\left(10\,Bab^3+9\,Ab^4\right)x^6+8\,Ba^4-18\,Aa^3b+3\left(Ba^2b^2+24\,Aab^3\right)x^4-\left(4\,Ba^3b-9\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(10*B*a*b^3 + 9*A*b^4)*x^6 + 8*B*a^4 - 18*A*a^3*b +
 3*(B*a^2*b^2 + 24*A*a*b^3)*x^4 - (4*B*a^3*b - 9*A*a^2*b^2)*x^2)*sqrt(b*x^2
 + a)/b^3
```

Sympy [A] time = 2.32155, size = 209, normalized size = 2.86

$$\begin{cases}-\frac{2Aa^3\sqrt{a+bx^2}}{35b^2}+\frac{Aa^2x^2\sqrt{a+bx^2}}{35b}+\frac{8Aax^4\sqrt{a+bx^2}}{35}+\frac{Abx^6\sqrt{a+bx^2}}{7}+\frac{8Ba^4\sqrt{a+bx^2}}{315b^3}-\frac{4Ba^3x^2\sqrt{a+bx^2}}{315b^2}+\frac{Ba^2x^4\sqrt{a+bx^2}}{105b}+\frac{10Bax^6\sqrt{a+bx^2}}{63}+\frac{Bbx}{} \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**3*sqrt(a + b*x**2)/(35*b**2) + A*a**2*x**2*sqrt(a + b*x*
*2)/(35*b) + 8*A*a*x**4*sqrt(a + b*x**2)/35 + A*b*x**6*sqrt(a + b*x**2)/7 +
 8*B*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*B*a**3*x**2*sqrt(a + b*x**2)/(315
*b**2) + B*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*B*a*x**6*sqrt(a + b*x**2
)/63 + B*b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**6
/6), True))
```

Giac [B] time = 1.12812, size = 247, normalized size = 3.38

$$\frac{\frac{21\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Aa}{b}+\frac{3\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Ba}{b^2}+\frac{3\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)A}{b}+\frac{\left(35\left(bx^2+a\right)\right.}{}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A*a/b + 3*(15*(b*x^
2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*B*a/b^2 +
 3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^
2)*A/b + (35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^
(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B/b^2)/b
```

3.524 $\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=155

$$\frac{a^2x\sqrt{a+bx^2}(8Ab-3aB)}{128b^2}-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{ax^3\sqrt{a+bx^2}(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-3a}{48b}$$

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3*a*B)*x^3*
Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(48*b) +
(B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.0673113, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^2x\sqrt{a+bx^2}(8Ab-3aB)}{128b^2}-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{ax^3\sqrt{a+bx^2}(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-3a}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3*a*B)*x^3*
Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(48*b) +
(B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx=\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{(-8Ab+3aB)\int x^2\left(a+bx^2\right)^{3/2}dx}{8b}$$
$$=\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{(a(8Ab-3aB))\int x^2\sqrt{a+bx^2}\,dx}{16b}$$
$$=\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\left(a^2(8Ab\right.}{}$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+$$

Mathematica [A] time = 0.249302, size = 130, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(6a^2b\left(4A+Bx^2\right)-9a^3B+8ab^2x^2\left(14A+9Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)+\frac{3a^{5/2}(3aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-9*a^3*B + 6*a^2*b*(4*A + B*x^2) + 16*b^3*x^4*
(4*A + 3*B*x^2) + 8*a*b^2*x^2*(14*A + 9*B*x^2)) + (3*a^(5/2)*(-8*A*b + 3*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(5/2))
```

Maple [A] time = 0.008, size = 177, normalized size = 1.1

$$\frac{Bx^3}{8b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bax}{16b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^2Bx}{64b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3Ba^3x}{128b^2}\sqrt{bx^2+a}+\frac{3Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Ax}{6b}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B/b^2*a*x*(b*x^2+a)^(5/2)+1/64*B/b^2*a^2*x
*(b*x^2+a)^(3/2)+3/128*B/b^2*a^3*x*(b*x^2+a)^(1/2)+3/128*B/b^(5/2)*a^4*ln(x
```

```
*b^(1/2)+(b*x^2+a)^(1/2))+1/6*A*x*(b*x^2+a)^(5/2)/b-1/24*A/b*a*x*(b*x^2+a)^
(3/2)-1/16*A/b*a^2*x*(b*x^2+a)^(1/2)-1/16*A/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78934, size = 597, normalized size = 3.85

$$\left[-\frac{3\left(3\,Ba^4-8\,Aa^3b\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Bb^4x^7+8\left(9\,Bab^3+8\,Ab^4\right)x^5+2\left(3\,Ba^2b^2+56\,A\right.\right.}{768\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*
b^2 + 56*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3
, -1/384*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - (48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*b^2 + 56*A*a*
b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] time = 22.2434, size = 287, normalized size = 1.85

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) -
3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(
1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b
*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.13432, size = 180, normalized size = 1.16

$$\frac{1}{384}\left(2\left(4\left(6\,Bbx^2+\frac{9\,Bab^6+8\,Ab^7}{b^6}\right)x^2+\frac{3\,Ba^2b^5+56\,Aab^6}{b^6}\right)x^2-\frac{3\left(3\,Ba^3b^4-8\,Aa^2b^5\right)}{b^6}\right)\sqrt{bx^2+a}x-\frac{\left(3\,Ba^4-8\,Aa^3b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b*x^2 + (9*B*a*b^6 + 8*A*b^7)/b^6)*x^2 + (3*B*a^2*b^5 + 56
*A*a*b^6)/b^6)*x^2 - 3*(3*B*a^3*b^4 - 8*A*a^2*b^5)/b^6)*sqrt(b*x^2 + a)*x -
 1/128*(3*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.525 $\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(7*b^2)
```

Rubi [A] time = 0.0324109, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(7*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^{3/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.0233831, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(-2aB+7Ab+5bBx^2\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

Maple [A] time = 0.003, size = 31, normalized size = 0.7

$$\frac{5\,bBx^{2}+7\,Ab-2\,Ba}{35\,b^{2}}\left(bx^{2}+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/35*(b*x^2+a)^(5/2)*(5*B*b*x^2+7*A*b-2*B*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62903, size = 161, normalized size = 3.5

$$\frac{\left(5\,Bb^{3}x^{6}+\left(8\,Bab^{2}+7\,Ab^{3}\right)x^{4}-2\,Ba^{3}+7\,Aa^{2}b+\left(Ba^{2}b+14\,Aab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*b^3*x^6 + (8*B*a*b^2 + 7*A*b^3)*x^4 - 2*B*a^3 + 7*A*a^2*b + (B*a^
2*b + 14*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 1.26381, size = 158, normalized size = 3.43

$$\begin{cases} \frac{Aa^{2}\sqrt{a+bx^{2}}}{5b}+\frac{2Aax^{2}\sqrt{a+bx^{2}}}{5}+\frac{Abx^{4}\sqrt{a+bx^{2}}}{5}-\frac{2Ba^{3}\sqrt{a+bx^{2}}}{35b^{2}}+\frac{Ba^{2}x^{2}\sqrt{a+bx^{2}}}{35b}+\frac{8Bax^{4}\sqrt{a+bx^{2}}}{35}+\frac{Bbx^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\: b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^{2}}{2}+\frac{Bx^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**2*sqrt(a + b*x**2)/(5*b) + 2*A*a*x**2*sqrt(a + b*x**2)/5 +
A*b*x**4*sqrt(a + b*x**2)/5 - 2*B*a**3*sqrt(a + b*x**2)/(35*b**2) + B*a**2*
```

```
x**2*sqrt(a + b*x**2)/(35*b) + 8*B*a*x**4*sqrt(a + b*x**2)/35 + B*b*x**6*sq
rt(a + b*x**2)/7, Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**4/4), True))
```

Giac [B] time = 1.09728, size = 162, normalized size = 3.52

$$\frac{35 \left(bx^2+a\right)^{\frac{3}{2}} Aa + 7 \left(3 \left(bx^2+a\right)^{\frac{5}{2}} - 5 \left(bx^2+a\right)^{\frac{3}{2}} a\right) A + \frac{7 \left(3 \left(bx^2+a\right)^{\frac{5}{2}} - 5 \left(bx^2+a\right)^{\frac{3}{2}} a\right) Ba}{b} + \frac{\left(15 \left(bx^2+a\right)^{\frac{7}{2}} - 42 \left(bx^2+a\right)^{\frac{5}{2}} a + 35 \left(bx^2+a\right)^{\frac{3}{2}} a}{b}}{105\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/105*(35*(b*x^2 + a)^(3/2)*A*a + 7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3
/2)*a)*A + 7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*B*a/b + (15*(b*x
^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*B/b)/b
```

3.526 $\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=118

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a + b*x^2)^(
3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*A*b - a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.040886, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a + b*x^2)^(
3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*A*b - a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx = \frac{Bx\left(a+bx^2\right)^{5/2}}{6b} - \frac{(-6Ab+aB)\int \left(a+bx^2\right)^{3/2}dx}{6b}$$
$$= \frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx\left(a+bx^2\right)^{5/2}}{6b} + \frac{(a(6Ab-aB))\int \sqrt{a+bx^2}dx}{8b}$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx\left(a+bx^2\right)^{5/2}}{6b} + \frac{\left(a^2(6Ab-aI\right.}{1}$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx\left(a+bx^2\right)^{5/2}}{6b} + \frac{\left(a^2(6Ab-aI\right.}{}$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx\left(a+bx^2\right)^{5/2}}{6b} + \frac{a^2(6Ab-aB}{1}$$

Mathematica [A] time = 0.210422, size = 109, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(3a^2B+2ab\left(15A+7Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)-\frac{3a^{3/2}(aB-6Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2*B + 4*b^2*x^2*(3*A + 2*B*x^2) + 2*a*b*(1
5*A + 7*B*x^2)) - (3*a^(3/2)*(-6*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

Maple [A] time = 0.003, size = 131, normalized size = 1.1

$$\frac{Bx}{6b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{Bax}{24b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{a^2Bx}{16b}\sqrt{bx^2+a} - \frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + \frac{Ax}{4}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3aAx}{8}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*B/b*a*x*(b*x^2+a)^(3/2)-1/16*B/b*a^2*x*(b*x^
2+a)^(1/2)-1/16*B/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*A*x*(b*x^2+
a)^(3/2)+3/8*A*a*x*(b*x^2+a)^(1/2)+3/8*A*a^2/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72901, size = 482, normalized size = 4.08

$$\left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,Bb^3x^5+2\left(7\,Bab^2+6\,Ab^3\right)x^3+3\left(Ba^2b+10\,Aab^2\right)x\right)\sqrt{b}}{96\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^3 + 3*(B*a^2*b + 10
*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^
3 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] time = 13.3764, size = 253, normalized size = 2.14

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{}}{24\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x/(
16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b*x**2/a)) + 11*
B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.11507, size = 138, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,Bbx^2+\frac{7\,Bab^4+6\,Ab^5}{b^4}\right)x^2+\frac{3\left(Ba^2b^3+10\,Aab^4\right)}{b^4}\right)\sqrt{bx^2+a}x+\frac{\left(Ba^3-6\,Aa^2b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^3 + 10*A*a
*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

3.527 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x} dx$

Optimal. Leaf size=76

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b)
 - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0533293, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b)
 - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}A\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}(aA)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}\left(a^2A\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{\left(a^2A\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.064648, size = 76, normalized size = 1.

$$\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\left(\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b) + a*A*(Sqrt[a + b*x^2
] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
```

Maple [A] time = 0.007, size = 70, normalized size = 0.9

$$\frac{B}{5\,b}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{A}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Aa^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+aA\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)+a*A*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59311, size = 408, normalized size = 5.37

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Bb^2x^4+3\,Ba^2+20\,Aab+\left(6\,Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{30\,b},\frac{15\,A\sqrt{-a}ab\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2
+ a))/b, 1/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*B*b^
2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 35.7311, size = 71, normalized size = 0.93

$$\frac{Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}+Aa\sqrt{a+bx^2}+\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a*sqrt(a + b*x**2) + A*
(a + b*x**2)**(3/2)/3 + B*(a + b*x**2)**(5/2)/(5*b)
```

Giac [A] time = 1.34425, size = 107, normalized size = 1.41

$$\frac{Aa^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^4+5\left(bx^2+a\right)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^2+a}Aab^5}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2 + a)^(5/2)
*B*b^4 + 5*(b*x^2 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^2 + a)*A*a*b^5)/b^5
```

3.528 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2} dx$

Optimal. Leaf size=109

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x^2)^(3/2))
/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0411167, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x^2)^(3/2))
/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^2}\,dx = -\frac{A\left(a+bx^2\right)^{5/2}}{ax} - \frac{(-4Ab-aB)\int\left(a+bx^2\right)^{3/2}\,dx}{a}$$
$$= \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{4}(3(4Ab+aB))\int\sqrt{a+bx^2}\,dx$$
$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{8}(3a(4Ab+a$$
$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{8}(3a(4Ab+a$$
$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{3a(4Ab+aB}{}$$

Mathematica [A] time = 0.190682, size = 87, normalized size = 0.8

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3\sqrt{a}(aB+4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} - \frac{8aA}{x} + 5aBx + 4Abx + 2bBx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-8*a*A)/x + 4*A*b*x + 5*a*B*x + 2*b*B*x^3 + (3*Sqrt[a]*(
4*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/
8
```

Maple [A] time = 0.008, size = 125, normalized size = 1.2

$$\frac{Bx}{4}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,Bax}{8}\sqrt{bx^2+a} + \frac{3\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{A}{ax}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Abx}{a}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,Abx}{2}\sqrt{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/4*x*B*(b*x^2+a)^(3/2)+3/8*B*a*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))-A*(b*x^2+a)^(5/2)/a/x+A*b/a*x*(b*x^2+a)^(3/2)+3/2*A*
b*x*(b*x^2+a)^(1/2)+3/2*A*b^(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6328, size = 424, normalized size = 3.89

$$\left[\frac{3\left(Ba^{2}+4\,Aab\right)\sqrt{b}x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\left(2\,Bb^{2}x^{4}-8\,Aab+\left(5\,Bab+4\,Ab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,bx},-\frac{3\left(Ba^{2}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2
+ a))/(b*x), -1/8*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - (2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2 +
a))/(b*x)]
```

Sympy [B] time = 8.71008, size = 216, normalized size = 1.98

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^{2}}{a}}}+\frac{A\sqrt{a}bx\sqrt{1+\frac{bx^{2}}{a}}}{2}-\frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3B\sqrt{a}bx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
+ B*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) +
 3*B*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(
a))/(8*sqrt(b)) + B*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.17245, size = 154, normalized size = 1.41

$$\frac{2\,Aa^{2}\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a}+\frac{1}{8}\left(2\,Bbx^{2}+\frac{5\,Bab^{2}+4\,Ab^{3}}{b^{2}}\right)\sqrt{bx^{2}+a}x-\frac{3\left(Ba^{2}\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*B*b*x^2 + (5
*B*a*b^2 + 4*A*b^3)/b^2)*sqrt(b*x^2 + a)*x - 3/16*(B*a^2*sqrt(b) + 4*A*a*b^
(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

3.529 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{\left(a+bx^2\right)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Ab)-\frac{1}{2}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2)^(3/2))/(
6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.0801891, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(a+bx^2\right)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Ab)-\frac{1}{2}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2)^(3/2))/(
6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)}{2a}$$

$$= \frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a} - \frac{A\left(a+bx^2\right)^{5/2}}{2ax^2} + \frac{1}{4}(3Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right.$$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a} - \frac{A\left(a+bx^2\right)^{5/2}}{2ax^2} + \frac{1}{4}(a(3Ab+2aB)$$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a} - \frac{A\left(a+bx^2\right)^{5/2}}{2ax^2} + \frac{(a(3Ab+2aB))}{}$$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a} - \frac{A\left(a+bx^2\right)^{5/2}}{2ax^2} - \frac{1}{2}\sqrt{a}(3Ab+2aB$$

Mathematica [A] time = 0.0497146, size = 80, normalized size = 0.73

$$\frac{1}{6}\left(\frac{\sqrt{a+bx^2}\left(-3aA+8aBx^2+6Abx^2+2bBx^4\right)}{x^2} - 3\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/x^2 - 3*Sqr
t[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/6
```

Maple [A] time = 0.008, size = 132, normalized size = 1.2

$$\frac{B}{3}\left(bx^2+a\right)^{\frac{3}{2}} - Ba^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + B\sqrt{bx^2+a}\,a - \frac{A}{2ax^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Ab}{2a}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{3Ab}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)-B*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*
x^2+a)^(1/2)*a-1/2*A*(b*x^2+a)^(5/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(3/2)-3/2*A*
```

```
b*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68712, size = 404, normalized size = 3.67

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bbx^4+2\,(4\,Ba+3\,Ab)x^2-3\,Aa\right)\sqrt{bx^2+a}}{12\,x^2},\ \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*a + 3*A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) + 2*(2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 +
a))/x^2, 1/6*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) + (2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 25.6166, size = 184, normalized size = 1.67

$$-\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(
a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt
(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A] time = 1.12249, size = 139, normalized size = 1.26

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb+6\sqrt{bx^2+a}Bab+6\sqrt{bx^2+a}Ab^2-\frac{3\sqrt{bx^2+a}Aab}{x^2}+\frac{3\left(2\,Ba^2b+3\,Aab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b*x^2 + a)^(3/2)*B*b + 6*sqrt(b*x^2 + a)*B*a*b + 6*sqrt(b*x^2 + a)*
A*b^2 - 3*sqrt(b*x^2 + a)*A*a*b/x^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.530 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(a+bx^2\right)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a + b*x^2)^
(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqrt[b]*(2*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.0472946, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$-\frac{\left(a+bx^2\right)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```


```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a + b*x^2)^
(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqrt[b]*(2*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx = -\frac{A(a+bx^2)^{5/2}}{3ax^3} - \frac{(-2Ab-3aB)\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx}{3a}$$

$$= -\frac{(2Ab+3aB)(a+bx^2)^{3/2}}{3ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} + \frac{(b(2Ab+3aB))\int \sqrt{a+bx^2}\,dx}{a}$$

$$= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a} - \frac{(2Ab+3aB)(a+bx^2)^{3/2}}{3ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}(b(2Ab+3aB$$

$$= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a} - \frac{(2Ab+3aB)(a+bx^2)^{3/2}}{3ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}(b(2Ab+3aB$$

$$= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a} - \frac{(2Ab+3aB)(a+bx^2)^{3/2}}{3ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}\sqrt{b}(2Ab+3a$$

Mathematica [C] time = 0.0767204, size = 83, normalized size = 0.7

$$\frac{\sqrt{a+bx^2}(-3aB-2Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{3x\sqrt{\frac{bx^2}{a}+1}} - \frac{A(a+bx^2)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(3*a*x^3) + ((-2*A*b - 3*a*B)*Sqrt[a + b*x^2]*Hyperg
eometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(3*x*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.007, size = 168, normalized size = 1.4

$$-\frac{A}{3ax^3}(bx^2+a)^{\frac{5}{2}} - \frac{2Ab}{3a^2x}(bx^2+a)^{\frac{5}{2}} + \frac{2b^2Ax}{3a^2}(bx^2+a)^{\frac{3}{2}} + \frac{b^2Ax}{a}\sqrt{bx^2+a} + Ab^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right) - \frac{B}{ax}(bx^2+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(5/2)/a/x^3-2/3*A*b/a^2/x*(b*x^2+a)^(5/2)+2/3*A*b^2/a^2*x*
(b*x^2+a)^(3/2)+A*b^2/a*x*(b*x^2+a)^(1/2)+A*b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))-B/a/x*(b*x^2+a)^(5/2)+B*b/a*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1
/2)+3/2*B*b^(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59618, size = 402, normalized size = 3.38

$$\left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(3\,Bbx^4-2\,(3\,Ba+4\,Ab)x^2-2\,Aa\right)\sqrt{bx^2+a}}{12\,x^3},-\frac{3\,(3\,Ba+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/
x^3, -1/6*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 5.48485, size = 202, normalized size = 1.7

$$-\frac{A\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Ab^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}bx\sqrt{1+\frac{bx^2}{a}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x
**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqrt(
a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)/(x*sqrt(1 + b*x**2
/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a
) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
```

Giac [B] time = 1.14788, size = 279, normalized size = 2.34

$$\frac{1}{2}\sqrt{bx^2+a}Bbx-\frac{1}{4}\left(3\,Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b*x - 1/4*(3*B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*
x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sqrt
(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2) - 6*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2
) + 3*B*a^4*sqrt(b) + 4*A*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^3
```

3.531 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5} dx$

Optimal. Leaf size=115

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b*x^2)^(3/2
))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(A*b + 4*a*B)*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] time = 0.0839492, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b*x^2)^(3/2
))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(A*b + 4*a*B)*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}+\frac{(Ab+4aB)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x^2\right)}{8a}\\
&= -\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}+\frac{(3b(Ab+4aB))\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)}{16a}\\
&= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a}-\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}+\frac{1}{16}(3b(Ab+4aB))\\
&= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a}-\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}+\frac{1}{8}(3(Ab+4aB))\,\mathrm{S}\\
&= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a}-\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}-\frac{3b(Ab+4aB)\tan}{8\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.0275285, size = 59, normalized size = 0.51

$$\frac{\left(a+bx^2\right)^{5/2}\left(bx^4(4aB+Ab)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)-5a^2A\right)}{20a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^4*Hypergeometric2F1[2, 5/2
, 7/2, 1 + (b*x^2)/a]))/(20*a^3*x^4)
```

Maple [A] time = 0.01, size = 184, normalized size = 1.6

$$-\frac{A}{4\,a{x}^{4}}\left(b{x}^{2}+a\right)^{\frac{5}{2}}-\frac{Ab}{8\,{a}^{2}{x}^{2}}\left(b{x}^{2}+a\right)^{\frac{5}{2}}+\frac{A{b}^{2}}{8\,{a}^{2}}\left(b{x}^{2}+a\right)^{\frac{3}{2}}-\frac{3\,A{b}^{2}}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{3\,A{b}^{2}}{8\,a}\sqrt{b{x}^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(5/2)/a/x^4-1/8*A*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*A*b^2/a^2*
(b*x^2+a)^(3/2)-3/8*A*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/8
*A*b^2/a*(b*x^2+a)^(1/2)-1/2*B/a/x^2*(b*x^2+a)^(5/2)+1/2*B*b/a*(b*x^2+a)^(3
/2)-3/2*B*b*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*B*b*(b*x^2+a)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64899, size = 436, normalized size = 3.79

$$\left[\frac{3\left(4\,Bab+Ab^{2}\right)\sqrt{a}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(8\,Babx^{4}-2\,Aa^{2}-\left(4\,Ba^{2}+5\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,ax^{4}},\frac{3\left(4\,Bab+Ab^{2}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(
b*x^2 + a))/(a*x^4), 1/8*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(b
*x^2 + a))/(a*x^4)]
```

Sympy [B] time = 56.6813, size = 216, normalized size = 1.88

$$-\frac{Aa^{2}}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}-\frac{3B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Ba\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*x**3*sqrt(
a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)/(8*x*
sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - 3
*B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(2*x) + B*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(b*x**
2) + 1)
```

Giac [A] time = 1.14449, size = 177, normalized size = 1.54

$$\frac{8\sqrt{bx^2+a}Bb^2 + \frac{3\left(4Bab^2+Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab^2 - 4\sqrt{bx^2+a}Ba^2b^2 + 5\left(bx^2+a\right)^{\frac{3}{2}}Ab^3 - 3\sqrt{bx^2+a}Aab^3}{b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^2 + a)
/sqrt(-a))/sqrt(-a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^
2*b^2 + 5*(b*x^2 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^2 + a)*A*a*b^3)/(b^2*x^4))/b
```

3.532 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6} dx$

Optimal. Leaf size=86

$$-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{bB\sqrt{a+bx^2}}{x}$$

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(a + b*x^2)
^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0346065, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{bB\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(a + b*x^2)
^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}dx = -\frac{A(a+bx^2)^{5/2}}{5ax^5} + B\int \frac{(a+bx^2)^{3/2}}{x^4}dx$$
$$= -\frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + (bB)\int \frac{\sqrt{a+bx^2}}{x^2}dx$$
$$= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + (b^2B)\int \frac{1}{\sqrt{a+bx^2}}dx$$
$$= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + (b^2B)\,\text{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+b}}\right.$$
$$= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.0355824, size = 76, normalized size = 0.88

$$-\frac{A(a+bx^2)^{5/2}}{5ax^5} - \frac{aB\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(5*a*x^5) - (a*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-
3/2, -3/2, -1/2, -((b*x^2)/a)])/(3*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.01, size = 115, normalized size = 1.3

$$-\frac{B}{3ax^3}(bx^2+a)^{\frac{5}{2}} - \frac{2Bb}{3a^2x}(bx^2+a)^{\frac{5}{2}} + \frac{2b^2Bx}{3a^2}(bx^2+a)^{\frac{3}{2}} + \frac{b^2Bx}{a}\sqrt{bx^2+a} + Bb^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right) - \frac{A}{5ax^5}(bx^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/3*B/a/x^3*(b*x^2+a)^(5/2)-2/3*B*b/a^2/x*(b*x^2+a)^(5/2)+2/3*B*b^2/a^2*x*
(b*x^2+a)^(3/2)+B*b^2/a*x*(b*x^2+a)^(1/2)+B*b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))-1/5*A*(b*x^2+a)^(5/2)/a/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63017, size = 439, normalized size = 5.1

$$\left[\frac{15\,Bab^{\frac{3}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(\left(20\,Bab+3\,Ab^2\right)x^4+3\,Aa^2+\left(5\,Ba^2+6\,Aab\right)x^2\right)\sqrt{bx^2+a}}{30\,ax^5},-\frac{15\,B}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
 2*((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt(b*x^
2 + a))/(a*x^5), -1/15*(15*B*a*sqrt(-b)*b*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) + ((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt
(b*x^2 + a))/(a*x^5)]
```

Sympy [B] time = 4.28487, size = 184, normalized size = 2.14

$$-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}-\frac{B\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*sqrt(a)*b/(x*sqrt(
1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqr
t(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(sqrt(
a)*sqrt(1 + b*x**2/a))
```

Giac [B] time = 1.14851, size = 319, normalized size = 3.71

$$-\frac{1}{2}\,Bb^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(30*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5
/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 110*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*
b^(5/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 20*B*a^5*b^(3/
2) + 3*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.533 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=120

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-6aB)}{24ax^4}+\frac{b\sqrt{a+bx^2}(Ab-6aB)}{16ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a + b*x^2)^(
3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b^2*(A*b - 6*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] time = 0.0961466, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-6aB)}{24ax^4}+\frac{b\sqrt{a+bx^2}(Ab-6aB)}{16ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a + b*x^2)^(
3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b^2*(A*b - 6*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,x,x^2\right)}{6a}$$
$$= \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{(b(Ab-6aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)}{16a}$$
$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{\left(b^2(Ab-6aB)\right)\,\text{S}}{}$$
$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{(b(Ab-6aB))\,\text{Su}}{}$$
$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} + \frac{b^2(Ab-6aB)\,\text{ta}}{16a^3}$$

Mathematica [A] time = 0.0804227, size = 119, normalized size = 0.99

$$\frac{-\left(a+bx^2\right)\left(4a^2\left(2A+3Bx^2\right)+2abx^2\left(7A+15Bx^2\right)+3Ab^2x^4\right)-3b^2x^6\sqrt{\frac{bx^2}{a}+1}(6aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{48ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (-((a + b*x^2)*(3*A*b^2*x^4 + 4*a^2*(2*A + 3*B*x^2) + 2*a*b*x^2*(7*A + 15*B
*x^2))) - 3*b^2*(-(A*b) + 6*a*B)*x^6*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (
b*x^2)/a]])/(48*a*x^6*Sqrt[a + b*x^2])
```

Maple [B] time = 0.011, size = 233, normalized size = 1.9

$$-\frac{A}{6\,ax^6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ab}{24\,a^2x^4}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ab^2}{48\,a^3x^2}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Ab^3}{48\,a^3}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(5/2)/a/x^6+1/24*A*b/a^2/x^4*(b*x^2+a)^(5/2)+1/48*A*b^2/a^
3/x^2*(b*x^2+a)^(5/2)-1/48*A*b^3/a^3*(b*x^2+a)^(3/2)+1/16*A*b^3/a^(3/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/16*A*b^3/a^2*(b*x^2+a)^(1/2)-1/4*B/a/x
^4*(b*x^2+a)^(5/2)-1/8*B*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*B*b^2/a^2*(b*x^2+a)^
(3/2)-3/8*B*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/8*B*b^2/a*(
b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70586, size = 506, normalized size = 4.22

$$\left[-\frac{3\left(6\,Bab^2-Ab^3\right)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(10\,Ba^2b+Aab^2\right)x^4+8\,Aa^3+2\left(6\,Ba^3+7\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{96\,a^2x^6},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(10*B*a^2*b + A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), 1/48*(3*(6*B*a*b^2 - A*b^3)*
sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(10*B*a^2*b + A*a*b^2)*x
^4 + 8*A*a^3 + 2*(6*B*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

Sympy [B] time = 82.4467, size = 253, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{11Aa\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{17Ab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}}-\frac{Ba^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ba\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)/(24*x**5*sqr
t(a/(b*x**2) + 1)) - 17*A*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(5
/2)/(16*a*x*sqrt(a/(b*x**2) + 1)) + A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a
**(3/2)) - B*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*a*sqrt(b)/(8*
x**3*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(
3/2)/(8*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sq
rt(a))
```

Giac [A] time = 1.12376, size = 215, normalized size = 1.79

$$\frac{\frac{3\left(6Bab^3-Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{30\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-48\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+18\sqrt{bx^2+a}Ba^3b^3+3\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+8\left(bx^2+a\right)^{\frac{3}{2}}Aab^4-3\sqrt{bx^2+a}Aa^2b^4}{ab^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) -
 (30*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 18*sqrt(b
*x^2 + a)*B*a^3*b^3 + 3*(b*x^2 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b
^4 - 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(a*b^3*x^6))/b
```

3.534 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

```
[Out] -(A*(a + b*x^2)^(5/2))/(7*a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(5/2))/(35*
a^2*x^5)
```

Rubi [A] time = 0.0221359, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{(a+bx^2)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(7*a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(5/2))/(35*
a^2*x^5)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx &= -\frac{A(a+bx^2)^{5/2}}{7ax^7}-\frac{(2Ab-7aB)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{7a}\\ &= -\frac{A(a+bx^2)^{5/2}}{7ax^7}+\frac{(2Ab-7aB)(a+bx^2)^{5/2}}{35a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.0170152, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{5/2}(5aA+7aBx^2-2Abx^2)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((a + b*x^2)^(5/2)*(5*a*A - 2*A*b*x^2 + 7*a*B*x^2))/(35*a^2*x^7)
```

Maple [A] time = 0.004, size = 37, normalized size = 0.7

$$-\frac{-2\,Abx^2+7\,Bax^2+5\,Aa}{35\,x^7a^2}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-2*A*b*x^2+7*B*a*x^2+5*A*a)/x^7/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63191, size = 173, normalized size = 3.26

$$-\frac{\left(\left(7\,Bab^2-2\,Ab^3\right)x^6+\left(14\,Ba^2b+Aab^2\right)x^4+5\,Aa^3+\left(7\,Ba^3+8\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*a*b^2 - 2*A*b^3)*x^6 + (14*B*a^2*b + A*a*b^2)*x^4 + 5*A*a^3 + (
7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^7)
```

Sympy [B] time = 4.67851, size = 518, normalized size = 9.77

$$-\frac{15Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(10
5*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(3/2)*s
qrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2)
+ 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a*sqrt(b)*sqrt(a/(b*x**2)
 + 1)/(5*x**4) - 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*sq
rt(a/(b*x**2) + 1)/(5*a)
```

Giac [B] time = 1.1331, size = 464, normalized size = 8.75

$$\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{7}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*B*a*b^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(7/2)
+ 105*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2) + 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*A*a*b^(7/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/
2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(7/2) + 77*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a^4*b^(5/2) + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b
^(7/2) - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 14*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 7*B*a^6*b^(5/2) - 2*A*a^5*b^(7/2))/((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.535 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=156

$$\frac{b^2\sqrt{a+bx^2}(3Ab-8aB)}{128a^2x^2}-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b\sqrt{a+bx^2}(3Ab-8aB)}{64ax^4}+\frac{(a+bx^2)^{3/2}(3Ab-8aB)}{48ax^6}-$$

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*a*B)*Sqrt[
a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2)^(3/2))/(48*a*x^6)
- (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A*b - 8*a*B)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rubi [A] time = 0.123087, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^2}(3Ab-8aB)}{128a^2x^2}-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b\sqrt{a+bx^2}(3Ab-8aB)}{64ax^4}+\frac{(a+bx^2)^{3/2}(3Ab-8aB)}{48ax^6}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*a*B)*Sqrt[
a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2)^(3/2))/(48*a*x^6)
- (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A*b - 8*a*B)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^9}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx,x,x^2\right)$$

$$=-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}+\frac{\left(-\frac{3Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x,x^2\right)}{8a}$$

$$=\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}-\frac{(b(3Ab-8aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)}{32a}$$

$$=\frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}-\frac{\left(b^2(3Ab-8aB)\right)}{}$$

$$=\frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+\right.}{8a}$$

$$=\frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+\right.}{8a}$$

$$=\frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+\right.}{8a}$$

Mathematica [C] time = 0.0244022, size = 62, normalized size = 0.4

$$-\frac{\left(a+bx^2\right)^{5/2}\left(5a^4A+b^3x^8(3Ab-8aB)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^2}{a}+1\right)\right)}{40a^5x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] -((a + b*x^2)^(5/2)*(5*a^4*A + b^3*(3*A*b - 8*a*B)*x^8*Hypergeometric2F1[5/
2, 4, 7/2, 1 + (b*x^2)/a]))/(40*a^5*x^8)
```

Maple [B] time = 0.017, size = 275, normalized size = 1.8

$$-\frac{A}{8\,a x^8}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{A b}{16\,a^2 x^6}\left(b x^2+a\right)^{\frac{5}{2}}-\frac{A b^2}{64\,a^3 x^4}\left(b x^2+a\right)^{\frac{5}{2}}-\frac{A b^3}{128\,a^4 x^2}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{A b^4}{128\,a^4}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{3\,A b^4}{128}\ln\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(5/2)/a/x^8+1/16*A*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*A*b^2/a^
3/x^4*(b*x^2+a)^(5/2)-1/128*A*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*A*b^4/a^4*(
b*x^2+a)^(3/2)-3/128*A*b^4/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/
128*A*b^4/a^3*(b*x^2+a)^(1/2)-1/6*B/a/x^6*(b*x^2+a)^(5/2)+1/24*B*b/a^2/x^4*
(b*x^2+a)^(5/2)+1/48*B*b^2/a^3/x^2*(b*x^2+a)^(5/2)-1/48*B*b^3/a^3*(b*x^2+a)
^(3/2)+1/16*B*b^3/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/16*B*b^3/
a^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7552, size = 621, normalized size = 3.98

$$\left[-\frac{3\left(8\,B a b^3-3\,A b^4\right)\sqrt{a}x^8\log\left(-\frac{b x^2-2\,\sqrt{b x^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(8\,B a^2 b^2-3\,A a b^3\right)x^6+48\,A a^4+2\left(56\,B a^3 b+3\,A a^2 b^2\right)x^4\right.}{768\,a^3 x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
56*B*a^3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a
))/(a^3*x^8), -1/384*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(56*B*a^
3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

Sympy [B] time = 136.73, size = 287, normalized size = 1.84

$$-\frac{A a^2}{8\sqrt{b}x^9\sqrt{\frac{a}{b x^2}+1}}-\frac{5 A a\sqrt{b}}{16 x^7\sqrt{\frac{a}{b x^2}+1}}-\frac{13 A b^{\frac{3}{2}}}{64 x^5\sqrt{\frac{a}{b x^2}+1}}+\frac{A b^{\frac{5}{2}}}{128 a x^3\sqrt{\frac{a}{b x^2}+1}}+\frac{3 A b^{\frac{7}{2}}}{128 a^2 x\sqrt{\frac{a}{b x^2}+1}}-\frac{3 A b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b x}}\right)}{128 a^{\frac{5}{2}}}-\frac{\ldots}{6\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*A*a*sqrt(b)/(16*x**7*sqrt
(a/(b*x**2) + 1)) - 13*A*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(5/
2)/(128*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(7/2)/(128*a**2*x*sqrt(a/(b*x
**2) + 1)) - 3*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2)) - B*a**2/(6
*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*B*a*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) - 17*B*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(5/2)/(16*a*
x*sqrt(a/(b*x**2) + 1)) + B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

Giac [A] time = 1.14682, size = 262, normalized size = 1.68

$$-\frac{\frac{3\left(8\,Bab^4-3\,Ab^5\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{24\left(bx^2+a\right)^{\frac{7}{2}}Bab^4+40\left(bx^2+a\right)^{\frac{5}{2}}Ba^2b^4-88\left(bx^2+a\right)^{\frac{3}{2}}Ba^3b^4+24\sqrt{bx^2+a}Ba^4b^4-9\left(bx^2+a\right)^{\frac{7}{2}}Ab^5+33\left(bx^2+a\right)^{\frac{5}{2}}Aab^5+33\,(b}{a^2b^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*
a^2) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 + 40*(b*x^2 + a)^(5/2)*B*a^2*b^4 - 88*
(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 9*(b*x^2 + a)^
(7/2)*A*b^5 + 33*(b*x^2 + a)^(5/2)*A*a*b^5 + 33*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 9*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^2*b^4*x^8))/b
```

3.536 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}} dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(5/2))/(9*a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(63*
a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rubi [A] time = 0.0361016, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(9*a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(63*
a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{10}}dx = -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9} - \frac{(4Ab-9aB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^8}dx}{9a}$$
$$= -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{63a^2x^7} + \frac{(2b(4Ab-9aB))\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}dx}{63a^2}$$
$$= -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{63a^2x^7} - \frac{2b(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{315a^3x^5}$$

Mathematica [A] time = 0.0330151, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{5/2}\left(-5a^2\left(7A+9Bx^2\right)+2abx^2\left(10A+9Bx^2\right)-8Ab^2x^4\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-8*A*b^2*x^4 - 5*a^2*(7*A + 9*B*x^2) + 2*a*b*x^2*(10*A
+ 9*B*x^2)))/(315*a^3*x^9)
```

Maple [A] time = 0.005, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-18\,Bx^4ab-20\,aAbx^2+45\,Bx^2a^2+35\,Aa^2}{315\,x^9a^3}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*A*b^2*x^4-18*B*a*b*x^4-20*A*a*b*x^2+45*B*a^2*x^2+
35*A*a^2)/x^9/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80658, size = 230, normalized size = 2.74

$$\frac{\left(2\left(9\,Bab^3-4\,Ab^4\right)x^8-\left(9\,Ba^2b^2-4\,Aab^3\right)x^6-35\,Aa^4-3\left(24\,Ba^3b+Aa^2b^2\right)x^4-5\left(9\,Ba^4+10\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*a*b^3 - 4*A*b^4)*x^8 - (9*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 35*A*a
^4 - 3*(24*B*a^3*b + A*a^2*b^2)*x^4 - 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^3*x^9)
```

Sympy [B] time = 6.33846, size = 1408, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 17*A*
a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a*b**(33/2)*x
**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*
a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 12*A*a*b**(19/2)*x**8*sqrt(a/(b*
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 8*A*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b
**5*x**8 + 105*a**3*b**6*x**10) - 15*B*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**5*
b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 17*B*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(1
05*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**3*b*
*(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 12*B*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105
*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a*b**(19/
2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(5/2)*
sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*
*2)
```

Giac [B] time = 1.18314, size = 540, normalized size = 6.43

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{7}{2}}+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*b^(7/2) - 315*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*B*a*b^(7/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(9
/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(7/2) + 1260*(sqrt(b)*x
- sqrt(b*x^2 + a))^10*A*a*b^(9/2) - 819*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a
^3*b^(7/2) + 1764*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) + 441*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(7/2) + 504*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*A*a^3*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2) + 144*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) + 81*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(7/2) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(9/2) - 9
*B*a^7*b^(7/2) + 4*A*a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.537 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=184

$$-\frac{3b^3\sqrt{a+bx^2}(Ab-2aB)}{256a^3x^2}+\frac{b^2\sqrt{a+bx^2}(Ab-2aB)}{128a^2x^4}+\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{32ax^6}+\frac{(a$$

```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)*Sqrt[a +
b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(256*a^3*x^2)
 + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^8) - (A*(a + b*x^2)^(5/2))/(10
*a*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7
/2))
```

Rubi [A] time = 0.145562, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{3b^3\sqrt{a+bx^2}(Ab-2aB)}{256a^3x^2}+\frac{b^2\sqrt{a+bx^2}(Ab-2aB)}{128a^2x^4}+\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{32ax^6}+\frac{(a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```


```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)*Sqrt[a +
b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(256*a^3*x^2)
 + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^8) - (A*(a + b*x^2)^(5/2))/(10
*a*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7
/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} + \frac{\left(-\frac{5Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,x,x^2\right)}{10a}$$

$$= \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} - \frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} - \frac{(3b(Ab-2aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)}{32a}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} - \frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} - \frac{\left(b^2(Ab-2aB)\right)\text{Sub}}{}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} + \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} - \frac{A\left(a+bx^2\right.}{10ax^{10}}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} + \frac{(Ab-2aB}{16}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} + \frac{(Ab-2aB}{16}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} + \frac{(Ab-2aB}{16}$$

Mathematica [C] time = 0.0267115, size = 61, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{5/2}\left(a^5A+b^4x^{10}(2aB-Ab)\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx^2}{a}+1\right)\right)}{10a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -((a + b*x^2)^(5/2)*(a^5*A + b^4*(-(A*b) + 2*a*B)*x^10*Hypergeometric2F1[5/
2, 5, 7/2, 1 + (b*x^2)/a]))/(10*a^6*x^10)
```

Maple [B] time = 0.027, size = 317, normalized size = 1.7

$$-\frac{A}{10\,a x^{10}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b}{16\,a^{2} x^{8}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{A b^{2}}{32\,a^{3} x^{6}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b^{3}}{128\,a^{4} x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b^{4}}{256\,a^{5} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{A b^{5}}{256\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(5/2)/a/x^10+1/16*A*b/a^2/x^8*(b*x^2+a)^(5/2)-1/32*A*b^2/
a^3/x^6*(b*x^2+a)^(5/2)+1/128*A*b^3/a^4/x^4*(b*x^2+a)^(5/2)+1/256*A*b^4/a^5
/x^2*(b*x^2+a)^(5/2)-1/256*A*b^5/a^5*(b*x^2+a)^(3/2)+3/256*A*b^5/a^(7/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-3/256*A*b^5/a^4*(b*x^2+a)^(1/2)-1/8*B/a
/x^8*(b*x^2+a)^(5/2)+1/16*B*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*B*b^2/a^3/x^4*(b
*x^2+a)^(5/2)-1/128*B*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*B*b^4/a^4*(b*x^2+a)
^(3/2)-3/128*B*b^4/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/128*B*b^
4/a^3*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0658, size = 722, normalized size = 3.92

$$\left[-\frac{15\left(2\,Bab^{4}-Ab^{5}\right)\sqrt{a}x^{10}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\left(15\left(2\,Ba^{2}b^{3}-Aab^{4}\right)x^{8}-10\left(2\,Ba^{3}b^{2}-Aa^{2}b^{3}\right)x^{6}-128\,Aa^{5}-\ldots\right.}{2560\,a^{4}x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*a*b^4 - A*b^5)*sqrt(a)*x^10*log(-(b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(a) + 2*a)/x^2) - 2*(15*(2*B*a^2*b^3 - A*a*b^4)*x^8 - 10*(2*B*a^3*b^
2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*b + A*a^3*b^2)*x^4 - 16*(10*B*
a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10), 1/1280*(15*(2*B*a*b^4 -
 A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(2*B*a^2*b^3 -
 A*a*b^4)*x^8 - 10*(2*B*a^3*b^2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*
b + A*a^3*b^2)*x^4 - 16*(10*B*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*
```

```
x^10)]
```

Sympy [B] time = 162.342, size = 345, normalized size = 1.88

$$-\frac{Aa^2}{10\sqrt{b}x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{19Aa\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23Ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{5}{2}}}{640ax^5\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{7}{2}}}{256a^2x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ab^{\frac{9}{2}}}{256a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{3A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

```
[Out] -A*a**2/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 19*A*a*sqrt(b)/(80*x**9*s
qrt(a/(b*x**2) + 1)) - 23*A*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) + A*b*
*(5/2)/(640*a*x**5*sqrt(a/(b*x**2) + 1)) - A*b**(7/2)/(256*a**2*x**3*sqrt(a
/(b*x**2) + 1)) - 3*A*b**(9/2)/(256*a**3*x*sqrt(a/(b*x**2) + 1)) + 3*A*b**5
*asinh(sqrt(a)/(sqrt(b)*x))/(256*a**(7/2)) - B*a**2/(8*sqrt(b)*x**9*sqrt(a/
(b*x**2) + 1)) - 5*B*a*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 13*B*b**(3/
2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(128*a*x**3*sqrt(a/(b*x**2)
+ 1)) + 3*B*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**4*asinh(sqr
t(a)/(sqrt(b)*x))/(128*a**(5/2))
```

Giac [A] time = 1.12723, size = 286, normalized size = 1.55

$$\frac{\frac{15\left(2Bab^5-Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{30\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-140\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+140\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-30\sqrt{bx^2+a}Ba^5b^5-15\left(bx^2+a\right)^{\frac{9}{2}}Ab^6+70\left(bx^2+a\right)^{\frac{7}{2}}Aab^6-128}{a^3b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*a*b^5 - A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (30*(b*x^2 + a)^(9/2)*B*a*b^5 - 140*(b*x^2 + a)^(7/2)*B*a^2*b^5 + 140
*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 30*sqrt(b*x^2 + a)*B*a^5*b^5 - 15*(b*x^2 + a
)^(9/2)*A*b^6 + 70*(b*x^2 + a)^(7/2)*A*a*b^6 - 128*(b*x^2 + a)^(5/2)*A*a^2*
b^6 - 70*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 15*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^3*b
^5*x^10))/b
```

3.538 $\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=103

$$\frac{a^2 \left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2} (Ab-3aB)}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2} (2Ab-3aB)}{9b^4}+\frac{B \left(a+bx^2\right)^{13/2}}{13b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(11*b^4) + (B*(a + b*
x^2)^(13/2))/(13*b^4)
```

Rubi [A] time = 0.0763276, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2 \left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2} (Ab-3aB)}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2} (2Ab-3aB)}{9b^4}+\frac{B \left(a+bx^2\right)^{13/2}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(11*b^4) + (B*(a + b*
x^2)^(13/2))/(13*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx = \frac{1}{2} \text{Subst}\left(\int x^2(a+bx)^{5/2}(A+Bx)\,dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{5/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{7/2}}{b^3}+\frac{(Ab-3aB}{b}\right.\right.$$
$$= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{11/2}}{11b^4}-$$

Mathematica [A] time = 0.0622146, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2b\left(13A+21Bx^2\right)-48a^3B-14ab^2x^2\left(26A+27Bx^2\right)+63b^3x^4\left(13A+11Bx^2\right)\right)}{9009b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-48*a^3*B + 63*b^3*x^4*(13*A + 11*B*x^2) + 8*a^2*b*(13*
A + 21*B*x^2) - 14*a*b^2*x^2*(26*A + 27*B*x^2)))/(9009*b^4)
```

Maple [A] time = 0.005, size = 77, normalized size = 0.8

$$\frac{693\,Bx^6b^3+819\,Ab^3x^4-378\,Bab^2x^4-364\,Aab^2x^2+168\,Ba^2bx^2+104\,Aa^2b-48\,Ba^3}{9009\,b^4}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/9009*(b*x^2+a)^(7/2)*(693*B*b^3*x^6+819*A*b^3*x^4-378*B*a*b^2*x^4-364*A*a
*b^2*x^2+168*B*a^2*b*x^2+104*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63686, size = 340, normalized size = 3.3

$$\frac{\left(693\,Bb^6x^{12}+63\left(27\,Bab^5+13\,Ab^6\right)x^{10}+7\left(159\,Ba^2b^4+299\,Aab^5\right)x^8-48\,Ba^6+104\,Aa^5b+\left(15\,Ba^3b^3+1469\,Aa^2b^4\right)x\right.}{9009\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*B*b^6*x^12 + 63*(27*B*a*b^5 + 13*A*b^6)*x^10 + 7*(159*B*a^2*b^4
 + 299*A*a*b^5)*x^8 - 48*B*a^6 + 104*A*a^5*b + (15*B*a^3*b^3 + 1469*A*a^2*b
^4)*x^6 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + 4*(6*B*a^5*b - 13*A*a^4*b^2)
*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 9.15945, size = 313, normalized size = 3.04

$$\begin{cases} \frac{8Aa^5\sqrt{a+bx^2}}{693b^3} - \frac{4Aa^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{Aa^3x^4\sqrt{a+bx^2}}{231b} + \frac{113Aa^2x^6\sqrt{a+bx^2}}{693} + \frac{23Aabx^8\sqrt{a+bx^2}}{99} + \frac{Ab^2x^{10}\sqrt{a+bx^2}}{11} - \frac{16Ba^6\sqrt{a+bx^2}}{3003b^4} + \frac{8Ba^5x^2\sqrt{a+bx^2}}{3003b^3} \\ a^{\frac{5}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*A*a**4*x**2*sqrt(a + b*
x**2)/(693*b**2) + A*a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*A*a**2*x**6*s
qrt(a + b*x**2)/693 + 23*A*a*b*x**8*sqrt(a + b*x**2)/99 + A*b**2*x**10*sqrt
(a + b*x**2)/11 - 16*B*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*B*a**5*x**2*sq
rt(a + b*x**2)/(3003*b**3) - 2*B*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5
*B*a**3*x**6*sqrt(a + b*x**2)/(3003*b) + 53*B*a**2*x**8*sqrt(a + b*x**2)/42
9 + 27*B*a*b*x**10*sqrt(a + b*x**2)/143 + B*b**2*x**12*sqrt(a + b*x**2)/13,
 Ne(b, 0)), (a**(5/2)*(A*x**6/6 + B*x**8/8), True))
```

Giac [B] time = 1.14551, size = 545, normalized size = 5.29

$$\frac{\frac{429\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa^2}{b^2}+\frac{143\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)Ba^2}{b^3}+\frac{286\left(35\left(bx^2+a\right)^{\frac{9}{2}}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/45045*(429*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a
)^(3/2)*a^2)*A*a^2/b^2 + 143*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*
a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B*a^2/b^3 + 286*
(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2
 - 105*(b*x^2 + a)^(3/2)*a^3)*A*a/b^2 + 26*(315*(b*x^2 + a)^(11/2) - 1540*(
b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^
3 + 1155*(b*x^2 + a)^(3/2)*a^4)*B*a/b^3 + 13*(315*(b*x^2 + a)^(11/2) - 1540
*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*
a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*A/b^2 + 5*(693*(b*x^2 + a)^(13/2) - 4095*
(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)^(9/2)*a^2 - 12870*(b*x^2 + a)^(7/2
)*a^3 + 9009*(b*x^2 + a)^(5/2)*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)*B/b^3)/b
```

3.539 $\int x^4 (a + bx^2)^{5/2} (A + Bx^2) \, dx$

Optimal. Leaf size=221

$$-\frac{a^4x\sqrt{a+bx^2}(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}(12Ab-5aB)}{1536b^2}+\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{a^2x^5\sqrt{a+bx^2}(12Ab-}{384b}$$

```
[Out] -(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(1024*b^3) + (a^3*(12*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*B)*x^5*Sqrt[a + b*x^
2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*x^2)^(3/2))/(192*b) + ((12*A*b
 - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(120*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b)
 + (a^5*(12*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2
))
```

Rubi [A] time = 0.10279, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^4x\sqrt{a+bx^2}(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}(12Ab-5aB)}{1536b^2}+\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{a^2x^5\sqrt{a+bx^2}(12Ab-}{384b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```


```
[Out] -(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(1024*b^3) + (a^3*(12*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*B)*x^5*Sqrt[a + b*x^
2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*x^2)^(3/2))/(192*b) + ((12*A*b
 - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(120*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b)
 + (a^5*(12*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2
))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx &= \frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}-\frac{(-12Ab+5aB)\int x^4\left(a+bx^2\right)^{5/2}dx}{12b}\\
&= \frac{(12Ab-5aB)x^5\left(a+bx^2\right)^{5/2}}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}+\frac{(a(12Ab-5aB))\int x^4\left(a+bx^2\right)^{3}}{24b}\\
&= \frac{a(12Ab-5aB)x^5\left(a+bx^2\right)^{3/2}}{192b}+\frac{(12Ab-5aB)x^5\left(a+bx^2\right)^{5/2}}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}+\\
&= \frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab-5aB)x^5\left(a+bx^2\right)^{3/2}}{192b}+\frac{(12Ab-5aB)x^5\left(a\right.}{120b}\\
&= \frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab-5aB)x^5\left(a+\right.}{192b}\\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a}}{384b}\\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a}}{384b}\\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a}}{384b}
\end{aligned}$$

Mathematica [A] time = 0.346228, size = 172, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(48a^2b^3x^4\left(62A+45Bx^2\right)+40a^3b^2x^2\left(3A+Bx^2\right)-10a^4b\left(18A+5Bx^2\right)+75a^5B+64ab^4x^6\left(63A+5\right.\right.\right.}{15360b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(75*a^5*B + 40*a^3*b^2*x^2*(3*A + B*x^2) + 256*
b^5*x^8*(6*A + 5*B*x^2) - 10*a^4*b*(18*A + 5*B*x^2) + 48*a^2*b^3*x^4*(62*A
+ 45*B*x^2) + 64*a*b^4*x^6*(63*A + 50*B*x^2)) - (15*a^(9/2)*(-12*A*b + 5*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(15360*b^(7/2))
```

Maple [A] time = 0.012, size = 257, normalized size = 1.2

$$\frac{Bx^5}{12\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Bax^3}{24\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2Bx}{64\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Ba^3x}{384\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,Ba^4x}{1536\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,Ba^5x}{1024\,b^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/12*B*x^5*(b*x^2+a)^(7/2)/b-1/24*B/b^2*a*x^3*(b*x^2+a)^(7/2)+1/64*B/b^3*a^
2*x*(b*x^2+a)^(7/2)-1/384*B/b^3*a^3*x*(b*x^2+a)^(5/2)-5/1536*B/b^3*a^4*x*(b
*x^2+a)^(3/2)-5/1024*B/b^3*a^5*x*(b*x^2+a)^(1/2)-5/1024*B/b^(7/2)*a^6*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))+1/10*A*x^3*(b*x^2+a)^(7/2)/b-3/80*A/b^2*a*x*(b*x^2
+a)^(7/2)+1/160*A/b^2*a^2*x*(b*x^2+a)^(5/2)+1/128*A/b^2*a^3*x*(b*x^2+a)^(3/
2)+3/256*A/b^2*a^4*x*(b*x^2+a)^(1/2)+3/256*A/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.452, size = 844, normalized size = 3.82

$$\left[-\frac{15\left(5Ba^6-12Aa^5b\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(1280Bb^6x^{11}+128\left(25Bab^5+12Ab^6\right)x^9+144\left(15Ba^2\right.\right.}{30720b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9 +
144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^5 -
 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*sqr
t(b*x^2 + a))/b^4, 1/15360*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9
 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^
5 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*
sqrt(b*x^2 + a))/b^4]
```

Sympy [A] time = 61.4179, size = 405, normalized size = 1.83

$$-\frac{3Aa^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Aa^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Aa^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29A\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^5\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Ab^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - A*a**(7/2)*x**3/(256*b*sqrt
(1 + b*x**2/a)) + 129*A*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*A*a**(3
/2)*b*x**7/(160*sqrt(1 + b*x**2/a)) + 29*A*sqrt(a)*b**2*x**9/(80*sqrt(1 + b
*x**2/a)) + 3*A*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + A*b**3*x**11
/(10*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(11/2)*x/(1024*b**3*sqrt(1 + b*x*
*2/a)) + 5*B*a**(9/2)*x**3/(3072*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**5
/(1536*b*sqrt(1 + b*x**2/a)) + 55*B*a**(5/2)*x**7/(384*sqrt(1 + b*x**2/a))
+ 67*B*a**(3/2)*b*x**9/(192*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)*b**2*x**11/(2
4*sqrt(1 + b*x**2/a)) - 5*B*a**6*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(7/2)) +
 B*b**3*x**13/(12*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.62696, size = 263, normalized size = 1.19

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bb^2x^2+\frac{25\,Bab^{11}+12\,Ab^{12}}{b^{10}}\right)x^2+\frac{9\left(15\,Ba^2b^{10}+28\,Aab^{11}\right)}{b^{10}}\right)x^2+\frac{5\,Ba^3b^9+372\,Aa^2b^{10}}{b^{10}}\right)x^2-\frac{5}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*b^2*x^2 + (25*B*a*b^11 + 12*A*b^12)/b^10)*x^2 + 9
*(15*B*a^2*b^10 + 28*A*a*b^11)/b^10)*x^2 + (5*B*a^3*b^9 + 372*A*a^2*b^10)/b
^10)*x^2 - 5*(5*B*a^4*b^8 - 12*A*a^3*b^9)/b^10)*x^2 + 15*(5*B*a^5*b^7 - 12*
A*a^4*b^8)/b^10)*sqrt(b*x^2 + a)*x + 1/1024*(5*B*a^6 - 12*A*a^5*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.540 $\int x^3 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{9/2}(Ab-2aB)}{9b^3}-\frac{a\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(9/
2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rubi [A] time = 0.0556794, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{9/2}(Ab-2aB)}{9b^3}-\frac{a\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(9/
2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^{5/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{7/2}}{b^2}+\frac{B(a+bx)^{9/2}}{b^2}\right)dx,x,\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.0424254, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2B-2ab\left(11A+14Bx^2\right)+7b^2x^2\left(11A+9Bx^2\right)\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(8*a^2*B + 7*b^2*x^2*(11*A + 9*B*x^2) - 2*a*b*(11*A + 14
*B*x^2)))/(693*b^3)
```

Maple [A] time = 0.005, size = 53, normalized size = 0.7

$$-\frac{-63\,b^2Bx^4-77\,Ab^2x^2+28\,Babx^2+22\,abA-8\,a^2B}{693\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] -1/693*(b*x^2+a)^(7/2)*(-63*B*b^2*x^4-77*A*b^2*x^2+28*B*a*b*x^2+22*A*a*b-8*
B*a^2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61244, size = 273, normalized size = 3.74

$$\frac{\left(63\,Bb^5x^{10}+7\left(23\,Bab^4+11\,Ab^5\right)x^8+\left(113\,Ba^2b^3+209\,Aab^4\right)x^6+8\,Ba^5-22\,Aa^4b+3\left(Ba^3b^2+55\,Aa^2b^3\right)x^4-\left(4\,B\right.\right.}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^10 + 7*(23*B*a*b^4 + 11*A*b^5)*x^8 + (113*B*a^2*b^3 + 209
*A*a*b^4)*x^6 + 8*B*a^5 - 22*A*a^4*b + 3*(B*a^3*b^2 + 55*A*a^2*b^3)*x^4 - (
4*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 5.9656, size = 260, normalized size = 3.56

$$\begin{cases} -\frac{2Aa^4\sqrt{a+bx^2}}{63b^2}+\frac{Aa^3x^2\sqrt{a+bx^2}}{63b}+\frac{5Aa^2x^4\sqrt{a+bx^2}}{21}+\frac{19Aabx^6\sqrt{a+bx^2}}{63}+\frac{Ab^2x^8\sqrt{a+bx^2}}{9}+\frac{8Ba^5\sqrt{a+bx^2}}{693b^3}-\frac{4Ba^4x^2\sqrt{a+bx^2}}{693b^2}+\frac{Ba^3x^4\sqrt{a+bx^2}}{231b}+\cdots \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**4*sqrt(a + b*x**2)/(63*b**2) + A*a**3*x**2*sqrt(a + b*x*
*2)/(63*b) + 5*A*a**2*x**4*sqrt(a + b*x**2)/21 + 19*A*a*b*x**6*sqrt(a + b*x
**2)/63 + A*b**2*x**8*sqrt(a + b*x**2)/9 + 8*B*a**5*sqrt(a + b*x**2)/(693*b
**3) - 4*B*a**4*x**2*sqrt(a + b*x**2)/(693*b**2) + B*a**3*x**4*sqrt(a + b*x
**2)/(231*b) + 113*B*a**2*x**6*sqrt(a + b*x**2)/693 + 23*B*a*b*x**8*sqrt(a
+ b*x**2)/99 + B*b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)*(A*x*
*4/4 + B*x**6/6), True))
```

Giac [B] time = 1.13096, size = 431, normalized size = 5.9

$$\frac{\frac{231\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Aa^2}{b}+\frac{33\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Ba^2}{b^2}+\frac{66\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa}{b}+\frac{22(3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3465*(231*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A*a^2/b + 33*(15*
(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*B*a^
2/b^2 + 66*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^
(3/2)*a^2)*A*a/b + 22*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189
*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B*a/b^2 + 11*(35*(b*x^2
 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*
x^2 + a)^(3/2)*a^3)*A/b + (315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*
a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 +
 a)^(3/2)*a^4)*B/b^2)/b
```

3.541 $\int x^2 \left(a + bx^2\right)^{5/2} \left(A + Bx^2\right) dx$

Optimal. Leaf size=188

$$\frac{a^3x\sqrt{a+bx^2}(10Ab-3aB)}{256b^2} - \frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}} + \frac{a^2x^3\sqrt{a+bx^2}(10Ab-3aB)}{128b} + \frac{ax^3\left(a+bx^2\right)^{3/2}(10A}{96b}$$

```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b - 3*a*B)*
x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(
96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/2))/(80*b) + (B*x^3*(a + b*x^2
)^(7/2))/(10*b) - (a^4*(10*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(256*b^(5/2))
```

Rubi [A] time = 0.08813, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^3x\sqrt{a+bx^2}(10Ab-3aB)}{256b^2} - \frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}} + \frac{a^2x^3\sqrt{a+bx^2}(10Ab-3aB)}{128b} + \frac{ax^3\left(a+bx^2\right)^{3/2}(10A}{96b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```


```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b - 3*a*B)*
x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(
96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/2))/(80*b) + (B*x^3*(a + b*x^2
)^(7/2))/(10*b) - (a^4*(10*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(256*b^(5/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx=\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{(-10Ab+3aB)\int x^2\left(a+bx^2\right)^{5/2}dx}{10b}$$

$$=\frac{(10Ab-3aB)x^3\left(a+bx^2\right)^{5/2}}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{(a(10Ab-3aB))\int x^2\left(a+bx^2\right)^{3/2}\ldots}{16b}$$

$$=\frac{a(10Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{96b}+\frac{(10Ab-3aB)x^3\left(a+bx^2\right)^{5/2}}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\ldots$$

$$=\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{96b}+\frac{(10Ab-3aB)x^3\left(a+b\ldots\right.}{80b}$$

$$=\frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\ldots\right.}{96b}$$

$$=\frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\ldots\right.}{96b}$$

$$=\frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\ldots\right.}{96b}$$

Mathematica [A] time = 0.295342, size = 151, normalized size = 0.8

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(4a^2b^2x^2\left(295A+186Bx^2\right)+30a^3b\left(5A+Bx^2\right)-45a^4B+16ab^3x^4\left(85A+63Bx^2\right)+96b^4x^6\left(5A+4Bx^2\ldots\right.\right.\right.}{3840b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-45*a^4*B + 30*a^3*b*(5*A + B*x^2) + 96*b^4*x^
6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(85*A + 63*B*x^2) + 4*a^2*b^2*x^2*(295*A +
 186*B*x^2)) + (15*a^(7/2)*(-10*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/
Sqrt[1 + (b*x^2)/a]))/(3840*b^(5/2))
```

Maple [A] time = 0.007, size = 215, normalized size = 1.1

$$\frac{Bx^3}{10b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3Bax}{80b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2Bx}{160b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ba^3x}{128b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3Ba^4x}{256b^2}\sqrt{bx^2+a}+\frac{3Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/10*B*x^3*(b*x^2+a)^(7/2)/b-3/80*B/b^2*a*x*(b*x^2+a)^(7/2)+1/160*B/b^2*a^2
*x*(b*x^2+a)^(5/2)+1/128*B/b^2*a^3*x*(b*x^2+a)^(3/2)+3/256*B/b^2*a^4*x*(b*x
^2+a)^(1/2)+3/256*B/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/8*A*x*(b*x^
2+a)^(7/2)/b-1/48*A/b*a*x*(b*x^2+a)^(5/2)-5/192*A/b*a^2*x*(b*x^2+a)^(3/2)-5
/128*A/b*a^3*x*(b*x^2+a)^(1/2)-5/128*A/b^(3/2)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99864, size = 732, normalized size = 3.89

$$\left[-\frac{15\left(3\,Ba^5-10\,Aa^4b\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(384\,Bb^5x^9+48\left(21\,Bab^4+10\,Ab^5\right)x^7+8\left(93\,Ba^2b^3\right.\right.}{7680\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(384*B*b^5*x^9 + 48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(9
3*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*
(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(3*B*a^5 -
10*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*B*b^5*x^9 +
48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3
*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x
^2 + a))/b^3]
```

Sympy [B] time = 40.913, size = 348, normalized size = 1.85

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) - 3*B*a**(9/2)*x/
(256*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a))
```

```
+ 129*B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*
sqrt(1 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*
a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [A] time = 1.15397, size = 223, normalized size = 1.19

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bb^2x^2+\frac{21\,Bab^9+10\,Ab^{10}}{b^8}\right)x^2+\frac{93\,Ba^2b^8+170\,Aab^9}{b^8}\right)x^2+\frac{5\left(3\,Ba^3b^7+118\,Aa^2b^8\right)}{b^8}\right)x^2-\frac{15\left(3\,Ba^4b^6\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*b^2*x^2 + (21*B*a*b^9 + 10*A*b^10)/b^8)*x^2 + (93*B*a^
2*b^8 + 170*A*a*b^9)/b^8)*x^2 + 5*(3*B*a^3*b^7 + 118*A*a^2*b^8)/b^8)*x^2 -
15*(3*B*a^4*b^6 - 10*A*a^3*b^7)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(3*B*a^5 - 1
0*A*a^4*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.542 $\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(9*b^2)
```

Rubi [A] time = 0.0345383, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(9*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{5/2}}{b}+\frac{B(a+bx)^{7/2}}{b}\right)dx,x,x^2\right)\\
&= \frac{(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.0258787, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(-2aB+9Ab+7bBx^2\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

Maple [A] time = 0.005, size = 31, normalized size = 0.7

$$\frac{7\, bBx^{2} + 9\, Ab - 2\, Ba}{63\, b^{2}} \left(bx^{2} + a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/63*(b*x^2+a)^(7/2)*(7*B*b*x^2+9*A*b-2*B*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62784, size = 211, normalized size = 4.59

$$\frac{\left(7\, Bb^{4}x^{8} + \left(19\, Bab^{3} + 9\, Ab^{4}\right)x^{6} - 2\, Ba^{4} + 9\, Aa^{3}b + 3\left(5\, Ba^{2}b^{2} + 9\, Aab^{3}\right)x^{4} + \left(Ba^{3}b + 27\, Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{2} + a}}{63\, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^4*x^8 + (19*B*a*b^3 + 9*A*b^4)*x^6 - 2*B*a^4 + 9*A*a^3*b + 3*(5
*B*a^2*b^2 + 9*A*a*b^3)*x^4 + (B*a^3*b + 27*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)
/b^2
```

Sympy [A] time = 3.57975, size = 209, normalized size = 4.54

$$\begin{cases} \frac{Aa^{3}\sqrt{a+bx^{2}}}{7b} + \frac{3Aa^{2}x^{2}\sqrt{a+bx^{2}}}{7} + \frac{3Aabx^{4}\sqrt{a+bx^{2}}}{7} + \frac{Ab^{2}x^{6}\sqrt{a+bx^{2}}}{7} - \frac{2Ba^{4}\sqrt{a+bx^{2}}}{63b^{2}} + \frac{Ba^{3}x^{2}\sqrt{a+bx^{2}}}{63b} + \frac{5Ba^{2}x^{4}\sqrt{a+bx^{2}}}{21} + \frac{19Babx^{6}\sqrt{a+bx^{2}}}{63} + \frac{Bb^{2}x^{8}\ldots}{\ldots} \\ a^{\frac{5}{2}}\left(\frac{Ax^{2}}{2} + \frac{Bx^{4}}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**3*sqrt(a + b*x**2)/(7*b) + 3*A*a**2*x**2*sqrt(a + b*x**2)/7
 + 3*A*a*b*x**4*sqrt(a + b*x**2)/7 + A*b**2*x**6*sqrt(a + b*x**2)/7 - 2*B*a
**4*sqrt(a + b*x**2)/(63*b**2) + B*a**3*x**2*sqrt(a + b*x**2)/(63*b) + 5*B*
a**2*x**4*sqrt(a + b*x**2)/21 + 19*B*a*b*x**6*sqrt(a + b*x**2)/63 + B*b**2*
x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**4/4), True))
```

Giac [B] time = 1.09716, size = 304, normalized size = 6.61

$$105\left(bx^2+a\right)^{\frac{3}{2}}Aa^2+42\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Aa+\frac{21\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Ba^2}{b}+3\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*A*a^2 + 42*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a
)^(3/2)*a)*A*a + 21*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*B*a^2/b +
 3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^
2)*A + 6*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3
/2)*a^2)*B*a/b + (35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x
^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B/b)/b
```

3.543 $\int \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=149

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab-aB)}{192b}$$

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a*B)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(5/2))/(48*b) + (B*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.0541756, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab-aB)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a*B)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(5/2))/(48*b) + (B*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx = \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} - \frac{(-8Ab+aB)\int \left(a+bx^2\right)^{5/2}dx}{8b}$$
$$= \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} + \frac{(5a(8Ab-aB))\int \left(a+bx^2\right)^{3/2}dx}{48b}$$
$$= \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} + \frac{(5a^2(8A\ldots}{}$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \ldots$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \ldots$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \ldots$$

Mathematica [A] time = 0.220048, size = 130, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(2a^2b\left(132A+59Bx^2\right)+15a^3B+8ab^2x^2\left(26A+17Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)-\frac{15a^{5/2}(aB-8Ab)\sinh^{-1}\left(\ldots\right.}{\sqrt{\frac{bx^2}{a}+1}}\right.}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*B + 16*b^3*x^4*(4*A + 3*B*x^2) + 8*a*b^
2*x^2*(26*A + 17*B*x^2) + 2*a^2*b*(132*A + 59*B*x^2)) - (15*a^(5/2)*(-8*A*b
 + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(3/2))
```

Maple [A] time = 0.005, size = 166, normalized size = 1.1

$$\frac{Bx}{8b}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{Bax}{48b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{5a^2Bx}{192b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5Ba^3x}{128b}\sqrt{bx^2+a} - \frac{5Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + \frac{Ax}{6}\left(bx^2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*B/b*a*x*(b*x^2+a)^(5/2)-5/192*B/b*a^2*x*(b*x
^2+a)^(3/2)-5/128*B/b*a^3*x*(b*x^2+a)^(1/2)-5/128*B/b^(3/2)*a^4*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))+1/6*A*x*(b*x^2+a)^(5/2)+5/24*A*a*x*(b*x^2+a)^(3/2)+5/16*
A*a^2*x*(b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81867, size = 603, normalized size = 4.05

$$\left[-\frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Bb^4x^7+8\left(17\,Bab^3+8\,Ab^4\right)x^5+2\left(59\,Ba^2b^2+104\,Aa\right.\right.}{768\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2
*b^2 + 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/
b^2, 1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2*b^2 + 104*
A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] time = 25.8584, size = 316, normalized size = 2.12

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{1}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(7/2)*x/(128*b*sqrt(1 + b*
x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**(3/2)*b*
x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/
a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.20882, size = 181, normalized size = 1.21

$$\frac{1}{384}\left(2\left(4\left(6\,Bb^2x^2+\frac{17\,Bab^7+8\,Ab^8}{b^6}\right)x^2+\frac{59\,Ba^2b^6+104\,Aab^7}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^5+88\,Aa^2b^6\right)}{b^6}\right)\sqrt{bx^2+ax}+\frac{5\left(Ba^4-8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*B*a^2*b^6
+ 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6)/b^6)*sqrt(b*x^2 +
a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

3.544 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=95

$$a^2A\sqrt{a+bx^2}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2)^(5/2))/5
 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0643206, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$a^2A\sqrt{a+bx^2}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2)^(5/2))/5
 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx, x, x^2\right)$$

$$= \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{2}A\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{3}aA\left(a+bx^2\right)^{3/2} + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{2}\left(a^2A\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,d\right.$$

$$= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA\left(a+bx^2\right)^{3/2} + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{2}\left(a^3A\right)\text{Subs}$$

$$= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA\left(a+bx^2\right)^{3/2} + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{\left(a^3A\right)\text{Subst}}{}$$

$$= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA\left(a+bx^2\right)^{3/2} + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} - a^{5/2}A\tanh^{-1}$$

Mathematica [A] time = 0.0932397, size = 88, normalized size = 0.93

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{1}{3}aA\left(4a+bx^2\right)\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + (B*(a + b*x^2)^(7/2))/(7*b) + (a*A*Sqrt[a + b*x^2
]*(4*a + b*x^2))/3 - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A] time = 0.007, size = 85, normalized size = 0.9

$$\frac{B}{7\,b}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{A}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Aa}{3}\left(bx^2+a\right)^{\frac{3}{2}} - Aa^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) + a^2A\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+1/5*A*(b*x^2+a)^(5/2)+1/3*a*A*(b*x^2+a)^(3/2)-A*a^(
5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+a^2*A*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67919, size = 529, normalized size = 5.57

$$\left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(15\,Bb^3x^6+3\left(15\,Bab^2+7\,Ab^3\right)x^4+15\,Ba^3+161\,Aa^2b+\left(45\,Ba^2b+77\,Aab^2\right)x^2\right.}{210\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b
+ (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/b, 1/105*(105*A*sqrt(-a)*
a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*
A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b
*x^2 + a))/b]
```

Sympy [A] time = 48.334, size = 88, normalized size = 0.93

$$\frac{Aa^3\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}+Aa^2\sqrt{a+bx^2}+\frac{Aa\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{5}+\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**3*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a**2*sqrt(a + b*x**2) +
 A*a*(a + b*x**2)**(3/2)/3 + A*(a + b*x**2)**(5/2)/5 + B*(a + b*x**2)**(7/2
)/(7*b)
```

Giac [A] time = 1.13819, size = 131, normalized size = 1.38

$$\frac{Aa^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}Bb^6+21\left(bx^2+a\right)^{\frac{5}{2}}Ab^7+35\left(bx^2+a\right)^{\frac{3}{2}}Aab^7+105\sqrt{bx^2+a}Aa^2b^7}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/105*(15*(b*x^2 + a)^(7/
2)*B*b^6 + 21*(b*x^2 + a)^(5/2)*A*b^7 + 35*(b*x^2 + a)^(3/2)*A*a*b^7 + 105*
sqrt(b*x^2 + a)*A*a^2*b^7)/b^7
```

3.545 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}(aB+6Ab)$$

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a + b*x^2)^(
3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) - (A*(a + b*x^2)^(7/2)
)/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sq
rt[b])
```

Rubi [A] time = 0.0532958, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}(aB+6Ab)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a + b*x^2)^(
3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) - (A*(a + b*x^2)^(7/2)
)/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sq
rt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^2}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{ax} - \frac{(-6Ab-aB)\int \left(a+bx^2\right)^{5/2}\,dx}{a}$$
$$= \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a} - \frac{A\left(a+bx^2\right)^{7/2}}{ax} + \frac{1}{6}(5(6Ab+aB))\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a} - \frac{A\left(a+bx^2\right)^{7/2}}{ax} + \frac{1}{8}(5a(6A\ldots$$
$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a}\ldots$$
$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a}\ldots$$
$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a}\ldots$$

Mathematica [A] time = 0.33446, size = 125, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(\frac{(aB+6Ab)\left(\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\left(33a^2+26abx^2+8b^2x^4\right)+15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} - \frac{48A\left(a+bx^2\right)^3}{x}\right)}{48a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-48*A*(a + b*x^2)^3)/x + ((6*A*b + a*B)*(Sqrt[b]*x*Sqrt[
1 + (b*x^2)/a]*(33*a^2 + 26*a*b*x^2 + 8*b^2*x^4) + 15*a^(5/2)*ArcSinh[(Sqrt
[b]*x)/Sqrt[a]]))/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/(48*a)
```

Maple [A] time = 0.008, size = 158, normalized size = 1.2

$$\frac{Bx}{6}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,Bax}{24}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2Bx}{16}\sqrt{bx^2+a} + \frac{5\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{A}{ax}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{Abx}{a}\left(bx^2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/6*x*B*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(b*x^2+a)^(
1/2)+5/16*B*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A*(b*x^2+a)^(7/2)/a/x
+A*b/a*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^(3/2)+15/8*A*b*a*x*(b*x^2+a)^(
1/2)+15/8*A*b^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66493, size = 547, normalized size = 4.02

$$\left[\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,Bb^3x^6+2\left(13\,Bab^2+6\,Ab^3\right)x^4-48\,Aa^2b+3\left(11\,Ba^2b\right.\right.}{96\,bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(8*B*b^3*x^6 + 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b +
 3*(11*B*a^2*b + 18*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(b*x), -1/48*(15*(B*a^3
+ 6*A*a^2*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^6 +
 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^
2)*sqrt(b*x^2 + a))/(b*x)]
```

Sympy [B] time = 16.8384, size = 306, normalized size = 2.25

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*B*a**(5/
2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a))
 + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*B*a**3*asinh(sqrt(b)*
x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.12696, size = 197, normalized size = 1.45

$$\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{48}\left(2\left(4\,Bb^2x^2+\frac{13\,Bab^5+6\,Ab^6}{b^4}\right)x^2+\frac{3\left(11\,Ba^2b^4+18\,Aab^5\right)}{b^4}\right)\sqrt{bx^2+a}x-\frac{5\left(Ba^3\sqrt{b}+6\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/48*(2*(4*B*b^2*x^
2 + (13*B*a*b^5 + 6*A*b^6)/b^4)*x^2 + 3*(11*B*a^2*b^4 + 18*A*a*b^5)/b^4)*sq
rt(b*x^2 + a)*x - 5/32*(B*a^3*sqrt(b) + 6*A*a^2*b^(3/2))*log((sqrt(b)*x - s
qrt(b*x^2 + a))^2)/b
```

3.546 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}(2aB+5$$

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x^2)^(3/2))
/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a + b*x^2)^(7/2))/(2*
a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.100338, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}(2aB+5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x^2)^(3/2))
/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a + b*x^2)^(7/2))/(2*
a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{2ax^2} + \frac{\left(\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)}{2a}$$

$$= \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a} - \frac{A\left(a+bx^2\right)^{7/2}}{2ax^2} + \frac{1}{4}(5Ab+2aB)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,\right.$$

$$= \frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2} + \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a} - \frac{A\left(a+bx^2\right)^{7/2}}{2ax^2} + \frac{1}{4}(a(5Ab+$$

$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2} + \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a} -$$

$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2} + \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a} -$$

$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2} + \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a} -$$

Mathematica [A] time = 0.0704792, size = 105, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(a^2\left(46Bx^2-15A\right)+a\left(70Abx^2+22bBx^4\right)+2b^2x^4\left(5A+3Bx^2\right)\right)}{30x^2} - \frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*b^2*x^4*(5*A + 3*B*x^2) + a^2*(-15*A + 46*B*x^2) + a*(7
0*A*b*x^2 + 22*b*B*x^4)))/(30*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/2
```

Maple [A] time = 0.008, size = 161, normalized size = 1.2

$$\frac{B}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Ba}{3}\left(bx^2+a\right)^{\frac{3}{2}} - Ba^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + B\sqrt{bx^2+a}\,a^2 - \frac{A}{2ax^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{Ab}{2a}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2)-B*a^(5/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2)*a^2-1/2*A*(b*x^2+a)^(7/2)/a/x^2+1/2*A*
b/a*(b*x^2+a)^(5/2)+5/6*A*b*(b*x^2+a)^(3/2)-5/2*A*b*a^(3/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)+5/2*A*b*a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65179, size = 527, normalized size = 3.9

$$\left[\frac{15\left(2\,Ba^2+5\,Aab\right)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^6+2\left(11\,Bab+5\,Ab^2\right)x^4-15\,Aa^2+2\left(23\,Ba^2+35\,Aab\right)x^2\right.}{60\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^6 + 2*(11*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2
 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^2, 1/30*(15*(2*B*a^2 + 5
*A*a*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (6*B*b^2*x^6 + 2*(1
1*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2
 + a))/x^2]
```

Sympy [A] time = 30.7282, size = 296, normalized size = 2.19

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a
**3/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1
) + 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b
), True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*s
qrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/
```

```
4, True))
```

Giac [A] time = 1.14378, size = 188, normalized size = 1.39

$$\frac{6\left(bx^2+a\right)^{\frac{5}{2}}Bb+10\left(bx^2+a\right)^{\frac{3}{2}}Bab+30\sqrt{bx^2+a}Ba^2b+10\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+60\sqrt{bx^2+a}Aab^2-\frac{15\sqrt{bx^2+a}Aa^2b}{x^2}+\frac{15(2}{}}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(b*x^2 + a)^(5/2)*B*b + 10*(b*x^2 + a)^(3/2)*B*a*b + 30*sqrt(b*x^2
+ a)*B*a^2*b + 10*(b*x^2 + a)^(3/2)*A*b^2 + 60*sqrt(b*x^2 + a)*A*a*b^2 - 15
*sqrt(b*x^2 + a)*A*a^2*b/x^2 + 15*(2*B*a^3*b + 5*A*a^2*b^2)*arctan(sqrt(b*x
^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.547 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4} dx$

Optimal. Leaf size=146

$$-\frac{(a+bx^2)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx(a+bx^2)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}(3aB+4Ab)+\frac{5}{8}a\sqrt{b}(3aB+4Ab)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.$$

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*x*(a + b*x
^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))/(3*a*x) - (A*(a + b
*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/8
```

Rubi [A] time = 0.0600457, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$-\frac{(a+bx^2)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx(a+bx^2)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}(3aB+4Ab)+\frac{5}{8}a\sqrt{b}(3aB+4Ab)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*x*(a + b*x
^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))/(3*a*x) - (A*(a + b
*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/8
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}dx = -\frac{A(a+bx^2)^{7/2}}{3ax^3} - \frac{(-4Ab-3aB)\int \frac{(a+bx^2)^{5/2}}{x^2}dx}{3a}$$
$$= -\frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax} - \frac{A(a+bx^2)^{7/2}}{3ax^3} + \frac{(5b(4Ab+3aB))\int (a+bx^2)^{3/2}dx}{3a}$$
$$= \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a} - \frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax} - \frac{A(a+bx^2)^{7/2}}{3ax^3} + \frac{1}{4}(5b(4A$$
$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2} + \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a} - \frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax}$$
$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2} + \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a} - \frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax}$$
$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2} + \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a} - \frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax}$$

Mathematica [C] time = 0.0366043, size = 84, normalized size = 0.58

$$\frac{a\sqrt{a+bx^2}(-3aB-4Ab)\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{3x\sqrt{\frac{bx^2}{a}+1}} - \frac{A(a+bx^2)^{7/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(3*a*x^3) + (a*(-4*A*b - 3*a*B)*Sqrt[a + b*x^2]*Hype
rgeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/(3*x*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.01, size = 204, normalized size = 1.4

$$-\frac{A}{3ax^3}(bx^2+a)^{\frac{7}{2}} - \frac{4Ab}{3a^2x}(bx^2+a)^{\frac{7}{2}} + \frac{4Ab^2x}{3a^2}(bx^2+a)^{\frac{5}{2}} + \frac{5Ab^2x}{3a}(bx^2+a)^{\frac{3}{2}} + \frac{5Ab^2x}{2}\sqrt{bx^2+a} + \frac{5Aa}{2}b^{\frac{3}{2}}\ln(x\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(7/2)/a/x^3-4/3*A*b/a^2/x*(b*x^2+a)^(7/2)+4/3*A*b^2/a^2*x*
(b*x^2+a)^(5/2)+5/3*A*b^2/a*x*(b*x^2+a)^(3/2)+5/2*A*b^2*x*(b*x^2+a)^(1/2)+5
/2*A*b^(3/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-B/a/x*(b*x^2+a)^(7/2)+B*b/a*x*
```

```
(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^(3/2)+15/8*B*b*a*x*(b*x^2+a)^(1/2)+15/8
*B*b^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66959, size = 514, normalized size = 3.52

$$\left[\frac{15\left(3\,Ba^2+4\,Aab\right)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Bb^2x^6+3\left(9\,Bab+4\,Ab^2\right)x^4-8\,Aa^2-8\left(3\,Ba^2+7\,A\right.\right.}{48\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(
3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3, -1/24*(15*(3*B*a^2 + 4*A*a*b)
*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^6 + 3*(9*B*a*
b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^
3]
```

Sympy [B] time = 10.4497, size = 299, normalized size = 2.05

$$-\frac{2Aa^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}b^2x\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{2A\sqrt{a}b^2x}{\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+\frac{5Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -2*A*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b**2*x*sqrt(1 + b*x**2/a
)/2 - 2*A*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - A*a**2*sqrt(b)*sqrt(a/(b*x**2
) + 1)/(3*x**2) - A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + 5*A*a*b**(3/2)*asin
h(sqrt(b)*x/sqrt(a))/2 - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3/2)*b*x
*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)
*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(
a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.14474, size = 321, normalized size = 2.2

$$\frac{1}{8}\left(2\,Bb^2x^2+\frac{9\,Bab^3+4\,Ab^4}{b^2}\right)\sqrt{bx^2+a}x-\frac{5}{16}\left(3\,Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*b^2*x^2 + (9*B*a*b^3 + 4*A*b^4)/b^2)*sqrt(b*x^2 + a)*x - 5/16*(3*B
*a^2*sqrt(b) + 4*A*a*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b) + 9*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*A*a^2*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b) - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/2) + 3*B*a^5*sqrt(b) + 7*A*a^4
*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.548 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}(4aB+3Ab)-\frac{5}{8}\sqrt{a}b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.$$

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a + b*x^2)^
(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*a*x^2) - (A*(a + b*x
^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/8
```

Rubi [A] time = 0.10115, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}(4aB+3Ab)-\frac{5}{8}\sqrt{a}b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a + b*x^2)^
(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*a*x^2) - (A*(a + b*x
^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/8
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4} + \frac{\left(\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx,x,x^2\right)}{4a}$$

$$= -\frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{7/2}}{4ax^4} + \frac{(5b(3Ab+4aB))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,\right.}{16a}$$

$$= \frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a} - \frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{7/2}}{4ax^4} + \frac{1}{16}(5b(3A$$

$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a} - \frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2} -$$

$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a} - \frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2} -$$

$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a} - \frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2} -$$

Mathematica [C] time = 0.0307477, size = 60, normalized size = 0.42

$$\frac{\left(a+bx^2\right)^{7/2}\left(bx^4(4aB+3Ab)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)-7a^2A\right)}{28a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-7*a^2*A + b*(3*A*b + 4*a*B)*x^4*Hypergeometric2F1[2, 7
/2, 9/2, 1 + (b*x^2)/a]))/(28*a^3*x^4)
```

Maple [A] time = 0.009, size = 213, normalized size = 1.5

$$-\frac{A}{4\,a x^{4}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{3\,A b}{8\,a^{2} x^{2}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{3\,A b^{2}}{8\,a^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{5\,A b^{2}}{8\,a}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{15\,A b^{2}}{8}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(7/2)/a/x^4-3/8*A*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*A*b^2/a^2*
(b*x^2+a)^(5/2)+5/8*A*b^2/a*(b*x^2+a)^(3/2)-15/8*A*b^2*a^(1/2)*ln((2*a+2*a^
(1/2)*(b*x^2+a)^(1/2))/x)+15/8*A*b^2*(b*x^2+a)^(1/2)-1/2*B/a/x^2*(b*x^2+a)^
(7/2)+1/2*B*b/a*(b*x^2+a)^(5/2)+5/6*B*b*(b*x^2+a)^(3/2)-5/2*B*b*a^(3/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/2*B*b*a*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72509, size = 516, normalized size = 3.61

$$\left[\frac{15\left(4\,Bab+3\,Ab^{2}\right)\sqrt{a}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(8\,Bb^{2}x^{6}+8\left(7\,Bab+3\,Ab^{2}\right)x^{4}-6\,Aa^{2}-3\left(4\,Ba^{2}+9\,Aab\right)x^{2}\right)\sqrt{b}}{48\,x^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*b^2)*x^4 - 6*A*a^2 -
 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^4, 1/24*(15*(4*B*a*b + 3*A*b
^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*B*b^2*x^6 + 8*(7*B*a
*b + 3*A*b^2)*x^4 - 6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x
^4]
```

Sympy [A] time = 61.9685, size = 279, normalized size = 1.95

$$-\frac{15A\sqrt{a}b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{Aa^{3}}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa^{2}\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{x}+\frac{7Aab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -15*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - A*a**3/(4*sqrt(b)*x**5*sq
rt(a/(b*x**2) + 1)) - 3*A*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*a*
b**(3/2)*sqrt(a/(b*x**2) + 1)/x + 7*A*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1))
 + A*b**(5/2)*x/sqrt(a/(b*x**2) + 1) - 5*B*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b
)*x))/2 - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*a**2*sqrt(b)/(x*s
qrt(a/(b*x**2) + 1)) + 2*B*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + B*b**2*Piece
wise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A] time = 1.15734, size = 231, normalized size = 1.62

$$\frac{8\left(bx^2+a\right)^{\frac{3}{2}}Bb^2+48\sqrt{bx^2+a}Bab^2+24\sqrt{bx^2+a}Ab^3+\frac{15\left(4Ba^2b^2+3Aab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\left(4\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^2-4\sqrt{bx^2+a}Ba^3b^2}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(b*x^2 + a)^(3/2)*B*b^2 + 48*sqrt(b*x^2 + a)*B*a*b^2 + 24*sqrt(b*x^
2 + a)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a)
)/sqrt(-a) - 3*(4*(b*x^2 + a)^(3/2)*B*a^2*b^2 - 4*sqrt(b*x^2 + a)*B*a^3*b^2
 + 9*(b*x^2 + a)^(3/2)*A*a*b^3 - 7*sqrt(b*x^2 + a)*A*a^2*b^3)/(b^2*x^4))/b
```

3.549 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=152

$$\frac{b^2x\sqrt{a+bx^2}(5aB+2Ab)}{2a}+\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2A$$

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B)*(a + b*x
^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2))/(15*a*x^3) - (A*(a
 + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/2
```

Rubi [A] time = 0.0624304, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$\frac{b^2x\sqrt{a+bx^2}(5aB+2Ab)}{2a}+\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2A$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B)*(a + b*x
^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2))/(15*a*x^3) - (A*(a
 + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/2
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^6}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5} - \frac{(-2Ab-5aB)\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx}{5a}$$

$$= -\frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} - \frac{A\left(a+bx^2\right)^{7/2}}{5ax^5} + \frac{(b(2Ab+5aB))\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx}{3a}$$

$$= -\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax} - \frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} - \frac{A\left(a+bx^2\right)^{7/2}}{5ax^5} + \frac{\left(b^2(2Ab\right.}{}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax} - \frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} -$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax} - \frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} -$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax} - \frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} -$$

Mathematica [C] time = 0.0364749, size = 84, normalized size = 0.55

$$\frac{a\sqrt{a+bx^2}(-5aB-2Ab)\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{15x^3\sqrt{\frac{bx^2}{a}+1}} - \frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(5*a*x^5) + (a*(-2*A*b - 5*a*B)*Sqrt[a + b*x^2]*Hype
rgeometric2F1[-5/2, -3/2, -1/2, -((b*x^2)/a)])/(15*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.011, size = 251, normalized size = 1.7

$$-\frac{B}{3\,a x^3}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{4\,Bb}{3\,a^2 x}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{4\,b^2 Bx}{3\,a^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,b^2 Bx}{3\,a}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,b^2 Bx}{2}\sqrt{bx^2+a} + \frac{5\,Ba}{2}b^{\frac{3}{2}}\ln\left(x\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/3*B/a/x^3*(b*x^2+a)^(7/2)-4/3*B*b/a^2/x*(b*x^2+a)^(7/2)+4/3*B*b^2/a^2*x*
(b*x^2+a)^(5/2)+5/3*B*b^2/a*x*(b*x^2+a)^(3/2)+5/2*B*b^2*x*(b*x^2+a)^(1/2)+5
/2*B*b^(3/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/5*A*(b*x^2+a)^(7/2)/a/x^5-2/
```

```
15*A*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*A*b^2/a^3/x*(b*x^2+a)^(7/2)+8/15*A*b^3/
a^3*x*(b*x^2+a)^(5/2)+2/3*A*b^3/a^2*x*(b*x^2+a)^(3/2)+A*b^3/a*x*(b*x^2+a)^(
1/2)+A*b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80469, size = 525, normalized size = 3.45

$$\left[\frac{15\left(5\,Bab+2\,Ab^2\right)\sqrt{b}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(15\,Bb^2x^6-2\left(35\,Bab+23\,Ab^2\right)x^4-6\,Aa^2-2\left(5\,Ba^2+\right.\right.}{60\,x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 -
2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^5, -1/30*(15*(5*B*a*b + 2*A*
b^2)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (15*B*b^2*x^6 - 2*(3
5*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2
+ a))/x^5]
```

Sympy [B] time = 7.30971, size = 292, normalized size = 1.92

$$-\frac{A\sqrt{a}b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Ab^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Ab^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{2Ba^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(5*x**4) - 11*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*A*b**(5/2)*
sqrt(a/(b*x**2) + 1)/15 + A*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - A*b**3*x/(s
qrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + B*sqrt
(a)*b**2*x*sqrt(1 + b*x**2/a)/2 - 2*B*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - B
*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 + 5*B*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/2
```

Giac [B] time = 1.16989, size = 433, normalized size = 2.85

$$\frac{1}{2}\sqrt{bx^2+a}Bb^2x-\frac{1}{4}\left(5\,Bab^{\frac{3}{2}}+2\,Ab^{\frac{5}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ba^2b^{\frac{3}{2}}+45\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b^2*x - 1/4*(5*B*a*b^(3/2) + 2*A*b^(5/2))*log((sqrt(b
)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*
b^(3/2) + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(5/2) - 150*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*B*a^3*b^(3/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^
2*b^(5/2) + 200*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(3/2) + 140*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(5/2) - 130*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*B*a^5*b^(3/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(5/2) + 35*B*a
^6*b^(3/2) + 23*A*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.550 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=149

$$\frac{5b^2\sqrt{a+bx^2}(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{(a+bx^2)^{5/2}(6aB+Ab)}{24ax^4}-\frac{5b(a+bx^2)^{3/2}(6aB+Ab)}{48ax^2}-\frac{A}{}$$

```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)*(a + b*x^
2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2))/(24*a*x^4) - (A*(a
 + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*Sqrt[a])
```

Rubi [A] time = 0.110362, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{5b^2\sqrt{a+bx^2}(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{(a+bx^2)^{5/2}(6aB+Ab)}{24ax^4}-\frac{5b(a+bx^2)^{3/2}(6aB+Ab)}{48ax^2}-\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```


```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)*(a + b*x^
2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2))/(24*a*x^4) - (A*(a
 + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*Sqrt[a])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}+\frac{(Ab+6aB)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx,x,x^2\right)}{12a}$$

$$= -\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}+\frac{(5b(Ab+6aB))\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,\right.}{48a}$$

$$= -\frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2}-\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}+\frac{\left(5b^2(Ab+\right.}{}$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a}-\frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2}-\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4}-\frac{A}{}$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a}-\frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2}-\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4}-\frac{A}{}$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a}-\frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2}-\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4}-\frac{A}{}$$

Mathematica [C] time = 0.0278098, size = 61, normalized size = 0.41

$$-\frac{\left(a+bx^2\right)^{7/2}\left(7a^3A+b^2x^6(6aB+Ab)\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)\right)}{42a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(7*a^3*A + b^2*(A*b + 6*a*B)*x^6*Hypergeometric2F1[3, 7
/2, 9/2, 1 + (b*x^2)/a]))/(42*a^4*x^6)
```

Maple [B] time = 0.012, size = 266, normalized size = 1.8

$$-\frac{A}{6ax^6}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Ab}{24a^2x^4}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Ab^2}{16a^3x^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ab^3}{16a^3}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5Ab^3}{48a^2}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5Ab^3}{16}\ln\left(\frac{1}{x}\left(2a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(7/2)/a/x^6-1/24*A*b/a^2/x^4*(b*x^2+a)^(7/2)-1/16*A*b^2/a^
3/x^2*(b*x^2+a)^(7/2)+1/16*A*b^3/a^3*(b*x^2+a)^(5/2)+5/48*A*b^3/a^2*(b*x^2+
a)^(3/2)-5/16*A*b^3/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/16*A*b^
3/a*(b*x^2+a)^(1/2)-1/4*B/a/x^4*(b*x^2+a)^(7/2)-3/8*B*b/a^2/x^2*(b*x^2+a)^(
7/2)+3/8*B*b^2/a^2*(b*x^2+a)^(5/2)+5/8*B*b^2/a*(b*x^2+a)^(3/2)-15/8*B*b^2*a
^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+15/8*B*b^2*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72656, size = 559, normalized size = 3.75

$$\left[\frac{15\left(6Bab^2+Ab^3\right)\sqrt{a}x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(48Bab^2x^6-3\left(18Ba^2b+11Aab^2\right)x^4-8Aa^3-2\left(6Ba^3+13Aa^2\right.\right.}{96ax^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(6*B*a*b^2 + A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(48*B*a*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 -
8*A*a^3 - 2*(6*B*a^3 + 13*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6), 1/48*(15*
(6*B*a*b^2 + A*b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (48*B*a
*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 + 13*A*a^
2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6)]
```

Sympy [B] time = 90.2878, size = 306, normalized size = 2.05

$$-\frac{Aa^3}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{17Aa^2\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35Aab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{3Ab^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}}-\frac{15B\sqrt{a}b^2\operatorname{asi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*A*a**2*sqrt(b)/(24*x**5*
sqrt(a/(b*x**2) + 1)) - 35*A*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*
b**(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*A*b**(5/2)/(16*x*sqrt(a/(b*x**2) +
1)) - 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a)) - 15*B*sqrt(a)*b**2*
asinh(sqrt(a)/(sqrt(b)*x))/8 - B*a**3/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1))
 - 3*B*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*a*b**(3/2)*sqrt(a/(b*
x**2) + 1)/x + 7*B*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)*x/sqr
t(a/(b*x**2) + 1)
```

Giac [A] time = 1.14836, size = 225, normalized size = 1.51

$$\frac{48\,\sqrt{bx^2+a}Bb^3 + \frac{15\left(6\,Bab^3+Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{54\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-96\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+42\sqrt{bx^2+a}Ba^3b^3+33\left(bx^2+a\right)^{\frac{5}{2}}Ab^4-40\left(bx^2+a\right)^{\frac{3}{2}}Aab}{b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(b*x^2 + a)*B*b^3 + 15*(6*B*a*b^3 + A*b^4)*arctan(sqrt(b*x^2 +
 a)/sqrt(-a))/sqrt(-a) - (54*(b*x^2 + a)^(5/2)*B*a*b^3 - 96*(b*x^2 + a)^(3/
2)*B*a^2*b^3 + 42*sqrt(b*x^2 + a)*B*a^3*b^3 + 33*(b*x^2 + a)^(5/2)*A*b^4 -
40*(b*x^2 + a)^(3/2)*A*a*b^4 + 15*sqrt(b*x^2 + a)*A*a^2*b^4)/(b^3*x^6))/b
```

3.551 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8} dx$

Optimal. Leaf size=108

$$-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}-\frac{b^2B\sqrt{a+bx^2}}{x}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}$$

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) - (B*(a + b*
x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) + b^(5/2)*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0459612, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}-\frac{b^2B\sqrt{a+bx^2}}{x}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) - (B*(a + b*
x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) + b^(5/2)*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^8}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + B\int \frac{\left(a+bx^2\right)^{5/2}}{x^6}\,dx$$

$$= -\frac{B\left(a+bx^2\right)^{5/2}}{5x^5} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + (bB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx$$

$$= -\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3} - \frac{B\left(a+bx^2\right)^{5/2}}{5x^5} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + \left(b^2B\right)\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$

$$= -\frac{b^2B\sqrt{a+bx^2}}{x} - \frac{bB\left(a+bx^2\right)^{3/2}}{3x^3} - \frac{B\left(a+bx^2\right)^{5/2}}{5x^5} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + \left(b^3B\right)\int \frac{1}{\sqrt{a+b}}$$

$$= -\frac{b^2B\sqrt{a+bx^2}}{x} - \frac{bB\left(a+bx^2\right)^{3/2}}{3x^3} - \frac{B\left(a+bx^2\right)^{5/2}}{5x^5} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + \left(b^3B\right)\text{Subst}\left(\int\right.$$

$$= -\frac{b^2B\sqrt{a+bx^2}}{x} - \frac{bB\left(a+bx^2\right)^{3/2}}{3x^3} - \frac{B\left(a+bx^2\right)^{5/2}}{5x^5} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7} + b^{5/2}B\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.0667457, size = 78, normalized size = 0.72

$$-\frac{a^2B\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}} - \frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(7*a*x^7) - (a^2*B*Sqrt[a + b*x^2]*Hypergeometric2F1
[-5/2, -5/2, -3/2, -((b*x^2)/a)])/(5*x^5*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.013, size = 155, normalized size = 1.4

$$-\frac{B}{5\,ax^5}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{2\,Bb}{15\,a^2x^3}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{8\,Bb^2}{15\,a^3x}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{8\,Bb^3x}{15\,a^3}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{2\,Bb^3x}{3\,a^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{Bb^3x}{a}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/5*B/a/x^5*(b*x^2+a)^(7/2)-2/15*B*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*B*b^2/a^
3/x*(b*x^2+a)^(7/2)+8/15*B*b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*B*b^3/a^2*x*(b*x^2
+a)^(3/2)+B*b^3/a*x*(b*x^2+a)^(1/2)+B*b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
-1/7*A*(b*x^2+a)^(7/2)/a/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80134, size = 560, normalized size = 5.19

$$\left[\frac{105\,Bab^{\frac{5}{2}}x^{7}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\left(\left(161\,Bab^{2}+15\,Ab^{3}\right)x^{6}+\left(77\,Ba^{2}b+45\,Aab^{2}\right)x^{4}+15\,Aa^{3}+3\left(7\,Ba^{3}\right.\right.}{210\,ax^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] [1/210*(105*B*a*b^(5/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a
^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7), -1/105*(105*B*
a*sqrt(-b)*b^2*x^7*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((161*B*a*b^2 + 15*
A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 15*A*a
^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7)]
```

Sympy [B] time = 6.80215, size = 592, normalized size = 5.48

$$-\frac{15Aa^{7}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{33Aa^{6}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{17Aa^{5}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**6*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 8*A*a**2*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 2*A*a*b**
(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15
*x**2) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a) - B*sqrt(a)*b**2/(x*sqrt(1
+ b*x**2/a)) - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*B*a*b**(3/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 +
B*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**3*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [B] time = 1.17736, size = 432, normalized size = 4.

$$-\frac{1}{2}\,Bb^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Bab^{\frac{5}{2}}+105\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Ab^{\frac{7}{2}}-1260\left(\sqrt{b}x-\sqrt{bx^{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(315*(sqrt(b)*x
 - sqrt(b*x^2 + a))^12*B*a*b^(5/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A
*b^(7/2) - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 2555*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) + 525*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*A*a^2*b^(7/2) - 3080*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 21
21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) + 315*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*A*a^4*b^(7/2) - 812*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^6*b^(5/
2) + 161*B*a^7*b^(5/2) + 15*A*a^6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^7
```

3.552 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9} dx$

Optimal. Leaf size=152

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}(Ab-8aB)}{128ax^2}+\frac{(a+bx^2)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b(a+bx^2)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A}{}$$

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8*a*B)*(a +
 b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)^(5/2))/(48*a*x^6) -
 (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*b - 8*a*B)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rubi [A] time = 0.11766, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}(Ab-8aB)}{128ax^2}+\frac{(a+bx^2)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b(a+bx^2)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8*a*B)*(a +
 b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)^(5/2))/(48*a*x^6) -
 (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*b - 8*a*B)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} + \frac{\left(-\frac{Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, x^2\right)}{8a}$$
$$= \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} - \frac{(5b(Ab-8aB))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right.}{96a}$$
$$= \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} - \frac{\left(5b^2(Ab-8\right.}{}$$
$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A}{}$$
$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A}{}$$
$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A}{}$$

Mathematica [A] time = 0.092213, size = 140, normalized size = 0.92

$$\frac{-\left(a+bx^2\right)\left(8a^2bx^2\left(17A+26Bx^2\right)+16a^3\left(3A+4Bx^2\right)+2ab^2x^4\left(59A+132Bx^2\right)+15Ab^3x^6\right)-15b^3x^8\sqrt{\frac{bx^2}{a}+1}(8a}{384ax^8\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (-((a + b*x^2)*(15*A*b^3*x^6 + 16*a^3*(3*A + 4*B*x^2) + 8*a^2*b*x^2*(17*A +
 26*B*x^2) + 2*a*b^2*x^4*(59*A + 132*B*x^2))) - 15*b^3*(-(A*b) + 8*a*B)*x^8
*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(384*a*x^8*Sqrt[a + b*x^
2])
```

Maple [B] time = 0.02, size = 311, normalized size = 2.1

$$-\frac{A}{8\,ax^8}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ab}{48\,a^2x^6}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ab^2}{192\,a^3x^4}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ab^3}{128\,a^4x^2}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Ab^4}{128\,a^4}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,Ab^4}{384\,a^3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(7/2)/a/x^8+1/48*A*b/a^2/x^6*(b*x^2+a)^(7/2)+1/192*A*b^2/a
^3/x^4*(b*x^2+a)^(7/2)+1/128*A*b^3/a^4/x^2*(b*x^2+a)^(7/2)-1/128*A*b^4/a^4*
(b*x^2+a)^(5/2)-5/384*A*b^4/a^3*(b*x^2+a)^(3/2)+5/128*A*b^4/a^(3/2)*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/128*A*b^4/a^2*(b*x^2+a)^(1/2)-1/6*B/a/x^6*
(b*x^2+a)^(7/2)-1/24*B*b/a^2/x^4*(b*x^2+a)^(7/2)-1/16*B*b^2/a^3/x^2*(b*x^2+
a)^(7/2)+1/16*B*b^3/a^3*(b*x^2+a)^(5/2)+5/48*B*b^3/a^2*(b*x^2+a)^(3/2)-5/16
*B*b^3/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/16*B*b^3/a*(b*x^2+a)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82248, size = 628, normalized size = 4.13

$$\left[-\frac{15\left(8\,Bab^3-Ab^4\right)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(88\,Ba^2b^2+5\,Aab^3\right)x^6+48\,Aa^4+2\left(104\,Ba^3b+59\,Aa^2b^2\right)x^4\right.}{768\,a^2x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*B*a*b^3 - A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
104*B*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2
+ a))/(a^2*x^8), 1/384*(15*(8*B*a*b^3 - A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) - (3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(104*B
*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2 + a))
/(a^2*x^8)]
```

Sympy [B] time = 159.292, size = 316, normalized size = 2.08

$$-\frac{Aa^3}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23Aa^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{127Aab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{133Ab^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}}+\frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{128a^{\frac{3}{2}}}-\frac{B}{6\sqrt{b}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*A*a**2*sqrt(b)/(48*x**7*
sqrt(a/(b*x**2) + 1)) - 127*A*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) -
133*A*b**(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a*x*sqrt
(a/(b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2)) - B*
a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*B*a**2*sqrt(b)/(24*x**5*sqr
t(a/(b*x**2) + 1)) - 35*B*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**
(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*B*b**(5/2)/(16*x*sqrt(a/(b*x**2) + 1))
 - 5*B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

Giac [A] time = 1.16437, size = 263, normalized size = 1.73

$$\frac{\frac{15\left(8\,Bab^4-Ab^5\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{264\left(bx^2+a\right)^{\frac{7}{2}}Bab^4-584\left(bx^2+a\right)^{\frac{5}{2}}Ba^2b^4+440\left(bx^2+a\right)^{\frac{3}{2}}Ba^3b^4-120\sqrt{bx^2+a}Ba^4b^4+15\left(bx^2+a\right)^{\frac{7}{2}}Ab^5+73\left(bx^2+a\right)^{\frac{5}{2}}Aab}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a)
 - (264*(b*x^2 + a)^(7/2)*B*a*b^4 - 584*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 440*(
b*x^2 + a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x^2 + a)*B*a^4*b^4 + 15*(b*x^2 + a)
^(7/2)*A*b^5 + 73*(b*x^2 + a)^(5/2)*A*a*b^5 - 55*(b*x^2 + a)^(3/2)*A*a^2*b^
5 + 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a*b^4*x^8))/b
```

3.553 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(7/2))/(9*a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(7/2))/(63*
a^2*x^7)
```

Rubi [A] time = 0.0215579, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{(a+bx^2)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(9*a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(7/2))/(63*
a^2*x^7)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx &= -\frac{A(a+bx^2)^{7/2}}{9ax^9}-\frac{(2Ab-9aB)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{9a}\\ &= -\frac{A(a+bx^2)^{7/2}}{9ax^9}+\frac{(2Ab-9aB)(a+bx^2)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.017225, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{7/2}(7aA+9aBx^2-2Abx^2)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(7*a*A - 2*A*b*x^2 + 9*a*B*x^2))/(63*a^2*x^9)
```

Maple [A] time = 0.004, size = 37, normalized size = 0.7

$$-\frac{-2\,Ab{x}^{2}+9\,Ba{x}^{2}+7\,Aa}{63\,{x}^{9}{a}^{2}} \left(b{x}^{2}+a \right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*A*b*x^2+9*B*a*x^2+7*A*a)/x^9/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86283, size = 223, normalized size = 4.21

$$-\frac{\left(\left(9\,Bab^{3}-2\,Ab^{4}\right)x^{8}+\left(27\,Ba^{2}b^{2}+Aab^{3}\right)x^{6}+7\,Aa^{4}+3\left(9\,Ba^{3}b+5\,Aa^{2}b^{2}\right)x^{4}+\left(9\,Ba^{4}+19\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{63\,a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*((9*B*a*b^3 - 2*A*b^4)*x^8 + (27*B*a^2*b^2 + A*a*b^3)*x^6 + 7*A*a^4 +
 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^4 + (9*B*a^4 + 19*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)/(a^2*x^9)
```

Sympy [B] time = 8.25986, size = 1489, normalized size = 28.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**8*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**7*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**6*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 30*A*a**6*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**5*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 66*A*a**5*b**(13/2)*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 34*A*
a**4*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 6*A*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a**2*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 24*A*a**2*b**(19/2)*x**8*sqrt
(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x
**10) - 16*A*a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 2
10*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/
(5*x**4) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*A*b**(9/2)*sqrt(
a/(b*x**2) + 1)/(15*a**2) - 15*B*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**6*b**(11
/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - 17*B*a**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**4*b**(15/2
)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*
a**3*b**6*x**10) - 12*B*a**3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a**2*b**(19/2)*
x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 2*B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*B*b**(5/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a)
```

Giac [B] time = 1.1743, size = 616, normalized size = 11.62

$$\frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}Bb^{\frac{7}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bab^{\frac{7}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Ab^{\frac{9}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*b^(7/2) - 126*(sqrt(b)*x - sqrt
(b*x^2 + a))^14*B*a*b^(7/2) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*b^(9/2
) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(7/2) + 210*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*A*a*b^(9/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^3
*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^2*b^(9/2) + 504*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^
8*A*a^3*b^(9/2) - 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2) + 378*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(9/2) + 198*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^6*b^(7/2) + 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^5*b^(9/2) -
```

```
18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7*b^(7/2) + 18*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*A*a^6*b^(9/2) + 9*B*a^8*b^(7/2) - 2*A*a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

3.554 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$\frac{b^3\sqrt{a+bx^2}(3Ab-10aB)}{256a^2x^2}-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^2\sqrt{a+bx^2}(3Ab-10aB)}{128ax^4}+\frac{b\left(a+bx^2\right)^{3/2}(3Ab-10aB)}{96ax^6}$$

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b - 10*a*B)*
Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a + b*x^2)^(3/2))/(96
*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2))/(80*a*x^8) - (A*(a + b*x^2)^
(7/2))/(10*a*x^10) - (b^4*(3*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(256*a^(5/2))
```

Rubi [A] time = 0.148023, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^3\sqrt{a+bx^2}(3Ab-10aB)}{256a^2x^2}-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^2\sqrt{a+bx^2}(3Ab-10aB)}{128ax^4}+\frac{b\left(a+bx^2\right)^{3/2}(3Ab-10aB)}{96ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b - 10*a*B)*
Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a + b*x^2)^(3/2))/(96
*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2))/(80*a*x^8) - (A*(a + b*x^2)^
(7/2))/(10*a*x^10) - (b^4*(3*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(256*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} + \frac{\left(-\frac{3Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx,x,x^2\right)}{10a}$$

$$= \frac{(3Ab-10aB)\left(a+bx^2\right)^{5/2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} - \frac{(b(3Ab-10aB))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x\right.}{32a}$$

$$= \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)\left(a+bx^2\right)^{5/2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} - \frac{\left(b^2(3Ab\right.}{}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)\left(a+bx^2\right)^{5/2}}{80ax^8}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6}$$

Mathematica [C] time = 0.026164, size = 62, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{7/2}\left(7a^5A+b^4x^{10}(10aB-3Ab)\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx^2}{a}+1\right)\right)}{70a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(7*a^5*A + b^4*(-3*A*b + 10*a*B)*x^10*Hypergeometric2F1
[7/2, 5, 9/2, 1 + (b*x^2)/a]))/(70*a^6*x^10)
```

Maple [B] time = 0.036, size = 353, normalized size = 1.9

$$-\frac{B}{8\,a x^8}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{B b}{48\,a^2 x^6}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{B b^2}{192\,a^3 x^4}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{B b^3}{128\,a^4 x^2}\left(b x^2+a\right)^{\frac{7}{2}}-\frac{B b^4}{128\,a^4}\left(b x^2+a\right)^{\frac{5}{2}}-\frac{5\,B b^4}{384\,a^3}\left(b x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/8*B/a/x^8*(b*x^2+a)^(7/2)+1/48*B*b/a^2/x^6*(b*x^2+a)^(7/2)+1/192*B*b^2/a
^3/x^4*(b*x^2+a)^(7/2)+1/128*B*b^3/a^4/x^2*(b*x^2+a)^(7/2)-1/128*B*b^4/a^4*
(b*x^2+a)^(5/2)-5/384*B*b^4/a^3*(b*x^2+a)^(3/2)+5/128*B*b^4/a^(3/2)*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/128*B*b^4/a^2*(b*x^2+a)^(1/2)-1/10*A*(b*x^
2+a)^(7/2)/a/x^10+3/80*A*b/a^2/x^8*(b*x^2+a)^(7/2)-1/160*A*b^2/a^3/x^6*(b*x
^2+a)^(7/2)-1/640*A*b^3/a^4/x^4*(b*x^2+a)^(7/2)-3/1280*A*b^4/a^5/x^2*(b*x^2
+a)^(7/2)+3/1280*A*b^5/a^5*(b*x^2+a)^(5/2)+1/256*A*b^5/a^4*(b*x^2+a)^(3/2)-
3/256*A*b^5/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/256*A*b^5/a^3*(
b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09433, size = 761, normalized size = 4.03

$$\left[-\frac{15\left(10\,B a b^4-3\,A b^5\right)\sqrt{a}x^{10}\log\left(-\frac{b x^2-2\,\sqrt{b x^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(15\left(10\,B a^2 b^3-3\,A a b^4\right)x^8+10\left(118\,B a^3 b^2+3\,A a^2 b^3\right)x^6+38\right.}{7680\,a^3 x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*
B*a^3*b^2 + 3*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x
^4 + 48*(10*B*a^5 + 21*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10), -1/3840*(
15*(10*B*a*b^4 - 3*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*B*a^3*b^2 + 3*A*a^2*b^3)*x^6 +
```

```
384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + 21*A*a^4*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17197, size = 311, normalized size = 1.65

$$-\frac{\frac{15\left(10\,Bab^5-3\,Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{150\left(bx^2+a\right)^{\frac{9}{2}}Bab^5+580\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5-1280\left(bx^2+a\right)^{\frac{5}{2}}Ba^3b^5+700\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-150\sqrt{bx^2+a}Ba^5b^5-45\left(bx^2\right.}{a^2b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 3*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^2) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 + 580*(b*x^2 + a)^(7/2)*B*a^2*b^5
- 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 + 700*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 45*(b*x^2 + a)^(9/2)*A*b^6 + 210*(b*x^2 + a)^(7
/2)*A*a*b^6 + 384*(b*x^2 + a)^(5/2)*A*a^2*b^6 - 210*(b*x^2 + a)^(3/2)*A*a^3
*b^6 + 45*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^2*b^5*x^10))/b
```

3.555 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\sqrt{a+bx^2}(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2
))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4) + (B*(a + b*x^2)^(7/
2))/(7*b^4)
```

Rubi [A] time = 0.0760649, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\sqrt{a+bx^2}(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2
))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4) + (B*(a + b*x^2)^(7/
2))/(7*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}}+\frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3}+\frac{(Ab-3aB)(a+bx)^{3/2}}{b^3}+\frac{B(a+bx)^{5/2}}{b^3}\right)\right.\\ &= \frac{a^2(Ab-aB)\sqrt{a+bx^2}}{b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.0547371, size = 78, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(8a^2b\left(7A+3Bx^2\right)-48a^3B-2ab^2x^2\left(14A+9Bx^2\right)+3b^3x^4\left(7A+5Bx^2\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^2) + 3*b^3*x^4*(7*A + 5*
B*x^2) - 2*a*b^2*x^2*(14*A + 9*B*x^2)))/(105*b^4)
```

Maple [A] time = 0.005, size = 77, normalized size = 0.8

$$\frac{15\,x^6Bb^3+21\,Ab^3x^4-18\,Bab^2x^4-28\,Aab^2x^2+24\,Ba^2bx^2+56\,Aa^2b-48\,Ba^3}{105\,b^4}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(15*B*b^3*x^6+21*A*b^3*x^4-18*B*a*b^2*x^4-28*A*a*b^2*
x^2+24*B*a^2*b*x^2+56*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59341, size = 173, normalized size = 1.73

$$\frac{\left(15\,Bb^3x^6-3\left(6\,Bab^2-7\,Ab^3\right)x^4-48\,Ba^3+56\,Aa^2b+4\left(6\,Ba^2b-7\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 - 3*(6*B*a*b^2 - 7*A*b^3)*x^4 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 1.37767, size = 172, normalized size = 1.72

$$\begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3}-\frac{4Aax^2\sqrt{a+bx^2}}{15b^2}+\frac{Ax^4\sqrt{a+bx^2}}{5b}-\frac{16Ba^3\sqrt{a+bx^2}}{35b^4}+\frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3}-\frac{6Bax^4\sqrt{a+bx^2}}{35b^2}+\frac{Bx^6\sqrt{a+bx^2}}{7b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**2
)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/(3
5*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a + b*
x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((A*x**6/6 + B*
x**8/8)/sqrt(a), True))
```

Giac [A] time = 1.10432, size = 140, normalized size = 1.4

$$\frac{15\left(bx^2+a\right)^{\frac{7}{2}}B-63\left(bx^2+a\right)^{\frac{5}{2}}Ba+105\left(bx^2+a\right)^{\frac{3}{2}}Ba^2-105\sqrt{bx^2+a}Ba^3+21\left(bx^2+a\right)^{\frac{5}{2}}Ab-70\left(bx^2+a\right)^{\frac{3}{2}}Aab+105}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2)*B - 63*(b*x^2 + a)^(5/2)*B*a + 105*(b*x^2 + a)^
(3/2)*B*a^2 - 105*sqrt(b*x^2 + a)*B*a^3 + 21*(b*x^2 + a)^(5/2)*A*b - 70*(b*
x^2 + a)^(3/2)*A*a*b + 105*sqrt(b*x^2 + a)*A*a^2*b)/b^4
```

3.556 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=122

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}(6Ab-5aB)}{24b^2}-\frac{ax\sqrt{a+bx^2}(6Ab-5aB)}{16b^3}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

```
[Out] -(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*A*b - 5*a*B)*x^3*Sqrt
[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) + (a^2*(6*A*b - 5*a*B
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rubi [A] time = 0.0529656, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 321, 217, 206}

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}(6Ab-5aB)}{24b^2}-\frac{ax\sqrt{a+bx^2}(6Ab-5aB)}{16b^3}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*A*b - 5*a*B)*x^3*Sqrt
[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) + (a^2*(6*A*b - 5*a*B
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}dx = \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(-6Ab+5aB)\int \frac{x^4}{\sqrt{a+bx^2}}dx}{6b}$$

$$= \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(a(6Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}dx}{8b^2}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{(a^2(6Ab-5aB))\int \frac{1}{\sqrt{a+bx^2}}}{16b^3}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{(a^2(6Ab-5aB))\,\text{Subst}\big(}{16b^3}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{a^2(6Ab-5aB)\tanh^{-1}\big(}{16b^{7/2}}$$

Mathematica [A] time = 0.0952979, size = 100, normalized size = 0.82

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^2B-2ab\left(9A+5Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)-3a^2(5aB-6Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*B + 4*b^2*x^2*(3*A + 2*B*x^2) - 2*a*b*(9
*A + 5*B*x^2)) - 3*a^2*(-6*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(48*b^(7/2))
```

Maple [A] time = 0.009, size = 143, normalized size = 1.2

$$\frac{x^5B}{6b}\sqrt{bx^2+a} - \frac{5Bax^3}{24b^2}\sqrt{bx^2+a} + \frac{5a^2Bx}{16b^3}\sqrt{bx^2+a} - \frac{5Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}} + \frac{Ax^3}{4b}\sqrt{bx^2+a} - \frac{3aAx}{8b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^5*(b*x^2+a)^(1/2)/b-5/24*B/b^2*a*x^3*(b*x^2+a)^(1/2)+5/16*B/b^3*a^2
*x*(b*x^2+a)^(1/2)-5/16*B/b^(7/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*A*x
^3/b*(b*x^2+a)^(1/2)-3/8*A/b^2*a*x*(b*x^2+a)^(1/2)+3/8*A/b^(5/2)*a^2*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71548, size = 490, normalized size = 4.02

$$\left[-\frac{3\left(5\,Ba^3-6\,Aa^2b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,Bb^3x^5-2\left(5\,Bab^2-6\,Ab^3\right)x^3+3\left(5\,Ba^2b-6\,Aab^2\right)\right.}{96\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^3)*x^3 + 3*(5*B*a^2*b
- 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/48*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^
3)*x^3 + 3*(5*B*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] time = 9.834, size = 235, normalized size = 1.93

$$-\frac{3Aa^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{A\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Ax^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x}{16b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{3}{2}}x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^5}{24b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - A*sqrt(a)*x**3/(8*b*sqrt(1 +
b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + A*x**5/(4*sqr
t(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x/(16*b**3*sqrt(1 + b*x**2/a)) + 5*
B*a**(3/2)*x**3/(48*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**5/(24*b*sqrt(1
+ b*x**2/a)) - 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + B*x**7/(6*
sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.13658, size = 144, normalized size = 1.18

$$\frac{1}{48}\left(2\left(\frac{4\,Bx^2}{b}-\frac{5\,Bab^3-6\,Ab^4}{b^5}\right)x^2+\frac{3\left(5\,Ba^2b^2-6\,Aab^3\right)}{b^5}\right)\sqrt{bx^2+a}x+\frac{\left(5\,Ba^3-6\,Aa^2b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2/b - (5*B*a*b^3 - 6*A*b^4)/b^5)*x^2 + 3*(5*B*a^2*b^2 - 6*A*
a*b^3)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.557 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{(a+bx^2)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}(Ab-aB)}{b^3}+\frac{B(a+bx^2)^{5/2}}{5b^3}$$

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/
(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rubi [A] time = 0.0549245, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{(a+bx^2)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}(Ab-aB)}{b^3}+\frac{B(a+bx^2)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/
(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx,x,x^2\right)\\ &= -\frac{a(Ab-aB)\sqrt{a+bx^2}}{b^3}+\frac{(Ab-2aB)(a+bx^2)^{3/2}}{3b^3}+\frac{B(a+bx^2)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0355832, size = 56, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(8a^2B-2ab(5A+2Bx^2)+b^2x^2(5A+3Bx^2)\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^2) + b^2*x^2*(5*A + 3*B*x^2)
))/(15*b^3)
```

Maple [A] time = 0.004, size = 53, normalized size = 0.8

$$-\frac{-3\,b^2Bx^4-5\,Ab^2x^2+4\,Babx^2+10\,abA-8\,a^2B}{15\,b^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(-3*B*b^2*x^4-5*A*b^2*x^2+4*B*a*b*x^2+10*A*a*b-8*B*a^
2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63704, size = 117, normalized size = 1.65

$$\frac{\left(3\,Bb^2x^4+8\,Ba^2-10\,Aab-\left(4\,Bab-5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x^
2 + a)/b^3
```

Sympy [A] time = 0.843588, size = 121, normalized size = 1.7

$$\begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{8Ba^2\sqrt{a+bx^2}}{15b^3}-\frac{4Bax^2\sqrt{a+bx^2}}{15b^2}+\frac{Bx^4\sqrt{a+bx^2}}{5b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b)
 + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*b*
*2) + B*x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/sqrt
(a), True))
```

Giac [A] time = 1.1498, size = 99, normalized size = 1.39

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}B-10\left(bx^2+a\right)^{\frac{3}{2}}Ba+15\sqrt{bx^2+a}Ba^2+5\left(bx^2+a\right)^{\frac{3}{2}}Ab-15\sqrt{bx^2+a}Aab}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 10*(b*x^2 + a)^(3/2)*B*a + 15*sqrt(b*x^2 + a)
*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 15*sqrt(b*x^2 + a)*A*a*b)/b^3
```

3.558 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{x\sqrt{a+bx^2}(4Ab-3aB)}{8b^2}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x^2])/(4*b)
 - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.035899, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}(4Ab-3aB)}{8b^2}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```



```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x^2])/(4*b)
 - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{\sqrt{a+bx^2}} \, dx = \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{(-4Ab+3aB)\int \frac{x^2}{\sqrt{a+bx^2}} \, dx}{4b}$$
$$= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{(a(4Ab-3aB))\int \frac{1}{\sqrt{a+bx^2}} \, dx}{8b^2}$$
$$= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{(a(4Ab-3aB))\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}$$
$$= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.0560067, size = 74, normalized size = 0.83

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(-3aB+4Ab+2bBx^2\right)+a(3aB-4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(4*A*b - 3*a*B + 2*b*B*x^2) + a*(-4*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.008, size = 101, normalized size = 1.1

$$\frac{x^3B}{4b}\sqrt{bx^2+a} - \frac{3Bax}{8b^2}\sqrt{bx^2+a} + \frac{3a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{Ax}{2b}\sqrt{bx^2+a} - \frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B/b^2*a*x*(b*x^2+a)^(1/2)+3/8*B/b^(5/2)*a^2
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*A*x/b*(b*x^2+a)^(1/2)-1/2*A*a/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62787, size = 382, normalized size = 4.29

$$\left[-\frac{\left(3\,Ba^2-4\,Aab\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(2\,Bb^2x^3-\left(3\,Bab-4\,Ab^2\right)x\right)\sqrt{bx^2+a}}{16\,b^3},-\frac{\left(3\,Ba^2-4\,Aab\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(2*B*b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1
/8*((3*B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*
b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 6.03696, size = 150, normalized size = 1.69

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**3/(8*b*sq
rt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + B*x**5
/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.13256, size = 101, normalized size = 1.13

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Bx^2}{b}-\frac{3\,Bab-4\,Ab^2}{b^3}\right)x-\frac{\left(3\,Ba^2-4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*B*x^2/b - (3*B*a*b - 4*A*b^2)/b^3)*x - 1/8*(3*B*a^2
- 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.559 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a+bx^2}(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

```
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.0335874, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\sqrt{a+bx^2}(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)\sqrt{a+bx^2}}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0217856, size = 33, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-2aB+3Ab+bBx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)
```

Maple [A] time = 0.003, size = 30, normalized size = 0.7

$$\frac{bBx^2 + 3\,Ab - 2\,Ba}{3\,b^2}\sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(B*b*x^2+3*A*b-2*B*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56539, size = 69, normalized size = 1.6

$$\frac{\left(Bbx^2 - 2\,Ba + 3\,Ab\right)\sqrt{bx^2 + a}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^2 - 2*B*a + 3*A*b)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 0.506212, size = 70, normalized size = 1.63

$$\begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2*
sqrt(a + b*x**2)/(3*b), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/sqrt(a), True))
```

Giac [A] time = 1.11937, size = 58, normalized size = 1.35

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}B-3\sqrt{bx^2+a}Ba+3\sqrt{bx^2+a}Ab}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 3*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 + a)*A*b)/b
^2
```

3.560 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.017414, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 217, 206}

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx &= \frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(-2Ab+aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(-2Ab+aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0172863, size = 57, normalized size = 0.98

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(aB-2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) - ((-2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.005, size = 62, normalized size = 1.1

$$\frac{Bx}{2b}\sqrt{bx^2+a}-\frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*x*(b*x^2+a)^(1/2)/b-1/2*B*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+A*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55744, size = 275, normalized size = 4.74

$$\left[\frac{2\sqrt{bx^2+a}Bbx-(Ba-2Ab)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4b^2},\frac{\sqrt{bx^2+a}Bbx+(Ba-2Ab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*B*b*x - (B*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*B*b*x + (B*a - 2*A*b)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] time = 2.49024, size = 126, normalized size = 2.17

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)
/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2))
```

Giac [A] time = 1.11632, size = 65, normalized size = 1.12

$$\frac{\sqrt{bx^2+a}Bx}{2b}+\frac{(Ba-2Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

3.561 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0331544, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{B\sqrt{a+bx^2}}{b} + \frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{B\sqrt{a+bx^2}}{b} + \frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= \frac{B\sqrt{a+bx^2}}{b} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0196275, size = 43, normalized size = 1.

$$\frac{B\sqrt{a+bx^2}}{b} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.008, size = 45, normalized size = 1.1

$$\frac{B}{b}\sqrt{bx^2+a} - A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*(b*x^2+a)^(1/2)/b-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58653, size = 246, normalized size = 5.72

$$\left[\frac{A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}Ba}{2\,ab}, \frac{A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}Ba}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*s
qrt(b*x^2 + a)*B*a)/(a*b), (A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
 sqrt(b*x^2 + a)*B*a)/(a*b)]
```

Sympy [A] time = 6.52413, size = 61, normalized size = 1.42

$$\frac{A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^2}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**2)))/(a*sqrt(-1/a)) - B*Piecewise((-x**2
/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**2)/b, True))/2
```

Giac [A] time = 1.13029, size = 51, normalized size = 1.19

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx^2+a}B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)*B/b
```

3.562 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

Rubi [A] time = 0.0176591, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {451, 217, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{ax} + B\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{A\sqrt{a+bx^2}}{ax} + B\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{A\sqrt{a+bx^2}}{ax} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0167366, size = 47, normalized size = 1.

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

Maple [A] time = 0.008, size = 41, normalized size = 0.9

$$B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{A}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58099, size = 255, normalized size = 5.43

$$\left[\frac{Ba\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,\sqrt{bx^2+a}Ab}{2\,abx}, -\frac{Ba\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}Ab}{abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqr
t(b*x^2 + a)*A*b)/(a*b*x), -(B*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + sqrt(b*x^2 + a)*A*b)/(a*b*x)]
```

Sympy [A] time = 1.21968, size = 99, normalized size = 2.11

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/
a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a
> 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0
)))
```

Giac [A] time = 1.1221, size = 78, normalized size = 1.66

$$-\frac{B\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2*A*sqrt(b)/((sqrt(b)
*x - sqrt(b*x^2 + a))^2 - a)
```

3.563 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(2*a^(3/2))
```

Rubi [A] time = 0.0458618, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(2*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} + \frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} + \frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0321569, size = 60, normalized size = 1.03

$$
\frac{1}{2}\left(-\frac{2\left(aB-\frac{Ab}{2}\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^2])/(a*x^2)) - (2*(-(A*b)/2 + a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/a^(3/2))/2
```

Maple [A] time = 0.007, size = 79, normalized size = 1.4

$$
-B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{A}{2ax^2}\sqrt{bx^2+a}+\frac{Ab}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*A*(b*x^2+a)^(1/2)/a/x^
2+1/2*A*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64842, size = 301, normalized size = 5.19

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}Aa}{4\,a^2x^2},\ \frac{(2\,Ba-Ab)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}Aa}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) + 2*sqrt(b*x^2 + a)*A*a)/(a^2*x^2), 1/2*((2*B*a - A*b)*sqrt(-a)*x
^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*A*a)/(a^2*x^2)]
```

Sympy [A] time = 13.1783, size = 66, normalized size = 1.14

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A] time = 1.11989, size = 84, normalized size = 1.45

$$\frac{\frac{\left(2\,Bab-Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{\sqrt{bx^2+a}Ab}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^2 + a)*A*b/(a*x^2))/b
```

3.564 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{a+bx^2}(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(3*a^2*x
)
```

Rubi [A] time = 0.0197902, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{\sqrt{a+bx^2}(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(3*a^2*x
)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{3ax^3}-\frac{(2Ab-3aB)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A] time = 0.0126149, size = 39, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}\left(a\left(A+3Bx^2\right)-2Abx^2\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/(3*a^2*x^3)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.7

$$-\frac{-2\,Ab{x}^{2}+3\,Ba{x}^{2}+Aa}{3\,{x}^{3}{a}^{2}}\sqrt{b{x}^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/x^3/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61191, size = 81, normalized size = 1.53

$$-\frac{{\left({\left(3\,B a - 2\,A b\right)} x^{2} + A a\right)} \sqrt{b x^{2} + a}}{3\,a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*B*a - 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] time = 1.68327, size = 70, normalized size = 1.32

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [B] time = 1.13835, size = 162, normalized size = 3.06

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3\,Ba^2\sqrt{b}-2\,Aab^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(3/2) + 3*B*a
^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.565 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a+bx^2}(3Ab-4aB)}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^2*x
^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] time = 0.0678825, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^2}(3Ab-4aB)}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^2*x
^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(b(3Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(3Ab-4aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a^2}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.156196, size = 83, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(-\frac{2a^2\left(A+2Bx^2\right)}{x^4}+\frac{b(4aB-3Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\frac{3aAb}{x^2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*((3*a*A*b)/x^2 - (2*a^2*(A + 2*B*x^2))/x^4 + (b*(-3*A*b +
4*a*B)*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a]))/(8*a^3)
```

Maple [A] time = 0.007, size = 119, normalized size = 1.3

$$-\frac{A}{4\,ax^4}\sqrt{bx^2+a}+\frac{3\,Ab}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,Ab^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{B}{2\,ax^2}\sqrt{bx^2+a}+\frac{Bb}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*A*b^2/a^(5
/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*B/a/x^2*(b*x^2+a)^(1/2)+1/2*B
*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70152, size = 406, normalized size = 4.51

$$\left[-\frac{\left(4\,Bab-3\,Ab^{2}\right)\sqrt{a}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(2\,Aa^{2}+\left(4\,Ba^{2}-3\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,a^{3}x^{4}},-\frac{\left(4\,Bab-3\,Ab^{2}\right)\sqrt{-a}x^{4}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(
a^3*x^4), -1/8*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) + (2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^4)]
```

Sympy [A] time = 28.9176, size = 150, normalized size = 1.67

$$-\frac{A}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{A\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3Ab^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2ax}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(s
qrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) +
 B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

Giac [A] time = 1.11397, size = 163, normalized size = 1.81

$$-\frac{\frac{\left(4\,Bab^{2}-3\,Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{4\left(bx^{2}+a\right)^{\frac{3}{2}}Bab^{2}-4\,\sqrt{bx^{2}+a}Ba^{2}b^{2}-3\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{3}+5\,\sqrt{bx^{2}+a}Aab^{3}}{a^{2}b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2)
 + (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 - 3*(b*x^2 +
a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*b^3)/(a^2*b^2*x^4))/b
```

3.566 $\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\sqrt{a+bx^2}(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rubi [A] time = 0.0346146, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\sqrt{a+bx^2}(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{5ax^5} - \frac{(4Ab-5aB)\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{5a}$$
$$= -\frac{A\sqrt{a+bx^2}}{5ax^5} + \frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3} + \frac{(2b(4Ab-5aB))\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}$$
$$= -\frac{A\sqrt{a+bx^2}}{5ax^5} + \frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3} - \frac{2b(4Ab-5aB)\sqrt{a+bx^2}}{15a^3x}$$

Mathematica [A] time = 0.0191678, size = 62, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}\left(a^2\left(3A+5Bx^2\right)-2abx^2\left(2A+5Bx^2\right)+8Ab^2x^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(8*A*b^2*x^4 - 2*a*b*x^2*(2*A + 5*B*x^2) + a^2*(3*A + 5*B
*x^2)))/(15*a^3*x^5)
```

Maple [A] time = 0.006, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-10\,Bx^4ab-4\,aAbx^2+5\,Bx^2a^2+3\,Aa^2}{15\,x^5a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(8*A*b^2*x^4-10*B*a*b*x^4-4*A*a*b*x^2+5*B*a^2*x^2+3*A
*a^2)/x^5/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71192, size = 130, normalized size = 1.55

$$\frac{\left(2\left(5\,Bab-4\,Ab^2\right)x^4-3\,Aa^2-\left(5\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 - (5*B*a^2 - 4*A*a*b)*x^2)*sqrt(b
*x^2 + a)/(a^3*x^5)
```

Sympy [B] time = 2.34485, size = 355, normalized size = 4.23

$$-\frac{3Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2Aa^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3Aa^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12Aa}{15a^5b^4x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(13/
2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - B*
sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

Giac [B] time = 1.14161, size = 238, normalized size = 2.83

$$\frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bb^{\frac{3}{2}}-35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Bab^{\frac{3}{2}}+40\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{5}{2}}+25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2b^{\frac{3}{2}}\cdots\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) - 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*B*a*b^(3/2) + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(5/2) + 2
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^(3/2) - 20*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a*b^(5/2) - 5*B*a^3*b^(3/2) + 4*A*a^2*b^(5/2))/((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^5
```

3.567 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=123

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2) + (b^2*(5*A*b - 6*a
*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rubi [A] time = 0.094098, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2) + (b^2*(5*A*b - 6*a
*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{\left(-\frac{5Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)}{6a}$$
$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} + \frac{(b(5Ab-6aB))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}$$
$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{\left(b^2(5Ab-6aB)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.}{32a^3}$$
$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{(b(5Ab-6aB))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{16a^3}$$
$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} + \frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Mathematica [C] time = 0.0199388, size = 61, normalized size = 0.5

$$-\frac{\sqrt{a+bx^2}\left(a^3A+b^2x^6(6aB-5Ab)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(a^3*A + b^2*(-5*A*b + 6*a*B)*x^6*Hypergeometric2F1[1/2,
3, 3/2, 1 + (b*x^2)/a]))/(6*a^4*x^6)
```

Maple [A] time = 0.01, size = 161, normalized size = 1.3

$$-\frac{A}{6\,ax^6}\sqrt{bx^2+a}+\frac{5\,Ab}{24\,a^2x^4}\sqrt{bx^2+a}-\frac{5\,Ab^2}{16\,a^3x^2}\sqrt{bx^2+a}+\frac{5\,Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{4\,ax^4}\sqrt{bx^2+a}+\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*A*(b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(b*x^2+a)^(1/2)-5/16*A*b^2/a^
3/x^2*(b*x^2+a)^(1/2)+5/16*A*b^3/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)-1/4*B/a/x^4*(b*x^2+a)^(1/2)+3/8*B*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*B*b^2/a
```

```
^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76422, size = 514, normalized size = 4.18

$$\left[-\frac{3\left(6\,Bab^2-5\,Ab^3\right)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(3\left(6\,Ba^2b-5\,Aab^2\right)x^4-8\,Aa^3-2\left(6\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{bx^2}}{96\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) - 2*(3*(6*B*a^2*b - 5*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*
a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6), 1/48*(3*(6*B*a*b^2 - 5*A*
b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(6*B*a^2*b - 5*A*a*
b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6
)]
```

Sympy [B] time = 48.6336, size = 235, normalized size = 1.91

$$-\frac{A}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}}{24ax^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{3}{2}}}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{5}{2}}}{16a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16a^{\frac{7}{2}}}-\frac{B}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*sqrt(a/(b*x
**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2
)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(7/2)) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*x**3
*sqrt(a/(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B
*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2))
```

Giac [A] time = 1.1274, size = 213, normalized size = 1.73

$$\frac{\frac{3\left(6\,Bab^3-5\,Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{18\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-48\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+30\sqrt{bx^2+a}Ba^3b^3-15\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+40\left(bx^2+a\right)^{\frac{3}{2}}Aab^4-33\sqrt{bx^2+a}Aa^2b^4}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - 5*A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
3) + (18*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 30*sq
rt(b*x^2 + a)*B*a^3*b^3 - 15*(b*x^2 + a)^(5/2)*A*b^4 + 40*(b*x^2 + a)^(3/2)
*A*a*b^4 - 33*sqrt(b*x^2 + a)*A*a^2*b^4)/(a^3*b^3*x^6))/b
```

3.568 $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2\sqrt{a+bx^2}(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (8*b^2*(6*A*b
- 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rubi [A] time = 0.0473258, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8b^2\sqrt{a+bx^2}(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (8*b^2*(6*A*b
- 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{7ax^7} - \frac{(6Ab-7aB)\int \frac{1}{x^6\sqrt{a+bx^2}}\,dx}{7a}$$
$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} + \frac{(4b(6Ab-7aB))\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}$$
$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} - \frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3} - \frac{\left(8b^2(6Ab-7aB)\right)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{105a^3}$$
$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} - \frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3} + \frac{8b^2(6Ab-7aB)\sqrt{a+bx^2}}{105a^4x}$$

Mathematica [A] time = 0.0289058, size = 84, normalized size = 0.72

$$\frac{\left(\frac{bx^2}{a}+1\right)\left(3a^2-4abx^2+8b^2x^4\right)(6Ab-7aB)}{105a^3x^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*(1 + (b*x^2)/a)*(3*a^2 -
4*a*b*x^2 + 8*b^2*x^4))/(105*a^3*x^5*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 83, normalized size = 0.7

$$-\frac{-48\,Ab^3x^6+56\,Bab^2x^6+24\,Aab^2x^4-28\,Ba^2bx^4-18\,Aa^2bx^2+21\,Ba^3x^2+15\,Aa^3}{105\,x^7a^4}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-48*A*b^3*x^6+56*B*a*b^2*x^6+24*A*a*b^2*x^4-28*B*a^
2*b*x^4-18*A*a^2*b*x^2+21*B*a^3*x^2+15*A*a^3)/x^7/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68952, size = 185, normalized size = 1.58

$$-\frac{\left(8\left(7\,Bab^2-6\,Ab^3\right)x^6-4\left(7\,Ba^2b-6\,Aab^2\right)x^4+15\,Aa^3+3\left(7\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*a*b^2 - 6*A*b^3)*x^6 - 4*(7*B*a^2*b - 6*A*a*b^2)*x^4 + 15*A*
a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*x^7)
```

Sympy [B] time = 3.31546, size = 819, normalized size = 7.

$$-\frac{5Aa^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{9Aa^5b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{}{35a^7b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*A*a**5*b**(21/2)*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**
7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x*
*12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 1
05*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 40*A*a*b
**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 16*A*b**(31/2)*x**12*sqrt
(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*
x**10 + 35*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a
**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*a**3*b**(11/2)
*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3
*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b**(15/2)*x**6*sqrt(a/
(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) -
 8*B*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8)
```

Giac [B] time = 1.11841, size = 313, normalized size = 2.68

$$\frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{7}{2}}+147\left(\sqrt{b}x-\sqrt{bx^2+a}\right)\right.}{105\left(\left(\sqrt{b}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(5/2) - 175*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(7/2)
 + 147*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(5/2) - 126*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*A*a*b^(7/2) - 49*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(5
/2) + 42*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(7/2) + 7*B*a^4*b^(5/2) -
6*A*a^3*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.569 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{5x^3\sqrt{a+bx^2}(6Ab-7aB)}{24b^3}-\frac{5ax\sqrt{a+bx^2}(6Ab-7aB)}{16b^4}+\frac{Bx^7}{6b\sqrt{a+b}}$$

```
[Out] -((6*A*b - 7*a*B)*x^5)/(6*b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqrt[a + b*x^
2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4) + (5*(6*A*b - 7*a*B)
*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rubi [A] time = 0.0676372, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{5x^3\sqrt{a+bx^2}(6Ab-7aB)}{24b^3}-\frac{5ax\sqrt{a+bx^2}(6Ab-7aB)}{16b^4}+\frac{Bx^7}{6b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```


```
[Out] -((6*A*b - 7*a*B)*x^5)/(6*b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqrt[a + b*x^
2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4) + (5*(6*A*b - 7*a*B)
*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*b^(9/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6 \left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{(-6Ab+7aB)\int \frac{x^6}{\left(a+bx^2\right)^{3/2}}\,dx}{6b}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{(5(6Ab-7aB))\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{6b^2}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} - \frac{(5a(6Ab-7aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} + \frac{(5a}{}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} + \frac{(5a}{}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} + \frac{5a^2}{}$$

Mathematica [A] time = 0.146117, size = 131, normalized size = 0.86

$$\frac{\sqrt{b}x\left(a^2\left(35bBx^2-90Ab\right)+105a^3B-2ab^2x^2\left(15A+7Bx^2\right)+4b^3x^4\left(3A+2Bx^2\right)\right)-15a^{5/2}\sqrt{\frac{bx^2}{a}+1}(7aB-6Ab)\,\mathrm{sinh}}{48b^{9/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(105*a^3*B + 4*b^3*x^4*(3*A + 2*B*x^2) - 2*a*b^2*x^2*(15*A + 7*B
*x^2) + a^2*(-90*A*b + 35*b*B*x^2)) - 15*a^(5/2)*(-6*A*b + 7*a*B)*Sqrt[1 +
(b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(48*b^(9/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.015, size = 185, normalized size = 1.2

$$\frac{x^7B}{6\,b}\frac{1}{\sqrt{bx^2+a}} - \frac{7\,Bax^5}{24\,b^2}\frac{1}{\sqrt{bx^2+a}} + \frac{35\,a^2Bx^3}{48\,b^3}\frac{1}{\sqrt{bx^2+a}} + \frac{35\,Ba^3x}{16\,b^4}\frac{1}{\sqrt{bx^2+a}} - \frac{35\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}} + \frac{Ax}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*B*x^7/b/(b*x^2+a)^(1/2)-7/24*B/b^2*a*x^5/(b*x^2+a)^(1/2)+35/48*B/b^3*a^
2*x^3/(b*x^2+a)^(1/2)+35/16*B/b^4*a^3*x/(b*x^2+a)^(1/2)-35/16*B/b^(9/2)*a^3
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*A*x^5/b/(b*x^2+a)^(1/2)-5/8*A/b^2*a*x^3/
(b*x^2+a)^(1/2)-15/8*A/b^3*a^2*x/(b*x^2+a)^(1/2)+15/8*A/b^(7/2)*a^2*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78727, size = 717, normalized size = 4.72

$$\left[-\frac{15\left(7\,Ba^4-6\,Aa^3b+\left(7\,Ba^3b-6\,Aa^2b^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,Bb^4x^7-2\left(7\,Bab^3-6\,Ab^4\right)x^5\right.}{96\left(b^6x^2+ab^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*B*b^4*x^7 - 2*(7*B*a*b
^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b^3)*x^3 + 15*(7*B*a^3*b - 6*A*a
^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5), 1/48*(15*(7*B*a^4 - 6*A*a^3*
b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (8*B*b^4*x^7 - 2*(7*B*a*b^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b
^3)*x^3 + 15*(7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5
)]
```

Sympy [A] time = 17.7981, size = 233, normalized size = 1.53

$$A\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{7\sqrt{a}}{24b^2\sqrt{1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt
(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*s
qrt(a)*b*sqrt(1 + b*x**2/a))) + B*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**2/a
)) + 35*a**(3/2)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**
2*sqrt(1 + b*x**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x*
*7/(6*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.12643, size = 184, normalized size = 1.21

$$\frac{\left(\left(2\left(\frac{4Bx^2}{b}-\frac{7Bab^5-6Ab^6}{b^7}\right)x^2+\frac{5\left(7Ba^2b^4-6Aab^5\right)}{b^7}\right)x^2+\frac{15\left(7Ba^3b^3-6Aa^2b^4\right)}{b^7}\right)x}{48\sqrt{bx^2+a}}+\frac{5\left(7Ba^3-6Aa^2b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*B*x^2/b - (7*B*a*b^5 - 6*A*b^6)/b^7)*x^2 + 5*(7*B*a^2*b^4 - 6*A
*a*b^5)/b^7)*x^2 + 15*(7*B*a^3*b^3 - 6*A*a^2*b^4)/b^7)*x/sqrt(b*x^2 + a) +
5/16*(7*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.570 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}(2Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*Sqrt[a + b*
x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) + (B*(a + b*x^2)^(5/2
))/(5*b^4)
```

Rubi [A] time = 0.0761042, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}(2Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*Sqrt[a + b*
x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) + (B*(a + b*x^2)^(5/2
))/(5*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}-\frac{a(2Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}
\end{aligned}$$

Mathematica [A] time = 0.0505392, size = 77, normalized size = 0.78

$$\frac{-8a^2b\left(5A-3Bx^2\right)+48a^3B-2ab^2x^2\left(10A+3Bx^2\right)+b^3x^4\left(5A+3Bx^2\right)}{15b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (48*a^3*B - 8*a^2*b*(5*A - 3*B*x^2) + b^3*x^4*(5*A + 3*B*x^2) - 2*a*b^2*x^2
*(10*A + 3*B*x^2))/(15*b^4*Sqrt[a + b*x^2])
```

Maple [A] time = 0.006, size = 77, normalized size = 0.8

$$-\frac{-3x^6Bb^3-5Ab^3x^4+6Bab^2x^4+20Aab^2x^2-24Ba^2bx^2+40Aa^2b-48Ba^3}{15b^4}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(-3*B*b^3*x^6-5*A*b^3*x^4+6*B*a*b^2*x^4+20*A*a*b^2*x^2-24*B*a^2*b*x^2
+40*A*a^2*b-48*B*a^3)/(b*x^2+a)^(1/2)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61189, size = 186, normalized size = 1.88

$$\frac{\left(3Bb^3x^6-\left(6Bab^2-5Ab^3\right)x^4+48Ba^3-40Aa^2b+4\left(6Ba^2b-5Aab^2\right)x^2\right)\sqrt{bx^2+a}}{15\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^3*x^6 - (6*B*a*b^2 - 5*A*b^3)*x^4 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 1.52397, size = 172, normalized size = 1.74

$$\begin{cases} -\frac{8Aa^2}{3b^3\sqrt{a+bx^2}} - \frac{4Aax^2}{3b^2\sqrt{a+bx^2}} + \frac{Ax^4}{3b\sqrt{a+bx^2}} + \frac{16Ba^3}{5b^4\sqrt{a+bx^2}} + \frac{8Ba^2x^2}{5b^3\sqrt{a+bx^2}} - \frac{2Bax^4}{5b^2\sqrt{a+bx^2}} + \frac{Bx^6}{5b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6} + \frac{Bx^8}{8}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b**2*sqrt(a
+ b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/(5*b**4*sqrt(a + b*x
**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*B*a*x**4/(5*b**2*sqrt(a
 + b*x**2)) + B*x**6/(5*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8
/8)/a**(3/2), True))
```

Giac [A] time = 1.1427, size = 131, normalized size = 1.32

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}B - 15\left(bx^2+a\right)^{\frac{3}{2}}Ba + 45\sqrt{bx^2+a}Ba^2 + 5\left(bx^2+a\right)^{\frac{3}{2}}Ab - 30\sqrt{bx^2+a}Aab + \frac{15\left(Ba^3-Aa^2b\right)}{\sqrt{bx^2+a}}}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 15*(b*x^2 + a)^(3/2)*B*a + 45*sqrt(b*x^2 + a)
*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 30*sqrt(b*x^2 + a)*A*a*b + 15*(B*a^3 - A
*a^2*b)/sqrt(b*x^2 + a))/b^4
```

3.571 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=119

$$-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}(4Ab-5aB)}{8b^3}-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

```
[Out] -((4*A*b - 5*a*B)*x^3)/(4*b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqrt[a + b*x^
2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (3*a*(4*A*b - 5*a*B)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] time = 0.0505837, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}(4Ab-5aB)}{8b^3}-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((4*A*b - 5*a*B)*x^3)/(4*b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqrt[a + b*x^
2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (3*a*(4*A*b - 5*a*B)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{Bx^5}{4b\sqrt{a+bx^2}}-\frac{(-4Ab+5aB)\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{4b}\\
&= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{(3(4Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b^2}\\
&= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{(3a(4Ab-5aB))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^3}\\
&= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{(3a(4Ab-5aB))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{8b^3}\\
&= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.119493, size = 108, normalized size = 0.91

$$\frac{\sqrt{b}x\left(-15a^2B+ab\left(12A-5Bx^2\right)+2b^2x^2\left(2A+Bx^2\right)\right)+3a^{3/2}\sqrt{\frac{bx^2}{a}+1}(5aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(-15*a^2*B + a*b*(12*A - 5*B*x^2) + 2*b^2*x^2*(2*A + B*x^2)) + 3
*a^(3/2)*(-4*A*b + 5*a*B)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])
/(8*b^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 141, normalized size = 1.2

$$\frac{x^5B}{4b}\frac{1}{\sqrt{bx^2+a}}-\frac{5Bax^3}{8b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15a^2Bx}{8b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{Ax^3}{2b}\frac{1}{\sqrt{bx^2+a}}+\frac{3aAx}{2b^2}\frac{1}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*B*x^5/b/(b*x^2+a)^(1/2)-5/8*B/b^2*a*x^3/(b*x^2+a)^(1/2)-15/8*B/b^3*a^2*
x/(b*x^2+a)^(1/2)+15/8*B/b^(7/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*A*x^
3/b/(b*x^2+a)^(1/2)+3/2*A/b^2*a*x/(b*x^2+a)^(1/2)-3/2*A/b^(5/2)*a*ln(x*b^(1
```

```
/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62042, size = 603, normalized size = 5.07

$$\left[-\frac{3\left(5\,Ba^3-4\,Aa^2b+\left(5\,Ba^2b-4\,Aab^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(2\,Bb^3x^5-\left(5\,Bab^2-4\,Ab^3\right)x^3\right.}{16\left(b^5x^2+ab^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b^3*x^5 - (5*B*a*b^2 -
4*A*b^3)*x^3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b
^4), -1/8*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b^3*x^5 - (5*B*a*b^2 - 4*A*b^3)*x^
3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 10.7782, size = 177, normalized size = 1.49

$$A\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{}{4\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(-15*a**(3/2)*x/(8
*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*
x**2/a)))
```

Giac [A] time = 1.11724, size = 140, normalized size = 1.18

$$\frac{\left(\left(\frac{2\,Bx^2}{b}-\frac{5\,Bab^3-4\,Ab^4}{b^5}\right)x^2-\frac{3\left(5\,Ba^2b^2-4\,Aab^3\right)}{b^5}\right)x}{8\,\sqrt{bx^2+a}}-\frac{3\left(5\,Ba^2-4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*B*x^2/b - (5*B*a*b^3 - 4*A*b^4)/b^5)*x^2 - 3*(5*B*a^2*b^2 - 4*A*a*b
^3)/b^5)*x/sqrt(b*x^2 + a) - 3/8*(5*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(7/2)
```

3.572 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{a+bx^2}(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b*x^2])/b^3
 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rubi [A] time = 0.0555836, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\sqrt{a+bx^2}(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b*x^2])/b^3
 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}}+\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{(Ab-2aB)\sqrt{a+bx^2}}{b^3}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.0320527, size = 55, normalized size = 0.82

$$\frac{-8a^2B + a\left(6Ab - 4bBx^2\right) + b^2x^2\left(3A + Bx^2\right)}{3b^3\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^2*B + b^2*x^2*(3*A + B*x^2) + a*(6*A*b - 4*b*B*x^2))/(3*b^3*Sqrt[a +
b*x^2])
```

Maple [A] time = 0.003, size = 52, normalized size = 0.8

$$\frac{b^2Bx^4 + 3\,Ab^2x^2 - 4\,Babx^2 + 6\,Aab - 8\,a^2B}{3\,b^3}\frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/3*(B*b^2*x^4+3*A*b^2*x^2-4*B*a*b*x^2+6*A*a*b-8*B*a^2)/(b*x^2+a)^(1/2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53889, size = 131, normalized size = 1.96

$$\frac{\left(Bb^2x^4 - 8\,Ba^2 + 6\,Aab - \left(4\,Bab - 3\,Ab^2\right)x^2\right)\sqrt{bx^2 + a}}{3\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^4 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x^2)*sqrt(b*x^2 +
a)/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 0.928273, size = 117, normalized size = 1.75

$$\begin{cases} \frac{2Aa}{b^2\sqrt{a+bx^2}} + \frac{Ax^2}{b\sqrt{a+bx^2}} - \frac{8Ba^2}{3b^3\sqrt{a+bx^2}} - \frac{4Bax^2}{3b^2\sqrt{a+bx^2}} + \frac{Bx^4}{3b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x**2)) - 8*
B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*sqrt(a + b*x**2)) + B
*x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(3/2), T
rue))
```

Giac [A] time = 1.12725, size = 88, normalized size = 1.31

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}B-6\sqrt{bx^2+a}Ba+3\sqrt{bx^2+a}Ab-\frac{3\left(Ba^2-Aab\right)}{\sqrt{bx^2+a}}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 6*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 + a)*A*b -
3*(B*a^2 - A*a*b)/sqrt(b*x^2 + a))/b^3
```

3.573 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/(2*b^2) +
((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] time = 0.0568336, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 388, 217, 206}

$$-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/(2*b^2) +
((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A + Bx^2\right)}{\left(a + bx^2\right)^{3/2}} dx = -\frac{(Ab - aB)x}{b^2\sqrt{a + bx^2}} - \frac{\int \frac{-Ab+aB-bBx^2}{\sqrt{a+bx^2}} dx}{b^2}$$

$$= -\frac{(Ab - aB)x}{b^2\sqrt{a + bx^2}} + \frac{Bx\sqrt{a + bx^2}}{2b^2} + \frac{(2Ab - 3aB)\int \frac{1}{\sqrt{a+bx^2}} dx}{2b^2}$$

$$= -\frac{(Ab - aB)x}{b^2\sqrt{a + bx^2}} + \frac{Bx\sqrt{a + bx^2}}{2b^2} + \frac{(2Ab - 3aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}$$

$$= -\frac{(Ab - aB)x}{b^2\sqrt{a + bx^2}} + \frac{Bx\sqrt{a + bx^2}}{2b^2} + \frac{(2Ab - 3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.0984349, size = 86, normalized size = 1.04

$$\frac{\sqrt{b}x\left(3aB - 2Ab + bBx^2\right) - \sqrt{a}\sqrt{\frac{bx^2}{a} + 1}(3aB - 2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(-2*A*b + 3*a*B + b*B*x^2) - Sqrt[a]*(-2*A*b + 3*a*B)*Sqrt[1 + (
b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.006, size = 97, normalized size = 1.2

$$\frac{x^3B}{2b}\frac{1}{\sqrt{bx^2 + a}} + \frac{3Bax}{2b^2}\frac{1}{\sqrt{bx^2 + a}} - \frac{3Ba}{2}\ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right)b^{-\frac{5}{2}} - \frac{Ax}{b}\frac{1}{\sqrt{bx^2 + a}} + A\ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B/b^2*a*x/(b*x^2+a)^(1/2)-3/2*B/b^(5/2)*a*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))-A*x/b/(b*x^2+a)^(1/2)+A/b^(3/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62837, size = 482, normalized size = 5.81

$$\left[-\frac{\left(3\,Ba^2-2\,Aab+\left(3\,Bab-2\,Ab^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(Bb^2x^3+\left(3\,Bab-2\,Ab^2\right)x\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(B*b^2*x^3 + (3*B*a*b - 2*A*b^2)*x)*s
qrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A
*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b^2*x^3 + (3*B*
a*b - 2*A*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 6.29665, size = 114, normalized size = 1.37

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.12964, size = 95, normalized size = 1.14

$$\frac{\left(\frac{Bx^2}{b}+\frac{3\,Bab-2\,Ab^2}{b^3}\right)x}{2\,\sqrt{bx^2+a}}+\frac{(3\,Ba-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(B*x^2/b + (3*B*a*b - 2*A*b^2)/b^3)*x/sqrt(b*x^2 + a) + 1/2*(3*B*a - 2*
A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.574 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

```
[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rubi [A] time = 0.0321411, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx,x,x^2\right)\\
&= -\frac{Ab-aB}{b^2\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0207583, size = 30, normalized size = 0.73

$$\frac{2aB-Ab+bBx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.003, size = 30, normalized size = 0.7

$$-\frac{-bBx^2 + Ab - 2Ba}{b^2}\frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -(-B*b*x^2+A*b-2*B*a)/(b*x^2+a)^(1/2)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54058, size = 80, normalized size = 1.95

$$\frac{\left(Bbx^2 + 2Ba - Ab\right)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (B*b*x^2 + 2*B*a - A*b)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.588677, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{A}{b\sqrt{a+bx^2}} + \frac{2Ba}{b^2\sqrt{a+bx^2}} + \frac{Bx^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)) + B*x**2
/(b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/a**(3/2), True))
```

Giac [A] time = 1.10764, size = 46, normalized size = 1.12

$$\frac{\sqrt{bx^2+a}B+\frac{Ba-Ab}{\sqrt{bx^2+a}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(b*x^2 + a)*B + (B*a - A*b)/sqrt(b*x^2 + a))/b^2
```

3.575 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^{3/2}} dx$

Optimal. Leaf size=54

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rubi [A] time = 0.0178983, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 217, 206}

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.053254, size = 70, normalized size = 1.3

$$\frac{a^{3/2}B\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x(Ab-aB)}{ab^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(A*b - a*B)*x + a^(3/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(a*b^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 54, normalized size = 1.

$$-\frac{Bx}{b}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Ax}{a}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+A*x/a/(b*x^2
+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53149, size = 370, normalized size = 6.85

$$\left[-\frac{2\left(Bab-Ab^2\right)\sqrt{bx^2+a}\,x-\left(Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\left(ab^3x^2+a^2b^2\right)}, -\frac{\left(Bab-Ab^2\right)\sqrt{bx^2+a}\,x+\left(Babx^2\right.}{ab^3x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(B*a*b - A*b^2)*sqrt(b*x^2 + a)*x - (B*a*b*x^2 + B*a^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), -((B*
a*b - A*b^2)*sqrt(b*x^2 + a)*x + (B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] time = 3.62923, size = 60, normalized size = 1.11

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.13518, size = 69, normalized size = 1.28

$$-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{(Ba-Ab)x}{\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/(sqrt(b*x
^2 + a)*a*b)
```

3.576 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Rubi [A] time = 0.0387111, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}} + \frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}} + \frac{A\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0326145, size = 53, normalized size = 1.

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Maple [A] time = 0.006, size = 60, normalized size = 1.1

$$-\frac{B}{b}\frac{1}{\sqrt{bx^2+a}} + \frac{A}{a}\frac{1}{\sqrt{bx^2+a}} - A\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] -B/b/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2
+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61111, size = 365, normalized size = 6.89

$$\left[\frac{\left(Ab^2x^2 + Aab\right)\sqrt{a}\log\left(-\frac{bx^2 - 2\sqrt{bx^2+a}\sqrt{a} + 2a}{x^2}\right) - 2\left(Ba^2 - Aab\right)\sqrt{bx^2 + a}}{2\left(a^2b^2x^2 + a^3b\right)}, \frac{\left(Ab^2x^2 + Aab\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - \left(Ba^2 - \right.}{a^2b^2x^2 + a^3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2*x^2 + A*a*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(B*a^2 - A*a*b)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b), ((A*
b^2*x^2 + A*a*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*a^2 - A*a*b
)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [A] time = 8.91535, size = 48, normalized size = 0.91

$$\frac{A\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{a\sqrt{-a}} - \frac{-Ab + Ba}{ab\sqrt{a + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a*sqrt(-a)) - (-A*b + B*a)/(a*b*sqrt(a +
 b*x**2))
```

Giac [A] time = 1.11949, size = 70, normalized size = 1.32

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{Ba - Ab}{\sqrt{bx^2 + a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (B*a - A*b)/(sqrt(b*x^2 +
 a)*a*b)
```

3.577 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0188691, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 191}

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)x}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0116595, size = 36, normalized size = 0.77

$$\frac{-aA+aBx^2-2Abx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$-\frac{2\,Abx^2-Bax^2+Aa}{xa^2}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*A*b*x^2-B*a*x^2+A*a)/(b*x^2+a)^(1/2)/x/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49501, size = 85, normalized size = 1.81

$$\frac{\left((Ba-2\,Ab)x^2-Aa\right)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] ((B*a - 2*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A] time = 4.80439, size = 68, normalized size = 1.45

$$A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.12052, size = 77, normalized size = 1.64

$$\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}+\frac{(Ba-Ab)x}{\sqrt{bx^2+a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + (B*a - A*b)*x/(sqrt
(b*x^2 + a)*a^2)
```

3.578 $\int \frac{A+Bx^2}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

```
[Out] -(3*A*b - 2*a*B)/(2*a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*x^2]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0671464, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(3*A*b - 2*a*B)/(2*a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*x^2]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{A}{2ax^2\sqrt{a+bx^2}} + \frac{\left(-\frac{3Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} - \frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} - \frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2b}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0184403, size = 57, normalized size = 0.66

$$
\frac{x^2(2aB-3Ab)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)-aA}{2a^2x^2\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) + (-3*A*b + 2*a*B)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/
a])/(2*a^2*x^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.009, size = 109, normalized size = 1.3

$$
\frac{B}{a}\frac{1}{\sqrt{bx^2+a}} - B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}} - \frac{A}{2ax^2}\frac{1}{\sqrt{bx^2+a}} - \frac{3Ab}{2a^2}\frac{1}{\sqrt{bx^2+a}} + \frac{3Ab}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] B/a/(b*x^2+a)^(1/2)-B/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*A/a
/x^2/(b*x^2+a)^(1/2)-3/2*A*b/a^2/(b*x^2+a)^(1/2)+3/2*A*b/a^(5/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7149, size = 506, normalized size = 5.88

$$\left[-\frac{\left(\left(2\,Bab-3\,Ab^{2}\right)x^{4}+\left(2\,Ba^{2}-3\,Aab\right)x^{2}\right)\sqrt{a}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(Aa^{2}-\left(2\,Ba^{2}-3\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{4\left(a^{3}bx^{4}+a^{4}x^{2}\right)}, \frac{((2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(a)*log(-(b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a^2 - (2*B*a^2 - 3*A*a*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2), 1/2*(((2*B*a*b - 3*A*b^2)*x^
4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (A
*a^2 - (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [B] time = 20.9669, size = 262, normalized size = 3.05

$$A\left(-\frac{1}{2a\sqrt{b}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(\frac{2a^{3}\sqrt{1+\frac{bx^{2}}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}+\frac{a^{3}\log\left(\frac{bx^{2}}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{2}}{a}}\right.}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(2*a**3*s
qrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*
a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/
2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(
7/2)*b*x**2))
```

Giac [A] time = 1.13625, size = 134, normalized size = 1.56

$$\frac{\left(2\,Ba-3\,Ab\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^{2}}+\frac{2\left(bx^{2}+a\right)Ba-2\,Ba^{2}-3\left(bx^{2}+a\right)Ab+2\,Aab}{2\left(\left(bx^{2}+a\right)^{\frac{3}{2}}-\sqrt{bx^{2}+a}a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*(
2*(b*x^2 + a)*B*a - 2*B*a^2 - 3*(b*x^2 + a)*A*b + 2*A*a*b)/(((b*x^2 + a)^(3
/2) - sqrt(b*x^2 + a)*a)*a^2)
```

3.579 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a + b*x^2]) +
(2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0311549, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a + b*x^2]) +
(2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{3ax^3\sqrt{a+bx^2}} - \frac{(4Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}$$
$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{(2b(4Ab-3aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{2b(4Ab-3aB)x}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0176366, size = 61, normalized size = 0.74

$$\frac{\left(a+2bx^2\right)(4Ab-3aB)}{3a^3x\sqrt{a+bx^2}} - \frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + ((4*A*b - 3*a*B)*(a + 2*b*x^2))/(3*a^3*x*Sqr
t[a + b*x^2])
```

Maple [A] time = 0.005, size = 58, normalized size = 0.7

$$-\frac{-8\,Ab^2x^4+6\,Bx^4ab-4\,aAbx^2+3\,Bx^2a^2+Aa^2}{3\,x^3a^3}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*A*b^2*x^4+6*B*a*b*x^4-4*A*a*b*x^2+3*B*a^2*x^2+A*a^2)/(b*x^2+a)^(1/
2)/x^3/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55447, size = 143, normalized size = 1.74

$$-\frac{\left(2\left(3\,Bab-4\,Ab^2\right)x^4+Aa^2+\left(3\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*a*b - 4*A*b^2)*x^4 + A*a^2 + (3*B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x
^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] time = 8.17542, size = 284, normalized size = 3.46

$$A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{}}{3a^5b^4x^2+6a^4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b
**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) +
8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
+ 3*a**3*b**6*x**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt
(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

Giac [B] time = 1.13369, size = 244, normalized size = 2.98

$$-\frac{\left(Bab-Ab^2\right)x}{\sqrt{bx^2+a}a^3}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+12\left(\sqrt{b}x\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*x/(sqrt(b*x^2 + a)*a^3) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3*a^2)
```

3.580 $\int \frac{A+Bx^2}{x^5(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{3b(5Ab-4aB)}{8a^3\sqrt{a+bx^2}}+\frac{5Ab-4aB}{8a^2x^2\sqrt{a+bx^2}}-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

```
[Out] (3*b*(5*A*b - 4*a*B))/(8*a^3*Sqrt[a + b*x^2]) - A/(4*a*x^4*Sqrt[a + b*x^2])
 + (5*A*b - 4*a*B)/(8*a^2*x^2*Sqrt[a + b*x^2]) - (3*b*(5*A*b - 4*a*B)*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] time = 0.0882975, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{3\sqrt{a+bx^2}(5Ab-4aB)}{8a^3x^2}-\frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}}-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^2]) - (5*A*b - 4*a*B)/(4*a^2*x^2*Sqrt[a + b*x^2])
+ (3*(5*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^3*x^2) - (3*b*(5*A*b - 4*a*B)*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} - \frac{(3(5Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{8a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3b(5Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a}\cdots}\right.}{16a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3(5Ab-4aB))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\cdots}\right.}{8a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} - \frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right.}{8a^{7/2}}$$

Mathematica [C] time = 0.0188725, size = 60, normalized size = 0.51

$$\frac{bx^4(5Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)-a^2A}{4a^3x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*A) + b*(5*A*b - 4*a*B)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^
2)/a])/(4*a^3*x^4*Sqrt[a + b*x^2])
```

Maple [A] time = 0.01, size = 153, normalized size = 1.3

$$-\frac{A}{4\,ax^4}\frac{1}{\sqrt{bx^2+a}}+\frac{5\,Ab}{8\,a^2x^2}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,Ab^2}{8\,a^3}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,Ab^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{2\,ax^2}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/4*A/a/x^4/(b*x^2+a)^(1/2)+5/8*A*b/a^2/x^2/(b*x^2+a)^(1/2)+15/8*A*b^2/a^3
/(b*x^2+a)^(1/2)-15/8*A*b^2/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1
/2*B/a/x^2/(b*x^2+a)^(1/2)-3/2*B*b/a^2/(b*x^2+a)^(1/2)+3/2*B*b/a^(5/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64314, size = 628, normalized size = 5.32

$$\left[-\frac{3\left(\left(4\,Bab^2-5\,Ab^3\right)x^6+\left(4\,Ba^2b-5\,Aab^2\right)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(4\,Ba^2b-5\,Aab^2\right)x^4+2\,Aa^3+\left(4\,Ba\right.\right.}{16\left(a^4bx^6+a^5x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^2)*x^4)*sqrt(a)
*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(4*B*a^2*b - 5*
A*a*b^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b
*x^6 + a^5*x^4), -1/8*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^
2)*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(4*B*a^2*b - 5*A*a*b
^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b*x^6
+ a^5*x^4)]
```

Sympy [A] time = 41.5856, size = 180, normalized size = 1.53

$$A\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqrt
(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b
*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a
)/(sqrt(b)*x))/(2*a**(5/2)))
```

Giac [A] time = 1.13359, size = 185, normalized size = 1.57

$$-\frac{3\left(4\,Bab-5\,Ab^{2}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^{3}}-\frac{Bab-Ab^{2}}{\sqrt{bx^{2}+a}a^{3}}-\frac{4\left(bx^{2}+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^{2}+a}Ba^{2}b-7\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}+9\,\sqrt{bx^{2}}}{8\,a^{3}b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
(B*a*b - A*b^2)/(sqrt(b*x^2 + a)*a^3) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*
sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^2 + a)*A*a
*b^2)/(a^3*b^2*x^4)
```

3.581 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a + b*x^2])
 - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (8*b^2*(6*A*b - 5*a*B
)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0447022, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a + b*x^2])
 - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (8*b^2*(6*A*b - 5*a*B
)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{5ax^5\sqrt{a+bx^2}} - \frac{(6Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^2}} + \frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}} + \frac{(4b(6Ab-5aB))\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{15a^2}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^2}} + \frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}} - \frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}} - \frac{\left(8b^2(6Ab-5aB)\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{15a^3}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^2}} + \frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}} - \frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}} - \frac{8b^2(6Ab-5aB)x}{15a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0230303, size = 60, normalized size = 0.52

$$\frac{x^2\left(a^2-4abx^2-8b^2x^4\right)(6Ab-5aB)-3a^3A}{15a^4x^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-3*a^3*A + (6*A*b - 5*a*B)*x^2*(a^2 - 4*a*b*x^2 - 8*b^2*x^4))/(15*a^4*x^5*
Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 83, normalized size = 0.7

$$-\frac{48\,Ab^3x^6-40\,Bab^2x^6+24\,Aab^2x^4-20\,Ba^2bx^4-6\,Aa^2bx^2+5\,Ba^3x^2+3\,Aa^3}{15\,x^5a^4}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(48*A*b^3*x^6-40*B*a*b^2*x^6+24*A*a*b^2*x^4-20*B*a^2*b*x^4-6*A*a^2*b*
x^2+5*B*a^3*x^2+3*A*a^3)/(b*x^2+a)^(1/2)/x^5/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57907, size = 194, normalized size = 1.69

$$\frac{\left(8\left(5\,Bab^2-6\,Ab^3\right)x^6+4\left(5\,Ba^2b-6\,Aab^2\right)x^4-3\,Aa^3-\left(5\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{15\left(a^4bx^7+a^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*a*b^2 - 6*A*b^3)*x^6 + 4*(5*B*a^2*b - 6*A*a*b^2)*x^4 - 3*A*a^3
 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b*x^7 + a^5*x^5)
```

Sympy [B] time = 14.5231, size = 593, normalized size = 5.16

$$A\left(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{30a^2}{5a^7b^9x^4+15a^6b^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x
**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a*
*7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10
) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b*
*10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sq
rt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x
**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*s
qrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**
5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*
b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
```

Giac [B] time = 1.15474, size = 397, normalized size = 3.45

$$\frac{\left(Bab^2-Ab^3\right)x}{\sqrt{bx^2+a}a^4}-\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+90\left(\sqrt{b}x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) - 2/15*(15*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*B*a*b^(3/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 90*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 24
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 110*(sqrt(b)*x - sqrt(b*x
```

```
^2 + a))^2*B*a^4*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2
) + 25*B*a^5*b^(3/2) - 33*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^5*a^3)
```

3.582 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{5b^2(7Ab-6aB)}{16a^4\sqrt{a+bx^2}}+\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b(7Ab-6aB)}{48a^3x^2\sqrt{a+bx^2}}+\frac{7Ab-6aB}{24a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

```
[Out] (-5*b^2*(7*A*b - 6*a*B))/(16*a^4*Sqrt[a + b*x^2]) - A/(6*a*x^6*Sqrt[a + b*x
^2]) + (7*A*b - 6*a*B)/(24*a^2*x^4*Sqrt[a + b*x^2]) - (5*b*(7*A*b - 6*a*B))
/(48*a^3*x^2*Sqrt[a + b*x^2]) + (5*b^2*(7*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/(16*a^(9/2))
```

Rubi [A] time = 0.116951, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b\sqrt{a+bx^2}(7Ab-6aB)}{16a^4x^2}+\frac{5\sqrt{a+bx^2}(7Ab-6aB)}{24a^3x^4}-\frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^2]) - (7*A*b - 6*a*B)/(6*a^2*x^4*Sqrt[a + b*x^2])
+ (5*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^3*x^4) - (5*b*(7*A*b - 6*a*B)*S
qrt[a + b*x^2])/(16*a^4*x^2) + (5*b^2*(7*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(16*a^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} + \frac{\left(-\frac{7Ab}{2}+3aB\right)\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} - \frac{(5(7Ab-6aB))\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{12a^2}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} + \frac{(5b(7Ab-6aB))\operatorname{Subst}\left(\int \frac{}{x^2\sqrt{}}\right.}{16a^3}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx^2}}{16a^4x^2} - \frac{(5b}{}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx^2}}{16a^4x^2} - \frac{(5b}{}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx^2}}{16a^4x^2} + \frac{5b^2}{}$$

Mathematica [C] time = 0.0207722, size = 62, normalized size = 0.41

$$\frac{b^2x^6(6aB-7Ab)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx^2}{a}+1\right)-a^3A}{6a^4x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^3*A) + b^2*(-7*A*b + 6*a*B)*x^6*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b
*x^2)/a])/(6*a^4*x^6*Sqrt[a + b*x^2])
```

Maple [A] time = 0.01, size = 197, normalized size = 1.3

$$-\frac{A}{6\,ax^6}\frac{1}{\sqrt{bx^2+a}}+\frac{7\,Ab}{24\,a^2x^4}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,Ab^2}{48\,a^3x^2}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,Ab^3}{16\,a^4}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6*A/a/x^6/(b*x^2+a)^(1/2)+7/24*A*b/a^2/x^4/(b*x^2+a)^(1/2)-35/48*A*b^2/a
^3/x^2/(b*x^2+a)^(1/2)-35/16*A*b^3/a^4/(b*x^2+a)^(1/2)+35/16*A*b^3/a^(9/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/4*B/a/x^4/(b*x^2+a)^(1/2)+5/8*B*b/a
^2/x^2/(b*x^2+a)^(1/2)+15/8*B*b^2/a^3/(b*x^2+a)^(1/2)-15/8*B*b^2/a^(7/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65938, size = 741, normalized size = 4.84

$$\left[-\frac{15\left(\left(6\,Bab^3 - 7\,Ab^4\right)x^8 + \left(6\,Ba^2b^2 - 7\,Aab^3\right)x^6\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) - 2\left(15\left(6\,Ba^2b^2 - 7\,Aab^3\right)x^6 - 8\,Aa^4 + 5\right.}{96\left(a^5bx^8 + a^6x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((6*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt
(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*(6*B*a^2*b^
2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4
 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x^6), 1/48*(15*((6*B*a
*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt(-a)*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (15*(6*B*a^2*b^2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*
a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a
^5*b*x^8 + a^6*x^6)]
```

Sympy [A] time = 72.196, size = 236, normalized size = 1.54

$$A\left(-\frac{1}{6a\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{7\sqrt{b}}{24a^2x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{35b^{\frac{3}{2}}}{48a^3x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{35b^{\frac{5}{2}}}{16a^4x\sqrt{\frac{a}{bx^2}+1}} + \frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{9}{2}}}\right) + B\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**2*x**5*sqr
t(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 35*b
```

```
**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35*b**3*asinh(sqrt(a)/(sqrt(b)*x
))/(16*a**(9/2))) + B*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(
b)/(8*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**
2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2)))
```

Giac [A] time = 1.1665, size = 243, normalized size = 1.59

$$\frac{5\left(6\,Bab^2-7\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}a^4}+\frac{Bab^2-Ab^3}{\sqrt{bx^2+a}a^4}+\frac{42\left(bx^2+a\right)^{\frac{5}{2}}Bab^2-96\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^2+54\,\sqrt{bx^2+a}Ba^3b^2-}{48\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 5/16*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4)
+ (B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^4) + 1/48*(42*(b*x^2 + a)^(5/2)*B*a*
b^2 - 96*(b*x^2 + a)^(3/2)*B*a^2*b^2 + 54*sqrt(b*x^2 + a)*B*a^3*b^2 - 57*(b
*x^2 + a)^(5/2)*A*b^3 + 136*(b*x^2 + a)^(3/2)*A*a*b^3 - 87*sqrt(b*x^2 + a)*
A*a^2*b^3)/(a^4*b^3*x^6)
```

3.583 $\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a + b*x^2])
 - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) + (8*b^2*(8*A*b - 7*a
*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*A*b - 7*a*B)*x)/(35*a^5*Sqrt[a
 + b*x^2])
```

Rubi [A] time = 0.0612387, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a + b*x^2])
 - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) + (8*b^2*(8*A*b - 7*a
*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*A*b - 7*a*B)*x)/(35*a^5*Sqrt[a
 + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^2}} - \frac{(8Ab-7aB)\int \frac{1}{x^6\left(a+bx^2\right)^{3/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} + \frac{(6b(8Ab-7aB))\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx}{35a^2}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} - \frac{\left(8b^2(8Ab-7aB)\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{35a^3}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} + \frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}} + \frac{\left(16b^3(8Ab-7aB\right.}{35}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} + \frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}} + \frac{16b^3(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0273841, size = 71, normalized size = 0.48

$$\frac{x^2\left(-2a^2bx^2+a^3+8ab^2x^4+16b^3x^6\right)(8Ab-7aB)-5a^4A}{35a^5x^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-5*a^4*A + (8*A*b - 7*a*B)*x^2*(a^3 - 2*a^2*b*x^2 + 8*a*b^2*x^4 + 16*b^3*x
^6))/(35*a^5*x^7*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 107, normalized size = 0.7

$$-\frac{-128\,Ab^4x^8+112\,Bab^3x^8-64\,Aab^3x^6+56\,Ba^2b^2x^6+16\,Aa^2b^2x^4-14\,Ba^3bx^4-8\,Aa^3bx^2+7\,Ba^4x^2+5\,Aa^4}{35\,x^7a^5}\frac{1}{\sqrt{bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(-128*A*b^4*x^8+112*B*a*b^3*x^8-64*A*a*b^3*x^6+56*B*a^2*b^2*x^6+16*A*
a^2*b^2*x^4-14*B*a^3*b*x^4-8*A*a^3*b*x^2+7*B*a^4*x^2+5*A*a^4)/(b*x^2+a)^(1/
2)/x^7/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77955, size = 246, normalized size = 1.66

$$-\frac{\left(16\left(7Bab^3-8Ab^4\right)x^8+8\left(7Ba^2b^2-8Aab^3\right)x^6+5Aa^4-2\left(7Ba^3b-8Aa^2b^2\right)x^4+\left(7Ba^4-8Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{35\left(a^5bx^9+a^6x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*a*b^3 - 8*A*b^4)*x^8 + 8*(7*B*a^2*b^2 - 8*A*a*b^3)*x^6 + 5*A
*a^4 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^4 + (7*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

Sympy [B] time = 25.4645, size = 1030, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b*
*17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**1
4) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a
**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**2
0*x**14) - 7*a**5*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 +
 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**
5*b**20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16
*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 +
 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a*
*9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19
*x**12 + 35*a**5*b**20*x**14) + 560*a**2*b**(43/2)*x**10*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a
**6*b**19*x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12*sqrt(a/(b*x*
*2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 +
 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(47/2)*x**14*sqrt(a/(
b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**
10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)) + B*(-a**5*b**(19/2)*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(5*a**
7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10)
 - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*
b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 40*a*b**(27/2)*x**8
*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**1
1*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(5*a
**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**1
0))
```

Giac [B] time = 1.1738, size = 549, normalized size = 3.71

$$-\frac{\left(Bab^3-Ab^4\right)x}{\sqrt{bx^2+a}a^5}+\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}}-35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}}-280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}}+280\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/35*(35*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*B*a*b^(5/2) - 35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 2
80*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 280*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*A*a*b^(7/2) + 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5
/2) - 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 1680*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a^3*b^(7/2) + 1337*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 1673*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(5/2) + 616*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(7/2) +
77*B*a^7*b^(5/2) - 93*A*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^7*a^4)
```

3.584 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{(a+bx^2)^{3/2}(Ab-4aB)}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B))/(b^5*Sq
rt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^5 + ((A*b - 4*a*B)*(
a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5/2))/(5*b^5)
```

Rubi [A] time = 0.101529, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{(a+bx^2)^{3/2}(Ab-4aB)}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B))/(b^5*Sq
rt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^5 + ((A*b - 4*a*B)*(
a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5/2))/(5*b^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3(-Ab+aB)}{b^4(a+bx)^{5/2}}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^{3/2}}+\frac{3a(-Ab+2aB)}{b^4\sqrt{a+bx}}+\frac{(Ab-4aB)\sqrt{a+bx}}{b^4}+\frac{B(a+\ldots}{b\ldots}\right.\right.\\ &= \frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a(Ab-2aB)\sqrt{a+bx^2}}{b^5}+\frac{(Ab-4aB)(a+bx^2)^{3/2}}{3b^5}+\frac{B(a+b\ldots}{5b^5\ldots}\end{aligned}$$

Mathematica [A] time = 0.0659652, size = 98, normalized size = 0.77

$$\frac{24a^2b^2x^2\left(2Bx^2-5A\right)+a^3\left(192bBx^2-80Ab\right)+128a^4B-2ab^3x^4\left(15A+4Bx^2\right)+b^4x^6\left(5A+3Bx^2\right)}{15b^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (128*a^4*B + 24*a^2*b^2*x^2*(-5*A + 2*B*x^2) + b^4*x^6*(5*A + 3*B*x^2) - 2*
a*b^3*x^4*(15*A + 4*B*x^2) + a^3*(-80*A*b + 192*b*B*x^2))/(15*b^5*(a + b*x^
2)^(3/2))
```

Maple [A] time = 0.006, size = 101, normalized size = 0.8

$$-\frac{-3\,x^8Bb^4-5\,Ab^4x^6+8\,Bab^3x^6+30\,Aab^3x^4-48\,Ba^2b^2x^4+120\,Aa^2b^2x^2-192\,Ba^3bx^2+80\,Aa^3b-128\,Ba^4}{15\,b^5}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(-3*B*b^4*x^8-5*A*b^4*x^6+8*B*a*b^3*x^6+30*A*a*b^3*x^4-48*B*a^2*b^2*x
^4+120*A*a^2*b^2*x^2-192*B*a^3*b*x^2+80*A*a^3*b-128*B*a^4)/(b*x^2+a)^(3/2)/
b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62329, size = 259, normalized size = 2.02

$$\frac{\left(3\,Bb^4x^8-\left(8\,Bab^3-5\,Ab^4\right)x^6+128\,Ba^4-80\,Aa^3b+6\left(8\,Ba^2b^2-5\,Aab^3\right)x^4+24\left(8\,Ba^3b-5\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{15\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^4*x^8 - (8*B*a*b^3 - 5*A*b^4)*x^6 + 128*B*a^4 - 80*A*a^3*b + 6*
(8*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 24*(8*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*x^
2 + a)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

Sympy [A] time = 2.93182, size = 437, normalized size = 3.41

$$\begin{cases} -\frac{80Aa^3b}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{120Aa^2b^2x^2}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{30Aab^3x^4}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{5Ab^4x^6}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{128Ba}{15ab^5\sqrt{a+bx^2}+15} \\ \frac{\frac{Ax^8}{8}+\frac{Bx^{10}}{10}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-80*A*a**3*b/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a +
 b*x**2)) - 120*A*a**2*b**2*x**2/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2
*sqrt(a + b*x**2)) - 30*A*a*b**3*x**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6
*x**2*sqrt(a + b*x**2)) + 5*A*b**4*x**6/(15*a*b**5*sqrt(a + b*x**2) + 15*b*
*6*x**2*sqrt(a + b*x**2)) + 128*B*a**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**
6*x**2*sqrt(a + b*x**2)) + 192*B*a**3*b*x**2/(15*a*b**5*sqrt(a + b*x**2) +
15*b**6*x**2*sqrt(a + b*x**2)) + 48*B*a**2*b**2*x**4/(15*a*b**5*sqrt(a + b*
x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 8*B*a*b**3*x**6/(15*a*b**5*sqrt(a
+ b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 3*B*b**4*x**8/(15*a*b**5*sqrt(
a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**8/8 + B*x**
10/10)/a**(5/2), True))
```

Giac [A] time = 1.13117, size = 167, normalized size = 1.3

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}B - 20\left(bx^2+a\right)^{\frac{3}{2}}Ba + 90\sqrt{bx^2+a}Ba^2 + 5\left(bx^2+a\right)^{\frac{3}{2}}Ab - 45\sqrt{bx^2+a}Aab + \frac{5\left(12\left(bx^2+a\right)Ba^3 - Ba^4 - 9\left(bx^2+a\right)Aa^2b\right.}{\left(bx^2+a\right)^{\frac{3}{2}}}}{15b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 20*(b*x^2 + a)^(3/2)*B*a + 90*sqrt(b*x^2 + a)
*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 45*sqrt(b*x^2 + a)*A*a*b + 5*(12*(b*x^2
+ a)*B*a^3 - B*a^4 - 9*(b*x^2 + a)*A*a^2*b + A*a^3*b)/(b*x^2 + a)^(3/2))/b^
5
```

3.585 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{x^5(4Ab-7aB)}{12b^2\left(a+bx^2\right)^{3/2}}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}(4Ab-7aB)}{8b^4}-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -((4*A*b - 7*a*B)*x^5)/(12*b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*(a + b*x^2
)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x^2]) + (5*(4*A*b - 7
*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] time = 0.0620309, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$-\frac{x^5(4Ab-7aB)}{12b^2\left(a+bx^2\right)^{3/2}}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}(4Ab-7aB)}{8b^4}-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -((4*A*b - 7*a*B)*x^5)/(12*b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*(a + b*x^2
)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x^2]) + (5*(4*A*b - 7
*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{(-4Ab+7aB)\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx}{4b}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} + \frac{(5(4Ab-7aB))\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{(5(4Ab-7aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b^3}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - \frac{(5a(4Ab-7}{}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - \frac{(5a(4Ab-7}{}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - \frac{5a(4Ab-7a}{}$$

Mathematica [A] time = 0.230948, size = 139, normalized size = 0.93

$$\frac{x\left(20a^2b\left(3A-7Bx^2\right)-105a^3B+ab^2x^2\left(80A-21Bx^2\right)+6b^3x^4\left(2A+Bx^2\right)\right)}{24b^4\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{a}\sqrt{a+bx^2}(7aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(-105*a^3*B + a*b^2*x^2*(80*A - 21*B*x^2) + 20*a^2*b*(3*A - 7*B*x^2) + 6
*b^3*x^4*(2*A + B*x^2)))/(24*b^4*(a + b*x^2)^(3/2)) + (5*Sqrt[a]*(-4*A*b +
7*a*B)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2)*Sqrt[1 + (b
*x^2)/a])
```

Maple [A] time = 0.017, size = 181, normalized size = 1.2

$$\frac{x^7B}{4b}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{7Bax^5}{8b^2}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{35a^2Bx^3}{24b^3}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{35a^2Bx}{8b^4}\frac{1}{\sqrt{bx^2+a}} + \frac{35a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/4*B*x^7/b/(b*x^2+a)^(3/2)-7/8*B/b^2*a*x^5/(b*x^2+a)^(3/2)-35/24*B/b^3*a^2
*x^3/(b*x^2+a)^(3/2)-35/8*B/b^4*a^2*x/(b*x^2+a)^(1/2)+35/8*B/b^(9/2)*a^2*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*A*x^5/b/(b*x^2+a)^(3/2)+5/6*A/b^2*a*x^3/(b*
x^2+a)^(3/2)+5/2*A/b^3*a*x/(b*x^2+a)^(1/2)-5/2*A/b^(7/2)*a*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75892, size = 856, normalized size = 5.74

$$\left[-\frac{15\left(7\,Ba^4-4\,Aa^3b+\left(7\,Ba^2b^2-4\,Aab^3\right)x^4+2\left(7\,Ba^3b-4\,Aa^2b^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(6\,B\right.}{48\left(b^7x^4+2\,ab^6x^2+a^2b^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^4 + 2*(7*B*a^
3*b - 4*A*a^2*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) - 2*(6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b^2 - 4*A
*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7*x^4 + 2
*a*b^6*x^2 + a^2*b^5), -1/24*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*
a*b^3)*x^4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b
^2 - 4*A*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7
*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [B] time = 29.4481, size = 804, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(7
9/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x
**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
```

```
))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*s
qrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b
*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47
/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*
x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sq
rt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))) + B*(105
*a**(157/2)*b**41*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(24*a**(153/2
)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x*
*2/a)) + 105*a**(155/2)*b**42*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(
a))/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x
**2*sqrt(1 + b*x**2/a)) - 105*a**78*b**(83/2)*x/(24*a**(153/2)*b**(91/2)*sq
rt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 140*a
**77*b**(85/2)*x**3/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(15
1/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 21*a**76*b**(87/2)*x**5/(24*a**(1
53/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 +
b*x**2/a)) + 6*a**75*b**(89/2)*x**7/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**
2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.15069, size = 200, normalized size = 1.34

$$\frac{\left(\left(3\left(\frac{2Bx^2}{b}-\frac{7Ba^2b^5-4Aab^6}{ab^7}\right)x^2-\frac{20\left(7Ba^3b^4-4Aa^2b^5\right)}{ab^7}\right)x^2-\frac{15\left(7Ba^4b^3-4Aa^3b^4\right)}{ab^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\left(7Ba^2-4Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*B*x^2/b - (7*B*a^2*b^5 - 4*A*a*b^6)/(a*b^7))*x^2 - 20*(7*B*a^3*
b^4 - 4*A*a^2*b^5)/(a*b^7))*x^2 - 15*(7*B*a^4*b^3 - 4*A*a^3*b^4)/(a*b^7))*x
/(b*x^2 + a)^(3/2) - 5/8*(7*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(9/2)
```

3.586 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

```
[Out] -(a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B))/(b^4*Sqr
t[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (B*(a + b*x^2)^(3/2))
/(3*b^4)
```

Rubi [A] time = 0.0759132, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B))/(b^4*Sqr
t[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (B*(a + b*x^2)^(3/2))
/(3*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{(Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.0498619, size = 73, normalized size = 0.75

$$\frac{8a^2b(A-3Bx^2)-16a^3B-6ab^2x^2(Bx^2-2A)+b^3x^4(3A+Bx^2)}{3b^4(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-16*a^3*B + 8*a^2*b*(A - 3*B*x^2) - 6*a*b^2*x^2*(-2*A + B*x^2) + b^3*x^4*(
3*A + B*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 76, normalized size = 0.8

$$\frac{x^6Bb^3+3Ab^3x^4-6Bab^2x^4+12Aab^2x^2-24Ba^2bx^2+8Aa^2b-16Ba^3}{3b^4}(bx^2+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(B*b^3*x^6+3*A*b^3*x^4-6*B*a*b^2*x^4+12*A*a*b^2*x^2-24*B*a^2*b*x^2+8*A*
a^2*b-16*B*a^3)/(b*x^2+a)^(3/2)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56345, size = 201, normalized size = 2.07

$$\frac{\left(Bb^3x^6-3\left(2Bab^2-Ab^3\right)x^4-16Ba^3+8Aa^2b-12\left(2Ba^2b-Aab^2\right)x^2\right)\sqrt{bx^2+a}}{3\left(b^6x^4+2ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^3*x^6 - 3*(2*B*a*b^2 - A*b^3)*x^4 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A] time = 1.82665, size = 337, normalized size = 3.47

$$\begin{cases} \frac{8Aa^2b}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{12Aab^2x^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{3Ab^3x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{16Ba^3}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{24Ba^2bx^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+b} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*A*a**2*b/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x
**2)) + 12*A*a*b**2*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) + 3*A*b**3*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) - 16*B*a**3/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x*
*2)) - 24*B*a**2*b*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)) - 6*B*a*b**2*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
 b*x**2)) + B*b**3*x**6/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(5/2), True))
```

Giac [A] time = 1.12941, size = 124, normalized size = 1.28

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}B - 9\sqrt{bx^2+a}Ba + 3\sqrt{bx^2+a}Ab - \frac{9\left(bx^2+a\right)Ba^2 - Ba^3 - 6\left(bx^2+a\right)Aab + Aa^2b}{\left(bx^2+a\right)^{\frac{3}{2}}}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 9*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 + a)*A*b -
(9*(b*x^2 + a)*B*a^2 - B*a^3 - 6*(b*x^2 + a)*A*a*b + A*a^2*b)/(b*x^2 + a)^(
3/2))/b^4
```

3.587 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)/(3*b^3*Sq
rt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*b - 5*a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] time = 0.0900452, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 1157, 388, 217, 206}

$$-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)/(3*b^3*Sq
rt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*b - 5*a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{a(Ab-aB)-3b(Ab-aB)x^2-3b^2Bx^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b^3}\\
&= \frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{\int \frac{3a(Ab-2aB)+3abBx^2}{\sqrt{a+bx^2}}\,dx}{3ab^3}\\
&= \frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}\\
&= \frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^3}\\
&= \frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.24877, size = 116, normalized size = 1.02

$$\frac{\sqrt{b}x\left(15a^2B+a\left(20bBx^2-6Ab\right)+b^2x^2\left(3Bx^2-8A\right)\right)-3\sqrt{a}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(5aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6b^{7/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(15*a^2*B + b^2*x^2*(-8*A + 3*B*x^2) + a*(-6*A*b + 20*b*B*x^2))
- 3*Sqrt[a]*(-2*A*b + 5*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[
b]*x)/Sqrt[a]])/(6*b^(7/2)*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.007, size = 134, normalized size = 1.2

$$\frac{x^5B}{2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5Bax^3}{6b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5Bax}{2b^3}\frac{1}{\sqrt{bx^2+a}}-\frac{5Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}-\frac{Ax^3}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*B*x^5/b/(b*x^2+a)^(3/2)+5/6*B/b^2*a*x^3/(b*x^2+a)^(3/2)+5/2*B/b^3*a*x/(
b*x^2+a)^(1/2)-5/2*B/b^(7/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/3*A*x^3/b/(b
*x^2+a)^(3/2)-A/b^2*x/(b*x^2+a)^(1/2)+A/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97516, size = 732, normalized size = 6.42

$$\left[-\frac{3\left(\left(5\,Bab^2-2\,Ab^3\right)x^4+5\,Ba^3-2\,Aa^2b+2\left(5\,Ba^2b-2\,Aab^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(3\,Bb^3x^5+\right.}{12\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((5*B*a*b^2 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b -
 2*A*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)
*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(3*((5*B*a*b^2 - 2
*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^2)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3
)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^
2 + a^2*b^4)]
```

Sympy [B] time = 17.8795, size = 675, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/
2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**
2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt
(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x
**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*
sqrt(1 + b*x**2/a))) + B*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt
(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(
```

```
53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**
(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/
2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**
2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) +
 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6
*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1
 + b*x**2/a)))
```

Giac [A] time = 1.13172, size = 151, normalized size = 1.32

$$\frac{\left(\left(\frac{3\,Bx^2}{b} + \frac{4\left(5\,Ba^2b^3 - 2\,Aab^4\right)}{ab^5}\right)x^2 + \frac{3\left(5\,Ba^3b^2 - 2\,Aa^2b^3\right)}{ab^5}\right)x}{6\left(bx^2 + a\right)^{\frac{3}{2}}} + \frac{(5\,Ba - 2\,Ab)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*B*x^2/b + 4*(5*B*a^2*b^3 - 2*A*a*b^4)/(a*b^5))*x^2 + 3*(5*B*a^3*b^2
 - 2*A*a^2*b^3)/(a*b^5))*x/(b*x^2 + a)^(3/2) + 1/2*(5*B*a - 2*A*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.588 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sqrt[a + b*x
^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rubi [A] time = 0.0535126, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sqrt[a + b*x
^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx,x,x^2\right)\\ &= \frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0346629, size = 54, normalized size = 0.79

$$\frac{8a^2B-2ab\left(A-6Bx^2\right)+3b^2x^2\left(Bx^2-A\right)}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2*B - 2*a*b*(A - 6*B*x^2) + 3*b^2*x^2*(-A + B*x^2))/(3*b^3*(a + b*x^2)
^(3/2))
```

Maple [A] time = 0.006, size = 53, normalized size = 0.8

$$-\frac{-3\,b^2Bx^4+3\,Ab^2x^2-12\,Babx^2+2\,Aab-8\,a^2B}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-3*B*b^2*x^4+3*A*b^2*x^2-12*B*a*b*x^2+2*A*a*b-8*B*a^2)/(b*x^2+a)^(3/2
)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76132, size = 155, normalized size = 2.28

$$\frac{\left(3\,Bb^2x^4+8\,Ba^2-2\,Aab+3\left(4\,Bab-Ab^2\right)x^2\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b^2*x^4 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x^2)*sqrt(b*x^2
+ a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A] time = 1.15936, size = 240, normalized size = 3.53

$$\begin{cases} -\frac{2Aab}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} - \frac{3Ab^2x^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{8Ba^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12Babx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3Bb^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*a*b/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) - 3*A*b**2*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) + 8*B*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 12*B*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 3*B*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(5/2), True))
```

Giac [A] time = 1.11266, size = 82, normalized size = 1.21

$$\frac{3\sqrt{bx^2+a}B + \frac{6\left(bx^2+a\right)Ba - Ba^2 - 3\left(bx^2+a\right)Ab + Aab}{\left(bx^2+a\right)^{\frac{3}{2}}}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(b*x^2 + a)*B + (6*(b*x^2 + a)*B*a - B*a^2 - 3*(b*x^2 + a)*A*b +
 A*a*b)/(b*x^2 + a)^(3/2))/b^3
```

3.589 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x^3(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a + b*x^2]) +
 (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.0303405, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {452, 288, 217, 206}

$$\frac{x^3(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a + b*x^2]) +
 (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}} + \frac{B\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b}$$

$$= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}} - \frac{Bx}{b^2\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$

$$= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}} - \frac{Bx}{b^2\sqrt{a+bx^2}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$

$$= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}} - \frac{Bx}{b^2\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.155562, size = 96, normalized size = 1.25

$$\frac{\sqrt{b}x\left(-3a^2B-4abBx^2+Ab^2x^2\right)+3a^{3/2}B\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3ab^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - 4*a*b*B*x^2) + 3*a^(3/2)*B*(a + b*x^2)*S
qrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(3*a*b^(5/2)*(a + b*x^2)^(
3/2))
```

Maple [A] time = 0.008, size = 92, normalized size = 1.2

$$-\frac{x^3B}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^2}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-\frac{Ax}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Ax}{3ab}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B*x^3/b/(b*x^2+a)^(3/2)-B*x/b^2/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))-1/3*A/b*x/(b*x^2+a)^(3/2)+1/3*A/b/a*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63859, size = 531, normalized size = 6.9

$$\left[\frac{3\left(Bab^2x^4+2\,Ba^2bx^2+Ba^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(3\,Ba^2bx+\left(4\,Bab^2-Ab^3\right)x^3\right)\sqrt{bx^2+a}}{6\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{3\left(B\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt
(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3), -1/3*(3*(B*a*b^2*x^4 +
2*B*a^2*b*x^2 + B*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*a
^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x
^2 + a^3*b^3)]
```

Sympy [B] time = 12.4323, size = 352, normalized size = 4.57

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{b}{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a*
*(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 +
b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt
(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2
*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 +
b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25
/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.12293, size = 93, normalized size = 1.21

$$-\frac{x\left(\frac{3\,Ba}{b^2}+\frac{\left(4\,Bab^2-Ab^3\right)x^2}{ab^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(3*B*a/b^2 + (4*B*a*b^2 - A*b^3)*x^2/(a*b^3))/(b*x^2 + a)^(3/2) - B*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.590 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0329654, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$-\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]

[Out] -(A*b - a*B)/(3*b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)\\
&= -\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0226032, size = 34, normalized size = 0.77

$$\frac{-2aB-Ab-3bBx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.7

$$-\frac{3\,bBx^2+Ab+2\,Ba}{3\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(3*B*b*x^2+A*b+2*B*a)/(b*x^2+a)^(3/2)/b^2
```

Maxima [A] time = 1.41398, size = 68, normalized size = 1.55

$$-\frac{Bx^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{2\,Ba}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

Fricas [A] time = 1.5358, size = 111, normalized size = 2.52

$$-\frac{\left(3\,Bbx^2+2\,Ba+Ab\right)\sqrt{bx^2+a}}{3\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*B*b*x^2 + 2*B*a + A*b)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2
*b^2)
```

Sympy [A] time = 1.06531, size = 143, normalized size = 3.25

$$\begin{cases} -\frac{Ab}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{2Ba}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3Bbx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-A*b/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 2*B*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*B*b*
x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)),
 ((A*x**2/2 + B*x**4/4)/a**(5/2), True))
```

Giac [A] time = 1.09832, size = 43, normalized size = 0.98

$$-\frac{3\left(bx^2+a\right)B-Ba+Ab}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*x^2 + a)*B - B*a + A*b)/((b*x^2 + a)^(3/2)*b^2)
```

3.591 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=47

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.0098935, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}} \, dx &= \frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx}{3a}\\ &= \frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.013434, size = 37, normalized size = 0.79

$$\frac{x\left(3aA+aBx^2+2Abx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$\frac{x\left(2\,Abx^{2}+Bax^{2}+3\,aA\right)}{3\,a^{2}}\left(bx^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*A*b*x^2+B*a*x^2+3*A*a)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 1.24358, size = 92, normalized size = 1.96

$$\frac{2\,Ax}{3\,\sqrt{bx^{2}+a}a^{2}}+\frac{Ax}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{Bx}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^{2}+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*x/((b
*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A] time = 1.54768, size = 115, normalized size = 2.45

$$\frac{\left((Ba+2\,Ab)x^{3}+3\,Aax\right)\sqrt{bx^{2}+a}}{3\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

Sympy [B] time = 10.2009, size = 144, normalized size = 3.06

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)+\frac{Bx^{3}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{3}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

Giac [A] time = 1.14196, size = 54, normalized size = 1.15

$$\frac{x\left(\frac{3A}{a} + \frac{(Bab+2Ab^2)x^2}{a^2b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.592 $\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=72

$$\frac{A}{a^2\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) - (A*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0492278, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{A}{a^2\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) - (A*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0202502, size = 61, normalized size = 0.85

$$\frac{a(Ab-aB)+3Ab\left(a+bx^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (a*(A*b - a*B) + 3*A*b*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x
^2)/a])/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.007, size = 75, normalized size = 1.

$$-\frac{B}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{A}{3a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{A}{a^2}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B/b/(b*x^2+a)^(3/2)+1/3*A/a/(b*x^2+a)^(3/2)+A/a^2/(b*x^2+a)^(1/2)-A/a^
(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98621, size = 521, normalized size = 7.24

$$\left[\frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Aab^2x^2-Ba^3+4\,Aa^2b\right)\sqrt{bx^2+a}}{6\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\ \frac{3\left(Ab^3x^4+2\,Aab^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*A*a*b^2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt
(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b), 1/3*(3*(A*b^3*x^4 + 2*A
*a*b^2*x^2 + A*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*A*a*b^
2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 +
a^5*b)]
```

Sympy [A] time = 17.7912, size = 66, normalized size = 0.92

$$\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}-\frac{-Ab+Ba}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A/(a**2*sqrt(a + b*x**2)) + A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a**2*sqrt(-a
)) - (-A*b + B*a)/(3*a*b*(a + b*x**2)**(3/2))
```

Giac [A] time = 1.12456, size = 89, normalized size = 1.24

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{Ba^2-3\left(bx^2+a\right)Ab-Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(B*a^2 - 3*(b*x^2 +
 a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a^2*b)
```

3.593 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^2)^(3/2))
- (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0265301, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 192, 191}

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^2)^(3/2))
- (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a}$$
$$= -\frac{A}{ax\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{(2(4Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{A}{ax\left(a+bx^2\right)^{3/2}} - \frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{2(4Ab-aB)x}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0203443, size = 60, normalized size = 0.78

$$\frac{-3a^2\left(A-Bx^2\right)+2abx^2\left(Bx^2-6A\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 - 3*a^2*(A - B*x^2) + 2*a*b*x^2*(-6*A + B*x^2))/(3*a^3*x*(a +
 b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 59, normalized size = 0.8

$$-\frac{8\,Ab^2x^4-2\,Bx^4ab+12\,aAbx^2-3\,Bx^2a^2+3\,Aa^2}{3\,xa^3}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(8*A*b^2*x^4-2*B*a*b*x^4+12*A*a*b*x^2-3*B*a^2*x^2+3*A*a^2)/(b*x^2+a)^(
3/2)/x/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83957, size = 161, normalized size = 2.09

$$\frac{\left(2\left(Bab-4\,Ab^2\right)x^4-3\,Aa^2+3\left(Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 + 3*(B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

Sympy [B] time = 23.1413, size = 265, normalized size = 3.44

$$A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(3*a*x/(3*a**(7
/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x**3/(
3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.14633, size = 136, normalized size = 1.77

$$\frac{x\left(\frac{\left(2\,Ba^3b^2-5\,Aa^2b^3\right)x^2}{a^5b}+\frac{3\left(Ba^4b-2\,Aa^3b^2\right)}{a^5b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((2*B*a^3*b^2 - 5*A*a^2*b^3)*x^2/(a^5*b) + 3*(B*a^4*b - 2*A*a^3*b^2)/
(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a^2)
```

3.594 $\int \frac{A+Bx^2}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2))
- (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.0865613, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2))
- (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{4a^2}$$
$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^3}$$
$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{2a^3b}$$
$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [C] time = 0.0210112, size = 57, normalized size = 0.5

$$\frac{x^2(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^2}{a}+1\right)-3aA}{6a^2x^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^2)
/a])/(6*a^2*x^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.01, size = 140, normalized size = 1.2

$$\frac{B}{3a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{B}{a^2}\frac{1}{\sqrt{bx^2+a}}-B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{A}{2ax^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{5Ab}{6a^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{5Ab}{2a^3}\frac{1}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)-1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A*b/a^2/(b*x^2+a)^(3/2)-
5/2*A*b/a^3/(b*x^2+a)^(1/2)+5/2*A*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70717, size = 756, normalized size = 6.69

$$\Bigg[-\frac{3\Big(\big(2\,Bab^2-5\,Ab^3\big)x^6+2\big(2\,Ba^2b-5\,Aab^2\big)x^4+\big(2\,Ba^3-5\,Aa^2b\big)x^2\Big)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\Big(3\big(2\,Ba^2b-}{12\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B
*a^3 - 5*A*a^2*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2
*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), 1/6*(3*((2
*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B*a^3 - 5*A*a^
2*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - 5*A*a
*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^
2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

Sympy [B] time = 49.8156, size = 1608, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
```

```
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*
a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3
*x**6) - 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x
**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt
(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4
+ 6*a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a*
*(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*
b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*
a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*
x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17
/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2
*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

Giac [A] time = 1.15418, size = 136, normalized size = 1.2

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^3}+\frac{3\left(bx^2+a\right)Ba+Ba^2-6\left(bx^2+a\right)Ab-Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^2+a}A}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 5*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 1/3*(
3*(b*x^2 + a)*B*a + B*a^2 - 6*(b*x^2 + a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a
^3) - 1/2*sqrt(b*x^2 + a)*A/(a^3*x^2)
```

3.595 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=108

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3(a+bx^2)^{3/2}}+\frac{2Ab-aB}{a^2x(a+bx^2)^{3/2}}-\frac{A}{3ax^3(a+bx^2)^{3/2}}$$

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)^(3/2)) +
(4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*(2*A*b - a*B)*x)/(3*
a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0487297, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3(a+bx^2)^{3/2}}+\frac{2Ab-aB}{a^2x(a+bx^2)^{3/2}}-\frac{A}{3ax^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)^(3/2)) +
(4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*(2*A*b - a*B)*x)/(3*
a^4*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}-\frac{(6Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{3a}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{(4b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a^2}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{(8b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^3}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{8b(2Ab-aB)x}{3a^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0245709, size = 79, normalized size = 0.73

$$\frac{6a^2bx^2\left(A-2Bx^2\right)-a^3\left(A+3Bx^2\right)-8ab^2x^4\left(Bx^2-3A\right)+16Ab^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (16*A*b^3*x^6 + 6*a^2*b*x^2*(A - 2*B*x^2) - 8*a*b^2*x^4*(-3*A + B*x^2) - a^
3*(A + 3*B*x^2))/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 82, normalized size = 0.8

$$-\frac{-16\,Ab^3x^6+8\,Bab^2x^6-24\,Aab^2x^4+12\,Ba^2bx^4-6\,Aa^2bx^2+3\,Ba^3x^2+Aa^3}{3\,x^3a^4}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*A*b^3*x^6+8*B*a*b^2*x^6-24*A*a*b^2*x^4+12*B*a^2*b*x^4-6*A*a^2*b*x
^2+3*B*a^3*x^2+A*a^3)/(b*x^2+a)^(3/2)/x^3/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63431, size = 209, normalized size = 1.94

$$-\frac{\left(8\left(Bab^2-2Ab^3\right)x^6+12\left(Ba^2b-2Aab^2\right)x^4+Aa^3+3\left(Ba^3-2Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*(B*a*b^2 - 2*A*b^3)*x^6 + 12*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 3*
(B*a^3 - 2*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x
^3)
```

Sympy [B] time = 43.6028, size = 524, normalized size = 4.85

$$A\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x}{3a^7b^9x^2+9a^6b^{10}x^4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x*
*4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/
(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3
*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**
9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 +
 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
*12*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4
*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

Giac [B] time = 1.15275, size = 302, normalized size = 2.8

$$-\frac{x\left(\frac{\left(5Ba^4b^3-8Aa^3b^4\right)x^2}{a^7b}+\frac{3\left(2Ba^5b^2-3Aa^4b^3\right)}{a^7b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx}\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*B*a^4*b^3 - 8*A*a^3*b^4)*x^2/(a^7*b) + 3*(2*B*a^5*b^2 - 3*A*a^4*
b^3)/(a^7*b))/(b*x^2 + a)^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*
a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^2*sqrt(b) + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(
```

```
3/2) + 3*B*a^3*sqrt(b) - 8*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^3*a^3)
```

3.596 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{5b(7Ab-4aB)}{8a^4\sqrt{a+bx^2}}+\frac{5b(7Ab-4aB)}{24a^3\left(a+bx^2\right)^{3/2}}+\frac{7Ab-4aB}{8a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

```
[Out] (5*b*(7*A*b - 4*a*B))/(24*a^3*(a + b*x^2)^(3/2)) - A/(4*a*x^4*(a + b*x^2)^(
3/2)) + (7*A*b - 4*a*B)/(8*a^2*x^2*(a + b*x^2)^(3/2)) + (5*b*(7*A*b - 4*a*B
))/(8*a^4*Sqrt[a + b*x^2]) - (5*b*(7*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(8*a^(9/2))
```

Rubi [A] time = 0.110352, antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{5\sqrt{a+bx^2}(7Ab-4aB)}{8a^4x^2}-\frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}}-\frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(4*a*x^4*(a + b*x^2)^(3/2)) - (7*A*b - 4*a*B)/(12*a^2*x^2*(a + b*x^2)^(3
/2)) - (5*(7*A*b - 4*a*B))/(12*a^3*x^2*Sqrt[a + b*x^2]) + (5*(7*A*b - 4*a*B
)*Sqrt[a + b*x^2])/(8*a^4*x^2) - (5*b*(7*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(8*a^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{7Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{(5(7Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)}{24a^2}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} - \frac{(5(7Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\right.}{8a^3}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} + \frac{(5b(7A}{}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} + \frac{(5(7Ab}{}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} - \frac{5b(7Ab}{}$$

Mathematica [C] time = 0.0214161, size = 60, normalized size = 0.41

$$\frac{bx^4(7Ab-4aB)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx^2}{a}+1\right)-3a^2A}{12a^3x^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A + b*(7*A*b - 4*a*B)*x^4*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x
^2)/a])/(12*a^3*x^4*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.011, size = 187, normalized size = 1.3

$$-\frac{A}{4ax^4}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{7Ab}{8a^2x^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{35Ab^2}{24a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{35Ab^2}{8a^4}\frac{1}{\sqrt{bx^2+a}}-\frac{35Ab^2}{8}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/4*A/a/x^4/(b*x^2+a)^(3/2)+7/8*A*b/a^2/x^2/(b*x^2+a)^(3/2)+35/24*A*b^2/a^
3/(b*x^2+a)^(3/2)+35/8*A*b^2/a^4/(b*x^2+a)^(1/2)-35/8*A*b^2/a^(9/2)*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*B/a/x^2/(b*x^2+a)^(3/2)-5/6*B*b/a^2/(b*x
^2+a)^(3/2)-5/2*B*b/a^3/(b*x^2+a)^(1/2)+5/2*B*b/a^(7/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73774, size = 880, normalized size = 6.03

$$\left[-\frac{15\left(\left(4Bab^3-7Ab^4\right)x^8+2\left(4Ba^2b^2-7Aab^3\right)x^6+\left(4Ba^3b-7Aa^2b^2\right)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(15\left(4Ba\right.\right.}{48\left(a^5b^2x^8+2a^6bx^6+a^7x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (
4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^4 + 20*(4*B*a^
3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5
*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4), -1/24*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2
*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^
4 + 20*(4*B*a^3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*
x^2 + a))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)]
```

Sympy [B] time = 106.168, size = 1323, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**(89/2)*b**75/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) +
24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) + 21*a**(87/2)*b**76*x**
2/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153
/2)*x**7*sqrt(a/(b*x**2) + 1)) + 140*a**(85/2)*b**77*x**4/(24*a**(93/2)*b**
(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*
x**2) + 1)) + 105*a**(83/2)*b**78*x**6/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a
/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) - 105*a
**42*b**(155/2)*x**5*sqrt(a/(b*x**2) + 1)*asinh(sqrt(a)/(sqrt(b)*x))/(24*a*
*(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7
*sqrt(a/(b*x**2) + 1)) - 105*a**41*b**(157/2)*x**7*sqrt(a/(b*x**2) + 1)*asi
nh(sqrt(a)/(sqrt(b)*x))/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1)
+ 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1))) + B*(-6*a**17*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x**2/a)/(12*a**(3
9/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**
3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b
*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x**2*
log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**2*x**4*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 30*
a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**14*b**3*x**6*lo
g(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) +
1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x**2 +
 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 3
0*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8))
```

Giac [A] time = 1.19665, size = 223, normalized size = 1.53

$$-\frac{5\left(4\,Bab-7\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^4}-\frac{6\left(bx^2+a\right)Bab+Ba^2b-9\left(bx^2+a\right)Ab^2-Aab^2}{3\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\sqrt{bx^2+a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) -
1/3*(6*(b*x^2 + a)*B*a*b + B*a^2*b - 9*(b*x^2 + a)*A*b^2 - A*a*b^2)/((b*x^2
 + a)^(3/2)*a^4) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*a^2
*b - 11*(b*x^2 + a)^(3/2)*A*b^2 + 13*sqrt(b*x^2 + a)*A*a*b^2)/(a^4*b^2*x^4)
```

3.597 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(3
/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)) - (8*b^2*(8*A*b -
5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^2*(8*A*b - 5*a*B)*x)/(15*a^5*S
qrt[a + b*x^2])
```

Rubi [A] time = 0.0568177, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(3
/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)) - (8*b^2*(8*A*b -
5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^2*(8*A*b - 5*a*B)*x)/(15*a^5*S
qrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} - \frac{(8Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} + \frac{(2b(8Ab-5aB))\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{5a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{\left(8b^2(8Ab-5aB)\right)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{5a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}} - \frac{\left(16b^2(8Ab -\right.}{}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}} - \frac{16b^2(8Ab-}{15a^5\sqrt{a+}}$$

Mathematica [A] time = 0.0376436, size = 72, normalized size = 0.49

$$\frac{ax^2\left(-6a^2bx^2+a^3-24ab^2x^4-16b^3x^6\right)(8Ab-5aB)-3a^5A}{15a^6x^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^5*A + a*(8*A*b - 5*a*B)*x^2*(a^3 - 6*a^2*b*x^2 - 24*a*b^2*x^4 - 16*b^
3*x^6))/(15*a^6*x^5*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 107, normalized size = 0.7

$$-\frac{128\,Ab^4x^8-80\,Bab^3x^8+192\,Aab^3x^6-120\,Ba^2b^2x^6+48\,Aa^2b^2x^4-30\,Ba^3bx^4-8\,Aa^3bx^2+5\,Ba^4x^2+3\,Aa^4}{15\,x^5a^5}\left(bx^2+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(128*A*b^4*x^8-80*B*a*b^3*x^8+192*A*a*b^3*x^6-120*B*a^2*b^2*x^6+48*A*
a^2*b^2*x^4-30*B*a^3*b*x^4-8*A*a^3*b*x^2+5*B*a^4*x^2+3*A*a^4)/(b*x^2+a)^(3/
2)/x^5/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73053, size = 267, normalized size = 1.83

$$\frac{\left(16\left(5\,Bab^3-8\,Ab^4\right)x^8+24\left(5\,Ba^2b^2-8\,Aab^3\right)x^6-3\,Aa^4+6\left(5\,Ba^3b-8\,Aa^2b^2\right)x^4-\left(5\,Ba^4-8\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{15\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*(5*B*a*b^3 - 8*A*b^4)*x^8 + 24*(5*B*a^2*b^2 - 8*A*a*b^3)*x^6 - 3*A
*a^4 + 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^4 - (5*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)
```

Sympy [B] time = 108.123, size = 944, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**6*b**(33/2)*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**
17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12) +
 2*a**5*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b
**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12)
 - 35*a**4*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**
8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**
12) - 280*a**3*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60
*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20
*x**12) - 560*a**2*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4
+ 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b
**20*x**12) - 448*a*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12) - 128*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12)) + B*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2
+ 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a*
*5*b**11*x**6 + 3*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
*12*x**8) + 40*a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*
a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**
8*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11
*x**6 + 3*a**4*b**12*x**8))
```

Giac [B] time = 1.16016, size = 454, normalized size = 3.11

$$\frac{x\left(\frac{(8Ba^5b^4-11Aa^4b^5)x^2}{a^9b}+\frac{3(3Ba^6b^3-4Aa^5b^4)}{a^9b}\right)}{3(bx^2+a)^{\frac{3}{2}}}-\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}-45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((8*B*a^5*b^4 - 11*A*a^4*b^5)*x^2/(a^9*b) + 3*(3*B*a^6*b^3 - 4*A*a^5*
b^4)/(a^9*b))/(b*x^2 + a)^(3/2) - 2/15*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
B*a*b^(3/2) - 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 150*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a*b^(5/2) + 250*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 490*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 170*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*B*a^4*b^(3/2) + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) + 40*B
*a^5*b^(3/2) - 73*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a
^4)
```

3.598 $\int x^5 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad}{5d^5}$$

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*(2*b*c - a
*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(c + d*
x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c + d*x^2)^(9/2))/(9*d^5) + (b^2*
(c + d*x^2)^(11/2))/(11*d^5)
```

Rubi [A] time = 0.129136, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 88}

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*(2*b*c - a
*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(c + d*
x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c + d*x^2)^(9/2))/(9*d^5) + (b^2*
(c + d*x^2)^(11/2))/(11*d^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{c^2(bc-ad)^2\sqrt{c+dx}}{d^4}+\frac{2c(bc-ad)(-2bc+ad)(c+dx)^{3/2}}{d^4}+\frac{\left(6b^2c^2-6a\right.}{}\right.\right.\\ &= \frac{c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^5}-\frac{2c(bc-ad)(2bc-ad)\left(c+dx^2\right)^{5/2}}{5d^5}+\frac{\left(6b^2c^2-6abcd+a^2d^2\right.}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.0921675, size = 132, normalized size = 0.84

$$\frac{\left(c+dx^2\right)^{3/2}\left(33a^2d^2\left(8c^2-12cdx^2+15d^2x^4\right)+22abd\left(24c^2dx^2-16c^3-30cd^2x^4+35d^3x^6\right)+b^2\left(240c^2d^2x^4-192c^3dx^2\right.}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(33*a^2*d^2*(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4) + 22*a*b*d
*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6) + b^2*(128*c^4 - 192*
c^3*d*x^2 + 240*c^2*d^2*x^4 - 280*c*d^3*x^6 + 315*d^4*x^8)))/(3465*d^5)
```

Maple [A] time = 0.008, size = 149, normalized size = 1.

$$\frac{315\,b^2x^8d^4+770\,abd^4x^6-280\,b^2cd^3x^6+495\,a^2d^4x^4-660\,abcd^3x^4+240\,b^2c^2d^2x^4-396\,a^2cd^3x^2+528\,abc^2d^2x^2-192\,b}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3465*(d*x^2+c)^(3/2)*(315*b^2*d^4*x^8+770*a*b*d^4*x^6-280*b^2*c*d^3*x^6+4
95*a^2*d^4*x^4-660*a*b*c*d^3*x^4+240*b^2*c^2*d^2*x^4-396*a^2*c*d^3*x^2+528*
a*b*c^2*d^2*x^2-192*b^2*c^3*d*x^2+264*a^2*c^2*d^2-352*a*b*c^3*d+128*b^2*c^4
)/d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61442, size = 398, normalized size = 2.54

$$\frac{\left(315\,b^2d^5x^{10}+35\left(b^2cd^4+22\,abd^5\right)x^8+128\,b^2c^5-352\,abc^4d+264\,a^2c^3d^2-5\left(8\,b^2c^2d^3-22\,abcd^4-99\,a^2d^5\right)x^6+3\,(16\right.}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 35*(b^2*c*d^4 + 22*a*b*d^5)*x^8 + 128*b^2*c^5 -
352*a*b*c^4*d + 264*a^2*c^3*d^2 - 5*(8*b^2*c^2*d^3 - 22*a*b*c*d^4 - 99*a^2*
d^5)*x^6 + 3*(16*b^2*c^3*d^2 - 44*a*b*c^2*d^3 + 33*a^2*c*d^4)*x^4 - 4*(16*b
^2*c^4*d - 44*a*b*c^3*d^2 + 33*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^5
```

Sympy [A] time = 2.74027, size = 389, normalized size = 2.48

$$\begin{cases} \frac{8a^2c^3\sqrt{c+dx^2}}{105d^3} - \frac{4a^2c^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{a^2cx^4\sqrt{c+dx^2}}{35d} + \frac{a^2x^6\sqrt{c+dx^2}}{7} - \frac{32abc^4\sqrt{c+dx^2}}{315d^4} + \frac{16abc^3x^2\sqrt{c+dx^2}}{315d^3} - \frac{4abc^2x^4\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^6\sqrt{c+dx^2}}{63d} + \frac{2a \\ \sqrt{c}\left(\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*a**2*c**2*x**2*sqrt(
c + d*x**2)/(105*d**2) + a**2*c*x**4*sqrt(c + d*x**2)/(35*d) + a**2*x**6*sq
rt(c + d*x**2)/7 - 32*a*b*c**4*sqrt(c + d*x**2)/(315*d**4) + 16*a*b*c**3*x*
*2*sqrt(c + d*x**2)/(315*d**3) - 4*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d**2
) + 2*a*b*c*x**6*sqrt(c + d*x**2)/(63*d) + 2*a*b*x**8*sqrt(c + d*x**2)/9 +
128*b**2*c**5*sqrt(c + d*x**2)/(3465*d**5) - 64*b**2*c**4*x**2*sqrt(c + d*x
**2)/(3465*d**4) + 16*b**2*c**3*x**4*sqrt(c + d*x**2)/(1155*d**3) - 8*b**2*
c**2*x**6*sqrt(c + d*x**2)/(693*d**2) + b**2*c*x**8*sqrt(c + d*x**2)/(99*d)
 + b**2*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (sqrt(c)*(a**2*x**6/6 + a*b*x
**8/4 + b**2*x**10/10), True))
```

Giac [A] time = 1.13152, size = 248, normalized size = 1.58

$$\frac{\frac{33\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)a^2}{d^2} + \frac{22\left(35\left(dx^2+c\right)^{\frac{9}{2}}-135\left(dx^2+c\right)^{\frac{7}{2}}c+189\left(dx^2+c\right)^{\frac{5}{2}}c^2-105\left(dx^2+c\right)^{\frac{3}{2}}c^3\right)ab}{d^3} + \frac{\left(315\left(dx^2+c\right)^{\frac{11}{2}}-1540\left(d\right.\right.}{}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^
(3/2)*c^2)*a^2/d^2 + 22*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 1
89*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*a*b/d^3 + (315*(d*x^2
 + c)^(11/2) - 1540*(d*x^2 + c)^(9/2)*c + 2970*(d*x^2 + c)^(7/2)*c^2 - 2772
*(d*x^2 + c)^(5/2)*c^3 + 1155*(d*x^2 + c)^(3/2)*c^4)*b^2/d^4)/d
```

3.599 $\int x^3 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^4) +
 (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rubi [A] time = 0.0959832, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^4) +
 (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{c(bc-ad)^2\sqrt{c+dx}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{3/2}}{d^3}-\frac{b(3bc-2ad)(c+d\right.\right.\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{5/2}}{5d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^4}+\end{aligned}$$

Mathematica [A] time = 0.0764489, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(21a^2d^2\left(3dx^2-2c\right)+6abd\left(8c^2-12cdx^2+15d^2x^4\right)+b^2\left(24c^2dx^2-16c^3-30cd^2x^4+35d^3x^6\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(21*a^2*d^2*(-2*c + 3*d*x^2) + 6*a*b*d*(8*c^2 - 12*c*d*x
^2 + 15*d^2*x^4) + b^2*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6)
))/(315*d^4)
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$-\frac{-35\,b^2x^6d^3-90\,abd^3x^4+30\,b^2cd^2x^4-63\,a^2d^3x^2+72\,abcd^2x^2-24\,b^2c^2dx^2+42\,a^2cd^2-48\,abc^2d+16\,b^2c^3}{315\,d^4}\left(dx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-35*b^2*d^3*x^6-90*a*b*d^3*x^4+30*b^2*c*d^2*x^4-63*
a^2*d^3*x^2+72*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+42*a^2*c*d^2-48*a*b*c^2*d+16*
b^2*c^3)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63593, size = 304, normalized size = 2.67

$$\frac{\left(35\,b^2d^4x^8+5\left(b^2cd^3+18\,abd^4\right)x^6-16\,b^2c^4+48\,abc^3d-42\,a^2c^2d^2-3\left(2\,b^2c^2d^2-6\,abcd^3-21\,a^2d^4\right)x^4+\left(8\,b^2c^3d-\right.\right.}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 5*(b^2*c*d^3 + 18*a*b*d^4)*x^6 - 16*b^2*c^4 + 48*a*
b*c^3*d - 42*a^2*c^2*d^2 - 3*(2*b^2*c^2*d^2 - 6*a*b*c*d^3 - 21*a^2*d^4)*x^4
 + (8*b^2*c^3*d - 24*a*b*c^2*d^2 + 21*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 1.59804, size = 308, normalized size = 2.7

$$\begin{cases} -\frac{2a^2c^2\sqrt{c+dx^2}}{15d^2} + \frac{a^2cx^2\sqrt{c+dx^2}}{15d} + \frac{a^2x^4\sqrt{c+dx^2}}{5} + \frac{16abc^3\sqrt{c+dx^2}}{105d^3} - \frac{8abc^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^4\sqrt{c+dx^2}}{35d} + \frac{2abx^6\sqrt{c+dx^2}}{7} - \frac{16b^2c^4\sqrt{c+dx^2}}{315d^4} + \frac{8b^2c^3x^2}{315} \\ \sqrt{c}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c**2*sqrt(c + d*x**2)/(15*d**2) + a**2*c*x**2*sqrt(c + d
*x**2)/(15*d) + a**2*x**4*sqrt(c + d*x**2)/5 + 16*a*b*c**3*sqrt(c + d*x**2)
/(105*d**3) - 8*a*b*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + 2*a*b*c*x**4*sq
rt(c + d*x**2)/(35*d) + 2*a*b*x**6*sqrt(c + d*x**2)/7 - 16*b**2*c**4*sqrt(c
 + d*x**2)/(315*d**4) + 8*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) - 2*b*
*2*c**2*x**4*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**6*sqrt(c + d*x**2)/(63
*d) + b**2*x**8*sqrt(c + d*x**2)/9, Ne(d, 0)), (sqrt(c)*(a**2*x**4/4 + a*b*
x**6/3 + b**2*x**8/8), True))
```

Giac [A] time = 1.1356, size = 192, normalized size = 1.68

$$\frac{\frac{21\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2}{d} + \frac{6\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)ab}{d^2} + \frac{\left(35\left(dx^2+c\right)^{\frac{9}{2}}-135\left(dx^2+c\right)^{\frac{7}{2}}c+189\left(dx^2+c\right)^{\frac{5}{2}}c^2-105\left(dx^2+c\right)^{\frac{3}{2}}c^3\right)b^2}{d^3}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a^2/d + 6*(15*(d*x^
2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a*b/d^2 +
 (35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^
2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2/d^3)/d
```

3.600 $\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(5
/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rubi [A] time = 0.0595577, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(5
/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{5/2}}{5d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}
\end{aligned}$$

Mathematica [A] time = 0.0419449, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(35a^2d^2+14abd\left(3dx^2-2c\right)+b^2\left(8c^2-12cdx^2+15d^2x^4\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x^2) + b^2*(8*c^2 - 1
2*c*d*x^2 + 15*d^2*x^4)))/(105*d^3)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.9

$$\frac{15\,b^{2}d^{2}x^{4}+42\,abd^{2}x^{2}-12\,b^{2}cdx^{2}+35\,a^{2}d^{2}-28\,cabd+8\,b^{2}c^{2}}{105\,d^{3}}\left(dx^{2}+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(3/2)*(15*b^2*d^2*x^4+42*a*b*d^2*x^2-12*b^2*c*d*x^2+35*a^2*
d^2-28*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59331, size = 225, normalized size = 2.92

$$\frac{\left(15\,b^{2}d^{3}x^{6}+8\,b^{2}c^{3}-28\,abc^{2}d+35\,a^{2}cd^{2}+3\left(b^{2}cd^{2}+14\,abd^{3}\right)x^{4}-\left(4\,b^{2}c^{2}d-14\,abcd^{2}-35\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^4 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x^2)*sqrt
(d*x^2 + c)/d^3
```

Sympy [A] time = 0.790295, size = 226, normalized size = 2.94

$$\begin{cases}\frac{a^{2}c\sqrt{c+dx^{2}}}{3d}+\frac{a^{2}x^{2}\sqrt{c+dx^{2}}}{3}-\frac{4abc^{2}\sqrt{c+dx^{2}}}{15d^{2}}+\frac{2abcx^{2}\sqrt{c+dx^{2}}}{15d}+\frac{2abx^{4}\sqrt{c+dx^{2}}}{5}+\frac{8b^{2}c^{3}\sqrt{c+dx^{2}}}{105d^{3}}-\frac{4b^{2}c^{2}x^{2}\sqrt{c+dx^{2}}}{105d^{2}}+\frac{b^{2}cx^{4}\sqrt{c+dx^{2}}}{35d}+\frac{b^{2}x^{6}\sqrt{c+dx^{2}}}{7}\\ \sqrt{c}\left(\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*c*sqrt(c + d*x**2)/(3*d) + a**2*x**2*sqrt(c + d*x**2)/3 - 4
*a*b*c**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*c*x**2*sqrt(c + d*x**2)/(15*d)
 + 2*a*b*x**4*sqrt(c + d*x**2)/5 + 8*b**2*c**3*sqrt(c + d*x**2)/(105*d**3)
- 4*b**2*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**4*sqrt(c + d*x**
2)/(35*d) + b**2*x**6*sqrt(c + d*x**2)/7, Ne(d, 0)), (sqrt(c)*(a**2*x**2/2
+ a*b*x**4/2 + b**2*x**6/6), True))
```

Giac [A] time = 1.54361, size = 130, normalized size = 1.69

$$\frac{35\left(dx^2+c\right)^{\frac{3}{2}}a^2+\frac{14\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)ab}{d}+\frac{\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)b^2}{d^2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(35*(d*x^2 + c)^(3/2)*a^2 + 14*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(
3/2)*c)*a*b/d + (15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2
+ c)^(3/2)*c^2)*b^2/d^2)/d
```

3.601 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=92

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2*(c
 + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.0841289, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2*(c
 + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b(bc-2ad)\sqrt{c+dx}}{d}+\frac{a^2\sqrt{c+dx}}{x}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx, x, x^2\right)$$
$$= -\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, x^2\right)$$
$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,d$$
$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{}\right.}{d}$$
$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.098136, size = 93, normalized size = 1.01

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b\left(c+dx^2\right)^{3/2}(2ad-bc)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2
*(c + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Maple [A] time = 0.01, size = 100, normalized size = 1.1

$$\frac{b^2x^2}{5\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{2\,b^2c}{15\,d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{2\,ab}{3\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)a^2+a^2\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] 1/5*b^2*x^2*(d*x^2+c)^(3/2)/d-2/15*b^2*c/d^2*(d*x^2+c)^(3/2)+2/3*a*b*(d*x^2
+c)^(3/2)/d-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*a^2+a^2*(d*x^2+c)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65494, size = 481, normalized size = 5.23

$$\left[\frac{15\,a^2\sqrt{c}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+\left(b^2cd+10\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{30\,d^2}, \frac{15\,a^2\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^
2) + 2*(3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + (b^2*c*d + 10
*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, 1/15*(15*a^2*sqrt(-c)*d^2*arctan(sqrt(
-c)/sqrt(d*x^2 + c)) + (3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2
 + (b^2*c*d + 10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A] time = 25.1065, size = 90, normalized size = 0.98

$$\frac{a^2c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+a^2\sqrt{c+dx^2}+\frac{b^2\left(c+dx^2\right)^{\frac{5}{2}}}{5d^2}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(4abd-2b^2c\right)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] a**2*c*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*sqrt(c + d*x**2) + b
**2*(c + d*x**2)**(5/2)/(5*d**2) + (c + d*x**2)**(3/2)*(4*a*b*d - 2*b**2*c)
/(6*d**2)
```

Giac [A] time = 1.12601, size = 136, normalized size = 1.48

$$\frac{a^2c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2d^8-5\left(dx^2+c\right)^{\frac{3}{2}}b^2cd^8+10\left(dx^2+c\right)^{\frac{3}{2}}abd^9+15\,\sqrt{dx^2+c}a^2d^{10}}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2 + c)^(5/2)
*b^2*d^8 - 5*(d*x^2 + c)^(3/2)*b^2*c*d^8 + 10*(d*x^2 + c)^(3/2)*a*b*d^9 + 1
5*sqrt(d*x^2 + c)*a^2*d^10)/d^10
```

3.602 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2(c+dx^2)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2(c+dx^2)^{3/2}}{3d}$$

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))/(3*d) - (
a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/(2*Sqrt[c])
```

Rubi [A] time = 0.0873646, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2(c+dx^2)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2(c+dx^2)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))/(3*d) - (
a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/(2*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+ad)+b^2cx\right)\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{4c}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{1}{4}(a(4bc+ad))\operatorname{Subst}\left(\int \frac{\ }{x\sqrt{\ }}\right.$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{2d}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} - \frac{a(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.0773352, size = 87, normalized size = 0.8

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^2}\left(-3a^2d+12abdx^2+2b^2x^2\left(c+dx^2\right)\right)}{dx^2} - \frac{3a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-3*a^2*d + 12*a*b*d*x^2 + 2*b^2*x^2*(c + d*x^2)))/(d*x^2
) - (3*a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/6
```

Maple [A] time = 0.01, size = 132, normalized size = 1.2

$$\frac{b^2}{3d}\left(dx^2+c\right)^{\frac{3}{2}}-2\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)ab+2\sqrt{dx^2+c}\,ab-\frac{a^2}{2cx^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2d}{2}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d-2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*a
*b+2*(d*x^2+c)^(1/2)*a*b-1/2*a^2*(d*x^2+c)^(3/2)/c/x^2-1/2*a^2*d/c^(1/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2*a^2*d/c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69558, size = 474, normalized size = 4.35

$$\left[\frac{3\left(4abcd+a^2d^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(2b^2cdx^4-3a^2cd+2\left(b^2c^2+6abcd\right)x^2\right)\sqrt{dx^2+c}}{12cdx^2},\frac{3\left(4abcd+a^2d^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a*b*c*d + a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(c) + 2*c)/x^2) + 2*(2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^2 + 6*a*b*c*d
)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2), 1/6*(3*(4*a*b*c*d + a^2*d^2)*sqrt(-c)*x^
2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^
2 + 6*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

Sympy [A] time = 30.8147, size = 148, normalized size = 1.36

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\sqrt{c}}-2ab\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{2abc}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{2ab\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}+b^2\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for}\ d=0\\ \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*asinh(sqrt(c)/(sqrt(d)*x)
)/(2*sqrt(c)) - 2*a*b*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c/(sqrt(d)
*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + b**2*Piec
ewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True))
```

Giac [A] time = 1.10147, size = 120, normalized size = 1.1

$$\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2+12\sqrt{dx^2+c}abd-\frac{3\sqrt{dx^2+c}a^2d}{x^2}+\frac{3\left(4abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x^2 + c)^(3/2)*b^2 + 12*sqrt(d*x^2 + c)*a*b*d - 3*sqrt(d*x^2 + c)
*a^2*d/x^2 + 3*(4*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(
-c))/d
```

3.603 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{\sqrt{c+dx^2}\left(ad(8bc-ad)+8b^2c^2\right)}{8c^2}-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8bc-ad}{8c^2x^2}$$

```
[Out] ((8*b^2*c^2 + a*d*(8*b*c - a*d))*Sqrt[c + d*x^2])/(8*c^2) - (a^2*(c + d*x^2
)^(3/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^2) - ((8*
b^2*c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rubi [A] time = 0.159906, antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{1}{8}\sqrt{c+dx^2}\left(\frac{ad(8bc-ad)}{c^2}+8b^2\right)-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8bc-ad}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] ((8*b^2 + (a*d*(8*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/8 - (a^2*(c + d*x^2)^(3
/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^2) - ((8*b^2*
c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc-ad)+2b^2cx\right)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\text{Subst}\left(\int \frac{\sqrt{c+}}{x}\right.$$

$$= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2} + \frac{1}{16}\left(c\left(8\right.\right.$$

$$= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2} + \frac{\left(c\left(8b^2\right.\right.}{}$$

$$= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2} - \frac{1}{8}\sqrt{c}\left(8b\right.$$

Mathematica [A] time = 0.074725, size = 104, normalized size = 0.73

$$\frac{\sqrt{c+dx^2}\left(-a^2\left(2c+dx^2\right)-8abcx^2+8b^2cx^4\right)}{8cx^4} - \frac{\left(-a^2d^2+8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a*b*c*x^2 + 8*b^2*c*x^4 - a^2*(2*c + d*x^2)))/(8*c*x^4
) - ((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8
*c^(3/2))
```

Maple [A] time = 0.01, size = 207, normalized size = 1.5

$$-\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)b^2+\sqrt{dx^2+c}b^2-\frac{a^2}{4cx^4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d}{8c^2x^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^2}{8}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*b^2+(d*x^2+c)^(1/2)*b^2-1/4*
a^2*(d*x^2+c)^(3/2)/c/x^4+1/8*a^2*d/c^2/x^2*(d*x^2+c)^(3/2)+1/8*a^2*d^2/c^(
3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/8*a^2*d^2/c^2*(d*x^2+c)^(1/2)-
a*b/c/x^2*(d*x^2+c)^(3/2)-a*b*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/
x)+a*b*d/c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68348, size = 497, normalized size = 3.48

$$\left[-\frac{\left(8b^2c^2+8abcd-a^2d^2\right)\sqrt{c}x^4\log\left(-\frac{dx^2+2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-2\left(8b^2c^2x^4-2a^2c^2-\left(8abc^2+a^2cd\right)x^2\right)\sqrt{dx^2+c}}{16c^2x^4},\ \frac{\left(8b^2c^2+8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 + 2*sqrt(
d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*b^2*c^2*x^4 - 2*a^2*c^2 - (8*a*b*c^2
+ a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4), 1/8*((8*b^2*c^2 + 8*a*b*c*d - a
^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c^2*x^4 - 2*
a^2*c^2 - (8*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4)]
```

Sympy [A] time = 73.1166, size = 219, normalized size = 1.53

$$-\frac{a^2c}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{3}{2}}}{8cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{3}{2}}}-\frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x}-\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}-b^2\sqrt{c}\operatorname{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -a**2*c/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*sqrt(d)/(8*x**3*sqrt
(c/(d*x**2) + 1)) - a**2*d**(3/2)/(8*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**2*
asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(3/2)) - a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/
x - a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) - b**2*sqrt(c)*asinh(sqrt(c)/(
sqrt(d)*x)) + b**2*c/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*sqrt(d)*x/sqrt
(c/(d*x**2) + 1)
```

Giac [A] time = 1.11195, size = 207, normalized size = 1.45

$$\frac{8\sqrt{dx^2+c}b^2d+\frac{\left(8b^2c^2d+8abcd^2-a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\sqrt{dx^2+c}abc^2d^2+\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3+\sqrt{dx^2+c}a^2cd^3}{cd^2x^4}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(d*x^2 + c)*b^2*d + (8*b^2*c^2*d + 8*a*b*c*d^2 - a^2*d^3)*arctan
(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 -
8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 + sqrt(d*x^2 + c)
*a^2*c*d^3)/(c*d^2*x^4))/d
```

3.604 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^7} dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4bc-$$

```
[Out] -((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(16*c^2*x^2) - (a^2*(c
 + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^4
) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
(16*c^(5/2))
```

Rubi [A] time = 0.148849, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 47, 63, 208}

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4bc-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] -((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(16*c^2*x^2) - (a^2*(c
 + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^4
) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
(16*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} + \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{3}{2}a(4bc-ad)+3b^2cx\right)\sqrt{c+dx}}{x^3}\,dx,x,x^2\right)}{6c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} + \frac{\left(8b^2c^2-4abcd+a^2d^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx\right.}{16c^2}$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} + \frac{\left(d\left(8b\right.\right.}{}$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} + \frac{\left(8b^2c^2\right.}{}$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} - \frac{d\left(8b^2\right.}{}$$

Mathematica [A] time = 0.117862, size = 142, normalized size = 0.95

$$\frac{-\left(c+dx^2\right)\left(a^2\left(8c^2+2cdx^2-3d^2x^4\right)+12abcx^2\left(2c+dx^2\right)+24b^2c^2x^4\right)-3dx^6\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2-4abcd+8b^2c^2\right)\tanh}{48c^2x^6\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] (-((c + d*x^2)*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c + d*x^2) + a^2*(8*c^2 +
2*c*d*x^2 - 3*d^2*x^4))) - 3*d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^6*Sqrt[1
 + (d*x^2)/c]*ArcTanh[Sqrt[1 + (d*x^2)/c]])/(48*c^2*x^6*Sqrt[c + d*x^2])
```

Maple [B] time = 0.013, size = 281, normalized size = 1.9

$$-\frac{a^2}{6\,cx^6}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d}{8\,c^2x^4}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2d^2}{16\,c^3x^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}+\frac{a^2d^3}{16\,c^3}\sqrt{dx^2+c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(3/2)/c/x^6+1/8*a^2*d/c^2/x^4*(d*x^2+c)^(3/2)-1/16*a^2*d
^2/c^3/x^2*(d*x^2+c)^(3/2)-1/16*a^2*d^3/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)
^(1/2))/x)+1/16*a^2*d^3/c^3*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c)^(3/2)+1
/4*a*b*d/c^2/x^2*(d*x^2+c)^(3/2)+1/4*a*b*d^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x
^2+c)^(1/2))/x)-1/4*a*b*d^2/c^2*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(3/
2)-1/2*b^2*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2*b^2*d/c*(d*x
^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74616, size = 608, normalized size = 4.08

$$\left[\frac{3\left(8\,b^2c^2d-4\,abcd^2+a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(8\,a^2c^3+3\left(8\,b^2c^3+4\,abc^2d-a^2cd^2\right)x^4+2\left(12\,abc^3+a^2c\right.\right.}{96\,c^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c
^2*d - a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c
^3*x^6), 1/48*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - (8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c^2*d - a^2*c
*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^6)]
```

Sympy [B] time = 88.9726, size = 291, normalized size = 1.95

$$-\frac{a^2c}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}}{48cx^3\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{5}{2}}}{16c^2x\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{5}{2}}}-\frac{abc}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

```
[Out] -a**2*c/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 5*a**2*sqrt(d)/(24*x**5*sqr
t(c/(d*x**2) + 1)) + a**2*d**(3/2)/(48*c*x**3*sqrt(c/(d*x**2) + 1)) + a**2*
d**(5/2)/(16*c**2*x*sqrt(c/(d*x**2) + 1)) - a**2*d**3*asinh(sqrt(c)/(sqrt(d
)*x))/(16*c**(5/2)) - a*b*c/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a*b*s
qrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)/(4*c*x*sqrt(c/(d*x**2)
+ 1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(3/2)) - b**2*sqrt(d)*sqr
t(c/(d*x**2) + 1)/(2*x) - b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(2*sqrt(c))
```

Giac [A] time = 1.15071, size = 300, normalized size = 2.01

$$\frac{\frac{3\left(8b^2c^2d^2-4abcd^3+a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2+12\left(dx^2+c\right)^{\frac{5}{2}}abcd^3-12\sqrt{dx^2+c}abc^3d^3-3\left(dx^2+c\right)}{c^2d^3x^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(8*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqrt
(-c))/(sqrt(-c)*c^2) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)^(
3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 12*(d*x^2 + c)^(5/2)*a*
b*c*d^3 - 12*sqrt(d*x^2 + c)*a*b*c^3*d^3 - 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*
(d*x^2 + c)^(3/2)*a^2*c*d^4 + 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^2*d^3*x^6))
/d
```

3.605 $\int x^2 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=191

$$-\frac{c^2\left(16a^2d^2+bc(5bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{x^3\sqrt{c+dx^2}\left(16a^2d^2+bc(5bc-16ad)\right)}{64d^2}+\frac{cx\sqrt{c+dx^2}\left(16a^2d^2+bc(5bc\right.}{128d^3}$$

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^3) + ((16*
a^2*d^2 + b*c*(5*b*c - 16*a*d))*x^3*Sqrt[c + d*x^2])/(64*d^2) - (b*(5*b*c -
 16*a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b^2*x^5*(c + d*x^2)^(3/2))/(8*d
) - (c^2*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d
*x^2]])/(128*d^(7/2))
```

Rubi [A] time = 0.186079, antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 321, 217, 206}

$$-\frac{c^2\left(16a^2d^2+bc(5bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{1}{64}x^3\sqrt{c+dx^2}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)+\frac{cx\sqrt{c+dx^2}\left(16a^2d^2+bc(5b\right.}{128d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^3) + ((16*
a^2 + (b*c*(5*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/64 - (b*(5*b*c - 16*
a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b^2*x^5*(c + d*x^2)^(3/2))/(8*d) -
(c^2*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(128*d^(7/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx=\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}+\frac{\int x^2\sqrt{c+dx^2}\left(8a^2d-b(5bc-16ad)x^2\right)dx}{8d}$$

$$=-\frac{b(5bc-16ad)x^3\left(c+dx^2\right)^{3/2}}{48d^2}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}+\frac{1}{16}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)\int x^2$$

$$=\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-16ad)x^3\left(c+dx^2\right)^{3/2}}{48d^2}+\frac{b^2x^5\left(c+dx\right.}{8d}$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-}{}$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-}{}$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-}{}$$

Mathematica [A] time = 0.101858, size = 157, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(c+2dx^2\right)+16abd\left(-3c^2+2cdx^2+8d^2x^4\right)+b^2\left(-10c^2dx^2+15c^3+8cd^2x^4+48d^3x^6\right)\right)-3c^2\left(\right.}{384d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(c + 2*d*x^2) + 16*a*b*d*(-3*c^2 + 2
*c*d*x^2 + 8*d^2*x^4) + b^2*(15*c^3 - 10*c^2*d*x^2 + 8*c*d^2*x^4 + 48*d^3*x
^6)) - 3*c^2*(5*b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]])/(384*d^(7/2))
```

Maple [A] time = 0.011, size = 259, normalized size = 1.4

$$\frac{b^2x^5}{8d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5b^2cx^3}{48d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{5b^2c^2x}{64d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5b^2c^3x}{128d^3}\sqrt{dx^2+c}-\frac{5b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*b^2*x^5*(d*x^2+c)^(3/2)/d-5/48*b^2*c/d^2*x^3*(d*x^2+c)^(3/2)+5/64*b^2*c
^2/d^3*x*(d*x^2+c)^(3/2)-5/128*b^2*c^3/d^3*x*(d*x^2+c)^(1/2)-5/128*b^2*c^4/
d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x^3*(d*x^2+c)^(3/2)/d-1/4*a*b
*c/d^2*x*(d*x^2+c)^(3/2)+1/8*a*b*c^2/d^2*x*(d*x^2+c)^(1/2)+1/8*a*b*c^3/d^(5
/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a^2*x*(d*x^2+c)^(3/2)/d-1/8*a^2*c/d*x
*(d*x^2+c)^(1/2)-1/8*a^2*c^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93364, size = 767, normalized size = 4.02

$$\left[\frac{3\left(5b^2c^4-16abc^3d+16a^2c^2d^2\right)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(48b^2d^4x^7+8\left(b^2cd^3+16abd^4\right)x^5-2\left(5b^2c\right.\right.}{768d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*
a*b*d^4)*x^5 - 2*(5*b^2*c^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2
*c^3*d - 16*a*b*c^2*d^2 + 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/384*(3*(
5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*a*b*d^4)*x^5 - 2*(5*b^2*c
^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 16*a*b*c^2*d^2 +
 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] time = 16.7008, size = 411, normalized size = 2.15

$$\frac{a^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}x^3}{8\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{a^2dx^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{3}{2}}x^3}{24d\sqrt{1+\frac{dx^2}{c}}}+\frac{5ab\sqrt{c}x^5}{12\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^3\operatorname{asi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*x**3/(8*sqrt(1 +
d*x**2/c)) - a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a**2*d*x**5/
(4*sqrt(c)*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)*x/(8*d**2*sqrt(1 + d*x**2/c))
 - a*b*c**(3/2)*x**3/(24*d*sqrt(1 + d*x**2/c)) + 5*a*b*sqrt(c)*x**5/(12*sqr
t(1 + d*x**2/c)) + a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a*b*d*x
**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d**3*sqrt(1 + d
*x**2/c)) + 5*b**2*c**(5/2)*x**3/(384*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(3
/2)*x**5/(192*d*sqrt(1 + d*x**2/c)) + 7*b**2*sqrt(c)*x**7/(48*sqrt(1 + d*x*
*2/c)) - 5*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(7/2)) + b**2*d*x**9/
(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.12663, size = 235, normalized size = 1.23

$$\frac{1}{384}\left(2\left(4\left(6\,b^2x^2+\frac{b^2cd^5+16\,abd^6}{d^6}\right)x^2-\frac{5\,b^2c^2d^4-16\,abcd^5-48\,a^2d^6}{d^6}\right)x^2+\frac{3\left(5\,b^2c^3d^3-16\,abc^2d^4+16\,a^2cd^5\right)}{d^6}\right)\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2 + (b^2*c*d^5 + 16*a*b*d^6)/d^6)*x^2 - (5*b^2*c^2*d^4
 - 16*a*b*c*d^5 - 48*a^2*d^6)/d^6)*x^2 + 3*(5*b^2*c^3*d^3 - 16*a*b*c^2*d^4
+ 16*a^2*c*d^5)/d^6)*sqrt(d*x^2 + c)*x + 1/128*(5*b^2*c^4 - 16*a*b*c^3*d +
16*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.606 $\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+b\right.}{}$$

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) - (b*(3*b*c
- 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(3/2)
)/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(16*d^(5/2))
```

Rubi [A] time = 0.0951957, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) - (b*(3*b*c
- 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(3/2)
)/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(16*d^(5/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{bx(a+bx^2)(c+dx^2)^{3/2}}{6d}+\frac{\int \sqrt{c+dx^2}\left(-a(bc-6ad)-b(3bc-8ad)x^2\right)dx}{6d}$$
$$= -\frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c+dx^2)^{3/2}}{6d}+\frac{(b^2c^2-4abcd+8a^2d^2)\int\sqrt{c+}}{8d^2}$$
$$= \frac{(b^2c^2-4abcd+8a^2d^2)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c+dx^2)}{6d}$$
$$= \frac{(b^2c^2-4abcd+8a^2d^2)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c+dx^2)}{6d}$$
$$= \frac{(b^2c^2-4abcd+8a^2d^2)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c+dx^2)}{6d}$$

Mathematica [A] time = 0.0664771, size = 122, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2+12abd(c+2dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4)\right)+3c(8a^2d^2-4abcd+b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}\right.}{48d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(c + 2*d*x^2) + b^2*(-3*c
^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*Log[d*
x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(5/2))
```

Maple [A] time = 0.007, size = 190, normalized size = 1.3

$$\frac{b^2x^3}{6\,d}(dx^2+c)^{\frac{3}{2}}-\frac{b^2cx}{8\,d^2}(dx^2+c)^{\frac{3}{2}}+\frac{b^2c^2x}{16\,d^2}\sqrt{dx^2+c}+\frac{b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{abx}{2\,d}(dx^2+c)^{\frac{3}{2}}-\frac{abcx}{4\,d}\sqrt{dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b^2*x^3*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d^2*x*(d*x^2+c)^(3/2)+1/16*b^2*c^2/
d^2*x*(d*x^2+c)^(1/2)+1/16*b^2*c^3/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/
2*a*b*x*(d*x^2+c)^(3/2)/d-1/4*a*b*c/d*x*(d*x^2+c)^(1/2)-1/4*a*b*c^2/d^(3/2)
*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a^2*x*(d*x^2+c)^(1/2)+1/2*a^2*c/d^(1/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71562, size = 585, normalized size = 3.93

$$\left[\frac{3\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(8\,b^2d^3x^5+2\left(b^2cd^2+12\,abd^3\right)x^3-3\left(b^2c^2d-4\,a\right.\right.}{96\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)
*x^3 - 3*(b^2*c^2*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/
48*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)*x^3 - 3*(b^2*c^2
*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [B] time = 10.1476, size = 291, normalized size = 1.95

$$\frac{a^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}}+\frac{abc^{\frac{3}{2}}x}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}x^3}{4\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{4d^{\frac{3}{2}}}+\frac{abdx^5}{2\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x}{16d^2\sqrt{1+\frac{dx^2}{c}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/2 + a**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sq
rt(d)) + a*b*c**(3/2)*x/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*sqrt(c)*x**3/(4*sq
rt(1 + d*x**2/c)) - a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(3/2)) + a*b*d*
x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x/(16*d**2*sqrt(1 + d*x
**2/c)) - b**2*c**(3/2)*x**3/(48*d*sqrt(1 + d*x**2/c)) + 5*b**2*sqrt(c)*x**
5/(24*sqrt(1 + d*x**2/c)) + b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(5/2)
) + b**2*d*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.11174, size = 173, normalized size = 1.16

$$\frac{1}{48}\left(2\left(4\,b^2x^2+\frac{b^2cd^3+12\,abd^4}{d^4}\right)x^2-\frac{3\left(b^2c^2d^2-4\,abcd^3-8\,a^2d^4\right)}{d^4}\right)\sqrt{dx^2+c}x-\frac{\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\log\left(\left|-\sqrt{d}x\right.\right.}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2 + (b^2*c*d^3 + 12*a*b*d^4)/d^4)*x^2 - 3*(b^2*c^2*d^2 - 4
*a*b*c*d^3 - 8*a^2*d^4)/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b^2*c^3 - 4*a*b*c^2*
d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.607 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}-\frac{\left(b^2c^2-8ad(ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{x\sqrt{c+dx^2}\left(b^2c^2-8ad(ad+bc)\right)}{8cd}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}$$

```
[Out] -((b^2*c^2 - 8*a*d*(b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c*d) - (a^2*(c + d*x^
2)^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b^2*c^2 - 8*a*d*(b*c
+ a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2))
```

Rubi [A] time = 0.086252, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}-\frac{\left(b^2c^2-8ad(ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{1}{8}x\sqrt{c+dx^2}\left(\frac{b^2c}{d}-\frac{8a(ad+bc)}{c}\right)+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((b^2*c)/d - (8*a*(b*c + a*d))/c)*x*Sqrt[c + d*x^2])/8 - (a^2*(c + d*x^2)
^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b^2*c^2 - 8*a*d*(b*c +
a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^2}\,dx = -\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{\int \left(2a(bc+ad)+b^2cx^2\right)\sqrt{c+dx^2}\,dx}{c}$$
$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{4}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)\int \sqrt{c+dx^2}\,dx$$
$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{8}\left(\frac{b^2c^2}{d}-8a(\right.$$
$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{8}\left(\frac{b^2c^2}{d}-8a(\right.$$
$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{\left(\frac{b^2c^2}{d}-8a(bc\right.}{}$$

Mathematica [A] time = 0.0908908, size = 99, normalized size = 0.74

$$\frac{\left(8a^2d^2+8abcd-b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{3/2}}+\sqrt{c+dx^2}\left(-\frac{a^2}{x}+abx+\frac{b^2x\left(c+2dx^2\right)}{8d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-(a^2/x) + a*b*x + (b^2*x*(c + 2*d*x^2))/(8*d)) + ((-(b^2*
c^2) + 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(3/2
))
```

Maple [A] time = 0.011, size = 163, normalized size = 1.2

$$\frac{b^2x}{4d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2cx}{8d}\sqrt{dx^2+c}-\frac{b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+abx\sqrt{dx^2+c}+abc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] 1/4*b^2*x*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d*x*(d*x^2+c)^(1/2)-1/8*b^2*c^2/d^(3/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+a*b*x*(d*x^2+c)^(1/2)+a*b*c/d^(1/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(3/2)/c/x+a^2*d/c*x*(d*x^2+c)^(1/2)+a
^2*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64763, size = 483, normalized size = 3.63

$$\left[-\frac{\left(b^2c^2 - 8\,abcd - 8\,a^2d^2\right)\sqrt{d}x\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) - 2\left(2\,b^2d^2x^4 - 8\,a^2d^2 + \left(b^2cd + 8\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{16\,d^2x}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^2*d^2*x^4 - 8*a^2*d^2 + (b^2*c*d + 8*a*b*
d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x), 1/8*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^4 - 8*a^2*d^2
+ (b^2*c*d + 8*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

Sympy [A] time = 6.55167, size = 219, normalized size = 1.65

$$-\frac{a^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}} + a^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{a^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + ab\sqrt{c}x\sqrt{1+\frac{dx^2}{c}} + \frac{abc\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}} + \frac{b^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}} + \frac{3b^2\sqrt{c}x^3}{8\sqrt{1+\frac{dx^2}{c}}} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -a**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*x*sqrt(1 + d*x**2/c
) + a*b*c*asinh(sqrt(d)*x/sqrt(c))/sqrt(d) + b**2*c**(3/2)*x/(8*d*sqrt(1 +
d*x**2/c)) + 3*b**2*sqrt(c)*x**3/(8*sqrt(1 + d*x**2/c)) - b**2*c**2*asinh(s
qrt(d)*x/sqrt(c))/(8*d**(3/2)) + b**2*d*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.12594, size = 170, normalized size = 1.28

$$\frac{2\,a^2c\sqrt{d}}{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c} + \frac{1}{8}\left(2\,b^2x^2 + \frac{b^2cd + 8\,abd^2}{d^2}\right)\sqrt{dx^2+c}x + \frac{\left(b^2c^2\sqrt{d} - 8\,abcd^{\frac{3}{2}} - 8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/8*(2*b^2*x^2 + (b
^2*c*d + 8*a*b*d^2)/d^2)*sqrt(d*x^2 + c)*x + 1/16*(b^2*c^2*sqrt(d) - 8*a*b*
c*d^(3/2) - 8*a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.608 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx$

Optimal. Leaf size=111

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2))/(3*c*x^
3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Rubi [A] time = 0.0654748, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```


```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2))/(3*c*x^
3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}dx = -\frac{a^2(c+dx^2)^{3/2}}{3cx^3}+\frac{\int \frac{(6abc+3b^2cx^2)\sqrt{c+dx^2}}{x^2}dx}{3c}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{(b(bc+4ad))\int \sqrt{c+dx^2}\,dx}{c}$$
$$= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\int \frac{1}{\sqrt{c+dx^2}}d$$
$$= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-}\right.$$
$$= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{b(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Mathematica [A] time = 0.0960524, size = 91, normalized size = 0.82

$$\sqrt{c+dx^2}\left(-\frac{a^2}{3x^3}-\frac{a(ad+6bc)}{3cx}+\frac{b^2x}{2}\right)+\frac{b(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (-a^2/(3*x^3) - (a*(6*b*c + a*d))/(3*c*x) + (b^2*x)/2)*Sqrt[c + d*x^2] + (b
*(b*c + 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Maple [A] time = 0.009, size = 122, normalized size = 1.1

$$\frac{xb^2}{2}\sqrt{dx^2+c}+\frac{b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{3cx^3}\left(dx^2+c\right)^{\frac{3}{2}}-2\frac{ab\left(dx^2+c\right)^{3/2}}{cx}+2\frac{abdx\sqrt{dx^2+c}}{c}+2ab\sqrt{d}\ln\left(x\sqrt{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x)
```

```
[Out] 1/2*x*b^2*(d*x^2+c)^(1/2)+1/2*b^2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1
/3*a^2*(d*x^2+c)^(3/2)/c/x^3-2*a*b*(d*x^2+c)^(3/2)/c/x+2*a*b*d/c*x*(d*x^2+c
)^(1/2)+2*a*b*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60159, size = 473, normalized size = 4.26

$$\left[\frac{3\left(b^2c^2+4\,abcd\right)\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(3\,b^2cdx^4-2\,a^2cd-2\left(6\,abcd+a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{12\,cdx^3},-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) + 2*(3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c*d + a^2*d^2)*x^
2)*sqrt(d*x^2 + c))/(c*d*x^3), -1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^3*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c
*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c*d*x^3)]
```

Sympy [A] time = 4.09598, size = 170, normalized size = 1.53

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{2ab\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+2ab\sqrt{d}\,\text{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{2abdx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{b^2c\,\text{asinh}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(3*c) - 2*a*b*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + 2*a*b*sqrt(d)*asinh(sq
rt(d)*x/sqrt(c)) - 2*a*b*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + b**2*sqrt(c)*x*
sqrt(1 + d*x**2/c)/2 + b**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sqrt(d))
```

Giac [B] time = 1.1244, size = 254, normalized size = 2.29

$$\frac{1}{2}\sqrt{dx^2+c}b^2x-\frac{\left(b^2c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}+\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)\right.}{3\left(\left(\sqrt{d}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x - 1/4*(b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*
sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(3/2) - 12*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 6*a*b*c^3*sqrt(d) + a^2*c^2*d^(3/2))/((
sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

3.609 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^6} \, dx$

Optimal. Leaf size=103

$$-\frac{a^2 \left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a \left(c+dx^2\right)^{3/2} (5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) - (2*a*(5*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]]
```

Rubi [A] time = 0.0580408, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 451, 277, 217, 206}

$$-\frac{a^2 \left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a \left(c+dx^2\right)^{3/2} (5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) - (2*a*(5*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^6}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}+\frac{\int \frac{\left(2a(5bc-ad)+5b^2cx^2\right)\sqrt{c+dx^2}}{x^4}\,dx}{5c}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\int \frac{\sqrt{c+dx^2}}{x^2}\,dx\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\right.\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.083774, size = 104, normalized size = 1.01

$$b^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2+cdx^2-2d^2x^4\right)+10abcx^2\left(c+dx^2\right)+15b^2c^2x^4\right)}{15c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c + d*x^2) + a^2*(3*c^2 +
 c*d*x^2 - 2*d^2*x^4)))/(15*c^2*x^5) + b^2*Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]]
```

Maple [A] time = 0.012, size = 123, normalized size = 1.2

$$-\frac{2\,ab}{3\,cx^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2}{5\,cx^5}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{2\,a^2d}{15\,c^2x^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2}{cx}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{b^2dx}{c}\sqrt{dx^2+c}+b^2\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x)
```

```
[Out] -2/3*a*b/c/x^3*(d*x^2+c)^(3/2)-1/5*a^2*(d*x^2+c)^(3/2)/c/x^5+2/15*a^2*d/c^2
/x^3*(d*x^2+c)^(3/2)-b^2/c/x*(d*x^2+c)^(3/2)+b^2*d/c*x*(d*x^2+c)^(1/2)+b^2*
d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65963, size = 512, normalized size = 4.97

$$\left[\frac{15\,b^2c^2\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(\left(15\,b^2c^2+10\,abcd-2\,a^2d^2\right)x^4+3\,a^2c^2+\left(10\,abc^2+a^2cd\right)x^2\right)\sqrt{d}}{30\,c^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*c^2*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - 2*((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a^2*c^2 + (10*a*b*c^2
 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5), -1/15*(15*b^2*c^2*sqrt(-d)*x^5
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2
)*x^4 + 3*a^2*c^2 + (10*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5)]
```

Sympy [B] time = 3.44587, size = 199, normalized size = 1.93

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}+\frac{2a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{b^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2\sqrt{d}\,\text{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(15*c*x**2) + 2*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - 2*a*b*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)
/(3*c) - b**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + b**2*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c)) - b**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [B] time = 1.16319, size = 544, normalized size = 5.28

$$-\frac{1}{2}\,b^2\sqrt{d}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c\sqrt{d}+30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abd^{\frac{3}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*b^2*c*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*
b*d^(3/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^2*sqrt(d) - 60*(sqrt(d
)*x - sqrt(d*x^2 + c))^6*a*b*c*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^3*sqrt(d) + 40*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*d^(3/2) + 10*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^4*sqrt(d) -
20*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a^2*c^2*d^(5/2) + 15*b^2*c^5*sqrt(d) + 10*a*b*c^4*d^(3/2) - 2*
a^2*c^3*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.610 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{\left(c+dx^2\right)^{3/2}\left(35b^2c^2-4ad(7bc-2ad)\right)}{105c^3x^3}-\frac{2a\left(c+dx^2\right)^{3/2}(7bc-2ad)}{35c^2x^5}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(7*c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*x^2)^(3/2)
)/(35*c^2*x^5) - ((35*b^2*c^2 - 4*a*d*(7*b*c - 2*a*d))*(c + d*x^2)^(3/2))/(
105*c^3*x^3)
```

Rubi [A] time = 0.0747952, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 264}

$$-\frac{\left(c+dx^2\right)^{3/2}\left(8a^2d^2-28abcd+35b^2c^2\right)}{105c^3x^3}-\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{2a\left(c+dx^2\right)^{3/2}(7bc-2ad)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(7*c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*x^2)^(3/2)
)/(35*c^2*x^5) - ((35*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2))/
(105*c^3*x^3)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} + \frac{\int \frac{(2a(7bc-2ad)+7b^2cx^2)\sqrt{c+dx^2}}{x^6}\,dx}{7c}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} - \frac{2a(7bc-2ad)(c+dx^2)^{3/2}}{35c^2x^5} - \frac{1}{35}\left(-35b^2+\frac{4ad(7bc-2ad)}{c^2}\right)\int \frac{\sqrt{c+dx^2}}{x^4}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} - \frac{2a(7bc-2ad)(c+dx^2)^{3/2}}{35c^2x^5} - \frac{\left(35b^2-\frac{4ad(7bc-2ad)}{c^2}\right)(c+dx^2)^{3/2}}{105cx^3}$$

Mathematica [A] time = 0.0257616, size = 76, normalized size = 0.77

$$-\frac{(c+dx^2)^{3/2}\left(a^2\left(15c^2-12cdx^2+8d^2x^4\right)+14abcx^2\left(3c-2dx^2\right)+35b^2c^2x^4\right)}{105c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(35*b^2*c^2*x^4 + 14*a*b*c*x^2*(3*c - 2*d*x^2) + a^2*(1
5*c^2 - 12*c*d*x^2 + 8*d^2*x^4)))/(105*c^3*x^7)
```

Maple [A] time = 0.004, size = 78, normalized size = 0.8

$$-\frac{8a^2d^2x^4-28abcdx^4+35b^2c^2x^4-12a^2cdx^2+42ac^2bx^2+15a^2c^2}{105x^7c^3}\left(dx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x)
```

```
[Out] -1/105*(d*x^2+c)^(3/2)*(8*a^2*d^2*x^4-28*a*b*c*d*x^4+35*b^2*c^2*x^4-12*a^2*
c*d*x^2+42*a*b*c^2*x^2+15*a^2*c^2)/x^7/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8319, size = 238, normalized size = 2.4

$$-\frac{\left(\left(35b^2c^2d-28abcd^2+8a^2d^3\right)x^6+15a^2c^3+\left(35b^2c^3+14abc^2d-4a^2cd^2\right)x^4+3\left(14abc^3+a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{105c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*((35*b^2*c^2*d - 28*a*b*c*d^2 + 8*a^2*d^3)*x^6 + 15*a^2*c^3 + (35*b^
2*c^3 + 14*a*b*c^2*d - 4*a^2*c*d^2)*x^4 + 3*(14*a*b*c^3 + a^2*c^2*d)*x^2)*s
qrt(d*x^2 + c)/(c^3*x^7)
```

Sympy [B] time = 3.62894, size = 510, normalized size = 5.15

$$-\frac{15a^2c^5d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{33a^2c^4d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{17a^2c^3d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

```
[Out] -15*a**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*
d**5*x**8 + 105*c**3*d**6*x**10) - 33*a**2*c**4*d**(11/2)*x**2*sqrt(c/(d*x*
*2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
17*a**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*
c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*a**2*c**2*d**(15/2)*x**6*sqrt(c/(
d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10
) - 12*a**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*a**2*d**(19/2)*x**10*sqrt(c/(d*x
**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a*b*d**(3/2)*sqrt(c/(d*x**
2) + 1)/(15*c*x**2) + 4*a*b*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - b**2*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/
(3*c)
```

Giac [B] time = 1.15813, size = 662, normalized size = 6.69

$$2\left(105\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2d^{\frac{3}{2}}-420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2cd^{\frac{3}{2}}+420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abd^{\frac{5}{2}}+665\left(\sqrt{d}x-\sqrt{dx^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*d^(3/2) - 420*(sqrt(d)*x -
sqrt(d*x^2 + c))^10*b^2*c*d^(3/2) + 420*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*
b*d^(5/2) + 665*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*d^(3/2) - 700*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(5/2) + 560*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a^2*d^(7/2) - 560*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*d^(3/2) + 280
*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(5/2) + 280*(sqrt(d)*x - sqrt(d*
x^2 + c))^6*a^2*c*d^(7/2) + 315*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*d^(
3/2) - 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(5/2) + 168*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a^2*c^2*d^(7/2) - 140*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b^2*c^5*d^(3/2) + 196*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(5/2) - 56
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(7/2) + 35*b^2*c^6*d^(3/2) - 28*
a*b*c^5*d^(5/2) + 8*a^2*c^4*d^(7/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^
7
```

3.611 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{105c^3x^5}-\frac{2a\left(c+dx^2\right)^{3/2}(3b}{21c^2x^7}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(9*c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^2)^(3/2))/
(21*c^2*x^7) - ((21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c + d*x^2)^(3/2))/(105*
c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c + d*x^2)^(3/2))/(315*
c^4*x^3)
```

Rubi [A] time = 0.131767, antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 264}

$$-\frac{\left(c+dx^2\right)^{3/2}\left(8a^2d^2-24abcd+21b^2c^2\right)}{105c^3x^5}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{2a\left(c+dx^2\right)^{3/2}(}{21c^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(9*c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^2)^(3/2))/
(21*c^2*x^7) - ((21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2))/(1
05*c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c + d*x^2)^(3/2))/(3
15*c^4*x^3)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{10}}\,dx = -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{\int \frac{\left(6a(3bc-ad)+9b^2cx^2\right)\sqrt{c+dx^2}}{x^8}\,dx}{9c}$$
$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{1}{21}\left(-21b^2+\frac{8ad(3bc-ad)}{c^2}\right)\int \frac{\sqrt{c+dx^2}}{x^6}$$
$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{105cx^5}-\frac{\left(2d\left(21\right.\right.}{}$$
$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{105cx^5}+\frac{2d\left(21\right.}{}$$

Mathematica [A] time = 0.0661442, size = 108, normalized size = 0.76

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(-30c^2dx^2+35c^3+24cd^2x^4-16d^3x^6\right)+6abcx^2\left(15c^2-12cdx^2+8d^2x^4\right)+21b^2c^2x^4\left(3c-2dx^2\right)\right)}{315c^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(21*b^2*c^2*x^4*(3*c - 2*d*x^2) + 6*a*b*c*x^2*(15*c^2 -
 12*c*d*x^2 + 8*d^2*x^4) + a^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d
^3*x^6)))/(315*c^4*x^9)
```

Maple [A] time = 0.007, size = 117, normalized size = 0.8

$$-\frac{-16\,x^6a^2d^3+48\,x^6abcd^2-42\,x^6b^2c^2d+24\,x^4a^2cd^2-72\,x^4abc^2d+63\,x^4b^2c^3-30\,x^2a^2c^2d+90\,x^2abc^3+35\,a^2c^3}{315\,x^9c^4}\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-16*a^2*d^3*x^6+48*a*b*c*d^2*x^6-42*b^2*c^2*d*x^6+2
4*a^2*c*d^2*x^4-72*a*b*c^2*d*x^4+63*b^2*c^3*x^4-30*a^2*c^2*d*x^2+90*a*b*c^3
*x^2+35*a^2*c^3)/x^9/c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.58041, size = 316, normalized size = 2.21

$$\frac{\left(2\left(21\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4\right)x^8-\left(21\,b^2c^3d-24\,abc^2d^2+8\,a^2cd^3\right)x^6-35\,a^2c^4-3\left(21\,b^2c^4+6\,abc^3d-2\,a^2c^2d^2\right)x^4-\right.}{315\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(21*b^2*c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)*x^8 - (21*b^2*c^3*d -
24*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^6 - 35*a^2*c^4 - 3*(21*b^2*c^4 + 6*a*b*c^3*
d - 2*a^2*c^2*d^2)*x^4 - 5*(18*a*b*c^4 + a^2*c^3*d)*x^2)*sqrt(d*x^2 + c)/(c
^4*x^9)
```

Sympy [B] time = 4.86183, size = 1061, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

```
[Out] -35*a**2*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 110*a**2*c**6
*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x
**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 114*a**2*c**5*d**(23/
2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 9
45*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 40*a**2*c**4*d**(25/2)*x**6*s
qrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d
**11*x**12 + 315*c**4*d**12*x**14) + 5*a**2*c**3*d**(27/2)*x**8*sqrt(c/(d*x
**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12
 + 315*c**4*d**12*x**14) + 30*a**2*c**2*d**(29/2)*x**10*sqrt(c/(d*x**2) + 1
)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c
**4*d**12*x**14) + 40*a**2*c*d**(31/2)*x**12*sqrt(c/(d*x**2) + 1)/(315*c**7
*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x
**14) + 16*a**2*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 30*a*
b*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x*
*8 + 105*c**3*d**6*x**10) - 66*a*b*c**4*d**(11/2)*x**2*sqrt(c/(d*x**2) + 1)
/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 34*a*b*c
**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5
*x**8 + 105*c**3*d**6*x**10) - 6*a*b*c**2*d**(15/2)*x**6*sqrt(c/(d*x**2) +
1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 24*a*b
*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*
x**8 + 105*c**3*d**6*x**10) - 16*a*b*d**(19/2)*x**10*sqrt(c/(d*x**2) + 1)/(
105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - b**2*sqrt(
d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*c
*x**2) + 2*b**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2)
```

Giac [B] time = 1.15167, size = 782, normalized size = 5.47

$$4\left(315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2d^{\frac{5}{2}}-1155\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2cd^{\frac{5}{2}}+1680\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abd^{\frac{7}{2}}+1575\left(\sqrt{d}x-\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*d^(5/2) - 1155*(sqrt(d)*x -
 sqrt(d*x^2 + c))^12*b^2*c*d^(5/2) + 1680*(sqrt(d)*x - sqrt(d*x^2 + c))^12*
a*b*d^(7/2) + 1575*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*c^2*d^(5/2) - 2520*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c*d^(7/2) + 2520*(sqrt(d)*x - sqrt(d*x
^2 + c))^10*a^2*d^(9/2) - 1071*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*d^(5
/2) + 504*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(7/2) + 1512*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^2*c*d^(9/2) + 609*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^2*c^4*d^(5/2) - 336*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(7/2) + 672*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(9/2) - 441*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b^2*c^5*d^(5/2) + 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^
(7/2) - 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(9/2) + 189*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^2*c^6*d^(5/2) - 216*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*b*c^5*d^(7/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(9/2) - 21
*b^2*c^7*d^(5/2) + 24*a*b*c^6*d^(7/2) - 8*a^2*c^5*d^(9/2))/((sqrt(d)*x - sq
rt(d*x^2 + c))^2 - c)^9
```

3.612 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{12}}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{1155c^4x^5}-\frac{\left(c+dx^2\right)}{}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(11*c*x^11) - (2*a*(11*b*c - 4*a*d)*(c + d*x^2)^(3
/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^(3/2
))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^(
3/2))/(1155*c^4*x^5) - (8*d^2*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*
x^2)^(3/2))/(3465*c^5*x^3)
```

Rubi [A] time = 0.170757, antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 264}

$$-\frac{\left(c+dx^2\right)^{3/2}\left(16a^2d^2-44abcd+33b^2c^2\right)}{231c^3x^7}-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(11*c*x^11) - (2*a*(11*b*c - 4*a*d)*(c + d*x^2)^(3
/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 44*a*b*c*d + 16*a^2*d^2)*(c + d*x^2)^(3/
2))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^
(3/2))/(1155*c^4*x^5) - (8*d^2*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d
*x^2)^(3/2))/(3465*c^5*x^3)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} + \frac{\int \frac{(2a(11bc-4ad)+11b^2cx^2)\sqrt{c+dx^2}}{x^{10}}\,dx}{11c}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{1}{33}\left(-33b^2+\frac{4ad(11bc-4ad)}{c^2}\right)\int \frac{\sqrt{c+}}{x}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+dx^2)^{3/2}}{231cx^7} - \frac{(4}{}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+dx^2)^{3/2}}{231cx^7} + \frac{4d}{}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+dx^2)^{3/2}}{231cx^7} + \frac{4d}{}$$

Mathematica [A] time = 0.0783732, size = 141, normalized size = 0.75

$$-\frac{(c+dx^2)^{3/2}\left(a^2(240c^2d^2x^4-280c^3dx^2+315c^4-192cd^3x^6+128d^4x^8)+22abcx^2(-30c^2dx^2+35c^3+24cd^2x^4-16\right.}{3465c^5x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(33*b^2*c^2*x^4*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4) + 22*
a*b*c*x^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d^3*x^6) + a^2*(315*c^
4 - 280*c^3*d*x^2 + 240*c^2*d^2*x^4 - 192*c*d^3*x^6 + 128*d^4*x^8)))/(3465*
c^5*x^11)
```

Maple [A] time = 0.008, size = 158, normalized size = 0.8

$$-\frac{128a^2d^4x^8-352abcd^3x^8+264b^2c^2d^2x^8-192a^2cd^3x^6+528abc^2d^2x^6-396b^2c^3dx^6+240a^2c^2d^2x^4-660abc^3dx^4}{3465x^{11}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x)
```

```
[Out] -1/3465*(d*x^2+c)^(3/2)*(128*a^2*d^4*x^8-352*a*b*c*d^3*x^8+264*b^2*c^2*d^2*
x^8-192*a^2*c*d^3*x^6+528*a*b*c^2*d^2*x^6-396*b^2*c^3*d*x^6+240*a^2*c^2*d^2
*x^4-660*a*b*c^3*d*x^4+495*b^2*c^4*x^4-280*a^2*c^3*d*x^2+770*a*b*c^4*x^2+31
5*a^2*c^4)/x^11/c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.33283, size = 410, normalized size = 2.17

$$-\frac{\left(8\left(33\,b^2c^2d^3-44\,abcd^4+16\,a^2d^5\right)x^{10}-4\left(33\,b^2c^3d^2-44\,abc^2d^3+16\,a^2cd^4\right)x^8+315\,a^2c^5+3\left(33\,b^2c^4d-44\,abc^3d^2+\right.}{3465\,c^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(33*b^2*c^2*d^3 - 44*a*b*c*d^4 + 16*a^2*d^5)*x^10 - 4*(33*b^2*c^
3*d^2 - 44*a*b*c^2*d^3 + 16*a^2*c*d^4)*x^8 + 315*a^2*c^5 + 3*(33*b^2*c^4*d
- 44*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x^6 + 5*(99*b^2*c^5 + 22*a*b*c^4*d - 8*a
^2*c^3*d^2)*x^4 + 35*(22*a*b*c^5 + a^2*c^4*d)*x^2)*sqrt(d*x^2 + c)/(c^5*x^1
1)
```

Sympy [B] time = 6.92099, size = 1856, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

```
[Out] -315*a**2*c**9*d**(33/2)*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1386
0*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465
*c**5*d**20*x**18) - 1295*a**2*c**8*d**(35/2)*x**2*sqrt(c/(d*x**2) + 1)/(34
65*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 138
60*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1990*a**2*c**7*d**(37/2)*x**
4*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20
790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 13
58*a**2*c**6*d**(39/2)*x**6*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1
3860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3
465*c**5*d**20*x**18) - 343*a**2*c**5*d**(41/2)*x**8*sqrt(c/(d*x**2) + 1)/(
3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 1
3860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 35*a**2*c**4*d**(43/2)*x**
10*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 2
80*a**2*c**3*d**(45/2)*x**12*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 +
13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 +
3465*c**5*d**20*x**18) - 560*a**2*c**2*d**(47/2)*x**14*sqrt(c/(d*x**2) + 1)
/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 +
 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 448*a**2*c*d**(49/2)*x**
16*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1
28*a**2*d**(51/2)*x**18*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860
*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*
c**5*d**20*x**18) - 70*a*b*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d*
*9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**1
```

```
4) - 220*a*b*c**6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 228*
a*b*c**5*d**(23/2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 80*a*b*c**4*d
**(25/2)*x**6*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**
10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 10*a*b*c**3*d**(27/2)*x
**8*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c
**5*d**11*x**12 + 315*c**4*d**12*x**14) + 60*a*b*c**2*d**(29/2)*x**10*sqrt(
c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11
*x**12 + 315*c**4*d**12*x**14) + 80*a*b*c*d**(31/2)*x**12*sqrt(c/(d*x**2) +
 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315
*c**4*d**12*x**14) + 32*a*b*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*
d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x*
*14) - 15*b**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 33*b**2*c**4*d**(11/2)*x**2*sqrt(c
/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**
10) - 17*b**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
+ 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*b**2*c**2*d**(15/2)*x**6*sq
rt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 12*b**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*b**2*d**(19/2)*x**10*sqrt(
c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x*
*10)
```

Giac [B] time = 1.19238, size = 902, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(d)*x - sqrt(d*x^2 + c))^16*b^2*d^(7/2) - 8085*(sqrt(d)*
x - sqrt(d*x^2 + c))^14*b^2*c*d^(7/2) + 13860*(sqrt(d)*x - sqrt(d*x^2 + c))
^14*a*b*d^(9/2) + 9933*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*c^2*d^(7/2) - 1
9404*(sqrt(d)*x - sqrt(d*x^2 + c))^12*a*b*c*d^(9/2) + 22176*(sqrt(d)*x - sq
rt(d*x^2 + c))^12*a^2*d^(11/2) - 5313*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*
c^3*d^(7/2) + 924*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c^2*d^(9/2) + 14784*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a^2*c*d^(11/2) + 2805*(sqrt(d)*x - sqrt(d*
x^2 + c))^8*b^2*c^4*d^(7/2) - 660*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^3*d
^(9/2) + 5280*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*c^2*d^(11/2) - 3135*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*b^2*c^5*d^(7/2) + 7260*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*a*b*c^4*d^(9/2) - 2640*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^3*d^(11
/2) + 1815*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^6*d^(7/2) - 2420*(sqrt(d)*
x - sqrt(d*x^2 + c))^4*a*b*c^5*d^(9/2) + 880*(sqrt(d)*x - sqrt(d*x^2 + c))^
4*a^2*c^4*d^(11/2) - 363*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^7*d^(7/2) +
484*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^6*d^(9/2) - 176*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a^2*c^5*d^(11/2) + 33*b^2*c^8*d^(7/2) - 44*a*b*c^7*d^(9/2) +
 16*a^2*c^6*d^(11/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^11
```

3.613 $\int x^4 (a+bx^2)^2 (c+dx^2)^{3/2}\,dx$

Optimal. Leaf size=281

$$\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1536d^3}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^4\left(24a^2d^2+bc(7bc-24ad)\right)\tanh}{1024d^{9/2}}$$

```
[Out] -(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^4) + (
c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(1536*d^3) + (
c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqrt[c + d*x^2])/(384*d^2) + ((24
*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*(c + d*x^2)^(3/2))/(192*d^2) - (b*(7*b
*c - 24*a*d)*x^5*(c + d*x^2)^(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))
/(12*d) + (c^4*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(1024*d^(9/2))
```

Rubi [A] time = 0.266162, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1536d^3}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^4\left(24a^2d^2+bc(7bc-24ad)\right)\tanh}{1024d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^4) + (
c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(1536*d^3) + (
c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqrt[c + d*x^2])/(384*d^2) + ((24
*a^2 + (b*c*(7*b*c - 24*a*d))/d^2)*x^5*(c + d*x^2)^(3/2))/192 - (b*(7*b*c -
 24*a*d)*x^5*(c + d*x^2)^(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))/(12
*d) + (c^4*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(9/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx=\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}+\frac{\int x^4\left(c+dx^2\right)^{3/2}\left(12a^2d-b(7bc-24ad)x^2\right)dx}{12d}$$

$$=-\frac{b(7bc-24ad)x^5\left(c+dx^2\right)^{5/2}}{120d^2}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}+\frac{1}{24}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)\int$$

$$=\frac{1}{192}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\left(c+dx^2\right)^{3/2}-\frac{b(7bc-24ad)x^5\left(c+dx^2\right)^{5/2}}{120d^2}+\frac{b^2x^7}{}$$

$$=\frac{1}{384}c\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\sqrt{c+dx^2}+\frac{1}{192}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\left(c+\right.$$

$$=\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{1}{384}c\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\sqrt{c+dx^2}+$$

$$=-\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{1}{384}c\left(\right.$$

$$=-\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{1}{384}c\left(\right.$$

$$=-\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{1}{384}c\left(\right.$$

Mathematica [A] time = 0.142605, size = 225, normalized size = 0.8

$$\sqrt{d}x\sqrt{c+dx^2}\left(120a^2d^2\left(2c^2dx^2-3c^3+24cd^2x^4+16d^3x^6\right)+24abd\left(8c^2d^2x^4-10c^3dx^2+15c^4+176cd^3x^6+128d^4x^8\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(120*a^2*d^2*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^
4 + 16*d^3*x^6) + 24*a*b*d*(15*c^4 - 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d
^3*x^6 + 128*d^4*x^8) + b^2*(-105*c^5 + 70*c^4*d*x^2 - 56*c^3*d^2*x^4 + 48*
c^2*d^3*x^6 + 1664*c*d^4*x^8 + 1280*d^5*x^10)) + 15*c^4*(7*b^2*c^2 - 24*a*b
*c*d + 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(9/2))
```

Maple [A] time = 0.02, size = 389, normalized size = 1.4

$$\frac{b^2x^7}{12\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{7\,b^2cx^5}{120\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{7\,x^3b^2c^2}{192\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{7\,b^2c^3x}{384\,d^4}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{7\,b^2c^4x}{1536\,d^4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{7\,b^2c^5x}{1024\,d^4}\sqrt{dx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/12*b^2*x^7*(d*x^2+c)^(5/2)/d-7/120*b^2*c/d^2*x^5*(d*x^2+c)^(5/2)+7/192*b^
2*c^2/d^3*x^3*(d*x^2+c)^(5/2)-7/384*b^2*c^3/d^4*x*(d*x^2+c)^(5/2)+7/1536*b^
2*c^4/d^4*x*(d*x^2+c)^(3/2)+7/1024*b^2*c^5/d^4*x*(d*x^2+c)^(1/2)+7/1024*b^2
*c^6/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/5*a*b*x^5*(d*x^2+c)^(5/2)/d-1/
8*a*b*c/d^2*x^3*(d*x^2+c)^(5/2)+1/16*a*b*c^2/d^3*x*(d*x^2+c)^(5/2)-1/64*a*b
*c^3/d^3*x*(d*x^2+c)^(3/2)-3/128*a*b*c^4/d^3*x*(d*x^2+c)^(1/2)-3/128*a*b*c^
5/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8*a^2*x^3*(d*x^2+c)^(5/2)/d-1/16*
a^2*c/d^2*x*(d*x^2+c)^(5/2)+1/64*a^2*c^2/d^2*x*(d*x^2+c)^(3/2)+3/128*a^2*c^
3/d^2*x*(d*x^2+c)^(1/2)+3/128*a^2*c^4/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.38041, size = 1116, normalized size = 3.97

$$\left[\frac{15\left(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(1280\,b^2d^6x^{11}+128\left(13\,b^2cd^5+24\,abd^6\right)x^9\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(13*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 88*a*b*c*d^5 + 40*a^2*d^6)*x^7
- 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2*c*d^5)*x^5 + 10*(7*b^2*c^4*d^
2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15*(7*b^2*c^5*d - 24*a*b*c^4*d^2
 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/15360*(15*(7*b^2*c^6 - 24*a*
b*c^5*d + 24*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (12
```

```
80*b^2*d^6*x^11 + 128*(13*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 8
8*a*b*c*d^5 + 40*a^2*d^6)*x^7 - 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2
*c*d^5)*x^5 + 10*(7*b^2*c^4*d^2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15
*(7*b^2*c^5*d - 24*a*b*c^4*d^2 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

Sympy [B] time = 62.2183, size = 598, normalized size = 2.13

$$-\frac{3a^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}} + \frac{13a^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}} + \frac{5a^2\sqrt{c}dx^7}{16\sqrt{1+\frac{dx^2}{c}}} + \frac{3a^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}} + \frac{a^2d^2x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{3abc^{\frac{9}{2}}x}{128d^3\sqrt{1+\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] -3*a**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - a**2*c**(5/2)*x**3/(128*
d*sqrt(1 + d*x**2/c)) + 13*a**2*c**(3/2)*x**5/(64*sqrt(1 + d*x**2/c)) + 5*a
**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c)) + 3*a**2*c**4*asinh(sqrt(d)*x/sq
rt(c))/(128*d**(5/2)) + a**2*d**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c)) + 3*a
*b*c**(9/2)*x/(128*d**3*sqrt(1 + d*x**2/c)) + a*b*c**(7/2)*x**3/(128*d**2*s
qrt(1 + d*x**2/c)) - a*b*c**(5/2)*x**5/(320*d*sqrt(1 + d*x**2/c)) + 23*a*b*
c**(3/2)*x**7/(80*sqrt(1 + d*x**2/c)) + 19*a*b*sqrt(c)*d*x**9/(40*sqrt(1 +
d*x**2/c)) - 3*a*b*c**5*asinh(sqrt(d)*x/sqrt(c))/(128*d**(7/2)) + a*b*d**2*
x**11/(5*sqrt(c)*sqrt(1 + d*x**2/c)) - 7*b**2*c**(11/2)*x/(1024*d**4*sqrt(1
 + d*x**2/c)) - 7*b**2*c**(9/2)*x**3/(3072*d**3*sqrt(1 + d*x**2/c)) + 7*b**
2*c**(7/2)*x**5/(7680*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**7/(1920*d
*sqrt(1 + d*x**2/c)) + 107*b**2*c**(3/2)*x**9/(960*sqrt(1 + d*x**2/c)) + 23
*b**2*sqrt(c)*d*x**11/(120*sqrt(1 + d*x**2/c)) + 7*b**2*c**6*asinh(sqrt(d)*
x/sqrt(c))/(1024*d**(9/2)) + b**2*d**2*x**13/(12*sqrt(c)*sqrt(1 + d*x**2/c)
)
```

Giac [A] time = 1.15109, size = 355, normalized size = 1.26

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2dx^2 + \frac{13\,b^2cd^{10} + 24\,abd^{11}}{d^{10}}\right)x^2 + \frac{3\left(b^2c^2d^9 + 88\,abcd^{10} + 40\,a^2d^{11}\right)}{d^{10}}\right)x^2 - \frac{7\,b^2c^3d^8 - 24\,abc^2d^9}{d^{10}}\ldots\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d*x^2 + (13*b^2*c*d^10 + 24*a*b*d^11)/d^10)*x^2
 + 3*(b^2*c^2*d^9 + 88*a*b*c*d^10 + 40*a^2*d^11)/d^10)*x^2 - (7*b^2*c^3*d^8
 - 24*a*b*c^2*d^9 - 360*a^2*c*d^10)/d^10)*x^2 + 5*(7*b^2*c^4*d^7 - 24*a*b*c
^3*d^8 + 24*a^2*c^2*d^9)/d^10)*x^2 - 15*(7*b^2*c^5*d^6 - 24*a*b*c^4*d^7 + 2
4*a^2*c^3*d^8)/d^10)*sqrt(d*x^2 + c)*x - 1/1024*(7*b^2*c^6 - 24*a*b*c^5*d +
 24*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.614 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(9/2))/(9*d^4) +
 (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rubi [A] time = 0.0896447, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(9/2))/(9*d^4) +
 (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2(c+dx)^{3/2}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{5/2}}{d^3}-\frac{b(3bc-2ad)(}{d^3}\right.\right.\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{7/2}}{7d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{9/}}{9d^4}\end{aligned}$$

Mathematica [A] time = 0.0885513, size = 100, normalized size = 0.88

$$\frac{\left(c+dx^2\right)^{5/2}\left(99a^2d^2\left(5dx^2-2c\right)+22abd\left(8c^2-20cdx^2+35d^2x^4\right)-3b^2\left(-40c^2dx^2+16c^3+70cd^2x^4-105d^3x^6\right)\right)}{3465d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(99*a^2*d^2*(-2*c + 5*d*x^2) + 22*a*b*d*(8*c^2 - 20*c*d*
x^2 + 35*d^2*x^4) - 3*b^2*(16*c^3 - 40*c^2*d*x^2 + 70*c*d^2*x^4 - 105*d^3*x
^6)))/(3465*d^4)
```

Maple [A] time = 0.006, size = 108, normalized size = 1.

$$-\frac{-315\,b^2x^6d^3-770\,abd^3x^4+210\,b^2cd^2x^4-495\,a^2d^3x^2+440\,abcd^2x^2-120\,b^2c^2dx^2+198\,a^2cd^2-176\,abc^2d+48\,b^2c}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3465*(d*x^2+c)^(5/2)*(-315*b^2*d^3*x^6-770*a*b*d^3*x^4+210*b^2*c*d^2*x^4
-495*a^2*d^3*x^2+440*a*b*c*d^2*x^2-120*b^2*c^2*d*x^2+198*a^2*c*d^2-176*a*b*
c^2*d+48*b^2*c^3)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5101, size = 398, normalized size = 3.49

$$\frac{\left(315\,b^2d^5x^{10}+70\left(6\,b^2cd^4+11\,abd^5\right)x^8-48\,b^2c^5+176\,abc^4d-198\,a^2c^3d^2+5\left(3\,b^2c^2d^3+220\,abcd^4+99\,a^2d^5\right)x^6-\right.}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 70*(6*b^2*c*d^4 + 11*a*b*d^5)*x^8 - 48*b^2*c^5 +
 176*a*b*c^4*d - 198*a^2*c^3*d^2 + 5*(3*b^2*c^2*d^3 + 220*a*b*c*d^4 + 99*a^
2*d^5)*x^6 - 6*(3*b^2*c^3*d^2 - 11*a*b*c^2*d^3 - 132*a^2*c*d^4)*x^4 + (24*b
^2*c^4*d - 88*a*b*c^3*d^2 + 99*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 4.1899, size = 384, normalized size = 3.37

$$\begin{cases} -\frac{2a^2c^3\sqrt{c+dx^2}}{35d^2} + \frac{a^2c^2x^2\sqrt{c+dx^2}}{35d} + \frac{8a^2cx^4\sqrt{c+dx^2}}{35} + \frac{a^2dx^6\sqrt{c+dx^2}}{7} + \frac{16abc^4\sqrt{c+dx^2}}{315d^3} - \frac{8abc^3x^2\sqrt{c+dx^2}}{315d^2} + \frac{2abc^2x^4\sqrt{c+dx^2}}{105d} + \frac{20abcx^6\sqrt{c+dx^2}}{63} + 2a \\ c^{\frac{3}{2}}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a**2*c**3*sqrt(c + d*x**2)/(35*d**2) + a**2*c**2*x**2*sqrt(c
+ d*x**2)/(35*d) + 8*a**2*c*x**4*sqrt(c + d*x**2)/35 + a**2*d*x**6*sqrt(c +
 d*x**2)/7 + 16*a*b*c**4*sqrt(c + d*x**2)/(315*d**3) - 8*a*b*c**3*x**2*sqrt
(c + d*x**2)/(315*d**2) + 2*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d) + 20*a*b
*c*x**6*sqrt(c + d*x**2)/63 + 2*a*b*d*x**8*sqrt(c + d*x**2)/9 - 16*b**2*c**
5*sqrt(c + d*x**2)/(1155*d**4) + 8*b**2*c**4*x**2*sqrt(c + d*x**2)/(1155*d*
*3) - 2*b**2*c**3*x**4*sqrt(c + d*x**2)/(385*d**2) + b**2*c**2*x**6*sqrt(c
+ d*x**2)/(231*d) + 4*b**2*c*x**8*sqrt(c + d*x**2)/33 + b**2*d*x**10*sqrt(c
 + d*x**2)/11, Ne(d, 0)), (c**(3/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
), True))
```

Giac [B] time = 1.12617, size = 437, normalized size = 3.83

$$\frac{\frac{231\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2c}{d} + \frac{66\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)abc}{d^2} + \frac{33\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)a^2}{d} + \frac{11\left(35\left(a\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(231*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a^2*c/d + 66*(15*
(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a*b*
c/d^2 + 33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^
(3/2)*c^2)*a^2/d + 11*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189
*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2*c/d^3 + 22*(35*(d*x
^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(
d*x^2 + c)^(3/2)*c^3)*a*b/d^2 + (315*(d*x^2 + c)^(11/2) - 1540*(d*x^2 + c)^
(9/2)*c + 2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2)*c^3 + 1155*(d
*x^2 + c)^(3/2)*c^4)*b^2/d^3)/d
```

3.615 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=235

$$\frac{c^2x\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}-\frac{c^3\left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{x^3\left(c+dx^2\right)^{3/2}\left(16a^2d^2+3bc\right.}{96d^2}$$

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d^3) + (c*(
16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/(128*d^2) + ((16*a^2
*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*(c + d*x^2)^(3/2))/(96*d^2) - (b*(b*c - 4*a
*d)*x^3*(c + d*x^2)^(5/2))/(16*d^2) + (b^2*x^5*(c + d*x^2)^(5/2))/(10*d) -
(c^3*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(256*d^(7/2))
```

Rubi [A] time = 0.217567, antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}-\frac{c^3\left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{1}{96}x^3\left(c+dx^2\right)^{3/2}\left(16a^2+\frac{3b}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d^3) + (c*(
16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/(128*d^2) + ((16*a^2
 + (3*b*c*(b*c - 4*a*d))/d^2)*x^3*(c + d*x^2)^(3/2))/96 - (b*(b*c - 4*a*d)*
x^3*(c + d*x^2)^(5/2))/(16*d^2) + (b^2*x^5*(c + d*x^2)^(5/2))/(10*d) - (c^3
*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
256*d^(7/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx &= \frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}+\frac{\int x^2\left(c+dx^2\right)^{3/2}\left(10a^2d-5b(bc-4ad)x^2\right)dx}{10d}\\
&= -\frac{b(bc-4ad)x^3\left(c+dx^2\right)^{5/2}}{16d^2}+\frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}+\frac{1}{16}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)\int x^2\left(c\right.\\
&= \frac{1}{96}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2}-\frac{b(bc-4ad)x^3\left(c+dx^2\right)^{5/2}}{16d^2}+\frac{b^2x^5\left(c+dx^2\right.}{10d}\\
&= \frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{96}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2}\\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{96}\left(16\right.\\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{96}\left(16\right.\\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{96}\left(16\right.
\end{aligned}$$

Mathematica [A] time = 0.116866, size = 193, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(80a^2d^2\left(3c^2+14cdx^2+8d^2x^4\right)+60abd\left(2c^2dx^2-3c^3+24cd^2x^4+16d^3x^6\right)+3b^2\left(8c^2d^2x^4-10c^3dx^2+15\right.\right.}{3840d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) + 6
0*a*b*d*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) + 3*b^2*(15*c^4
- 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d^3*x^6 + 128*d^4*x^8)) - 15*c^3*(3*
```

```
b^2*c^2 - 12*a*b*c*d + 16*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(384
0*d^(7/2))
```

Maple [A] time = 0.007, size = 321, normalized size = 1.4

$$\frac{b^2x^5}{10\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2cx^3}{16\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2c^2x}{32\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2c^3x}{128\,d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3\,b^2c^4x}{256\,d^3}\sqrt{dx^2+c}-\frac{3\,b^2c^5}{256}\ln\left(x\sqrt{d}+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/10*b^2*x^5*(d*x^2+c)^(5/2)/d-1/16*b^2*c/d^2*x^3*(d*x^2+c)^(5/2)+1/32*b^2*
c^2/d^3*x*(d*x^2+c)^(5/2)-1/128*b^2*c^3/d^3*x*(d*x^2+c)^(3/2)-3/256*b^2*c^4
/d^3*x*(d*x^2+c)^(1/2)-3/256*b^2*c^5/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+
1/4*a*b*x^3*(d*x^2+c)^(5/2)/d-1/8*a*b*c/d^2*x*(d*x^2+c)^(5/2)+1/32*a*b*c^2/
d^2*x*(d*x^2+c)^(3/2)+3/64*a*b*c^3/d^2*x*(d*x^2+c)^(1/2)+3/64*a*b*c^4/d^(5/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/6*a^2*x*(d*x^2+c)^(5/2)/d-1/24*a^2*c/d*x
*(d*x^2+c)^(3/2)-1/16*a^2*c^2/d*x*(d*x^2+c)^(1/2)-1/16*a^2*c^3/d^(3/2)*ln(x
*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96273, size = 953, normalized size = 4.06

$$\left[\frac{15\left(3\,b^2c^5-12\,abc^4d+16\,a^2c^3d^2\right)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(384\,b^2d^5x^9+48\left(11\,b^2cd^4+20\,abd^5\right)x^7\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(11*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 -
10*(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d -
 12*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/3840*(15*(3*b^
2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^
2 + c)) + (384*b^2*d^5*x^9 + 48*(11*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*
c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 - 10*(3*b^2*c^3*d^2 - 12*a*b*c^2*
d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d - 12*a*b*c^3*d^2 + 16*a^2*c^2*d^
3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] time = 41.3064, size = 505, normalized size = 2.15

$$\frac{a^2c^{\frac{5}{2}}x}{16d\sqrt{1+\frac{dx^2}{c}}}+\frac{17a^2c^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{11a^2\sqrt{c}dx^5}{24\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16d^{\frac{3}{2}}}+\frac{a^2d^2x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3abc^{\frac{7}{2}}x}{64d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x^3}{64d\sqrt{1+\frac{dx^2}{c}}}+\frac{13a}{32\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*a**2*c**(3/2)*x**3/(48*sqrt(
1 + d*x**2/c)) + 11*a**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - a**2*c**3
*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + a**2*d**2*x**7/(6*sqrt(c)*sqrt(1
+ d*x**2/c)) - 3*a*b*c**(7/2)*x/(64*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)
*x**3/(64*d*sqrt(1 + d*x**2/c)) + 13*a*b*c**(3/2)*x**5/(32*sqrt(1 + d*x**2/
c)) + 5*a*b*sqrt(c)*d*x**7/(8*sqrt(1 + d*x**2/c)) + 3*a*b*c**4*asinh(sqrt(d
)*x/sqrt(c))/(64*d**(5/2)) + a*b*d**2*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 3*b**2*c**(9/2)*x/(256*d**3*sqrt(1 + d*x**2/c)) + b**2*c**(7/2)*x**3/(256*
d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**5/(640*d*sqrt(1 + d*x**2/c)) +
23*b**2*c**(3/2)*x**7/(160*sqrt(1 + d*x**2/c)) + 19*b**2*sqrt(c)*d*x**9/(80
*sqrt(1 + d*x**2/c)) - 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(7/2))
+ b**2*d**2*x**11/(10*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.11371, size = 296, normalized size = 1.26

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2dx^2+\frac{11\,b^2cd^8+20\,abd^9}{d^8}\right)x^2+\frac{3\,b^2c^2d^7+180\,abcd^8+80\,a^2d^9}{d^8}\right)x^2-\frac{5\left(3\,b^2c^3d^6-12\,abc^2d^7-112\,a^2c\right.}{d^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d*x^2 + (11*b^2*c*d^8 + 20*a*b*d^9)/d^8)*x^2 + (3*b^
2*c^2*d^7 + 180*a*b*c*d^8 + 80*a^2*d^9)/d^8)*x^2 - 5*(3*b^2*c^3*d^6 - 12*a*
b*c^2*d^7 - 112*a^2*c*d^8)/d^8)*x^2 + 15*(3*b^2*c^4*d^5 - 12*a*b*c^3*d^6 +
16*a^2*c^2*d^7)/d^8)*sqrt(d*x^2 + c)*x + 1/256*(3*b^2*c^5 - 12*a*b*c^4*d +
16*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.616 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(7
/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rubi [A] time = 0.0599617, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(7
/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^2(c+dx)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx,x
\right.\\
&= \frac{(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{7/2}}{7d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.0463802, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{5/2}\left(63a^2d^2+18abd\left(5dx^2-2c\right)+b^2\left(8c^2-20cdx^2+35d^2x^4\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x^2) + b^2*(8*c^2 - 2
0*c*d*x^2 + 35*d^2*x^4)))/(315*d^3)
```

Maple [A] time = 0.005, size = 69, normalized size = 0.9

$$\frac{35\,b^2d^2x^4+90\,abd^2x^2-20\,b^2cdx^2+63\,a^2d^2-36\,cabd+8\,b^2c^2}{315\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/315*(d*x^2+c)^(5/2)*(35*b^2*d^2*x^4+90*a*b*d^2*x^2-20*b^2*c*d*x^2+63*a^2*
d^2-36*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39242, size = 305, normalized size = 3.96

$$\frac{\left(35\,b^2d^4x^8+10\left(5\,b^2cd^3+9\,abd^4\right)x^6+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+3\left(b^2c^2d^2+48\,abcd^3+21\,a^2d^4\right)x^4-2\left(2\,b^2c^3d-9\right.}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 10*(5*b^2*c*d^3 + 9*a*b*d^4)*x^6 + 8*b^2*c^4 - 36*a
*b*c^3*d + 63*a^2*c^2*d^2 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^4
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 2.38344, size = 303, normalized size = 3.94

$$\begin{cases} \frac{a^2c^2\sqrt{c+dx^2}}{5d}+\frac{2a^2cx^2\sqrt{c+dx^2}}{5}+\frac{a^2dx^4\sqrt{c+dx^2}}{5}-\frac{4abc^3\sqrt{c+dx^2}}{35d^2}+\frac{2abc^2x^2\sqrt{c+dx^2}}{35d}+\frac{16abcx^4\sqrt{c+dx^2}}{35}+\frac{2abdx^6\sqrt{c+dx^2}}{7}+\frac{8b^2c^4\sqrt{c+dx^2}}{315d^3}-\frac{4b^2c^3x^2\sqrt{}}{315c} \\ c^{\frac{3}{2}}\left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((a**2*c**2*sqrt(c + d*x**2)/(5*d) + 2*a**2*c*x**2*sqrt(c + d*x**2
)/5 + a**2*d*x**4*sqrt(c + d*x**2)/5 - 4*a*b*c**3*sqrt(c + d*x**2)/(35*d**2
) + 2*a*b*c**2*x**2*sqrt(c + d*x**2)/(35*d) + 16*a*b*c*x**4*sqrt(c + d*x**2
)/35 + 2*a*b*d*x**6*sqrt(c + d*x**2)/7 + 8*b**2*c**4*sqrt(c + d*x**2)/(315*
d**3) - 4*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**2) + b**2*c**2*x**4*sqrt(
c + d*x**2)/(105*d) + 10*b**2*c*x**6*sqrt(c + d*x**2)/63 + b**2*d*x**8*sqrt
(c + d*x**2)/9, Ne(d, 0)), (c**(3/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/
6), True))
```

Giac [B] time = 1.12525, size = 315, normalized size = 4.09

$$\frac{105\left(dx^2+c\right)^{\frac{3}{2}}a^2c+21\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2+\frac{42\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)abc}{d}+\frac{3\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2\right.\right.}{d^2}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(105*(d*x^2 + c)^(3/2)*a^2*c + 21*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c
)^(3/2)*c)*a^2 + 42*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a*b*c/d +
 3*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^
2)*b^2*c/d^2 + 6*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2
 + c)^(3/2)*c^2)*a*b/d + (35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c +
189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2/d^2)/d
```

3.617 $\int (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

Optimal. Leaf size=196

$$\frac{x(c+dx^2)^{3/2}(48a^2d^2-16abcd+3b^2c^2)}{192d^2}+\frac{cx\sqrt{c+dx^2}(48a^2d^2-16abcd+3b^2c^2)}{128d^2}+\frac{c^2(48a^2d^2-16abcd+3b^2c^2)\tanh}{128d^{5/2}}$$

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*d^2) + ((3
*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2))/(192*d^2) - (b*(3*
b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(128*d^(5/2))
```

Rubi [A] time = 0.124054, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x(c+dx^2)^{3/2}(48a^2d^2-16abcd+3b^2c^2)}{192d^2}+\frac{cx\sqrt{c+dx^2}(48a^2d^2-16abcd+3b^2c^2)}{128d^2}+\frac{c^2(48a^2d^2-16abcd+3b^2c^2)\tanh}{128d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*d^2) + ((3
*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2))/(192*d^2) - (b*(3*
b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(128*d^(5/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx = \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d} + \frac{\int \left(c+dx^2\right)^{3/2}\left(-a(bc-8ad)-b(3bc-10ad)x^2\right) dx}{8d}$$

$$= -\frac{b(3bc-10ad)x\left(c+dx^2\right)^{5/2}}{48d^2} + \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d} - \frac{(-bc(3bc-10ad)+6ad(bc}{48}$$

$$= \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{192d^2} - \frac{b(3bc-10ad)x\left(c+dx^2\right)^{5/2}}{48d^2} + \frac{bx\left(a+bx^2\right)}{8}$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{192d^2} -$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{192d^2} -$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{192d^2} -$$

Mathematica [A] time = 0.0923715, size = 159, normalized size = 0.81

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(5c+2dx^2\right)+16abd\left(3c^2+14cdx^2+8d^2x^4\right)+b^2\left(6c^2dx^2-9c^3+72cd^2x^4+48d^3x^6\right)\right)+3c^2\left(48\right.}{384d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(5*c + 2*d*x^2) + 16*a*b*d*(3*c^2 +
14*c*d*x^2 + 8*d^2*x^4) + b^2*(-9*c^3 + 6*c^2*d*x^2 + 72*c*d^2*x^4 + 48*d^3
*x^6)) + 3*c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]])/(384*d^(5/2))
```

Maple [A] time = 0.008, size = 249, normalized size = 1.3

$$\frac{b^2x^3}{8\,d}\left(dx^2+c\right)^{\frac{5}{2}} - \frac{b^2cx}{16\,d^2}\left(dx^2+c\right)^{\frac{5}{2}} + \frac{b^2c^2x}{64\,d^2}\left(dx^2+c\right)^{\frac{3}{2}} + \frac{3\,b^2c^3x}{128\,d^2}\sqrt{dx^2+c} + \frac{3\,b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}} + \frac{abx}{3\,d}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*b^2*x^3*(d*x^2+c)^(5/2)/d-1/16*b^2*c/d^2*x*(d*x^2+c)^(5/2)+1/64*b^2*c^2
/d^2*x*(d*x^2+c)^(3/2)+3/128*b^2*c^3/d^2*x*(d*x^2+c)^(1/2)+3/128*b^2*c^4/d^
```

```
(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)/d-1/12*a*b*c/
d*x*(d*x^2+c)^(3/2)-1/8*a*b*c^2/d*x*(d*x^2+c)^(1/2)-1/8*a*b*c^3/d^(3/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a^2*x*(d*x^2+c)^(3/2)+3/8*a^2*c*x*(d*x^2+c)^
(1/2)+3/8*a^2*c^2/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8286, size = 776, normalized size = 3.96

$$\left[\frac{3\left(3b^2c^4-16abc^3d+48a^2c^2d^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(48b^2d^4x^7+8\left(9b^2cd^3+16abd^4\right)x^5+2\left(3b\right.\right.}{768d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 1
6*a*b*d^4)*x^5 + 2*(3*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*
b^2*c^3*d - 16*a*b*c^2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/384*
(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - (48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 16*a*b*d^4)*x^5 + 2*(3
*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*b^2*c^3*d - 16*a*b*c^
2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [B] time = 26.2115, size = 440, normalized size = 2.24

$$\frac{a^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{a^2d^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^{\frac{5}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{17abc^{\frac{3}{2}}x^3}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{11ab\sqrt{}}{12\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + a**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c
)) + 3*a**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(
d)*x/sqrt(c))/(8*sqrt(d)) + a**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 a*b*c**(5/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 17*a*b*c**(3/2)*x**3/(24*sqrt(1
+ d*x**2/c)) + 11*a*b*sqrt(c)*d*x**5/(12*sqrt(1 + d*x**2/c)) - a*b*c**3*asi
nh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a*b*d**2*x**7/(3*sqrt(c)*sqrt(1 + d*x*
*2/c)) - 3*b**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x*
```

```
*3/(128*d*sqrt(1 + d*x**2/c)) + 13*b**2*c**(3/2)*x**5/(64*sqrt(1 + d*x**2/c
)) + 5*b**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c)) + 3*b**2*c**4*asinh(sqrt
(d)*x/sqrt(c))/(128*d**(5/2)) + b**2*d**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c
))
```

Giac [A] time = 1.11977, size = 236, normalized size = 1.2

$$\frac{1}{384}\left(2\left(4\left(6\,b^2dx^2+\frac{9\,b^2cd^6+16\,abd^7}{d^6}\right)x^2+\frac{3\,b^2c^2d^5+112\,abcd^6+48\,a^2d^7}{d^6}\right)x^2-\frac{3\left(3\,b^2c^3d^4-16\,abc^2d^5-80\,a^2cd^6\right)}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (9*b^2*c*d^6 + 16*a*b*d^7)/d^6)*x^2 + (3*b^2*c^2
*d^5 + 112*a*b*c*d^6 + 48*a^2*d^7)/d^6)*x^2 - 3*(3*b^2*c^3*d^4 - 16*a*b*c^2
*d^5 - 80*a^2*c*d^6)/d^6)*sqrt(d*x^2 + c)*x - 1/128*(3*b^2*c^4 - 16*a*b*c^3
*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.618 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x} \, dx$

Optimal. Leaf size=111

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a*d)*(c + d
*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2) - a^2*c^(3/2)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.0936338, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a*d)*(c + d
*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2) - a^2*c^(3/2)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)(c+dx)^{3/2}}{d} + \frac{a^2(c+dx)^{3/2}}{x} + \frac{b^2(c+dx)^{5/2}}{d}\right) dx, x, x^2\right)$$
$$= -\frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x} dx, x, x^2\right)$$
$$= \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{1}{2}(a^2c)\,\text{Subst}\left(\int \frac{\sqrt{c\cdots}}{\cdots}\right.$$
$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{1}{2}(a^2c^2\cdots$$
$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{(a^2c^2)\cdots}{\cdots}$$
$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} - a^2c^{3/2}\,\text{ta}\cdots$$

Mathematica [A] time = 0.0760643, size = 110, normalized size = 0.99

$$\frac{1}{3}a^2(c+dx^2)^{3/2} + a^2c\left(\sqrt{c+dx^2} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right) + \frac{b(c+dx^2)^{5/2}(2ad-bc)}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] (a^2*(c + d*x^2)^(3/2))/3 + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(5/2))/(5*d^2)
+ (b^2*(c + d*x^2)^(7/2))/(7*d^2) + a^2*c*(Sqrt[c + d*x^2] - Sqrt[c]*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])
```

Maple [A] time = 0.01, size = 115, normalized size = 1.

$$\frac{b^2x^2}{7d}(dx^2+c)^{\frac{5}{2}} - \frac{2b^2c}{35d^2}(dx^2+c)^{\frac{5}{2}} + \frac{2ab}{5d}(dx^2+c)^{\frac{5}{2}} + \frac{a^2}{3}(dx^2+c)^{\frac{3}{2}} - a^2c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right) + a^2c\sqrt{dx^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x)
```

```
[Out] 1/7*b^2*x^2*(d*x^2+c)^(5/2)/d-2/35*b^2*c/d^2*(d*x^2+c)^(5/2)+2/5*a*b*(d*x^2
+c)^(5/2)/d+1/3*a^2*(d*x^2+c)^(3/2)-a^2*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)
^(1/2))/x)+a^2*c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45481, size = 648, normalized size = 5.84

$$\left[\frac{105\,a^2c^{\frac{3}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\left(4\,b^2cd^2+7\,abd^3\right)x^4+\left(3\,b^2c^2d+\right.\right.}{210\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^2*c^(3/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6*(4*
b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*x^2)
*sqrt(d*x^2 + c))/d^2, 1/105*(105*a^2*sqrt(-c)*c*d^2*arctan(sqrt(-c)/sqrt(d
*x^2 + c)) + (15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6
*(4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*
x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A] time = 59.4763, size = 109, normalized size = 0.98

$$\frac{a^2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+a^2c\sqrt{c+dx^2}+\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3}+\frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d^2}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(4abd-2b^2c\right)}{10d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

```
[Out] a**2*c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c*sqrt(c + d*x**2
) + a**2*(c + d*x**2)**(3/2)/3 + b**2*(c + d*x**2)**(7/2)/(7*d**2) + (c + d
*x**2)**(5/2)*(4*a*b*d - 2*b**2*c)/(10*d**2)
```

Giac [A] time = 1.12203, size = 163, normalized size = 1.47

$$\frac{a^2c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2d^{12}-21\left(dx^2+c\right)^{\frac{5}{2}}b^2cd^{12}+42\left(dx^2+c\right)^{\frac{5}{2}}abd^{13}+35\left(dx^2+c\right)^{\frac{3}{2}}a^2d^{14}+105\sqrt{dx^2}}{105\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/105*(15*(d*x^2 + c)^(
7/2)*b^2*d^12 - 21*(d*x^2 + c)^(5/2)*b^2*c*d^12 + 42*(d*x^2 + c)^(5/2)*a*b*
d^13 + 35*(d*x^2 + c)^(3/2)*a^2*d^14 + 105*sqrt(d*x^2 + c)*a^2*c*d^14)/d^14
```

3.619 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=175

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\left(c+dx^2\right)^{3/2}\left(b^2c^2-12ad(2ad+bc)\right)}{24cd}-\frac{x\sqrt{c+dx^2}\left(b^2c^2\right.}{}$$

```
[Out] -((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/(16*d) - ((b^2*c^2 -
12*a*d*(b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c*d) - (a^2*(c + d*x^2)^(5/2
))/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) - (c*(b^2*c^2 - 12*a*d*(b*c + 2*
a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(3/2))
```

Rubi [A] time = 0.118555, antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\sqrt{c+dx^2}\left(b^2c^2-12ad(2ad+bc)\right)}{16d}-\frac{1}{24}x\left(c+dx^2\right)^{3/2}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] -((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/(16*d) - (((b^2*c)/d
- (12*a*(b*c + 2*a*d))/c)*x*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))
/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) - (c*(b^2*c^2 - 12*a*d*(b*c + 2*a*
d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^2}\,dx = -\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{\int \left(2a(bc+2ad)+b^2cx^2\right)\left(c+dx^2\right)^{3/2}\,dx}{c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(bc+2ad)\right)\int \left(c+dx^2\right)^{3/2}\,dx}{6cd}$$

$$= -\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2\right.}{}$$

$$= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(\right.}{}$$

$$= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(\right.}{}$$

$$= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(\right.}{}$$

Mathematica [A] time = 0.14645, size = 135, normalized size = 0.77

$$\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2+20abcd+b^2c^2\right)}{16d}-\frac{a^2c}{x}+\frac{1}{24}bx^3(12ad+7bc)+\frac{1}{6}b^2dx^5\right)-\frac{c\left(-24a^2d^2-12abcd+b^2c^2\right)\log\left(\sqrt{d}\right.}{16d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c)/x) + ((b^2*c^2 + 20*a*b*c*d + 8*a^2*d^2)*x)/(16*
d) + (b*(7*b*c + 12*a*d)*x^3)/24 + (b^2*d*x^5)/6) - (c*(b^2*c^2 - 12*a*b*c*
d - 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(16*d^(3/2))
```

Maple [A] time = 0.012, size = 221, normalized size = 1.3

$$\frac{b^2x}{6\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2cx}{24\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2c^2x}{16\,d}\sqrt{dx^2+c}-\frac{b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+\frac{abx}{2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,abcx}{4}\sqrt{dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x)
```

```
[Out] 1/6*b^2*x*(d*x^2+c)^(5/2)/d-1/24*b^2*c/d*x*(d*x^2+c)^(3/2)-1/16*b^2*c^2/d*x
*(d*x^2+c)^(1/2)-1/16*b^2*c^3/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a*b
```

```
*x*(d*x^2+c)^(3/2)+3/4*a*b*c*x*(d*x^2+c)^(1/2)+3/4*a*b*c^2/d^(1/2)*ln(x*d^(
1/2)+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(5/2)/c/x+a^2*d/c*x*(d*x^2+c)^(3/2)+3/2
*a^2*d*x*(d*x^2+c)^(1/2)+3/2*a^2*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38773, size = 660, normalized size = 3.77

$$\left[-\frac{3\left(b^2c^3-12\,abc^2d-24\,a^2cd^2\right)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(8\,b^2d^3x^6-48\,a^2cd^2+2\left(7\,b^2cd^2+12\,abd^3\right)x\right.}{96\,d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*sqrt(d)*x*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^2*d^3*x^6 - 48*a^2*c*d^2 + 2*(7*b
^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(d^2*x), 1/48*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*s
qrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^6 - 48*a^2*c*d^
2 + 2*(7*b^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*
d^3)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

Sympy [B] time = 16.6259, size = 367, normalized size = 2.1

$$-\frac{a^2c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{a^2\sqrt{c}dx\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{a^2\sqrt{c}dx}{\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2}+abc^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}+\frac{abc^{\frac{3}{2}}x}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}dx^3}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

```
[Out] -a**2*c**(3/2)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)
/2 - a**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + 3*a**2*c*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c))/2 + a*b*c**(3/2)*x*sqrt(1 + d*x**2/c) + a*b*c**(3/2)*x/(4*sqrt(1
+ d*x**2/c)) + 3*a*b*sqrt(c)*d*x**3/(4*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asi
nh(sqrt(d)*x/sqrt(c))/(4*sqrt(d)) + a*b*d**2*x**5/(2*sqrt(c)*sqrt(1 + d*x**
2/c)) + b**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*b**2*c**(3/2)*x**3/(
48*sqrt(1 + d*x**2/c)) + 11*b**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - b
**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + b**2*d**2*x**7/(6*sqrt(c)
*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.13756, size = 234, normalized size = 1.34

$$\frac{2\,a^2c^2\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{48}\left(2\left(4\,b^2dx^2+\frac{7\,b^2cd^4+12\,abd^5}{d^4}\right)x^2+\frac{3\left(b^2c^2d^3+20\,abcd^4+8\,a^2d^5\right)}{d^4}\right)\sqrt{dx^2+cx}+\frac{\left(b^2c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/48*(2*(4*b^2*d*
x^2 + (7*b^2*c*d^4 + 12*a*b*d^5)/d^4)*x^2 + 3*(b^2*c^2*d^3 + 20*a*b*c*d^4 +
 8*a^2*d^5)/d^4)*sqrt(d*x^2 + c)*x + 1/32*(b^2*c^3*sqrt(d) - 12*a*b*c^2*d^(
3/2) - 24*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.620 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=136

$$-\frac{a^2 (c+dx^2)^{5/2}}{2cx^2}+\frac{a (c+dx^2)^{3/2} (3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}(3ad+4bc)-\frac{1}{2}a\sqrt{c}(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2 (c+$$

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d*x^2)^(3/2
))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*x^2)^(5/2))/(2*c*x^2
) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.108765, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2 (c+dx^2)^{5/2}}{2cx^2}+\frac{a (c+dx^2)^{3/2} (3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}(3ad+4bc)-\frac{1}{2}a\sqrt{c}(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2 (c+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d*x^2)^(3/2
))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*x^2)^(5/2))/(2*c*x^2
) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+3ad)+b^2cx\right)(c+dx)^{3/2}}{x}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{(a(4bc+3ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, x^2\right)}{4c}$$

$$= \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{1}{4}(a(4bc+3ad))\,\text{Subst}$$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/}}{2cx^2}$$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/}}{2cx^2}$$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/}}{2cx^2}$$

Mathematica [A] time = 0.087984, size = 108, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(-15a^2d\left(c-2dx^2\right)+20abdx^2\left(4c+dx^2\right)+6b^2x^2\left(c+dx^2\right)^2\right)}{30dx^2} - \frac{1}{2}a\sqrt{c}(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d*(c - 2*d*x^2) + 6*b^2*x^2*(c + d*x^2)^2 + 20*a*
b*d*x^2*(4*c + d*x^2)))/(30*d*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqr
```

```
t[c + d*x^2]/Sqrt[c]])/2
```

Maple [A] time = 0.011, size = 161, normalized size = 1.2

$$\frac{b^2}{5d}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2ab}{3}\left(dx^2+c\right)^{\frac{3}{2}}-2ab\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)c^{3/2}+2ab\sqrt{dx^2+c}c-\frac{a^2}{2cx^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d}{2c}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x)
```

```
[Out] 1/5*b^2*(d*x^2+c)^(5/2)/d+2/3*a*b*(d*x^2+c)^(3/2)-2*a*b*ln((2*c+2*c^(1/2)*(
d*x^2+c)^(1/2))/x)*c^(3/2)+2*a*b*(d*x^2+c)^(1/2)*c-1/2*a^2*(d*x^2+c)^(5/2)/
c/x^2+1/2*a^2*d/c*(d*x^2+c)^(3/2)-3/2*a^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^
2+c)^(1/2))/x)+3/2*a^2*d*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41225, size = 610, normalized size = 4.49

$$\left[\frac{15\left(4abcd+3a^2d^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(6b^2d^2x^6+4\left(3b^2cd+5abd^2\right)x^4-15a^2cd+2\left(3b^2c^2+40abcd\right.\right.}{60dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(c) + 2*c)/x^2) + 2*(6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 -
 15*a^2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(d*x^2), 1/30*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqr
t(d*x^2 + c)) + (6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 - 15*a^2*c*d
 + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

Sympy [A] time = 40.1501, size = 303, normalized size = 2.23

$$-\frac{3a^2\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2}-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{a^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-2abc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{2abc^2}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abc\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

```
[Out] -3*a**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c*sqrt(d)*sqrt(c/(d*x
**2) + 1)/(2*x) + a**2*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + a**2*d**(3/2)*x
/sqrt(c/(d*x**2) + 1) - 2*a*b*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c
**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c*sqrt(d)*x/sqrt(c/(d*x**2) +
1) + 2*a*b*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*
d), True)) + b**2*c*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3
/2)/(3*d), True)) + b**2*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) +
c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(
c)*x**4/4, True))
```

Giac [A] time = 1.13866, size = 170, normalized size = 1.25

$$\frac{6\left(dx^2+c\right)^{\frac{5}{2}}b^2+20\left(dx^2+c\right)^{\frac{3}{2}}abd+60\sqrt{dx^2+c}abcd+30\sqrt{dx^2+c}a^2d^2-\frac{15\sqrt{dx^2+c}a^2cd}{x^2}+\frac{15\left(4abc^2d+3a^2cd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(d*x^2 + c)^(5/2)*b^2 + 20*(d*x^2 + c)^(3/2)*a*b*d + 60*sqrt(d*x^2
+ c)*a*b*c*d + 30*sqrt(d*x^2 + c)*a^2*d^2 - 15*sqrt(d*x^2 + c)*a^2*c*d/x^2
+ 15*(4*a*b*c^2*d + 3*a^2*c*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c))
/d
```

3.621 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4} dx$

Optimal. Leaf size=184

$$-\frac{a^2 \left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{x \left(c+dx^2\right)^{3/2} \left(8ad(ad+3bc)+3b^2c^2\right)}{12c^2}+\frac{x\sqrt{c+dx^2} \left(8ad(ad+3bc)+3b^2c^2\right)}{8c}+\frac{\left(8ad(ad+3bc)+3b^2c\right.}{8\sqrt{}}$$

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3*b^2*c^2 +
 8*a*d*(3*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - (a^2*(c + d*x^2)^(5/2
))/(3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)^(5/2))/(3*c^2*x) + ((3*b^2*c^
2 + 8*a*d*(3*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rubi [A] time = 0.129846, antiderivative size = 181, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2 \left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{1}{12}x \left(c+dx^2\right)^{3/2} \left(\frac{8ad(ad+3bc)}{c^2}+3b^2\right)+\frac{x\sqrt{c+dx^2} \left(8ad(ad+3bc)+3b^2c^2\right)}{8c}+\frac{\left(8ad(ad+3bc)+3b\right.}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3*b^2 + (8*
a*d*(3*b*c + a*d))/c^2)*x*(c + d*x^2)^(3/2))/12 - (a^2*(c + d*x^2)^(5/2))/(
3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)^(5/2))/(3*c^2*x) + ((3*b^2*c^2 +
8*a*d*(3*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4} dx = -\frac{a^2 (c+dx^2)^{5/2}}{3cx^3} + \frac{\int \frac{(2a(3bc+ad)+3b^2cx^2)(c+dx^2)^{3/2}}{x^2} dx}{3c}$$

$$= -\frac{a^2 (c+dx^2)^{5/2}}{3cx^3} - \frac{2a(3bc+ad)(c+dx^2)^{5/2}}{3c^2x} - \frac{1}{3}\left(-3b^2 - \frac{8ad(3bc+ad)}{c^2}\right)\int (c+dx^2)$$

$$= \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x(c+dx^2)^{3/2} - \frac{a^2 (c+dx^2)^{5/2}}{3cx^3} - \frac{2a(3bc+ad)(c+dx^2)^{5/2}}{3c^2x}$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x(c+dx^2)^{3/2} - \frac{a^2(}{}$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x(c+dx^2)^{3/2} - \frac{a^2(}{}$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right)x(c+dx^2)^{3/2} - \frac{a^2(}{}$$

Mathematica [A] time = 0.0916018, size = 118, normalized size = 0.64

$$\frac{1}{24}\left(\frac{3(8a^2d^2+24abcd+3b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}} + \frac{\sqrt{c+dx^2}\left(-8a^2c+3bx^4(8ad+5bc)-16ax^2(2ad+3bc)+\right.}{x^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*c - 16*a*(3*b*c + 2*a*d)*x^2 + 3*b*(5*b*c + 8*a*d
)*x^4 + 6*b^2*d*x^6))/x^3 + (3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*Log[d*x
 + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/24
```

Maple [A] time = 0.011, size = 241, normalized size = 1.3

$$\frac{xb^2}{4}(dx^2+c)^{\frac{3}{2}} + \frac{3b^2cx}{8}\sqrt{dx^2+c} + \frac{3b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}} - \frac{a^2}{3cx^3}(dx^2+c)^{\frac{5}{2}} - \frac{2a^2d}{3c^2x}(dx^2+c)^{\frac{5}{2}} + \frac{2a^2d^2x}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x)
```

```
[Out] 1/4*x*b^2*(d*x^2+c)^(3/2)+3/8*b^2*c*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2))-1/3*a^2*(d*x^2+c)^(5/2)/c/x^3-2/3*a^2*d/c^2/x*
(d*x^2+c)^(5/2)+2/3*a^2*d^2/c^2*x*(d*x^2+c)^(3/2)+a^2*d^2/c*x*(d*x^2+c)^(1/
2)+a^2*d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(5/2)+2*a*
b*d/c*x*(d*x^2+c)^(3/2)+3*a*b*d*x*(d*x^2+c)^(1/2)+3*a*b*d^(1/2)*c*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38383, size = 603, normalized size = 3.28

$$\left[\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(6\,b^2d^2x^6+3\left(5\,b^2cd+8\,abd^2\right)x^4-8\,a^2cd-16\right.}{48\,dx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^3*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^
2)*x^4 - 8*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^
3), -1/24*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^3*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^2)*x^4 - 8
*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

Sympy [B] time = 10.932, size = 352, normalized size = 1.91

$$-\frac{a^2\sqrt{c}d}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+a^2d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{a^2d^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}dx\sqrt{1+\frac{dx^2}{c}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

```
[Out] -a**2*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1
)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + a**2*d**(3/2)*asinh(sqr
t(d)*x/sqrt(c)) - a**2*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 2*a*b*c**(3/2)
/(x*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*d*x*sqrt(1 + d*x**2/c) - 2*a*b*sqrt(c
)*d*x/sqrt(1 + d*x**2/c) + 3*a*b*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + b**2*
c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + b**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c)) +
3*b**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*b**2*c**2*asinh(sqrt(d)*x/
```

```
sqrt(c))/(8*sqrt(d)) + b**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.15275, size = 354, normalized size = 1.92

$$\frac{1}{8}\left(2\,b^2dx^2+\frac{5\,b^2cd^2+8\,abd^3}{d^2}\right)\sqrt{dx^2+c}x-\frac{\left(3\,b^2c^2\sqrt{d}+24\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,d}+\frac{4\left(3\left(\sqrt{d}x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d*x^2 + (5*b^2*c*d^2 + 8*a*b*d^3)/d^2)*sqrt(d*x^2 + c)*x - 1/16*
(3*b^2*c^2*sqrt(d) + 24*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x - sqr
t(d*x^2 + c))^2)/d + 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*sqrt(d)
 + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a*b*c^3*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(
3/2) + 3*a*b*c^4*sqrt(d) + 2*a^2*c^3*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c)
)^2 - c)^3
```

3.622 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{a^2(c+dx^2)^{5/2}}{4cx^4}+\frac{(c+dx^2)^{3/2}(3ad(ad+8bc)+8b^2c^2)}{24c^2}+\frac{\sqrt{c+dx^2}(3ad(ad+8bc)+8b^2c^2)}{8c}-\frac{(3ad(ad+8bc)+8b^2c^2)}{8\sqrt{c}}$$

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b^2*c^2 + 3
*a*d*(8*b*c + a*d))*(c + d*x^2)^(3/2))/(24*c^2) - (a^2*(c + d*x^2)^(5/2))/(
4*c*x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2))/(8*c^2*x^2) - ((8*b^2*c^2 +
3*a*d*(8*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rubi [A] time = 0.207048, antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2(c+dx^2)^{5/2}}{4cx^4}+\frac{1}{24}(c+dx^2)^{3/2}\left(\frac{3ad(ad+8bc)}{c^2}+8b^2\right)+\frac{\sqrt{c+dx^2}(3ad(ad+8bc)+8b^2c^2)}{8c}-\frac{(3ad(ad+8bc)+8b^2c^2}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b^2 + (3*a*
d*(8*b*c + a*d))/c^2)*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))/(4*c*
x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2))/(8*c^2*x^2) - ((8*b^2*c^2 + 3*a*
d*(8*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+ad)+2b^2cx\right)(c+dx)^{3/2}}{x^2}\,dx, x, x^2\right)}{4c}\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4} - \frac{a(8bc+ad)\left(c+dx^2\right)^{5/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\text{Subst}\left(\int \frac{(c
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} - \frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4} - \frac{a(8bc+ad)\left(c+dx^2\right)^{5/2}}{8c^2x^2} +\\
&= \frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} - \frac{a^2\left(c-\right.}{4}\\
&= \frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} - \frac{a^2\left(c-\right.}{4}\\
&= \frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} - \frac{a^2\left(c-\right.}{4}
\end{aligned}$$

Mathematica [A] time = 0.0969294, size = 116, normalized size = 0.64

$$\frac{1}{24}\left(\frac{\sqrt{c+dx^2}\left(-3a^2\left(2c+5dx^2\right)-24abx^2\left(c-2dx^2\right)+8b^2x^4\left(4c+dx^2\right)\right)}{x^4} - \frac{3\left(3a^2d^2+24abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c}}\right.}{\sqrt{c}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-24*a*b*x^2*(c - 2*d*x^2) + 8*b^2*x^4*(4*c + d*x^2) - 3*
a^2*(2*c + 5*d*x^2)))/x^4 - (3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/24
```

Maple [A] time = 0.011, size = 256, normalized size = 1.4

$$\frac{b^2}{3}\left(dx^2+c\right)^{\frac{3}{2}}-b^2c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)+b^2\sqrt{dx^2+c}c-\frac{a^2}{4cx^4}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2d}{8c^2x^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d^2}{8c^2}\left(dx^2+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x)
```

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)-b^2*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+b
^2*(d*x^2+c)^(1/2)*c-1/4*a^2*(d*x^2+c)^(5/2)/c/x^4-1/8*a^2*d/c^2/x^2*(d*x^2
+c)^(5/2)+1/8*a^2*d^2/c^2*(d*x^2+c)^(3/2)-3/8*a^2*d^2/c^(1/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)+3/8*a^2*d^2/c*(d*x^2+c)^(1/2)-a*b/c/x^2*(d*x^2+c)^
(5/2)+a*b*d/c*(d*x^2+c)^(3/2)-3*a*b*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)+3*a*b*d*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51112, size = 605, normalized size = 3.34

$$\left[\frac{3\left(8b^2c^2+24abcd+3a^2d^2\right)\sqrt{c}x^4\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(8b^2cdx^6+16\left(2b^2c^2+3abcd\right)x^4-6a^2c^2-3\left(8abc^2+\right.\right.}{48cx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a
*b*c*d)*x^4 - 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(
c*x^4), 1/24*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^4*arctan(sq
rt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a*b*c*d)*x^4 -
 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^4)]
```

Sympy [A] time = 89.4264, size = 332, normalized size = 1.83

$$-\frac{a^2c^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2c\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{a^2d^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8\sqrt{c}}-3ab\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)-\frac{abc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

```
[Out] -a**2*c**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c*sqrt(d)/(8*x**3
*sqrt(c/(d*x**2) + 1)) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*
*(3/2)/(8*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*d**2*asinh(sqrt(c)/(sqrt(d)*x))/
(8*sqrt(c)) - 3*a*b*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x)) - a*b*c*sqrt(d)*sq
rt(c/(d*x**2) + 1)/x + 2*a*b*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a*b*d**
(3/2)*x/sqrt(c/(d*x**2) + 1) - b**2*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + b
**2*c**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c*sqrt(d)*x/sqrt(c/(d*x**2
) + 1) + b**2*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/
(3*d), True))
```

Giac [A] time = 1.16235, size = 246, normalized size = 1.36

$$\frac{8\left(dx^2+c\right)^{\frac{3}{2}}b^2d+24\sqrt{dx^2+c}b^2cd+48\sqrt{dx^2+c}abd^2+\frac{3\left(8b^2c^2d+24abcd^2+3a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\left(8\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\sqrt{dx^2+}\right.}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(d*x^2 + c)^(3/2)*b^2*d + 24*sqrt(d*x^2 + c)*b^2*c*d + 48*sqrt(d*x^
2 + c)*a*b*d^2 + 3*(8*b^2*c^2*d + 24*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x
^2 + c)/sqrt(-c))/sqrt(-c) - 3*(8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^
2 + c)*a*b*c^2*d^2 + 5*(d*x^2 + c)^(3/2)*a^2*d^3 - 3*sqrt(d*x^2 + c)*a^2*c*
d^3)/(d^2*x^4))/d
```

3.623 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} dx$

Optimal. Leaf size=147

$$-\frac{a^2 (c+dx^2)^{5/2}}{5cx^5} - \frac{2ab (c+dx^2)^{5/2}}{3cx^3} - \frac{b (c+dx^2)^{3/2} (4ad+3bc)}{3cx} + \frac{bdx\sqrt{c+dx^2}(4ad+3bc)}{2c} + \frac{1}{2}b\sqrt{d}(4ad+3bc)\tanh^{-1}\Bigg($$

```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d)*(c + d*x
^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5) - (2*a*b*(c + d*x^2)
^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/2
```

Rubi [A] time = 0.0897115, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 277, 195, 217, 206}

$$-\frac{a^2 (c+dx^2)^{5/2}}{5cx^5} - \frac{2ab (c+dx^2)^{5/2}}{3cx^3} - \frac{b (c+dx^2)^{3/2} (4ad+3bc)}{3cx} + \frac{bdx\sqrt{c+dx^2}(4ad+3bc)}{2c} + \frac{1}{2}b\sqrt{d}(4ad+3bc)\tanh^{-1}\Bigg($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```


```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d)*(c + d*x
^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5) - (2*a*b*(c + d*x^2)
^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/2
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^6}\,dx &= -\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}+\frac{\int \frac{\left(10abc+5b^2cx^2\right)\left(c+dx^2\right)^{3/2}}{x^4}\,dx}{5c}\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{(b(3bc+4ad))\int \frac{\left(c+dx^2\right)^{3/2}}{x^2}\,dx}{3c}\\
&= -\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx}-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{(bd(3bc+4ad))\int \sqrt{}}{c}\\
&= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx}-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right.}{3cx^3}\\
&= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx}-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right.}{3cx^3}\\
&= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx}-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right.}{3cx^3}
\end{aligned}$$

Mathematica [A] time = 0.104643, size = 113, normalized size = 0.77

$$\frac{1}{2}b\sqrt{d}(4ad+3bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-\frac{\sqrt{c+dx^2}\left(6a^2\left(c+dx^2\right)^2+20abcx^2\left(c+4dx^2\right)+15b^2cx^4\left(2c-dx^2\right)\right)}{30cx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c*x^4*(2*c - d*x^2) + 6*a^2*(c + d*x^2)^2 + 20*a*
b*c*x^2*(c + 4*d*x^2)))/(30*c*x^5) + (b*Sqrt[d]*(3*b*c + 4*a*d)*Log[d*x + S
qrt[d]*Sqrt[c + d*x^2]])/2
```

Maple [A] time = 0.012, size = 203, normalized size = 1.4

$$-\frac{2\,ab}{3\,c{x}^{3}}\left(d{x}^{2}+c\right)^{\frac{5}{2}}-\frac{4\,dab}{3\,{c}^{2}x}\left(d{x}^{2}+c\right)^{\frac{5}{2}}+\frac{4\,ab{d}^{2}x}{3\,{c}^{2}}\left(d{x}^{2}+c\right)^{\frac{3}{2}}+2\,\frac{ab{d}^{2}x\sqrt{d{x}^{2}+c}}{c}+2\,ab{d}^{3/2}\ln\left(x\sqrt{d}+\sqrt{d{x}^{2}+c}\right)-\frac{{a}^{2}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x)
```

```
[Out] -2/3*a*b*(d*x^2+c)^(5/2)/c/x^3-4/3*a*b*d/c^2/x*(d*x^2+c)^(5/2)+4/3*a*b*d^2/
c^2*x*(d*x^2+c)^(3/2)+2*a*b*d^2/c*x*(d*x^2+c)^(1/2)+2*a*b*d^(3/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))-1/5*a^2*(d*x^2+c)^(5/2)/c/x^5-b^2/c/x*(d*x^2+c)^(5/2)+
b^2*d/c*x*(d*x^2+c)^(3/2)+3/2*b^2*d*x*(d*x^2+c)^(1/2)+3/2*b^2*d^(1/2)*c*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50456, size = 609, normalized size = 4.14

$$\left[\frac{15\left(3\,b^{2}c^{2}+4\,abcd\right)\sqrt{d}x^{5}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+2\left(15\,b^{2}cdx^{6}-2\left(15\,b^{2}c^{2}+40\,abcd+3\,a^{2}d^{2}\right)x^{4}-6\,a^{2}c^{2}-\right.}{60\,cx^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 +
 c)*sqrt(d)*x - c) + 2*(15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2
*d^2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*
x^5), -1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^
4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^5)]
```

Sympy [B] time = 7.24789, size = 304, normalized size = 2.07

$$-\frac{a^{2}c\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{5x^{4}}-\frac{2a^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{5x^{2}}-\frac{a^{2}d^{\frac{5}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{5c}-\frac{2ab\sqrt{c}d}{x\sqrt{1+\frac{dx^{2}}{c}}}-\frac{2abc\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{3x^{2}}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3}+2abd^{\frac{3}{2}}\operatorname{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

```
[Out] -a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(5*x**2) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(5*c) - 2*a*b*sqrt(
c)*d/(x*sqrt(1 + d*x**2/c)) - 2*a*b*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2)
 - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 2*a*b*d**(3/2)*asinh(sqrt(d)*x/s
```

```
qrt(c)) - 2*a*b*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)/(x*sqrt
(1 + d*x**2/c)) + b**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)/2 - b**2*sqrt(c)*d*x/
sqrt(1 + d*x**2/c) + 3*b**2*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/2
```

Giac [B] time = 1.21198, size = 549, normalized size = 3.73

$$\frac{1}{2}\sqrt{dx^2+c}b^2dx - \frac{1}{4}\left(3\,b^2c\sqrt{d} + 4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right) + \frac{2\left(15\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 b^2c^2\sqrt{d} + 60\left(\sqrt{d}x - \sqrt{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*d*x - 1/4*(3*b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sq
rt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^
2*c^2*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 15*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*b^2*c^3*sqrt(d) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) + 90
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) + 220*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*a*b*c^3*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) - 140*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) + 15*b^2*c^6*sqrt(d) + 40*a*b*c^5*d^(
3/2) + 3*a^2*c^4*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.624 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{3/2}\left(ad(12bc-ad)+24b^2c^2\right)}{48c^2x^2}+\frac{d\sqrt{c+dx^2}\left(ad(12bc-ad)+24b^2c^2\right)}{16c^2}-\frac{d\left(ad(12bc-ad)+24\right.}{16c}$$

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) - ((24*b^2*c
^2 + a*d*(12*b*c - a*d))*(c + d*x^2)^(3/2))/(48*c^2*x^2) - (a^2*(c + d*x^2)
^(5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*x^2)^(5/2))/(24*c^2*x^4) - (d*
(24*b^2*c^2 + a*d*(12*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(
3/2))
```

Rubi [A] time = 0.222079, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 89, 78, 47, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{3/2}\left(\frac{ad(12bc-ad)}{c^2}+24b^2\right)}{48x^2}+\frac{d\sqrt{c+dx^2}\left(ad(12bc-ad)+24b^2c^2\right)}{16c^2}-\frac{d\left(ad(12bc-ad)+24b^2c^2\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) - ((24*b^2 +
 (a*d*(12*b*c - a*d))/c^2)*(c + d*x^2)^(3/2))/(48*x^2) - (a^2*(c + d*x^2)^(
5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*x^2)^(5/2))/(24*c^2*x^4) - (d*(2
4*b^2*c^2 + a*d*(12*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/
2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc-ad)+3b^2cx\right)(c+dx)^{3/2}}{x^3}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6} - \frac{a(12bc-ad)\left(c+dx^2\right)^{5/2}}{24c^2x^4} + \frac{1}{48}\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\text{Subst}\left(\int\right.$$

$$= -\frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2} - \frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6} - \frac{a(12bc-ad)\left(c+dx^2\right)^{5/2}}{24c^2x^4} + \frac{1}{32}\left(\right.$$

$$= \frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2} - \frac{a^2\left(c+dx^2\right.}{6cx^6}$$

$$= \frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2} - \frac{a^2\left(c+dx^2\right.}{6cx^6}$$

$$= \frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2} - \frac{a^2\left(c+dx^2\right.}{6cx^6}$$

Mathematica [C] time = 0.0467302, size = 92, normalized size = 0.49

$$-\frac{\left(c+dx^2\right)^{5/2}\left(dx^6\left(a^2d^2-12abcd-24b^2c^2\right){}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{dx^2}{c}+1\right)+5ac^2\left(4ac-adx^2+12bcx^2\right)\right)}{120c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] -((c + d*x^2)^(5/2)*(5*a*c^2*(4*a*c + 12*b*c*x^2 - a*d*x^2) + d*(-24*b^2*c^
2 - 12*a*b*c*d + a^2*d^2)*x^6*Hypergeometric2F1[2, 5/2, 7/2, 1 + (d*x^2)/c]
))/(120*c^4*x^6)
```

Maple [B] time = 0.013, size = 335, normalized size = 1.8

$$-\frac{a^2}{6\,cx^6}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d}{24\,c^2x^4}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d^2}{48\,c^3x^2}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2d^3}{48\,c^3}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(5/2)/c/x^6+1/24*a^2*d/c^2/x^4*(d*x^2+c)^(5/2)+1/48*a^2*
d^2/c^3/x^2*(d*x^2+c)^(5/2)-1/48*a^2*d^3/c^3*(d*x^2+c)^(3/2)+1/16*a^2*d^3/c
^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/16*a^2*d^3/c^2*(d*x^2+c)^(1/
2)-1/2*a*b/c/x^4*(d*x^2+c)^(5/2)-1/4*a*b*d/c^2/x^2*(d*x^2+c)^(5/2)+1/4*a*b*
d^2/c^2*(d*x^2+c)^(3/2)-3/4*a*b*d^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/
2))/x)+3/4*a*b*d^2/c*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(5/2)+1/2*b^2*
d/c*(d*x^2+c)^(3/2)-3/2*b^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
+3/2*b^2*d*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4945, size = 674, normalized size = 3.6

$$\left[-\frac{3\left(24\,b^2c^2d+12\,abcd^2-a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(48\,b^2c^2dx^6-8\,a^2c^3-3\left(8\,b^2c^3+20\,abc^2d+a^2cd^2\right)x^4\right.}{96\,c^2x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(48*b^2*c^2*d*x^6 - 8*a^2*c^3 -
3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 + 7*a^2*c^2*d)
*x^2)*sqrt(d*x^2 + c))/(c^2*x^6), 1/48*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^
2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (48*b^2*c^2*d*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 +
7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^6)]
```

Sympy [B] time = 118.042, size = 367, normalized size = 1.96

$$-\frac{a^2c^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{11a^2c\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{17a^2d^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{5}{2}}}{16cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{3}{2}}}-\frac{abc^2}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3ab}{4x^3\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

```
[Out] -a**2*c**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 11*a**2*c*sqrt(d)/(24*x*
*5*sqrt(c/(d*x**2) + 1)) - 17*a**2*d**(3/2)/(48*x**3*sqrt(c/(d*x**2) + 1))
- a**2*d**(5/2)/(16*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**3*asinh(sqrt(c)/(sq
rt(d)*x))/(16*c**(3/2)) - a*b*c**2/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) -
3*a*b*c*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)*sqrt(c/(d*x**2
) + 1)/x - a*b*d**(3/2)/(4*x*sqrt(c/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(
c)/(sqrt(d)*x))/(4*sqrt(c)) - 3*b**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2
 - b**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) + b**2*c*sqrt(d)/(x*sqrt(c/(d*
x**2) + 1)) + b**2*d**(3/2)*x/sqrt(c/(d*x**2) + 1)
```

Giac [A] time = 1.16873, size = 350, normalized size = 1.87

$$\frac{48\sqrt{dx^2+c}b^2d^2+\frac{3\left(24b^2c^2d^2+12abcd^3-a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2+60\left(dx^2+c\right)^{\frac{5}{2}}abcd^3-\cdots}{\cdots}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(d*x^2 + c)*b^2*d^2 + 3*(24*b^2*c^2*d^2 + 12*a*b*c*d^3 - a^2*d
^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (24*(d*x^2 + c)^(5/2)*b
^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*
d^2 + 60*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 3
6*sqrt(d*x^2 + c)*a*b*c^3*d^3 + 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2 + c)
^(3/2)*a^2*c*d^4 - 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c*d^3*x^6))/d
```

3.625 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(11/2))/(11*d^4)
 + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rubi [A] time = 0.0897296, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(11/2))/(11*d^4)
 + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^{5/2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2(c+dx)^{5/2}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{7/2}}{d^3}-\frac{b(3bc-2ad)(}{d^3}\right.\right.\\
&= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{9/2}}{9d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{11}}{11d^4}
\end{aligned}$$

Mathematica [A] time = 0.0982566, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(143a^2d^2\left(7dx^2-2c\right)+26abd\left(8c^2-28cdx^2+63d^2x^4\right)+b^2\left(168c^2dx^2-48c^3-378cd^2x^4+693d^3x^6\right)\right)}{9009d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(143*a^2*d^2*(-2*c + 7*d*x^2) + 26*a*b*d*(8*c^2 - 28*c*d
*x^2 + 63*d^2*x^4) + b^2*(-48*c^3 + 168*c^2*d*x^2 - 378*c*d^2*x^4 + 693*d^3
*x^6)))/(9009*d^4)
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$-\frac{-693\,b^2x^6d^3-1638\,abd^3x^4+378\,b^2cd^2x^4-1001\,a^2d^3x^2+728\,abcd^2x^2-168\,b^2c^2dx^2+286\,a^2cd^2-208\,abc^2d+48}{9009\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] -1/9009*(d*x^2+c)^(7/2)*(-693*b^2*d^3*x^6-1638*a*b*d^3*x^4+378*b^2*c*d^2*x^
4-1001*a^2*d^3*x^2+728*a*b*c*d^2*x^2-168*b^2*c^2*d*x^2+286*a^2*c*d^2-208*a*
b*c^2*d+48*b^2*c^3)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.43238, size = 490, normalized size = 4.3

$$\frac{\left(693\,b^2d^6x^{12}+63\left(27\,b^2cd^5+26\,abd^6\right)x^{10}+7\left(159\,b^2c^2d^4+598\,abcd^5+143\,a^2d^6\right)x^8-48\,b^2c^6+208\,abc^5d-286\,a^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*b^2*d^6*x^12 + 63*(27*b^2*c*d^5 + 26*a*b*d^6)*x^10 + 7*(159*b^2
*c^2*d^4 + 598*a*b*c*d^5 + 143*a^2*d^6)*x^8 - 48*b^2*c^6 + 208*a*b*c^5*d -
286*a^2*c^4*d^2 + (15*b^2*c^3*d^3 + 2938*a*b*c^2*d^4 + 2717*a^2*c*d^5)*x^6
- 3*(6*b^2*c^4*d^2 - 26*a*b*c^3*d^3 - 715*a^2*c^2*d^4)*x^4 + (24*b^2*c^5*d
- 104*a*b*c^4*d^2 + 143*a^2*c^3*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 9.42471, size = 468, normalized size = 4.11

$$\begin{cases} -\frac{2a^2c^4\sqrt{c+dx^2}}{63d^2} + \frac{a^2c^3x^2\sqrt{c+dx^2}}{63d} + \frac{5a^2c^2x^4\sqrt{c+dx^2}}{21} + \frac{19a^2cdx^6\sqrt{c+dx^2}}{63} + \frac{a^2d^2x^8\sqrt{c+dx^2}}{9} + \frac{16abc^5\sqrt{c+dx^2}}{693d^3} - \frac{8abc^4x^2\sqrt{c+dx^2}}{693d^2} + \frac{2abc^3x^4\sqrt{c+dx^2}}{231d} + \\ c^{\frac{5}{2}}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c**4*sqrt(c + d*x**2)/(63*d**2) + a**2*c**3*x**2*sqrt(c
+ d*x**2)/(63*d) + 5*a**2*c**2*x**4*sqrt(c + d*x**2)/21 + 19*a**2*c*d*x**6*
sqrt(c + d*x**2)/63 + a**2*d**2*x**8*sqrt(c + d*x**2)/9 + 16*a*b*c**5*sqrt(
c + d*x**2)/(693*d**3) - 8*a*b*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + 2*a*
b*c**3*x**4*sqrt(c + d*x**2)/(231*d) + 226*a*b*c**2*x**6*sqrt(c + d*x**2)/6
93 + 46*a*b*c*d*x**8*sqrt(c + d*x**2)/99 + 2*a*b*d**2*x**10*sqrt(c + d*x**2
)/11 - 16*b**2*c**6*sqrt(c + d*x**2)/(3003*d**4) + 8*b**2*c**5*x**2*sqrt(c
+ d*x**2)/(3003*d**3) - 2*b**2*c**4*x**4*sqrt(c + d*x**2)/(1001*d**2) + 5*b
**2*c**3*x**6*sqrt(c + d*x**2)/(3003*d) + 53*b**2*c**2*x**8*sqrt(c + d*x**2
)/429 + 27*b**2*c*d*x**10*sqrt(c + d*x**2)/143 + b**2*d**2*x**12*sqrt(c + d
*x**2)/13, Ne(d, 0)), (c**(5/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8), T
rue))
```

Giac [B] time = 1.14423, size = 749, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*(3003*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a^2*c^2/d + 858
*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)
*a*b*c^2/d^2 + 858*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x
^2 + c)^(3/2)*c^2)*a^2*c/d + 143*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7
/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2*c^2/d^3
+ 572*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/
2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*a*b*c/d^2 + 143*(35*(d*x^2 + c)^(9/2) -
 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2
)*c^3)*a^2/d + 26*(315*(d*x^2 + c)^(11/2) - 1540*(d*x^2 + c)^(9/2)*c + 2970
*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2)*c^3 + 1155*(d*x^2 + c)^(3/2
)*c^4)*b^2*c/d^3 + 26*(315*(d*x^2 + c)^(11/2) - 1540*(d*x^2 + c)^(9/2)*c +
2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2)*c^3 + 1155*(d*x^2 + c)^
(3/2)*c^4)*a*b/d^2 + 5*(693*(d*x^2 + c)^(13/2) - 4095*(d*x^2 + c)^(11/2)*c
+ 10010*(d*x^2 + c)^(9/2)*c^2 - 12870*(d*x^2 + c)^(7/2)*c^3 + 9009*(d*x^2 +
 c)^(5/2)*c^4 - 3003*(d*x^2 + c)^(3/2)*c^5)*b^2/d^3)/d
```

3.626 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx$

Optimal. Leaf size=281

$$\frac{c^2x^3\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{512d^2}+\frac{c^3x\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\text{ta}}{1024d^{7/2}}$$

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^3) + (c
^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(512*d^2) + (c*
(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c + d*x^2)^(3/2))/(384*d^2) + ((40
*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c + d*x^2)^(5/2))/(320*d^2) - (b*(5*b
*c - 24*a*d)*x^3*(c + d*x^2)^(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))
/(12*d) - (c^4*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(1024*d^(7/2))
```

Rubi [A] time = 0.257678, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x^3\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{512d^2}+\frac{c^3x\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\text{ta}}{1024d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^3) + (c
^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(512*d^2) + (c*
(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c + d*x^2)^(3/2))/(384*d^2) + ((40
*a^2 + (b*c*(5*b*c - 24*a*d))/d^2)*x^3*(c + d*x^2)^(5/2))/320 - (b*(5*b*c -
 24*a*d)*x^3*(c + d*x^2)^(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))/(12
*d) - (c^4*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(7/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}dx &= \frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}+\frac{\int x^2\left(c+dx^2\right)^{5/2}\left(12a^2d-b(5bc-24ad)x^2\right)dx}{12d}\\
&= -\frac{b(5bc-24ad)x^3\left(c+dx^2\right)^{7/2}}{120d^2}+\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}+\frac{1}{40}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)\int x^2\\
&= \frac{1}{320}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{5/2}-\frac{b(5bc-24ad)x^3\left(c+dx^2\right)^{7/2}}{120d^2}+\frac{b^2x^5(c}{1}\\
&= \frac{1}{384}c\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2}+\frac{1}{320}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3(c+\\
&= \frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{384}c\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3(c+d\\
&= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{384}\\
&= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{384}\\
&= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{384}\end{aligned}$$

Mathematica [A] time = 0.146804, size = 226, normalized size = 0.8

$$\sqrt{d}x\sqrt{c+dx^2}\left(40a^2d^2\left(118c^2dx^2+15c^3+136cd^2x^4+48d^3x^6\right)+24abd\left(248c^2d^2x^4+10c^3dx^2-15c^4+336cd^3x^6+128d^4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(40*a^2*d^2*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*
x^4 + 48*d^3*x^6) + 24*a*b*d*(-15*c^4 + 10*c^3*d*x^2 + 248*c^2*d^2*x^4 + 33
6*c*d^3*x^6 + 128*d^4*x^8) + 5*b^2*(15*c^5 - 10*c^4*d*x^2 + 8*c^3*d^2*x^4 +
 432*c^2*d^3*x^6 + 640*c*d^4*x^8 + 256*d^5*x^10)) - 15*c^4*(5*b^2*c^2 - 24*
a*b*c*d + 40*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(7/2))
```

Maple [A] time = 0.014, size = 383, normalized size = 1.4

$$\frac{b^2x^5}{12\,d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{b^2cx^3}{24\,d^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{b^2c^2x}{64\,d^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{b^2c^3x}{384\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{5\,b^2c^4x}{1536\,d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,b^2c^5x}{1024\,d^3}\sqrt{dx^2+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*b^2*x^5*(d*x^2+c)^(7/2)/d-1/24*b^2*c/d^2*x^3*(d*x^2+c)^(7/2)+1/64*b^2*
c^2/d^3*x*(d*x^2+c)^(7/2)-1/384*b^2*c^3/d^3*x*(d*x^2+c)^(5/2)-5/1536*b^2*c^
4/d^3*x*(d*x^2+c)^(3/2)-5/1024*b^2*c^5/d^3*x*(d*x^2+c)^(1/2)-5/1024*b^2*c^6
/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/5*a*b*x^3*(d*x^2+c)^(7/2)/d-3/40*a
*b*c/d^2*x*(d*x^2+c)^(7/2)+1/80*a*b*c^2/d^2*x*(d*x^2+c)^(5/2)+1/64*a*b*c^3/
d^2*x*(d*x^2+c)^(3/2)+3/128*a*b*c^4/d^2*x*(d*x^2+c)^(1/2)+3/128*a*b*c^5/d^(
5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8*a^2*x*(d*x^2+c)^(7/2)/d-1/48*a^2*c/d
*x*(d*x^2+c)^(5/2)-5/192*a^2*c^2/d*x*(d*x^2+c)^(3/2)-5/128*a^2*c^3/d*x*(d*x
^2+c)^(1/2)-5/128*a^2*c^4/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.72963, size = 1131, normalized size = 4.02

$$\left[\frac{15\left(5\,b^2c^6-24\,abc^5d+40\,a^2c^4d^2\right)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(1280\,b^2d^6x^{11}+128\left(25\,b^2cd^5+24\,abd^6\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(25*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*
x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c
^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c
^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/15360*(15*(5*b^2*c^6 -
24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
```

```
+ (1280*b^2*d^6*x^11 + 128*(25*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2
*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4
 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^
4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 +
 c))/d^4]
```

Sympy [B] time = 77.3702, size = 602, normalized size = 2.14

$$\frac{5a^2c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{133a^2c^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{dx^2}{c}}}+\frac{127a^2c^{\frac{3}{2}}dx^5}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{23a^2\sqrt{c}d^2x^7}{48\sqrt{1+\frac{dx^2}{c}}}-\frac{5a^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}}+\frac{a^2d^3x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3abc^{\frac{9}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] 5*a**2*c**(7/2)*x/(128*d*sqrt(1 + d*x**2/c)) + 133*a**2*c**(5/2)*x**3/(384*
sqrt(1 + d*x**2/c)) + 127*a**2*c**(3/2)*d*x**5/(192*sqrt(1 + d*x**2/c)) + 2
3*a**2*sqrt(c)*d**2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*a**2*c**4*asinh(sqrt(d
)*x/sqrt(c))/(128*d**(3/2)) + a**2*d**3*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
 - 3*a*b*c**(9/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(7/2)*x**3/(128*
d*sqrt(1 + d*x**2/c)) + 129*a*b*c**(5/2)*x**5/(320*sqrt(1 + d*x**2/c)) + 73
*a*b*c**(3/2)*d*x**7/(80*sqrt(1 + d*x**2/c)) + 29*a*b*sqrt(c)*d**2*x**9/(40
*sqrt(1 + d*x**2/c)) + 3*a*b*c**5*asinh(sqrt(d)*x/sqrt(c))/(128*d**(5/2)) +
 a*b*d**3*x**11/(5*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(11/2)*x/(1024*d
**3*sqrt(1 + d*x**2/c)) + 5*b**2*c**(9/2)*x**3/(3072*d**2*sqrt(1 + d*x**2/c
)) - b**2*c**(7/2)*x**5/(1536*d*sqrt(1 + d*x**2/c)) + 55*b**2*c**(5/2)*x**7
/(384*sqrt(1 + d*x**2/c)) + 67*b**2*c**(3/2)*d*x**9/(192*sqrt(1 + d*x**2/c)
) + 7*b**2*sqrt(c)*d**2*x**11/(24*sqrt(1 + d*x**2/c)) - 5*b**2*c**6*asinh(s
qrt(d)*x/sqrt(c))/(1024*d**(7/2)) + b**2*d**3*x**13/(12*sqrt(c)*sqrt(1 + d*
x**2/c))
```

Giac [A] time = 1.1303, size = 358, normalized size = 1.27

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2d^2x^2+\frac{25\,b^2cd^{11}+24\,abd^{12}}{d^{10}}\right)x^2+\frac{3\left(45\,b^2c^2d^{10}+168\,abcd^{11}+40\,a^2d^{12}\right)}{d^{10}}\right)x^2+\frac{5\,b^2c^3d^9+744\,ab}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^2*x^2 + (25*b^2*c*d^11 + 24*a*b*d^12)/d^10)*x
^2 + 3*(45*b^2*c^2*d^10 + 168*a*b*c*d^11 + 40*a^2*d^12)/d^10)*x^2 + (5*b^2*
c^3*d^9 + 744*a*b*c^2*d^10 + 680*a^2*c*d^11)/d^10)*x^2 - 5*(5*b^2*c^4*d^8 -
 24*a*b*c^3*d^9 - 472*a^2*c^2*d^10)/d^10)*x^2 + 15*(5*b^2*c^5*d^7 - 24*a*b*
c^4*d^8 + 40*a^2*c^3*d^9)/d^10)*sqrt(d*x^2 + c)*x + 1/1024*(5*b^2*c^6 - 24*
a*b*c^5*d + 40*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.627 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(9
/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)
```

Rubi [A] time = 0.059001, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(9
/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)^{5/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx,x\right.\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{9/2}}{9d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [A] time = 0.0498562, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(99a^2d^2+22abd\left(7dx^2-2c\right)+b^2\left(8c^2-28cdx^2+63d^2x^4\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x^2) + b^2*(8*c^2 - 2
8*c*d*x^2 + 63*d^2*x^4)))/(693*d^3)
```

Maple [A] time = 0.003, size = 69, normalized size = 0.9

$$\frac{63\,b^2d^2x^4+154\,abd^2x^2-28\,b^2cdx^2+99\,a^2d^2-44\,cabd+8\,b^2c^2}{693\,d^3}\left(dx^2+c\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/693*(d*x^2+c)^(7/2)*(63*b^2*d^2*x^4+154*a*b*d^2*x^2-28*b^2*c*d*x^2+99*a^2
*d^2-44*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.40111, size = 389, normalized size = 5.05

$$\frac{\left(63\,b^2d^5x^{10}+7\left(23\,b^2cd^4+22\,abd^5\right)x^8+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+\left(113\,b^2c^2d^3+418\,abcd^4+99\,a^2d^5\right)x^6+3\left(b^2c^3\right.\right.}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^2*d^5*x^10 + 7*(23*b^2*c*d^4 + 22*a*b*d^5)*x^8 + 8*b^2*c^5 - 44
*a*b*c^4*d + 99*a^2*c^3*d^2 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5
)*x^6 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^4 - (4*b^2*c^4*d
 - 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 6.13842, size = 384, normalized size = 4.99

$$\begin{cases} \frac{a^2c^3\sqrt{c+dx^2}}{7d}+\frac{3a^2c^2x^2\sqrt{c+dx^2}}{7}+\frac{3a^2cdx^4\sqrt{c+dx^2}}{7}+\frac{a^2d^2x^6\sqrt{c+dx^2}}{7}-\frac{4abc^4\sqrt{c+dx^2}}{63d^2}+\frac{2abc^3x^2\sqrt{c+dx^2}}{63d}+\frac{10abc^2x^4\sqrt{c+dx^2}}{21}+\frac{38abcdx^6\sqrt{c+dx^2}}{63}+2 \\ c^{\frac{5}{2}}\left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((a**2*c**3*sqrt(c + d*x**2)/(7*d) + 3*a**2*c**2*x**2*sqrt(c + d*x
**2)/7 + 3*a**2*c*d*x**4*sqrt(c + d*x**2)/7 + a**2*d**2*x**6*sqrt(c + d*x**
2)/7 - 4*a*b*c**4*sqrt(c + d*x**2)/(63*d**2) + 2*a*b*c**3*x**2*sqrt(c + d*x
**2)/(63*d) + 10*a*b*c**2*x**4*sqrt(c + d*x**2)/21 + 38*a*b*c*d*x**6*sqrt(c
 + d*x**2)/63 + 2*a*b*d**2*x**8*sqrt(c + d*x**2)/9 + 8*b**2*c**5*sqrt(c + d
*x**2)/(693*d**3) - 4*b**2*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + b**2*c**
3*x**4*sqrt(c + d*x**2)/(231*d) + 113*b**2*c**2*x**6*sqrt(c + d*x**2)/693 +
 23*b**2*c*d*x**8*sqrt(c + d*x**2)/99 + b**2*d**2*x**10*sqrt(c + d*x**2)/11
, Ne(d, 0)), (c**(5/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6), True))
```

Giac [B] time = 1.1466, size = 564, normalized size = 7.32

$$\frac{1155\left(dx^2+c\right)^{\frac{3}{2}}a^2c^2+462\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2c+\frac{462\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)abc^2}{d}+33\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3465*(1155*(d*x^2 + c)^(3/2)*a^2*c^2 + 462*(3*(d*x^2 + c)^(5/2) - 5*(d*x^
2 + c)^(3/2)*c)*a^2*c + 462*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a
*b*c^2/d + 33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 +
c)^(3/2)*c^2)*a^2 + 33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*
(d*x^2 + c)^(3/2)*c^2)*b^2*c^2/d^2 + 132*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2
+ c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a*b*c/d + 22*(35*(d*x^2 + c)^(9/2)
 - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3
/2)*c^3)*b^2*c/d^2 + 22*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 1
89*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*a*b/d + (315*(d*x^2 +
 c)^(11/2) - 1540*(d*x^2 + c)^(9/2)*c + 2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(
d*x^2 + c)^(5/2)*c^3 + 1155*(d*x^2 + c)^(3/2)*c^4)*b^2/d^2)/d
```

3.628 $\int (a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

Optimal. Leaf size=240

$$\frac{x(c+dx^2)^{5/2}(80a^2d^2-20abcd+3b^2c^2)}{480d^2}+\frac{cx(c+dx^2)^{3/2}(80a^2d^2-20abcd+3b^2c^2)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}(80a^2d^2-20abcd}{256d^2}$$

```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(256*d^2) + (
c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(3/2))/(384*d^2) + ((
3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(5/2))/(480*d^2) - (3*b*
(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(7/2))/(10*d) + (c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(256*d^(5/2))
```

Rubi [A] time = 0.152393, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x(c+dx^2)^{5/2}(80a^2d^2-20abcd+3b^2c^2)}{480d^2}+\frac{cx(c+dx^2)^{3/2}(80a^2d^2-20abcd+3b^2c^2)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}(80a^2d^2-20abcd}{256d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```


```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(256*d^2) + (
c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(3/2))/(384*d^2) + ((
3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(5/2))/(480*d^2) - (3*b*
(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(7/2))/(10*d) + (c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(256*d^(5/2))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} dx = \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}{10d} + \frac{\int \left(c+dx^2\right)^{5/2}\left(-a(bc-10ad)-3b(bc-4ad)x^2\right) dx}{10d}$$

$$= -\frac{3b(bc-4ad)x\left(c+dx^2\right)^{7/2}}{80d^2} + \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}{10d} - \frac{(8ad(bc-10ad)-3bc(bc-}{80d^2}$$

$$= \frac{\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{5/2}}{480d^2} - \frac{3b(bc-4ad)x\left(c+dx^2\right)^{7/2}}{80d^2} + \frac{bx\left(a+bx^2\right)}{10}$$

$$= \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{384d^2} + \frac{\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{5/2}}{480d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{384d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{384d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{384d^2}$$

Mathematica [A] time = 0.122641, size = 192, normalized size = 0.8

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(80a^2d^2\left(33c^2+26cdx^2+8d^2x^4\right)+20abd\left(118c^2dx^2+15c^3+136cd^2x^4+48d^3x^6\right)+b^2\left(744c^2d^2x^4+30\right.\right.}{3840d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) +
20*a*b*d*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d^3*x^6) + b^2*(-45*c
^4 + 30*c^3*d*x^2 + 744*c^2*d^2*x^4 + 1008*c*d^3*x^6 + 384*d^4*x^8)) + 15*c
^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]
)/(3840*d^(5/2))
```

Maple [A] time = 0.008, size = 308, normalized size = 1.3

$$\frac{b^2x^3}{10\,d}\left(dx^2+c\right)^{\frac{7}{2}} - \frac{3\,b^2cx}{80\,d^2}\left(dx^2+c\right)^{\frac{7}{2}} + \frac{b^2c^2x}{160\,d^2}\left(dx^2+c\right)^{\frac{5}{2}} + \frac{b^2c^3x}{128\,d^2}\left(dx^2+c\right)^{\frac{3}{2}} + \frac{3\,b^2c^4x}{256\,d^2}\sqrt{dx^2+c} + \frac{3\,b^2c^5}{256}\ln\left(x\sqrt{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/10*b^2*x^3*(d*x^2+c)^(7/2)/d-3/80*b^2*c/d^2*x*(d*x^2+c)^(7/2)+1/160*b^2*c
^2/d^2*x*(d*x^2+c)^(5/2)+1/128*b^2*c^3/d^2*x*(d*x^2+c)^(3/2)+3/256*b^2*c^4/
d^2*x*(d*x^2+c)^(1/2)+3/256*b^2*c^5/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1
/4*a*b*x*(d*x^2+c)^(7/2)/d-1/24*a*b*c/d*x*(d*x^2+c)^(5/2)-5/96*a*b*c^2/d*x*
(d*x^2+c)^(3/2)-5/64*a*b*c^3/d*x*(d*x^2+c)^(1/2)-5/64*a*b*c^4/d^(3/2)*ln(x*
d^(1/2)+(d*x^2+c)^(1/2))+1/6*a^2*x*(d*x^2+c)^(5/2)+5/24*a^2*c*x*(d*x^2+c)^(
3/2)+5/16*a^2*c^2*x*(d*x^2+c)^(1/2)+5/16*a^2*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^
2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91466, size = 963, normalized size = 4.01

$$\left[\frac{15\left(3\,b^2c^5 - 20\,abc^4d + 80\,a^2c^3d^2\right)\sqrt{d}\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\left(384\,b^2d^5x^9 + 48\left(21\,b^2cd^4 + 20\,abd^5\right)x^7 + 8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(21*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(93*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 +
 10*(3*b^2*c^3*d^2 + 236*a*b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d
 - 20*a*b*c^3*d^2 - 176*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/3840*(15*(
3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (384*b^2*d^5*x^9 + 48*(21*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(93
*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 + 10*(3*b^2*c^3*d^2 + 236*a*
b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d - 20*a*b*c^3*d^2 - 176*a^2
*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [B] time = 50.0722, size = 537, normalized size = 2.24

$$\frac{a^2c^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2} + \frac{3a^2c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^2}{c}}} + \frac{35a^2c^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{dx^2}{c}}} + \frac{17a^2\sqrt{c}d^2x^5}{24\sqrt{1+\frac{dx^2}{c}}} + \frac{5a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16\sqrt{d}} + \frac{a^2d^3x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{5abc^{\frac{7}{2}}x}{64d\sqrt{1+\frac{dx^2}{c}}} + \frac{13}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*a**2*c**(5/2)*x/(16*sqrt(1 + d*x**
2/c)) + 35*a**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*a**2*sqrt(c)*d
**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*a**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16
*sqrt(d)) + a**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(7/2)*
x/(64*d*sqrt(1 + d*x**2/c)) + 133*a*b*c**(5/2)*x**3/(192*sqrt(1 + d*x**2/c)
) + 127*a*b*c**(3/2)*d*x**5/(96*sqrt(1 + d*x**2/c)) + 23*a*b*sqrt(c)*d**2*x
**7/(24*sqrt(1 + d*x**2/c)) - 5*a*b*c**4*asinh(sqrt(d)*x/sqrt(c))/(64*d**(3
/2)) + a*b*d**3*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) - 3*b**2*c**(9/2)*x/(25
6*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(7/2)*x**3/(256*d*sqrt(1 + d*x**2/c))
+ 129*b**2*c**(5/2)*x**5/(640*sqrt(1 + d*x**2/c)) + 73*b**2*c**(3/2)*d*x**7
/(160*sqrt(1 + d*x**2/c)) + 29*b**2*sqrt(c)*d**2*x**9/(80*sqrt(1 + d*x**2/c
)) + 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(5/2)) + b**2*d**3*x**11/
(10*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.16371, size = 298, normalized size = 1.24

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{21\,b^2cd^9+20\,abd^{10}}{d^8}\right)x^2+\frac{93\,b^2c^2d^8+340\,abcd^9+80\,a^2d^{10}}{d^8}\right)x^2+\frac{5\left(3\,b^2c^3d^7+236\,abc^2d^8\right.}{d^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (21*b^2*c*d^9 + 20*a*b*d^10)/d^8)*x^2 + (9
3*b^2*c^2*d^8 + 340*a*b*c*d^9 + 80*a^2*d^10)/d^8)*x^2 + 5*(3*b^2*c^3*d^7 +
236*a*b*c^2*d^8 + 208*a^2*c*d^9)/d^8)*x^2 - 15*(3*b^2*c^4*d^6 - 20*a*b*c^3*
d^7 - 176*a^2*c^2*d^8)/d^8)*sqrt(d*x^2 + c)*x - 1/256*(3*b^2*c^5 - 20*a*b*c
^4*d + 80*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.629 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$a^2c^2\sqrt{c+dx^2}-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\,(c+dx}{9d^2}$$

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c + d*x^2)^(5
/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) + (b^2*(c + d*x^2)^(9/
2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.110641, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$a^2c^2\sqrt{c+dx^2}-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\,(c+dx}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c + d*x^2)^(5
/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) + (b^2*(c + d*x^2)^(9/
2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x}dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x}dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)(c+dx)^{5/2}}{d} + \frac{a^2(c+dx)^{5/2}}{x} + \frac{b^2(c+dx)^{7/2}}{d}\right)dx, x, x^2\right)$$

$$= -\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x}dx, x, x^2\right)$$

$$= \frac{1}{5}a^2\left(c+dx^2\right)^{5/2} - \frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2} + \frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int \frac{(c+\right.$$

$$= \frac{1}{3}a^2c\left(c+dx^2\right)^{3/2} + \frac{1}{5}a^2\left(c+dx^2\right)^{5/2} - \frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2} + \frac{1}{2}\left(\right.$$

$$= a^2c^2\sqrt{c+dx^2} + \frac{1}{3}a^2c\left(c+dx^2\right)^{3/2} + \frac{1}{5}a^2\left(c+dx^2\right)^{5/2} - \frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(\right.}{}$$

$$= a^2c^2\sqrt{c+dx^2} + \frac{1}{3}a^2c\left(c+dx^2\right)^{3/2} + \frac{1}{5}a^2\left(c+dx^2\right)^{5/2} - \frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(\right.}{}$$

$$= a^2c^2\sqrt{c+dx^2} + \frac{1}{3}a^2c\left(c+dx^2\right)^{3/2} + \frac{1}{5}a^2\left(c+dx^2\right)^{5/2} - \frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2} + \frac{b^2\left(\right.}{}$$

Mathematica [A] time = 0.108364, size = 123, normalized size = 0.93

$$\frac{1}{3}a^2c\left(\sqrt{c+dx^2}\left(4c+dx^2\right)-3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{b\left(c+dx^2\right)^{7/2}(2ad-bc)}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(c + d*x^2)^(5/2))/5 + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2)
+ (b^2*(c + d*x^2)^(9/2))/(9*d^2) + (a^2*c*(Sqrt[c + d*x^2]*(4*c + d*x^2) -
 3*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/3
```

Maple [A] time = 0.009, size = 132, normalized size = 1.

$$\frac{b^2x^2}{9\,d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{2\,b^2c}{63\,d^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{2\,ab}{7\,d}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{a^2}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2c}{3}\left(dx^2+c\right)^{\frac{3}{2}}-a^2c^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x)
```

```
[Out] 1/9*b^2*x^2*(d*x^2+c)^(7/2)/d-2/63*b^2*c/d^2*(d*x^2+c)^(7/2)+2/7*a*b*(d*x^2
+c)^(7/2)/d+1/5*a^2*(d*x^2+c)^(5/2)+1/3*a^2*c*(d*x^2+c)^(3/2)-a^2*c^(5/2)*l
n((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+a^2*c^2*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45754, size = 824, normalized size = 6.24

$$\left[\frac{315\,a^2c^{\frac{5}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(35\,b^2d^4x^8+5\left(19\,b^2cd^3+18\,abd^4\right)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\left(25\right.\right.}{630\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^2*c^(5/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 +
 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2
*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2
 + c))/d^2, 1/315*(315*a^2*sqrt(-c)*c^2*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 + 90*a
*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2*d^4)
*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)
)/d^2]
```

Sympy [A] time = 85.5675, size = 128, normalized size = 0.97

$$\frac{a^2c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+a^2c^2\sqrt{c+dx^2}+\frac{a^2c\left(c+dx^2\right)^{\frac{3}{2}}}{3}+\frac{a^2\left(c+dx^2\right)^{\frac{5}{2}}}{5}+\frac{b^2\left(c+dx^2\right)^{\frac{9}{2}}}{9d^2}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}\left(4abd-2b^2c\right)}{14d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

```
[Out] a**2*c**3*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c**2*sqrt(c + d*x
**2) + a**2*c*(c + d*x**2)**(3/2)/3 + a**2*(c + d*x**2)**(5/2)/5 + b**2*(c
+ d*x**2)**(9/2)/(9*d**2) + (c + d*x**2)**(7/2)*(4*a*b*d - 2*b**2*c)/(14*d*
*2)
```

Giac [A] time = 1.14128, size = 190, normalized size = 1.44

$$\frac{a^2c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{35\left(dx^2+c\right)^{\frac{9}{2}}b^2d^{16}-45\left(dx^2+c\right)^{\frac{7}{2}}b^2cd^{16}+90\left(dx^2+c\right)^{\frac{7}{2}}abd^{17}+63\left(dx^2+c\right)^{\frac{5}{2}}a^2d^{18}+105\left(\right.}{315\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/315*(35*(d*x^2 + c)^(
9/2)*b^2*d^16 - 45*(d*x^2 + c)^(7/2)*b^2*c*d^16 + 90*(d*x^2 + c)^(7/2)*a*b*
d^17 + 63*(d*x^2 + c)^(5/2)*a^2*d^18 + 105*(d*x^2 + c)^(3/2)*a^2*c*d^18 + 3
15*sqrt(d*x^2 + c)*a^2*c^2*d^18)/d^18
```

3.630 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=217

$$-\frac{a^2 \left(c+dx^2\right)^{7/2}}{cx}-\frac{5c^2 \left(b^2c^2-16ad(3ad+bc)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{x \left(c+dx^2\right)^{5/2} \left(b^2c^2-16ad(3ad+bc)\right)}{48cd}-\frac{5x \left(c+dx^2\right)^{3/}}{}$$

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d) - (5*(b^2
*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*d) - ((b^2*c^2 - 16*
a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(5/2))/(48*c*d) - (a^2*(c + d*x^2)^(7/2))/
(c*x) + (b^2*x*(c + d*x^2)^(7/2))/(8*d) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3
*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Rubi [A] time = 0.141456, antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2 \left(c+dx^2\right)^{7/2}}{cx}-\frac{5c^2 \left(b^2c^2-16ad(3ad+bc)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{5x \left(c+dx^2\right)^{3/2} \left(b^2c^2-16ad(3ad+bc)\right)}{192d}-\frac{5cx\sqrt{c+dx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d) - (5*(b^2
*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*d) - (((b^2*c)/d - (
16*a*(b*c + 3*a*d))/c)*x*(c + d*x^2)^(5/2))/48 - (a^2*(c + d*x^2)^(7/2))/(c
*x) + (b^2*x*(c + d*x^2)^(7/2))/(8*d) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3*a
*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} dx = -\frac{a^2 (c+dx^2)^{7/2}}{cx} + \frac{\int (2a(bc+3ad)+b^2cx^2)(c+dx^2)^{5/2} dx}{c}$$
$$= -\frac{a^2 (c+dx^2)^{7/2}}{cx} + \frac{b^2x (c+dx^2)^{7/2}}{8d} - \frac{(b^2c^2-16ad(bc+3ad)) \int (c+dx^2)^{5/2} dx}{8cd}$$
$$= -\frac{1}{48}\left(\frac{b^2c}{d} - \frac{16a(bc+3ad)}{c}\right) x (c+dx^2)^{5/2} - \frac{a^2 (c+dx^2)^{7/2}}{cx} + \frac{b^2x (c+dx^2)^{7/2}}{8d} - \frac{(5(b}{}$$
$$= -\frac{5(b^2c^2-16ad(bc+3ad)) x (c+dx^2)^{3/2}}{192d} - \frac{1}{48}\left(\frac{b^2c}{d} - \frac{16a(bc+3ad)}{c}\right) x (c+dx^2)^{5/2} -$$
$$= -\frac{5c(b^2c^2-16ad(bc+3ad)) x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad)) x (c+dx^2)^{3/2}}{192d} - \frac{1}{4}$$
$$= -\frac{5c(b^2c^2-16ad(bc+3ad)) x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad)) x (c+dx^2)^{3/2}}{192d} - \frac{1}{4}$$
$$= -\frac{5c(b^2c^2-16ad(bc+3ad)) x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad)) x (c+dx^2)^{3/2}}{192d} - \frac{1}{4}$$

Mathematica [A] time = 0.116308, size = 174, normalized size = 0.8

$$\sqrt{c+dx^2}\left(\frac{1}{192}x^3 (48a^2d^2+208abcd+59b^2c^2) + \frac{cx(144a^2d^2+176abcd+5b^2c^2)}{128d} - \frac{a^2c^2}{x} + \frac{1}{48}bdx^5(16ad+17bc) + \frac{1}{8}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c^2)/x) + (c*(5*b^2*c^2 + 176*a*b*c*d + 144*a^2*d^2
)*x)/(128*d) + ((59*b^2*c^2 + 208*a*b*c*d + 48*a^2*d^2)*x^3)/192 + (b*d*(17
*b*c + 16*a*d)*x^5)/48 + (b^2*d^2*x^7)/8) - (5*c^2*(b^2*c^2 - 16*a*b*c*d -
48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Maple [A] time = 0.009, size = 278, normalized size = 1.3

$$\frac{b^2x}{8d}(dx^2+c)^{\frac{7}{2}} - \frac{b^2cx}{48d}(dx^2+c)^{\frac{5}{2}} - \frac{5b^2c^2x}{192d}(dx^2+c)^{\frac{3}{2}} - \frac{5b^2c^3x}{128d}\sqrt{dx^2+c} - \frac{5b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}} + \frac{abx}{3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x)
```

```
[Out] 1/8*b^2*x*(d*x^2+c)^(7/2)/d-1/48*b^2*c/d*x*(d*x^2+c)^(5/2)-5/192*b^2*c^2/d*
x*(d*x^2+c)^(3/2)-5/128*b^2*c^3/d*x*(d*x^2+c)^(1/2)-5/128*b^2*c^4/d^(3/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)+5/12*a*b*c*x*(d*x^2+
c)^(3/2)+5/8*a*b*c^2*x*(d*x^2+c)^(1/2)+5/8*a*b*c^3/d^(1/2)*ln(x*d^(1/2)+(d*
x^2+c)^(1/2))-a^2*(d*x^2+c)^(7/2)/c/x+a^2*d/c*x*(d*x^2+c)^(5/2)+5/4*a^2*d*x
*(d*x^2+c)^(3/2)+15/8*a^2*d*c*x*(d*x^2+c)^(1/2)+15/8*a^2*d^(1/2)*c^2*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71392, size = 855, normalized size = 3.94

$$\left[-\frac{15\left(b^2c^4-16\,abc^3d-48\,a^2c^2d^2\right)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(48\,b^2d^4x^8+8\left(17\,b^2cd^3+16\,abd^4\right)x^6-38\right.}{768\,d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*sqrt(d)*x*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(48*b^2*d^4*x^8 + 8*(17*b^2*c*d^3
+ 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*d^3 + 4
8*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x^2)*sqr
t(d*x^2 + c))/(d^2*x), 1/384*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (48*b^2*d^4*x^8 + 8*(17*b^2
*c*d^3 + 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*
d^3 + 48*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x
^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

Sympy [B] time = 33.549, size = 496, normalized size = 2.29

$$-\frac{a^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+a^2c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7a^2c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}d^2x^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{15a^2c^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8}+\frac{a^2d^3x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+abc^{\frac{5}{2}}x\sqrt{1+\frac{dx}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + a**2*c**(3/2)*d*x*sqrt(1 + d*x**2/c
) - 7*a**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*d**2*x**3/(
8*sqrt(1 + d*x**2/c)) + 15*a**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/8 + a
**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*c**(5/2)*x*sqrt(1 + d*x*
*2/c) + 3*a*b*c**(5/2)*x/(8*sqrt(1 + d*x**2/c)) + 35*a*b*c**(3/2)*d*x**3/(2
4*sqrt(1 + d*x**2/c)) + 17*a*b*sqrt(c)*d**2*x**5/(12*sqrt(1 + d*x**2/c)) +
5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*sqrt(d)) + a*b*d**3*x**7/(3*sqrt(c)*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d*sqrt(1 + d*x**2/c)) + 133*b*
*2*c**(5/2)*x**3/(384*sqrt(1 + d*x**2/c)) + 127*b**2*c**(3/2)*d*x**5/(192*s
qrt(1 + d*x**2/c)) + 23*b**2*sqrt(c)*d**2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*
b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(3/2)) + b**2*d**3*x**9/(8*sqrt(
c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.12851, size = 296, normalized size = 1.36

$$\frac{2\,a^2c^3\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{384}\left(2\left(4\left(6\,b^2d^2x^2+\frac{17\,b^2cd^7+16\,abd^8}{d^6}\right)x^2+\frac{59\,b^2c^2d^6+208\,abcd^7+48\,a^2d^8}{d^6}\right)x^2+\frac{3\left(5\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^3*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/384*(2*(4*(6*b^
2*d^2*x^2 + (17*b^2*c*d^7 + 16*a*b*d^8)/d^6)*x^2 + (59*b^2*c^2*d^6 + 208*a*
b*c*d^7 + 48*a^2*d^8)/d^6)*x^2 + 3*(5*b^2*c^3*d^5 + 176*a*b*c^2*d^6 + 144*a
^2*c*d^7)/d^6)*sqrt(d*x^2 + c)*x + 5/256*(b^2*c^4*sqrt(d) - 16*a*b*c^3*d^(3
/2) - 48*a^2*c^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.631 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3} dx$

Optimal. Leaf size=162

$$-\frac{a^2 (c+dx^2)^{7/2}}{2cx^2} - \frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) + \frac{a(c+dx^2)^{5/2}(5ad+4bc)}{10c} + \frac{1}{6}a(c+dx^2)^{3/2}(5ad+4bc) + \frac{1}{2}a$$

```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(3
/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) + (b^2*(c + d*x^2)^(7
/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2) - (a*c^(3/2)*(4*b*c + 5*a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.132013, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2 (c+dx^2)^{7/2}}{2cx^2} - \frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) + \frac{a(c+dx^2)^{5/2}(5ad+4bc)}{10c} + \frac{1}{6}a(c+dx^2)^{3/2}(5ad+4bc) + \frac{1}{2}a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```


```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(3
/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) + (b^2*(c + d*x^2)^(7
/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2) - (a*c^(3/2)*(4*b*c + 5*a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+5ad)+b^2cx\right)(c+dx)^{5/2}}{x}\,dx,x,x^2\right)}{2c}$$

$$= \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} - \frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{(a(4bc+5ad))\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x}\,dx,x,x^2\right)}{4c}$$

$$= \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} - \frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{1}{4}(a(4bc+5ad))\,\text{Subst}$$

$$= \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} - \frac{a^2\left(c+dx^2\right.}{2cx^2}$$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + $$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + $$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + $$

Mathematica [A] time = 0.187495, size = 122, normalized size = 0.75

$$-\frac{\frac{a^2\left(c+dx^2\right)^{7/2}}{x^2} + \frac{1}{15}a(5ad+4bc)\left(15c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) - \sqrt{c+dx^2}\left(23c^2+11cdx^2+3d^2x^4\right)\right) - \frac{2b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] -((-2*b^2*c*(c + d*x^2)^(7/2))/(7*d) + (a^2*(c + d*x^2)^(7/2))/x^2 + (a*(4*
b*c + 5*a*d)*(-(Sqrt[c + d*x^2]*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)) + 15*c^(
5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/15)/(2*c)
```

Maple [A] time = 0.01, size = 193, normalized size = 1.2

$$\frac{b^2}{7d}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{2ab}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2cab}{3}\left(dx^2+c\right)^{\frac{3}{2}}-2abc^{5/2}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)+2ab\sqrt{dx^2+c}c^2-\frac{a^2}{2cx^2}\left(dx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x)
```

```
[Out] 1/7*b^2*(d*x^2+c)^(7/2)/d+2/5*a*b*(d*x^2+c)^(5/2)+2/3*a*b*c*(d*x^2+c)^(3/2)
-2*a*b*c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+2*a*b*(d*x^2+c)^(1/2)*
c^2-1/2*a^2*(d*x^2+c)^(7/2)/c/x^2+1/2*a^2*d/c*(d*x^2+c)^(5/2)+5/6*a^2*d*(d*
x^2+c)^(3/2)-5/2*a^2*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+5/2*a^
2*d*c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46548, size = 807, normalized size = 4.98

$$\left[\frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(30b^2d^3x^8+6\left(15b^2cd^2+14abd^3\right)x^6-105a^2c^2d+2\left(45b^2c^2\right.\right.}{420dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/420*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b
*d^3)*x^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x
^4 + 2*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(
d*x^2), 1/210*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + (30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b*d^3)*x^6 - 1
05*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x^4 + 2*(15*b^
2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

Sympy [A] time = 50.4034, size = 518, normalized size = 3.2

$$-\frac{5a^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{2a^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{2a^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}+a^2d^2\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for } d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)-2abc^{\frac{5}{2}}\operatorname{asinh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -5*a**2*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c**2*sqrt(d)*sqrt(c/
(d*x**2) + 1)/(2*x) + 2*a**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a**2
*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + a**2*d**2*Piecewise((sqrt(c)*x**2/2, E
q(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) - 2*a*b*c**(5/2)*asinh(sqrt(c)
/(sqrt(d)*x)) + 2*a*b*c**3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c**2*sq
rt(d)*x/sqrt(c/(d*x**2) + 1) + 4*a*b*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0
)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*a*b*d**2*Piecewise((-2*c**2*sqrt
(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x
**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*c**2*Piecewise((sqrt(c)*x
**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*b**2*c*d*Piecewise
((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**
4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*d**2*Piecew
ise((8*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*c**2*x**2*sqrt(c + d*x**2)/(105
*d**2) + c*x**4*sqrt(c + d*x**2)/(35*d) + x**6*sqrt(c + d*x**2)/7, Ne(d, 0)
), (sqrt(c)*x**6/6, True))
```

Giac [A] time = 1.13621, size = 223, normalized size = 1.38

$$\frac{30\left(dx^2+c\right)^{\frac{7}{2}}b^2+84\left(dx^2+c\right)^{\frac{5}{2}}abd+140\left(dx^2+c\right)^{\frac{3}{2}}abcd+420\sqrt{dx^2+c}abc^2d+70\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2+420\sqrt{dx^2+c}a}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/210*(30*(d*x^2 + c)^(7/2)*b^2 + 84*(d*x^2 + c)^(5/2)*a*b*d + 140*(d*x^2 +
 c)^(3/2)*a*b*c*d + 420*sqrt(d*x^2 + c)*a*b*c^2*d + 70*(d*x^2 + c)^(3/2)*a^
2*d^2 + 420*sqrt(d*x^2 + c)*a^2*c*d^2 - 105*sqrt(d*x^2 + c)*a^2*c^2*d/x^2 +
 105*(4*a*b*c^3*d + 5*a^2*c^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c
))/d
```

3.632 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} dx$

Optimal. Leaf size=223

$$-\frac{a^2 (c+dx^2)^{7/2}}{3cx^3}+\frac{x (c+dx^2)^{5/2} (4ad(2ad+3bc)+b^2c^2)}{6c^2}+\frac{5x (c+dx^2)^{3/2} (4ad(2ad+3bc)+b^2c^2)}{24c}+\frac{5}{16}x\sqrt{c+dx^2} (4ac$$

```
[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(b^2*c^2 +
4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((b^2*c^2 + 4*a*d*(3*b
*c + 2*a*d))*x*(c + d*x^2)^(5/2))/(6*c^2) - (a^2*(c + d*x^2)^(7/2))/(3*c*x^
3) - (2*a*(3*b*c + 2*a*d)*(c + d*x^2)^(7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*
a*d*(3*b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

Rubi [A] time = 0.174919, antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2 (c+dx^2)^{7/2}}{3cx^3}+\frac{1}{6}x (c+dx^2)^{5/2} \left(\frac{4ad(2ad+3bc)}{c^2}+b^2\right)+\frac{5x (c+dx^2)^{3/2} (4ad(2ad+3bc)+b^2c^2)}{24c}+\frac{5}{16}x\sqrt{c+dx^2} (4ac$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(b^2*c^2 +
4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((b^2 + (4*a*d*(3*b*c
+ 2*a*d))/c^2)*x*(c + d*x^2)^(5/2))/6 - (a^2*(c + d*x^2)^(7/2))/(3*c*x^3) -
 (2*a*(3*b*c + 2*a*d)*(c + d*x^2)^(7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*a*d*
(3*b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx = -\frac{a^2(c+dx^2)^{7/2}}{3cx^3} + \frac{\int \frac{(2a(3bc+2ad)+3b^2cx^2)(c+dx^2)^{5/2}}{x^2}\,dx}{3c}$$

$$= -\frac{a^2(c+dx^2)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)(c+dx^2)^{7/2}}{3c^2x} + \left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)\int (c+dx^2)^{5/}$$

$$= \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{5/2} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)(c+dx^2)^{7/2}}{3c^2x}$$

$$= \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{3/2} + \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{5/2} -$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{3/2} +$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{3/2} +$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{3/2} +$$

Mathematica [A] time = 0.119293, size = 155, normalized size = 0.7

$$\frac{1}{48}\left(\frac{\sqrt{c+dx^2}\left(-8a^2(2c^2+14cdx^2-3d^2x^4)+12abx^2(-8c^2+9cdx^2+2d^2x^4)+b^2x^4(33c^2+26cdx^2+8d^2x^4)\right)}{x^3}+\frac{15c}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*(2*c^2 + 14*c*d*x^2 - 3*d^2*x^4) + 12*a*b*x^2*(-8
*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + b^2*x^4*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4)))
/x^3 + (15*c*(b^2*c^2 + 12*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c +
d*x^2]])/Sqrt[d])/48
```

Maple [A] time = 0.012, size = 298, normalized size = 1.3

$$\frac{xb^2}{6}(dx^2+c)^{\frac{5}{2}} + \frac{5b^2cx}{24}(dx^2+c)^{\frac{3}{2}} + \frac{5b^2c^2x}{16}\sqrt{dx^2+c} + \frac{5b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}} - \frac{a^2}{3cx^3}(dx^2+c)^{\frac{7}{2}} - \frac{4a^2d}{3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x)
```

```
[Out] 1/6*x*b^2*(d*x^2+c)^(5/2)+5/24*b^2*c*x*(d*x^2+c)^(3/2)+5/16*b^2*c^2*x*(d*x^
2+c)^(1/2)+5/16*b^2*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/3*a^2*(d*x^
2+c)^(7/2)/c/x^3-4/3*a^2*d/c^2/x*(d*x^2+c)^(7/2)+4/3*a^2*d^2/c^2*x*(d*x^2+c
)^(5/2)+5/3*a^2*d^2/c*x*(d*x^2+c)^(3/2)+5/2*a^2*d^2*x*(d*x^2+c)^(1/2)+5/2*a
^2*d^(3/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(7/2)+2*a*b*
d/c*x*(d*x^2+c)^(5/2)+5/2*a*b*d*x*(d*x^2+c)^(3/2)+15/4*a*b*d*c*x*(d*x^2+c)^
(1/2)+15/4*a*b*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52664, size = 776, normalized size = 3.48

$$\left[\frac{15\left(b^2c^3+12\,abc^2d+8\,a^2cd^2\right)\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(8\,b^2d^3x^8+2\left(13\,b^2cd^2+12\,abd^3\right)x^6-16\,a^2c^{2}\cdots\right.}{96\,dx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*x^3*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12
*a*b*d^3)*x^6 - 16*a^2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*
x^4 - 16*(6*a*b*c^2*d + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3), -1/48*(
15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12*a*b*d^3)*x^6 - 16*a^
2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*x^4 - 16*(6*a*b*c^2*d
 + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

Sympy [B] time = 20.9042, size = 490, normalized size = 2.2

$$-\frac{2a^2c^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{a^2\sqrt{c}d^2x\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{2a^2\sqrt{c}d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+\frac{5a^2cd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2}-\frac{2abc^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -2*a**2*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d**2*x*sqrt(1 + d*
x**2/c)/2 - 2*a**2*sqrt(c)*d**2*x/sqrt(1 + d*x**2/c) - a**2*c**2*sqrt(d)*sq
rt(c/(d*x**2) + 1)/(3*x**2) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*
*2*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(c))/2 - 2*a*b*c**(5/2)/(x*sqrt(1 + d*x**
2/c)) + 2*a*b*c**(3/2)*d*x*sqrt(1 + d*x**2/c) - 7*a*b*c**(3/2)*d*x/(4*sqrt(
1 + d*x**2/c)) + 3*a*b*sqrt(c)*d**2*x**3/(4*sqrt(1 + d*x**2/c)) + 15*a*b*c*
*2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/4 + a*b*d**3*x**5/(2*sqrt(c)*sqrt(1 + d
*x**2/c)) + b**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*b**2*c**(5/2)*x/(16*sq
rt(1 + d*x**2/c)) + 35*b**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*b*
*2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*b**2*c**3*asinh(sqrt(d)*x/
sqrt(c))/(16*sqrt(d)) + b**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.18788, size = 414, normalized size = 1.86

$$\frac{1}{48}\left(2\left(4\,b^2d^2x^2+\frac{13\,b^2cd^5+12\,abd^6}{d^4}\right)x^2+\frac{3\left(11\,b^2c^2d^4+36\,abcd^5+8\,a^2d^6\right)}{d^4}\right)\sqrt{dx^2+cx}-\frac{5\left(b^2c^3\sqrt{d}+12\,abc^2d^{\frac{3}{2}}+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*d^2*x^2 + (13*b^2*c*d^5 + 12*a*b*d^6)/d^4)*x^2 + 3*(11*b^2*c
^2*d^4 + 36*a*b*c*d^5 + 8*a^2*d^6)/d^4)*sqrt(d*x^2 + c)*x - 5/32*(b^2*c^3*s
qrt(d) + 12*a*b*c^2*d^(3/2) + 8*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*sqrt(d) + 9*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 +
 c))^2*a*b*c^4*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(3/2)
 + 6*a*b*c^5*sqrt(d) + 7*a^2*c^4*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2
- c)^3
```

3.633 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=222

$$-\frac{a^2(c+dx^2)^{7/2}}{4cx^4}+\frac{(c+dx^2)^{5/2}(5ad(3ad+8bc)+8b^2c^2)}{40c^2}+\frac{(c+dx^2)^{3/2}(5ad(3ad+8bc)+8b^2c^2)}{24c}+\frac{1}{8}\sqrt{c+dx^2}\,(5ad(3ac$$

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2*c^2 + 5*a
*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2*c^2 + 5*a*d*(8*b*c
+ 3*a*d))*(c + d*x^2)^(5/2))/(40*c^2) - (a^2*(c + d*x^2)^(7/2))/(4*c*x^4) -
 (a*(8*b*c + 3*a*d)*(c + d*x^2)^(7/2))/(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 +
5*a*d*(8*b*c + 3*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.253353, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2(c+dx^2)^{7/2}}{4cx^4}+\frac{1}{40}(c+dx^2)^{5/2}\left(\frac{5ad(3ad+8bc)}{c^2}+8b^2\right)+\frac{(c+dx^2)^{3/2}(5ad(3ad+8bc)+8b^2c^2)}{24c}+\frac{1}{8}\sqrt{c+dx^2}\,(5ad(3a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2*c^2 + 5*a
*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2 + (5*a*d*(8*b*c + 3
*a*d))/c^2)*(c + d*x^2)^(5/2))/40 - (a^2*(c + d*x^2)^(7/2))/(4*c*x^4) - (a*
(8*b*c + 3*a*d)*(c + d*x^2)^(7/2))/(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 + 5*a*
d*(8*b*c + 3*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^5}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^3}\,dx,x,x^2\right)$$

$$=-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+3ad)+2b^2cx\right)(c+dx)^{5/2}}{x^2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}-\frac{a(8bc+3ad)\left(c+dx^2\right)^{7/2}}{8c^2x^2}+\frac{1}{16}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\text{Subst}\left(\int$$

$$=\frac{1}{40}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}-\frac{a(8bc+3ad)\left(c+dx^2\right)^{7/2}}{8c^2x^2}$$

$$=\frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}+\frac{1}{40}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}-$$

$$=\frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2}+\frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}+$$

$$=\frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2}+\frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}+$$

$$=\frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2}+\frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}+$$

Mathematica [A] time = 0.103731, size = 153, normalized size = 0.69

$$\frac{\sqrt{c+dx^2}\left(-15a^2\left(2c^2+9cdx^2-8d^2x^4\right)+40abx^2\left(-3c^2+14cdx^2+2d^2x^4\right)+8b^2x^4\left(23c^2+11cdx^2+3d^2x^4\right)\right)}{120x^4}-\frac{1}{8}\sqrt{c}\,($$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) + 40*a*b*x^2*(-3*
c^2 + 14*c*d*x^2 + 2*d^2*x^4) + 8*b^2*x^4*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)
))/(120*x^4) - (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/8
```

Maple [A] time = 0.01, size = 305, normalized size = 1.4

$$\frac{b^2}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2c}{3}\left(dx^2+c\right)^{\frac{3}{2}}-b^2c^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)+b^2\sqrt{dx^2+c}c^2-\frac{a^2}{4cx^4}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{3a^2d}{8c^2x^2}\left(dx^2+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x)
```

```
[Out] 1/5*b^2*(d*x^2+c)^(5/2)+1/3*b^2*c*(d*x^2+c)^(3/2)-b^2*c^(5/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)+b^2*(d*x^2+c)^(1/2)*c^2-1/4*a^2*(d*x^2+c)^(7/2)/c/
x^4-3/8*a^2*d/c^2/x^2*(d*x^2+c)^(7/2)+3/8*a^2*d^2/c^2*(d*x^2+c)^(5/2)+5/8*a
^2*d^2/c*(d*x^2+c)^(3/2)-15/8*a^2*d^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)+15/8*a^2*d^2*(d*x^2+c)^(1/2)-a*b/c/x^2*(d*x^2+c)^(7/2)+a*b*d/c*(d*
x^2+c)^(5/2)+5/3*a*b*d*(d*x^2+c)^(3/2)-5*a*b*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d
*x^2+c)^(1/2))/x)+5*a*b*d*c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44407, size = 740, normalized size = 3.33

$$\left[\frac{15\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c}x^4\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(24b^2d^2x^8+8\left(11b^2cd+10abd^2\right)x^6+8\left(23b^2c^2+70\right.\right.}{240x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(24*b^2*d^2*x^8 + 8*(11*b^2*c*d +
 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^
2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^4, 1/120*(15*(8*b^2*
```

```
c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (24*b^2*d^2*x^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*
a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sq
rt(d*x^2 + c))/x^4]
```

Sympy [A] time = 98.836, size = 473, normalized size = 2.13

$$-\frac{15a^2\sqrt{c}d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8}-\frac{a^2c^3}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2c^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{7a^2cd^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-5abc^{\frac{3}{2}}d\operatorname{as}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -15*a**2*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/8 - a**2*c**3/(4*sqrt(d)*x
**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c**2*sqrt(d)/(8*x**3*sqrt(c/(d*x**2) + 1
)) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a**2*c*d**(3/2)/(8*x*sqrt(c
/(d*x**2) + 1)) + a**2*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*a*b*c**(3/2)*d*a
sinh(sqrt(c)/(sqrt(d)*x)) - a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/x + 4*a*b
*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 4*a*b*c*d**(3/2)*x/sqrt(c/(d*x**2)
 + 1) + 2*a*b*d**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/
2)/(3*d), True)) - b**2*c**(5/2)*asinh(sqrt(c)/(sqrt(d)*x)) + b**2*c**3/(sq
rt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c**2*sqrt(d)*x/sqrt(c/(d*x**2) + 1) +
2*b**2*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d)
, True)) + b**2*d**2*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2
*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**
4/4, True))
```

Giac [A] time = 1.14426, size = 327, normalized size = 1.47

$$\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2d+40\left(dx^2+c\right)^{\frac{3}{2}}b^2cd+120\sqrt{dx^2+c}b^2c^2d+80\left(dx^2+c\right)^{\frac{3}{2}}abd^2+480\sqrt{dx^2+c}abcd^2+120\sqrt{dx^2+c}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/120*(24*(d*x^2 + c)^(5/2)*b^2*d + 40*(d*x^2 + c)^(3/2)*b^2*c*d + 120*sqrt
(d*x^2 + c)*b^2*c^2*d + 80*(d*x^2 + c)^(3/2)*a*b*d^2 + 480*sqrt(d*x^2 + c)*
a*b*c*d^2 + 120*sqrt(d*x^2 + c)*a^2*d^3 + 15*(8*b^2*c^3*d + 40*a*b*c^2*d^2
+ 15*a^2*c*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - 15*(8*(d*x^2 +
c)^(3/2)*a*b*c^2*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 9*(d*x^2 + c)^(3/2)*
a^2*c*d^3 - 7*sqrt(d*x^2 + c)*a^2*c^2*d^3)/(d^2*x^4))/d
```

3.634 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=228

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{15c^2x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx^2}\left(8ad(\right.}{}$$

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + (d*(15*b^2
*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - ((15*b^2*c^2 +
8*a*d*(5*b*c + a*d))*(c + d*x^2)^(5/2))/(15*c^2*x) - (a^2*(c + d*x^2)^(7/2)
)/(5*c*x^5) - (2*a*(5*b*c + a*d)*(c + d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]
*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rubi [A] time = 0.160411, antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 277, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(\frac{8ad(ad+5bc)}{c^2}+15b^2\right)}{15x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx^2}\left(8ad(ad+\right.}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + (d*(15*b^2
*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - ((15*b^2 + (8*a
*d*(5*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(15*x) - (a^2*(c + d*x^2)^(7/2))/
(5*c*x^5) - (2*a*(5*b*c + a*d)*(c + d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]*(
15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^6}\,dx = -\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}+\frac{\int \frac{\left(2a(5bc+ad)+5b^2cx^2\right)\left(c+dx^2\right)^{5/2}}{x^4}\,dx}{5c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{2a(5bc+ad)\left(c+dx^2\right)^{7/2}}{15c^2x^3}-\frac{1}{15}\left(-15b^2-\frac{8ad(5bc+ad)}{c^2}\right)\int \frac{\left(c+d\right.}{x^2}$$

$$= -\frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{15x}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{2a(5bc+ad)\left(c+dx^2\right)^{7/2}}{15c^2x^3}+\frac{1}{3}\Big($$

$$= \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-\frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{15x}-\frac{a^2\left(c\right.}{5}$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-$$

Mathematica [A] time = 0.154735, size = 158, normalized size = 0.69

$$\sqrt{c+dx^2}\left(\frac{-23a^2d^2-70abcd-15b^2c^2}{15x}-\frac{a^2c^2}{5x^5}-\frac{ac(11ad+10bc)}{15x^3}+\frac{1}{8}bdx(8ad+9bc)+\frac{1}{4}b^2d^2x^3\right)+\frac{1}{8}\sqrt{d}\left(8a^2d^2+40a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] Sqrt[c + d*x^2]*(-(a^2*c^2)/(5*x^5) - (a*c*(10*b*c + 11*a*d))/(15*x^3) + (-
15*b^2*c^2 - 70*a*b*c*d - 23*a^2*d^2)/(15*x) + (b*d*(9*b*c + 8*a*d)*x)/8 +
(b^2*d^2*x^3)/4) + (Sqrt[d]*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*Log[d*x +
```

```
Sqrt[d]*Sqrt[c + d*x^2]])/8
```

Maple [A] time = 0.014, size = 369, normalized size = 1.6

$$-\frac{2ab}{3cx^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{8abd}{3c^2x}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{8abd^2x}{3c^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{10abd^2x}{3c}\left(dx^2+c\right)^{\frac{3}{2}}+5abd^2x\sqrt{dx^2+c}+5abd^{3/2}c\ln\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x)
```

```
[Out] -2/3*a*b/c/x^3*(d*x^2+c)^(7/2)-8/3*a*b*d/c^2/x*(d*x^2+c)^(7/2)+8/3*a*b*d^2/
c^2*x*(d*x^2+c)^(5/2)+10/3*a*b*d^2/c*x*(d*x^2+c)^(3/2)+5*a*b*d^2*x*(d*x^2+c
)^(1/2)+5*a*b*d^(3/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/5*a^2*(d*x^2+c)^(7/
2)/c/x^5-2/15*a^2*d/c^2/x^3*(d*x^2+c)^(7/2)-8/15*a^2*d^2/c^3/x*(d*x^2+c)^(7
/2)+8/15*a^2*d^3/c^3*x*(d*x^2+c)^(5/2)+2/3*a^2*d^3/c^2*x*(d*x^2+c)^(3/2)+a^
2*d^3/c*x*(d*x^2+c)^(1/2)+a^2*d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-b^2/c/x
*(d*x^2+c)^(7/2)+b^2*d/c*x*(d*x^2+c)^(5/2)+5/4*b^2*d*x*(d*x^2+c)^(3/2)+15/8
*b^2*d*c*x*(d*x^2+c)^(1/2)+15/8*b^2*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52209, size = 738, normalized size = 3.24

$$\left[\frac{15\left(15b^2c^2+40abcd+8a^2d^2\right)\sqrt{d}x^5\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(30b^2d^2x^8+15\left(9b^2cd+8abd^2\right)x^6-8\left(15b\right.\right.}{240x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^5*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a
*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8
*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^5, -1/120*(15*(15*b^2*c^
2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
 - (30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*
b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqr
t(d*x^2 + c))/x^5]
```

Sympy [B] time = 13.6701, size = 474, normalized size = 2.08

$$-\frac{a^2\sqrt{c}d^2}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4} - \frac{11a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15x^2} - \frac{8a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15} + a^2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{a^2d^3x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{4abc^{\frac{3}{2}}\cdots}{x\sqrt{1+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(c)*d**2/(x*sqrt(1 + d*x**2/c)) - a**2*c**2*sqrt(d)*sqrt(c/(d*x**
2) + 1)/(5*x**4) - 11*a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*x**2) - 8*a*
*2*d**(5/2)*sqrt(c/(d*x**2) + 1)/15 + a**2*d**(5/2)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d**3*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 4*a*b*c**(3/2)*d/(x*sqrt(1 +
 d*x**2/c)) + a*b*sqrt(c)*d**2*x*sqrt(1 + d*x**2/c) - 4*a*b*sqrt(c)*d**2*x/
sqrt(1 + d*x**2/c) - 2*a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a
*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*b*c*d**(3/2)*asinh(sqrt(d)*x/sqr
t(c)) - b**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + b**2*c**(3/2)*d*x*sqrt(1 + d
*x**2/c) - 7*b**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*b**2*sqrt(c)*d**2
*x**3/(8*sqrt(1 + d*x**2/c)) + 15*b**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
)/8 + b**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [B] time = 1.15609, size = 689, normalized size = 3.02

$$\frac{1}{8}\left(2\,b^2d^2x^2+\frac{9\,b^2cd^3+8\,abd^4}{d^2}\right)\sqrt{dx^2+c}x - \frac{1}{16}\left(15\,b^2c^2\sqrt{d}+40\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right) + \frac{2\left(15\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d^2*x^2 + (9*b^2*c*d^3 + 8*a*b*d^4)/d^2)*sqrt(d*x^2 + c)*x - 1/1
6*(15*b^2*c^2*sqrt(d) + 40*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x -
sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*sqrt
(d) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(3/2) + 45*(sqrt(d)*x -
sqrt(d*x^2 + c))^8*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c
^4*sqrt(d) - 300*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(3/2) - 90*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c
))^4*b^2*c^5*sqrt(d) + 400*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^(3/2)
+ 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(5/2) - 60*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*b^2*c^6*sqrt(d) - 260*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c
^5*d^(3/2) - 70*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(5/2) + 15*b^2*c^
7*sqrt(d) + 70*a*b*c^6*d^(3/2) + 23*a^2*c^5*d^(5/2))/((sqrt(d)*x - sqrt(d*x
^2 + c))^2 - c)^5
```

3.635 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7} dx$

Optimal. Leaf size=222

$$-\frac{a^2(c+dx^2)^{7/2}}{6cx^6} - \frac{(c+dx^2)^{5/2}(ad(ad+12bc)+8b^2c^2)}{16c^2x^2} + \frac{5d(c+dx^2)^{3/2}(ad(ad+12bc)+8b^2c^2)}{48c^2} + \frac{5d\sqrt{c+dx^2}\,(ad(ad}{16c}$$

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (5*d*(8*b^2
*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2) - ((8*b^2*c^2 + a*d*
(12*b*c + a*d))*(c + d*x^2)^(5/2))/(16*c^2*x^2) - (a^2*(c + d*x^2)^(7/2))/(
6*c*x^6) - (a*(12*b*c + a*d)*(c + d*x^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*
c^2 + a*d*(12*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rubi [A] time = 0.252158, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 89, 78, 47, 50, 63, 208}

$$-\frac{a^2(c+dx^2)^{7/2}}{6cx^6} - \frac{(c+dx^2)^{5/2}\left(\frac{ad(ad+12bc)}{c^2}+8b^2\right)}{16x^2} + \frac{5d(c+dx^2)^{3/2}(ad(ad+12bc)+8b^2c^2)}{48c^2} + \frac{5d\sqrt{c+dx^2}\,(ad(ad+12bc}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (5*d*(8*b^2
*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2) - ((8*b^2 + (a*d*(12
*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(16*x^2) - (a^2*(c + d*x^2)^(7/2))/(6*
c*x^6) - (a*(12*b*c + a*d)*(c + d*x^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*c^
2 + a*d*(12*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} dx = \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^4} dx, x, x^2\right)$$

$$= -\frac{a^2 (c+dx^2)^{7/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc+ad)+3b^2cx\right)(c+dx)^{5/2}}{x^3} dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2 (c+dx^2)^{7/2}}{6cx^6} - \frac{a(12bc+ad)(c+dx^2)^{7/2}}{24c^2x^4} + \frac{1}{16}\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right) \text{Subst}\left(\int \frac{(c+\right.$$

$$= -\frac{\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{5/2}}{16x^2} - \frac{a^2 (c+dx^2)^{7/2}}{6cx^6} - \frac{a(12bc+ad)(c+dx^2)^{7/2}}{24c^2x^4} + \frac{1}{32}\left(5d\right.$$

$$= \frac{5}{48} d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{5/2}}{16x^2} - \frac{a^2 (c+dx^2)^{7/}}{6cx^6}$$

$$= \frac{5}{16} cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48} d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{(8b^2}{}$$

$$= \frac{5}{16} cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48} d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{(8b^2}{}$$

$$= \frac{5}{16} cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48} d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{(8b^2}{}$$

Mathematica [C] time = 0.0598314, size = 92, normalized size = 0.41

$$\frac{(c+dx^2)^{7/2}\left(3dx^6\left(a^2d^2+12abcd+8b^2c^2\right) {}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{dx^2}{c}+1\right) - 7ac^2\left(4ac+adx^2+12bcx^2\right)\right)}{168c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] ((c + d*x^2)^(7/2)*(-7*a*c^2*(4*a*c + 12*b*c*x^2 + a*d*x^2) + 3*d*(8*b^2*c^
2 + 12*a*b*c*d + a^2*d^2)*x^6*Hypergeometric2F1[2, 7/2, 9/2, 1 + (d*x^2)/c]
))/(168*c^4*x^6)
```

Maple [A] time = 0.014, size = 387, normalized size = 1.7

$$-\frac{a^2}{6cx^6}(dx^2+c)^{\frac{7}{2}} - \frac{a^2d}{24c^2x^4}(dx^2+c)^{\frac{7}{2}} - \frac{a^2d^2}{16c^3x^2}(dx^2+c)^{\frac{7}{2}} + \frac{a^2d^3}{16c^3}(dx^2+c)^{\frac{5}{2}} + \frac{5a^2d^3}{48c^2}(dx^2+c)^{\frac{3}{2}} - \frac{5a^2d^3}{16}\ln\left(\frac{1}{x}\left(2c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(7/2)/c/x^6-1/24*a^2*d/c^2/x^4*(d*x^2+c)^(7/2)-1/16*a^2*
d^2/c^3/x^2*(d*x^2+c)^(7/2)+1/16*a^2*d^3/c^3*(d*x^2+c)^(5/2)+5/48*a^2*d^3/c
^2*(d*x^2+c)^(3/2)-5/16*a^2*d^3/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/
x)+5/16*a^2*d^3/c*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c)^(7/2)-3/4*a*b*d/c
^2/x^2*(d*x^2+c)^(7/2)+3/4*a*b*d^2/c^2*(d*x^2+c)^(5/2)+5/4*a*b*d^2/c*(d*x^2
```

```
+c)^(3/2)-15/4*a*b*d^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+15/4*a
*b*d^2*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(7/2)+1/2*b^2*d/c*(d*x^2+c)^
(5/2)+5/6*b^2*d*(d*x^2+c)^(3/2)-5/2*b^2*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+
c)^(1/2))/x)+5/2*b^2*d*c*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48991, size = 778, normalized size = 3.5

$$\left[\frac{15\left(8 b^2c^2d + 12 abcd^2 + a^2d^3\right)\sqrt{c}x^6 \log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right) + 2\left(16 b^2cd^2x^8 + 16\left(7 b^2c^2d + 6 abcd^2\right)x^6 - 8 a^2c^3 - 3\left(\right.\right.}{96 cx^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2
*d + 6*a*b*c*d^2)*x^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*
d^2)*x^4 - 2*(12*a*b*c^3 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6), 1/4
8*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) + (16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2*d + 6*a*b*c*d^2)*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*d^2)*x^4 - 2*(12*a*b*c^3
 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6)]
```

Sympy [A] time = 131.077, size = 468, normalized size = 2.11

$$-\frac{a^2c^3}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}} - \frac{17a^2c^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{35a^2cd^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{3a^2d^{\frac{5}{2}}}{16x\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^3 \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16\sqrt{c}} - \frac{15ab\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

```
[Out] -a**2*c**3/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 17*a**2*c**2*sqrt(d)/(24
*x**5*sqrt(c/(d*x**2) + 1)) - 35*a**2*c*d**(3/2)/(48*x**3*sqrt(c/(d*x**2) +
 1)) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(2*x) - 3*a**2*d**(5/2)/(16*x*sqr
t(c/(d*x**2) + 1)) - 5*a**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*sqrt(c)) -
15*a*b*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/4 - a*b*c**3/(2*sqrt(d)*x**5
*sqrt(c/(d*x**2) + 1)) - 3*a*b*c**2*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) -
```

```
2*a*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a*b*c*d**(3/2)/(4*x*sqrt(c/(d*
x**2) + 1)) + 2*a*b*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*b**2*c**(3/2)*d*asi
nh(sqrt(c)/(sqrt(d)*x))/2 - b**2*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) +
2*b**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*b**2*c*d**(3/2)*x/sqrt(c/(
d*x**2) + 1) + b**2*d**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2
)**(3/2)/(3*d), True))
```

Giac [A] time = 1.18783, size = 386, normalized size = 1.74

$$\frac{16\left(dx^2+c\right)^{\frac{3}{2}}b^2d^2+96\sqrt{dx^2+c}b^2cd^2+96\sqrt{dx^2+c}abd^3+\frac{15\left(8b^2c^2d^2+12abcd^3+a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+\right.}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(16*(d*x^2 + c)^(3/2)*b^2*d^2 + 96*sqrt(d*x^2 + c)*b^2*c*d^2 + 96*sqrt
(d*x^2 + c)*a*b*d^3 + 15*(8*b^2*c^2*d^2 + 12*a*b*c*d^3 + a^2*d^4)*arctan(sq
rt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(
d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 108*(d*x^2
+ c)^(5/2)*a*b*c*d^3 - 192*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 84*sqrt(d*x^2 +
c)*a*b*c^3*d^3 + 33*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*
d^4 + 15*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(d^3*x^6))/d
```

3.636 $\int \frac{x^4 (a+bx^2)^2}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=194

$$\frac{c^2 \left(48a^2d^2+5bc(7bc-16ad)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}+\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2+5bc(7bc-16ad)\right)}{192d^3}-\frac{cx\sqrt{c+dx^2}\left(48a^2d^2+5b\right.}{128d^4}$$

```
[Out] -(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^4) + ((
48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x^3*Sqrt[c + d*x^2])/(192*d^3) - (b*(7
*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2) + (b^2*x^7*Sqrt[c + d*x^2])/(8
*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(128*d^(9/2))
```

Rubi [A] time = 0.154555, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {464, 459, 321, 217, 206}

$$\frac{c^2 \left(48a^2d^2+5bc(7bc-16ad)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}+\frac{x^3\sqrt{c+dx^2}\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)}{192d}-\frac{cx\sqrt{c+dx^2}\left(48a^2d^2+5bc(7bc\right.}{128d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^4) + ((
48*a^2 + (5*b*c*(7*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/(192*d) - (b*(7
*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2) + (b^2*x^7*Sqrt[c + d*x^2])/(8
*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(128*d^(9/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{b^2x^7\sqrt{c+dx^2}}{8d} + \frac{\int \frac{x^4\left(8a^2d-b(7bc-16ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{8d}$$

$$= -\frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2} + \frac{b^2x^7\sqrt{c+dx^2}}{8d} - \frac{1}{48}\left(-48a^2 - \frac{5bc(7bc-16ad)}{d^2}\right)\int \frac{x^4}{\sqrt{c+dx^2}}\,dx$$

$$= \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2} + \frac{b^2x^7\sqrt{c+dx^2}}{8d} - \frac{\left(c\left(48a^2+\frac{5bc(7bc}{d}\right.\right.}{6}$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-16ad)x^5\sqrt{c+dx}}{48d^2}$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-16ad)x^5\sqrt{c+dx}}{48d^2}$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-16ad)x^5\sqrt{c+dx}}{48d^2}$$

Mathematica [A] time = 0.113913, size = 159, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(2dx^2-3c\right)+16abd\left(15c^2-10cdx^2+8d^2x^4\right)+b^2\left(70c^2dx^2-105c^3-56cd^2x^4+48d^3x^6\right)\right)+3c^2\left(\right.}{384d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(-3*c + 2*d*x^2) + 16*a*b*d*(15*c^2
- 10*c*d*x^2 + 8*d^2*x^4) + b^2*(-105*c^3 + 70*c^2*d*x^2 - 56*c*d^2*x^4 + 4
8*d^3*x^6)) + 3*c^2*(35*b^2*c^2 - 80*a*b*c*d + 48*a^2*d^2)*Log[d*x + Sqrt[d
]*Sqrt[c + d*x^2]])/(384*d^(9/2))
```

Maple [A] time = 0.016, size = 265, normalized size = 1.4

$$\frac{b^2x^7}{8d}\sqrt{dx^2+c} - \frac{7b^2cx^5}{48d^2}\sqrt{dx^2+c} + \frac{35x^3b^2c^2}{192d^3}\sqrt{dx^2+c} - \frac{35b^2c^3x}{128d^4}\sqrt{dx^2+c} + \frac{35b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}} + \frac{abx^5}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*b^2*x^7*(d*x^2+c)^(1/2)/d-7/48*b^2*c/d^2*x^5*(d*x^2+c)^(1/2)+35/192*b^2
*c^2/d^3*x^3*(d*x^2+c)^(1/2)-35/128*b^2*c^3/d^4*x*(d*x^2+c)^(1/2)+35/128*b^
2*c^4/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x^5/d*(d*x^2+c)^(1/2)-5
/12*a*b*c/d^2*x^3*(d*x^2+c)^(1/2)+5/8*a*b*c^2/d^3*x*(d*x^2+c)^(1/2)-5/8*a*b
*c^3/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a^2*x^3/d*(d*x^2+c)^(1/2)-3/
8*a^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8*a^2*c^2/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62515, size = 782, normalized size = 4.03

$$\left[\frac{3\left(35\,b^2c^4 - 80\,abc^3d + 48\,a^2c^2d^2\right)\sqrt{d}\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\left(48\,b^2d^4x^7 - 8\left(7\,b^2cd^3 - 16\,abd^4\right)x^5 + 2\right.}{768\,d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 -
16*a*b*d^4)*x^5 + 2*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3
5*b^2*c^3*d - 80*a*b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/38
4*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*
x/sqrt(d*x^2 + c)) - (48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 - 16*a*b*d^4)*x^5 + 2
*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 80*a*
b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

Sympy [B] time = 18.3569, size = 422, normalized size = 2.18

$$-\frac{3a^2c^{\frac{3}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2\sqrt{c}x^3}{8d\sqrt{1+\frac{dx^2}{c}}} + \frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}} + \frac{a^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{5abc^{\frac{5}{2}}x}{8d^3\sqrt{1+\frac{dx^2}{c}}} + \frac{5abc^{\frac{3}{2}}x^3}{24d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{ab\sqrt{c}x^5}{12d\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**(3/2)*x/(8*d**2*sqrt(1 + d*x**2/c)) - a**2*sqrt(c)*x**3/(8*d*sqr
t(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a**2
*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(5/2)*x/(8*d**3*sqrt(1 + d*
x**2/c)) + 5*a*b*c**(3/2)*x**3/(24*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x
```

```
**5/(12*d*sqrt(1 + d*x**2/c)) - 5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(
7/2)) + a*b*x**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) - 35*b**2*c**(7/2)*x/(128*d
**4*sqrt(1 + d*x**2/c)) - 35*b**2*c**(5/2)*x**3/(384*d**3*sqrt(1 + d*x**2/c
)) + 7*b**2*c**(3/2)*x**5/(192*d**2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**7
/(48*d*sqrt(1 + d*x**2/c)) + 35*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**
(9/2)) + b**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.13077, size = 240, normalized size = 1.24

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,b^2x^2}{d}-\frac{7\,b^2cd^5-16\,abd^6}{d^7}\right)x^2+\frac{35\,b^2c^2d^4-80\,abcd^5+48\,a^2d^6}{d^7}\right)x^2-\frac{3\left(35\,b^2c^3d^3-80\,abc^2d^4+48\,a^2cd^5\right)}{d^7}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2/d - (7*b^2*c*d^5 - 16*a*b*d^6)/d^7)*x^2 + (35*b^2*c^
2*d^4 - 80*a*b*c*d^5 + 48*a^2*d^6)/d^7)*x^2 - 3*(35*b^2*c^3*d^3 - 80*a*b*c^
2*d^4 + 48*a^2*c*d^5)/d^7)*sqrt(d*x^2 + c)*x - 1/128*(35*b^2*c^4 - 80*a*b*c
^3*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.637 $\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=112

$$-\frac{b\left(c+dx^2\right)^{5/2}(3bc-2ad)}{5d^4}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}(bc-ad)^2}{d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(5/2))/(5*d^4) + (b^
2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rubi [A] time = 0.0889361, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{5/2}(3bc-2ad)}{5d^4}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}(bc-ad)^2}{d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(5/2))/(5*d^4) + (b^
2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{c(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{(bc-ad)(3bc-ad)\sqrt{c+dx}}{d^3}-\frac{b(3bc-2ad)(c+dx)^{3/2}}{d^3}+\frac{b^2(c+d\cdots}{d^3}\right.\right.$$

$$=-\frac{c(bc-ad)^2\sqrt{c+dx^2}}{d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{3/2}}{3d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^4}+\frac{b^2\left(c+\cdots\right.}{7d\cdots}$$

Mathematica [A] time = 0.0708376, size = 99, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}\left(35a^2d^2\left(dx^2-2c\right)+14abd\left(8c^2-4cdx^2+3d^2x^4\right)-3b^2\left(-8c^2dx^2+16c^3+6cd^2x^4-5d^3x^6\right)\right)}{105d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(35*a^2*d^2*(-2*c + d*x^2) + 14*a*b*d*(8*c^2 - 4*c*d*x^2 +
 3*d^2*x^4) - 3*b^2*(16*c^3 - 8*c^2*d*x^2 + 6*c*d^2*x^4 - 5*d^3*x^6)))/(105
*d^4)
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$-\frac{-15\,b^2x^6d^3-42\,abd^3x^4+18\,b^2cd^2x^4-35\,a^2d^3x^2+56\,abcd^2x^2-24\,b^2c^2dx^2+70\,a^2cd^2-112\,abc^2d+48\,b^2c^3}{105\,d^4}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/105*(d*x^2+c)^(1/2)*(-15*b^2*d^3*x^6-42*a*b*d^3*x^4+18*b^2*c*d^2*x^4-35*
a^2*d^3*x^2+56*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+70*a^2*c*d^2-112*a*b*c^2*d+48
*b^2*c^3)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33319, size = 231, normalized size = 2.06

$$\frac{\left(15\,b^2d^3x^6-48\,b^2c^3+112\,abc^2d-70\,a^2cd^2-6\left(3\,b^2cd^2-7\,abd^3\right)x^4+\left(24\,b^2c^2d-56\,abcd^2+35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 - 48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2 - 6*(3*b^
2*c*d^2 - 7*a*b*d^3)*x^4 + (24*b^2*c^2*d - 56*a*b*c*d^2 + 35*a^2*d^3)*x^2)*
sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 1.48448, size = 240, normalized size = 2.14

$$\begin{cases} -\frac{2a^2c\sqrt{c+dx^2}}{3d^2} + \frac{a^2x^2\sqrt{c+dx^2}}{3d} + \frac{16abc^2\sqrt{c+dx^2}}{15d^3} - \frac{8abcx^2\sqrt{c+dx^2}}{15d^2} + \frac{2abx^4\sqrt{c+dx^2}}{5d} - \frac{16b^2c^3\sqrt{c+dx^2}}{35d^4} + \frac{8b^2c^2x^2\sqrt{c+dx^2}}{35d^3} - \frac{6b^2cx^4\sqrt{c+dx^2}}{35d^2} + \frac{b^2x^6}{} \\ \frac{\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}}{\sqrt{c}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c*sqrt(c + d*x**2)/(3*d**2) + a**2*x**2*sqrt(c + d*x**2)
/(3*d) + 16*a*b*c**2*sqrt(c + d*x**2)/(15*d**3) - 8*a*b*c*x**2*sqrt(c + d*x
**2)/(15*d**2) + 2*a*b*x**4*sqrt(c + d*x**2)/(5*d) - 16*b**2*c**3*sqrt(c +
d*x**2)/(35*d**4) + 8*b**2*c**2*x**2*sqrt(c + d*x**2)/(35*d**3) - 6*b**2*c*
x**4*sqrt(c + d*x**2)/(35*d**2) + b**2*x**6*sqrt(c + d*x**2)/(7*d), Ne(d, 0
)), ((a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/sqrt(c), True))
```

Giac [A] time = 1.13078, size = 203, normalized size = 1.81

$$\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2 - 63\left(dx^2+c\right)^{\frac{5}{2}}b^2c + 105\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2 - 105\sqrt{dx^2+c}b^2c^3 + 42\left(dx^2+c\right)^{\frac{5}{2}}abd - 140\left(dx^2+c\right)^{\frac{3}{2}}abc}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(d*x^2 + c)^(7/2)*b^2 - 63*(d*x^2 + c)^(5/2)*b^2*c + 105*(d*x^2 +
 c)^(3/2)*b^2*c^2 - 105*sqrt(d*x^2 + c)*b^2*c^3 + 42*(d*x^2 + c)^(5/2)*a*b*
d - 140*(d*x^2 + c)^(3/2)*a*b*c*d + 210*sqrt(d*x^2 + c)*a*b*c^2*d + 35*(d*x
^2 + c)^(3/2)*a^2*d^2 - 105*sqrt(d*x^2 + c)*a^2*c*d^2)/d^4
```

3.638 $\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2+bc(5bc-12ad)\right)}{16d^3}-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

```
[Out] ((8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*x*Sqrt[c + d*x^2])/(16*d^3) - (b*(5*b*c
 - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d)
- (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(16*d^(7/2))
```

Rubi [A] time = 0.140198, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {464, 459, 321, 217, 206}

$$\frac{x\sqrt{c+dx^2}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)}{16d}-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*a^2 + (b*c*(5*b*c - 12*a*d))/d^2)*x*Sqrt[c + d*x^2])/(16*d) - (b*(5*b*c
 - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d)
- (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(16*d^(7/2))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{b^2x^5\sqrt{c+dx^2}}{6d} + \frac{\int \frac{x^2\left(6a^2d-b(5bc-12ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{6d}$$
$$= -\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} + \frac{1}{8}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{\left(c\left(5b^2c^2-12abc\right.\right.}{}$$
$$= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{\left(c\left(5b^2c^2-12abc\right.\right.}{}$$
$$= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{c\left(5b^2c^2-12abcd\right.}{}$$

Mathematica [A] time = 0.0859012, size = 125, normalized size = 0.86

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2+12abd\left(2dx^2-3c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-3c\left(8a^2d^2-12abcd+5b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+\right.}{48d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(-3*c + 2*d*x^2) + b^2*(1
5*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - 3*c*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)
*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(7/2))
```

Maple [A] time = 0.009, size = 197, normalized size = 1.4

$$\frac{b^2x^5}{6\,d}\sqrt{dx^2+c} - \frac{5\,b^2cx^3}{24\,d^2}\sqrt{dx^2+c} + \frac{5\,b^2c^2x}{16\,d^3}\sqrt{dx^2+c} - \frac{5\,b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}} + \frac{abx^3}{2\,d}\sqrt{dx^2+c} - \frac{3\,abcx}{4\,d^2}\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b^2*x^5*(d*x^2+c)^(1/2)/d-5/24*b^2*c/d^2*x^3*(d*x^2+c)^(1/2)+5/16*b^2*c
^2/d^3*x*(d*x^2+c)^(1/2)-5/16*b^2*c^3/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
+1/2*a*b*x^3/d*(d*x^2+c)^(1/2)-3/4*a*b*c/d^2*x*(d*x^2+c)^(1/2)+3/4*a*b*c^2/
d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a^2*x/d*(d*x^2+c)^(1/2)-1/2*a^2*c
/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5535, size = 605, normalized size = 4.14

$$\left[\frac{3\left(5\,b^2c^3 - 12\,abc^2d + 8\,a^2cd^2\right)\sqrt{d}\log\left(-2\,dx^2 + 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\left(8\,b^2d^3x^5 - 2\left(5\,b^2cd^2 - 12\,abd^3\right)x^3 + 3\left(5\,b^2c^2d\right.\right.}{96\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b
*d^3)*x^3 + 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/
d^4, 1/48*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b*d^3)*x^3
+ 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] time = 11.6463, size = 301, normalized size = 2.06

$$\frac{a^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2d} - \frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}} - \frac{3abc^{\frac{3}{2}}x}{4d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{ab\sqrt{c}x^3}{4d\sqrt{1+\frac{dx^2}{c}}} + \frac{3abc^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{4d^{\frac{5}{2}}} + \frac{abx^5}{2\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{5b^2c^{\frac{5}{2}}x}{16d^3\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - a**2*c*asinh(sqrt(d)*x/sqrt(c))/(
2*d**(3/2)) - 3*a*b*c**(3/2)*x/(4*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x*
*3/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(5/
2)) + a*b*x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(5/2)*x/(16*d**3*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(3/2)*x**3/(48*d**2*sqrt(1 + d*x**2/c)) - b
**2*sqrt(c)*x**5/(24*d*sqrt(1 + d*x**2/c)) - 5*b**2*c**3*asinh(sqrt(d)*x/sq
rt(c))/(16*d**(7/2)) + b**2*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A] time = 1.15804, size = 182, normalized size = 1.25

$$\frac{1}{48}\left(2\left(\frac{4\,b^2x^2}{d} - \frac{5\,b^2cd^3 - 12\,abd^4}{d^5}\right)x^2 + \frac{3\left(5\,b^2c^2d^2 - 12\,abcd^3 + 8\,a^2d^4\right)}{d^5}\right)\sqrt{dx^2+cx} + \frac{\left(5\,b^2c^3 - 12\,abc^2d + 8\,a^2cd^2\right)\log\left(\right.}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2/d - (5*b^2*c*d^3 - 12*a*b*d^4)/d^5)*x^2 + 3*(5*b^2*c^2*d
^2 - 12*a*b*c*d^3 + 8*a^2*d^4)/d^5)*sqrt(d*x^2 + c)*x + 1/16*(5*b^2*c^3 - 1
2*a*b*c^2*d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.639 $\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2)^(3/2))/(
3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rubi [A] time = 0.0535002, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2)^(3/2))/(
3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2\sqrt{c+dx^2}}{d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{3/2}}{3d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}
\end{aligned}$$

Mathematica [A] time = 0.0406491, size = 66, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abd\left(dx^2-2c\right)+b^2\left(8c^2-4cdx^2+3d^2x^4\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(8*c^2 - 4*c*d
*x^2 + 3*d^2*x^4)))/(15*d^3)
```

Maple [A] time = 0.006, size = 69, normalized size = 0.9

$$\frac{3\,b^2d^2x^4+10\,abd^2x^2-4\,b^2cdx^2+15\,a^2d^2-20\,cabd+8\,b^2c^2}{15\,d^3}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*b^2*d^2*x^4+10*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^2
-20*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35014, size = 151, normalized size = 2.04

$$\frac{\left(3\,b^2d^2x^4+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\left(2\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 0.895434, size = 158, normalized size = 2.14

$$\begin{cases} \frac{a^2\sqrt{c+dx^2}}{d}-\frac{4abc\sqrt{c+dx^2}}{3d^2}+\frac{2abx^2\sqrt{c+dx^2}}{3d}+\frac{8b^2c^2\sqrt{c+dx^2}}{15d^3}-\frac{4b^2cx^2\sqrt{c+dx^2}}{15d^2}+\frac{b^2x^4\sqrt{c+dx^2}}{5d} & \text{for}\ d\neq 0 \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*sqrt(c + d*x**2)/d - 4*a*b*c*sqrt(c + d*x**2)/(3*d**2) + 2*
a*b*x**2*sqrt(c + d*x**2)/(3*d) + 8*b**2*c**2*sqrt(c + d*x**2)/(15*d**3) -
4*b**2*c*x**2*sqrt(c + d*x**2)/(15*d**2) + b**2*x**4*sqrt(c + d*x**2)/(5*d)
, Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/sqrt(c), True))
```

Giac [A] time = 1.12304, size = 132, normalized size = 1.78

$$\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2-10\left(dx^2+c\right)^{\frac{3}{2}}b^2c+15\sqrt{dx^2+c}b^2c^2+10\left(dx^2+c\right)^{\frac{3}{2}}abd-30\sqrt{dx^2+c}abcd+15\sqrt{dx^2+c}a^2d^2}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b^2 - 10*(d*x^2 + c)^(3/2)*b^2*c + 15*sqrt(d*x^2
+ c)*b^2*c^2 + 10*(d*x^2 + c)^(3/2)*a*b*d - 30*sqrt(d*x^2 + c)*a*b*c*d + 15
*sqrt(d*x^2 + c)*a^2*d^2)/d^3
```

3.640 $\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=107

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}$$

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)*Sqrt[c +
d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(8*d^(5/2))
```

Rubi [A] time = 0.0594074, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 217, 206}

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)*Sqrt[c +
d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(8*d^(5/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} + \frac{\int \frac{-a(bc-4ad)-3b(bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{4d}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{8d^2}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\right.}{8d^2}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} + \frac{\left(3b^2c^2-8abcd+8a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}$$

Mathematica [A] time = 0.0515335, size = 91, normalized size = 0.85

$$\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+b\sqrt{d}x\sqrt{c+dx^2}\left(8ad-3bc+2bdx^2\right)}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(-3*b*c + 8*a*d + 2*b*d*x^2) + (3*b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(5/2))
```

Maple [A] time = 0.007, size = 131, normalized size = 1.2

$$\frac{b^2x^3}{4d}\sqrt{dx^2+c} - \frac{3b^2cx}{8d^2}\sqrt{dx^2+c} + \frac{3b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}} + \frac{abx}{d}\sqrt{dx^2+c} - abc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}} + a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*b^2*x^3/d*(d*x^2+c)^(1/2)-3/8*b^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2/d
^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+a*b*x/d*(d*x^2+c)^(1/2)-a*b*c/d^(3/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))+a^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38382, size = 440, normalized size = 4.11

$$\left[\frac{\left(3\,b^2c^2-8\,abcd+8\,a^2d^2\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(2\,b^2d^2x^3-\left(3\,b^2cd-8\,abd^2\right)x\right)\sqrt{dx^2+c}}{16\,d^3},-\frac{\left(3\,b^2c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)*sqr
t(d*x^2 + c))/d^3, -1/8*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)
*sqrt(d*x^2 + c))/d^3]
```

Sympy [A] time = 6.20184, size = 238, normalized size = 2.22

$$a^2\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0\end{cases}\right)+\frac{ab\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{d}-\frac{abc\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{\frac{3}{2}}}-\frac{3b^2c^{\frac{3}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}x^3}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*Piecewise((sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)),
(sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acos
h(x*sqrt(-d/c))/sqrt(-c), (d > 0) & (c < 0))) + a*b*sqrt(c)*x*sqrt(1 + d*x*
*2/c)/d - a*b*c*asinh(sqrt(d)*x/sqrt(c))/d**(3/2) - 3*b**2*c**(3/2)*x/(8*d*
*2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**3/(8*d*sqrt(1 + d*x**2/c)) + 3*b**
2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + b**2*x**5/(4*sqrt(c)*sqrt(1
+ d*x**2/c))
```

Giac [A] time = 1.12756, size = 123, normalized size = 1.15

$$\frac{1}{8}\left(\frac{2\,b^2x^2}{d}-\frac{3\,b^2cd-8\,abd^2}{d^3}\right)\sqrt{dx^2+c}x-\frac{\left(3\,b^2c^2-8\,abcd+8\,a^2d^2\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*x^2/d - (3*b^2*c*d - 8*a*b*d^2)/d^3)*sqrt(d*x^2 + c)*x - 1/8*(3*
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(
5/2)
```

3.641 $\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2))/(3*d^2)
- (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.0706001, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2))/(3*d^2)
- (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b(bc-2ad)}{d\sqrt{c+dx}} + \frac{a^2}{x\sqrt{c+dx}} + \frac{b^2\sqrt{c+dx}}{d}\right)dx, x, x^2\right)$$
$$= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2} + \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d}$$
$$= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.0624067, size = 63, normalized size = 0.84

$$\frac{b\sqrt{c+dx^2}\left(6ad-2bc+bdx^2\right)}{3d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2]*(-2*b*c + 6*a*d + b*d*x^2))/(3*d^2) - (a^2*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Maple [A] time = 0.009, size = 87, normalized size = 1.2

$$\frac{b^2x^2}{3\,d}\sqrt{dx^2+c} - \frac{2\,b^2c}{3\,d^2}\sqrt{dx^2+c} + 2\,\frac{\sqrt{dx^2+c}ab}{d} - a^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*b^2*x^2/d*(d*x^2+c)^(1/2)-2/3*b^2*c/d^2*(d*x^2+c)^(1/2)+2*a*b/d*(d*x^2+
c)^(1/2)-a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33734, size = 365, normalized size = 4.87

$$\left[\frac{3\,a^2\sqrt{c}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\left(b^2cdx^2-2\,b^2c^2+6\,abcd\right)\sqrt{dx^2+c}}{6\,cd^2},\ \frac{3\,a^2\sqrt{-c}d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(b^2cdx^2-2\,b^2c\right.}{3\,cd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2)
 + 2*(b^2*c*d*x^2 - 2*b^2*c^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2), 1/3*(3
*a^2*sqrt(-c)*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c*d*x^2 - 2*b^2*c
^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2)]
```

Sympy [A] time = 25.2199, size = 76, normalized size = 1.01

$$\frac{a^2\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+dx^2}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d^2}+\frac{b\sqrt{c+dx^2}\,(2ad-bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*atan(1/(sqrt(-1/c)*sqrt(c + d*x**2)))/(c*sqrt(-1/c)) + b**2*(c + d*x**
2)**(3/2)/(3*d**2) + b*sqrt(c + d*x**2)*(2*a*d - b*c)/d**2
```

Giac [A] time = 1.13875, size = 111, normalized size = 1.48

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^2+c}b^2cd^4+6\,\sqrt{dx^2+c}abd^5}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/3*((d*x^2 + c)^(3/2)*b^2*
d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 + 6*sqrt(d*x^2 + c)*a*b*d^5)/d^6
```

3.642 $\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) - (b*(b*c -
4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Rubi [A] time = 0.045298, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 388, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) - (b*(b*c -
4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^2\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{\int \frac{2abc+b^2cx^2}{\sqrt{c+dx^2}}\,dx}{c}\\ &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d}\\ &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2d}\\ &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0637257, size = 76, normalized size = 0.93

$$\sqrt{c+dx^2}\left(\frac{b^2x}{2d}-\frac{a^2}{cx}\right)-\frac{b(bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a^2/(c*x)) + (b^2*x)/(2*d))*Sqrt[c + d*x^2] - (b*(b*c - 4*a*d)*Log[d*x +
 Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Maple [A] time = 0.009, size = 88, normalized size = 1.1

$$\frac{b^2x}{2\,d}\sqrt{dx^2+c}-\frac{b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+2\,\frac{ab\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{\sqrt{d}}-\frac{a^2}{cx}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*b^2*x*(d*x^2+c)^(1/2)/d-1/2*b^2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
+2*a*b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-a^2*(d*x^2+c)^(1/2)/c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33145, size = 374, normalized size = 4.56

$$\left[-\frac{\left(b^2c^2-4\,abcd\right)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(b^2cdx^2-2\,a^2d^2\right)\sqrt{dx^2+c}}{4\,cd^2x},\ \frac{\left(b^2c^2-4\,abcd\right)\sqrt{-d}x\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 4*a*b*c*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) - 2*(b^2*c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x), 1/2*(
(b^2*c^2 - 4*a*b*c*d)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*
c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x)]
```

Sympy [A] time = 3.34884, size = 155, normalized size = 1.89

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+2ab\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for } d>0\wedge c<0\end{cases}\right)+\frac{b^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + 2*a*b*Piecewise((sqrt(-c/d)*asin(x*s
qrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(
c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))/sqrt(-c), (d > 0) &
 (c < 0))) + b**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - b**2*c*asinh(sqrt(d)
*x/sqrt(c))/(2*d**(3/2))
```

Giac [A] time = 1.13686, size = 126, normalized size = 1.54

$$\frac{\sqrt{dx^2+c}b^2x}{2\,d}+\frac{2\,a^2\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}+\frac{\left(b^2c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d + 2*a^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^
2 - c) + 1/4*(b^2*c*sqrt(d) - 4*a*b*d^(3/2))*log((sqrt(d)*x - sqrt(d*x^2 +
c))^2)/d^2
```

3.643 $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4*b*c - a*d
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.0676449, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4*b*c - a*d
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}a(4bc-ad)+b^2cx}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{4c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2cd}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0532708, size = 77, normalized size = 0.96

$$\frac{\frac{\sqrt{c}\sqrt{c+dx^2}\left(2b^2cx^2-a^2d\right)}{dx^2}+a(ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[c]*(-(a^2*d) + 2*b^2*c*x^2)*Sqrt[c + d*x^2])/(d*x^2) + a*(-4*b*c + a
*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Maple [A] time = 0.01, size = 100, normalized size = 1.3

$$\frac{b^2}{d}\sqrt{dx^2+c}-2\,\frac{ab}{\sqrt{c}}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)-\frac{a^2}{2\,cx^2}\sqrt{dx^2+c}+\frac{a^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*(d*x^2+c)^(1/2)/d-2*a*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1
/2*a^2*(d*x^2+c)^(1/2)/c/x^2+1/2*a^2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^
(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46884, size = 387, normalized size = 4.84

$$\left[-\frac{\left(4\,abcd-a^2d^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(2\,b^2c^2x^2-a^2cd\right)\sqrt{dx^2+c}}{4\,c^2dx^2},\ \frac{\left(4\,abcd-a^2d^2\right)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+}{2\,c^2dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b*c*d - a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) - 2*(2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2
), 1/2*((4*a*b*c*d - a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + (2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2)]
```

Sympy [A] time = 43.0845, size = 99, normalized size = 1.24

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx}+\frac{a^2d\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2c^{\frac{3}{2}}}-\frac{2ab\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}+b^2\left(\begin{cases}\frac{x^2}{2\sqrt{c}} & \text{for } d=0\\ \frac{\sqrt{c+dx^2}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + a**2*d*asinh(sqrt(c)/(sqrt(d)*
x))/(2*c**(3/2)) - 2*a*b*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) + b**2*Piecewis
e((x**2/(2*sqrt(c)), Eq(d, 0)), (sqrt(c + d*x**2)/d, True))
```

Giac [A] time = 1.12102, size = 109, normalized size = 1.36

$$\frac{2\,\sqrt{dx^2+c}b^2-\frac{\sqrt{dx^2+c}a^2d}{cx^2}+\frac{\left(4\,abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b^2 - sqrt(d*x^2 + c)*a^2*d/(c*x^2) + (4*a*b*c*d - a
^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c))/d
```

3.644 $\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*x^2])/(3*c
^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d]
```

Rubi [A] time = 0.0496488, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 451, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*x^2])/(3*c
^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^4\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{2a(3bc-ad)+3b^2cx^2}{x^2\sqrt{c+dx^2}}\,dx}{3c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.0707773, size = 72, normalized size = 0.86

$$
\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-\frac{a\sqrt{c+dx^2}\left(a\left(c-2dx^2\right)+6bcx^2\right)}{3c^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[c + d*x^2]*(6*b*c*x^2 + a*(c - 2*d*x^2)))/(3*c^2*x^3) + (b^2*Log[d
*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d]
```

Maple [A] time = 0.009, size = 85, normalized size = 1.

$$
b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{3\,cx^3}\sqrt{dx^2+c}+\frac{2\,a^2d}{3\,c^2x}\sqrt{dx^2+c}-2\,\frac{\sqrt{dx^2+c}\,ab}{cx}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/3*a^2*(d*x^2+c)^(1/2)/c/x^3+2/3
*a^2*d/c^2/x*(d*x^2+c)^(1/2)-2*a*b/c/x*(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31875, size = 390, normalized size = 4.64

$$\left[\frac{3\,b^2c^2\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(a^2cd+2\left(3\,abcd-a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{6\,c^2dx^3},-\frac{3\,b^2c^2\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)\ldots}{\ldots}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^2*c^2*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - 2*(a^2*c*d + 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3),
-1/3*(3*b^2*c^2*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d
+ 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3)]
```

Sympy [A] time = 2.20859, size = 158, normalized size = 1.88

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+b^2\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(3*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + b**2*Piecewise(
(sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asin
h(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))
/sqrt(-c), (d > 0) & (c < 0)))
```

Giac [B] time = 1.16775, size = 211, normalized size = 2.51

$$-\frac{b^2\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,\sqrt{d}}+\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ab\sqrt{d}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2\ldots\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d) + 4/3*(3*(sqrt(d)*x -
 sqrt(d*x^2 + c))^4*a*b*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*s
qrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) + 3*a*b*c^2*sqrt(d)
- a^2*c*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

3.645 $\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{8c^2x^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(4*c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*x^2])/(8*c
^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(8*c^(5/2))
```

Rubi [A] time = 0.105793, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$-\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(4*c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*x^2])/(8*c
^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(8*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-3ad)+2b^2cx}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{1}{16}\left(8b^2 - \frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{\left(8b^2 - \frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{8d}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} - \frac{\left(8b^2c^2 - 8abcd + 3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Mathematica [A] time = 0.08643, size = 92, normalized size = 0.87

$$-\frac{\left(3a^2d^2 - 8abcd + 8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{a\sqrt{c+dx^2}\left(2ac - 3adx^2 + 8bcx^2\right)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[c + d*x^2]*(2*a*c + 8*b*c*x^2 - 3*a*d*x^2))/(8*c^2*x^4) - ((8*b^2*
c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5/2))
```

Maple [A] time = 0.012, size = 157, normalized size = 1.5

$$-b^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}} - \frac{a^2}{4cx^4}\sqrt{dx^2+c} + \frac{3a^2d}{8c^2x^2}\sqrt{dx^2+c} - \frac{3a^2d^2}{8}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -b^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/4*a^2*(d*x^2+c)^(1/2)/
c/x^4+3/8*a^2*d/c^2/x^2*(d*x^2+c)^(1/2)-3/8*a^2*d^2/c^(5/2)*ln((2*c+2*c^(1/
```

```
2)*(d*x^2+c)^(1/2))/x)-a*b/c/x^2*(d*x^2+c)^(1/2)+a*b*d/c^(3/2)*ln((2*c+2*c^
(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42033, size = 463, normalized size = 4.37

$$\left[\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\sqrt{c}x^4\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(2\,a^2c^2+\left(8\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{16\,c^3x^4},\ \frac{\left(8\,b^2c^2-8\,abcd+3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^2*c^2 + (8*a*b*c^2 - 3*a^2*c*d)*x^
2)*sqrt(d*x^2 + c))/(c^3*x^4), 1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqr
t(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2*c^2 + (8*a*b*c^2 - 3*a^
2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^4)]
```

Sympy [A] time = 77.5791, size = 178, normalized size = 1.68

$$-\frac{a^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2\sqrt{d}}{8cx^3\sqrt{\frac{c}{dx^2}+1}}+\frac{3a^2d^{\frac{3}{2}}}{8c^2x\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{5}{2}}}-\frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{cx}+\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{c^{\frac{3}{2}}}-\frac{b^2\operatorname{asin}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(8*c*x**3*sqrt(c
/(d*x**2) + 1)) + 3*a**2*d**(3/2)/(8*c**2*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*
d**2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(5/2)) - a*b*sqrt(d)*sqrt(c/(d*x**2)
+ 1)/(c*x) + a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/c**(3/2) - b**2*asinh(sqrt(c)
/(sqrt(d)*x))/sqrt(c)
```

Giac [A] time = 1.13172, size = 189, normalized size = 1.78

$$\frac{\frac{\left(8\,b^2c^2d-8\,abcd^2+3\,a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\,\sqrt{dx^2+c}abc^2d^2-3\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3+5\,\sqrt{dx^2+c}a^2cd^3}{c^2d^2x^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((8*b^2*c^2*d - 8*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(sqrt(-c)*c^2) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*b*
c^2*d^2 - 3*(d*x^2 + c)^(3/2)*a^2*d^3 + 5*sqrt(d*x^2 + c)*a^2*c*d^3)/(c^2*d
^2*x^4))/d
```

3.646 $\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{\sqrt{c+dx^2}\left(15b^2c^2-4ad(5bc-2ad)\right)}{15c^3x}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{15c^2x^3}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(1
5*c^2*x^3) - ((15*b^2*c^2 - 4*a*d*(5*b*c - 2*a*d))*Sqrt[c + d*x^2])/(15*c^3
*x)
```

Rubi [A] time = 0.0717884, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 264}

$$-\frac{\sqrt{c+dx^2}\left(8a^2d^2-20abcd+15b^2c^2\right)}{15c^3x}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(1
5*c^2*x^3) - ((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Sqrt[c + d*x^2])/(15*c^
3*x)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{5cx^5} + \frac{\int \frac{2a(5bc-2ad)+5b^2cx^2}{x^4\sqrt{c+dx^2}}\,dx}{5c}$$
$$= -\frac{a^2\sqrt{c+dx^2}}{5cx^5} - \frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3} - \frac{1}{15}\left(-15b^2 + \frac{4ad(5bc-2ad)}{c^2}\right)\int \frac{1}{x^2\sqrt{c+dx^2}}\,dx$$
$$= -\frac{a^2\sqrt{c+dx^2}}{5cx^5} - \frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3} - \frac{\left(15b^2 - \frac{4ad(5bc-2ad)}{c^2}\right)\sqrt{c+dx^2}}{15cx}$$

Mathematica [A] time = 0.0225581, size = 74, normalized size = 0.75

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2-4cdx^2+8d^2x^4\right)+10abcx^2\left(c-2dx^2\right)+15b^2c^2x^4\right)}{15c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*(3*c^2
 - 4*c*d*x^2 + 8*d^2*x^4)))/(15*c^3*x^5)
```

Maple [A] time = 0.004, size = 78, normalized size = 0.8

$$-\frac{8a^2d^2x^4-20abcdx^4+15b^2c^2x^4-4a^2cdx^2+10ac^2bx^2+3a^2c^2}{15x^5c^3}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(8*a^2*d^2*x^4-20*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*
d*x^2+10*a*b*c^2*x^2+3*a^2*c^2)/x^5/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36814, size = 163, normalized size = 1.65

$$-\frac{\left(\left(15b^2c^2-20abcd+8a^2d^2\right)x^4+3a^2c^2+2\left(5abc^2-2a^2cd\right)x^2\right)\sqrt{dx^2+c}}{15c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*x^4 + 3*a^2*c^2 + 2*(5*a*b*c^2
 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^5)
```

Sympy [B] time = 3.16681, size = 391, normalized size = 3.95

$$-\frac{3a^2c^4d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2a^2c^3d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{3a^2c^2d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{12a^2cd}{15c^5d^4x^4+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**4*d**(9/2)*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**
5*x**6 + 15*c**3*d**6*x**8) - 2*a**2*c**3*d**(11/2)*x**2*sqrt(c/(d*x**2) +
1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 3*a**2*c**
2*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**
6 + 15*c**3*d**6*x**8) - 12*a**2*c*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(15*
c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 8*a**2*d**(17/2)*
x**8*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*
d**6*x**8) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 4*a*b*d**(3/2)
*sqrt(c/(d*x**2) + 1)/(3*c**2) - b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c
```

Giac [B] time = 1.15187, size = 421, normalized size = 4.25

$$\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abd^{\frac{3}{2}}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*sqrt(d) - 60*(sqrt(d)*x - sqrt
(d*x^2 + c))^6*b^2*c*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*d^(3/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 140*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c*d^(3/2) + 80*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d
^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 100*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*c*d^(5/2) + 15*b^2*c^4*sqrt(d) - 20*a*b*c^3*d^(3/2) + 8*a^2*c^2*d^(5/2
))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.647 $\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{c+dx^2}\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}+\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}(12bc}{24c^2x^4}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(6*c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d*x^2])/(24
*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(16*c^3*
x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(16*c^(7/2))
```

Rubi [A] time = 0.161291, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{\sqrt{c+dx^2}\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}+\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}(12bc}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(6*c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d*x^2])/(24
*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(16*c^3*
x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(16*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^7\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^4\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{6cx^6}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}a(12bc-5ad)+3b^2cx}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)}{6c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4}+\frac{1}{16}\left(8b^2-\frac{ad(12bc-5ad)}{c^2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,x^2\right.\\
&= -\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4}-\frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2}+\frac{\left(d\left(-8b^2+\frac{ad(12bc-5}{c^2}\right.\right.}{}\\
&= -\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4}-\frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2}+\frac{\left(-8b^2+\frac{ad(12bc-5ad)}{c^2}\right.}{}\\
&= -\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4}-\frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2}+\frac{d\left(8b^2-\frac{ad(12bc-5ad)}{c^2}\right.}{16c}
\end{aligned}
$$

Mathematica [A] time = 0.23156, size = 135, normalized size = 0.89

$$
\frac{\sqrt{c+dx^2}\left(\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)}{\sqrt{\frac{dx^2}{c}+1}}-\frac{c\left(a^2\left(8c^2-10cdx^2+15d^2x^4\right)+12abcx^2\left(2c-3dx^2\right)+24b^2c^2x^4\right)}{x^6}\right)}{48c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-((c*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c - 3*d*x^2) + a^2
*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4)))/x^6) + (3*d*(8*b^2*c^2 - 12*a*b*c*d +
5*a^2*d^2)*ArcTanh[Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^2)/c]))/(48*c^4)
```

Maple [A] time = 0.012, size = 224, normalized size = 1.5

$$-\frac{a^2}{6cx^6}\sqrt{dx^2+c}+\frac{5a^2d}{24c^2x^4}\sqrt{dx^2+c}-\frac{5a^2d^2}{16c^3x^2}\sqrt{dx^2+c}+\frac{5a^2d^3}{16}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{7}{2}}-\frac{ab}{2cx^4}\sqrt{dx^2+c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(1/2)/c/x^6+5/24*a^2*d/c^2/x^4*(d*x^2+c)^(1/2)-5/16*a^2*
d^2/c^3/x^2*(d*x^2+c)^(1/2)+5/16*a^2*d^3/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c
)^(1/2))/x)-1/2*a*b/c/x^4*(d*x^2+c)^(1/2)+3/4*a*b*d/c^2/x^2*(d*x^2+c)^(1/2)
-3/4*a*b*d^2/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*b^2/c/x^2*(d
*x^2+c)^(1/2)+1/2*b^2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5465, size = 630, normalized size = 4.17

$$\left[\frac{3\left(8b^2c^2d-12abcd^2+5a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2+2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-2\left(8a^2c^3+3\left(8b^2c^3-12abc^2d+5a^2cd^2\right)x^4+2\left(12a\right.\right.}{96c^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a
*b*c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2
+ c))/(c^4*x^6), -1/48*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(-c)
*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a*b*
c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
))/(c^4*x^6)]
```

Sympy [B] time = 140.095, size = 301, normalized size = 1.99

$$-\frac{a^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2\sqrt{d}}{24cx^5\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2d^{\frac{3}{2}}}{48c^2x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2d^{\frac{5}{2}}}{16c^3x\sqrt{\frac{c}{dx^2}+1}}+\frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{7}{2}}}-\frac{ab}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{}{4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(24*c*x**5*sqrt(
c/(d*x**2) + 1)) - 5*a**2*d**(3/2)/(48*c**2*x**3*sqrt(c/(d*x**2) + 1)) - 5*
a**2*d**(5/2)/(16*c**3*x*sqrt(c/(d*x**2) + 1)) + 5*a**2*d**3*asinh(sqrt(c)/
(sqrt(d)*x))/(16*c**(7/2)) - a*b/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a*
b*sqrt(d)/(4*c*x**3*sqrt(c/(d*x**2) + 1)) + 3*a*b*d**(3/2)/(4*c**2*x*sqrt(c
/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(5/2)) - b**2
*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(
2*c**(3/2))
```

Giac [A] time = 1.17442, size = 325, normalized size = 2.15

$$-\frac{\frac{3\left(8b^2c^2d^2-12abcd^3+5a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2-36\left(dx^2+c\right)^{\frac{5}{2}}abcd^3+96\left(dx^2+c\right)^{\frac{3}{2}}abc^2d^3-60\sqrt{d}}{c^3d^3x^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 5*a^2*d^4)*arctan(sqrt(d*x^2 + c)/
sqrt(-c))/(sqrt(-c)*c^3) + (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 +
c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 - 36*(d*x^2 + c)^(5/2
)*a*b*c*d^3 + 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 - 60*sqrt(d*x^2 + c)*a*b*c^3
*d^3 + 15*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 + 33*s
qrt(d*x^2 + c)*a^2*c^2*d^4)/(c^3*d^3*x^6))/d
```

3.648 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{x^3\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}+\frac{x\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\,\mathrm{t}}{16d^{9/2}}$$

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a*b*c*d + 2
4*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 - 60*a*b*c*d + 24*a^2
*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d^2) -
 (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(16*d^(9/2))
```

Rubi [A] time = 0.149479, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 459, 321, 217, 206}

$$-\frac{x^3\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}+\frac{x\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\,\mathrm{t}}{16d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a*b*c*d + 2
4*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 - 60*a*b*c*d + 24*a^2
*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d^2) -
 (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(16*d^(9/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^4\left(-a^2d^2+5(bc-ad)^2-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)\int \frac{x^4}{\sqrt{c+dx^2}}\,dx}{6cd^2} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x^3\sqrt{c+dx^2}}{24cd^3} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} + \frac{\left(35b^2c^2-60abcd+24\right.}{8d^3} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x^3\sqrt{c+dx^2}}{24cd^3} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x^3\sqrt{c+dx^2}}{24cd^3} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x^3\sqrt{c+dx^2}}{24cd^3}
\end{aligned}$$

Mathematica [A] time = 0.145878, size = 158, normalized size = 0.8

$$\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2-28abcd+19b^2c^2\right)}{16d^4} - \frac{bx^3(11bc-12ad)}{24d^3} + \frac{cx(bc-ad)^2}{d^4\left(c+dx^2\right)} + \frac{b^2x^5}{6d^2}\right) - \frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\log\left(\right.}{16d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(((19*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*x)/(16*d^4) - (b*(1
1*b*c - 12*a*d)*x^3)/(24*d^3) + (b^2*x^5)/(6*d^2) + (c*(b*c - a*d)^2*x)/(d^
4*(c + d*x^2))) - (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*Log[d*x + Sqrt[
d]*Sqrt[c + d*x^2]])/(16*d^(9/2))
```

Maple [A] time = 0.015, size = 263, normalized size = 1.3

$$\frac{b^2x^7}{6\,d}\frac{1}{\sqrt{dx^2+c}} - \frac{7\,b^2cx^5}{24\,d^2}\frac{1}{\sqrt{dx^2+c}} + \frac{35\,x^3b^2c^2}{48\,d^3}\frac{1}{\sqrt{dx^2+c}} + \frac{35\,b^2c^3x}{16\,d^4}\frac{1}{\sqrt{dx^2+c}} - \frac{35\,b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}} + \frac{abx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/6*b^2*x^7/d/(d*x^2+c)^(1/2)-7/24*b^2*c/d^2*x^5/(d*x^2+c)^(1/2)+35/48*b^2*
c^2/d^3*x^3/(d*x^2+c)^(1/2)+35/16*b^2*c^3/d^4*x/(d*x^2+c)^(1/2)-35/16*b^2*c
^3/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a*b*x^5/d/(d*x^2+c)^(1/2)-5/4*
a*b*c/d^2*x^3/(d*x^2+c)^(1/2)-15/4*a*b*c^2/d^3*x/(d*x^2+c)^(1/2)+15/4*a*b*c
^2/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a^2*x^3/d/(d*x^2+c)^(1/2)+3/2*
a^2*c/d^2*x/(d*x^2+c)^(1/2)-3/2*a^2*c/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69173, size = 950, normalized size = 4.82

$$\left[\frac{3\left(35\,b^2c^4 - 60\,abc^3d + 24\,a^2c^2d^2 + \left(35\,b^2c^3d - 60\,abc^2d^2 + 24\,a^2cd^3\right)x^2\right)\sqrt{d}\log\left(-2\,dx^2 + 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\left(\right.}{96\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(35*b^2*c^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a
*b*c^2*d^2 + 24*a^2*c*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) + 2*(8*b^2*d^4*x^7 - 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^
2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d
^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^6*x^2 + c*d^5), 1/48*(3*(35*b^2*c
^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^
2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^4*x^7
- 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^
2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2
 + c))/(d^6*x^2 + c*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(a + bx^2\right)^2}{\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A] time = 1.13322, size = 236, normalized size = 1.2

$$\frac{\left(\left(2\left(\frac{4b^2x^2}{d}-\frac{7b^2cd^5-12abd^6}{d^7}\right)x^2+\frac{35b^2c^2d^4-60abcd^5+24a^2d^6}{d^7}\right)x^2+\frac{3\left(35b^2c^3d^3-60abc^2d^4+24a^2cd^5\right)}{d^7}\right)x}{48\sqrt{dx^2+c}}+\frac{\left(35b^2c^3-60abc^2d+24a^2cd^2\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*b^2*x^2/d - (7*b^2*c*d^5 - 12*a*b*d^6)/d^7)*x^2 + (35*b^2*c^2*d
^4 - 60*a*b*c*d^5 + 24*a^2*d^6)/d^7)*x^2 + 3*(35*b^2*c^3*d^3 - 60*a*b*c^2*d
^4 + 24*a^2*c*d^5)/d^7)*x/sqrt(d*x^2 + c) + 1/16*(35*b^2*c^3 - 60*a*b*c^2*d
 + 24*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.649 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{b\left(c+dx^2\right)^{3/2}(3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a*d)*Sqrt[c
 + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^4) + (b^2*(c +
d*x^2)^(5/2))/(5*d^4)
```

Rubi [A] time = 0.0859802, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{3/2}(3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a*d)*Sqrt[c
 + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^4) + (b^2*(c +
d*x^2)^(5/2))/(5*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3(c+dx)^{3/2}}+\frac{(bc-ad)(3bc-ad)}{d^3\sqrt{c+dx}}-\frac{b(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{b^2(c+dx)^{3/2}}{d^3}\right)dx\right.\\
&= \frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{(bc-ad)(3bc-ad)\sqrt{c+dx^2}}{d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}
\end{aligned}$$

Mathematica [A] time = 0.0608721, size = 97, normalized size = 0.9

$$\frac{15a^2d^2(2c+dx^2)+10abd(-8c^2-4cdx^2+d^2x^4)+3b^2(8c^2dx^2+16c^3-2cd^2x^4+d^3x^6)}{15d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (15*a^2*d^2*(2*c + d*x^2) + 10*a*b*d*(-8*c^2 - 4*c*d*x^2 + d^2*x^4) + 3*b^2
*(16*c^3 + 8*c^2*d*x^2 - 2*c*d^2*x^4 + d^3*x^6))/(15*d^4*Sqrt[c + d*x^2])
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$\frac{3\,b^2x^6d^3+10\,abd^3x^4-6\,b^2cd^2x^4+15\,a^2d^3x^2-40\,abcd^2x^2+24\,b^2c^2dx^2+30\,a^2cd^2-80\,abc^2d+48\,b^2c^3}{15\,d^4}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(3*b^2*d^3*x^6+10*a*b*d^3*x^4-6*b^2*c*d^2*x^4+15*a^2*d^3*x^2-40*a*b*c*
d^2*x^2+24*b^2*c^2*d*x^2+30*a^2*c*d^2-80*a*b*c^2*d+48*b^2*c^3)/(d*x^2+c)^(1
/2)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3194, size = 246, normalized size = 2.28

$$\frac{\left(3\,b^2d^3x^6+48\,b^2c^3-80\,abc^2d+30\,a^2cd^2-2\left(3\,b^2cd^2-5\,abd^3\right)x^4+\left(24\,b^2c^2d-40\,abcd^2+15\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{15\left(d^5x^2+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^3*x^6 + 48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 - 2*(3*b^2*c
*d^2 - 5*a*b*d^3)*x^4 + (24*b^2*c^2*d - 40*a*b*c*d^2 + 15*a^2*d^3)*x^2)*sqr
t(d*x^2 + c)/(d^5*x^2 + c*d^4)
```

Sympy [A] time = 1.74068, size = 236, normalized size = 2.19

$$\begin{cases} \frac{2a^2c}{d^2\sqrt{c+dx^2}} + \frac{a^2x^2}{d\sqrt{c+dx^2}} - \frac{16abc^2}{3d^3\sqrt{c+dx^2}} - \frac{8abcx^2}{3d^2\sqrt{c+dx^2}} + \frac{2abx^4}{3d\sqrt{c+dx^2}} + \frac{16b^2c^3}{5d^4\sqrt{c+dx^2}} + \frac{8b^2c^2x^2}{5d^3\sqrt{c+dx^2}} - \frac{2b^2cx^4}{5d^2\sqrt{c+dx^2}} + \frac{b^2x^6}{5d\sqrt{c+dx^2}} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((2*a**2*c/(d**2*sqrt(c + d*x**2)) + a**2*x**2/(d*sqrt(c + d*x**2)
) - 16*a*b*c**2/(3*d**3*sqrt(c + d*x**2)) - 8*a*b*c*x**2/(3*d**2*sqrt(c + d
*x**2)) + 2*a*b*x**4/(3*d*sqrt(c + d*x**2)) + 16*b**2*c**3/(5*d**4*sqrt(c +
 d*x**2)) + 8*b**2*c**2*x**2/(5*d**3*sqrt(c + d*x**2)) - 2*b**2*c*x**4/(5*d
**2*sqrt(c + d*x**2)) + b**2*x**6/(5*d*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2
*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/c**(3/2), True))
```

Giac [A] time = 1.13426, size = 180, normalized size = 1.67

$$\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2 - 15\left(dx^2+c\right)^{\frac{3}{2}}b^2c + 45\sqrt{dx^2+c}b^2c^2 + 10\left(dx^2+c\right)^{\frac{3}{2}}abd - 60\sqrt{dx^2+c}abcd + 15\sqrt{dx^2+c}a^2d^2 + \frac{15(}{}}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b^2 - 15*(d*x^2 + c)^(3/2)*b^2*c + 45*sqrt(d*x^2
+ c)*b^2*c^2 + 10*(d*x^2 + c)^(3/2)*a*b*d - 60*sqrt(d*x^2 + c)*a*b*c*d + 15
*sqrt(d*x^2 + c)*a^2*d^2 + 15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/sqrt(d*x^
2 + c))/d^4
```

3.650 $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{x\sqrt{c+dx^2}\left(8a^2d^2-24abcd+15b^2c^2\right)}{8cd^3}+\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a*b*c*d + 8
*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[c + d*x^2])/(4*d^2)
+ ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(8*d^(7/2))
```

Rubi [A] time = 0.117052, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 459, 321, 217, 206}

$$-\frac{x\sqrt{c+dx^2}\left(8a^2d^2-24abcd+15b^2c^2\right)}{8cd^3}+\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a*b*c*d + 8
*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[c + d*x^2])/(4*d^2)
+ ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(8*d^(7/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^2\left(-a^2d^2+3(bc-ad)^2-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)\int \frac{x^2}{\sqrt{c+dx^2}}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abcd+8\ldots\right.}{8d^3}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abcd+8\ldots\right.}{\ldots}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abcd+8\ldots\right.}{8\ldots}$$

Mathematica [A] time = 0.111202, size = 124, normalized size = 0.82

$$\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{7/2}} + \sqrt{c+dx^2}\left(-\frac{x(ad-bc)^2}{d^3\left(c+dx^2\right)} - \frac{bx(7bc-8ad)}{8d^3} + \frac{b^2x^3}{4d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(b*(7*b*c - 8*a*d)*x)/(8*d^3) + (b^2*x^3)/(4*d^2) - ((-(b
*c) + a*d)^2*x)/(d^3*(c + d*x^2))) + ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)
*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(7/2))
```

Maple [A] time = 0.008, size = 192, normalized size = 1.3

$$\frac{b^2x^5}{4d}\frac{1}{\sqrt{dx^2+c}} - \frac{5b^2cx^3}{8d^2}\frac{1}{\sqrt{dx^2+c}} - \frac{15b^2c^2x}{8d^3}\frac{1}{\sqrt{dx^2+c}} + \frac{15b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}} + \frac{abx^3}{d}\frac{1}{\sqrt{dx^2+c}} + 3\,\frac{\ldots}{d^2\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4*b^2*x^5/d/(d*x^2+c)^(1/2)-5/8*b^2*c/d^2*x^3/(d*x^2+c)^(1/2)-15/8*b^2*c^
2/d^3*x/(d*x^2+c)^(1/2)+15/8*b^2*c^2/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+
```

```
a*b*x^3/d/(d*x^2+c)^(1/2)+3*a*b*c/d^2*x/(d*x^2+c)^(1/2)-3*a*b*c/d^(5/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))-a^2*x/d/(d*x^2+c)^(1/2)+a^2/d^(3/2)*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50125, size = 759, normalized size = 4.99

$$\left[\frac{\left(15\,b^2c^3-24\,abc^2d+8\,a^2cd^2+\left(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3\right)x^2\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(2\,b^2d^3x^5-\left(\cdots\right.\right.}{16\left(d^5x^2+cd^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*
d^2 + 8*a^2*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(2*b^2*d^3*x^5 - (5*b^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*
a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/(d^5*x^2 + c*d^4), -1/8*((15*b^2
*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^
3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^3*x^5 - (5*b
^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^3)*x)*sq
rt(d*x^2 + c))/(d^5*x^2 + c*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A] time = 1.12833, size = 177, normalized size = 1.16

$$\frac{\left(\left(\frac{2\,b^2x^2}{d}-\frac{5\,b^2cd^3-8\,abd^4}{d^5}\right)x^2-\frac{15\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4}{d^5}\right)x}{8\,\sqrt{dx^2+c}}-\frac{\left(15\,b^2c^2-24\,abcd+8\,a^2d^2\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^2*x^2/d - (5*b^2*c*d^3 - 8*a*b*d^4)/d^5)*x^2 - (15*b^2*c^2*d^2 -
24*a*b*c*d^3 + 8*a^2*d^4)/d^5)*x/sqrt(d*x^2 + c) - 1/8*(15*b^2*c^2 - 24*a*b
*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.651 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c + d*x^2])/
d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rubi [A] time = 0.0556265, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c + d*x^2])/
d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx, x, x^2\right)\\
&= -\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}-\frac{2b(bc-ad)\sqrt{c+dx^2}}{d^3}+\frac{b^2(c+dx^2)^{3/2}}{3d^3}
\end{aligned}$$

Mathematica [A] time = 0.0382335, size = 65, normalized size = 0.89

$$\frac{-3a^2d^2 + 6abd\left(2c + dx^2\right) + b^2\left(-8c^2 - 4cdx^2 + d^2x^4\right)}{3d^3\sqrt{c + dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*d^2 + 6*a*b*d*(2*c + d*x^2) + b^2*(-8*c^2 - 4*c*d*x^2 + d^2*x^4))/(
3*d^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.007, size = 69, normalized size = 1.

$$-\frac{-b^2d^2x^4 - 6\,abd^2x^2 + 4\,b^2cdx^2 + 3\,a^2d^2 - 12\,cabd + 8\,b^2c^2}{3\,d^3}\frac{1}{\sqrt{dx^2 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-b^2*d^2*x^4-6*a*b*d^2*x^2+4*b^2*c*d*x^2+3*a^2*d^2-12*a*b*c*d+8*b^2*c
^2)/(d*x^2+c)^(1/2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35808, size = 165, normalized size = 2.26

$$\frac{\left(b^2d^2x^4 - 8\,b^2c^2 + 12\,abcd - 3\,a^2d^2 - 2\left(2\,b^2cd - 3\,abd^2\right)x^2\right)\sqrt{dx^2 + c}}{3\left(d^4x^2 + cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^2*x^4 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^4*x^2 + c*d^3)
```

Sympy [A] time = 1.01098, size = 155, normalized size = 2.12

$$\begin{cases} -\frac{a^2}{d\sqrt{c+dx^2}} + \frac{4abc}{d^2\sqrt{c+dx^2}} + \frac{2abx^2}{d\sqrt{c+dx^2}} - \frac{8b^2c^2}{3d^3\sqrt{c+dx^2}} - \frac{4b^2cx^2}{3d^2\sqrt{c+dx^2}} + \frac{b^2x^4}{3d\sqrt{c+dx^2}} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-a**2/(d*sqrt(c + d*x**2)) + 4*a*b*c/(d**2*sqrt(c + d*x**2)) + 2
*a*b*x**2/(d*sqrt(c + d*x**2)) - 8*b**2*c**2/(3*d**3*sqrt(c + d*x**2)) - 4*
b**2*c*x**2/(3*d**2*sqrt(c + d*x**2)) + b**2*x**4/(3*d*sqrt(c + d*x**2)), N
e(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(3/2), True))
```

Giac [A] time = 1.13702, size = 108, normalized size = 1.48

$$\frac{\left(dx^2 + c\right)^{\frac{3}{2}} b^2 - 6\sqrt{dx^2 + c} b^2 c + 6\sqrt{dx^2 + c} abd - \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)}{\sqrt{dx^2 + c}}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^2 + c)^(3/2)*b^2 - 6*sqrt(d*x^2 + c)*b^2*c + 6*sqrt(d*x^2 + c)*a*
b*d - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/sqrt(d*x^2 + c))/d^3
```

3.652 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{bx\sqrt{c+dx^2}(3bc-2ad)}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c - 2*a*d)*x
*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*d^(5/2))
```

Rubi [A] time = 0.0539976, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 388, 217, 206}

$$\frac{bx\sqrt{c+dx^2}(3bc-2ad)}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c - 2*a*d)*x
*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*d^(5/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{\int \frac{abc+b(3bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0998388, size = 93, normalized size = 0.88

$$\sqrt{c+dx^2}\left(\frac{x(bc-ad)^2}{cd^2\left(c+dx^2\right)}+\frac{b^2x}{2d^2}\right)-\frac{b(3bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*((b^2*x)/(2*d^2) + ((b*c - a*d)^2*x)/(c*d^2*(c + d*x^2))) -
 (b*(3*b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(5/2))
```

Maple [A] time = 0.005, size = 123, normalized size = 1.2

$$\frac{b^2x^3}{2d}\frac{1}{\sqrt{dx^2+c}}+\frac{3b^2cx}{2d^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}-2\frac{abx}{d\sqrt{dx^2+c}}+2\frac{ab\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{3/2}}+\frac{a^2x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*b^2*x^3/d/(d*x^2+c)^(1/2)+3/2*b^2*c/d^2*x/(d*x^2+c)^(1/2)-3/2*b^2*c/d^(
5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-2*a*b*x/d/(d*x^2+c)^(1/2)+2*a*b/d^(3/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))+a^2*x/c/(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36837, size = 595, normalized size = 5.61

$$\left[-\frac{\left(3 b^2 c^3 - 4 abc^2 d + \left(3 b^2 c^2 d - 4 abcd^2\right) x^2\right) \sqrt{d} \log\left(-2 dx^2 - 2 \sqrt{dx^2 + c} \sqrt{d} x - c\right) - 2 \left(b^2 cd^2 x^3 + \left(3 b^2 c^2 d - 4 abcd^2 + \cdots\right.\right.}{4 \left(cd^4 x^2 + c^2 d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(d)*
log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(b^2*c*d^2*x^3 + (3*b^2
*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)
, 1/2*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c*d^2*x^3 + (3*b^2*c^2*d - 4*a*b
*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{\left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A] time = 1.13102, size = 124, normalized size = 1.17

$$\frac{\left(\frac{b^2 x^2}{d} + \frac{3 b^2 c^2 d - 4 abcd^2 + 2 a^2 d^3}{cd^3}\right) x}{2 \sqrt{dx^2 + c}} + \frac{\left(3 b^2 c - 4 abd\right) \log\left(\left|-\sqrt{d} x + \sqrt{dx^2 + c}\right|\right)}{2 d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2/d + (3*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)/(c*d^3))*x/sqrt(d*
x^2 + c) + 1/2*(3*b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d
^(5/2)
```

3.653 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2 - (a^2*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Rubi [A] time = 0.0750599, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```


```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2 - (a^2*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{3/2}} + \frac{b^2}{d\sqrt{c+dx}} + \frac{a^2}{cx\sqrt{c+dx}}\right) dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}$$
$$= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd}$$
$$= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

Mathematica [C] time = 0.040323, size = 62, normalized size = 0.83

$$\frac{a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right) + bc\left(-2ad+2bc+bdx^2\right)}{cd^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c*(2*b*c - 2*a*d + b*d*x^2) + a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ (d*x^2)/c])/(c*d^2*Sqrt[c + d*x^2])
```

Maple [A] time = 0.009, size = 102, normalized size = 1.4

$$\frac{b^2x^2}{d}\frac{1}{\sqrt{dx^2+c}} + 2\frac{b^2c}{d^2\sqrt{dx^2+c}} - 2\frac{ab}{d\sqrt{dx^2+c}} + \frac{a^2}{c}\frac{1}{\sqrt{dx^2+c}} - a^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x)
```

```
[Out] b^2*x^2/d/(d*x^2+c)^(1/2)+2*b^2*c/d^2/(d*x^2+c)^(1/2)-2*a*b/d/(d*x^2+c)^(1/
2)+a^2/c/(d*x^2+c)^(1/2)-a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34391, size = 489, normalized size = 6.52

$$\left[\frac{\left(a^2d^3x^2+a^2cd^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(b^2c^2dx^2+2b^2c^3-2abc^2d+a^2cd^2\right)\sqrt{dx^2+c}}{2\left(c^2d^3x^2+c^3d^2\right)}, \frac{\left(a^2d^3x^2+a^2cd^2\right)\sqrt{-c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(c) + 2*c)/x^2) + 2*(b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2), ((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2)]
```

Sympy [A] time = 17.9242, size = 70, normalized size = 0.93

$$\frac{a^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{c\sqrt{-c}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{(ad-bc)^2}{cd^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c*sqrt(-c)) + b**2*sqrt(c + d*x**2)/d
**2 + (a*d - b*c)**2/(c*d**2*sqrt(c + d*x**2))
```

Giac [A] time = 1.13746, size = 111, normalized size = 1.48

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{\sqrt{dx^2+c}b^2}{d^2}+\frac{b^2c^2-2abcd+a^2d^2}{\sqrt{dx^2+c}cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) + sqrt(d*x^2 + c)*b^2/d^2
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c*d^2)
```

3.654 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(b^2c^2-2ad(bc-ad)\right)}{c^2d\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2*c^2 - 2*a*d*(b*c - a*d))*x)/(c^2*d*Sqr
t[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(3/2)
```

Rubi [A] time = 0.0634586, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 385, 217, 206}

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2/d - (2*a*(b*c - a*d))/c^2)*x)/Sqrt[c +
 d*x^2] + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(3/2)
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = -\frac{a^2}{cx\sqrt{c+dx^2}} + \frac{\int \frac{2a(bc-ad)+b^2cx^2}{(c+dx^2)^{3/2}}\,dx}{c}$$

$$= -\frac{a^2}{cx\sqrt{c+dx^2}} - \frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}} + \frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d}$$

$$= -\frac{a^2}{cx\sqrt{c+dx^2}} - \frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{d}$$

$$= -\frac{a^2}{cx\sqrt{c+dx^2}} - \frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Mathematica [A] time = 0.0851169, size = 81, normalized size = 0.89

$$\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}} - \frac{\sqrt{c+dx^2}\left(a^2+\frac{x^2(bc-ad)^2}{d(c+dx^2)}\right)}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[c + d*x^2]*(a^2 + ((b*c - a*d)^2*x^2)/(d*(c + d*x^2))))/(c^2*x)) +
(b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(3/2)
```

Maple [A] time = 0.009, size = 99, normalized size = 1.1

$$-\frac{b^2x}{d}\frac{1}{\sqrt{dx^2+c}} + b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}} + 2\frac{abx}{c\sqrt{dx^2+c}} - \frac{a^2}{cx}\frac{1}{\sqrt{dx^2+c}} - 2\frac{a^2dx}{c^2\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2*x/d/(d*x^2+c)^(1/2)+b^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+2*a*b*x/
c/(d*x^2+c)^(1/2)-a^2/c/x/(d*x^2+c)^(1/2)-2*a^2*d/c^2*x/(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37992, size = 504, normalized size = 5.54

$$\left[\frac{\left(b^2c^2dx^3 + b^2c^3x\right)\sqrt{d}\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) - 2\left(a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + 2\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{2\left(c^2d^3x^3 + c^3d^2x\right)}, -\frac{\left(b^2c^2d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(d)*x - c) - 2*(a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 2*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x), -((b^2*c^2*d*x^3 + b^2*c^3*x)*s
qrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d^2 + (b^2*c^2*d - 2*a*
b*c*d^2 + 2*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^2\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.1504, size = 140, normalized size = 1.54

$$-\frac{b^2\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{2\,d^{\frac{3}{2}}} + \frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c\right)c} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{\sqrt{dx^2+c}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2) + 2*a^2*sqrt(d)/(((sq
rt(d)*x - sqrt(d*x^2 + c))^2 - c)*c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(s
qrt(d*x^2 + c)*c^2*d)
```

3.655 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{-\frac{3a^2d}{c}+4ab-\frac{2b^2c}{d}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

```
[Out] (4*a*b - (2*b^2*c)/d - (3*a^2*d)/c)/(2*c*Sqrt[c + d*x^2]) - a^2/(2*c*x^2*Sq
rt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^
(5/2))
```

Rubi [A] time = 0.0976093, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$\frac{-\frac{3a^2d}{c}+4ab-\frac{2b^2c}{d}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (3*a^2*d)/c)/(2*c*Sqrt[c + d*x^2]) - a^2/(2*c*x^2*Sq
rt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^
(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-3ad)+b^2cx}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{2c}$$

$$= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4c^2}$$

$$= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2c^2d}$$

$$= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} - \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Mathematica [A] time = 0.0826468, size = 96, normalized size = 0.93

$$\frac{a(3ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}} - \frac{a^2d\left(c+3dx^2\right)-4abcdx^2+2b^2c^2x^2}{2c^2dx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(c + 3*d*x^2))/(2*c^2*d*x^2*Sqrt[c
+ d*x^2]) + (a*(-4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(5/2
))
```

Maple [A] time = 0.009, size = 135, normalized size = 1.3

$$-\frac{b^2}{d}\frac{1}{\sqrt{dx^2+c}}+2\,\frac{ab}{c\sqrt{dx^2+c}}-2\,\frac{ab}{c^{3/2}}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)-\frac{a^2}{2\,cx^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3\,a^2d}{2\,c^2}\frac{1}{\sqrt{dx^2+c}}+\frac{3\,a^2d}{2}\ln\left(\frac{1}{x}\left(2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2/d/(d*x^2+c)^(1/2)+2*a*b/c/(d*x^2+c)^(1/2)-2*a*b/c^(3/2)*ln((2*c+2*c^(1
/2)*(d*x^2+c)^(1/2))/x)-1/2*a^2/c/x^2/(d*x^2+c)^(1/2)-3/2*a^2*d/c^2/(d*x^2+
c)^(1/2)+3/2*a^2*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40858, size = 620, normalized size = 6.02

$$\left[-\frac{\left(\left(4\,abcd^2 - 3\,a^2d^3\right)x^4 + \left(4\,abc^2d - 3\,a^2cd^2\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right) + 2\,\left(a^2c^2d + \left(2\,b^2c^3 - 4\,abc^2d + 3\,a^2cd^2\right)x^2\right)}{4\,\left(c^3d^2x^4 + c^4dx^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c^2*d +
(2*b^2*c^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4
+ c^4*d*x^2), 1/2*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*
d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (a^2*c^2*d + (2*b^2*c
^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4 + c^4*d*
x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^3\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.15972, size = 189, normalized size = 1.83

$$\frac{(4\,abc - 3\,a^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}c^2} - \frac{2\,(dx^2 + c)b^2c^2 - 2\,b^2c^3 - 4\,(dx^2 + c)abcd + 4\,abc^2d + 3\,(dx^2 + c)a^2d^2 - 2\,a^2cd^2}{2\left((dx^2 + c)^{\frac{3}{2}} - \sqrt{dx^2 + c}c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1
/2*(2*(d*x^2 + c)*b^2*c^2 - 2*b^2*c^3 - 4*(d*x^2 + c)*a*b*c*d + 4*a*b*c^2*d
 + 3*(d*x^2 + c)*a^2*d^2 - 2*a^2*c*d^2)/(((d*x^2 + c)^(3/2) - sqrt(d*x^2 +
c)*c)*c^2*d)
```

3.656 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{x\left(3b^2c^2-4ad(3bc-2ad)\right)}{3c^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

```
[Out] -a^2/(3*c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*Sqrt[c + d*
x^2]) + ((3*b^2*c^2 - 4*a*d*(3*b*c - 2*a*d))*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0720084, antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 191}

$$\frac{x\left(8a^2d^2-12abcd+3b^2c^2\right)}{3c^3\sqrt{c+dx^2}}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(3*c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*Sqrt[c + d*
x^2]) + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = -\frac{a^2}{3cx^3\sqrt{c+dx^2}} + \frac{\int \frac{2a(3bc-2ad)+3b^2cx^2}{x^2(c+dx^2)^{3/2}}\,dx}{3c}$$
$$= -\frac{a^2}{3cx^3\sqrt{c+dx^2}} - \frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}} - \frac{1}{3}\left(-3b^2+\frac{4ad(3bc-2ad)}{c^2}\right)\int \frac{1}{(c+dx^2)^{3/2}}\,dx$$
$$= -\frac{a^2}{3cx^3\sqrt{c+dx^2}} - \frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}} + \frac{\left(3b^2-\frac{4ad(3bc-2ad)}{c^2}\right)x}{3c\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0236606, size = 74, normalized size = 0.76

$$\frac{a^2(-c^2+4cdx^2+8d^2x^4)-6abcx^2(c+2dx^2)+3b^2c^2x^4}{3c^3x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] (3*b^2*c^2*x^4 - 6*a*b*c*x^2*(c + 2*d*x^2) + a^2*(-c^2 + 4*c*d*x^2 + 8*d^2*
x^4))/(3*c^3*x^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.004, size = 77, normalized size = 0.8

$$-\frac{-8a^2d^2x^4+12abcdx^4-3b^2c^2x^4-4a^2cdx^2+6ac^2bx^2+a^2c^2}{3x^3c^3}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-8*a^2*d^2*x^4+12*a*b*c*d*x^4-3*b^2*c^2*x^4-4*a^2*c*d*x^2+6*a*b*c^2*x
^2+a^2*c^2)/x^3/(d*x^2+c)^(1/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36997, size = 173, normalized size = 1.78

$$\frac{\left((3b^2c^2-12abcd+8a^2d^2)x^4-a^2c^2-2(3abc^2-2a^2cd)x^2\right)\sqrt{dx^2+c}}{3(c^3dx^5+c^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^4 - a^2*c^2 - 2*(3*a*b*c^2 - 2*
a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d*x^5 + c^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^4 \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

Giac [B] time = 1.13776, size = 269, normalized size = 2.77

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{\sqrt{dx^2 + c}c^3} + \frac{2\left(6\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 abc\sqrt{d} - 3\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 a^2d^{\frac{3}{2}} - 12\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 abc^2\sqrt{d} + \right.}{3\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c\right)^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^3) + 2/3*(6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 12*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 6*a*b*c^3*sqrt(d) - 5*a^2*c^2*d^(3/
2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^2)
```

3.657 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{8b^2c^2-3ad(8bc-5ad)}{8c^3\sqrt{c+dx^2}}-\frac{\left(8b^2c^2-3ad(8bc-5ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

```
[Out] (8*b^2*c^2 - 3*a*d*(8*b*c - 5*a*d))/(8*c^3*Sqrt[c + d*x^2]) - a^2/(4*c*x^4*
Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[c + d*x^2]) - ((8*b^
2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2)
)
```

Rubi [A] time = 0.167187, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}}-\frac{\left(8b^2c^2-3ad(8bc-5ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (8*b^2 - (3*a*d*(8*b*c - 5*a*d))/c^2)/(8*c*Sqrt[c + d*x^2]) - a^2/(4*c*x^4*
Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[c + d*x^2]) - ((8*b^
2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}a(8bc-5ad)+2b^2cx}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{1}{16}\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\operatorname{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.}{16c}$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,\right.}{8cd}$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} - \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Mathematica [C] time = 0.0341562, size = 89, normalized size = 0.61

$$\frac{x^4\left(15a^2d^2 - 24abcd + 8b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right) + ac\left(-2ac + 5adx^2 - 8bcx^2\right)}{8c^3x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*c*(-2*a*c - 8*b*c*x^2 + 5*a*d*x^2) + (8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^
2)*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(8*c^3*x^4*Sqrt[c +
d*x^2])
```

Maple [A] time = 0.011, size = 211, normalized size = 1.5

$$\frac{b^2}{c}\frac{1}{\sqrt{dx^2+c}} - b^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}} - \frac{a^2}{4cx^4}\frac{1}{\sqrt{dx^2+c}} + \frac{5a^2d}{8c^2x^2}\frac{1}{\sqrt{dx^2+c}} + \frac{15a^2d^2}{8c^3}\frac{1}{\sqrt{dx^2+c}} - \frac{15a^2d^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x)
```

```
[Out] b^2/c/(d*x^2+c)^(1/2)-b^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/4
*a^2/c/x^4/(d*x^2+c)^(1/2)+5/8*a^2*d/c^2/x^2/(d*x^2+c)^(1/2)+15/8*a^2*d^2/c
^3/(d*x^2+c)^(1/2)-15/8*a^2*d^2/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/
x)-a*b/c/x^2/(d*x^2+c)^(1/2)-3*a*b*d/c^2/(d*x^2+c)^(1/2)+3*a*b*d/c^(5/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41592, size = 782, normalized size = 5.39

$$\left[\frac{\left(\left(8b^2c^2d - 24abcd^2 + 15a^2d^3\right)x^6 + \left(8b^2c^3 - 24abc^2d + 15a^2cd^2\right)x^4\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right) - 2\left(2a^2c^3 - \left(8b^2c\right.\right.}{16\left(c^4dx^6 + c^5x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b
*c^2*d + 15*a^2*c*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) - 2*(2*a^2*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4
 + (8*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4), 1
/8*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b*c^
2*d + 15*a^2*c*d^2)*x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2
*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4 + (8*a*b*c^3 - 5*a^2*c
^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^5 \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.17151, size = 220, normalized size = 1.52

$$\frac{\left(8\,b^2c^2 - 24\,abcd + 15\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^3} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{\sqrt{dx^2 + c}c^3} - \frac{8\left(dx^2 + c\right)^{\frac{3}{2}}abcd - 8\,\sqrt{dx^2 + c}abc^2d - 7\left(dx^2 + c\right)}{8\,c^3d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c^3) - 1/
8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 7*(d*x^2 + c
)^(3/2)*a^2*d^2 + 9*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^3*d^2*x^4)
```

3.658 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}} \, dx$

Optimal. Leaf size=141

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{15b^2c^2-8ad(5bc-3ad)}{15c^3x\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

```
[Out] -a^2/(5*c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^3*Sqrt[c +
 d*x^2]) - (15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))/(15*c^3*x*Sqrt[c + d*x^2])
- (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x)/(15*c^4*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.113882, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 191}

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^3*Sqrt[c +
 d*x^2]) - (15*b^2 - (8*a*d*(5*b*c - 3*a*d))/c^2)/(15*c*x*Sqrt[c + d*x^2])
- (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x)/(15*c^4*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}dx = -\frac{a^2}{5cx^5\sqrt{c+dx^2}} + \frac{\int \frac{2a(5bc-3ad)+5b^2cx^2}{x^4(c+dx^2)^{3/2}}dx}{5c}$$
$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{1}{15}\left(-15b^2+\frac{8ad(5bc-3ad)}{c^2}\right)\int \frac{1}{x^2(c+dx^2)^{3/2}}dx$$
$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{\left(2d\left(15b^2-\frac{8ad(5bc-3ad)}{c^2}\right)\right)\int \frac{1}{(c+dx^2)^{3/2}}dx}{15c}$$
$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{2d\left(15b^2-\frac{8ad(5bc-3ad)}{c^2}\right)x}{15c^2\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0806853, size = 105, normalized size = 0.74

$$\sqrt{c+dx^2}\left(\frac{-33a^2d^2+50abcd-15b^2c^2}{15c^4x} - \frac{a^2}{5c^2x^5} + \frac{a(9ad-10bc)}{15c^3x^3} - \frac{dx(bc-ad)^2}{c^4(c+dx^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-a^2/(5*c^2*x^5) + (a*(-10*b*c + 9*a*d))/(15*c^3*x^3) + (-
15*b^2*c^2 + 50*a*b*c*d - 33*a^2*d^2)/(15*c^4*x) - (d*(b*c - a*d)^2*x)/(c^4
*(c + d*x^2)))
```

Maple [A] time = 0.007, size = 117, normalized size = 0.8

$$-\frac{48x^6a^2d^3-80x^6abcd^2+30x^6b^2c^2d+24x^4a^2cd^2-40x^4abc^2d+15x^4b^2c^3-6x^2a^2c^2d+10x^2abc^3+3a^2c^3}{15x^5c^4}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/15*(48*a^2*d^3*x^6-80*a*b*c*d^2*x^6+30*b^2*c^2*d*x^6+24*a^2*c*d^2*x^4-40
*a*b*c^2*d*x^4+15*b^2*c^3*x^4-6*a^2*c^2*d*x^2+10*a*b*c^3*x^2+3*a^2*c^3)/x^5
/(d*x^2+c)^(1/2)/c^4
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50501, size = 258, normalized size = 1.83

$$-\frac{\left(2\left(15\,b^2c^2d-40\,abcd^2+24\,a^2d^3\right)x^6+3\,a^2c^3+\left(15\,b^2c^3-40\,abc^2d+24\,a^2cd^2\right)x^4+2\left(5\,abc^3-3\,a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{15\left(c^4dx^7+c^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(15*b^2*c^2*d - 40*a*b*c*d^2 + 24*a^2*d^3)*x^6 + 3*a^2*c^3 + (15*b
^2*c^3 - 40*a*b*c^2*d + 24*a^2*c*d^2)*x^4 + 2*(5*a*b*c^3 - 3*a^2*c^2*d)*x^2
)*sqrt(d*x^2 + c)/(c^4*d*x^7 + c^5*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

Giac [B] time = 1.17203, size = 610, normalized size = 4.33

$$-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x}{\sqrt{dx^2+c}c^4}+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c^2\sqrt{d}-30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abcd^{\frac{3}{2}}+15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x/(sqrt(d*x^2 + c)*c^4) + 2/15*(15*(sq
rt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 30*(sqrt(d)*x - sqrt(d*x^2 +
 c))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 180*(sqrt(d)*x - sqrt(d*x
^2 + c))^6*a*b*c^2*d^(3/2) - 90*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c*d^(5/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 320*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 240*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^
2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) + 220*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 150*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 50*a*b*c^5*d^(3/2) + 33*a^2
*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^3)
```

3.659 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{d(24b^2c^2-5ad(12bc-7ad))}{16c^4\sqrt{c+dx^2}}-\frac{24b^2c^2-5ad(12bc-7ad)}{48c^3x^2\sqrt{c+dx^2}}+\frac{d(24b^2c^2-5ad(12bc-7ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}$$

```
[Out] -(d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d)))/(16*c^4*Sqrt[c + d*x^2]) - a^2/(
6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4*Sqrt[c + d*x^2]
) - (24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))/(48*c^3*x^2*Sqrt[c + d*x^2]) + (d
*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(1
6*c^(9/2))
```

Rubi [A] time = 0.218087, antiderivative size = 193, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 51, 63, 208}

$$\frac{35a^2d^2-60abcd+24b^2c^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}(24b^2c^2-5ad(12bc-7ad))}{16c^4x^2}+\frac{d(24b^2c^2-5ad(12bc-7ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4*Sqrt[c +
d*x^2]) + (24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)/(24*c^3*x^2*Sqrt[c + d*x^2
]) - ((24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*Sqrt[c + d*x^2])/(16*c^4*x^2) +
 (d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(16*c^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(12bc-7ad)+3b^2cx}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{1}{48}\left(24b^2 - \frac{5ad(12bc-7ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,d\right.$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} + \frac{\left(24b^2c^2-60abcd+35a^2d^2\right.}{1}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{\left(24b^2c^2-60abcd+35a^2d^2\right.}{16c^4x^2}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{\left(24b^2c^2-60abcd+35a^2d^2\right.}{16c^4x^2}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{\left(24b^2c^2-60abcd+35a^2d^2\right.}{16c^4x^2}$$

Mathematica [C] time = 0.037015, size = 92, normalized size = 0.48

$$\frac{dx^6\left(-35a^2d^2+60abcd-24b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac^2\left(-4ac+7adx^2-12bcx^2\right)}{24c^4x^6\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*c^2*(-4*a*c - 12*b*c*x^2 + 7*a*d*x^2) + d*(-24*b^2*c^2 + 60*a*b*c*d - 35
*a^2*d^2)*x^6*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (d*x^2)/c])/(24*c^4*x^6*S
qrt[c + d*x^2])
```

Maple [A] time = 0.012, size = 281, normalized size = 1.5

$$-\frac{a^2}{6\,cx^6}\frac{1}{\sqrt{dx^2+c}}+\frac{7\,a^2d}{24\,c^2x^4}\frac{1}{\sqrt{dx^2+c}}-\frac{35\,a^2d^2}{48\,c^3x^2}\frac{1}{\sqrt{dx^2+c}}-\frac{35\,a^2d^3}{16\,c^4}\frac{1}{\sqrt{dx^2+c}}+\frac{35\,a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/6*a^2/c/x^6/(d*x^2+c)^(1/2)+7/24*a^2*d/c^2/x^4/(d*x^2+c)^(1/2)-35/48*a^2
*d^2/c^3/x^2/(d*x^2+c)^(1/2)-35/16*a^2*d^3/c^4/(d*x^2+c)^(1/2)+35/16*a^2*d^
3/c^(9/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*a*b/c/x^4/(d*x^2+c)^(1/
2)+5/4*a*b*d/c^2/x^2/(d*x^2+c)^(1/2)+15/4*a*b*d^2/c^3/(d*x^2+c)^(1/2)-15/4*
a*b*d^2/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*b^2/c/x^2/(d*x^2+
c)^(1/2)-3/2*b^2*d/c^2/(d*x^2+c)^(1/2)+3/2*b^2*d/c^(5/2)*ln((2*c+2*c^(1/2)*
(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55133, size = 976, normalized size = 5.14

$$\left[\frac{3\left(\left(24\,b^2c^2d^2-60\,abcd^3+35\,a^2d^4\right)x^8+\left(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3\right)x^6\right)\sqrt{c}\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(3\left(24\,b^2c^3\right.\right.}{96\left(c^5dx^8+c^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d
- 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 +
c)*sqrt(c) + 2*c)/x^2) - 2*(3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3
)*x^6 + 8*a^2*c^4 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(1
2*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6), -1/48
```

```
*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*
a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
+ (3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + 8*a^2*c^4 + (24*b
^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(12*a*b*c^4 - 7*a^2*c^3*d)*
x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**7*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.15284, size = 360, normalized size = 1.89

$$-\frac{\left(24\,b^2c^2d-60\,abcd^2+35\,a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{16\,\sqrt{-c}c^4}-\frac{b^2c^2d-2\,abcd^2+a^2d^3}{\sqrt{dx^2+c}c^4}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqr
t(-c))/(sqrt(-c)*c^4) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)/(sqrt(d*x^2 + c
)*c^4) - 1/48*(24*(d*x^2 + c)^(5/2)*b^2*c^2*d - 48*(d*x^2 + c)^(3/2)*b^2*c^
3*d + 24*sqrt(d*x^2 + c)*b^2*c^4*d - 84*(d*x^2 + c)^(5/2)*a*b*c*d^2 + 192*(
d*x^2 + c)^(3/2)*a*b*c^2*d^2 - 108*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 57*(d*x^2
+ c)^(5/2)*a^2*d^3 - 136*(d*x^2 + c)^(3/2)*a^2*c*d^3 + 87*sqrt(d*x^2 + c)*a
^2*c^2*d^3)/(c^4*d^3*x^6)
```

3.660 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=202

$$\frac{x^3(8a^2d^2-40abcd+35b^2c^2)}{12cd^3\sqrt{c+dx^2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-40abcd+35b^2c^2)}{8cd^4}+\frac{(8a^2d^2-40abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}+$$

```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d
 + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^5)/(4*d^2*Sqrt[c + d
*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^4
) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(9/2))
```

Rubi [A] time = 0.154685, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {463, 459, 288, 321, 217, 206}

$$\frac{x^3(8a^2d^2-40abcd+35b^2c^2)}{12cd^3\sqrt{c+dx^2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-40abcd+35b^2c^2)}{8cd^4}+\frac{(8a^2d^2-40abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d
 + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^5)/(4*d^2*Sqrt[c + d
*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^4
) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(9/2))
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{x^4\left(-3a^2d^2+5(bc-ad)^2-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)\int \frac{x^4}{\left(c+dx^2\right)^{3/2}}\,dx}{12cd^2}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right.}{4cd^3}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right.}{8cd^4}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right.}{8cd^4}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right.}{8cd^4}$$

Mathematica [A] time = 0.130485, size = 156, normalized size = 0.77

$$\frac{x\left(-8a^2d^2\left(3c+4dx^2\right)+8abd\left(15c^2+20cdx^2+3d^2x^4\right)+b^2\left(-\left(140c^2dx^2+105c^3+21cd^2x^4-6d^3x^6\right)\right)\right)}{24d^4\left(c+dx^2\right)^{3/2}} + \frac{\left(8a^2d^2-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(-8*a^2*d^2*(3*c + 4*d*x^2) + 8*a*b*d*(15*c^2 + 20*c*d*x^2 + 3*d^2*x^4)
- b^2*(105*c^3 + 140*c^2*d*x^2 + 21*c*d^2*x^4 - 6*d^3*x^6)))/(24*d^4*(c + d
*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqr
t[c + d*x^2]])/(8*d^(9/2))
```

Maple [A] time = 0.019, size = 255, normalized size = 1.3

$$\frac{b^2x^7}{4d}\left(dx^2+c\right)^{-\frac{3}{2}} - \frac{7\,b^2cx^5}{8\,d^2}\left(dx^2+c\right)^{-\frac{3}{2}} - \frac{35\,x^3b^2c^2}{24\,d^3}\left(dx^2+c\right)^{-\frac{3}{2}} - \frac{35\,b^2c^2x}{8\,d^4}\frac{1}{\sqrt{dx^2+c}} + \frac{35\,b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/4*b^2*x^7/d/(d*x^2+c)^(3/2)-7/8*b^2*c/d^2*x^5/(d*x^2+c)^(3/2)-35/24*b^2*c
^2/d^3*x^3/(d*x^2+c)^(3/2)-35/8*b^2*c^2/d^4*x/(d*x^2+c)^(1/2)+35/8*b^2*c^2/
d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+a*b*x^5/d/(d*x^2+c)^(3/2)+5/3*a*b*c/d
^2*x^3/(d*x^2+c)^(3/2)+5*a*b*c/d^3*x/(d*x^2+c)^(1/2)-5*a*b*c/d^(7/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))-1/3*a^2*x^3/d/(d*x^2+c)^(3/2)-a^2/d^2*x/(d*x^2+c)^(
1/2)+a^2/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58405, size = 1127, normalized size = 5.58

$$\left[\frac{3\left(35\,b^2c^4 - 40\,abc^3d + 8\,a^2c^2d^2 + \left(35\,b^2c^2d^2 - 40\,abcd^3 + 8\,a^2d^4\right)x^4 + 2\left(35\,b^2c^3d - 40\,abc^2d^2 + 8\,a^2cd^3\right)x^2\right)\sqrt{d}\log\left(-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*b^2*c^4 - 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*
a*b*c*d^3 + 8*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3
)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d
^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5 - 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3
 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt
(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d^5), -1/24*(3*(35*b^2*c^4 - 40*a
*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^4
+ 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5
- 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*
```

```
a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2
*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [A] time = 1.1684, size = 257, normalized size = 1.27

$$\frac{\left(\left(3\left(\frac{2b^2x^2}{d}-\frac{7b^2c^2d^5-8abcd^6}{cd^7}\right)x^2-\frac{4\left(35b^2c^3d^4-40abc^2d^5+8a^2cd^6\right)}{cd^7}\right)x^2-\frac{3\left(35b^2c^4d^3-40abc^3d^4+8a^2c^2d^5\right)}{cd^7}\right)x}{24\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{\left(35b^2c^2-40abcd+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*b^2*x^2/d - (7*b^2*c^2*d^5 - 8*a*b*c*d^6)/(c*d^7))*x^2 - 4*(35*
b^2*c^3*d^4 - 40*a*b*c^2*d^5 + 8*a^2*c*d^6)/(c*d^7))*x^2 - 3*(35*b^2*c^4*d^
3 - 40*a*b*c^3*d^4 + 8*a^2*c^2*d^5)/(c*d^7))*x/(d*x^2 + c)^(3/2) - 1/8*(35*
b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^
(9/2)
```

3.661 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c - a*d))/(
d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/d^4 + (b^2*(c +
d*x^2)^(3/2))/(3*d^4)
```

Rubi [A] time = 0.0895646, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c - a*d))/(
d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/d^4 + (b^2*(c +
d*x^2)^(3/2))/(3*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{5/2}} \, dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3(c+dx)^{5/2}} + \frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^{3/2}} - \frac{b(3bc-2ad)}{d^3\sqrt{c+dx}} + \frac{b^2\sqrt{c+dx}}{d^3}\right) dx, x, x^2\right)$$
$$= \frac{c(bc-ad)^2}{3d^4\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}} - \frac{b(3bc-2ad)\sqrt{c+dx^2}}{d^4} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d^4}$$

Mathematica [A] time = 0.0590191, size = 98, normalized size = 0.89

$$\frac{-a^2d^2\left(2c+3dx^2\right)+2abd\left(8c^2+12cdx^2+3d^2x^4\right)+b^2\left(-24c^2dx^2-16c^3-6cd^2x^4+d^3x^6\right)}{3d^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2*(2*c + 3*d*x^2)) + 2*a*b*d*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4) + b^
2*(-16*c^3 - 24*c^2*d*x^2 - 6*c*d^2*x^4 + d^3*x^6))/(3*d^4*(c + d*x^2)^(3/2
))
```

Maple [A] time = 0.005, size = 108, normalized size = 1.

$$-\frac{-b^2x^6d^3-6\,abd^3x^4+6\,b^2cd^2x^4+3\,a^2d^3x^2-24\,abcd^2x^2+24\,b^2c^2dx^2+2\,a^2cd^2-16\,abc^2d+16\,b^2c^3}{3\,d^4}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-b^2*d^3*x^6-6*a*b*d^3*x^4+6*b^2*c*d^2*x^4+3*a^2*d^3*x^2-24*a*b*c*d^2
*x^2+24*b^2*c^2*d*x^2+2*a^2*c*d^2-16*a*b*c^2*d+16*b^2*c^3)/(d*x^2+c)^(3/2)/
d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37507, size = 252, normalized size = 2.29

$$\frac{\left(b^2d^3x^6-16\,b^2c^3+16\,abc^2d-2\,a^2cd^2-6\left(b^2cd^2-abd^3\right)x^4-3\left(8\,b^2c^2d-8\,abcd^2+a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^6 - 16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 6*(b^2*c*d^2 -
 a*b*d^3)*x^4 - 3*(8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(d*x^2 + c
)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [A] time = 1.97742, size = 454, normalized size = 4.13

$$\begin{cases} -\frac{2a^2cd^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{3a^2d^3x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{16abc^2d}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{24abcd^2x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{6abd^3x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c*d**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c +
 d*x**2)) - 3*a**2*d**3*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
c + d*x**2)) + 16*a*b*c**2*d/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
c + d*x**2)) + 24*a*b*c*d**2*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*
sqrt(c + d*x**2)) + 6*a*b*d**3*x**4/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**
2*sqrt(c + d*x**2)) - 16*b**2*c**3/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2
*sqrt(c + d*x**2)) - 24*b**2*c**2*d*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**
5*x**2*sqrt(c + d*x**2)) - 6*b**2*c*d**2*x**4/(3*c*d**4*sqrt(c + d*x**2) +
3*d**5*x**2*sqrt(c + d*x**2)) + b**2*d**3*x**6/(3*c*d**4*sqrt(c + d*x**2) +
 3*d**5*x**2*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**4/4 + a*b*x**6/3 + b**
2*x**8/8)/c**(5/2), True))
```

Giac [A] time = 1.15051, size = 173, normalized size = 1.57

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2 - 9\sqrt{dx^2+c}b^2c + 6\sqrt{dx^2+c}abd - \frac{9\left(dx^2+c\right)b^2c^2 - b^2c^3 - 12\left(dx^2+c\right)abcd + 2abc^2d + 3\left(dx^2+c\right)a^2d^2 - a^2cd^2}{\left(dx^2+c\right)^{\frac{3}{2}}}}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^2 + c)^(3/2)*b^2 - 9*sqrt(d*x^2 + c)*b^2*c + 6*sqrt(d*x^2 + c)*a*
b*d - (9*(d*x^2 + c)*b^2*c^2 - b^2*c^3 - 12*(d*x^2 + c)*a*b*c*d + 2*a*b*c^2
*d + 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/(d*x^2 + c)^(3/2))/d^4
```

3.662 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d)*x)/(d^3*
Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b*(5*b*c - 4*a*d)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Rubi [A] time = 0.108313, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 455, 388, 217, 206}

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d)*x)/(d^3*
Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b*(5*b*c - 4*a*d)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= \frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{x^2\left(3bc(bc-2ad)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}\\
&= \frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{\int \frac{-6bcd(bc-ad)+3b^2cd^2x^2}{\sqrt{c+dx^2}}\,dx}{3cd^4}\\
&= \frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{(b(5bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^3}\\
&= \frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{(b(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2d^3}\\
&= \frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.111844, size = 118, normalized size = 0.98

$$\frac{x\left(2a^2d^3x^2-4abcd\left(3c+4dx^2\right)+b^2c\left(15c^2+20cdx^2+3d^2x^4\right)\right)}{6cd^3\left(c+dx^2\right)^{3/2}}+\frac{b(4ad-5bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a^2*d^3*x^2 - 4*a*b*c*d*(3*c + 4*d*x^2) + b^2*c*(15*c^2 + 20*c*d*x^2
+ 3*d^2*x^4)))/(6*c*d^3*(c + d*x^2)^(3/2)) + (b*(-5*b*c + 4*a*d)*Log[d*x +
Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Maple [A] time = 0.008, size = 185, normalized size = 1.5

$$\frac{b^2x^5}{2d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{5b^2cx^3}{6d^2}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{5b^2cx}{2d^3}\frac{1}{\sqrt{dx^2+c}}-\frac{5b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}-\frac{2abx^3}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}-2\frac{}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/2*b^2*x^5/d/(d*x^2+c)^(3/2)+5/6*b^2*c/d^2*x^3/(d*x^2+c)^(3/2)+5/2*b^2*c/d
^3*x/(d*x^2+c)^(1/2)-5/2*b^2*c/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-2/3*a*
b*x^3/d/(d*x^2+c)^(3/2)-2*a*b/d^2*x/(d*x^2+c)^(1/2)+2*a*b/d^(5/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))-1/3*a^2/d*x/(d*x^2+c)^(3/2)+1/3*a^2/c/d*x/(d*x^2+c)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52046, size = 869, normalized size = 7.18

$$\left[-\frac{3\left(5b^2c^4-4abc^3d+\left(5b^2c^2d^2-4abcd^3\right)x^4+2\left(5b^2c^3d-4abc^2d^2\right)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-2(3}{12\left(cd^6x^4+2c^2d^5x^2+c^3d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2*
(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) - 2*(3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^
2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(c*d^6*x^4
 + 2*c^2*d^5*x^2 + c^3*d^4), 1/6*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d
^2 - 4*a*b*c*d^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*
b*c*d^3 + a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c)
)/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [A] time = 1.13243, size = 176, normalized size = 1.45

$$\frac{\left(\left(\frac{3\,b^2x^2}{d}+\frac{2\left(10\,b^2c^2d^3-8\,abcd^4+a^2d^5\right)}{cd^5}\right)x^2+\frac{3\left(5\,b^2c^3d^2-4\,abc^2d^3\right)}{cd^5}\right)x}{6\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\left(5\,b^2c-4\,abd\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{2\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*b^2*x^2/d + 2*(10*b^2*c^2*d^3 - 8*a*b*c*d^4 + a^2*d^5)/(c*d^5))*x^2
 + 3*(5*b^2*c^3*d^2 - 4*a*b*c^2*d^3)/(c*d^5))*x/(d*x^2 + c)^(3/2) + 1/2*(5*
b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.663 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

```
[Out] -(b*c - a*d)^2/(3*d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3*Sqrt[c +
d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rubi [A] time = 0.0581318, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] -(b*c - a*d)^2/(3*d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3*Sqrt[c +
d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right)dx,x,x^2\right)\\
&= -\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.0379859, size = 67, normalized size = 0.93

$$\frac{-a^2d^2-2abd\left(2c+3dx^2\right)+b^2\left(8c^2+12cdx^2+3d^2x^4\right)}{3d^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(2*c + 3*d*x^2) + b^2*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4
))/(3*d^3*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 68, normalized size = 0.9

$$-\frac{-3\,b^2d^2x^4+6\,abd^2x^2-12\,b^2cdx^2+a^2d^2+4\,cabd-8\,b^2c^2}{3\,d^3}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-3*b^2*d^2*x^4+6*a*b*d^2*x^2-12*b^2*c*d*x^2+a^2*d^2+4*a*b*c*d-8*b^2*c
^2)/(d*x^2+c)^(3/2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3561, size = 182, normalized size = 2.53

$$\frac{\left(3\,b^2d^2x^4+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d^3)
```

Sympy [A] time = 1.27729, size = 303, normalized size = 4.21

$$\begin{cases} -\frac{a^2d^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{4abcd}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{6abd^2x^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} + \frac{8b^2c^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} + \frac{12b^2cdx^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+} \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-a**2*d**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x
**2)) - 4*a*b*c*d/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) - 6*a*b*d**2*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**
2)) + 8*b**2*c**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) + 12*b**2*c*d*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x*
*2)) + 3*b**2*d**2*x**4/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d
*x**2)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(5/2), Tru
e))
```

Giac [A] time = 1.16322, size = 105, normalized size = 1.46

$$\frac{3\sqrt{dx^2+c}b^2 + \frac{6\left(dx^2+c\right)b^2c - b^2c^2 - 6\left(dx^2+c\right)abd + 2\,abcd - a^2d^2}{\left(dx^2+c\right)^{\frac{3}{2}}}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(d*x^2 + c)*b^2 + (6*(d*x^2 + c)*b^2*c - b^2*c^2 - 6*(d*x^2 + c)
*a*b*d + 2*a*b*c*d - a^2*d^2)/(d*x^2 + c)^(3/2))/d^3
```

3.664 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(3*c*d*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*
c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/d^(5/2)
```

Rubi [A] time = 0.0497085, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 385, 217, 206}

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(3*c*d*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*
c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/d^(5/2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{a(bc+2ad)+3b^2cx^2}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.122668, size = 101, normalized size = 0.96

$$\frac{x\left(a^2d^2\left(3c+2dx^2\right)+2abcd^2x^2-b^2c^2\left(3c+4dx^2\right)\right)}{3c^2d^2\left(c+dx^2\right)^{3/2}}+\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*b*c*d^2*x^2 + a^2*d^2*(3*c + 2*d*x^2) - b^2*c^2*(3*c + 4*d*x^2)))/(
3*c^2*d^2*(c + d*x^2)^(3/2)) + (b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(
5/2)
```

Maple [A] time = 0.005, size = 136, normalized size = 1.3

$$-\frac{b^2x^3}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{b^2x}{d^2}\frac{1}{\sqrt{dx^2+c}}+b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}-\frac{2abx}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{2abx}{3cd}\frac{1}{\sqrt{dx^2+c}}+\frac{a^2x}{3c}\left(dx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2*x^3/d/(d*x^2+c)^(3/2)-b^2/d^2*x/(d*x^2+c)^(1/2)+b^2/d^(5/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))-2/3*a*b/d*x/(d*x^2+c)^(3/2)+2/3*a*b/c/d*x/(d*x^2+c)
^(1/2)+1/3*a^2*x/c/(d*x^2+c)^(3/2)+2/3*a^2/c^2*x/(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37971, size = 655, normalized size = 6.24

$$\left[\frac{3\left(b^2c^2d^2x^4 + 2\,b^2c^3dx^2 + b^2c^4\right)\sqrt{d}\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) - 2\left(2\left(2\,b^2c^2d^2 - abcd^3 - a^2d^4\right)x^3 + 3\left(b^2c^3d - a^2\right.\right.}{6\left(c^2d^5x^4 + 2\,c^3d^4x^2 + c^4d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*
d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*x^4 + 2*c
^3*d^4*x^2 + c^4*d^3), -1/3*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4
)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*(2*b^2*c^2*d^2 - a*b*c*d
^3 - a^2*d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*
x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [A] time = 1.15513, size = 142, normalized size = 1.35

$$-\frac{x\left(\frac{2\left(2\,b^2c^2d^2-abcd^3-a^2d^4\right)x^2}{c^2d^3}+\frac{3\left(b^2c^3d-a^2cd^3\right)}{c^2d^3}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^2/(c^2*d^3) + 3*(b^2*c^3*
d - a^2*c*d^3)/(c^2*d^3))/(d*x^2 + c)^(3/2) - b^2*log(abs(-sqrt(d)*x + sqrt
(d*x^2 + c)))/d^(5/2)
```

3.665 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/Sqrt[c + d*
x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rubi [A] time = 0.0920913, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/Sqrt[c + d*
x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{5/2}} + \frac{b^2c^2-a^2d^2}{c^2d(c+dx)^{3/2}} + \frac{a^2}{c^2x\sqrt{c+dx}}\right)dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c^2}$$
$$= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{c^2d}$$
$$= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

Mathematica [C] time = 0.0446822, size = 67, normalized size = 0.76

$$\frac{a^2d^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right) - bc\left(2ad+2bc+3bdx^2\right)}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*(2*b*c + 2*a*d + 3*b*d*x^2)) + a^2*d^2*Hypergeometric2F1[-3/2, 1, -1
/2, 1 + (d*x^2)/c])/(3*c*d^2*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.011, size = 120, normalized size = 1.4

$$-\frac{b^2x^2}{d}\left(dx^2+c\right)^{-\frac{3}{2}} - \frac{2\,b^2c}{3\,d^2}\left(dx^2+c\right)^{-\frac{3}{2}} - \frac{2\,ab}{3\,d}\left(dx^2+c\right)^{-\frac{3}{2}} + \frac{a^2}{3\,c}\left(dx^2+c\right)^{-\frac{3}{2}} + \frac{a^2}{c^2}\frac{1}{\sqrt{dx^2+c}} - a^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x)
```

```
[Out] -b^2*x^2/d/(d*x^2+c)^(3/2)-2/3*b^2*c/d^2/(d*x^2+c)^(3/2)-2/3*a*b/d/(d*x^2+c
)^(3/2)+1/3*a^2/c/(d*x^2+c)^(3/2)+a^2/c^2/(d*x^2+c)^(1/2)-a^2/c^(5/2)*ln((2
*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46352, size = 651, normalized size = 7.4

$$\left[\frac{3\left(a^2d^4x^4+2a^2cd^3x^2+a^2c^2d^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-2\left(2b^2c^4+2abc^3d-4a^2c^2d^2+3\left(b^2c^3d-a^2cd^3\right)x^2\right)\sqrt{}}{6\left(c^3d^4x^4+2c^4d^3x^2+c^5d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b^2*c^4 + 2*a*b*c^3*d - 4*a^2
*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4 + 2
*c^4*d^3*x^2 + c^5*d^2), 1/3*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^
2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b^2*c^4 + 2*a*b*c^3*d - 4
*a^2*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4
 + 2*c^4*d^3*x^2 + c^5*d^2)]
```

Sympy [A] time = 23.7789, size = 87, normalized size = 0.99

$$\frac{a^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{c^2\sqrt{-c}}+\frac{(ad-bc)^2}{3cd^2\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{(ad-bc)(ad+bc)}{c^2d^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c**2*sqrt(-c)) + (a*d - b*c)**2/(3*c*
d**2*(c + d*x**2)**(3/2)) + (a*d - b*c)*(a*d + b*c)/(c**2*d**2*sqrt(c + d*x
**2))
```

Giac [A] time = 1.1615, size = 138, normalized size = 1.57

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{3\left(dx^2+c\right)b^2c^2-b^2c^3+2abc^2d-3\left(dx^2+c\right)a^2d^2-a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/3*(3*(d*x^2 + c)*b^
2*c^2 - b^2*c^3 + 2*a*b*c^2*d - 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2
+ c)^(3/2)*c^2*d^2)
```

3.666 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^2))/(3*c^2
*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0457317, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 378, 191}

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^2))/(3*c^2
*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{cx(c+dx^2)^{3/2}} + \frac{\int \frac{2a(bc-2ad)+b^2cx^2}{(c+dx^2)^{5/2}}\,dx}{c}$$

$$= -\frac{a^2}{cx(c+dx^2)^{3/2}} + \frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2(c+dx^2)^{3/2}} + \frac{(4a(bc-2ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= -\frac{a^2}{cx(c+dx^2)^{3/2}} + \frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2(c+dx^2)^{3/2}} + \frac{4a(bc-2ad)x}{3c^3\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0250743, size = 76, normalized size = 0.84

$$\frac{-a^2(3c^2+12cdx^2+8d^2x^4)+2abcx^2(3c+2dx^2)+b^2c^2x^4}{3c^3x(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4 + 2*a*b*c*x^2*(3*c + 2*d*x^2) - a^2*(3*c^2 + 12*c*d*x^2 + 8*d^
2*x^4))/(3*c^3*x*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 78, normalized size = 0.9

$$-\frac{8a^2d^2x^4-4abcdx^4-b^2c^2x^4+12a^2cdx^2-6ac^2bx^2+3a^2c^2}{3xc^3}(dx^2+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(8*a^2*d^2*x^4-4*a*b*c*d*x^4-b^2*c^2*x^4+12*a^2*c*d*x^2-6*a*b*c^2*x^2+
3*a^2*c^2)/x/(d*x^2+c)^(3/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30322, size = 188, normalized size = 2.09

$$\frac{((b^2c^2+4abcd-8a^2d^2)x^4-3a^2c^2+6(abc^2-2a^2cd)x^2)\sqrt{dx^2+c}}{3(c^3d^2x^5+2c^4dx^3+c^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*x^4 - 3*a^2*c^2 + 6*(a*b*c^2 - 2*a^2
*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d^2*x^5 + 2*c^4*d*x^3 + c^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(5/2)), x)
```

Giac [A] time = 1.17338, size = 158, normalized size = 1.76

$$\frac{x\left(\frac{\left(b^2c^4d+4\,abc^3d^2-5\,a^2c^2d^3\right)x^2}{c^5d}+\frac{6\left(abc^4d-a^2c^3d^2\right)}{c^5d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((b^2*c^4*d + 4*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x^2/(c^5*d) + 6*(a*b*c^4
*d - a^2*c^3*d^2)/(c^5*d))/(d*x^2 + c)^(3/2) + 2*a^2*sqrt(d)/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)*c^2)
```

3.667 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

```
[Out] (4*a*b - (2*b^2*c)/d - (5*a^2*d)/c)/(6*c*(c + d*x^2)^(3/2)) - a^2/(2*c*x^2*
(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c + d*x^2]) - (a*(4*b*
c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

Rubi [A] time = 0.117171, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (5*a^2*d)/c)/(6*c*(c + d*x^2)^(3/2)) - a^2/(2*c*x^2*
(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c + d*x^2]) - (a*(4*b*
c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3 (c+dx^2)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{5/2}}\,dx, x, x^2\right)\\
&= -\frac{a^2}{2cx^2 (c+dx^2)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-5ad)+b^2cx}{x(c+dx)^{5/2}}\,dx, x, x^2\right)}{2c}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4c^3}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2c^3d}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} - \frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0452251, size = 105, normalized size = 0.8

$$\frac{-c\left(a^2d\left(3c+5dx^2\right)-4abcdx^2+2b^2c^2x^2\right)-3adx^2\left(c+dx^2\right)(5ad-4bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)}{6c^3dx^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(c*(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(3*c + 5*d*x^2))) - 3*a*d*(-4*b
*c + 5*a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])
/(6*c^3*d*x^2*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.012, size = 169, normalized size = 1.3

$$-\frac{b^2}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{2ab}{3c}\left(dx^2+c\right)^{-\frac{3}{2}}+2\frac{ab}{c^2\sqrt{dx^2+c}}-2\frac{ab}{c^{5/2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)-\frac{a^2}{2cx^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{5a^2d}{6c^2}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2/d/(d*x^2+c)^(3/2)+2/3*a*b/c/(d*x^2+c)^(3/2)+2*a*b/c^2/(d*x^2+c)^(1
/2)-2*a*b/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*a^2/c/x^2/(d*x^
2+c)^(3/2)-5/6*a^2*d/c^2/(d*x^2+c)^(3/2)-5/2*a^2*d/c^3/(d*x^2+c)^(1/2)+5/2*
a^2*d/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44415, size = 894, normalized size = 6.82

$$\left[-\frac{3\left(\left(4\,abcd^3-5\,a^2d^4\right)x^6+2\left(4\,abc^2d^2-5\,a^2cd^3\right)x^4+\left(4\,abc^3d-5\,a^2c^2d^2\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2+2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(3\,a^2c^3\right.}{12\left(c^4d^3x^6+2\,c^5d^2x^4+c^6dx^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*
x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2
 + c)*sqrt(c) + 2*c)/x^2) + 2*(3*a^2*c^3*d - 3*(4*a*b*c^2*d^2 - 5*a^2*c*d^3
)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c
^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2), 1/6*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*
x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x
^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a^2*c^3*d - 3*(4*a*b*c^2
*d^2 - 5*a^2*c*d^3)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*s
qrt(d*x^2 + c))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(5/2)), x)
```

Giac [A] time = 1.1442, size = 173, normalized size = 1.32

$$\frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}c^3}-\frac{\sqrt{dx^2+c}a^2}{2\,c^3x^2}-\frac{b^2c^3-6\,(dx^2+c)abcd-2\,abc^2d+6\,(dx^2+c)a^2d^2+a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1
/2*sqrt(d*x^2 + c)*a^2/(c^3*x^2) - 1/3*(b^2*c^3 - 6*(d*x^2 + c)*a*b*c*d - 2
*a*b*c^2*d + 6*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c^3*d)
```

3.668 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}+\frac{x\left(b^2c^2-8ad(bc-ad)\right)}{3c^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

```
[Out] -a^2/(3*c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c + d*x^2)^(3/
2)) + ((b^2*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^3*(c + d*x^2)^(3/2)) + (2*(b^2
*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.124814, antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 192, 191}

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}+\frac{x\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)}{3c\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(3*c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c + d*x^2)^(3/
2)) + ((b^2 - (8*a*d*(b*c - a*d))/c^2)*x)/(3*c*(c + d*x^2)^(3/2)) + (2*(b^2
*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx &= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}+\frac{\int \frac{6a(bc-ad)+3b^2cx^2}{x^2(c+dx^2)^{5/2}}\,dx}{3c}\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}-\left(-b^2+\frac{8ad(bc-ad)}{c^2}\right)\int \frac{1}{(c+dx^2)^{5/2}}\,dx\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c(c+dx^2)^{3/2}}+\frac{\left(2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)\right)\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{3c}\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c(c+dx^2)^{3/2}}+\frac{2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c^2\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0715314, size = 107, normalized size = 0.82

$$\frac{a^2\left(6c^2dx^2-c^3+24cd^2x^4+16d^3x^6\right)-2abcx^2\left(3c^2+12cdx^2+8d^2x^4\right)+b^2c^2x^4\left(3c+2dx^2\right)}{3c^4x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4*(3*c + 2*d*x^2) - 2*a*b*c*x^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4)
 + a^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*c^4*x^3*(c + d*
x^2)^(3/2))
```

Maple [A] time = 0.005, size = 116, normalized size = 0.9

$$-\frac{-16\,x^6a^2d^3+16\,x^6abcd^2-2\,x^6b^2c^2d-24\,x^4a^2cd^2+24\,x^4abc^2d-3\,x^4b^2c^3-6\,x^2a^2c^2d+6\,x^2abc^3+a^2c^3}{3\,x^3c^4}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-16*a^2*d^3*x^6+16*a*b*c*d^2*x^6-2*b^2*c^2*d*x^6-24*a^2*c*d^2*x^4+24*
a*b*c^2*d*x^4-3*b^2*c^3*x^4-6*a^2*c^2*d*x^2+6*a*b*c^3*x^2+a^2*c^3)/x^3/(d*x
^2+c)^(3/2)/c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55648, size = 258, normalized size = 1.97

$$\frac{\left(2\left(b^2c^2d-8\,abcd^2+8\,a^2d^3\right)x^6-a^2c^3+3\left(b^2c^3-8\,abc^2d+8\,a^2cd^2\right)x^4-6\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^4d^2x^7+2\,c^5dx^5+c^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(b^2*c^2*d - 8*a*b*c*d^2 + 8*a^2*d^3)*x^6 - a^2*c^3 + 3*(b^2*c^3 - 8
*a*b*c^2*d + 8*a^2*c*d^2)*x^4 - 6*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 1.15766, size = 348, normalized size = 2.66

$$\frac{x\left(\frac{2\left(b^2c^5d^2-5\,abc^4d^3+4\,a^2c^3d^4\right)x^2}{c^7d}+\frac{3\left(b^2c^6d-4\,abc^5d^2+3\,a^2c^4d^3\right)}{c^7d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-\cdots\right.}{3\left(\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^2*c^5*d^2 - 5*a*b*c^4*d^3 + 4*a^2*c^3*d^4)*x^2/(c^7*d) + 3*(b^2
*c^6*d - 4*a*b*c^5*d^2 + 3*a^2*c^4*d^3)/(c^7*d))/(d*x^2 + c)^(3/2) + 4/3*(3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 9
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 3*a*b*c^3*sqrt(d) - 4*a^2*
c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^3)
```

3.669 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{24c^3\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}-\ldots$$

```
[Out] (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(24*c^3*(c + d*x^2)^(3/2)) - a^2/(4*c*x
^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*(c + d*x^2)^(3/2)) +
 (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*Sqrt[c + d*x^2]) - ((8*b^2*c^2
- 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rubi [A] time = 0.215523, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}-\frac{a(8bc\ldots}{8c^2x^2\left(c\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (8*b^2 - (5*a*d*(8*b*c - 7*a*d))/c^2)/(24*c*(c + d*x^2)^(3/2)) - a^2/(4*c*x
^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*(c + d*x^2)^(3/2)) +
 (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*Sqrt[c + d*x^2]) - ((8*b^2*c^2
- 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-7ad)+2b^2cx}{x^2(c+dx)^{5/2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{1}{16}\left(8b^2 - \frac{5ad(8bc-7ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x(c+dx)^{5/2}}\right.$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{\left(8b^2-\frac{5ad(8bc-7ad)}{c^2}\right)\text{Subst}\left(\int \frac{}{x(c+}\right.}{16c}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{5ad(8bc}{c^2}\right.}{}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{5ad(8bc}{c^2}\right.}{}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} - \frac{\left(8b^2-\frac{5ad(8bc}{c^2}\right.}{}$$

Mathematica [C] time = 0.0363152, size = 90, normalized size = 0.49

$$\frac{x^4\left(35a^2d^2-40abcd+8b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-3ac\left(2ac-7adx^2+8bcx^2\right)}{24c^3x^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a*c*(2*a*c + 8*b*c*x^2 - 7*a*d*x^2) + (8*b^2*c^2 - 40*a*b*c*d + 35*a^2*
d^2)*x^4*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/(24*c^3*x^4*(c +
d*x^2)^(3/2))
```

Maple [A] time = 0.013, size = 265, normalized size = 1.4

$$\frac{b^2}{3c}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{b^2}{c^2}\frac{1}{\sqrt{dx^2+c}}-b^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}-\frac{a^2}{4cx^4}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{7a^2d}{8c^2x^2}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{35a^2d^2}{24c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*b^2/c/(d*x^2+c)^(3/2)+b^2/c^2/(d*x^2+c)^(1/2)-b^2/c^(5/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)-1/4*a^2/c/x^4/(d*x^2+c)^(3/2)+7/8*a^2*d/c^2/x^2/(d
*x^2+c)^(3/2)+35/24*a^2*d^2/c^3/(d*x^2+c)^(3/2)+35/8*a^2*d^2/c^4/(d*x^2+c)^
(1/2)-35/8*a^2*d^2/c^(9/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-a*b/c/x^2/
(d*x^2+c)^(3/2)-5/3*a*b*d/c^2/(d*x^2+c)^(3/2)-5*a*b*d/c^3/(d*x^2+c)^(1/2)+5
*a*b*d/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53703, size = 1150, normalized size = 6.22

$$\left[\frac{3\left(\left(8b^2c^2d^2-40abcd^3+35a^2d^4\right)x^8+2\left(8b^2c^3d-40abc^2d^2+35a^2cd^3\right)x^6+\left(8b^2c^4-40abc^3d+35a^2c^2d^2\right)x^4\right)\sqrt{c}\log}{48}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d
- 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c
^2*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
2*(3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b
^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x
^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 + 2*c^6*d*x^6 + c^7*x^4), 1/24*(3*((8*b^2
```

```
*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*c^2*d^2
 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4)*sqr
t(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 +
 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 +
2*c^6*d*x^6 + c^7*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(5/2)), x)
```

Giac [A] time = 1.17544, size = 284, normalized size = 1.54

$$\frac{\left(8\,b^2c^2-40\,abcd+35\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^4}+\frac{3\left(dx^2+c\right)b^2c^2+b^2c^3-12\left(dx^2+c\right)abcd-2\,abc^2d+9\left(dx^2+c\right)a^2d^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^4) + 1/3*(3*(d*x^2 + c)*b^2*c^2 + b^2*c^3 - 12*(d*x^2 + c)*a*b*
c*d - 2*a*b*c^2*d + 9*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c
^4) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 11*(
d*x^2 + c)^(3/2)*a^2*d^2 + 13*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^4*d^2*x^4)
```

3.670 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{8dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^5\sqrt{c+dx^2}}-\frac{4dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{5b^2c^2-4ad(5bc-4ad)}{5c^3x\left(c+dx^2\right)^{3/2}}-\frac{2a(5b}{15c^2x^3(}$$

```
[Out] -a^2/(5*c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*x^3*(c + d
*x^2)^(3/2)) - (5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))/(5*c^3*x*(c + d*x^2)^(3/
2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^4*(c + d*x^2)^(3/2)
) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.165987, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 271, 192, 191}

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{8dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^5\sqrt{c+dx^2}}-\frac{4dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4a}{15c^2x^3(c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*x^3*(c + d
*x^2)^(3/2)) - (5*b^2 - (4*a*d*(5*b*c - 4*a*d))/c^2)/(5*c*x*(c + d*x^2)^(3/
2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^4*(c + d*x^2)^(3/2)
) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} + \frac{\int \frac{2a(5bc-4ad)+5b^2cx^2}{x^4(c+dx^2)^{5/2}}\,dx}{5c}$$

$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} - \frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}} - \frac{1}{5}\left(-5b^2+\frac{4ad(5bc-4ad)}{c^2}\right)\int \frac{1}{x^2(c+dx^2)^{5/2}}\,dx$$

$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} - \frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}} - \frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}} - \frac{\left(4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)\right)\int \frac{}{(c+d}}{5c}$$

$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} - \frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}} - \frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}} - \frac{4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)x}{15c^2(c+dx^2)^{3/2}} - \frac{(8d}{}$$

$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} - \frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}} - \frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}} - \frac{4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)x}{15c^2(c+dx^2)^{3/2}} - \frac{8d(}{}$$

Mathematica [A] time = 0.0990494, size = 142, normalized size = 0.78

$$\frac{-a^2(48c^2d^2x^4-8c^3dx^2+3c^4+192cd^3x^6+128d^4x^8)+10abcx^2(6c^2dx^2-c^3+24cd^2x^4+16d^3x^6)-5b^2c^2x^4(3c^2+1}{15c^5x^5(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*b^2*c^2*x^4*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 10*a*b*c*x^2*(-c^3 + 6*c
^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) - a^2*(3*c^4 - 8*c^3*d*x^2 + 48*c^2*d
^2*x^4 + 192*c*d^3*x^6 + 128*d^4*x^8))/(15*c^5*x^5*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 158, normalized size = 0.9

$$-\frac{128a^2d^4x^8-160abcd^3x^8+40b^2c^2d^2x^8+192a^2cd^3x^6-240abc^2d^2x^6+60b^2c^3dx^6+48a^2c^2d^2x^4-60abc^3dx^4+15}{15x^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/15*(128*a^2*d^4*x^8-160*a*b*c*d^3*x^8+40*b^2*c^2*d^2*x^8+192*a^2*c*d^3*x
^6-240*a*b*c^2*d^2*x^6+60*b^2*c^3*d*x^6+48*a^2*c^2*d^2*x^4-60*a*b*c^3*d*x^4
+15*b^2*c^4*x^4-8*a^2*c^3*d*x^2+10*a*b*c^4*x^2+3*a^2*c^4)/x^5/(d*x^2+c)^(3/
2)/c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7727, size = 359, normalized size = 1.96

$$-\frac{\left(8\left(5b^2c^2d^2-20abcd^3+16a^2d^4\right)x^8+12\left(5b^2c^3d-20abc^2d^2+16a^2cd^3\right)x^6+3a^2c^4+3\left(5b^2c^4-20abc^3d+16a^2c^2d^2\right.}{15\left(c^5d^2x^9+2c^6dx^7+c^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*b^2*c^2*d^2 - 20*a*b*c*d^3 + 16*a^2*d^4)*x^8 + 12*(5*b^2*c^3*d
- 20*a*b*c^2*d^2 + 16*a^2*c*d^3)*x^6 + 3*a^2*c^4 + 3*(5*b^2*c^4 - 20*a*b*c^
3*d + 16*a^2*c^2*d^2)*x^4 + 2*(5*a*b*c^4 - 4*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c
)/(c^5*d^2*x^9 + 2*c^6*d*x^7 + c^7*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 1.19317, size = 687, normalized size = 3.75

$$-\frac{x\left(\frac{\left(5b^2c^6d^3-16abc^5d^4+11a^2c^4d^5\right)x^2}{c^9d}+\frac{6\left(b^2c^7d^2-3abc^6d^3+2a^2c^5d^4\right)}{c^9d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*b^2*c^6*d^3 - 16*a*b*c^5*d^4 + 11*a^2*c^4*d^5)*x^2/(c^9*d) + 6*(
b^2*c^7*d^2 - 3*a*b*c^6*d^3 + 2*a^2*c^5*d^4)/(c^9*d))/(d*x^2 + c)^(3/2) + 2
/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 60*(sqrt(d)*x - s
qrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 45*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 300*(sqrt(d)*x
 - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) - 240*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*c*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 500*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 490*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt
(d) + 340*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 320*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 80*a*b*c^5*d^(3
/2) + 73*a^2*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^4)
```

3.671 $\int \frac{x^5}{\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.0256232, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 302, 205}

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx = \frac{x\int \frac{x^4}{a+bx^2}\,dx}{\sqrt{dx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{\sqrt{dx^2}}$$
$$= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{\left(a^2x\right)\int \frac{1}{a+bx^2}\,dx}{b^2\sqrt{dx^2}}$$
$$= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}$$

Mathematica [A] time = 0.0240915, size = 56, normalized size = 0.78

$$\frac{x\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b
^(5/2)*Sqrt[d*x^2])
```

Maple [A] time = 0.007, size = 53, normalized size = 0.7

$$\frac{x}{3\,b^2}\left(\sqrt{ab}x^3b-3\sqrt{ab}xa+3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3*x*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(b*x/(a*b)^(1/2)))/(
d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33494, size = 315, normalized size = 4.38

$$\left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2+2\,\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\left(bx^2-3\,a\right)\sqrt{dx^2}}{6\,b^2d},\frac{3\,ad\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)+\left(bx^2-3\,a\right)\sqrt{dx^2}}{3\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*d*sqrt(-a/(b*d))*log((b*x^2 + 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)
/(b*x^2 + a)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d), 1/3*(3*a*d*sqrt(a/(b*
d))*arctan(sqrt(d*x^2)*b*sqrt(a/(b*d))/a) + (b*x^2 - 3*a)*sqrt(d*x^2))/(b^2
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A] time = 1.12183, size = 95, normalized size = 1.32

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b^2}+\frac{\sqrt{dx^2}b^2d^5x^2-3\,\sqrt{dx^2}abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + 1/3*(sqrt(d*x^2)*
b^2*d^5*x^2 - 3*sqrt(d*x^2)*a*b*d^5)/(b^3*d^6)
```

3.672 $\int \frac{x^3}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=52

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

```
[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt
[d*x^2])
```

Rubi [A] time = 0.0157799, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 321, 205}

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt
[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx = \frac{x\int \frac{x^2}{a+bx^2}\,dx}{\sqrt{dx^2}}$$
$$= \frac{x^2}{b\sqrt{dx^2}} - \frac{(ax)\int \frac{1}{a+bx^2}\,dx}{b\sqrt{dx^2}}$$
$$= \frac{x^2}{b\sqrt{dx^2}} - \frac{\sqrt{a}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Mathematica [A] time = 0.0156811, size = 44, normalized size = 0.85

$$\frac{x\left(\sqrt{b}x-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sqrt[d*x^2])
```

Maple [A] time = 0.004, size = 38, normalized size = 0.7

$$\frac{x}{b}\left(x\sqrt{ab}-a\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] x*(x*(a*b)^(1/2)-a*arctan(b*x/(a*b)^(1/2)))/(d*x^2)^(1/2)/b/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24982, size = 257, normalized size = 4.94

$$\left[\frac{d\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2-2\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\sqrt{dx^2}}{2\,bd}, -\frac{d\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)-\sqrt{dx^2}}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/(b*d))*log((b*x^2 - 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)/(b*
x^2 + a)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)*b*sq
rt(a/(b*d))/a) - sqrt(d*x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{dx^2}\left(a + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A] time = 1.1171, size = 62, normalized size = 1.19

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b} - \frac{\sqrt{dx^2}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)/b)/d
```

3.673 $\int \frac{x}{\sqrt{dx^2}(a+bx^2)} dx$

Optimal. Leaf size=34

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rubi [A] time = 0.0082262, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 205}

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{dx^2}\left(a+bx^2\right)} dx &= \frac{x\int \frac{1}{a+bx^2} dx}{\sqrt{dx^2}}\\ &= \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0072771, size = 34, normalized size = 1.

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Maple [A] time = 0.004, size = 24, normalized size = 0.7

$$x \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{dx^2}} \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35109, size = 205, normalized size = 6.03

$$\left[-\frac{\sqrt{-abd} \log\left(\frac{bdx^2-ad-2\sqrt{-abd}\sqrt{dx^2}}{bx^2+a}\right)}{2\,abd}, \frac{\sqrt{abd} \arctan\left(\frac{\sqrt{abd}\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b*d)*log((b*d*x^2 - a*d - 2*sqrt(-a*b*d)*sqrt(d*x^2))/(b*x^2
+ a))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{dx^2}\left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A] time = 1.12092, size = 31, normalized size = 0.91

$$\frac{\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

3.674 $\int \frac{1}{x\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqr
t[d*x^2])
```

Rubi [A] time = 0.0165593, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 325, 205}

$$-\frac{\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqr
t[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}\,dx = \frac{x\int \frac{1}{x^2(a+bx^2)}\,dx}{\sqrt{dx^2}}$$
$$= -\frac{1}{a\sqrt{dx^2}} - \frac{(bx)\int \frac{1}{a+bx^2}\,dx}{a\sqrt{dx^2}}$$
$$= -\frac{1}{a\sqrt{dx^2}} - \frac{\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}$$

Mathematica [A] time = 0.0163537, size = 46, normalized size = 0.92

$$-\frac{dx^2\left(\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((d*x^2*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*(d*x^2
)^(3/2)))
```

Maple [A] time = 0.006, size = 36, normalized size = 0.7

$$-\frac{1}{a}\left(b\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x+\sqrt{ab}\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] -(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28342, size = 273, normalized size = 5.46

$$\left[\frac{dx^2\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2-2\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)-2\sqrt{dx^2}}{2\,adx^2}, -\frac{dx^2\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+\sqrt{dx^2}}{adx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*x^2*sqrt(-b/(a*d))*log((b*x^2 - 2*sqrt(d*x^2)*a*sqrt(-b/(a*d)) - a)
/(b*x^2 + a)) - 2*sqrt(d*x^2))/(a*d*x^2), -(d*x^2*sqrt(b/(a*d))*arctan(sqrt
(d*x^2)*sqrt(b/(a*d))) + sqrt(d*x^2))/(a*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A] time = 1.09595, size = 65, normalized size = 1.3

$$-d\left(\frac{b\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}ad}+\frac{1}{\sqrt{dx^2}ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -d*(b*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a*d) + 1/(sqrt(d*x^2)*
a*d))
```

3.675 $\int \frac{1}{x^3\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=68

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.0240175, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 325, 205}

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{dx^2}\left(a+bx^2\right)}\,dx = \frac{x\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{\sqrt{dx^2}}$$
$$= -\frac{1}{3ax^2\sqrt{dx^2}} - \frac{(bx)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a\sqrt{dx^2}}$$
$$= \frac{b}{a^2\sqrt{dx^2}} - \frac{1}{3ax^2\sqrt{dx^2}} + \frac{\left(b^2x\right)\int \frac{1}{a+bx^2}\,dx}{a^2\sqrt{dx^2}}$$
$$= \frac{b}{a^2\sqrt{dx^2}} - \frac{1}{3ax^2\sqrt{dx^2}} + \frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}$$

Mathematica [A] time = 0.0215465, size = 58, normalized size = 0.85

$$\frac{d\left(3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\left(a-3bx^2\right)\right)}{3a^{5/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (d*(-(Sqrt[a]*(a - 3*b*x^2)) + 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/
(3*a^(5/2)*(d*x^2)^(3/2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.9

$$\frac{1}{3a^2x^2}\left(3b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3+3bx^2\sqrt{ab}-a\sqrt{ab}\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3/x^2*(3*b^2*arctan(b*x/(a*b)^(1/2))*x^3+3*b*x^2*(a*b)^(1/2)-a*(a*b)^(1/2
))/(d*x^2)^(1/2)/a^2/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36295, size = 331, normalized size = 4.87

$$\left[\frac{3\,bdx^4\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2+2\,\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)+2\left(3\,bx^2-a\right)\sqrt{dx^2}}{6\,a^2dx^4},\ \frac{3\,bdx^4\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+\left(3\,bx^2-a\right)\sqrt{dx^2}}{3\,a^2dx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((b*x^2 + 2*sqrt(d*x^2)*a*sqrt(-b/(a*d))
- a)/(b*x^2 + a)) + 2*(3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4), 1/3*(3*b*d*x^
4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + (3*b*x^2 - a)*sqrt(d*x^
2))/(a^2*d*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{dx^2}\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A] time = 1.11755, size = 92, normalized size = 1.35

$$\frac{1}{3}\,d^2\left(\frac{3\,b^2\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}a^2d^2}+\frac{3\,bdx^2-ad}{\sqrt{dx^2}a^2d^3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*d^2*(3*b^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a^2*d^2) + (3
*b*d*x^2 - a*d)/(sqrt(d*x^2)*a^2*d^3*x^2))
```

3.676 $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{a^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2])/(4*b) +
 (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(8*b^3*d^(3/2))
```

Rubi [A] time = 0.231156, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {478, 582, 523, 217, 206, 377, 205}

$$-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{a^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2])/(4*b) +
 (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(8*b^3*d^(3/2))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\int \frac{x^2\left(3ac+(-bc+4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\int \frac{-ac(bc-4ad)+\left(-b^2c^2-4abcd+8a^2d^2\right)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8b^2d}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\left(a^2(bc-ad)\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^3}-\frac{\left(b^2c^2+4abcd-8a^2d^2\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{8b^3d}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(b^2c^2+4abcd-8a^2d^2\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{8b^3d}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{a^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(b^2c^2+4abcd-8a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.211981, size = 148, normalized size = 0.94

$$\frac{-\left(-8a^2d^2+4abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+8a^{3/2}d^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+b\sqrt{d}x\sqrt{c+dx^2}\left(-4ad+bc+2bdx^2\right)}{8b^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(b*c - 4*a*d + 2*b*d*x^2) + 8*a^(3/2)*d^(3/2)*
Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] - (b^
2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^3*d
^(3/2))
```

Maple [B] time = 0.019, size = 1088, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)/d-1/8/b*c/d*x*(d*x^2+c)^(1/2)-1/8/b*c^2/d^(3/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))-1/2/b^2*a*x*(d*x^2+c)^(1/2)-1/2/b^2*a*c/d^(1/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/2/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*a^
2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-
1/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*d+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x+1/b*(-a*b)^(1/2)))*c+1/2/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*a^2*d^
(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b
^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))*d-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.39072, size = 1885, normalized size = 12.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (b^
2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
```

```
t(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/
(b^3*d^2), 1/8*(2*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x
^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*
d^2), 1/16*(8*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x
)*sqrt(d*x^2 + c))/(b^3*d^2), 1/8*(4*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*s
qrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^
2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a
*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [A] time = 1.15251, size = 254, normalized size = 1.62

$$\frac{1}{8}\sqrt{dx^2+c}\,x\left(\frac{2\,x^2}{b}+\frac{b^5cd-4\,ab^4d^2}{b^6d^2}\right)-\frac{\left(a^2bc\sqrt{d}-a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b^3}+\frac{\left(b^2c^2\sqrt{d}+4\,abcd^{\frac{3}{2}}-8\,a^2d^{\frac{5}{2}}\right)}{16\,b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*x*(2*x^2/b + (b^5*c*d - 4*a*b^4*d^2)/(b^6*d^2)) - (a^2*
b*c*sqrt(d) - a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) + 1/16*
(b^2*c^2*sqrt(d) + 4*a*b*c*d^(3/2) - 8*a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d
*x^2 + c))^2)/(b^3*d^2)
```

3.677 $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[b*c - a*d]
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rubi [A] time = 0.0812728, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[b*c - a*d]
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} + \frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0641972, size = 85, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\left(b\left(c+dx^2\right)-3ad\right)}{3b^2d} + \frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a*d + b*(c + d*x^2)))/(3*b^2*d) + (a*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Maple [B] time = 0.011, size = 963, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(3/2)/b/d-1/2/b^2*a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*a*d^(1/2)*(-a*b)^(1/2)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b^3*a^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/2/b^2*a/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
```

```
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/2/b^2*a*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/2/b^3*a*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))*d+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53402, size = 636, normalized size = 7.23

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(bdx^2+bc-3\,ad\right)\sqrt{dx^2+c}}{12\,b^2d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*
sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b*d*
x^2 + b*c - 3*a*d)*sqrt(d*x^2 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b*d*x^2 + b*c - 3*a*d)*sqrt(d*x
^2 + c))/(b^2*d)]
```

Sympy [A] time = 5.63263, size = 87, normalized size = 0.99

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^2}}{2b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{6b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**2)/(2*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**2)/sqrt((a*d - b*c)/b))/(2*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**2)**(3
/2)/(6*b))/d**2
```

Giac [A] time = 1.13647, size = 130, normalized size = 1.48

$$-\frac{\frac{3\left(abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2-3\sqrt{dx^2+c}abd}{b^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/3*(3*(a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/
(sqrt(-b^2*c + a*b*d)*b^2) - ((d*x^2 + c)^(3/2)*b^2 - 3*sqrt(d*x^2 + c)*a*b
*d)/b^3)/d
```

3.678 $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rubi [A] time = 0.0962023, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{x\sqrt{c+dx^2}}{2b}-\frac{\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{x\sqrt{c+dx^2}}{2b}+\frac{(bc-2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2}-\frac{(a(bc-ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{x\sqrt{c+dx^2}}{2b}+\frac{(bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}-\frac{(a(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}\\
&= \frac{x\sqrt{c+dx^2}}{2b}-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.123648, size = 108, normalized size = 0.96

$$
\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-2\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+bx\sqrt{c+dx^2}}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])] + ((b*c - 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x
^2]])/Sqrt[d])/(2*b^2)
```

Maple [B] time = 0.01, size = 1010, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b+1/2/b*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a
/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)-1/2*a/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
```

```
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-1/2*a/(-a*b)^(1/2)/b/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/2*a/(-a*b)^(1/2)/
b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)-1/2*a/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))-1/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98577, size = 1544, normalized size = 13.79

$$\left[\frac{2\sqrt{dx^2+c}bdx-(bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3ab}{}\right.}{4b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt
(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a*b*c
*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(b*c - 2*a*d)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*sqrt(a*b*c
- a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (b*c - 2*a*d)
*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d), 1/2*(sqr
t(d*x^2 + c)*b*d*x - sqrt(a*b*c - a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*(
(b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^
2 - a^2*c*d)*x)) - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [A] time = 1.15268, size = 185, normalized size = 1.65

$$\frac{\sqrt{dx^2+c}x}{2b}+\frac{\left(abc\sqrt{d}-a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b^2}-\frac{(bc-2ad)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*x/b + (a*b*c*sqrt(d) - a^2*d^(3/2))*arctan(1/2*((sqrt(d
)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*
b*c*d - a^2*d^2)*b^2) - 1/4*(b*c - 2*a*d)*log((sqrt(d)*x - sqrt(d*x^2 + c))
^2)/(b^2*sqrt(d))
```

3.679 $\int \frac{x\sqrt{c+dx^2}}{a+bx^2} \, dx$

Optimal. Leaf size=65

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

Rubi [A] time = 0.0544701, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{c+dx^2}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}$$
$$= \frac{\sqrt{c+dx^2}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{bd}$$
$$= \frac{\sqrt{c+dx^2}}{b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0255953, size = 65, normalized size = 1.

$$\frac{\sqrt{c+dx^2}}{b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

Maple [B] time = 0.007, size = 936, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))*c+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln(
(d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d-1/2/b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43189, size = 539, normalized size = 8.29

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\sqrt{dx^2+c}}{4\,b}, -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(d*x^2 +
 c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2
*sqrt(d*x^2 + c))/b]
```

Sympy [A] time = 3.42645, size = 61, normalized size = 0.94

$$\frac{2\left(\frac{d\sqrt{c+dx^2}}{2b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**2)/(2*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**2)/sqrt((a*d
 - b*c)/b))/(2*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.11184, size = 86, normalized size = 1.32

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}+\frac{\sqrt{dx^2+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + sqrt(d*x^2 + c)/b
```

3.680 $\int \frac{\sqrt{c+dx^2}}{a+bx^2} dx$

Optimal. Leaf size=81

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rubi [A] time = 0.0443272, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}-\frac{(-bc+ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\ &= \frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}-\frac{(-bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}\\ &= \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0322809, size = 84, normalized size = 1.04

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b
```

Maple [B] time = 0.008, size = 948, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/2/(-a*b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/
2*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))*a*d-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72887, size = 1288, normalized size = 15.9

$$\left[\frac{2\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2 \ldots}}{b^2x^4+2\,abx^2+a^2}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-(b*
c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b, -1/4*(4*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*
c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/b, 1/2*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/
b, -1/2*(2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt((b*c - a*d)/a
)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/
((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [A] time = 1.14559, size = 151, normalized size = 1.86

$$-\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b}-\frac{\sqrt{d}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b) - 1/2*sqr
t(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b
```

3.681 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rubi [A] time = 0.0733895, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad}$$
$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}$$

Mathematica [A] time = 0.0343538, size = 78, normalized size = 0.98

$$\frac{\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b])/a
```

Maple [B] time = 0.01, size = 984, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a),x)
```

```
[Out] -1/a*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a*(d*x^2+c)^(1/2)-1/2/
a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)+1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))*c-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b
)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
```

```
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 1.82432, size = 1261, normalized size = 15.76

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)}{4\,a}, \frac{4\sqrt{\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(c)*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/4*(4*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) + sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*
b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 +
a^2)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqr
t(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) +
sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/2*(sqrt(-
(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)))/a]
```

Sympy [A] time = 6.25726, size = 78, normalized size = 0.98

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.13864, size = 117, normalized size = 1.46

$$-d\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -d*((b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*a*d) - c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.682 $\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rubi [A] time = 0.0506841, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx = -\frac{\sqrt{c+dx^2}}{ax} + \frac{\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^2}}{ax} + \frac{(-bc+ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^2}}{ax} + \frac{(-bc+ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{a}$$

$$= -\frac{\sqrt{c+dx^2}}{ax} - \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.018657, size = 51, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c + d*x^2]*Hypergeometric2F1[-1/2, 1, 1/2, ((-(b*c) + a*d)*x^2)/(a*
(c + d*x^2))])/(a*x))
```

Maple [B] time = 0.017, size = 1017, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x)
```

```
[Out] 1/2*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/2*b/a/(-a*b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)-1/2/a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*d+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*c-1/a/c/x*(d*x^2+c)^(3/2)+1/a*d/c*x*(d*x^2+c)^(1/2)+1/a
```

```
*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 1.39687, size = 570, normalized size = 8.14

$$\left[\frac{x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+c}}{4\,ax}, -\frac{x\sqrt{\frac{bc-ad}{a}}\,\mathrm{arct}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sq
rt(d*x^2 + c))/(a*x), -1/2*(x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + 2*sqrt(d*x^2 + c))/(a*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)), x)
```

Giac [B] time = 1.90558, size = 158, normalized size = 2.26

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a}+\frac{2\,c\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + 2*c*sqrt
(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a)
```

3.683 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=113

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

```
[Out] -Sqrt[c + d*x^2]/(2*a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/a^2
```

Rubi [A] time = 0.120464, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/a^2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\ &= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2}-\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2}\\ &= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d}-\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{\right.}{2a^2d}\\ &= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.122045, size = 107, normalized size = 0.95

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(2*a^2)
```

Maple [B] time = 0.013, size = 1054, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x)
```

```
[Out] b/a^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2*(d*x^2+c)^(1/2)+1
/2*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/
```

```
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/2*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)^(1
/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/a/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d-1/2*b/a^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/2/a/c/x^2*(d*
x^2+c)^(3/2)-1/2/a*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2/a*d/
c*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 1.95258, size = 1578, normalized size = 13.96

$$\left[\frac{\sqrt{b^2c - abd}cx^2 \log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) - (2\,bc - ad)\sqrt{c}x^2 \log\left(-\frac{dx^2-2 \cdots}{\cdots}\right) \cdots}{4\,a^2cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^
2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b*c - a*d)*
sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*
x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*(2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) - sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*
c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*sqrt(-b^2*c + a*b*d)*c*x^2*ar
ctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(c)*x^2*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a
```

```
^2*c*x^2), -1/2*(sqrt(-b^2*c + a*b*d)*c*x^2*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)), x)
```

Giac [A] time = 1.12783, size = 163, normalized size = 1.44

$$\frac{1}{2}\,d^2\left(\frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2d^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,d^2}-\frac{\sqrt{dx^2+c}}{ad^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*(b^2*c - a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-
c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^2 + c)/(a*d^2*x^2))
```

3.684 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{c+dx^2}(3bc-ad)}{3a^2cx}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

```
[Out] -Sqrt[c + d*x^2]/(3*a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a^2*c*x) +
(b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a
^(5/2)
```

Rubi [A] time = 0.119622, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}(3bc-ad)}{3a^2cx}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a^2*c*x) +
(b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a
^(5/2)
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-3bc+ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}-\frac{\int -\frac{3bc(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 5.1209, size = 93, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(3bcx^2-a\left(c+dx^2\right)\right)}{3a^2cx^3}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + d*x^2)))/(3*a^2*c*x^3) + (b*Sqrt[b*c -
 a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(5/2)
```

Maple [B] time = 0.013, size = 1059, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x)
```

```
[Out] -1/3/a/c/x^3*(d*x^2+c)^(3/2)-1/2*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*d
^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2
*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
```

```
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))*c+1/2*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*d^(1/2)*ln(
(d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*b/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d-1/2*b
^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))*c+b/a^2/c/x*(d*x^2+c)^(3/2)-b/a^2*d/c*x*(d*x^2+c)^(1/2)-b/a^2*d^(1/2)
*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 1.68074, size = 678, normalized size = 6.46

$$\left[\frac{3\,bcx^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,((3\,bc-ad)x^2-ac)\sqrt{}}{12\,a^2cx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b*c*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*((3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3), 1/6*(3*b*c*x
^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((3*b*c
 - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^4\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)), x)
```

Giac [B] time = 2.18278, size = 290, normalized size = 2.76

$$-\frac{\left(b^2c\sqrt{d} - abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2} - \frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 bc\sqrt{d} - 3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}} - 6\left(\sqrt{d}x - \right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 - c\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) - 2
/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 3*
b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2
)
```

3.685 $\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\cdots\right.}{b^4}$$

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d) + ((7*b*c
 - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c + d*x^2])/(6*b) + (
a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(16*b^4*d^(3/2))
```

Rubi [A] time = 0.402702, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {477, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\cdots\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d) + ((7*b*c
 - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c + d*x^2])/(6*b) + (
a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(16*b^4*d^(3/2))
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{\int \frac{x^4\left(c(6bc-5ad)+d(7bc-6ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6b}\\
&= \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}-\frac{\int \frac{x^2\left(3acd(7bc-6ad)-3d\left(b^2c^2-10abcd+8a^2d^2\right)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{24b^2d}\\
&= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d}+\frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{\int \frac{-3acd\left(b^2c^2-10abc\right.}{}}{}\\
&= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d}+\frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{\left(a^2(bc-ad)^2\right)\int}{b}\\
&= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d}+\frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{\left(a^2(bc-ad)^2\right)\mathrm{S}}{}\\
&= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d}+\frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{a^{3/2}(bc-ad)^{3/2}\,\mathrm{t}}{b}
\end{aligned}$$

Mathematica [A] time = 0.209171, size = 196, normalized size = 0.93

$$\frac{b\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(5c+2dx^2\right)+b^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3\left(-24a^2bcd^2+16a^3d^3+6ab^2c^2d+b^3c^3\right)\log}{48b^4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(5*c + 2*d*x^2) + b^2*(3
*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) + 48*a^(3/2)*d^(3/2)*(b*c - a*d)^(3/2)*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] - 3*(b^3*c^3 + 6*a*b^2*c^
2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*
b^4*d^(3/2))
```

Maple [B] time = 0.019, size = 2081, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*d^2+1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*d^2+1/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-1/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2/b^2*a^2/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4
/b^3*a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x+3/4/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/b^3*a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b
+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^3*a^3/(-a*b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*d-1/2/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/4/b^3*a^2*d*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/24
/b*c/d*x*(d*x^2+c)^(3/2)-1/16/b*c^2/d*x*(d*x^2+c)^(1/2)-3/8/b^2*a*c*x*(d*x^
2+c)^(1/2)-3/8/b^2*a*c^2/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/6/b*x*(d*x
^2+c)^(5/2)/d-1/16/b*c^3/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/4/b^2*a*x*
(d*x^2+c)^(3/2)+1/6/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/b^4*a^3*d^(3/2)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/6/b^2*a^2/(-a
*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-1/2/b^4*a^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)
^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b^3*a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2/b^2*
a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.03, size = 2427, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*lo
g(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(a*b*c*d^2 - a^2*d^3)*sq
rt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c +
a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(8*b^3*d^3*x^5 + 2
*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*
d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a
^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 12*(
a*b*c*d^2 - a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2
*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
- a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a
*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/96*(48*(a*b*c*d^
2 - a^2*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*
d)*x)) + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*
log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^3*d^3*x^5 + 2*(7*b
^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*
x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(24*(a*b*c*d^2 - a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(b^3*c^3 + 6*a*
b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*
x^2 + c)) + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2
*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(c + dx^2\right)^{\frac{3}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [A] time = 1.15657, size = 358, normalized size = 1.7

$$\frac{1}{48}\left(2\left(\frac{4\,dx^2}{b}+\frac{7\,b^9cd^4-6\,ab^8d^5}{b^{10}d^4}\right)x^2+\frac{3\left(b^9c^2d^3-10\,ab^8cd^4+8\,a^2b^7d^5\right)}{b^{10}d^4}\right)\sqrt{dx^2+cx}-\frac{\left(a^2b^2c^2\sqrt{d}-2\,a^3bcd^{\frac{3}{2}}+a^4d^{\frac{5}{2}}\right)}{\sqrt{abcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d*x^2/b + (7*b^9*c*d^4 - 6*a*b^8*d^5)/(b^10*d^4))*x^2 + 3*(b^9*c
^2*d^3 - 10*a*b^8*c*d^4 + 8*a^2*b^7*d^5)/(b^10*d^4))*sqrt(d*x^2 + c)*x - (a
^2*b^2*c^2*sqrt(d) - 2*a^3*b*c*d^(3/2) + a^4*d^(5/2))*arctan(1/2*((sqrt(d)*
x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*
c*d - a^2*d^2)*b^4) + 1/32*(b^3*c^3*sqrt(d) + 6*a*b^2*c^2*d^(3/2) - 24*a^2*
b*c*d^(5/2) + 16*a^3*d^(7/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*d^2
)
```

3.686 $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{c+dx^2}(bc-ad)}{b^3}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3*b^2) + (c
 + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.105579, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{c+dx^2}(bc-ad)}{b^3}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3*b^2) + (c
 + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{\left(c+dx^2\right)^{5/2}}{5bd} - \frac{a\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= -\frac{a\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5bd} - \frac{\left(a(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,\right.}{2b^3}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5bd} - \frac{\left(a(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{b^3d}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5bd} + \frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.091145, size = 108, normalized size = 0.94

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2-5abd\left(4c+dx^2\right)+3b^2\left(c+dx^2\right)^2\right)}{15b^3d}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 3*b^2*(c + d*x^2)^2 - 5*a*b*d*(4*c + d*x^2))
)/(15*b^3*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[
b*c - a*d]])/b^(7/2)
```

Maple [B] time = 0.013, size = 1897, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/5*(d*x^2+c)^(5/2)/b/d-1/6/b^2*a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^3*a*d*(-a*b)^(1/2)*((x+1/
```

```
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x+3/4/b^3*a*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^3*a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2/b^2*a*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-
1/2/b^4*a^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))+1/2/b^4*a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*d^2-1/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*d*c+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*c^2-1/6/b^2*a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b^3*a*d*(-a*b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x-3/4/b^3*a*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^3*a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2/b^2*a*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+
1/2/b^4*a^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+1/2/b^4*a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*d^2-1/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*d*c+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82952, size = 846, normalized size = 7.36

$$\left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(3\,b^2d^2x^4+3\,b^2c^2\cdots\right.}{60\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2
*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 +
 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) - 4*(3*b^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^3*d), 1/30*(15*(a*b*c*d - a^2*d^2
)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(3*b^2*d^2*
x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^2)*sq
rt(d*x^2 + c))/(b^3*d)]
```

Sympy [A] time = 30.7343, size = 104, normalized size = 0.9

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}}{3b^2}-\frac{a\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{5bd}+\frac{\sqrt{c+dx^2}\left(a^2d-abc\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(3/2)/(3*b**2) - a*(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(5/2)/(5*b*d)
 + sqrt(c + d*x**2)*(a**2*d - a*b*c)/b**3
```

Giac [A] time = 1.61985, size = 204, normalized size = 1.77

$$-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^2+c\right)^{\frac{3}{2}}ab^3d^5-15\sqrt{dx^2+c}ab^3cd^5+15\sqrt{dx}}{15\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5/2)*b^4*d^4 - 5
*(d*x^2 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^2 + c)*a*b^3*c*d^5 + 15*sqrt(d*x
^2 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

3.687 $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=158

$$\frac{(8a^2d^2-12abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}+\frac{x\sqrt{c+dx^2}(5bc-4ad)}{8b^2}-\frac{\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x^2])/(4*b)
 - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

Rubi [A] time = 0.251982, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {477, 582, 523, 217, 206, 377, 205}

$$\frac{(8a^2d^2-12abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}+\frac{x\sqrt{c+dx^2}(5bc-4ad)}{8b^2}-\frac{\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```


```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x^2])/(4*b)
 - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{dx^3\sqrt{c+dx^2}}{4b}+\frac{\int \frac{x^2\left(c(4bc-3ad)+d(5bc-4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b}\\
&= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{\int \frac{acd(5bc-4ad)-d\left(3b^2c^2-12abcd+8a^2d^2\right)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8b^2d}\\
&= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{\left(a(bc-ad)^2\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^3}+\frac{\left(3b^2c^2-12abcd+\right.}{8}\\
&= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{\left(a(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^3}+\frac{(}{}\\
&= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{\left(3b^2c^2-12abcd\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.1632, size = 139, normalized size = 0.88

$$\frac{\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+bx\sqrt{c+dx^2}\left(-4ad+5bc+2bdx^2\right)-8\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(5*b*c - 4*a*d + 2*b*d*x^2) - 8*Sqrt[a]*(b*c - a*d)^(3
/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + ((3*b^2*c^2 - 1
```

```
2*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(8*b^3)
```

Maple [B] time = 0.012, size = 1973, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)+3/8/b*c*x*(d*x^2+c)^(1/2)+3/8/b*c^2/d^(1/2)*ln(x*d^
(1/2)+(d*x^2+c)^(1/2))+1/6*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*a/b^2*d*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-3/4*a/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))*c-1/2*a^2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2*a/(-a*b)^(1/2)/b*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*c+1/2*a^2/b^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))-1/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2+a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/6*a/(-a*b)^(1/2)/b
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)-1/4*a/b^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4*a/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(
x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2*a^2/(-a*b)^(1/2)/b^2*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*d-1/2*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2*a^2/b^3*d^(3/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a^3/(-a*b)^(1/2)/b^3/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-a^2/(-a*b)^(
1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+1
/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.88555, size = 1959, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 4*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sq
rt(d*x^2 + c))/(b^3*d), -1/8*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 2*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d
^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/16*(8*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^
2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (3*b^2*c^2 - 12*a*b*c
*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2
*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/8
*(4*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + (3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*
x^2 + c))/(b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(c + dx^2\right)^{\frac{3}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [A] time = 1.13095, size = 267, normalized size = 1.69

$$\frac{1}{8}\sqrt{dx^2 + c}\left(\frac{2\,dx^2}{b} + \frac{5\,b^5cd^2 - 4\,ab^4d^3}{b^6d^2}\right)x + \frac{\left(ab^2c^2\sqrt{d} - 2\,a^2bcd^{\frac{3}{2}} + a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}b^3} - \frac{\left(3\,b^2c^2 - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d*x^2/b + (5*b^5*c*d^2 - 4*a*b^4*d^3)/(b^6*d^2))*x +
 (a*b^2*c^2*sqrt(d) - 2*a^2*b*c*d^(3/2) + a^3*d^(5/2))*arctan(1/2*((sqrt(d)
*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b
*c*d - a^2*d^2)*b^3) - 1/16*(3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*log((sqrt(
d)*x - sqrt(d*x^2 + c))^2)/(b^3*sqrt(d))
```

3.688 $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{c+dx^2}(bc-ad)}{b^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{(c+dx^2)^{3/2}}{3b}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b*c - a*d)^
(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rubi [A] time = 0.076083, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}(bc-ad)}{b^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{(c+dx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b*c - a*d)^
(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)$$
$$=\frac{\left(c+dx^2\right)^{3/2}}{3b}+\frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{2b}$$
$$=\frac{(bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3b}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2b^2}$$
$$=\frac{(bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3b}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{b^2d}$$
$$=\frac{(bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3b}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.067103, size = 83, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)}{3b^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) - ((b*c - a*d)^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Maple [B] time = 0.011, size = 1856, normalized size = 20.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/6/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)-1/4/b^2*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b^2*d^(1/2)*(-a*b)^(
1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2
/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*a*d+1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b^3*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-
a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/b^3/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2+1/b^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
```

```
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c-1/2/b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+1/6/b*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b
^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/b^2*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d+1/2
/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*c-1/2/b^3*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a
*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2+1/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77045, size = 655, normalized size = 7.2

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(bdx^2+4\,bc-3\,ad\right)}{12\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2, -1/6*(3*(b*c - a*d)*sqr
t(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(b*d*x^2 + 4*b*c
 - 3*a*d)*sqrt(d*x^2 + c))/b^2]
```

Sympy [A] time = 17.6166, size = 80, normalized size = 0.88

$$\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^{2}}\,(-ad+bc)}{b^{2}}+\frac{(ad-bc)^{2}\operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{3}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d + b*c)/b**2 + (a*d - b*c
)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.11273, size = 151, normalized size = 1.66

$$\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}+3\,\sqrt{dx^{2}+c}b^{2}c-3\,\sqrt{dx^{2}+c}abd}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2 + 3*sqrt(d*x^2
+ c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3
```

3.689 $\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}+\frac{\sqrt{d}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(3*b*c - 2*a*d)*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rubi [A] time = 0.0955573, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 217, 206, 377, 205}

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}+\frac{\sqrt{d}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(3*b*c - 2*a*d)*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{\int \frac{c(2bc-ad)+d(3bc-2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2}+\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\right.}{b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}+\frac{\sqrt{d}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.162279, size = 110, normalized size = 0.97

$$\frac{\sqrt{d}(3bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}+bdx\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2] + (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(3*b*c - 2*a*d)*Log[d*x + Sqrt[
d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.009, size = 1875, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/6/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*d/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*d^(1/2)*ln((-d*(-a*b)
^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(1/2)/b*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
```

```
)^(1/2)*a*d-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln((-d*(-a*b)^(1/2
)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a+1/2/(-a*b)^(1/2)/b^2/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2-1/(-a*b)^(1
/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c+1/
2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*c^2+1/6/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*d/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*d^(1/2)*ln((d*(-a
*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/(-a*b)^(1/2)/b
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*a*d+1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/(-a*b)^(1/2)/b^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2+1/(-a*b)^
(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d*c-
1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76252, size = 1582, normalized size = 14.

$$\left[\frac{2\sqrt{dx^2+c}bdx - (3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right) - (bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-\cdots}{\cdots}\right.}{4b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*sqrt(-(b*c - a*d)/a)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(3*b*c
- 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*c - a*d)*sqrt(-(b
*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c
)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^
2 + c)*b*d*x + 2*(b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^
2 - a*c*d)*x)) - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(d)*x - c))/b^2, 1/2*(sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(-d)*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d -
 a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [A] time = 1.13906, size = 205, normalized size = 1.81

$$\frac{\sqrt{dx^2+c}dx}{2\,b}-\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^2}-\frac{\left(b^2c^2\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d*x/b - 1/4*(3*b*c*sqrt(d) - 2*a*d^(3/2))*log((sqrt(d)*
x - sqrt(d*x^2 + c))^2)/b^2 - (b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5
/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*
c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2)
```

3.690 $\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)} dx$

Optimal. Leaf size=96

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*
c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(3/
2))
```

Rubi [A] time = 0.106707, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 84, 156, 63, 208}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*
c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(3/
2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{d\sqrt{c+dx^2}}{b} + \frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}$$
$$= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ab}$$
$$= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{abd}$$
$$= \frac{d\sqrt{c+dx^2}}{b} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

Mathematica [A] time = 0.06233, size = 102, normalized size = 1.06

$$\frac{a\sqrt{b}d\sqrt{c+dx^2}+(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)-b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (a*Sqrt[b]*d*Sqrt[c + d*x^2] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]] + (b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(a*b^(3/2))
```

Maple [B] time = 0.011, size = 1919, normalized size = 20.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a),x)
```

```
[Out] 1/3/a*(d*x^2+c)^(3/2)-1/a*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a
*(d*x^2+c)^(1/2)*c-1/6/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a*d*(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a
/b*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
```

```
*c)/b)^(1/2))*c+1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*(-a*b)^
(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*
a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2-1/b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/2/a/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/6/a*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)-1/4/a*d*(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a/b*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a
*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-
1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*c+1/2/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))*d*c+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 3.85178, size = 1493, normalized size = 15.55

$$\left[\frac{2\,bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+4\,\sqrt{dx^2+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\right.}{b^2x^4+2\,abx^2+a^2}\right.}{4\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*s
qrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(a*b), 1/4*(4*b*sqrt(-c)*c*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 4
*sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x
^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b
*x^2 + a^2)))/(a*b), 1/2*(b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b), 1/2*(2*b*sqrt(-c)*c*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b)]
```

Sympy [A] time = 18.1705, size = 92, normalized size = 0.96

$$\frac{d\sqrt{c+dx^2}}{b}+\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{(ad-bc)^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

```
[Out] d*sqrt(c + d*x**2)/b + c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) -
(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt((a*d
 - b*c)/b))
```

Giac [A] time = 1.15973, size = 158, normalized size = 1.65

$$d\left(\frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}+\frac{\sqrt{dx^2+c}}{b}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}abd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*(c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d) + sqrt(d*x^2 + c)/b
- (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*
b*d))/(sqrt(-b^2*c + a*b*d)*a*b*d))
```

3.691 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=102

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/b
```

Rubi [A] time = 0.0914889, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {474, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/b
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)}\,dx &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{\int \frac{-c(bc-2ad)+ad^2x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}-\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{ab}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}-\frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{ab}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.133246, size = 105, normalized size = 1.03

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]])/b
```

Maple [B] time = 0.011, size = 1956, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x)
```

```
[Out] 1/6*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*d^(1/2)*ln((-d*(-a
*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/(-a*b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
```

```
b)^(1/2)*d+1/2*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln((-d*(-a*b)^(1/
2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b*a/(-a*b)^(1/2)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2+1/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2*b/a/(-a
*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-
1/6*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*d^(1/2)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*d-1/2*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln((d*(-a*b)^(1/2)
/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b*a/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/(-a*b)^(1/2)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+1/2*b/a/(-a*b
)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/
a/c/x*(d*x^2+c)^(5/2)+1/a*d/c*x*(d*x^2+c)^(3/2)+3/2/a*d*x*(d*x^2+c)^(1/2)+3
/2/a*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 2.49242, size = 1553, normalized size = 15.23

$$\left[\frac{2\,ad^{\frac{3}{2}}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-(ab\cdots\right.}{b^2x^4+2\,abx^2+a^2}\cdots\right.}{4\,abx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c
- a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^
3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*s
qrt(d*x^2 + c)*b*c)/(a*b*x), -1/4*(4*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 4*sqrt(d*x^2 + c)*b*c)/(a*b*x), 1/2*(a*d^(3/2)*x*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*x*sqrt((b*c - a*d)/a)*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b
*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*sqrt(d*x^2 + c)*b*c)/(a*b*x), -
1/2*(2*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*x*sq
rt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt
((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*sqrt(d*x^2 +
 c)*b*c)/(a*b*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)), x)
```

Giac [A] time = 1.18765, size = 220, normalized size = 2.16

$$-\frac{d^{\frac{3}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}+\frac{2\,c^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}+\frac{\left(b^2c^2\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b + 2*c^2*sqrt(d)/(((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)*a) + (b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a
^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*b)
```

3.692 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)} dx$

Optimal. Leaf size=114

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

```
[Out] -(c*Sqrt[c + d*x^2])/(2*a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rubi [A] time = 0.140629, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(2*a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx,x,x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{4a^2} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2a^2d} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x\right.}{a^2d}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} + \frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}$$

Mathematica [A] time = 0.11092, size = 108, normalized size = 0.95

$$\frac{-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}+\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{ac\sqrt{c+dx^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^2])/x^2) + Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]] - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/Sqrt[b])/(2*a^2)
```

Maple [B] time = 0.012, size = 2003, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x)
```

```
[Out] 1/6*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
```

```
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/6*b/a^2*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
-1/3*b/a^2*(d*x^2+c)^(3/2)+3/2/a*d*(d*x^2+c)^(1/2)-1/2*b/a^2/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/4/a^2*d^(1/2)*(-a*b)^(
1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/
b/a*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+1/4/a^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/a^2*d*(-a*b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x-3/4/a^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b/a*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1
/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/2/a/c/x^2*(d*x^2+c)^(5/2)+1/2/a*d/
c*(d*x^2+c)^(3/2)-3/2/a*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+b/a
^2*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2*(d*x^2+c)^(1/2)*c+1/
2*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))*d^2+1/2*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 3.75458, size = 1628, normalized size = 14.28

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+(2\,bc-3\,ad)\sqrt{c}x^2\log\left(-\right.}{4\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
 - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*
c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4*(2*(2*b*c - 3*a*d)*sqrt(-c
)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c - a*d)*x^2*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4
*(2*(b*c - a*d)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/2*((b*c - a*d)*x^2*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b*c - 3*a*d)*sqrt(-
c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a*c)/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)), x)
```

Giac [A] time = 1.14877, size = 182, normalized size = 1.6

$$\frac{1}{2}\,d^2\left(\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{\left(2\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}-\frac{\sqrt{dx^2+c}c}{ad^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c^2 - 3*a*c*d)*arctan(
```

```
sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^2 + c)*c/(a*d^2*x^2
))
```

3.693 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{c+dx^2}(3bc-4ad)}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

```
[Out] -(c*Sqrt[c + d*x^2])/(3*a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2])/(3*a^2*x
) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/a^(5/2)
```

Rubi [A] time = 0.126267, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {474, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}(3bc-4ad)}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(3*a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2])/(3*a^2*x
) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/a^(5/2)
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-c(3bc-4ad)-d(2bc-3ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}-\frac{\int -\frac{3c(bc-ad)^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0166746, size = 53, normalized size = 0.52

$$-\frac{\left(c+dx^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -((c + d*x^2)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, ((-(b*c) + a*d)*x^2)/(
a*(c + d*x^2))])/(3*a*x^3)
```

Maple [B] time = 0.013, size = 2089, normalized size = 20.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x)
```

```
[Out] -1/2/a*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))+1/a*d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/2/a*d^(3/2)*ln((-d*(-a*b)^(
1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4*b/a^2*d^(1/2)*ln((d*(
-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d
```

```
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2*b/a/(-a*b)^(
1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*d+1/2*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2*b/a/(-a*b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*d-1/2*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/4*b/a^2*d*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/
4*b/a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))*c-2/3/a*d/c^2/x*(d*x^2+c)^(5/2)+2/3/a*d^2/c^2*x*(d*x^2+c)^(3/2)+1/a*d
^2/c*x*(d*x^2+c)^(1/2)-3/2*b/a^2*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/
4*b/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x-b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))*d*c+b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-b/a^2*d/c*x*(d*x^2+c)^(3/2)-1/2*b^2/a^2
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*
c^2+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*c^2-1/6*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2-1/2/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+1/6*b^2
/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)-1/3/a/c/x^3*(d*x^2+c)^(5/2)+b/a^2/c/x*(d*x^2+c)
^(5/2)-3/2*b/a^2*d*x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 1.82501, size = 701, normalized size = 6.87

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(\left(3\,bc-4\,ad\right)x^2\right.}{12\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3), 1/6
*(3*(b*c - a*d)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) + 2*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^{\frac{3}{2}}}{x^4\left(a + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)), x)
```

Giac [B] time = 2.86217, size = 346, normalized size = 3.39

$$-\frac{\left(b^2c^2\sqrt{d} - 2\,abcd^{\frac{3}{2}} + a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}a^2} - \frac{2\left(3\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 bc^2\sqrt{d} - 6\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x -
 sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d
 - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c^2*sqrt(d) - 6
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c^3*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^2*d^(3/2) + 3*b
*c^4*sqrt(d) - 4*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^
2)
```

3.694 $\int \frac{x^4 (c+dx^2)^{5/2}}{a+bx^2} dx$

Optimal. Leaf size=291

$$\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{192b^3}+\frac{x\sqrt{c+dx^2}\left(144a^2bcd^2-64a^3d^3-88ab^2c^2d+5b^3c^3\right)}{128b^4d}-\frac{\left(-240a^2b^2c^2d^2+32\right.}{}$$

```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sqrt[c + d*x
^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*d^2)*x^3*Sqrt[c + d*
x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqrt[c + d*x^2])/(48*b^2) + (d*x^
5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 24
0*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(128*b^5*d^(3/2))
```

Rubi [A] time = 0.574524, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {477, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{192b^3}+\frac{x\sqrt{c+dx^2}\left(144a^2bcd^2-64a^3d^3-88ab^2c^2d+5b^3c^3\right)}{128b^4d}-\frac{\left(-240a^2b^2c^2d^2+32\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```


```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sqrt[c + d*x
^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*d^2)*x^3*Sqrt[c + d*
x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqrt[c + d*x^2])/(48*b^2) + (d*x^
5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 24
0*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(128*b^5*d^(3/2))
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
```

```
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}dx = \frac{dx^5\left(c+dx^2\right)^{3/2}}{8b} + \frac{\int \frac{x^4\sqrt{c+dx^2}\left(c(8bc-5ad)+d(11bc-8ad)x^2\right)}{a+bx^2}dx}{8b}$$

$$= \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2} + \frac{dx^5\left(c+dx^2\right)^{3/2}}{8b} + \frac{\int \frac{x^4\left(c\left(48b^2c^2-85abcd+40a^2d^2\right)+d\left(59b^2c^2-104abcd+48a^2d^2\right)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{48b^2}$$

$$= \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3} + \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2} + \frac{dx^5\left(c+dx^2\right)^{3/2}}{8b} - \frac{\int \frac{x^2(}{}}{}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3}$$

Mathematica [A] time = 0.244448, size = 247, normalized size = 0.85

$$\frac{\frac{bx\sqrt{c+dx^2}\left(48a^2bd^2\left(9c+2dx^2\right)-192a^3d^3-8ab^2d\left(33c^2+26cdx^2+8d^2x^4\right)+b^3\left(118c^2dx^2+15c^3+136cd^2x^4+48d^3x^6\right)\right)}{d} + \frac{3\left(240a^2b^2c^2d^2-320a^3bcd^3+128a^4d^4-40ab^3c\right.}{d^{3/2}}}{384b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(-192*a^3*d^3 + 48*a^2*b*d^2*(9*c + 2*d*x^2) - 8*a*b^
2*d*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^3*(15*c^3 + 118*c^2*d*x^2 + 136*c
*d^2*x^4 + 48*d^3*x^6)))/d + 384*a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(-5*b^4*c^4 - 40*a*b^3*c^3*d + 2
40*a^2*b^2*c^2*d^2 - 320*a^3*b*c*d^3 + 128*a^4*d^4)*Log[d*x + Sqrt[d]*Sqrt[
c + d*x^2]])/d^(3/2))/(384*b^5)
```

Maple [B] time = 0.019, size = 3373, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 3/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*d^2*c-3/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+3/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
```

1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2+1/8/b^3*a^2*d*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*
x-1/48/b*c/d*x*(d*x^2+c)^(5/2)-1/2/b^5*a^5/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/2/b^2*a^2/(-
a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3
-1/b^3*a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/b^5*a^5/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3+7/16/b^3*a^2*d*c*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x+7/16/b^3*a^2*d*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/b^3*a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-3/
2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))*d*c^2+1/10/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/2/b^5*a^4*d^(5/2)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/10/b^2*a^2/(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(5/2)+1/2/b^5*a^4*d^(5/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/8/b*x*(d*x^2+c)^(7/2)/d-5/128/b*c^4/d^(3/2
)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/6/b^2*a*x*(d*x^2+c)^(5/2)+1/2/b^2*a^2/(-a
b)^(1/2)((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*c^2-1/6/b^3*a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/6/b^2*a^2/(-
a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)*c+15/16/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/6/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-
1/4/b^4*a^3*d^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b^4*a^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/b^4*a^4/(-a*b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
d^2-1/2/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/8/b^3*a^2*d*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16/
b^3*a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))*c^2-1/4/b^4*a^3*d^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b^4*a^3*d^(3/2)*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^4*a^4/(-a*b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*d^2-5/192/b*c^2/d*x*(d*x^2+c)^(3/2)-5/128/b*c^3/d*x*(d*x^2+c)^(1/2

```
)-5/24/b^2*a*c*x*(d*x^2+c)^(3/2)-5/16/b^2*a*c^2*x*(d*x^2+c)^(1/2)-5/16/b^2*
a*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/6/b^3*a^3/(-a*b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 48.843, size = 3162, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*
d^3 - 128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
- 192*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3
*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(
5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(
d*x^2 + c))/(b^5*d^2), 1/384*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c
^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + 96*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8
*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3
 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)
*sqrt(d*x^2 + c))/(b^5*d^2), 1/768*(384*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^
3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^
2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))
 - 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 -
128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(4
8*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 10
4*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 1
44*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(b^5*d^2), 1/384*(192*
(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sq
rt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2
*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a
^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2
*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d -
 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(
```

```
b^5*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(c + dx^2\right)^{\frac{5}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [A] time = 1.68015, size = 483, normalized size = 1.66

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,d^2x^2}{b} + \frac{17\,b^{14}cd^7 - 8\,ab^{13}d^8}{b^{15}d^6}\right)x^2 + \frac{59\,b^{14}c^2d^6 - 104\,ab^{13}cd^7 + 48\,a^2b^{12}d^8}{b^{15}d^6}\right)x^2 + \frac{3\left(5\,b^{14}c^3d^5 - 88\,ab^{13}c^2d^6\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^2*x^2/b + (17*b^14*c*d^7 - 8*a*b^13*d^8)/(b^15*d^6))*x^2 +
 (59*b^14*c^2*d^6 - 104*a*b^13*c*d^7 + 48*a^2*b^12*d^8)/(b^15*d^6))*x^2 + 3
*(5*b^14*c^3*d^5 - 88*a*b^13*c^2*d^6 + 144*a^2*b^12*c*d^7 - 64*a^3*b^11*d^8
)/(b^15*d^6))*sqrt(d*x^2 + c)*x - (a^2*b^3*c^3*sqrt(d) - 3*a^3*b^2*c^2*d^(3
/2) + 3*a^4*b*c*d^(5/2) - a^5*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
b^5) + 1/256*(5*b^4*c^4*sqrt(d) + 40*a*b^3*c^3*d^(3/2) - 240*a^2*b^2*c^2*d^
(5/2) + 320*a^3*b*c*d^(7/2) - 128*a^4*d^(9/2))*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/(b^5*d^2)
```

3.695 $\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=144

$$-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\sqrt{c+dx^2}(bc-ad)^2}{b^4}+\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*x^2)^(3/2)
)/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)^(7/2)/(7*b*d) + (a*
(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(9/
2)
```

Rubi [A] time = 0.15003, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\sqrt{c+dx^2}(bc-ad)^2}{b^4}+\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*x^2)^(3/2)
)/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)^(7/2)/(7*b*d) + (a*
(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(9/
2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{\left(c+dx^2\right)^{7/2}}{7bd} - \frac{a\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{a\left(c+dx^2\right)^{5/2}}{5b^2} + \frac{\left(c+dx^2\right)^{7/2}}{7bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b^2}$$
$$= -\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3} - \frac{a\left(c+dx^2\right)^{5/2}}{5b^2} + \frac{\left(c+dx^2\right)^{7/2}}{7bd} - \frac{\left(a(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, \right.}{2b^3}$$
$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3} - \frac{a\left(c+dx^2\right)^{5/2}}{5b^2} + \frac{\left(c+dx^2\right)^{7/2}}{7bd} - \frac{\left(a(bc-ad)^3\right.}{}$$
$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3} - \frac{a\left(c+dx^2\right)^{5/2}}{5b^2} + \frac{\left(c+dx^2\right)^{7/2}}{7bd} - \frac{\left(a(bc-ad)^3\right.}{}$$
$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3} - \frac{a\left(c+dx^2\right)^{5/2}}{5b^2} + \frac{\left(c+dx^2\right)^{7/2}}{7bd} + \frac{a(bc-ad)^{5/}}{}$$

Mathematica [A] time = 0.278928, size = 136, normalized size = 0.94

$$-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2} - \frac{a(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right) - 3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{9/2}} + \frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] -(a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)^(7/2)/(7*b*d) - (a*(b*c - a*d)
*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2) - 3*(b*c - a*d)^(3/2)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]))/(3*b^(9/2))
```

Maple [B] time = 0.011, size = 3127, normalized size = 21.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x)

[Out] 7/16/b^3*a*d*(-a*b)^(1/2)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/10/b^2*a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/10/b^2*a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/2/b^4*a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c-3/2/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2+1/4/b^4*a^2*d^2*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/b^4*a^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+15/16/b^3*a*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/b^3*a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-1/2/b^5*a^3*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-7/16/b^3*a*d*(-a*b)^(1/2)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/7*(d*x^2+c)^(7/2)/b/d-1/6/b^2*a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2/b^4*a^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/2/b^2*a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/b^3*a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6/b^2*a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2/b^4*a^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/2/b^2*a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/b^3*a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/2/b^5*a^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/b^3*a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/b^5*a^3*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b^5*a^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-5/4/b^4*a^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-15/16/b^3*a*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/8/b^3*a*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/4/b^4*a^2*d^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/2/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(

```
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-1/8/b^3
*a*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/2/b^4*a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88092, size = 1137, normalized size = 7.9

$$\left[\frac{105\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/420*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*b^2*
c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*d^3)*x^4
+ (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 + c))/(b^4
*d), 1/210*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 1
61*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*
d^3)*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 +
 c))/(b^4*d)]
```

Sympy [A] time = 62.0938, size = 144, normalized size = 1.

$$-\frac{a\left(c+dx^2\right)^{\frac{5}{2}}}{5b^2}+\frac{a\left(ad-bc\right)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^5\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}}{7bd}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(a^2d-abc\right)}{3b^3}+\frac{\sqrt{c+dx^2}\left(-a^3d^2+2a^2bcd-ab\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(5/2)/(5*b**2) + a*(a*d - b*c)**3*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**5*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(7/2)/(7*b*d)
 + (c + d*x**2)**(3/2)*(a**2*d - a*b*c)/(3*b**3) + sqrt(c + d*x**2)*(-a**3*
d**2 + 2*a**2*b*c*d - a*b**2*c**2)/b**4
```

Giac [A] time = 1.13966, size = 308, normalized size = 2.14

$$-\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^4}+\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^6d^6-21\left(dx^2+c\right)^{\frac{5}{2}}ab^5d^7-35\left(dx^2+c\right)^{\frac{3}{2}}ab^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(sqrt(d*x^2
+ c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/105*(15*(d*x^2
+ c)^(7/2)*b^6*d^6 - 21*(d*x^2 + c)^(5/2)*a*b^5*d^7 - 35*(d*x^2 + c)^(3/2)*
a*b^5*c*d^7 - 105*sqrt(d*x^2 + c)*a*b^5*c^2*d^7 + 35*(d*x^2 + c)^(3/2)*a^2*
b^4*d^8 + 210*sqrt(d*x^2 + c)*a^2*b^4*c*d^8 - 105*sqrt(d*x^2 + c)*a^3*b^3*d
^9)/(b^7*d^7)
```

3.696 $\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=217

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(40a^2bcd^2-16a^3d^3-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}(3b\cdots}{8b^2}$$

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3) + (d*(3*
b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c + d*x^2)^(3/2))/(6*b)
 - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rubi [A] time = 0.392116, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {477, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(40a^2bcd^2-16a^3d^3-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}(3b\cdots}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3) + (d*(3*
b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c + d*x^2)^(3/2))/(6*b)
 - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx = \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} + \frac{\int \frac{x^2\sqrt{c+dx^2}\left(3c(2bc-ad)+3d(3bc-2ad)x^2\right)}{a+bx^2}\,dx}{6b}$$

$$= \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} + \frac{\int \frac{x^2\left(3c\left(8b^2c^2-13abcd+6a^2d^2\right)+3d\left(11b^2c^2-18abcd+8a^2d^2\right)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{24b^2}$$

$$= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} - \frac{\int \frac{3acd\left(11b^2\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} - \frac{\left(a(bc-ad\cdots\right.}{\cdots}$$

$$= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} - \frac{\left(a(bc-ad\cdots\right.}{\cdots}$$

$$= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b} - \frac{\sqrt{a}(bc-a\cdots}{\cdots}$$

Mathematica [A] time = 0.144749, size = 187, normalized size = 0.86

$$\frac{bx\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(9c+2dx^2\right)+b^2\left(33c^2+26cdx^2+8d^2x^4\right)\right)+\frac{3\left(40a^2bcd^2-16a^3d^3-30ab^2c^2d+5b^3c^3\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right.}{\sqrt{d}}}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(9*c + 2*d*x^2) + b^2*(33*c^2 +
26*c*d*x^2 + 8*d^2*x^4)) - 48*Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^
2*b*c*d^2 - 16*a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^
4)
```

Maple [B] time = 0.012, size = 3235, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 3/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*d^2*c-3/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-3/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+3/2*a^2/(-a*b)^(1/2)/b^2
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2+1/6/b*x
*(d*x^2+c)^(5/2)-1/2*a^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d^3+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-7/16*a/b^2*d*c*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x-a^2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-7/16*a/b^2*d*c*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+a^2/(-a
*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2*a^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3-1/8*a/b^2*d*((x+1/b*(-a*b)^(1/2)
```

```
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/16*a/
b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)*c^2-1/10*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/2*a^3/b^4*d^(5/2)*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/10*a/(-a*b)^(1/2)/b*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5
/2)-1/2*a^3/b^4*d^(5/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+5/24/b*c*x*(d*x^2+c)^(3/2)+5/16/b*c^2*x*(d*x^2+c)^(1/2)+5/16/
b*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-15/16*a/b^2*d^(1/2)*ln((d*(-a*b
)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/6*a^2/(-a*b)^(1/
2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)*d-1/6*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/4*a^2/b^3*d^2*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x+5/4*a^2/b^3*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c+1/4*a^2/b^3*d^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^3*d^(3/2)*ln((-d*(-a*
b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2*a^3/(-a*b)^(1/2
)/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*d^2+1/2*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/2*a/(-a*b)^(1/2)/b
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*c^2-1/2*a^3/(-a*b)^(1/2)/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/8*a/b^2*d*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
*x+1/6*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/6*a^2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d
```

Maximа [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.7969, size = 2535, normalized size = 11.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(b^2*c^2*d - 2*a*b*c
```

```
*d^2 + a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c
*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*
(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*
a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/48*(3*(5*b^3*c^3
 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(-a*b*c + a^
2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*
x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 -
 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt
(d*x^2 + c))/(b^4*d), -1/96*(48*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*
b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^
3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b
^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2
 + c))/(b^4*d), -1/48*(24*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^3 - 30
*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*
b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [A] time = 1.14543, size = 373, normalized size = 1.72

$$\frac{1}{48}\left(2\left(\frac{4\,d^2x^2}{b}+\frac{13\,b^9cd^5-6\,ab^8d^6}{b^{10}d^4}\right)x^2+\frac{3\left(11\,b^9c^2d^4-18\,ab^8cd^5+8\,a^2b^7d^6\right)}{b^{10}d^4}\right)\sqrt{dx^2+c}\,x+\frac{\left(ab^3c^3\sqrt{d}-3\,a^2b^2c^2d^{\frac{3}{2}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2/b + (13*b^9*c*d^5 - 6*a*b^8*d^6)/(b^10*d^4))*x^2 + 3*(11
*b^9*c^2*d^4 - 18*a*b^8*c*d^5 + 8*a^2*b^7*d^6)/(b^10*d^4))*sqrt(d*x^2 + c)*
x + (a*b^3*c^3*sqrt(d) - 3*a^2*b^2*c^2*d^(3/2) + 3*a^3*b*c*d^(5/2) - a^4*d^
(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*
b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^4) - 1/32*(5*b^3*c^3 - 30*a*b^
2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)
/(b^4*sqrt(d))
```

3.697 $\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{c+dx^2}(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3/2))/(3*b^
2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.106765, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3/2))/(3*b^
2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx,x,x^2\right)$$
$$= \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{2b}$$
$$= \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{2b^2}$$
$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^3\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{2b^3}$$
$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^3\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{b^3d}$$
$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} - \frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.139983, size = 113, normalized size = 0.95

$$\frac{(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}} + \frac{\left(c+dx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (c + d*x^2)^(5/2)/(5*b) + ((b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*
a*d + b*d*x^2) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]]))/(3*b^(7/2))
```

Maple [B] time = 0.009, size = 3078, normalized size = 25.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -7/16/b^2*d*(-a*b)^(1/2)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^3*d^2*(-a*b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
*a+5/4/b^3*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))*a*c+3/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2)))*a*d*c^2-3/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
```

```
(x+1/b*(-a*b)^(1/2)))*a^2*d^2*c-1/6/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*a*d+1/2/b^3*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a^2
*d^2-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3
-1/6/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*a*d+1/2/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a^2*d^2+1/10/b*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/10/b*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(5/2)+1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/6/b*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/2/b*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*c^2-15/16/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)
^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/8/b^2*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/2/b^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d*c^2-1/2/
b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/2/b^4*
d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))*a^2+1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))*a^3*d^3+1/8/b^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/b^2*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c-1/
2/b^4*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))*a^2+1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*a^3*d^3+15/16/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+
(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/b^2*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c-3/2/b^
3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2*c+7/
16/b^2*d*(-a*b)^(1/2)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b^3*d^2*(-a*b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*a-
5/4/b^3*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))*a*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88861, size = 873, normalized size = 7.34

$$\left[\frac{15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}-4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(3\,b^{2}d^{2}\right.}{60\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d
^2 + (11*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3, -1/30*(15*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
 a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)
*x^2)) - 2*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3]
```

Sympy [A] time = 37.2889, size = 117, normalized size = 0.98

$$\frac{\left(c+dx^{2}\right)^{\frac{5}{2}}}{5b}+\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}\left(-ad+bc\right)}{3b^{2}}+\frac{\sqrt{c+dx^{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{b^{3}}-\frac{\left(ad-bc\right)^{3}\operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{4}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(5/2)/(5*b) + (c + d*x**2)**(3/2)*(-a*d + b*c)/(3*b**2) + sqr
t(c + d*x**2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/b**3 - (a*d - b*c)**3*ata
n(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.1404, size = 248, normalized size = 2.08

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{3}}+\frac{3\left(dx^{2}+c\right)^{\frac{5}{2}}b^{4}+5\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}c+15\sqrt{dx^{2}+c}b^{4}c^{2}-5\left(dx\right.}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5
/2)*b^4 + 5*(d*x^2 + c)^(3/2)*b^4*c + 15*sqrt(d*x^2 + c)*b^4*c^2 - 5*(d*x^2
 + c)^(3/2)*a*b^3*d - 30*sqrt(d*x^2 + c)*a*b^3*c*d + 15*sqrt(d*x^2 + c)*a^2
*b^2*d^2)/b^5
```

3.698 $\int \frac{(c+dx^2)^{5/2}}{a+bx^2} dx$

Optimal. Leaf size=156

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{8b^2}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3}+\frac{dx\left(c+\right.}{4b}$$

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^(3/2))/(4*
b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3)
```

Rubi [A] time = 0.179535, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {416, 528, 523, 217, 206, 377, 205}

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{8b^2}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3}+\frac{dx\left(c+\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^(3/2))/(4*
b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{dx\,(c+dx^2)^{3/2}}{4b} + \frac{\int \frac{\sqrt{c+dx^2}\,(c(4bc-ad)+d(7bc-4ad)x^2)}{a+bx^2}\,dx}{4b}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx\,(c+dx^2)^{3/2}}{4b} + \frac{\int \frac{c(8b^2c^2-9abcd+4a^2d^2)+d(15b^2c^2-20abcd+8a^2d^2)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{8b^2}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx\,(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^3} + \frac{(d(15b^2c^2-20abcd+8}{8b^3}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx\,(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^3} + \frac{(d(15b}{}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx\,(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3} + \frac{\sqrt{d}\,(15b^2c^2-20abcd+}{8}$$

Mathematica [A] time = 0.102895, size = 140, normalized size = 0.9

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bdx\sqrt{c+dx^2}\left(-4ad+9bc+2bdx^2\right)+\frac{8(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2]*(9*b*c - 4*a*d + 2*b*d*x^2) + (8*(b*c - a*d)^(5/2)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(15
*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b
^3)
```

Maple [B] time = 0.011, size = 3101, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*c^3+1/10/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/10/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/2/(-a*b)^(
1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c^2
-3/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))*a^2*d^2*c+3/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))*a*d*c^2+3/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2*c-5/4/b^2*d^(3/2)*ln((-d*(-a*
b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a*c-1/4/b^2*d^2*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x*a+1/2/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))*a^3*d^3-1/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c+1/(-a*b)^(1/2)/b*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*a*d*c-1/2/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))*a^3*d^3-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/6/(-a*b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)*c+1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/(-a*b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+
7/16*d/b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x-1/6/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*a*d-1/4/b^2*d^2*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x*a-5/4/b^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))*a*c+1/2/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a^2*d^2-1/2/(-a*b)^(1/2)/b^2
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*a^2*d^2+7/16*d/b*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/6/(-a*b)^(1/2)/b*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*a*d+
1/8*d/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)*x+15/16/b*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2
```

```
))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))*c^2+1/2/b^3*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a^2-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/8*d/b*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16/b*d^
(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+
1/2/b^3*d^(5/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.13208, size = 2022, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (9*b^2
*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d +
 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - (2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, 1/16
*(8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c -
2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3
+ (b*c^2 - a*c*d)*x)) + (15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-
2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (9*b^2*c*d
- 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d + 8*a^
2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 4*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d
*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) -
(2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [A] time = 1.14431, size = 290, normalized size = 1.86

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b}+\frac{9\,b^5cd^3-4\,ab^4d^4}{b^6d^2}\right)x-\frac{\left(15\,b^2c^2\sqrt{d}-20\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^3}-\frac{\left(b^3c^3\sqrt{d}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b + (9*b^5*c*d^3 - 4*a*b^4*d^4)/(b^6*d^2))*x
 - 1/16*(15*b^2*c^2*sqrt(d) - 20*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/b^3 - (b^3*c^3*sqrt(d) - 3*a*b^2*c^2*d^(3/2) + 3*
a^2*b*c*d^(5/2) - a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3)
```

3.699 $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{d\sqrt{c+dx^2}(2bc-ad)}{b^2}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*b) - (c^(5
/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

Rubi [A] time = 0.190523, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 84, 154, 156, 63, 208}

$$\frac{d\sqrt{c+dx^2}(2bc-ad)}{b^2}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*b) - (c^(5
/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)}\,dx,x,x^2\right)\\
&= \frac{d\left(c+dx^2\right)^{3/2}}{3b}+\frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(bc^2+d(2bc-ad)x\right)}{x(a+bx)}\,dx,x,x^2\right)}{2b}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}+\frac{\text{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{2}d\left(3b^2c^2-3abcd+a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{b^2}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}+\frac{c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{2ab^2}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}+\frac{c^3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad}-\frac{(bc-ad)^3\,\text{Subst}}{}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.110131, size = 114, normalized size = 0.92

$$\frac{d\sqrt{c+dx^2}\left(-3ad+7bc+bdx^2\right)}{3b^2}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2]*(7*b*c - 3*a*d + b*d*x^2))/(3*b^2) - (c^(5/2)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

Maple [B] time = 0.012, size = 3148, normalized size = 25.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a),x)
```

```
[Out] -7/16/a*d*(-a*b)^(1/2)/b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+7/16/a*d*(-a*b)^(1/2)/b*c*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+1/5/a*(d*x^2+c)^(5/2)-1/10/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/10/a*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-15/16/a/b*d^(1/2)
*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))*c^2-1/8/a*d*(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6/b*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6/a*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
*c-1/2/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*c^2+1/3/a*c*(d*x^2+c)^(3/2)-1/a*c^(5/2)*ln((2*c+2*c^(1/2
)*(d*x^2+c)^(1/2))/x)+1/a*(d*x^2+c)^(1/2)*c^2+1/6/b*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6/a*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
3/2)*c-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*c^2-1/2*a/b^3*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-3/2/b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2-1/4/b^2*d^2*(-a*b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x-5/4/b^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/
2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))*c+1/2*a/b^3*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1
/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3-1/2*a/b^2*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+
1/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*d*c+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))*c^3-1/2*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/a/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/4/b^2*d^2*(-a*b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
```

```
c)/b)^(1/2)*x+5/4/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+1/8/a*d*(-a*b)^(1/2)/b*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15
/16/a/b*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 9.03999, size = 1829, normalized size = 14.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^
2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b
^2), 1/12*(12*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2
*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), 1/6*
(3*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*
d^2)*x^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^
2), 1/6*(6*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2)]
```

Sympy [A] time = 39.2436, size = 119, normalized size = 0.96

$$\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^2}\left(-ad^2+2bcd\right)}{b^2}+\frac{c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

```
[Out] d*(c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d**2 + 2*b*c*d)/b**2 + c
**3*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) + (a*d - b*c)**3*atan(sqrt
(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**3*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.14521, size = 227, normalized size = 1.83

$$\frac{1}{3}\left(\frac{3\,c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2+6\sqrt{dx^2+c}b^2c-3\sqrt{dx^2+c}abd}{b^3}-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\operatorname{arctan}}{\sqrt{-b^2c+abd}ab^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*(3*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d) + ((d*x^2 + c)^(
3/2)*b^2 + 6*sqrt(d*x^2 + c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3 - 3*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b^2*d))*d
```

3.700 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(a*x) -
 ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(2*b^2)
```

Rubi [A] time = 0.197359, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {474, 528, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(a*x) -
 ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(2*b^2)
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)}dx &= -\frac{c\left(c+dx^2\right)^{3/2}}{ax}+\frac{\int \frac{\sqrt{c+dx^2}\left(-c(bc-4ad)+d(2bc+ad)x^2\right)}{a+bx^2}dx}{a}\\
&= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}+\frac{\int \frac{-c\left(2b^2c^2-6abcd+a^2d^2\right)+ad^2(5bc-2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}dx}{2ab}\\
&= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}+\frac{\left(d^2(5bc-2ad)\right)\int \frac{1}{\sqrt{c+dx^2}}dx}{2b^2}-\frac{(bc-ad)^3\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}d}{ab^2}\\
&= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}+\frac{\left(d^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{1-dx^2}dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}-\frac{(bc-ad)^3}{}\\
&= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{}{}\right.}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0892338, size = 132, normalized size = 0.91

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}+\sqrt{c+dx^2}\left(\frac{d^2x}{2b}-\frac{c^2}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (-(c^2/(a*x)) + (d^2*x)/(2*b))*Sqrt[c + d*x^2] - ((b*c - a*d)^(5/2)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b^2) + (d^(3/2)*(5
*b*c - 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.013, size = 3191, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x)
```

```
[Out] -7/16/a*d*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x+1/6*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/6/(-a*b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)*d-1/6/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/a*d/c*x*(d*x^2+c)^(5/2)+15/8/a*d*
c*x*(d*x^2+c)^(1/2)-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-1/2/b^2*a*d^(5/
2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/10*b/
a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(5/2)-1/a/c/x*(d*x^2+c)^(7/2)+5/4/a*d*x*(d*x^2+c)^(3/2)+
15/8/a*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/10*b/a/(-a*b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(5/2)-1/8/a*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*x-15/16/a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/b*d^(3/2)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/(-a*b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*d*c-1/2/b^2*a*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-1/8/a*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/16/a*d^(1/2)*ln((-d*(-a*b)^(1
/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
5/4/b*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))*c+1/2*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-7/16/a*d*c*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/6*b/a/(-a
*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)*c-1/2/b*a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/2*b/a/(-a*b)^(1/
2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*c^2-3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*d*c^2+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2+1/2/b*a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2*b
/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
```

```
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)*c^3-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))*d^3+3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c-3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+1/2/b^2*a^2/(-a*b)^(1/2)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 5.54583, size = 1901, normalized size = 13.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(d)*x - c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)/a)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*
d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^
2 + c))/(a*b^2*x), -1/4*(2*(5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)
/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(
b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*s
qrt(d*x^2 + c))/(a*b^2*x), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt((
b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*
c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (5*a*b*c*d - 2*a^2
*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(a*b*d^
2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x), -1/2*((5*a*b*c*d - 2*a^2*d^2
)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (a*
b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)), x)
```

Giac [A] time = 1.14109, size = 279, normalized size = 1.92

$$\frac{\sqrt{dx^2+c}\,d^2x}{2\,b}+\frac{2\,c^3\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^2}+\frac{\left(b^3c^3\sqrt{d}-3\,ab^2c^2d^{\frac{3}{2}}+3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d^2*x/b + 2*c^3*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))
^2 - c)*a) - 1/4*(5*b*c*d^(3/2) - 2*a*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/b^2 + (b^3*c^3*sqrt(d) - 3*a*b^2*c^2*d^(3/2) + 3*a^2*b*c*d^(5/2) -
 a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*b^2)
```

3.701 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(2*a*x^2)
 + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b
*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*b^
(3/2))
```

Rubi [A] time = 0.243589, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 98, 154, 156, 63, 208}

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(2*a*x^2)
 + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b
*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*b^
(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
```

```
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(\frac{1}{2}c(2bc-5ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)}\,dx, x, x^2\right)}{2a}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{4}bc^2(2bc-5ad)+\frac{1}{4}d\left(b^2c^2-6abcd+2a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{ab}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\left(c^2(2bc-5ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2} + \frac{(bc-ad)^3}{\cdots}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\left(c^2(2bc-5ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^2d} + \frac{(bc\cdots}{\cdots}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} + \frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{5/2}\tanh^{-1}\left(\cdots\right.}{a^2b^{3/2}}$$

Mathematica [A] time = 0.130054, size = 125, normalized size = 0.87

$$\frac{-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^2}\left(2ad^2x^2-bc^2\right)}{bx^2} + c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(b*c^2) + 2*a*d^2*x^2))/(b*x^2) + c^(3/2)*(2*b*c - 5*
a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/(2*a^2)
```

Maple [B] time = 0.012, size = 3247, normalized size = 22.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3
+3/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-3
/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+1/2
/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3-7/1
6/a^2*d*(-a*b)^(1/2)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b/a*d^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4
/b/a*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*c+7/16/a^2*d*(-a*b)^(1/2)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b/a*d^2*(-a*b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x-5/4/b/a*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/10*b/a^2*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5
/2)-1/6/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)*d-1/5*b/a^2*(d*x^2+c)^(5/2)+1/10*b/a^2*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/6/a*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)*d+1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+5/6/a*d*(d*x^2+c)^(3/2)-1/2/a/c/x^2*(d*x^2+
c)^(7/2)+1/2/a*d/c*(d*x^2+c)^(5/2)-5/2/a*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2
+c)^(1/2))/x)+5/2/a*d*c*(d*x^2+c)^(1/2)+1/6*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/a*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*d*c+1/2*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/2/b^2*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/6*b/a^2*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/a*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*d*c+1/2*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/2/b^2*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a
*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/3*b/a^2*c*(d*x^2+c
```

```
)^(3/2)+b/a^2*c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2*(d*x^2+c)
^(1/2)*c^2+15/16/a^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+3/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))*d*c^2-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/8/a^2*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/16/a^2
*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))*c^2+1/8/a^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 8.53475, size = 1908, normalized size = 13.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - (2*b^2*c^2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2
 + c))/(a^2*b*x^2), -1/4*(2*(2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*
d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
 a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d
*x^2 + c))/(a^2*b*x^2), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b^2*c^2 - 5*a*b*c*d
```

```
)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^
2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*
sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
 + (2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)), x)
```

Giac [A] time = 1.13786, size = 231, normalized size = 1.6

$$\frac{1}{2}\,d^2\left(\frac{2\,\sqrt{dx^2+c}}{b}-\frac{\sqrt{dx^2+c}\,c^2}{ad^2x^2}-\frac{\left(2\,bc^3-5\,ac^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}+\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2}}{\sqrt{-b^2c}}\right.}{\sqrt{-b^2c+abd}a^2bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*sqrt(d*x^2 + c)/b - sqrt(d*x^2 + c)*c^2/(a*d^2*x^2) - (2*b*c^3 -
 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) + 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b*d^2))
```

3.702 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=130

$$\frac{c\sqrt{c+dx^2}(bc-2ad)}{a^2x}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))/(3*a*x^3)
 + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rubi [A] time = 0.178362, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {474, 580, 523, 217, 206, 377, 205}

$$\frac{c\sqrt{c+dx^2}(bc-2ad)}{a^2x}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))/(3*a*x^3)
 + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)}\,dx = -\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{\int \frac{\sqrt{c+dx^2}\left(-3c(bc-2ad)+3ad^2x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{3a}$$

$$=\frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{\int \frac{3c\left(b^2c^2-3abcd+3a^2d^2\right)+3a^2d^3x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2}$$

$$=\frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}+\frac{(bc-ad)^3\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2b}$$

$$=\frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{}{a-(-}\right.}{a^2b}$$

$$=\frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Mathematica [A] time = 0.096201, size = 125, normalized size = 0.96

$$\frac{c\sqrt{c+dx^2}\left(3bcx^2-a\left(c+7dx^2\right)\right)}{3a^2x^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 7*d*x^2)))/(3*a^2*x^3) + ((b*c - a*d
)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*b)
+ (d^(5/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b
```

Maple [B] time = 0.013, size = 3346, normalized size = 25.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*c^3+4/3/a*d^2/c^2*x*(d*x^2+c)^(5/2)-4/3/a*d/c^2/x*(d*x^2+c)^(7/2
)-15/8*b/a^2*d*c*x*(d*x^2+c)^(1/2)-b/a^2*d/c*x*(d*x^2+c)^(5/2)+b/a/(-a*b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*d*c-1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))*d^3-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2/b*d^(5/2)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*d^2+1/2/b*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/b*a/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3-1/4/a*d^2
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x-5/4/a*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*c-1/10*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/4/a*d^2*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
-5/4/a*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))*c-1/3/a/c/x^3*(d*x^2+c)^(7/2)+5/2/a*d^2*x*(d*x^2+c)^(1/2)+5/2/a*d^(3/2
)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/10*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^
2*c+1/8*b/a^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16*b/a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/6*b/a/(-a*b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-
1/6*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2
+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d^2*c+b/a^2/c/x*(d*x^2+c)^(7/2)-5/4*b/a^2*d*x*(d*x^2+c)^(3/2)-15/8*b/a^
2*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+5/3/a*d^2/c*x*(d*x^2+c)^(3/2)+1
/8*b/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)*x+15/16*b/a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
```

```
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/6*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/6*b^
2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/2*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+7/16*
b/a^2*d*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x+7/16*b/a^2*d*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/2*b/a/(-a*b)^(1/2)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2+3/2*b/a/(-a*b
)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-
1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 3.50601, size = 1932, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a
^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*
x^2 + c))/(a^2*b*x^3), -1/12*(12*a^2*sqrt(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqr
t(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*
log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
- a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c
*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), 1/6*(3*a^2*d^(5/2)*x^3*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*
sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sq
rt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*(a*b*c^2 -
 (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), -1/6*(6*a^2*sqr
t(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d +
```

```
 a^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
+ 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^4\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)), x)
```

Giac [B] time = 1.2001, size = 410, normalized size = 3.15

$$-\frac{d^{\frac{5}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}-\frac{\left(b^3c^3\sqrt{d}-3\,ab^2c^2d^{\frac{3}{2}}+3\,a^2bcd^{\frac{5}{2}}-a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2b}-\frac{2\left(3\left(\sqrt{d}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^(5/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b - (b^3*c^3*sqrt(d) - 3*
a*b^2*c^2*d^(3/2) + 3*a^2*b*c*d^(5/2) - a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c
*d - a^2*d^2)*a^2*b) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c^3*sqrt(d)
 - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c^4*sqrt(d) + 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^3*d^(3/2
) + 3*b*c^5*sqrt(d) - 7*a*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 -
c)^3*a^2)
```

3.703 $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(ad+bc)}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}$$

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c
- a*d])
```

Rubi [A] time = 0.106297, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(ad+bc)}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c
- a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^2\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.118738, size = 89, normalized size = 0.89

$$
\frac{\sqrt{c+dx^2}\left(-3ad-2bc+bdx^2\right)}{3b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.015, size = 362, normalized size = 3.6

$$
\frac{x^2}{3\,bd}\sqrt{dx^2+c}-\frac{2\,c}{3\,bd^2}\sqrt{dx^2+c}-\frac{a}{b^2d}\sqrt{dx^2+c}-\frac{a^2}{2\,b^3}\ln\left(\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\right.}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3/b*x^2/d*(d*x^2+c)^(1/2)-2/3/b*c/d^2*(d*x^2+c)^(1/2)-1/b^2*a/d*(d*x^2+c)
^(1/2)-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.43324, size = 807, normalized size = 8.07

$$\left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\right.}{12\left(b^4cd^2-ab^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c
*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sq
rt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^3*
c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(b^4*c*d^2 - a*b^3*d^3), -1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(-1/2*
(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 -
 (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.6414, size = 142, normalized size = 1.42

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^2+c}b^2cd^4-3\sqrt{dx^2+c}abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^
2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 - 3*sqrt(
d*x^2 + c)*a*b*d^5)/(b^3*d^6)
```

3.704 $\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0639165, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{c+dx^2}}{bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2b}$$
$$=\frac{\sqrt{c+dx^2}}{bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{bd}$$
$$=\frac{\sqrt{c+dx^2}}{bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0540831, size = 68, normalized size = 1.

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.011, size = 318, normalized size = 4.7

$$\frac{1}{bd}\sqrt{dx^2+c}+\frac{a}{2b^2}\ln\left(\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)/b/d+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.5331, size = 648, normalized size = 9.53

$$\left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2+4\,\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\left(b^2c-abd\right)\sqrt{dx^2+c}}{4\,\left(b^3cd-ab^2d^2\right)}, \sqrt{-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*
c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c - a*b*d
)*sqrt(d*x^2 + c))/(b^3*c*d - a*b^2*d^2), 1/2*(sqrt(-b^2*c + a*b*d)*a*d*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*
c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c))/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.12499, size = 86, normalized size = 1.26

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^2+c}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - sqrt(d*x^2 + c)/b)/d
```

3.705 $\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

Rubi [A] time = 0.0427684, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0146924, size = 49, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

Maple [B] time = 0.008, size = 300, normalized size = 6.1

$$-\frac{1}{2b}\ln\left(\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/b/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.38258, size = 498, normalized size = 10.16

$$\left[\frac{\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,\sqrt{b^2c-abd}}, -\frac{\sqrt{-b^2c+abd}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right.\right.}\right.}{2\left(b^2c-abd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(b^2*c - a*b*d), -1/2*sqrt(-b^2*c + a*b
*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c
)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2))/(b^2*c - a*b*d)]
```

Sympy [A] time = 3.76659, size = 36, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.13582, size = 53, normalized size = 1.08

$$\frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.706 $\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0737381, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}dx, x, x^2\right)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}dx, x, x^2\right)}{2a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}dx, x, \sqrt{c+dx^2}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0747263, size = 78, normalized size = 0.98

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/a
```

Maple [B] time = 0.01, size = 331, normalized size = 4.1

$$-\frac{1}{a}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}} + \frac{1}{2a}\ln\left(\left(\left(-2\frac{ad-bc}{b} - 2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right) + 2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d - 2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/a/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2/a/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x), x)
```

Fricas [A] time = 1.8067, size = 1315, normalized size = 16.44

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\,\sqrt{c}\log\left(-\frac{dx^2-2\,\sqrt{d}\right.}{4\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
(b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 2*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d
+ a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/
(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*a
rctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d
*x^2 + b*c)) - 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a*c)]
```

Sympy [A] time = 6.80179, size = 63, normalized size = 0.79

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*sqrt((a*d - b*c)/b)) + atan(
sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c))
```

Giac [A] time = 1.11664, size = 107, normalized size = 1.34

$$-d\left(\frac{b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -d*(b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
a*d) - arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.707 $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

```
[Out] -Sqrt[c + d*x^2]/(2*a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.115032, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2c}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{\ldots}\right)}{2a^2cd}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.274271, size = 109, normalized size = 0.95

$$\frac{-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{a\sqrt{c+dx^2}}{cx^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/(c*x^2)) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/c^(3/2) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/Sqrt[b*c - a*d])/(2*a^2)
```

Maple [B] time = 0.011, size = 385, normalized size = 3.4

$$\frac{b}{a^2}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}} - \frac{b}{2a^2}\ln\left(\left(-2\frac{ad-bc}{b} - 2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right) + 2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d - 2 \ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] b/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*b/a^2/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d
```

```
-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2*b/a^2/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*(d*x^2+c)^(1/2)/a/c/x^2+1/2
/a*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+cx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3), x)
```

Fricas [A] time = 2.22528, size = 1620, normalized size = 14.09

$$\left[\frac{bc^2x^2\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+(2\,bc+ad)\sqrt{c}x}{4\,a^2c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^
2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(b
*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2
*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2
*a*b*x^2 + a^2)) - 2*(2*b*c + a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2
+ c)) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(2*b*c^2*x^2*sqrt(-b/(b*c
 - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c -
a*d))/(b*d*x^2 + b*c)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x
^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/2*(b*
c^2*x^2*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
+ c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (2*b*c + a*d)*sqrt(-c)*x^2*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.12779, size = 159, normalized size = 1.38

$$\frac{1}{2}\,d^2\left(\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}cd^2}-\frac{\sqrt{dx^2+c}}{acd^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqr
t(-c)*c*d^2) - sqrt(d*x^2 + c)/(a*c*d^2*x^2))
```

3.708 $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rubi [A] time = 0.0939242, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{x\sqrt{c+dx^2}}{2bd} - \frac{\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2bd} \\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(bc+2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2d} \\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^2d} \\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.115657, size = 112, normalized size = 0.98

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}} + \frac{bx\sqrt{c+dx^2}}{d}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]])/d^(3/2))/(2*b^2)
```

Maple [B] time = 0.012, size = 386, normalized size = 3.4

$$\frac{x}{2\,bd}\sqrt{dx^2+c} - \frac{c}{2\,b}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}} - \frac{a}{b^2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}} + \frac{a^2}{2\,b^2}\ln\left(\left(\left(-2\,\frac{ad-bc}{b} - 2\,\frac{d\sqrt{-ab}}{b}\left(x + \frac{\sqrt{-ab}}{b}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b/d-1/2/b*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/b
^2*a*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*
```

```
(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/b^2*a^2/(-a*b)^(1
/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06082, size = 1593, normalized size = 13.97

$$\left[\frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{dx^2+c}}{4\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + (b*c + 2*a*d)*sqrt(
d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), 1/4*(a*d^2*s
qrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 -
(a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a
*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(
sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2), -1/4*(2*a*d^2*sqrt(a/(b*c - a*d))*a
rctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a
*d*x^3 + a*c*x)) - 2*sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(d)*log(-2*d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), -1/2*(a*d^2*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.1516, size = 184, normalized size = 1.61

$$-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b^2}+\frac{\sqrt{dx^2+c}x}{2\,bd}+\frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) + 1/2*sqrt(d*x^2 + c)
*x/(b*d) + 1/4*(b*c*sqrt(d) + 2*a*d^(3/2))*log((sqrt(d)*x - sqrt(d*x^2 + c)
)^2)/(b^2*d^2)
```

3.709 $\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

Rubi [A] time = 0.0497784, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b} - \frac{a\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}$$

Mathematica [A] time = 0.0478693, size = 85, normalized size = 1.04

$$\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

Maple [B] time = 0.01, size = 337, normalized size = 4.1

$$\frac{1}{b}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}} - \frac{a}{2b}\ln\left(\left(-2\frac{ad-bc}{b} - 2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right) + 2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d - 2\frac{d\sqrt{-ab}}{b}\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2*a/(-a*b)^(1/2)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87811, size = 1337, normalized size = 16.3

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((b^2c^2-3\,abcd+2\,a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{d}\log\left(-2\right.}{4\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d
)*x - c))/(b*d), 1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*
b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)))/(b*d), 1/2*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + sqrt(d)
*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b*d), 1/2*(d*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - 2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))
/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.12899, size = 138, normalized size = 1.68

$$\frac{a\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b}-\frac{\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt
(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b) - 1/2*log((sqrt(d)*x - sqr
t(d*x^2 + c))^2)/(b*sqrt(d))
```

3.710 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.019298, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0126275, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Maple [B] time = 0.01, size = 306, normalized size = 6.2

$$\frac{1}{2}\ln\left(\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x+\frac{1}{b}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72325, size = 513, normalized size = 10.47

$$\left[-\frac{\sqrt{-abc+a^{2}d}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left(\left(bc-2\,ad\right)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{4\left(abc-a^{2}d\right)},\ \frac{\arctan\left(\frac{\sqrt{abc-a^{2}d}\left(\left(bc-2\,ad\right)x^{2}-a\right.}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}\right.\right.}\right.}{2\sqrt{abc-a^{2}d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.13969, size = 95, normalized size = 1.94

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.711 $\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0533914, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2) \sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{acx} - \frac{\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^2}}{acx} - \frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^2}}{acx} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{a}$$

$$= -\frac{\sqrt{c+dx^2}}{acx} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 3.15722, size = 177, normalized size = 2.39

$$-\frac{\left(\frac{dx^2}{c}+1\right)\left(\frac{4x^2(c+dx^2)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)}{3c^2(a+bx^2)} + \frac{(c+2dx^2)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)}{c\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}}\right)}{x(a+bx^2)\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((1 + (d*x^2)/c)*(((c + 2*d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))]])/(c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]) + (4
*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^2)
/(c*(a + b*x^2))])/(3*c^2*(a + b*x^2))))/(x*(a + b*x^2)*Sqrt[c + d*x^2]))
```

Maple [B] time = 0.011, size = 334, normalized size = 4.5

$$-\frac{b}{2a}\ln\left(\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))-(d*x^2+c)^(1/2)/a/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [B] time = 1.76213, size = 682, normalized size = 9.22

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(abc-a^2d\right)\sqrt{dx^2+c}}{4\left(a^2bc^2-a^3cd\right)x},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c + a^2*d)*b*c*x*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)
*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*
b*c - a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x), -1/2*(sqrt(a*b*c -
 a^2*d)*b*c*x*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b*c -
 a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.14061, size = 150, normalized size = 2.03

$$d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}ad}+\frac{2}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] d^(3/2)*(b*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)*a*d))
```

3.712 $\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

```
[Out] -Sqrt[c + d*x^2]/(3*a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/(3*a^2*c^2
*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*
Sqrt[b*c - a*d])
```

Rubi [A] time = 0.122726, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/(3*a^2*c^2
*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*
Sqrt[b*c - a*d])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3ac}\\ &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}-\frac{\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c^2}\\ &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\ &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\ &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 5.10724, size = 96, normalized size = 0.87

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\left(-ac+2adx^2+3bcx^2\right)}{3a^2c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3) + (b^2*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*Sqrt[b*c - a
*d])
```

Maple [B] time = 0.012, size = 379, normalized size = 3.5

$$-\frac{1}{3\,acx^3}\sqrt{dx^2+c}+\frac{2\,d}{3\,ac^2x}\sqrt{dx^2+c}+\frac{b^2}{2\,a^2}\ln\left(\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)/a/c/x^3+2/3/a*d/c^2/x*(d*x^2+c)^(1/2)+1/2*b^2/a^2/(-a*
b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
```

```
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2*b
^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))+b/a^2/c/x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [B] time = 1.99592, size = 849, normalized size = 7.72

$$\left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^3\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(a^2bc^2-a^3cd\right.}{12\left(a^3bc^3-a^4c^2d\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^3), 1/6*(3*sqrt(a*b*c - a^2*d)*b^2
*c^2*x^3*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b*c^2 -
a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3
*b*c^3 - a^4*c^2*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.89253, size = 263, normalized size = 2.39

$$-\frac{1}{3}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2}+\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2
*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 6*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x - sqr
t(d*x^2 + c))^2 - c)^3*a^2*d^2))
```

3.713 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=109

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rubi [A] time = 0.100527, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {470, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\int \frac{ac+(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{d(bc-ad)}\\ &= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{bd}+\frac{a^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b(bc-ad)}\\ &= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b(bc-ad)}\\ &= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.166266, size = 111, normalized size = 1.02

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{cx}{d\sqrt{c+dx^2}(ad-bc)}+\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x)/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + Log[d*x + Sqrt[d]*S
qrt[c + d*x^2]]/(b*d^(3/2))
```

Maple [B] time = 0.016, size = 720, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b*x/d/(d*x^2+c)^(1/2)+1/b/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/b^2*a*
x/c/(d*x^2+c)^(1/2)+1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^
```

```
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^2*a^2/
(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x*d-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*
c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)+1/2/b^2*a^2/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/b*a^2/(-a*b)^(1/2)/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.00503, size = 2066, normalized size = 18.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*b*c*d*x - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*
sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*d^3*x^2 + a*c*
d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*
x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)
, -1/4*(4*sqrt(d*x^2 + c)*b*c*d*x + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d^2)*sqrt(-
a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*
c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2), -1/2*(2*s
qrt(d*x^2 + c)*b*c*d*x + (a*d^3*x^2 + a*c*d^2)*sqrt(a/(b*c - a*d))*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3
+ a*c*x)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b
*d^4)*x^2), -1/2*(2*sqrt(d*x^2 + c)*b*c*d*x + 2*(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*
c*d^3 - a*b*d^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.14158, size = 201, normalized size = 1.84

$$-\frac{b^2cx}{\left(b^3cd-ab^2d^2\right)\sqrt{dx^2+c}}-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\left(b^2c-abd\right)}-\frac{\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,bd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b^2*c*x/((b^3*c*d - a*b^2*d^2)*sqrt(d*x^2 + c)) - a^2*sqrt(d)*arctan(1/2*(
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(
sqrt(a*b*c*d - a^2*d^2)*(b^2*c - a*b*d)) - 1/2*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/(b*d^(3/2))
```

3.714 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0717906, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)}$$
$$= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.0611258, size = 86, normalized size = 1.12

$$\frac{c}{d\sqrt{c+dx^2}(ad-bc)} + \frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(-(b*c) + a*d)^2)
```

Maple [B] time = 0.012, size = 653, normalized size = 8.5

$$-\frac{1}{bd}\frac{1}{\sqrt{dx^2+c}} + \frac{a}{2\,(ad-bc)\,b}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d - 2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right) - \frac{ad-bc}{b}}} + \frac{adx}{2\,b^2\,(ad-bc)\,c}\sqrt{-ab}\frac{}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b/d/(d*x^2+c)^(1/2)+1/2/b*a/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^2*a*(-a*b)^(1/2)/(
a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d-1/2/b*a/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/b*a/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b^2*a*(-a*b)^(
1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/b*a/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55666, size = 892, normalized size = 11.58

$$\left[-\frac{\left(ad^2x^2+acd\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(b^2c^2-abcd\right)}{4\left(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+\left(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*d^2*x^2 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 4*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2), 1/2*((a*d^2*x^2 +
 a*c*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*
c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) -
 2*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.13728, size = 105, normalized size = 1.36

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^2+c}(bc-ad)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + c/(sqrt(d*x^2 + c)*(b*c - a*d)))/d
```

3.715 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=74

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rubi [A] time = 0.049263, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {471, 12, 377, 205}

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{x}{(bc-ad)\sqrt{c+dx^2}} - \frac{\int \frac{a}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}$$

$$= \frac{x}{(bc-ad)\sqrt{c+dx^2}} - \frac{a\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}$$

$$= \frac{x}{(bc-ad)\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}$$

$$= \frac{x}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.320419, size = 111, normalized size = 1.5

$$\frac{x^2(bc-ad)+ac\sqrt{\frac{dx^2}{c}+1}\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{x\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*x^2 + a*c*Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[1 + (d*x^2)/c]*ArcTanh
[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/((b*c - a*d)^2*x*Sqrt[c + d
*x^2])
```

Maple [B] time = 0.011, size = 653, normalized size = 8.8

$$\frac{x}{bc}\frac{1}{\sqrt{dx^2+c}} - \frac{a}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d - 2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right) - \frac{ad-bc}{b}}} - \frac{adx}{2\,(ad-bc)\,bc}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d - 2 \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b*x/c/(d*x^2+c)^(1/2)-1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*a/b/(a*d
-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*x*d+1/2*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-1/2*a/b/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88482, size = 698, normalized size = 9.43

$$\left[-\frac{\left(dx^2+c\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{}}{4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d
^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d
 + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a
*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)
) + 2*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.18859, size = 139, normalized size = 1.88

$$-\frac{a\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}+\frac{x}{\sqrt{dx^2+c}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -a*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + x/(sqrt(d*x^
2 + c)*(b*c - a*d))
```

3.716 $\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=72

$$\frac{1}{\sqrt{c+dx^2}(bc-ad)} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rubi [A] time = 0.0563638, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{1}{\sqrt{c+dx^2}(bc-ad)} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)}$$
$$= \frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.015186, size = 50, normalized size = 0.69

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(dx^2+c)}{bc-ad}\right)}{\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]/((-(b*c) + a
*d)*Sqrt[c + d*x^2]))
```

Maple [B] time = 0.01, size = 618, normalized size = 8.6

$$-\frac{1}{2\,ad-2\,bc}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}-\frac{dx}{2\,(ad-bc)\,bc}\sqrt{-ab}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-a}}{b}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)-1/2/b*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/(
a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+1/2/b*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39954, size = 678, normalized size = 9.42

$$\left[-\frac{\left(dx^2+c\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+}}{4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b
*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*sqrt(d*
x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [A] time = 8.34151, size = 61, normalized size = 0.85

$$-\frac{1}{\sqrt{c+dx^2}\,(ad-bc)}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -1/(sqrt(c + d*x**2)*(a*d - b*c)) - atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/
b))/(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A] time = 1.13494, size = 96, normalized size = 1.33

$$\frac{b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{1}{\sqrt{dx^2+c}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 1/(sqrt(d*x^2 + c)*(b*c - a*d))
```

3.717 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0369599, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Mathematica [C] time = 1.56943, size = 236, normalized size = 2.99

$$-\frac{\frac{15c(3c+2dx^2)\left(c(a+bx^2)\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}-a(c+dx^2)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)\right)}{\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}}+\frac{4x^4(c+dx^2)(bc-ad)^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)}{a+bx^2}}{15c^3x(a+bx^2)\sqrt{c+dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((15*c*(3*c + 2*d*x^2)*(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))
/(c^2*(a + b*x^2)^2)] - a*(c + d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a +
 b*x^2))]]))/Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)] + (4
*(b*c - a*d)^2*x^4*(c + d*x^2)*Hypergeometric2F1[2, 2, 7/2, ((b*c - a*d)*x^
2)/(c*(a + b*x^2))])/(a + b*x^2))/(15*c^3*(-(b*c) + a*d)*x*(a + b*x^2)*Sqrt
[c + d*x^2])
```

Maple [B] time = 0.009, size = 628, normalized size = 8.

$$\frac{b}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{dx}{(2\,ad-2\,bc)\,c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-
a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))-1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
*d+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1
```

```
/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01076, size = 914, normalized size = 11.57

$$\left[-\frac{4\left(abcd-a^2d^2\right)\sqrt{dx^2+c}x-\left(bcdx^2+bc^2\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)}{b^2x^4+2\,abx^2+a^2}\right.}{4\left(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.14587, size = 144, normalized size = 1.82

$$\frac{b\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}-\frac{dx}{\left(bc^2-acd\right)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

3.718 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*
c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*
(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.112166, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 85, 156, 63, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*
c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*
(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$=-\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2c(bc-ad)}$$

$$=-\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ac}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a(bc-ad)}$$

$$=-\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{acd}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{}\right.}{ad(bc-ad)}$$

$$=-\frac{d}{c(bc-ad)\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.0293199, size = 87, normalized size = 0.81

$$\frac{(bc-ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)-bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{ac\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]) + (b*c
 - a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(a*c*(b*c - a*d)*Sq
rt[c + d*x^2])
```

Maple [B] time = 0.012, size = 681, normalized size = 6.4

$$\frac{1}{ac}\frac{1}{\sqrt{dx^2+c}}-\frac{1}{a}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}+\frac{b}{2\,a\,(ad-bc)}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{}{2\,a\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/a/c/(d*x^2+c)^(1/2)-1/a/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2
/a/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
```

```
1/2))-(a*d-b*c)/b)^(1/2)+1/2/a*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a
/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1
/2/a/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2
/a/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [B] time = 3.32774, size = 2030, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c^2 -
 a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/4
*(4*sqrt(d*x^2 + c)*a*c*d - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b*c^
4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2*sqrt(d*x^2 + c)*a*c
*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
+ 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2
*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)]
```

Sympy [A] time = 10.612, size = 94, normalized size = 0.88

$$\frac{d}{c\sqrt{c+dx^2}\,(ad-bc)}+\frac{b\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*sqrt(c + d*x**2)*(a*d - b*c)) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b*
c)/b))/(a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + atan(sqrt(c + d*x**2)/sqrt(-c)
)/(a*c*sqrt(-c))
```

Giac [A] time = 1.12866, size = 158, normalized size = 1.48

$$-\left(\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(abcd-a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{1}{\left(bc^2-acd\right)\sqrt{dx^2+c}}-\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}cd}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c*d - a^2*d^2)*s
qrt(-b^2*c + a*b*d)) + 1/((b*c^2 - a*c*d)*sqrt(d*x^2 + c)) - arctan(sqrt(d*
x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c*d))*d
```

3.719 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=124

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c + d*x^2])/(a
*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.116737, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c + d*x^2])/(a
*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}+\frac{\int \frac{bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{c(bc-ad)}\\ &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{\int \frac{b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{ac^2(bc-ad)}\\ &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a(bc-ad)}\\ &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a(bc-ad)}\\ &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 5.18765, size = 102, normalized size = 0.82

$$\frac{\frac{d^2x^2}{bc-ad}-\frac{c+dx^2}{a}}{c^2x\sqrt{c+dx^2}}-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d^2*x^2)/(b*c - a*d) - (c + d*x^2)/a)/(c^2*x*Sqrt[c + d*x^2]) - (b^2*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(3/
2))
```

Maple [B] time = 0.012, size = 695, normalized size = 5.6

$$-\frac{b^2}{2\,a\,(ad-bc)}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}-\frac{bdx}{2\,a\,(ad-bc)\,c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a/(a*d-b*c)/c/((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
*d+1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))+1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a/(a
*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/a/c/x/(d*x^2+c)^(1/2)-2/a*d/c^
2*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [B] time = 2.61655, size = 1139, normalized size = 9.19

$$\left[\frac{\left(b^2c^2dx^3+b^2c^3x\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(ab^2c\right.}{4\left(\left(a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3\right)x^3+\left(a^2b^2c^5-2\,a^3bc^4d+a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*
b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c
- 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^4*d - 2*a^3*b*c
^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a^4*c^3*d^2)*x),
 -1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*
c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x
^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^
4*d - 2*a^3*b*c^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a
^4*c^3*d^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^2\right) \left(c + dx^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 2.46304, size = 205, normalized size = 1.65

$$-\frac{b^2\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\left(abc-a^2d\right)}+\frac{d^2x}{\left(bc^3-ac^2d\right)\sqrt{dx^2+c}}+\frac{2\,\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(a*b*c - a^2*d)) + d^2*x/
((b*c^3 - a*c^2*d)*sqrt(d*x^2 + c)) + 2*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^2 - c)*a*c)
```

3.720 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

```
[Out] -(d*(b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*Sqr
t[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c
^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2
*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.216236, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*Sqr
t[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c
^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2
*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$=-\frac{1}{2acx^2\sqrt{c+dx^2}}-\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)}{2ac}$$

$$=-\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}}-\frac{1}{2acx^2\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(bc-3ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,\right.}{ac^2(bc-ad)}$$

$$=-\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}}-\frac{1}{2acx^2\sqrt{c+dx^2}}+\frac{b^3\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2(bc-ad)}-\frac{(2bc}{}$$

$$=-\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}}-\frac{1}{2acx^2\sqrt{c+dx^2}}+\frac{b^3\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{a^2d(bc-ad)}-$$

$$=-\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}}-\frac{1}{2acx^2\sqrt{c+dx^2}}+\frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{b^{5/2}\tanh^{}}{a^2(bc}$$

Mathematica [C] time = 0.046871, size = 117, normalized size = 0.75

$$\frac{2b^2c^2x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+(ad-bc)\left(x^2(3ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac\right)}{2a^2c^2x^2\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b^2*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]
 + (-(b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (d*x^2)/c]))/(2*a^2*c^2*(b*c - a*d)*x^2*Sqrt[c + d*x^2])
```

Maple [B] time = 0.012, size = 763, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -b/a^2/c/(d*x^2+c)^(1/2)+b/a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
)-1/2*b^2/a^2/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x*d+1/2*b^2/a^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))-1/2*b^2/a^2/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*(-a*b)^(1/2)/(a*d
-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*x*d+1/2*b^2/a^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/a/c/x^2/(d*x^2+c)^(1/2)-3/2/a*d/c^2/(d*x^
2+c)^(1/2)+3/2/a*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [B] time = 5.52174, size = 2649, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 +
 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2
*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(
b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - ((2*b^2*c^2*d + a*b*c*d^2 -
```

```
3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*log(-(d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c^3 - a^2*c^2*d + (a*
b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x
^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), -1/4*(2*((2*b^2*c^2*d + a*b*c*d^2 - 3*a^
2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(a*b*c^3
 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*
d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/4*(2*(b^2*c^3*d*x^4
+ b^2*c^4*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + ((2*b^2*c^2*d + a*b*c*d
^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*lo
g(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b*c^3 - a^2*c^2*d
+ (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d
^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/2*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*
sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqr
t(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)
*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/
sqrt(d*x^2 + c)) - (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a + bx^2\right) \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.15079, size = 248, normalized size = 1.59

$$\frac{1}{2} \left(\frac{2\, b^3 \arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2 - a^3d^3\right)\sqrt{-b^2c + abd}} - \frac{\left(dx^2 + c\right)bc - 3\left(dx^2 + c\right)ad + 2\,acd}{\left(abc^3d - a^2c^2d^2\right)\left(\left(dx^2 + c\right)^{\frac{3}{2}} - \sqrt{dx^2 + c}c\right)} - \frac{\left(2\,bc + 3\,ad\right) \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2d^2} \right) d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c*d^2 - a
^3*d^3)*sqrt(-b^2*c + a*b*d)) - ((d*x^2 + c)*b*c - 3*(d*x^2 + c)*a*d + 2*a*
c*d)/((a*b*c^3*d - a^2*c^2*d^2)*((d*x^2 + c)^(3/2) - sqrt(d*x^2 + c)*c)) -
(2*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2*d^2))*d^
2
```

3.721 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c + d*x^2])/
(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a*d)*Sqrt[c + d*x^2])
/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.218351, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c + d*x^2])/
(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a*d)*Sqrt[c + d*x^2])
/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}+\frac{\int \frac{bc-4ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}-\frac{\int \frac{(3bc-4ad)(bc+2ad)+2bd(bc-4ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3ac^2(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{\int \frac{}{(a}}{3a}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{b^3\int}{}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{b^3\,\mathrm{S}}{}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{b^3\,\mathrm{ta}}{a^5}$$

Mathematica [A] time = 5.20593, size = 124, normalized size = 0.7

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(\frac{x^2(5ad+3bc)}{a^2}+\frac{3d^3x^4}{\left(c+dx^2\right)(ad-bc)}-\frac{c}{a}\right)}{3c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 5*a*d)*x^2)/a^2 + (3*d^3*x^4)/((-(b*c)
 + a*d)*(c + d*x^2))))/(3*c^3*x^3) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.014, size = 762, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3/a/c/x^3/(d*x^2+c)^(1/2)+4/3/a*d/c^2/x/(d*x^2+c)^(1/2)+8/3/a*d^2/c^3*x/
(d*x^2+c)^(1/2)+1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a
*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-
b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^3/a^2/(-a*b)^(1/
2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+b
/a^2/c/x/(d*x^2+c)^(1/2)+2*b/a^2*d/c^2*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [B] time = 3.30226, size = 1412, normalized size = 8.02

$$\left[\frac{3\left(b^3c^3dx^5+b^3c^4x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(a^2 \cdots\right.}{12\left(\left(a^3b^2c^5d-2\,a^4bc^4d^2+a^5 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(
(b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^
3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4
- 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((
a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b
*c^5*d + a^5*c^4*d^2)*x^3), 1/6*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(a*b*c
 - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x
^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*
b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2
 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a
^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b*c^5*d + a^5*c^4*d^2)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 3.69682, size = 371, normalized size = 2.11

$$\frac{b^3\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bc-a^3d\right)\sqrt{abcd-a^2d^2}}-\frac{d^3x}{\left(bc^4-ac^3d\right)\sqrt{dx^2+c}}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a*b*c*d - a^2*d^2)) - d^3*x
/((b*c^4 - a*c^3*d)*sqrt(d*x^2 + c)) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
c*d^(3/2) + 3*b*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 +
c))^2 - c)^3*a^2*c^2)
```

3.722 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -(c*x)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3*d*(b*c -
a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.112787, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(c*x)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3*d*(b*c -
a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{\int \frac{ac+(bc-3ad)x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3d(bc-ad)}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{\int \frac{3a^2cd}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3cd(bc-ad)^2}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{(bc-ad)^2}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.303118, size = 160, normalized size = 1.37

$$\frac{x^2\left(a^2d\left(3c+4dx^2\right)-abc\left(3c+5dx^2\right)+b^2c^2x^2\right)-\frac{3a^2(c+dx^2)^2\sqrt{\frac{x^2(ad-bc)}{ac}}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{dx^2}{c}+1}}}{3x\left(c+dx^2\right)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^2*(b^2*c^2*x^2 + a^2*d*(3*c + 4*d*x^2) - a*b*c*(3*c + 5*d*x^2)) - (3*a^2
*Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]*(c + d*x^2)^2*ArcTanh[Sqrt[(-(b/a) + d/c)
*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^2)/c])/(3*(b*c - a*d)^3*x*(c + d*
x^2)^(3/2))
```

Maple [B] time = 0.017, size = 1207, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d*x/(d*x^2+c)^(3/2)+1/3/b/c/d*x/(d*x^2+c)^(1/2)-1/3/b^2*a*x/c/(d*x^2
+c)^(3/2)-2/3/b^2*a/c^2*x/(d*x^2+c)^(1/2)+1/6/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3/b^2*a^2*d/(a*d-b*c)/c^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x-1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b*a^2/(a*d-b*c)
^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x*d+1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3/b^2*a^2*d/(a*d-b*c)/c^2/((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x+1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b*a^2/(a*d-b*c)^2/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x*d-1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.19198, size = 1088, normalized size = 9.3

$$\left[\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x
^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((b*c - 4*a*
d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
```

```
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a
^2*c*d^3)*x^2), -1/6*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(a/(b*c - a*d
))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d)
)/(a*d*x^3 + a*c*x)) - 2*((b*c - 4*a*d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 1.11974, size = 410, normalized size = 3.5

$$-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(b^3c^4d-6\,ab^2c^3d^2+9\,a^2bc^2d^3-4\,a^3cd^4\right)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}-\frac{3\left(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3\right)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) + 1/3*((b^3*c^4*d - 6*a*b^2*c^3*d^2 + 9*a^2*b*c^2*d^3 - 4*a^3*c*d^4)
*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a
^4*c*d^5) - 3*(a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)/(b^4*c^5*d - 4*
a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2
+ c)^(3/2)
```

3.723 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{\sqrt{c+dx^2}(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] -c/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c + d*x^2])
+ (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d
)^(5/2)
```

Rubi [A] time = 0.0920235, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$-\frac{a}{\sqrt{c+dx^2}(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```


```
[Out] -c/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c + d*x^2])
+ (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d
)^(5/2)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)\\
&= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)}\\
&= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)^2}\\
&= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)^2}\\
&= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0287797, size = 77, normalized size = 0.75

$$\frac{c(ad-bc) - 3ad\left(c+dx^2\right)\, {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(dx^2+c)}{bc-ad}\right)}{3d\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (c*(-(b*c) + a*d) - 3*a*d*(c + d*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c
 + d*x^2))/(b*c - a*d)])/(3*d*(b*c - a*d)^2*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.013, size = 1123, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d/(d*x^2+c)^(3/2)+1/6/b*a/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/b^2*a*d*(-a*b)^(1/
2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)*x+1/3/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-1/2*a/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b*a/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x*d+1/2*a/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))+1/6/b*a/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)/c/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)*x-1/3/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*a/(a*d-b*
c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+1/2/b*a/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*a/(a*d-
b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86032, size = 1106, normalized size = 10.74

$$\left[\frac{3\left(ad^{3}x^{4}+2\,acd^{2}x^{2}+ac^{2}d\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(2\,b^{2}c^{2}-3\,abcd+a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}\sqrt{}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right.}{12\left(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3}+\left(b^{2}c^{2}d^{3}-2\,abcd^{4}+a^{2}d^{5}\right)x^{4}+2\left(b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(b/(b*c - a*d))*log((b^2
*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
+ 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*d^2*x^2 + b
*c^2 + 2*a*c*d)*sqrt(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 +
 (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/6*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(-b/(
b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c
 - a*d))/(b*d*x^2 + b*c)) + 2*(3*a*d^2*x^2 + b*c^2 + 2*a*c*d)*sqrt(d*x^2 +
c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 +
```

```
a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [A] time = 1.14421, size = 171, normalized size = 1.66

$$-\frac{\frac{3\,abd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{bc^2+3\,(dx^2+c)ad-acd}{(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (b*c^2 + 3*(d*x^2 + c)*a*d - a*c
*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2)))/d
```

3.724 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}(bc-ad)^2}+\frac{x}{3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c - a*d)^2*
Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.0875847, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}(bc-ad)^2}+\frac{x}{3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c - a*d)^2*
Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} - \frac{\int \frac{a-2bx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3(bc-ad)} \\
&= \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\int \frac{3abc}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3c(bc-ad)^2} \\
&= \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} \\
&= \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{(bc-ad)^2} \\
&= \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\sqrt{ab}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 2.83197, size = 257, normalized size = 2.23

$$\frac{12x^6(c+dx^2)(bc-ad)^3\,{}_2F_1\left(2,2;\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right) - \frac{35c(a+bx^2)(5c+2dx^2)\left(-3a^2(c+dx^2)^2\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c(a+bx^2)}}\right)-c(a+bx^2)\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}(-3ac-4a\ldots\right.}{\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}}}}{315c^3x(a+bx^2)^2(c+dx^2)^{3/2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((-35*c*(a + b*x^2)*(5*c + 2*d*x^2)*(-(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^
2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]*(-3*a*c + b*c*x^2 - 4*a*d*x^2)) - 3*a^2
*(c + d*x^2)^2*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]))/Sqrt[(a*(b
*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)] + 12*(b*c - a*d)^3*x^6*(c +
 d*x^2)*Hypergeometric2F1[2, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(3
15*c^3*(b*c - a*d)^2*x*(a + b*x^2)^2*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.011, size = 1134, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3/b*x/c/(d*x^2+c)^(3/2)+2/3/b/c^2*x/(d*x^2+c)^(1/2)-1/6*a/(-a*b)^(1/2)/(a
*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)-1/6*a/b*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*a/b*d/(a*d-b*c)/c^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x+1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*a/(a*d-b*c)^2/c
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x*d-1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/6*a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/
6*a/b*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*a/b*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*a/(
-a*b)^(1/2)*b/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*a
/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.68625, size = 1130, normalized size = 9.83

$$\left[\frac{3\left(bcd^2x^4+2\,bc^2dx^2+bc^3\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^4+2\left(b^2c^4d-2\,abc^3d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a/(b*c - a*d))*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x
^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt
(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*c^2
*x + (2*b*c*d + a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c
^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a
```

```
*b*c^3*d^2 + a^2*c^2*d^3)*x^2), 1/6*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3
)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*(3*b*c^2*x + (2*b*c*d + a*d^2)*
x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 -
 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^
3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 1.14976, size = 393, normalized size = 3.42

$$\frac{ab\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(2\,b^3c^3d^2-3\,ab^2c^2d^3+a^3d^5\right)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}+\frac{3\left(b^3c^4d-2\,ab^2c^3d^2+a^2bc^2d^3\right)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a*b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2*
d^2)) + 1/3*((2*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + a^3*d^5)*x^2/(b^4*c^5*d - 4
*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5) + 3*(b^3*
c^4*d - 2*a*b^2*c^3*d^2 + a^2*b*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a
^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^(3/2)
```

3.725 $\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d*x^2]) - (
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rubi [A] time = 0.0848251, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d*x^2]) - (
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)}\\
&= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)^2}\\
&= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)^2}\\
&= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.021596, size = 52, normalized size = 0.53

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(dx^2+c)}{bc-ad}\right)}{3(c+dx^2)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)]/(3*(b*c - a*d
)*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.009, size = 1086, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)-1/6/b*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3/b
*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(
a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1
```

```
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6/(a*d-b*c)/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/b*d
*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3/b*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x*d-1/2*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86772, size = 1061, normalized size = 10.83

$$\left[\frac{3\left(bd^2x^4+2\,bcdx^2+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-a}}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)
*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*d*x^2 + 4*b*c -
 a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^
2), 1/6*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d -
2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [A] time = 13.8985, size = 85, normalized size = 0.87

$$\frac{b}{\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{1}{3\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] b/(sqrt(c + d*x**2)*(a*d - b*c)**2) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b
*c)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 1/(3*(c + d*x**2)**(3/2)*(a*
d - b*c))
```

Giac [A] time = 1.10842, size = 159, normalized size = 1.62

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\left(dx^2+c\right)b+bc-ad}{3\left(b^2c^2-2\,abcd+a^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(3*(d*x^2 + c)*b + b*c - a*d)/((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2))
```

3.726 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=122

$$\frac{b^2 \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}} - \frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}(bc-ad)^2} - \frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x)/(3*c^2*(
b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.100544, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}} - \frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}(bc-ad)^2} - \frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x)/(3*c^2*(
b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} + \frac{\int \frac{3bc-2ad-2bdx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\int \frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3c^2(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}$$

Mathematica [C] time = 5.46133, size = 1385, normalized size = 11.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(-1575*Sqrt[(a*(b*c - a*d)*x^2*(c
+ d*x^2))/(c^2*(a + b*x^2)^2)] - (2100*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d
*x^2))/(c^2*(a + b*x^2)^2)])/c - (840*d^2*x^4*Sqrt[(a*(b*c - a*d)*x^2*(c +
d*x^2))/(c^2*(a + b*x^2)^2)])/c^2 + 2100*(((b*c - a*d)*x^2)/(c*(a + b*x^2))
)^(3/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))] + (2800*d*x^2*(((b*c - a*d)*x
^2)/(c*(a + b*x^2)))^(3/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])/c + (1120
*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2)*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))])/c^2 + 1575*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] +
 (2100*d*x^2*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c + (840*d^2*
x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c^2 + (1575*(b*c - a*d
)^2*x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)^2
) + (2100*d*(b*c - a*d)^2*x^6*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))
]])/(c^3*(a + b*x^2)^2) + (840*d^2*(b*c - a*d)^2*x^8*ArcSin[Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2) - (3150*(b*c - a*d)*x^2*ArcS
in[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (4200*d*(-(b
*c) + a*d)*x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b
*x^2)) + (1680*d^2*(-(b*c) + a*d)*x^6*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a +
 b*x^2))]])/(c^3*(a + b*x^2)) + 96*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2
```

```
)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2, 2, 9/2, ((b*c
- a*d)*x^2)/(c*(a + b*x^2))] + (168*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)
))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2, 2, 9/2,
 ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/c + (72*d^2*x^4*(((b*c - a*d)*x^2)/(c*
(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1
[2, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/c^2 + 24*(((b*c - a*d)*x^2)
/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometri
cPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + (48*d*x^2*((
(b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2)
)]*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))
])/c + (24*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)
*x^2)/(c*(a + b*x^2))])/c^2))/(315*a*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5
/2)*(c + d*x^2)^(5/2))
```

Maple [B] time = 0.01, size = 1086, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/6/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*d/
(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x-1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/
(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6/(-a*b)^(1/2)/(a*d-b*c)*b
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)+1/6*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/
2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-
1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.22251, size = 1530, normalized size = 12.54

$$\left[-\frac{3\left(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+\cdots}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(ab^3c^7-3\,a^2b^2c^6d+3\,a^3bc^5d^2-a^4c^4d^3+\left(ab^3c^5d^2-3\,a^2b^2c^4d^3+3\,a^3bc^3d^4-a^4c^2\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((5*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 +
 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x)*sqrt(d
*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4*d^3 + (
a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*x^4 + 2*
(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x^2), 1/6
*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c
*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((5*a*b^2*c^2*d^2 - 7*a^2*b
*c*d^3 + 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4
*d^3 + (a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*
x^4 + 2*(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 1.1623, size = 433, normalized size = 3.55

$$-\frac{b^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\frac{\left(5\,b^3c^3d^3-12\,ab^2c^2d^4+9\,a^2bcd^5-2\,a^3d^6\right)x^2}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}+\frac{3\left(2\,b^3c^4d^2-5\,ab^2c^3d^3+4\,a^2bc^2d^4-a^3cd^5\right)}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) - 1/3*((5*b^3*c^3*d^3 - 12*a*b^2*c^2*d^4 + 9*a^2*b*c*d^5 - 2*a^3*d^6
)*x^2/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5) + 3*(2*b^3*c^4*d^2 - 5*a*b^2*c^3*d^3 + 4*a^2*b*c^2*d^4 - a^3*c
*d^5)/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5))*x/(d*x^2 + c)^(3/2)
```

3.727 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

```
[Out] -d/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(b*c - a*d)
^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2)) + (b^(5/
2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*(b*c - a*d)^(5/2)
)
```

Rubi [A] time = 0.184983, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 85, 152, 156, 63, 208}

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(b*c - a*d)
^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2)) + (b^(5/
2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*(b*c - a*d)^(5/2)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)}{2c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)^2+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,d\right.}{c^2(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ac^2} - \frac{b^3\,\mathrm{S}}{}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ac^2d}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{b^{5/2}\tanh^{-1}\left(\right.}{a(bc-a}$$

Mathematica [C] time = 0.0353496, size = 90, normalized size = 0.62

$$\frac{(bc-ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{3ac\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)]) + (b*
c - a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/(3*a*c*(b*c - a*d
)*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.011, size = 1186, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3/a/c/(d*x^2+c)^(3/2)+1/a/c^2/(d*x^2+c)^(1/2)-1/a/c^(5/2)*ln((2*c+2*c^(1/
2)*(d*x^2+c)^(1/2))/x)+1/6/a/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/a*d*(-a*b)^(1/2)/(a*
d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x+1/3/a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a*b
^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1
/2/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))+1/6/a/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6/a*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*x-1/3/a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a*b^2/(a*d-b*c)^2/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+1/2/a*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/a*b^2/(a*d-b*
c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{5}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [B] time = 9.38594, size = 3514, normalized size = 24.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*(b^2*c^4
 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 +
2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b
*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^
3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*
c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), 1/12*(12*(b^2*c^4 - 2*a*b*c^3*
d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d
- 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + 3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*log(
(b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*
x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*
x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(7*a*b*c^3*d
 - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b
^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a
^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/
6*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arc
tan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x
^2 + b*c)) - 3*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(
c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(7*a*b*c^3*d - 4
*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c
^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c
^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/6*(3
*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 6*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^
3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*
arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*
b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a
^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2
*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2)]
```

Sympy [A] time = 17.0752, size = 133, normalized size = 0.92

$$\frac{d}{3c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d(ad-2bc)}{c^2\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{b^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(a*d - 2*b*c)/(c**2*sqrt(c + d*
x**2)*(a*d - b*c)**2) - b**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*
sqrt((a*d - b*c)/b)*(a*d - b*c)**2) + atan(sqrt(c + d*x**2)/sqrt(-c))/(a*c*
*2*sqrt(-c))
```

Giac [A] time = 1.17202, size = 242, normalized size = 1.67

$$-\frac{1}{3}\left(\frac{3\,b^3\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{6\left(dx^2+c\right)bc+bc^2-3\left(dx^2+c\right)ad-acd}{\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}c^2d}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b^2*c^2*d -
2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b^2*c + a*b*d)) + (6*(d*x^2 + c)*b*c + b*c^2
 - 3*(d*x^2 + c)*a*d - a*c*d)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2
 + c)^(3/2)) - 3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c^2*d))*d
```

3.728 $\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -d/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3*c^2*(b*c
- a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2*a*d)*Sqrt[c + d*x^2
])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqr
t[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.236014, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```



```
[Out] -d/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3*c^2*(b*c
- a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2*a*d)*Sqrt[c + d*x^2
])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqr
t[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{3bc-4ad-4bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}+\frac{\int \frac{(bc-4ad)(3bc-2ad)-2bd(7bc-4ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,d}{3c^2(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}-\frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}-\frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}-\frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}-\frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x}$$

Mathematica [A] time = 5.25171, size = 143, normalized size = 0.8

$$\frac{\sqrt{c+dx^2}\left(\frac{d^2x^2(8bc-5ad)}{\left(c+dx^2\right)(bc-ad)^2}+\frac{cd^2x^2}{\left(c+dx^2\right)^2(bc-ad)}-\frac{3}{a}\right)}{3c^3x}-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3/a + (c*d^2*x^2)/((b*c - a*d)*(c + d*x^2)^2) + (d^2*(8*
b*c - 5*a*d)*x^2)/((b*c - a*d)^2*(c + d*x^2))))/(3*c^3*x) - (b^3*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.014, size = 1192, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6*b/a*d/(a*d-b*c)/c/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
*x-1/3*b/a*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+1/2*b^2/a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a/(-a*b)^(1/2)/(a*d-
b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/6*b^2
/a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6*b/a*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*b
/a*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+1/2*b^2/a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/a/c/x/(d*x^2+
c)^(3/2)-4/3/a*d/c^2*x/(d*x^2+c)^(3/2)-8/3/a*d/c^3*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [B] time = 5.05336, size = 1868, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-a*b*c + a^2
*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x
^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d +
9*a^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 2
2*a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*
a^3*b*c^2*d^3 - 4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^
3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a
^3*b^2*c^6*d^2 + 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*
b^2*c^7*d + 3*a^4*b*c^6*d^2 - a^5*c^5*d^3)*x), -1/6*(3*(b^3*c^3*d^2*x^5 + 2
*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2
*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a
*b*c^2 - a^2*c*d)*x)) + 2*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d + 9*a^3*b*c^3*d^2
- 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 22*a^3*b*c*d^4 -
8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*a^3*b*c^2*d^3 -
4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^5*d^3 +
3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a^3*b^2*c^6*d^2 +
 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*b^2*c^7*d + 3*a^
4*b*c^6*d^2 - a^5*c^5*d^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a+bx^2\right) \left(c+dx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 3.22824, size = 494, normalized size = 2.78

$$\frac{b^3\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(8\,b^3c^5d^4-21\,ab^2c^4d^5+18\,a^2bc^3d^6-5\,a^3c^2d^7\right)x^2}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}+\frac{3\left(3\,b^3c^6d^3-8\,ab^2c^5d^4+7\,a^2bc^4d^5-2\,a^3c^3d^6\right)}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + 1/3*((8*b^3*c^5*d^4 - 21*a*b^2*c^4*d^5 + 18*a^2*b*c^3*d^6 - 5*a
^3*c^2*d^7)*x^2/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^3*b*
c^6*d^4 + a^4*c^5*d^5) + 3*(3*b^3*c^6*d^3 - 8*a*b^2*c^5*d^4 + 7*a^2*b*c^4*d
^5 - 2*a^3*c^3*d^6)/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^
3*b*c^6*d^4 + a^4*c^5*d^5))*x/(d*x^2 + c)^(3/2) + 2*sqrt(d)/(((sqrt(d)*x -
sqrt(d*x^2 + c))^2 - c)*a*c^2)
```

3.729 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}(bc-ad)^2}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{}{2}$$

```
[Out] -(d*(3*b*c - 5*a*d))/(6*a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x^2
*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2))/(2*a*c^3*(b*c -
 a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a^2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.32226, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}(bc-ad)^2}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*(3*b*c - 5*a*d))/(6*a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x^2
*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2))/(2*a*c^3*(b*c -
 a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a^2*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(2bc+5ad)+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{3}{4}(bc-ad)(2bc+5ad)-\frac{3}{4}bd(3bc-5a}{x(a+bx)(c+dx)^{3/2}}\right.}{3ac^2(bc-ad)}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} - \frac{2\,\mathrm{Subst}}{}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^4\,\mathrm{Subs}}{}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^4\,\mathrm{Subs}}{}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} + \frac{(2bc+5}{}$$

Mathematica [C] time = 0.0529899, size = 118, normalized size = 0.56

$$\frac{2b^2c^2x^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(dx^2+c)}{bc-ad}\right)+(ad-bc)\left(x^2(5ad+2bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)+3ac\right)}{6a^2c^2x^2\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b^2*c^2*x^2*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)
] + (-(b*c) + a*d)*(3*a*c + (2*b*c + 5*a*d)*x^2*Hypergeometric2F1[-3/2, 1,
-1/2, 1 + (d*x^2)/c]))/(6*a^2*c^2*(b*c - a*d)*x^2*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.013, size = 1289, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b/a^2/c/(d*x^2+c)^(3/2)-b/a^2/c^2/(d*x^2+c)^(1/2)+b/a^2/c^(5/2)*ln((2*
c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/6*b^2/a^2/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6*b/a^2*d
*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^
2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x+1/2*b^3/a^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)^2*(-a
*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6*b^2/a^2/(a*d-b*c)/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6*b/a
^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c
)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x+1/2*b^3/a^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b^2/a^2/(a*d-b*c)^2
*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/a/c/x^2/(d*x^2+c)^(3/2)-5/6/a*d
/c^2/(d*x^2+c)^(3/2)-5/2/a*d/c^3/(d*x^2+c)^(1/2)+5/2/a*d/c^(7/2)*ln((2*c+2*
c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [B] time = 19.0385, size = 4504, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*((2*
b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4
*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^
2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c
^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2
*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2
*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*
c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^
2), -1/12*(6*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x
^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 +
(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-c)*a
rctan(sqrt(-c)/sqrt(d*x^2 + c)) - 3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^
3*c^6*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*
b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^
2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a
^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*
c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2), 1/12*(6*(b
^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(-b/(b*c - a*d))*arctan
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^
6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (
2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log
(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b
*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*
x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2
 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*
c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d +
 a^4*c^6*d^2)*x^2), 1/6*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2
)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*s
qrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 -
8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c
^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*
a^3*c^2*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a*b^2*c^5
- 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^
3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*
```

```
(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*
b*c^7*d + a^4*c^6*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a+bx^2\right) \left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [A] time = 1.15693, size = 294, normalized size = 1.39

$$\frac{1}{6}\left(\frac{6\,b^4\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^2c^2d^2-2\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{2\left(9\left(dx^2+c\right)bc+bc^2-6\left(dx^2+c\right)ad-acd\right)}{\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{3\,(2\,bc+5\,ad)\arctan}{a^2\sqrt{-c}c^3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^4*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^2*c^2*d^2
 - 2*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)) + 2*(9*(d*x^2 + c)*b*c +
b*c^2 - 6*(d*x^2 + c)*a*d - a*c*d)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(
d*x^2 + c)^(3/2)) - 3*(2*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2
*sqrt(-c)*c^3*d^2) - 3*sqrt(d*x^2 + c)/(a*c^3*d^2*x^2))*d^2
```

3.730 $\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}+\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}-\frac{d(3bc}{c^2x^3\sqrt{c+d}}$$

```
[Out] -d/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/(c^2*(b*c
- a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*Sqrt[c
+ d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c - 2*a*d)*(3*b^2*c^2 + 8*a*b*c
*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.360926, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}+\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}-\frac{d(3bc}{c^2x^3\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/(c^2*(b*c
- a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*Sqrt[c
+ d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c - 2*a*d)*(3*b^2*c^2 + 8*a*b*c
*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{3(bc-2ad)-6bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}+\frac{\int \frac{3\left(b^2c^2-12abcd+8a^2d^2\right)-12bd(3b}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}}{3c^2(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c+d}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c+d}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c+d}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c+d}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c+d}}{3c^2(bc-ad)^2x^3}$$

Mathematica [A] time = 5.34409, size = 160, normalized size = 0.65

$$\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}\left(\frac{x^2(8ad+3bc)}{a^2}+\frac{d^3x^4(8ad-11bc)}{\left(c+dx^2\right)(bc-ad)^2}-\frac{cd^3x^4}{\left(c+dx^2\right)^2(bc-ad)}-\frac{c}{a}\right)}{3c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 8*a*d)*x^2)/a^2 - (c*d^3*x^4)/((b*c -
a*d)*(c + d*x^2)^2) + (d^3*(-11*b*c + 8*a*d)*x^4)/((b*c - a*d)^2*(c + d*x^2
))))/(3*c^4*x^3) + (b^4*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.017, size = 1285, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/a/c/x^3/(d*x^2+c)^(3/2)+2/a*d/c^2/x/(d*x^2+c)^(3/2)+8/3/a*d^2/c^3*x/(d
*x^2+c)^(3/2)+16/3/a*d^2/c^4*x/(d*x^2+c)^(1/2)+1/6*b^3/a^2/(-a*b)^(1/2)/(a*
d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)+1/6*b^2/a^2*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b^2/a^2*d/(a*d-b
*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*x-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b^3/
a^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6*b^3/a^2/(-a*b)^(
1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)+1/6*b^2/a^2*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b^2/a^2*
d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
1/2*b^3/a^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+b/a^2/c/x/(d
*x^2+c)^(3/2)+4/3*b/a^2*d/c^2*x/(d*x^2+c)^(3/2)+8/3*b/a^2*d/c^3*x/(d*x^2+c)
^(1/2)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [B] time = 7.20433, size = 2218, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d
+ 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a
^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*
b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*
(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^
2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*
b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5
*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^
7*d^2 - a^6*c^6*d^3)*x^3), 1/6*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*
c^6*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x
)) - 2*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*
a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*
a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20
*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^
2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^
3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3
*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*
b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^7*d^2 - a^6*c^6*d^3)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] time = 3.80566, size = 662, normalized size = 2.7

$$-\frac{b^{4}\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\frac{\left(11\,b^{3}c^{6}d^{5}-30\,ab^{2}c^{5}d^{6}+27\,a^{2}bc^{4}d^{7}-8\,a^{3}c^{3}d^{8}\right)x^{2}}{b^{4}c^{11}d-4\,ab^{3}c^{10}d^{2}+6\,a^{2}b^{2}c^{9}d^{3}-4\,a^{3}bc^{8}d^{4}+a^{4}c^{7}d^{5}}+\frac{3\left(4\,b^{3}c^{7}d^{4}-11\,ab^{2}c^{6}d^{5}+10\,a^{2}bc^{5}d^{6}-3\,a^{3}\cdots\right.}{b^{4}c^{11}d-4\,ab^{3}c^{10}d^{2}+6\,a^{2}b^{2}c^{9}d^{3}-4\,a^{3}bc^{8}d^{4}+\cdots}}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^4*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b*c*d
 - a^2*d^2)) - 1/3*((11*b^3*c^6*d^5 - 30*a*b^2*c^5*d^6 + 27*a^2*b*c^4*d^7 -
 8*a^3*c^3*d^8)*x^2/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*d^3 - 4*
a^3*b*c^8*d^4 + a^4*c^7*d^5) + 3*(4*b^3*c^7*d^4 - 11*a*b^2*c^6*d^5 + 10*a^2
*b*c^5*d^6 - 3*a^3*c^4*d^7)/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*
d^3 - 4*a^3*b*c^8*d^4 + a^4*c^7*d^5))*x/(d*x^2 + c)^(3/2) - 2/3*(3*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d
^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 18*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d) + 8*a*c^2*d^(3/2))/(((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2*c^3)
```

3.731 $\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{a}(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) - (Sqrt[a
]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
)/(2*b^3*Sqrt[d])
```

Rubi [A] time = 0.168776, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 582, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) - (Sqrt[a
]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
)/(2*b^3*Sqrt[d])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{x^2\left(3c+4dx^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}-\frac{\int \frac{4acd-2d(bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b^2d}\\
&= \frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{(bc-4ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3}-\frac{(a(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^3}\\
&= \frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{(bc-4ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{}{a-(\cdot}\right.}{2b^3}\\
&= \frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}-\frac{\sqrt{a}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.174694, size = 134, normalized size = 0.89

$$\frac{\frac{bx\left(2a+bx^2\right)\sqrt{c+dx^2}}{a+bx^2}+\frac{(bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{a}(4ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*(2*a + b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + (Sqrt[a]*(-3*b*c + 4*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d] +
((b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^3)
```

Maple [B] time = 0.022, size = 2615, normalized size = 17.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b^2+1/2/b^2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1
/4/b^2*a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b^3*a*d*(-a*b)^(1/2)/
(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)-1/4/b^3*a^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b^4*a^2*d^2*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/b^3*a*
d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*c+1/4/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*a*d^(1/2)/(a*d-b*c)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4/b^2*
a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*
c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)-1/4/b^3*a^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^4*a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^3*a*d*(-a*b)
^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))*c+1/4/b^2*a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*a*d^(1/2)/(a*d-b*c)*ln((d*(-
a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/b^2*a/(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-3/4/b^3*a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))+3/4/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-3/4/b^2*a/(-a*b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)-3/4/b^3*a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2
```

```
)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))-3/4/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))*d+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + cx^4}}{\left(bx^2 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 2.77704, size = 2172, normalized size = 14.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2
*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a
*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a
^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(
b^4*d*x^2 + a*b^3*d), -1/8*(4*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqr
t(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*
c*d - 4*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*
c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c
- a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^
2 + c))/(b^4*d*x^2 + a*b^3*d), 1/4*((3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4
*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sq
rt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - (a*b*c - 4*a^2*d + (
b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(b^4*d*x^2 + a*b^3*d), -1/4
*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a*b*d^2)*x^2)*sqrt
(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 +
c))/(b^4*d*x^2 + a*b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.20954, size = 389, normalized size = 2.59

$$\frac{\sqrt{dx^2+c}\,x}{2\,b^2}+\frac{\left(3\,abc\sqrt{d}-4\,a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^3}-\frac{\left(bc\sqrt{d}-4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^3d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*x/b^2 + 1/2*(3*a*b*c*sqrt(d) - 4*a^2*d^(3/2))*arctan(1/
2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)
)/(sqrt(a*b*c*d - a^2*d^2)*b^3) - 1/4*(b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/(b^3*d) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*
c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d)
)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

3.732 $\int \frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{c+dx^2}(2bc-3ad)}{2b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(3/2
))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.110655, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}(2bc-3ad)}{2b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(3/2
))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{4b(bc-ad)}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{4b^2}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2b^2d}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.151748, size = 117, normalized size = 0.86

$$\frac{\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^2}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}}+\frac{a\left(c+dx^2\right)^{3/2}}{a+bx^2}}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((a*(c + d*x^2)^(3/2))/(a + b*x^2) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x
^2] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]))/
b^(3/2))/(2*b*(b*c - a*d))
```

Maple [B] time = 0.011, size = 2543, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2*a*d/
(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)-1/4/b^3*(-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)^
(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^3*a^2*d^2/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/b^2*a*d/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/
4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*(-a*b)^(1/2)*d^(1/2)/(a*
d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1
/2/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)-1/2/b^3*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/2/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*d^(1/2)*(-a*b)^(1/2
)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^3/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d-1/2/b^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4/b^2*(-a*b)^
(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2*a*d/(a*d-b*c)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)+1/4/b^3*(-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^3*a^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^2*a*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4/b^2*(-a*b)^(1/2
)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b^2*(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83694, size = 921, normalized size = 6.77

$$\left[-\frac{\left(2\,abc-3\,a^{2}d+\left(2\,b^{2}c-3\,abd\right)x^{2}\right)\sqrt{b^{2}c-abd}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(bdx^{2}+2\,bc-ad\right)\sqrt{b^{2}c-abd}\sqrt{dx^{2}+}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right.}{8\left(ab^{4}c-a^{2}b^{3}d+\left(b^{5}c-ab^{4}d\right)x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2
)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x
^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^2), -1/4*((
2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c -
 a*b^2*d)*x^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\sqrt{c+dx^{2}}}{\left(a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.15483, size = 150, normalized size = 1.1

$$\frac{\frac{\sqrt{dx^{2}+c}ad^{2}}{\left(\left(dx^{2}+c\right)b-bc+ad\right)b^{2}}+\frac{2\,\sqrt{dx^{2}+c}d}{b^{2}}+\frac{\left(2\,bcd-3\,ad^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/(((d*x^2 + c)*b - b*c + a*d)*b^2) + 2*sqrt(d*x^2
 + c)*d/b^2 + (2*b*c*d - 3*a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*
b*d))/(sqrt(-b^2*c + a*b*d)*b^2))/d
```

3.733 $\int \frac{x^2\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=120

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] -(x*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt[b*c - a*d]) + (Sqrt
[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rubi [A] time = 0.0833367, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {467, 523, 217, 206, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt[b*c - a*d]) + (Sqrt
[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{c+2dx^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{(bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0803817, size = 118, normalized size = 0.98

$$\frac{-\frac{bx\sqrt{c+dx^2}}{a+bx^2}+\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}+2\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2])/(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[b*c - a*d]) + 2*Sqrt[d]*Lo
g[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.011, size = 2547, normalized size = 21.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b^2*d*(-a*b)^(1/2)/(a*d-
```

```
b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+1/4/b^2*d^(3/2)*a/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a+1/4/b^2*d*(-a*b)^(1/
2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
-1/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b
+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/b/(a*d-b*c)/(x+1/b*(-a*b)
^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)+1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(3/2
)*a/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))+1/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))*a-1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/b*d/(a*d-b*c)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-1/4/b*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))*c-1/4/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*ln((-d*(-a*
b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b^2/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*b)
^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4
/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + cx^2}}{\left(bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2, x)
```

Fricas [B] time = 2.50309, size = 2260, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - 4*(a^2*b*c - a^3*d + (a*b^
2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^
2), -1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + 8*(a^2*b*c - a^3*d + (a
*b^2*c - a^2*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c
 - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*
c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*
a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^2), -1/4*
(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^2*d)*(a*b*c - 2*a
^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
x)) - 2*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b
^3*d)*x^2), -1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^
2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*
d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 4*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-
d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c -
a^2*b^3*d)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c + dx^2}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)

[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.16533, size = 339, normalized size = 2.82

$$-\frac{\left(bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^2}-\frac{\sqrt{d}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) -
 1/2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^2)
```

3.734 $\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0620141, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b}$$
$$= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b}$$
$$= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0817497, size = 80, normalized size = 1.

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[-(b*c) + a*d]])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] time = 0.01, size = 1617, normalized size = 20.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d
-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/4*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b
+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/b*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4*(-a*b)^(1/2
)/a/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*ln((d*
(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4*(-a*b)^(1/2
```

```
)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/4*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*
c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x-1/4*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b
+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62071, size = 747, normalized size = 9.34

$$\left[\frac{\left(bdx^2 + ad\right)\sqrt{b^2c - abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) - 4\left(b^2c - abd\right)\sqrt{dx^2}}{8\left(ab^3c - a^2b^2d + \left(b^4c - ab^3d\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*
d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b
^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2
), -1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c
- a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d -
 a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d +
 (b^4*c - a*b^3*d)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c + dx^2}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.16134, size = 107, normalized size = 1.34

$$\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^2+c}}{\left(\left(dx^2+c\right)b-bc+ad\right)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*(arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c + a*d)*b))
```

3.735 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0345822, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}$$

$$= \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a}$$

$$= \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.291004, size = 112, normalized size = 1.37

$$\frac{x\left(\frac{a\left(c+dx^2\right)}{a+bx^2}+\frac{c\sqrt{\frac{dx^2}{c}+1}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{2a^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*((a*(c + d*x^2))/(a + b*x^2) + (c*Sqrt[1 + (d*x^2)/c]*ArcTanh[Sqrt[(-(b/
a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(2*
a^2*Sqrt[c + d*x^2])
```

Maple [B] time = 0.009, size = 2559, normalized size = 31.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b/a*d*(-a*b)^(1/2)/(a*d
-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b/a*d*(-a*b)^(1/2)/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4/
a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x+1/4/a*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1
/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2
))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
```

```
*c)/b)^(3/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-
b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b
^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))+1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/a*d^
(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))*c-1/4/(-a*b)^(1/2)/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+
(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/4/(-a*b)^(1/2)/a/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/(-a*b)^(1/2)/a
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+1/4/a*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))*d-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2, x)
```

Fricas [B] time = 2.12999, size = 765, normalized size = 9.33

$$\left[\frac{4\left(abc-a^2d\right)\sqrt{dx^2+c}\,x-\left(bcx^2+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+}}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(a^3bc-a^4d+\left(a^2b^2c-a^3bd\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c - a^4*d + (a^2*b^2*c - a^
3*b*d)*x^2), 1/4*(2*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c)*sqr
t(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^3*b*c
 - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [B] time = 3.04043, size = 294, normalized size = 3.59

$$-\frac{c\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,a}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) - ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a*b)
```

3.736 $\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=119

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.111746, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```



```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\operatorname{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^2d}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.220515, size = 112, normalized size = 0.94

$$\frac{\frac{a\sqrt{c+dx^2}}{a+bx^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2])/(a + b*x^2) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]] + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(Sq
rt[b]*Sqrt[b*c - a*d]))/(2*a^2)
```

Maple [B] time = 0.015, size = 2585, normalized size = 21.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/a^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a^2*(d*x^2+c)^(1/2)+
1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a*d/(a*d-
b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+1/4/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a*d/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/(-a*b)^(1/2)/a*d/(a
*d-b*c)*b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln((-d*(-a*b)^
(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4/(-a*b)^(1/2)/a/(a*
d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/(-a
*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))-1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1
/4/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)^(1/2)
/b*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/a/b
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/2/a^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/2/a^2*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-1/2/a/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))*d+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x), x)
```

Fricas [B] time = 2.62733, size = 2221, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c
)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c - a^2*b*
d)*sqrt(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/
8*(8*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) - (2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*
b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a*b*c - a^2*d +
(2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c)*log(-
(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b^2*c - a^2*b*d)*sqrt
(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d
*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d
 + (b^2*c*d - a*b*d^2)*x^2)) + 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2
)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*
x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{x\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)**2), x)
```

Giac [A] time = 1.1411, size = 170, normalized size = 1.43

$$\frac{1}{2}d^2\left(\frac{\sqrt{dx^2+c}}{\left(\left(dx^2+c\right)b-bc+ad\right)ad}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^2*(sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c + a*d)*a*d) - (2*b*c - a*d)*
arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^
2) + 2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2))
```

3.737 $\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^2)) - ((3*
b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5
/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.110186, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {469, 583, 12, 377, 205}

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^2)) - ((3*
b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5
/2)*Sqrt[b*c - a*d])
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{\int \frac{-3c-2dx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{\int \frac{c(3bc-2ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2c}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 5.07074, size = 101, normalized size = 0.89

$$\left(-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}\right)\sqrt{c+dx^2}+\frac{(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2))) + ((-3*b*c + 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*Sqrt[
b*c - a*d])
```

Maple [B] time = 0.013, size = 2618, normalized size = 23.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a*d^2*(-a*b)^(1/2)/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a
```

```
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/a*d^2*(-a*b)^(1/2)/(a*d-b*c)
/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/4*b/a^2/(
-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-1/a^2/c/x*(d*x^2+c)^(3/2)+1/4/a*d^(3/2)/(a*d-b*c)*ln(
(-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4*b/a^2/(-a
*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+1/4/a*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a
*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4/a^2*d/(a*d-b*c)*b*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-1/4/a^2*d^(1/2)/(a*d-b*c)*b*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))*c-3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/a^2*d/(a*d-b*c)*b*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/a^2
*d^(1/2)/(a*d-b*c)*b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))*c-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a^2*d^(1/2)*ln((
-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4/a^2*d^(1/2
)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a^2*d^(
1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)+1/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a^2*d*(-a
*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-3/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/a^2*d/c*x*(d*x^2+c)^(1/2)+1/4
/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2), x)
```

Fricas [B] time = 2.34814, size = 956, normalized size = 8.46

$$\left[\frac{\left(\left(3\,b^2c-2\,abd\right)x^3+\left(3\,abc-2\,a^2d\right)x\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+}}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(\left(a^3b^2c-a^4bd\right)x^3+\left(a^4bc-a^5d\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b*c - 2*a^3*d + 3*(a*b^2*c -
 a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d)*x^3 + (a^4*b*c - a^5
*d)*x), -1/4*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(a*b*c
- a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b*c -
2*a^3*d + 3*(a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d
)*x^3 + (a^4*b*c - a^5*d)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)**2), x)
```

Giac [B] time = 3.68658, size = 444, normalized size = 3.93

$$\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2}+\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
```

```
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 10*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d))/(((sqrt(d)*x
 - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2)
```

3.738 $\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x^2*(a + b*
x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*Sqrt[c]) -
(Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^3*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.208962, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 99, 151, 156, 63, 208}

$$-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```


```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x^2*(a + b*
x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*Sqrt[c]) -
(Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^3*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2(bc-ad)}\\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(b(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^3} - \frac{(4bc-ad)\operatorname{Subs}}{}\\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(b(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^3d} - \frac{(4bc-ad)\,\mathrm{S}}{}\\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.229626, size = 190, normalized size = 1.19

$$
\frac{\sqrt{c}\left(a\left(a+2bx^2\right)\sqrt{c+dx^2}(bc-ad)+\sqrt{b}x^2\left(a+bx^2\right)(4bc-3ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)-x^2\left(a+bx^2\right)\left(a^2d^2-5ab\right.}{2a^3\sqrt{c}x^2\left(a+bx^2\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^2*(a + b*x^2)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^2)*Sqrt[c + d*x^2] + Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^2*(a + b*x^2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]]))/(2*a^3*Sqrt[c]*(-(b*c) + a*d)*x^2*(a + b*x^2))
```

Maple [B] time = 0.014, size = 2669, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/a^3*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))+1/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))*d-1/2/a^2/c/x^2*(d*x^2+c)^(3/2)-1/2/a^2*d/c^(1/2)*ln((2*c+2*c^(1/2)
*(d*x^2+c)^(1/2))/x)+1/2/a^2*d/c*(d*x^2+c)^(1/2)-1/a^3*d^(1/2)*(-a*b)^(1/2)
*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+2*b/a^3*c^(1/2)
*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(
x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((d*
(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*b/a/(-a*b)^(1
/2)*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)-1/4/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)
^(1/2)))-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*c-2*b/a^3*(d*x^2+c)^(1/2)+b/a^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+b/a^3*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-b/a^3/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4*b/a^2*d/(a*d-b*c)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)-1/4*b/a^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4*b/a^2*d/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4*b^2/a^2/(-a*b
```

```
)^(1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*
ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4*b/a^2
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
+1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^2/(-a*b)^(1/2)*
d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 3.14152, size = 2222, normalized size = 13.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 +
 c))/(a^3*b*c*x^4 + a^4*c*x^2), -1/8*(4*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c -
 a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c^2 - 3*a*
b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(
2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*
sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b*c*x^2 + a^2*c)
*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*
x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2
 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((4
*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a
^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3
*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^2*c - a*b*d)*x^
4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(2
*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{x^3 \left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)**2), x)
```

Giac [A] time = 1.17364, size = 258, normalized size = 1.62

$$-\frac{1}{2}\,d^3\left(\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b - 2\sqrt{dx^2+c}\,bc + \sqrt{dx^2+c}\,ad}{\left(\left(dx^2+c\right)^2 b - 2\left(dx^2+c\right)bc + bc^2 + \left(dx^2+c\right)ad - acd\right)a^2d^2} - \frac{\left(4\,b^2c - 3\,abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^3d^3} + \frac{(4\,bc - ad}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*((2*(d*x^2 + c)^(3/2)*b - 2*sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*
a*d)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*
d)*a^2*d^2) - (4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*
b*d))/(sqrt(-b^2*c + a*b*d)*a^3*d^3) + (4*b*c - a*d)*arctan(sqrt(d*x^2 + c)
/sqrt(-c))/(a^3*sqrt(-c)*d^3))
```

3.739 $\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=147

$$\frac{\sqrt{c+dx^2}(15bc-2ad)}{6a^3cx}+\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^
3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*b*c - 4*a*d)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.202642, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {469, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}(15bc-2ad)}{6a^3cx}+\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^
3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*b*c - 4*a*d)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int \frac{-5c-4dx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{\int \frac{-c(15bc-2ad)-10bcdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2c}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int -\frac{3bc^2(5bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^3c^2}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^3}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,d}{2a^3}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{b(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 5.13965, size = 120, normalized size = 0.82

$$\frac{\sqrt{c+dx^2}\left(3bx^2\left(\frac{bx^2}{a+bx^2}+4\right)-\frac{2a\left(c+dx^2\right)}{c}\right)}{6a^3x^3} + \frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a*(c + d*x^2))/c + 3*b*x^2*(4 + (b*x^2)/(a + b*x^2)))
)/(6*a^3*x^3) + (b*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2]])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.017, size = 2667, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -2*b/a^3*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-5/4*b^2/a^3/(-a*b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)+5/4*b/a^3*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*b/a
^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))-1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+5/4*b^2/a^3/(-a*b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+5/4*b/a^3*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-1/3/a^2/c/x^3*(d*x^2+c)^(3/2)-1/4*b^2/a^3/(a*d-b*c)
/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*b/a^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+2*b/a^3/c/x*(d*x^2+c)^(3/2
)-1/4/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))-2*b/a^3*d/c*x*(d*x^2+c)^(1/2)+1/4*b^2/a^3*d^(1/2)/(a
*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1
/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a^2*d^2*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4*b^2/a^
3*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x+1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*
b^2/a^3*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^3*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)
^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-5/4*b/a^2/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+5/4*b^2/a^
3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*c+5/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*d-5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4), x)
```

Fricas [B] time = 2.60957, size = 1238, normalized size = 8.42

$$\left[\frac{3\left(\left(5\,b^3c^2-4\,ab^2cd\right)x^5+\left(5\,ab^2c^2-4\,a^2bcd\right)x^3\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-\cdots\right.}{b^2x^4+2\,abx^2+a^2}\right.}{24\left(\left(a^4b^2c^2-a^5bc\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
+ a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b*c^2 - 2
*a^4*c*d - (15*a*b^3*c^2 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2
*c^2 - 6*a^3*b*c*d + a^4*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c
*d)*x^5 + (a^5*b*c^2 - a^6*c*d)*x^3), 1/12*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^
5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b*c^2 - 2*a^4*c*d - (15*a*b^3*c^2
 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4
*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c*d)*x^5 + (a^5*b*c^2 - a
^6*c*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)**2), x)
```

Giac [B] time = 6.57034, size = 487, normalized size = 3.31

$$-\frac{\left(5\,b^{2}c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}a^{3}}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-}{\Big(\Big(\sqrt{d}x-\sqrt{dx^{2}+c}\Big)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\,\Big(\sqrt{d}x-\sqrt{dx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c*sqrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c
))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*d + b*c^2)*a^3) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt
(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c^2*sqrt(d) + 6*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.740 $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=197

$$\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}+\frac{3x\sqrt{c+dx^2}(3bc-4ad)}{8b^3}-\frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4}-\frac{x^3}{2}$$

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c + d*x^2])/(
4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 2*a*
d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(8*b^4*Sqrt[d])
```

Rubi [A] time = 0.337645, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {467, 581, 582, 523, 217, 206, 377, 205}

$$\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}+\frac{3x\sqrt{c+dx^2}(3bc-4ad)}{8b^3}-\frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4}-\frac{x^3}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c + d*x^2])/(
4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 2*a*
d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(8*b^4*Sqrt[d])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2} dx = -\frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\sqrt{c+dx^2}(3c+6dx^2)}{a+bx^2} dx}{2b}$$

$$= \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(6c(2bc-3ad)+6d(3bc-4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}} dx}{8b^2}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{\int \frac{6acd(3bc-4ad)-6d\left(b^2c^2-8abcd+8a^2d^2\right)x}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{16b^3d}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{(3a(bc-2ad)(bc-ad)) \int \frac{1}{\left(a+bx^2\right)}}{2b^4}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{(3a(bc-2ad)(bc-ad)) \operatorname{Subst}\left(\int\right.}{2b^4}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3 \left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\right.}{2b^4}$$

Mathematica [A] time = 0.169172, size = 192, normalized size = 0.97

$$\frac{\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}} - \frac{12\sqrt{a}\left(2a^2d^2-3abcd+b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}} + \frac{b\sqrt{c+dx^2}\left(-12a^2dx+ab\left(9cx-6dx^3\right)+b^2x^3\left(5c+2dx^2\right)\right)}{a+bx^2}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x^2]*(-12*a^2*d*x + b^2*x^3*(5*c + 2*d*x^2) + a*b*(9*c*x - 6
*d*x^3)))/(a + b*x^2) - (12*Sqrt[a]*(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d] + (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/
(8*b^4)
```

Maple [B] time = 0.021, size = 4795, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/b^2*a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/b^4*a^3*d^(5/2)/(a*
d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/
b^2*a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/b^4*a^3*d^(5/2)/(a*d-b*c
)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-9/8/b^3*
```

```
a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
+3/4/b^3*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-3/4/b^2*a/(-a*b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/
8/b^3*a*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x+3/4/b^2*a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/8/b^3*a*d*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x-9/8/b^3*a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c+3/4/b^5*a^3*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/8/b^2*a*d/(a*d-b*c)*c*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x-3/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4/b^3*a^2/(-a*b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*d-3/4/b^4*a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d/(a*
d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)*x+3/8/b^2*a*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/8/b^2*c*x*(d*x^2+c)^(1/2)+3/8/b^2
*c^2/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/4/b^2*a/(-a*b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
+3/4/b^4*a^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))+1/4/b^2*a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b^4*a^2*d^(3/2)*ln((-d*(-a*b
)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2*x*(d*x^2+c)^(3
/2)+3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*d*c+3/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4/b^5*a
^3*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))+3/8/b^2*a*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/2/b^4*a^2*d^2*(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)*c+3/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))*c^2+3/2/b^4*a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4/b^3*a*d*(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)*c^2-3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
```

```
b*(-a*b)^(1/2)))*d*c+1/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/8/b^3*a
^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/b^3*a^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/b^4*a^3/(-a*b)^(1/2)/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+3/4/b^2*a/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
^2-3/4/b^4*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(
-a*b)^(1/2)))*d^2-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c^2-9/8/b^3*a^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^
(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/b^4*a^2*d^2*(-a*b)
^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8/b^2*a*d^(1
/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))-1/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/8/b^3*a^2*d^2/(a*d-b
*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}x^4}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 3.48654, size = 2678, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*
a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 6
*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*
```

```
d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^
2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b
^4*d), -1/8*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*
d + 8*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 3*(a*b*
c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^2)*x^3 +
 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b^4*d), -
1/16*(12*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a*b*c - a^2
*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c
)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 3*(a*b^2*c^2 - 8*a^2
*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(d)*log
(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^3*d^2*x^5 + (5*b^3*c*
d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b
^5*d*x^2 + a*b^4*d), -1/8*(6*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x
^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d
^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^3*d^2*x^5 + (5*
b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 +
c))/(b^5*d*x^2 + a*b^4*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.19291, size = 532, normalized size = 2.7

$$\frac{1}{8}\sqrt{dx^2+cx}\left(\frac{2\,dx^2}{b^2}+\frac{5\,b^7cd^2-8\,ab^6d^3}{b^9d^2}\right)+\frac{3\left(ab^2c^2\sqrt{d}-3\,a^2bcd^{\frac{3}{2}}+2\,a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}b^4}-\frac{3\left(b^2c^2\sqrt{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*x*(2*d*x^2/b^2 + (5*b^7*c*d^2 - 8*a*b^6*d^3)/(b^9*d^2))
 + 3/2*(a*b^2*c^2*sqrt(d) - 3*a^2*b*c*d^(3/2) + 2*a^3*d^(5/2))*arctan(1/2*(
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(
sqrt(a*b*c*d - a^2*d^2)*b^4) - 3/16*(b^2*c^2*sqrt(d) - 8*a*b*c*d^(3/2) + 8*
a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*d) - ((sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2
```

```
*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(5/2) - a*b^2*c^3*sq
rt(d) + a^2*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2
)*b^4)
```

3.741 $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\left(c+dx^2\right)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{\sqrt{c+dx^2}(2bc-5ad)}{2b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{a\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c + d*x^2)^(3
/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(b*c - a*d)*(a + b*x^
2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[b*c - a*d]])/(2*b^(7/2))
```

Rubi [A] time = 0.135738, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^2\right)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{\sqrt{c+dx^2}(2bc-5ad)}{2b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{a\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```


```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c + d*x^2)^(3
/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(b*c - a*d)*(a + b*x^
2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[b*c - a*d]])/(2*b^(7/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx,x,x^2\right)$$

$$= \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{4b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{((2bc-5ad)(bc-a}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{((2bc-5ad)(bc-a}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-5ad)\sqrt{bc-a}}{2}$$

Mathematica [A] time = 0.105191, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^2}\left(-15a^2d+ab\left(11c-10dx^2\right)+2b^2x^2\left(4c+dx^2\right)\right)}{6b^3\left(a+bx^2\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(11*c - 10*d*x^2) + 2*b^2*x^2*(4*c + d*x^
2)))/(6*b^3*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Maple [B] time = 0.016, size = 4673, normalized size = 28.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))*c^2-1/2/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-3/4/b^3*d^(1/2)*(-a*b)^(1/2)*ln((-
d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-9/8/b^3*(-a*b
)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))*c-3/8/b^3*(-a*b)^(1/2)*d^2*a/(a*d-b*c)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b^
2*a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)*c^2-3/8/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/2/b^3*a^2*d^2/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/6/b^2*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)+1/6/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4/b^2*a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/2/b^3*a^2*d^2/(a*d-b*c)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+3/8/b^3*(-a*b)^(1/2)*
d^2*a/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8/b^3*(-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((d*
(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/8/b^2*(-a*b)^
(1/2)*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b^2*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/4
/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4/b^3*a^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^2*(-
a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/b^3*d^(1/2)*(-a*b)^
(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2
/b^4*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*a-1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))*a^2*d^2+1/4/b^3*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*(-a*b)^(1/2)
/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
```

```
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4/b^2*a*d/(a*d-b*c)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3
/4/b^3*a^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b
)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/b^4/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2-1/4/b^3*d*(-a*b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+3/4/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4/b^4*(-a*b)^(1/2)*d^(
5/2)*a^2/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))-3/4/b^4*a^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))+1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8/b^2*(
-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))+1/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))*a*d*c-3/4/b^4*(-a*b)^(1/2)*d^(5/2)*a^2/(a*d-b*c)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4/b^4*a^3*d^3/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^2*(
-a*b)^(1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/b^2*(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*
c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/b^3/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c+3/4/b^2*a
*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87574, size = 888, normalized size = 5.45

$$\left[-\frac{3\left(2\,abc-5\,a^2d+\left(2\,b^2c-5\,abd\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right.}{24\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt((b*c - a*d)/b)
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*
d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)
/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d +
2*(4*b^2*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/12*(3*(2*
a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2
*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*
d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d + 2*(4*b^2
*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13813, size = 234, normalized size = 1.44

$$\frac{\left(2\,b^2c^2 - 7\,abcd + 5\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}b^3} + \frac{\sqrt{dx^2+c}abcd - \sqrt{dx^2+c}a^2d^2}{2\left(\left(dx^2+c\right)b - bc + ad\right)b^3} + \frac{\left(dx^2+c\right)^{\frac{3}{2}}b^4 + 3\,\sqrt{dx^2+c}b^4c - 6\,\sqrt{dx^2+c}ab^3d}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/2*(sqrt(d*x^2 + c)*a*b*c*d - sqr
t(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^3) + 1/3*((d*x^2 + c)^
(3/2)*b^4 + 3*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6
```

3.742 $\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=149

$$\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}+\frac{\sqrt{d}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) + ((b*c
 - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*b^3)
```

Rubi [A] time = 0.163972, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 528, 523, 217, 206, 377, 205}

$$\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}+\frac{\sqrt{d}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) + ((b*c
 - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*b^3)
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{c+dx^2}\left(c+4dx^2\right)}{a+bx^2}\,dx}{2b}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{2c(bc-2ad)+2d(3bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(d(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3}+\frac{((bc-4ad)(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(d(3bc-4ad))\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^3}+\frac{((bc-4ad)(bc-a\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}+\frac{\sqrt{d}(3bc-4ad)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.165094, size = 155, normalized size = 1.04

$$\frac{\frac{\left(4a^2d^2-5abcd+b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}\left(b\left(c-dx^2\right)-2ad\right)}{a+bx^2}+\sqrt{d}(3bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2]*(-2*a*d + b*(c - d*x^2)))/(a + b*x^2)) + ((b^2*c^2
- 5*a*b*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
```

```
])])/(Sqrt[a]*Sqrt[b*c - a*d]) + Sqrt[d]*(3*b*c - 4*a*d)*Log[d*x + Sqrt[d]*
Sqrt[c + d*x^2]])/(2*b^3)
```

Maple [B] time = 0.012, size = 4685, normalized size = 31.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -3/4/b^3*d^(5/2)*a^2/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))-1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c+1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d*c+3/4/b^4*d^3*(-a*b)^(1/2)/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2+1/8
/b^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x+3/8/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/b^3*d^(3/2)*ln
((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a+1/8/b^2*d*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x+3/8/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))*c-1/4/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/b^3*d^(3/2)*ln((-d*(-
a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a+1/4/b/(a*d-b*c)/(
x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2
)+9/8/b^2*d^(3/2)*a/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))*c+3/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a-3/4/b^2
*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/8/b*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b
^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/8/b*d/(a*d-b*c)*c*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4
/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*a^2*d^2+3/8/b^2*d^2*a/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*d*(-a*b)^(1/2)/(a*d
-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
```

```
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/4/(-a*b)^(1/2)/b^2*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*a*d+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*c^2-1/4/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d-1/4/(-a*b)^(1/2)/b/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/4/(-a*b)^(1/2)
/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2-3
/4/b^4*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2)))*a^2-3/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/4/b^2*d*(-a*b)^(1/2)/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/12/
(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)-1/12/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/8/b*d^(1/2)/(a*d-b
*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/
8/b*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))-1/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*
c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)-3/4/b^3*d^(5/2)*a^2/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/2/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*c+3/2/b^3*d^2*(-a*b)
^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*a*c+9/8/b^2*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2
))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))*c-3/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a+3
/8/b^2*d^2*a/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A] time = 2.73047, size = 2142, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2
 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*
x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*
d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/8*(4*(3
*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-(b*c - a*d)/
a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 - (b^2*c - 2*a*b*
d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), 1/4*((a*b*c - 4*a^2*d + (b^2*c -
4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
- (3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x
^2 + c))/(b^4*x^2 + a*b^3), -1/4*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d
)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c - 4*a^2*d + (b^
2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(c + dx^2\right)^{\frac{3}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.19297, size = 454, normalized size = 3.05

$$\frac{\sqrt{dx^2 + c}dx}{2\,b^2} - \frac{\left(3\,bc\sqrt{d} - 4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{4\,b^3} - \frac{\left(b^2c^2\sqrt{d} - 5\,abcd^{\frac{3}{2}} + 4\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2}{2\,\sqrt{abcd - a^2d^2}}\right.}{2\,\sqrt{abcd - a^2d^2}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d*x/b^2 - 1/4*(3*b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^2*c^2*sqrt(d) - 5*a*b*c*d^(3/2) + 4*
a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(
3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*
b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

3.743 $\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=99

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2)) - (3*d*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5
/2))
```

Rubi [A] time = 0.0774449, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 47, 50, 63, 208}

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2)) - (3*d*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5
/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^2\right)\\
&= -\frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3d)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3d(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0208049, size = 54, normalized size = 0.55

$$\frac{d\left(c+dx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{5(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(c + d*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^2))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^2)
```

Maple [B] time = 0.011, size = 2821, normalized size = 28.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -9/8*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
```

```
)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/8*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^2*l
n((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/2*a/b^2*d
^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
-3/2*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*c+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8*(-a*b)^(1/2)/
a/b*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))+3/8*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8*(-a*b)^(1
/2)/b^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*c-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/4*a/b^2*d^
2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-3/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/8*(-a*b)^(1/2)/a/b*d/
(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x-3/8*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*
a/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/b*d/(a*d-b*
c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+
1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+3/4*(-a*b)^(1/2)*a/b^3*d^(5
/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/8*(-a*b)
^(1/2)/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-3/4*(-a*b)^(1/2)*a/b^3*d^(5/2)
/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+3/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78823, size = 701, normalized size = 7.08

$$\left[\frac{3\left(bdx^2+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(2\,bdx^2-bc+3\,ad\right)\sqrt{d}}{8\left(b^3x^2+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d*x^2 + a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*
c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2), -1/4*(3
*(b*d*x^2 + a*d)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
- 2*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.13787, size = 161, normalized size = 1.63

$$\frac{1}{2}\,d\left(\frac{3\left(bc-ad\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{2\sqrt{dx^2+c}}{b^2}-\frac{\sqrt{dx^2+c}bc-\sqrt{dx^2+c}ad}{\left(\left(dx^2+c\right)b-bc+ad\right)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*(3*(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2*sqrt(d*x^2 + c)/b^2 - (sqrt(d*x^2 + c)*b*c - sqrt(d
*x^2 + c)*a*d)/(((d*x^2 + c)*b - b*c + a*d)*b^2))
```

3.744 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab\left(a+bx^2\right)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c - a*d]*(b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)
*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rubi [A] time = 0.0887277, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 523, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab\left(a+bx^2\right)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c - a*d]*(b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)
*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab(a+bx^2)}+\frac{\int \frac{c(bc+ad)+2ad^2x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab(a+bx^2)}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{((bc-ad)(bc+2ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab^2}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab(a+bx^2)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{((bc-ad)(bc+2ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{2ab^2}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab(a+bx^2)}+\frac{\sqrt{bc-ad}(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.152078, size = 141, normalized size = 1.08

$$\frac{\frac{(-2a^2d^2+abcd+b^2c^2)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}+\frac{bx\sqrt{c+dx^2}(bc-ad)}{a(a+bx^2)}+2d^{3/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*(b*c - a*d)*x*Sqrt[c + d*x^2])/(a*(a + b*x^2)) + ((b^2*c^2 + a*b*c*d -
2*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*
Sqrt[b*c - a*d]) + 2*d^(3/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.013, size = 4689, normalized size = 35.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
```

d(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/4/(-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+3/4/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b^2*a*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/8/b*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/b*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8/a*d/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/8/a/b*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/12/(-a*b)^(1/2)/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/12/(-a*b)^(1/2)/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4/a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/8/a*d/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/8/a/b*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/4/b^2*a*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8/a*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/8/a*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/8/b*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/b*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+3/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/

```
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/4/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/4/(-a*b
)^(1/2)/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*c+1/4/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/4/(-a*b)^(1/2)/a*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*c+3/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/2/b^2*d^2*(-a*b)^(1
/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*
c-3/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4/b^3*a*d^3*(-a*b)^(1/2)/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/b
/a*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/8/a*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b
/a*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/8/a*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/(
-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d^2+3/2/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4/b^3*a*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2, x)
```

Fricas [A] time = 2.58328, size = 1936, normalized size = 14.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)*sqrt(d)*l
og(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c + 2*a^2*d + (b^2*c
```

```
+ 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
))/(a*b^3*x^2 + a^2*b^2), 1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x - 8*(a*b
*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^
2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
(a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c)*x + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d)*log(-2*d
*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2
*c - a*b*d)*sqrt(d*x^2 + c)*x - 4*(a*b*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*
c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
- a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/(a*b^3*x^2 + a^2*b^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.19073, size = 425, normalized size = 3.24

$$-\frac{d^{\frac{3}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}-\frac{\left(b^2c^2\sqrt{d}+abcd^{\frac{3}{2}}-2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}ab^2}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^2\ldots}{\left(\left(\sqrt{d}x-\sqrt{d\ldots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 - 1/2*(b^2*c^2*sqrt(d
) + a*b*c*d^(3/2) - 2*a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*b^
2) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2
) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b
- 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a*d + b*c^2)*a*b^2)
```

3.745 $\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3/2))
```

Rubi [A] time = 0.142303, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```


```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{4a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+}\right.}{2a^2bd}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}$$

Mathematica [A] time = 0.162929, size = 122, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^2}(bc-ad)}{b\left(a+bx^2\right)} - 2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^2])/(b*(a + b*x^2)) - 2*c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/(2*a^2)
```

Maple [B] time = 0.013, size = 4718, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/a/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/2
/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/2/a
/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*d+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))*d^2+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))*c^2+1/4/a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4*d^2/(a*d-b*c)/b*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*
d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)-3/4*d^2/(a*d-b*c)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a/b*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2
/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+3/8/(
-a*b)^(1/2)*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln
((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/a*d/(a
*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*c-9/8/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b
+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/a*d/(a*d-b*c)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c
-1/2/a/b^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))-1/6/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/3/a^2*(d*x^2+c)
^(3/2)-3/4/(-a*b)^(1/2)*a*d^(5/2)/b/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4*a*d^3/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/2*d^2/(a*d-b*c)/b/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+3/4/a^2/b*d^(1/2)*(-a
*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*
c-3/8/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a^2*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/a^2*c^(3
/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a^2*(d*x^2+c)^(1/2)*c-1/2/a^2*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*c-3/4*a*d^3/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))+3/2*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
```

```
/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4/a^2*d*(-a*b)^(1/2)/b*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1
/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/(-a*b)^(1/
2)*a*d^(5/2)/b/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+3/8/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*c*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/8/(-a*b)^(1/
2)/a*d/(a*d-b*c)*b*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/a/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b
)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/8/(-a*b)^(1/2)/a*d^(
1/2)/(a*d-b*c)*b*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))-3/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(
-a*b)^(1/2)))*c^2+1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
5/2)-1/4/a^2*d*(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a^2/b*d^(1/2)*(-a*b)^(1/2)*ln
((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/a/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-1/4/(-a*b)^(1/2
)/a*d/(a*d-b*c)*b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*c^2*ln
((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(
1/2)/a*d/(a*d-b*c)*b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x), x)
```

Fricas [A] time = 3.71917, size = 1901, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^
2*b^2*x^2 + a^3*b), 1/8*(8*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqr
t(d*x^2 + c)) + (2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/
b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*
d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a
^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^2*c*x^2 + a*b*c)*
sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c - a^
2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2
*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
+ 4*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b*
c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^{\frac{3}{2}}}{x\left(a + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x*(a + b*x**2)**2), x)
```

Giac [A] time = 1.14234, size = 223, normalized size = 1.73

$$\frac{1}{2}\,d^2\left(\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2} + \frac{\sqrt{dx^2+c}bc - \sqrt{dx^2+c}ad}{\left(\left(dx^2+c\right)b - bc + ad\right)abd} - \frac{\left(2\,b^2c^2 - abcd - a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2bd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) + (sqrt(
d*x^2 + c)*b*c - sqrt(d*x^2 + c)*a*d)/(((d*x^2 + c)*b - b*c + a*d)*a*b*d) -
 (2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/(sqrt(-b^2*c + a*b*d)*a^2*b*d^2))
```

3.746 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\left(a+bx^2\right)}$$

```
[Out] -((3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*Sqrt[c + d*x^2]
)/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rubi [A] time = 0.124316, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {468, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -((3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*Sqrt[c + d*x^2]
)/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)}-\frac{\int \frac{-c(3bc-ad)-2bcdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}\\
&= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)}+\frac{\int -\frac{3bc^2(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2bc}\\
&= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)}-\frac{(3c(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2}\\
&= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)}-\frac{(3c(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a^2}\\
&= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)}-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0153216, size = 52, normalized size = 0.41

$$-\frac{c\sqrt{c+dx^2}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^2]*Hypergeometric2F1[-1/2, 2, 1/2, ((-(b*c) + a*d)*x^2)/(
a*(c + d*x^2))])/(a^2*x))
```

Maple [B] time = 0.013, size = 4764, normalized size = 37.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] 3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
```

```
1/b*(-a*b)^(1/2)))*c^2-3/8/a^2*d/(a*d-b*c)*b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a^2*d*(-a*b)
^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*c^2+3/4/a*d^2*(-a*b)^(1/2)/(a*d-b*c)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a^2*d*(-a*b)^(1/2)/
(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*c+3/4*d^3*(-a*b)^(1/2)/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/8/a*d^2/(a*d-b*c)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
9/8/a*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2
)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))*c+1/a^2*d/c*x*(d*x^2+c)^(3/2)+3/4*b/a^2/(-a*b)^(1/2)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*c-3/4/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))*d^2-3/4*b/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4/b/(-a*b)^(1/2)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+1/4/a^2/(a*d-b*c)*b
/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
3/8/a*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8/a*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)
/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/2/a/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-3/4/a*d^2*(-a*b)^(
1/2)/(a*d-b*c)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4
*d^3*(-a*b)^(1/2)/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))-1/4/a^2*d/(a*d-b*c)*b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/a^2*d^(1/2)/(a*d-b*
c)*b*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1
/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/2/a*d^2*(-a*b)^(1/2)/(a
*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/
4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/2/a*d^2*(-a*b)^(1/2)/(a*d-b*c)/b
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4*d^(5/2)
/b/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)-9/8/a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
```

```
(1/2))*c+3/4/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+3/4/b/a*d^(3/2)*ln((d*(-a*b)^(1/2)/
b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*b/a^2/(-a*b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)-3/8/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x-3/4/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+3/4/b/a*d^(3/2)*ln(
(-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*b/a^2/(-a
*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-3/8/a^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/a^2*d^(1/2)*ln((-d*(-a*b)^(1/
2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/a^2/c/x*(d*x^2+c)^(5/2)
+3/2/a^2*d*x*(d*x^2+c)^(1/2)+3/2/a^2*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)-3/4*d^(5/2)/b/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))+3/2/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*d*c-3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/4/a^2*d/(a*d-b*c)*b*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8
/a^2*d^(1/2)/(a*d-b*c)*b*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))+3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*c^2-3/8/a^2*d/(a*d-b*c)*b*c*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A] time = 1.81013, size = 732, normalized size = 5.72

$$\left[\frac{3\left(bcx^3+acx\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left((3\,bc-ad)x^2+\right.}{8\left(a^2bx^3+a^3x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*x^3 + a*c*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3 + a
^3*x), -1/4*(3*(b*c*x^3 + a*c*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (
b*c^2 - a*c*d)*x)) + 2*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*
x^3 + a^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)**2), x)
```

Giac [B] time = 3.78937, size = 556, normalized size = 4.34

$$\frac{3\left(bc^2\sqrt{d}-acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2}+\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abcd^{\frac{3}{2}}+2\left(\sqrt{\right.}\cdots}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*(b*c^2*sqrt(d) - a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(5/2)
 - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 12*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^
(5/2) + 3*b^2*c^4*sqrt(d) - a*b*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 +
 c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*c*d - b*c^3)*a^2*b)
```

3.747 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{c+dx^2}(2bc-ad)}{2a^2(a+bx^2)}+\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*(a + b*x^2)) - (c*Sqrt[c + d*x^2])/
(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b])
```

Rubi [A] time = 0.255223, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^2}(2bc-ad)}{2a^2(a+bx^2)}+\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```


```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*(a + b*x^2)) - (c*Sqrt[c + d*x^2])/
(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx,x,x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{4a^3} + \frac{((bc-ad)\cdots}{\cdots}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2a^3d} + \frac{((b\cdots}{\cdots}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{\sqrt{bc-ad}(4bc-ad\cdots}{2a^3\cdots}$$

Mathematica [A] time = 0.221665, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^2}\left(-ac+adx^2-2bcx^2\right)}{x^2\left(a+bx^2\right)}+\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(a*c) - 2*b*c*x^2 + a*d*x^2))/(x^2*(a + b*x^2)) + Sqr
t[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4
*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b])/(2
*a^3)
```

Maple [B] time = 0.013, size = 4820, normalized size = 28.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/4
*b/a^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/a^3*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/2/a
^3*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c+1/b/a^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/b/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*d^2+2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))*d*c-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))*c^2-1/4*b/a^2*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/b/a^2*d^(3/2
)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))-1/b/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+
2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+3/8*
b/a/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8*b/a/(-a*b)^(1/2)*d^(3/2)/(a*
d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/
4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
```

```
/2)))*c^2-1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8*b^2/a^2/(-a*
b)^(1/2)*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)
/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))+1/2/a^3*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/2/a^3*d^(1/2)*(-a*
b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-
3/2/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))*c-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3
/4*b/a^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*
(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(5/2)-3/4*b/a^2*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/2/a*d^2/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+3/8*b^2/a
^2/(-a*b)^(1/2)*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/8*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b
*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x+3/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/8*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*c^
2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*b^2/
a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b/a^3*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/a^2*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*d+3/2/a^2*d*(d*x^2+c)^(1/2)+1/3*b/a^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/a^2*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-2/3*b/a
^3*(d*x^2+c)^(3/2)-2*b/a^3*(d*x^2+c)^(1/2)*c+3/4/a*d^2/(a*d-b*c)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
3/4/(-a*b)^(1/2)*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))+b/a^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/a^2/c/x^2*(d*x^2+c)^(5/2)+1/2
/a^2*d/c*(d*x^2+c)^(3/2)-3/2/a^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2
))/x)+b/a^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*c+3/4/a*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/(-a*b)^(1/2)*d^(5
/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))+2*b/a^3*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-3/8*b/a/(-a*b)^(1
/2)*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((-d
*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 3.68468, size = 2195, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt((b*c - a*d)/b)*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c -
3*a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
 4*(a^2*c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2),
-1/8*(4*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*arctan
(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2
)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2
+ c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*c + (2*a*b*
c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c - a
*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^
2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c
*d - a*d^2)*x^2)) + ((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqr
t(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c + (2*a*
b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c -
 a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*
x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b
*c*d - a*d^2)*x^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*c + (2*a*b*c - a^2*d)*x
^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)**2), x)
```

Giac [A] time = 1.149, size = 300, normalized size = 1.76

$$-\frac{1}{2}d^3\left(\frac{2\left(dx^2+c\right)^{\frac{3}{2}}bc-2\sqrt{dx^2+c}bc^2-\left(dx^2+c\right)^{\frac{3}{2}}ad+2\sqrt{dx^2+c}acd}{\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-acd\right)a^2d^2}-\frac{\left(4b^2c^2-5abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^3d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*((2*(d*x^2 + c)^(3/2)*b*c - 2*sqrt(d*x^2 + c)*b*c^2 - (d*x^2 + c)^
(3/2)*a*d + 2*sqrt(d*x^2 + c)*a*c*d)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c
+ b*c^2 + (d*x^2 + c)*a*d - a*c*d)*a^2*d^2) - (4*b^2*c^2 - 5*a*b*c*d + a^2*
d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a
^3*d^3) + (4*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c
)*d^3))
```

3.748 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2} dx$

Optimal. Leaf size=166

$$\frac{\sqrt{c+dx^2}(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

```
[Out] -((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b*c - 11*a*d)*Sqrt[
c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x^3*(a + b*x^2
)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(7/2))
```

Rubi [A] time = 0.239563, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {468, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b*c - 11*a*d)*Sqrt[
c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x^3*(a + b*x^2
)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(7/2))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{-c(5bc-3ad)-2d(2bc-ad)x^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{\int \frac{-bc^2(15bc-11ad)-2bcd(5bc-3ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2bc}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} - \frac{\int -\frac{3bc^2(5bc-2ad)(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,d}{6a^3bc^2}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{((5bc-2ad)(bc-ad))}{2a^3}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{((5bc-2ad)(bc-ad))}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{(5bc-2ad)\sqrt{bc-ad}\,\mathrm{t}}{2a^{7/2}}$$

Mathematica [A] time = 5.12802, size = 131, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(-2a^2\left(c+4dx^2\right)+abx^2\left(10c-11dx^2\right)+15b^2cx^4\right)}{6a^3x^3\left(a+bx^2\right)} + \frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(10*c - 11*d*x^2) - 2*a^2*(c + 4*d
*x^2)))/(6*a^3*x^3*(a + b*x^2)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Maple [B] time = 0.014, size = 4908, normalized size = 29.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x)

[Out] -9/8*b/a^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))*c+3/4/a*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+5/12*b^2/a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12
*b^2/a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/a*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/
b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/3/a^2/c/x^3*(d*x^2+c)^(5/2)-
5/4/a^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))-5/4/a^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))+1/a^2*d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+3/4*b/a^3*d*(-a*b)^(
1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*c^2-3/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4/b/a*d^3*(-a*b)^(1/2
)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/
2/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))*c-3/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+3/4*b/a^3*d*(-a*b)^(1/2)/(a*d
-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*c-3/4/b/a*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/2/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+5/4*b^2/a^3/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^
2+3/8*b^2/a^3*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/2*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+1/4*b^2/a^3*d/(a*d-b*c
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(3/2)*x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(3/2)-3/8*b/a^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-
a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(5/2)-3/4/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+15/8*b/a
^3*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-
```

```
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
*c+5/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))*d^2+5/8*b/a^3*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3*b/a^3*d*x*(d*x^2+c)^(1/2)-3*b/a^3*d^(
1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+5/4*b/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-5/4
*b^2/a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2)
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(5/2)+3/4/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*b/a^3/c/x*(d*x^2
+c)^(5/2)-5/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))*d^2-2/3/a^2*d/c^2/x*(d*x^2+c)^(5/2)+2/3/a^2*d^2/c^2*x*(d*x^2
+c)^(3/2)+1/a^2*d^2/c*x*(d*x^2+c)^(1/2)+5/8*b/a^3*d*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+15/8*b/a^3*
d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-5
/4*b/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+5/4*b^2/a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/8*
b/a^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*b/a^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4*b^2/a^3*d/(a*d-b*c)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)*x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-2*b/a^3*d/c*x*(d*x^2+c)^(
3/2)+1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/8*b^2/a^3*d/(a*d-b*c)*c
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x-5/2*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))*d*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A] time = 2.34436, size = 934, normalized size = 5.63

$$\left[-\frac{3\left(\left(5\,b^2c-2\,abd\right)x^5+\left(5\,abc-2\,a^2d\right)x^3\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\cdots}{b^2x^4+2\,abx^2+a^2}\right)\cdots}{24\left(a^3bx^5+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c - 11*a*b*d)*
x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^
4*x^3), 1/12*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((15*b^2*c - 11*a
*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^
5 + a^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)**2), x)
```

Giac [B] time = 6.47274, size = 597, normalized size = 3.6

$$-\frac{\left(5\,b^2c^2\sqrt{d}-7\,abcd^{\frac{3}{2}}+2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^3}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\cdots}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c^2*sqrt(d) - 7*a*b*c*d^(3/2) + 2*a^2*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d)
 - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 +
```

```
c))^4*b*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*b*c^3*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*c^2*d^(3/2) + 3*b*c^4*sqrt(d) - 2*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)^3*a^3)
```

3.749 $\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=258

$$\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(120a^2bcd^2-64a^3d^3-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}(7bc}{8b^3}$$

```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^4) + (d*(7
*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(c + d*x^2)^(3/2))/(3
*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 8*a*d
)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*b^5) + ((5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```

Rubi [A] time = 0.448375, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {467, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(120a^2bcd^2-64a^3d^3-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}(7bc}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```


```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^4) + (d*(7
*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(c + d*x^2)^(3/2))/(3
*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 8*a*d
)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*b^5) + ((5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```


Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
```

```
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}dx = -\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(c+dx^2\right)^{3/2}\left(3c+8dx^2\right)}{a+bx^2}dx}{2b}$$

$$= \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\sqrt{c+dx^2}\left(6c(3bc-4ad)+6d(7bc-8ad)x^2\right)}{a+bx^2}dx}{12b^2}$$

$$= \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(6c\left(12b^2c^2-37abcd+24a^2d^2\right)+6d\left(19b\right.\right.}{\left(a+bx^2\right)\sqrt{c+dx}}}{48b^3}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+\right.}{2b\left(a-\right.}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+\right.}{2b\left(a-\right.}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+\right.}{2b\left(a-\right.}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+\right.}{2b\left(a-\right.}$$

Mathematica [A] time = 0.279052, size = 219, normalized size = 0.85

$$\frac{bx\sqrt{c+dx^2}\left(72a^2d^2+2bdx^2(13bc-12ad)+\frac{24a(bc-ad)^2}{a+bx^2}-108abcd+33b^2c^2+8b^2d^2x^4\right)+\frac{3\left(120a^2bcd^2-64a^3d^3-60ab^2c^2d+5b^3c^3\right)\mathrm{l}}{\sqrt{d}}}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(33*b^2*c^2 - 108*a*b*c*d + 72*a^2*d^2 + 2*b*d*(13*b*c
 - 12*a*d)*x^2 + 8*b^2*d^2*x^4 + (24*a*(b*c - a*d)^2)/(a + b*x^2)) + 24*Sqr
t[a]*(b*c - a*d)^(3/2)*(-3*b*c + 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*
a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^5)
```

Maple [B] time = 0.022, size = 7611, normalized size = 29.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^4}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 17.5238, size = 3663, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 +
(5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 12*(3*a*b^2*c^2*d - 11*
a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
 + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(8*b^4*d^3*x^7
+ 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*
a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*
sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(3*(5*a*b^3*c^3 - 60*a^2*b^2*
c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^
2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) - 6*(3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2
*c*d^2 + 8*a^2*b*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*
d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c
*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/96*(24*(
3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2
+ 8*a^2*b*d^3)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + 3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^
4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2
)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^4*d^3*x^
7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 4
8*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x
)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(12*(3*a*b^2*c^2*d - 11*a^2
*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*a
*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 6
0*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x
^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2
*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^
```

```
5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23077, size = 703, normalized size = 2.72

$$\frac{1}{48}\left(2\left(\frac{4\,d^2x^2}{b^2}+\frac{13\,b^{12}cd^5-12\,ab^{11}d^6}{b^{14}d^4}\right)x^2+\frac{3\left(11\,b^{12}c^2d^4-36\,ab^{11}cd^5+24\,a^2b^{10}d^6\right)}{b^{14}d^4}\right)\sqrt{dx^2+cx}+\frac{\left(3\,ab^3c^3\sqrt{d}-14\,a^2b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2/b^2 + (13*b^12*c*d^5 - 12*a*b^11*d^6)/(b^14*d^4))*x^2 +
3*(11*b^12*c^2*d^4 - 36*a*b^11*c*d^5 + 24*a^2*b^10*d^6)/(b^14*d^4))*sqrt(d*
x^2 + c)*x + 1/2*(3*a*b^3*c^3*sqrt(d) - 14*a^2*b^2*c^2*d^(3/2) + 19*a^3*b*c
*d^(5/2) - 8*a^4*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^5) - 1/32*(
5*b^3*c^3*sqrt(d) - 60*a*b^2*c^2*d^(3/2) + 120*a^2*b*c*d^(5/2) - 64*a^3*d^(
7/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^5*d) - ((sqrt(d)*x - sqrt(d*x
^2 + c))^2*a*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^
2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(5/2) - 2*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a^4*d^(7/2) - a*b^3*c^4*sqrt(d) + 2*a^2*b^2*c^3*d^(3
/2) - a^3*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*
b^5)
```

3.750 $\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=198

$$\frac{(c+dx^2)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{(c+dx^2)^{3/2}(2bc-7ad)}{6b^3}+\frac{\sqrt{c+dx^2}(2bc-7ad)(bc-ad)}{2b^4}-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}}{2b^{9/2}}$$

```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c - 7*a*d)*(c
 + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)^(5/2))/(10*b^2*(b*c
 - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c -
7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*b^(9/2))
```

Rubi [A] time = 0.183271, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{(c+dx^2)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{(c+dx^2)^{3/2}(2bc-7ad)}{6b^3}+\frac{\sqrt{c+dx^2}(2bc-7ad)(bc-ad)}{2b^4}-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```


```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c - 7*a*d)*(c
 + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)^(5/2))/(10*b^2*(b*c
 - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c -
7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*b^(9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+dx)^{5/2}}{(a+bx)^2}\,dx,x,x^2\right)$$

$$=\frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)}+\frac{(2bc-7ad)\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx,x,x^2\right)}{4b(bc-ad)}$$

$$=\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)}+\frac{(2bc-7ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{4b^2}$$

$$=\frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3}+\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)}+\frac{((2bc-7ad)(bc-a}{}$$

$$=\frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4}+\frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3}+\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a\left(c+\right.}{2b(bc-ad}$$

$$=\frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4}+\frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3}+\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a\left(c+\right.}{2b(bc-ad}$$

$$=\frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4}+\frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3}+\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a\left(c+\right.}{2b(bc-ad}$$

Mathematica [A] time = 0.399523, size = 164, normalized size = 0.83

$$\frac{\left(bc-\frac{7ad}{2}\right)\left(\frac{2(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)+\frac{a\left(c+dx^2\right)^{7/2}}{a+bx^2}}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((a*(c + d*x^2)^(7/2))/(a + b*x^2) + (b*c - (7*a*d)/2)*((2*(c + d*x^2)^(5/2
))/(5*b) + (2*(b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2
```

```
) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])
)/(3*b^(7/2))))/(2*b*(b*c - a*d))
```

Maple [B] time = 0.014, size = 7443, normalized size = 37.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14066, size = 1223, normalized size = 6.18

$$\left[\frac{15 \left(2\,ab^2c^2 - 9\,a^2bcd + 7\,a^3d^2 + \left(2\,b^3c^2 - 9\,ab^2cd + 7\,a^2bd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\,(\cdots}{b^2x^4+2\,abx^2+a^2}\cdots\right.}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/120*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*
d + 7*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(6*b^3*d^2*x^6 + 61*a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3
*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)
*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/60*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d +
 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(6*b^3*d^2*x^6 + 61*a*b^2*c^2
- 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^
3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1618, size = 356, normalized size = 1.8

$$\frac{\left(2\,b^3c^3-11\,ab^2c^2d+16\,a^2bcd^2-7\,a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}b^4}+\frac{\sqrt{dx^2+c}ab^2c^2d-2\,\sqrt{dx^2+c}a^2bcd^2+\sqrt{dx^2+c}a^3d^3}{2\left(\left(dx^2+c\right)b-bc+ad\right)b^4}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^3 - 11*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(sqrt(d
*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/2*(sqrt(d*
x^2 + c)*a*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a^2*b*c*d^2 + sqrt(d*x^2 + c)*a^3*
d^3)/(((d*x^2 + c)*b - b*c + a*d)*b^4) + 1/15*(3*(d*x^2 + c)^(5/2)*b^8 + 5*
(d*x^2 + c)^(3/2)*b^8*c + 15*sqrt(d*x^2 + c)*b^8*c^2 - 10*(d*x^2 + c)^(3/2)
*a*b^7*d - 60*sqrt(d*x^2 + c)*a*b^7*c*d + 45*sqrt(d*x^2 + c)*a^2*b^6*d^2)/b
^10
```

3.751 $\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{8b^3}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4}$$

```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x^2)^(3/2))
/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((b*c - 6*a*d)*(b*c -
a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a
]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(8*b^4)
```

Rubi [A] time = 0.244147, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 528, 523, 217, 206, 377, 205}

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{8b^3}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```


```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x^2)^(3/2))
/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((b*c - 6*a*d)*(b*c -
a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a
]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(8*b^4)
```


Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(c+6dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{c+dx^2}\left(2c(2bc-3ad)+2d(11bc-12ad)x^2\right)}{a+bx^2}\,dx}{8b^2}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{2c\left(4b^2c^2-17abcd+12a^2d^2\right)+2d\left(15b^2c^2-\cdots\right.}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{16b^3}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\left((bc-6ad)(bc-ad)^2\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{\cdots}}}{2b^4}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\left((bc-6ad)(bc-ad)^2\right)\text{Subst}\left(\int\cdots\right.}{2b^4}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b\cdots}}{\sqrt{a}\cdots}\right.}{2\sqrt{a}b^4}$$

Mathematica [A] time = 0.221138, size = 173, normalized size = 0.89

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bx\sqrt{c+dx^2}\left(-\frac{4(bc-ad)^2}{a+bx^2}+d(9bc-8ad)+2bd^2x^2\right)+\frac{4(bc-6ad)(bc-\cdots}{\cdots}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d*(9*b*c - 8*a*d) + 2*b*d^2*x^2 - (4*(b*c - a*d)^2)/(
a + b*x^2)) + (4*(b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 2
4*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^4)
```

Maple [B] time = 0.013, size = 7459, normalized size = 38.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A] time = 8.82906, size = 2938, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c
*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x
- c) + 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*
a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*
b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/8*((15*a*
b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b^2*c^2 - 7*a^2
*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt(-(b*c
- a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*s
qrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + 3*(3*b
```

```
^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sq
rt(d*x^2 + c))/(b^5*x^2 + a*b^4), 1/16*(4*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*
d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*
d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*
b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c)
)/(b^5*x^2 + a*b^4), -1/8*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*
b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*
c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - (2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 -
 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.19869, size = 602, normalized size = 3.09

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b^2}+\frac{9\,b^7cd^3-8\,ab^6d^4}{b^9d^2}\right)x-\frac{\left(15\,b^2c^2\sqrt{d}-40\,abcd^{\frac{3}{2}}+24\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4}-\frac{\left(b^3c^3\sqrt{d}-8\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b^2 + (9*b^7*c*d^3 - 8*a*b^6*d^4)/(b^9*d^2))
*x - 1/16*(15*b^2*c^2*sqrt(d) - 40*a*b*c*d^(3/2) + 24*a^2*d^(5/2))*log((sqr
t(d)*x - sqrt(d*x^2 + c))^2)/b^4 - 1/2*(b^3*c^3*sqrt(d) - 8*a*b^2*c^2*d^(3/
2) + 13*a^2*b*c*d^(5/2) - 6*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2
)*b^4) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*b^4)
```

3.752 $\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=126

$$\frac{5d\sqrt{c+dx^2}(bc-ad)}{2b^3} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2))/(6*b^2)
 - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rubi [A] time = 0.102801, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 47, 50, 63, 208}

$$\frac{5d\sqrt{c+dx^2}(bc-ad)}{2b^3} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2))/(6*b^2)
 - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx,x,x^2\right)$$
$$= -\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{(5d)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{4b}$$
$$= \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{(5d(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{4b^2}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left(5d(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x\right.}{4b^3}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{2b^3}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

Mathematica [C] time = 0.0200995, size = 54, normalized size = 0.43

$$\frac{d\left(c+dx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{7(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(c + d*x^2)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(c + d*x^2))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^2)
```

Maple [B] time = 0.011, size = 4363, normalized size = 34.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+5/12*a/b^2*d^2/(a*
d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)-5/12/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+5/2*a/b^2*d^2/(a*d-b*c)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*c+5/4*(-a*b)^(1/2)*a^2/b^4*d^(7/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(
x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-5/4*a^3/b^4*d^4/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/16*(-a*b)^(1/2)/b^2
*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)*x-75/32*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((d*(
-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+5/2*a/b^2*d^2/
(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*c-5/4*(-a*b)^(1/2)*a^2/b^4*d^(7/2)/(a*d-b*c)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-5/4*a^3/b^4*d^4/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+5/4/b*d/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1
/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(7/2)+5/16*(-a*b)^(1
/2)/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+75/32*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/4*(-
a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(7/2)+5/4/b*d/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-5/4*a^2/b^3
*d^3/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)-5/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+5/12*a/b^2*d^2/(a*
d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)-5/12/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-5/4*a^2/b^3*d^3/(a*d-b*c)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)-5/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d-b*
c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x-25/8*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2
)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+15/4*a^2/b^3*d^3/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+15/4*a^2/b^
3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
```

```
)*c-15/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*c^2-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)*x-15/32*(-a*b
)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^3*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-15/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(5/2)*x+15/32*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^3*ln((d*(-a*b)^(1/2)/b+
(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-35/32*(-a*b)^(1/2)/b^2*d^2/(a*d-
b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x+5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d-b*c)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+25/8*(-
a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*c+35/32*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*c*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+5/16*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/32*(-a*b)^(1/2)/a/b*d/
(a*d-b*c)*c^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x-5/16*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1
5/32*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06908, size = 950, normalized size = 7.54

$$\left[-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4}{24\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
```

```
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*a*b*
c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2
 + a*b^3), -1/12*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(-(b
*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d^2*x^4 - 3*b^
2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(b^4*x^2 + a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1492, size = 257, normalized size = 2.04

$$\frac{1}{6}d\left(\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3}-\frac{3\left(\sqrt{dx^2+c}b^2c^2-2\,\sqrt{dx^2+c}abcd+\sqrt{dx^2+c}a^2d^2\right)}{\left(\left(dx^2+c\right)b-bc+ad\right)b^3}+\frac{2\left(\left(dx^2+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 3*(sqrt(d*x^2 + c)*b^2*c^2 - 2*s
qrt(d*x^2 + c)*a*b*c*d + sqrt(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a
*d)*b^3) + 2*((d*x^2 + c)^(3/2)*b^4 + 6*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^
2 + c)*a*b^3*d)/b^6)
```

3.753 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=174

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/2}(bc-}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)
^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*b^3) + (d^(3/2)*(5*b*
c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```

Rubi [A] time = 0.21934, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 528, 523, 217, 206, 377, 205}

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/2}(bc-}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```


```
[Out] -(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)
^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*b^3) + (d^(3/2)*(5*b*
c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\int \frac{\sqrt{c+dx^2}\left(c(bc+ad)-2d(bc-2ad)x^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\int \frac{2c\left(b^2c^2+2abcd-2a^2d^2\right)+2ad^2(5bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(d^2(5bc-4ad)\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3} + \frac{\left((bc-ad)^2(b\right.}{}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right.}{2b^3}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3} + \frac{d}{}$$

Mathematica [A] time = 0.187233, size = 143, normalized size = 0.82

$$\frac{\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}} + bx\sqrt{c+dx^2}\left(\frac{(bc-ad)^2}{a\left(a+bx^2\right)}+d^2\right) + d^{3/2}(5bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d^2 + (b*c - a*d)^2/(a*(a + b*x^2))) + ((b*c - a*d)^(
3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a
```

```
^(3/2) + d^(3/2)*(5*b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^3
)
```

Maple [B] time = 0.012, size = 7451, normalized size = 42.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2, x)
```

Fricas [A] time = 6.08956, size = 2585, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b^2*c^2 + 3*a^2*b*c*
d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d
)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-
(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2
- 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/
8*(4*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 +
(b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d +
2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), 1/4*((a*b^2*c^2 + 3
*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (5*a^2*b*c*d - 4*a^
3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^
2 + c)*sqrt(d)*x - c) + 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b
```

```
*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/4*(2*(5*a^2*b*c*d - 4*a
^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d
*x^2 + c)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d
- 4*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*
x^2 + c))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [B] time = 1.1818, size = 549, normalized size = 3.16

$$\frac{\sqrt{dx^2+c}\,d^2x}{2\,b^2} - \frac{\left(5\,bcd^{\frac{3}{2}} - 4\,ad^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{4\,b^3} - \frac{\left(b^3c^3\sqrt{d} + 2\,ab^2c^2d^{\frac{3}{2}} - 7\,a^2bcd^{\frac{5}{2}} + 4\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x\right.}{}\right.}{2\,\sqrt{abcd-a^2d^2}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d^2*x/b^2 - 1/4*(5*b*c*d^(3/2) - 4*a*d^(5/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^3*c^3*sqrt(d) + 2*a*b^2*c^2*d^(3/2
) - 7*a^2*b*c*d^(5/2) + 4*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a*b^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*a*b^3)
```

3.754 $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=160

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*(c + d*x^2)^(3/
2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2 +
((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*a^2*b^(5/2))
```

Rubi [A] time = 0.218048, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 98, 154, 156, 63, 208}

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] -(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*(c + d*x^2)^(3/
2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2 +
((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*a^2*b^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}\left(bc^2-\frac{1}{2}d(bc-3ad)x\right)}{x(a+bx)}\,dx, x, x^2\right)}{2ab}\\
&= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{4}d\left(b^2c^2+4abcd-3a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{ab^2}\\
&= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{c^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{\left((bc-ad)^2(2bc\right.}{}\\
&= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{c^3\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{\left((bc-a\right.}{}\\
&= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{(bc-ad)^{3/2}(2bc+3ad)}{2a^2b^5}
\end{aligned}
$$

Mathematica [A] time = 0.196237, size = 158, normalized size = 0.99

$$
\frac{\frac{a\sqrt{c+dx^2}\left(3a^2d^2+2abd\left(dx^2-c\right)+b^2c^2\right)}{b^2\left(a+bx^2\right)} + \frac{\sqrt{bc-ad}\left(-3a^2d^2+abcd+2b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}} - 2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(b^2*c^2 + 3*a^2*d^2 + 2*a*b*d*(-c + d*x^2)))/(b^2*(a +
 b*x^2)) - 2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2
*b^2*c^2 + a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/b^(5/2))/(2*a^2)
```

Maple [B] time = 0.016, size = 7477, normalized size = 46.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x), x)
```

Fricas [A] time = 10.9943, size = 2377, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
+ a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)
*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^3
*c^2*x^2 + a*b^2*c^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c
)/x^2) - 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x
^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/8*(8*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)
*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 +
(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3
*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c
*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*
```

```
d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)
/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*b*d^2*
x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 +
a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*
d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 4*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + 2*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 +
c))/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15474, size = 290, normalized size = 1.81

$$\frac{1}{2}d^2\left(\frac{2\,c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}+\frac{2\sqrt{dx^2+c}}{b^2}+\frac{\sqrt{dx^2+c}b^2c^2-2\sqrt{dx^2+c}abcd+\sqrt{dx^2+c}a^2d^2}{\left(\left(dx^2+c\right)b-bc+ad\right)ab^2d}-\frac{\left(2\,b^3c^3-ab^2c^2d-4\,a^2\right.}{\sqrt{-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) + 2*sqrt
(d*x^2 + c)/b^2 + (sqrt(d*x^2 + c)*b^2*c^2 - 2*sqrt(d*x^2 + c)*a*b*c*d + sq
rt(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a*d)*a*b^2*d) - (2*b^3*c^3 -
 a*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b^2*d^2))
```

3.755 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\left(a+bx^2\right)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] -(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*(c + d*x^2)^(
3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rubi [A] time = 0.188748, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {468, 580, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\left(a+bx^2\right)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*(c + d*x^2)^(
3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}-\frac{\int \frac{\sqrt{c+dx^2}\left(-c(3bc-ad)-2ad^2x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{2ab}\\
&=-\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}-\frac{\int \frac{c\left(3b^2c^2-4abcd-a^2d^2\right)-2a^2d^3x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2b}\\
&=-\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}+\frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}-\frac{\left((bc-ad)^2(3bc+2ad)\right)\int \frac{}{(a}}{2a^2b^2}\\
&=-\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}-\frac{\left((bc-ad)^2\right.}{}\\
&=-\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}-\frac{(bc-ad)^{3/2}(3bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}+\frac{d^5}{}
\end{aligned}$$

Mathematica [A] time = 0.14581, size = 150, normalized size = 0.89

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}+\sqrt{c+dx^2}\left(-\frac{x(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{a^2x}\right)+\frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(c^2/(a^2*x)) - ((b*c - a*d)^2*x)/(2*a^2*b*(a + b*x^2)))
- ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^
2]])/b^2
```

Maple [B] time = 0.014, size = 7529, normalized size = 44.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A] time = 4.51749, size = 2458, normalized size = 14.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(d)*x - c) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^
2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
 (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) - 4*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^
2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/8*(8*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 - a*b^2*c
*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c
 - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b^2*c^2 + (3*b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b
^2*x), -1/4*(((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^
2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x
```

```
^2 + c)*sqrt(d)*x - c) + 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/4*(4*(a^2*b*d^2*x^
3 + a^3*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 -
a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqr
t((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
(b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(2*a*b^2*c^2
+ (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3
+ a^3*b^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)**2), x)
```

Giac [B] time = 1.21701, size = 740, normalized size = 4.4

$$-\frac{d^{\frac{5}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}+\frac{\left(3\,b^3c^3\sqrt{d}-4\,ab^2c^2d^{\frac{3}{2}}-a^2bcd^{\frac{5}{2}}+2\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2b^2}+\frac{3\left(\sqrt{d}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^(5/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 + 1/2*(3*b^3*c^3*sqrt
(d) - 4*a*b^2*c^2*d^(3/2) - a^2*b*c*d^(5/2) + 2*a^3*d^(7/2))*arctan(1/2*((s
qrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sq
rt(a*b*c*d - a^2*d^2)*a^2*b^2) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^3*c^3
*sqrt(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4
*a^3*d^(7/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^4*sqrt(d) + 14*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*a^2*b*c^2*d^(5/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*c*d^(7/2) +
 3*b^3*c^5*sqrt(d) - 2*a*b^2*c^4*d^(3/2) + a^2*b*c^3*d^(5/2))/(((sqrt(d)*x
- sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2*b^2)
```

3.756 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}(bc-ad)(2bc-ad)}{2a^2b\left(a+bx^2\right)}-\frac{c\left(c+dx\right.}{2ax^2\left(a+\right.}$$

```
[Out] -((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*(a + b*x^2)) - (c*(c
+ d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*b^(3/2))
```

Rubi [A] time = 0.269985, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 98, 149, 156, 63, 208}

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}(bc-ad)(2bc-ad)}{2a^2b\left(a+bx^2\right)}-\frac{c\left(c+dx\right.}{2ax^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*(a + b*x^2)) - (c*(c
+ d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*b^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
```

```
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)^2}\,dx, x, x^2\right)$$

$$= -\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(\frac{1}{2}c(4bc-5ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^2}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bc^2(4bc-5ad)-\frac{1}{2}d\left(2b^2c^2-2abcd-a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,d\right.}{2a^2b}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\left(c^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^3} +$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\left(c^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\right.}{2a^3d}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} + \frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{(bc-ad)^{3}}{}$$

Mathematica [A] time = 0.432793, size = 175, normalized size = 0.97

$$-\frac{\frac{a\sqrt{c+dx^2}\left(a^2d^2x^2+abc\left(c-2dx^2\right)+2b^2c^2x^2\right)}{bx^2\left(a+bx^2\right)} + \frac{\sqrt{bc-ad}\left(-a^2d^2-3abcd+4b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + c^{3/2}(-(4bc-5ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((a*Sqrt[c + d*x^2]*(2*b^2*c^2*x^2 + a^2*d^2*x^2 + a*b*c*(c - 2*d*x^2)))/(
b*x^2*(a + b*x^2)) - c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]] + (Sqrt[b*c - a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/(2*a^3)
```

Maple [B] time = 0.017, size = 7590, normalized size = 42.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 12.609, size = 2638, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*
c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c -
a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)
*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b*c^2 + (2*a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b
*x^2), -1/8*(4*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)
*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^3*c^2 - 3*a*b^2*c*d
 - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt((b*c -
a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*
a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2), -1/4*
```

```
(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d -
a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (
(4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)*lo
g(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*b*c^2 + (2*a*b^2
*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^
2), -1/4*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2
*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*
x^2)) + 2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*b*c^2 + (2*a*b^2*c^2 -
2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15505, size = 390, normalized size = 2.17

$$-\frac{1}{2}\,d^3\left(\frac{\left(4\,bc^3-5\,ac^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^3\sqrt{-c}d^3}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2-2\,\sqrt{dx^2+c}b^2c^3-2\left(dx^2+c\right)^{\frac{3}{2}}abcd+3\,\sqrt{dx^2+c}abc^2d+}{\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*((4*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(
-c)*d^3) + (2*(d*x^2 + c)^(3/2)*b^2*c^2 - 2*sqrt(d*x^2 + c)*b^2*c^3 - 2*(d*
x^2 + c)^(3/2)*a*b*c*d + 3*sqrt(d*x^2 + c)*a*b*c^2*d + (d*x^2 + c)^(3/2)*a^
2*d^2 - sqrt(d*x^2 + c)*a^2*c*d^2)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c +
b*c^2 + (d*x^2 + c)*a*d - a*c*d)*a^2*b*d^2) - (4*b^3*c^3 - 7*a*b^2*c^2*d +
2*a^2*b*c*d^2 + a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sq
rt(-b^2*c + a*b*d)*a^3*b*d^3))
```

3.757 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{c+dx^2}\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}-\frac{c\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

```
[Out] -(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b^2*c^2 - 20*a*b*
c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^(3
/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rubi [A] time = 0.247435, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {468, 580, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}-\frac{c\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```


```
[Out] -(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b^2*c^2 - 20*a*b*
c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^(3
/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```


Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{\sqrt{c+dx^2}\left(-c(5bc-3ad)-2bcdx^2\right)}{x^4\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{c\left(15b^2c^2-20abcd+3a^2d^2\right)+2bcd(5bc-6ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2b}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} + \cdots$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} + \cdots$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} + \cdots$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} + \cdots$$

Mathematica [C] time = 0.0174576, size = 54, normalized size = 0.31

$$-\frac{c\left(c+dx^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c*(c + d*x^2)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, ((-(b*c) + a*d)*x^2)
/(a*(c + d*x^2))])/(3*a^2*x^3)
```

Maple [B] time = 0.018, size = 7705, normalized size = 43.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A] time = 2.64601, size = 996, normalized size = 5.66

$$\left[-\frac{15\left(\left(b^2c^2-abcd\right)x^5+\left(abc^2-a^2cd\right)x^3\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-}}{b^2x^4+2\,abx^2+a^2}\right.}{24\left(a^3bx^5+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sqrt(-(b*c -
 a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sq
rt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c^2 - 20*a*b*
c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^
2 + c))/(a^3*b*x^5 + a^4*x^3), 1/12*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2
 - a^2*c*d)*x^3)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x
)) + 2*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^
2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)**2), x)
```

Giac [B] time = 6.27901, size = 670, normalized size = 3.81

$$-\frac{5\left(b^2c^3\sqrt{d}-2\,abc^2d^{\frac{3}{2}}+a^2cd^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^3}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\cdots}{\left(\left(\sqrt{d}x-\sqrt{dx^2+\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/2*(b^2*c^3*sqrt(d) - 2*a*b*c^2*d^(3/2) + a^2*c*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(
7/2) - b^3*c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d
)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3*b) - 2/3*(6*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b*c^3*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c^2*d^(3/
2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^4*sqrt(d) + 12*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*c^3*d^(3/2) + 6*b*c^5*sqrt(d) - 7*a*c^4*d^(3/2))/(((sqrt(
d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.758 $\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^2*(b*c - a*
d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b^2*Sqrt[d])
```

Rubi [A] time = 0.111655, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^2*(b*c - a*
d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b^2*Sqrt[d])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b(bc-ad)}\\ &= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(a(3bc-2ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^2(bc-ad)}\\ &= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+}\right.}{2b^2(bc-ad)}\\ &= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.189716, size = 129, normalized size = 0.98

$$\frac{\frac{abx\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2
) + (2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^2)
```

Maple [B] time = 0.019, size = 846, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/4/b^2*a/(a*d-b*c)/(x+1/b*(-a*
b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
```

```
-(a*d-b*c)/b)^(1/2)-1/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4/b^2*a/(-a*b)^(1/2)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^2*a/(a*d-b
*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^3*a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [A] time = 4.88907, size = 2236, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*d*x + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*s
qrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 2*a^2
*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^
2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)
*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*c*d - a^2*b^2*d
^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*d*x - 8*(a*b*c
- a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d)
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d
)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a
*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)
*a*b*d*x + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*
c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
 - a*d))/(a*d*x^3 + a*c*x)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(
d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*c*d - a^2*b^2*d^
2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*d*x - 4*(a*b*c -
 a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) +
```

```
 (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*
arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(
a*d*x^3 + a*c*x)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.18719, size = 383, normalized size = 2.9

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\left(b^3c-ab^2d\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+}\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2
 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqrt(
a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/2*log((sqrt(d)*x
 - sqrt(d*x^2 + c))^2)/(b^2*sqrt(d))
```

3.759 $\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.084961, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```


```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$=\frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{4b(bc-ad)}$$
$$=\frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2bd(bc-ad)}$$
$$=\frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.0896682, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*b^(
3/2))
```

Maple [B] time = 0.011, size = 807, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/
b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/b^2/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/b^2*(-a*b)^(
1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
```

```
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15612, size = 936, normalized size = 9.45

$$\left[\frac{\left(2\,abc - a^2d + \left(2\,b^2c - abd\right)x^2\right)\sqrt{b^2c - abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) + 4\,(}{8\left(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2 + \left(b^5c^2 - 2\,ab^4cd + a^2b^3d^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a*b^4*c^2 - 2
*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2), -1
/4*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
 a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c
))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.12643, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^2+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^2+c\right)b-bc+ad\right)} + \frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^2 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.760 $\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2(a+bx^2)(bc-ad)}$$

```
[Out] -(x*Sqrt[c + d*x^2])/(2*(b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0533037, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*(b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} + \frac{\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} + \frac{c\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.424221, size = 124, normalized size = 1.39

$$\frac{\sqrt{c+dx^2}\left(-\frac{x^2(bc-ad)}{a+bx^2}-\frac{c\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{dx^2}{c}+1}}\right)}{2x(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(((b*c - a*d)*x^2)/(a + b*x^2)) - (c*Sqrt[(-(b/a) + d/c)
*x^2]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^
2)/c]))/(2*(b*c - a*d)^2*x)
```

Maple [B] time = 0.01, size = 817, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d*(-a*b)^(1/2)/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/(-a*b
)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/
(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
+1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^2*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
```

```
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] time = 2.92807, size = 871, normalized size = 9.79

$$\left[-\frac{4\left(abc-a^2d\right)\sqrt{dx^2+c}\,x-\left(bcx^2+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2}}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c +
a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(
d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), -1/4*(2*(a*b*c - a^2*d)
*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 3.1892, size = 312, normalized size = 3.51

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}(bc-ad)}+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + ((sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(
b^2*c - a*b*d))
```

3.761 $\int \frac{x}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(2*(b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0678864, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*(b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0881002, size = 85, normalized size = 0.98

$$
\frac{1}{2}\left(\frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]/((-(b*c) + a*d)*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/2
```

Maple [B] time = 0.009, size = 513, normalized size = 5.9

$$
-\frac{1}{4\,ab\,(ad-bc)}\sqrt{-ab}\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\left(x-\frac{1}{b}\sqrt{-ab}\right)^{-1}-\frac{d}{4\,(ad-bc)\,b}\ln\left(\left(-2\,\frac{ad}{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4*(-a*
b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04122, size = 849, normalized size = 9.76

$$\left[-\frac{\left(bdx^2+ad\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(b^2c-abd\right)\sqrt{dx^2+}}{8\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a
*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(
b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c
+ a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^
2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(b^2*c - a*b*d)*s
qrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.12665, size = 124, normalized size = 1.43

$$-\frac{1}{2}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{\sqrt{dx^2+c}}{\left(\left(dx^2+c\right)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d*(arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) + sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c + a*d)*(b*c - a*d)))
```

3.762 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rubi [A] time = 0.0529042, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$
$$= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}$$
$$= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.766238, size = 405, normalized size = 4.05

$$\frac{x\sqrt{c+dx^2}\left(-30dx^2\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}} - 45c\sqrt{\frac{ax^2(c+dx^2)(bc-ad)}{c^2(a+bx^2)^2}} + 16dx^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{5/2} {}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right) + 1\right.}{30c^2(a+bx^2)^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*(-45*c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*
x^2)^2)] - 30*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2
)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 30*d*x^2*ArcSin
[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16*c*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2,
3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 16*d*x^2*(((b*c - a*d)*x^2)/(c
*(a + b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(30*c^2*(((b*c - a*d)*x^2
)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^2*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))
])
```

Maple [B] time = 0.01, size = 823, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(a
*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
```

```
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] time = 3.06731, size = 954, normalized size = 9.54

$$\left[\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\,(\left(\right.}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.14046, size = 304, normalized size = 3.04

$$-\frac{1}{2}\, d^{\frac{3}{2}} \left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc+4\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/((((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

3.763 $\int \frac{1}{x(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.135801, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```



```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c-}}\right.}{4a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int -\right.}{2a^2d(bc}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.234552, size = 123, normalized size = 0.95

$$\frac{\frac{ab\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - (2*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*a^2)
```

Maple [B] time = 0.011, size = 838, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/a^2/c^
(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/
(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x), x)
```

Fricas [A] time = 6.06707, size = 2201, normalized size = 16.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*
b^2*c^2 - a^3*b*c*d)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*c + 8*(a*b*c - a^2*d
+ (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*a*b*c
^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*
d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
 c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c^2 - a^4*c*d
 + (a^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^
2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*
```

```
b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^2 - 3*a^2*c*d + (2*b^2
*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 4*(a*b*c - a^2*
d + (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a^3*b*
c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.13377, size = 207, normalized size = 1.59

$$-\frac{1}{2}\,d^2\left(\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx^2+c}b}{\left(abcd-a^2d^2\right)\left(\left(dx^2+c\right)b-bc+ad\right)}-\frac{2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d)
)/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x^2 + c)*b/((a*b*
c*d - a^2*d^2)*((d*x^2 + c)*b - b*c + a*d)) - 2*arctan(sqrt(d*x^2 + c)/sqrt
(-c))/(a^2*sqrt(-c)*d^2))
```

3.764 $\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{c+dx^2}(3bc-2ad)}{2a^2cx(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*x) + (b*Sqrt[c + d*
x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.135748, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}(3bc-2ad)}{2a^2cx(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*x) + (b*Sqrt[c + d*
x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{(b(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x}\right.}{2a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 5.15409, size = 116, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(\frac{b^2x^2}{\left(a+bx^2\right)(ad-bc)}-\frac{2}{c}\right)}{2a^2x} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/c + (b^2*x^2)/((-(b*c) + a*d)*(a + b*x^2))))/(2*a^2*x)
 - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.013, size = 841, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a^2*d*(-a*b)^(1/2)/(
```

```
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a
^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a^2*d*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/4*b/a^2/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3
/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))-1/a^2/c/x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2 \sqrt{dx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [B] time = 3.36674, size = 1238, normalized size = 8.42

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^3+\left(3\,ab^2c^2-4\,a^2bcd\right)x\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^3+\left(a^4b^2c^3-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt
(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^
2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b^2*c^2 - 4*a
^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2
*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x), -1/4*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3
+ (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c
- a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (
3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*
c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a
^6*c*d^2)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 3.66341, size = 535, normalized size = 3.64

$$\frac{1}{2}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abd-6\left(\sqrt{a}\right.\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}x\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt(a
*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c - 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*b*d - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2
+ 14*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d - 8*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/((((sqrt(d)*x - sqrt(d*x^2 + c))^6*
b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

3.765 $\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}(2bc-ad)}{2a^2c\left(a+bx^2\right)(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*(a + b*x^2)) - Sqrt
[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.243484, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}(2bc-ad)}{2a^2c\left(a+bx^2\right)(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*(a + b*x^2)) - Sqrt
[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx\right.}{2a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{4a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{2a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}} - \frac{b^{3/2}(4bc}{}$$

Mathematica [A] time = 0.551765, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^2}\left(a^2d+ab\left(dx^2-c\right)-2b^2cx^2\right)}{x^2\left(a+bx^2\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(a^2*d - 2*b^2*c*x^2 + a*b*(-c + d*x^2)))/((b*c - a*d)*
x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(2*a^3*c)
```

Maple [B] time = 0.013, size = 899, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 2*b/a^3/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/4*b^2/a^2/(-a*b)^(1
/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-b/a^3/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/a^2/c/x^2*(d*x^2+c)^(1/2)+1/2/a^2*
d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*b^2/a^2/(-a*b)^(1/2)/(a
*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2 \sqrt{dx^2+c}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3), x)
```

Fricas [A] time = 10.8053, size = 2869, normalized size = 15.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)
*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2
*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*
d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*
a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) +
 2*c)/x^2) - 4*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d
*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2),
 -1/8*(4*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*
b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - ((4*b^3*c
^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(b/(b*c -
a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^
2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a
^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*
b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2), 1/4*(((4*b^3*c^3
 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(-b/(b*c - a
*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d)
)/(b*d*x^2 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*
c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(c) + 2*c)/x^2) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2
)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*
d)*x^2), 1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2
*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arcta
n(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b
*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 -
 a^5*c^2*d)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.1305, size = 362, normalized size = 1.96

$$\frac{1}{2}d^3\left(\frac{\left(4b^3c-5ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2c-2\sqrt{dx^2+c}b^2c^2-\left(dx^2+c\right)^{\frac{3}{2}}abd+2\sqrt{dx^2+c}abcd-\sqrt{}}{\left(a^2bc^2d^2-a^3cd^3\right)\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*(d*x^2 + c)^(3/2)*b^
2*c - 2*sqrt(d*x^2 + c)*b^2*c^2 - (d*x^2 + c)^(3/2)*a*b*d + 2*sqrt(d*x^2 +
```

```
c)*a*b*c*d - sqrt(d*x^2 + c)*a^2*d^2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^2
+ c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - (4*b*c +
 a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.766 $\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt{c+dx^2}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(bc-}$$

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^2*c*(b*c - a*d)*x^3) + ((15*b^2*c^2
 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(b*c - a*d)*x) + (b*S
qrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^2*(5*b*c - 6*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)
^(3/2))
```

Rubi [A] time = 0.252386, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^2*c*(b*c - a*d)*x^3) + ((15*b^2*c^2
 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(b*c - a*d)*x) + (b*S
qrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^2*(5*b*c - 6*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)
^(3/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} - \frac{\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-2ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} -$$

Mathematica [A] time = 5.23383, size = 136, normalized size = 0.66

$$\frac{\sqrt{c+dx^2}\left(\frac{3b^3x^4}{\left(a+bx^2\right)(bc-ad)}+\frac{4x^2(ad+3bc)}{c^2}-\frac{2a}{c}\right)}{6a^3x^3}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a)/c + (4*(3*b*c + a*d)*x^2)/c^2 + (3*b^3*x^4)/((b*c
- a*d)*(a + b*x^2))))/(6*a^3*x^3) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.015, size = 893, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3/a^2/c/x^3*(d*x^2+c)^(1/2)+2/3/a^2*d/c^2/x*(d*x^2+c)^(1/2)-1/4*b^2/a^3/
(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4*b^2/a^3/(
a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+5/4*b^2/a^3/(-
a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4
*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))+2*b/a^3/c/x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2 \sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [A] time = 5.08281, size = 1536, normalized size = 7.46

$$\left[-\frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^5+\left(5ab^3c^3-6a^2b^2c^2d\right)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left((bc-2\right.}{b^2x^4+2abx^2+a^2}\right.}{24\left(\left(a^4b^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d
)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(
-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^3*b^
2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*
a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4
*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d
 + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^3), 1
/12*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x
```

```
^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) -
2*(2*a^3*b^2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3
*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*
c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^
5*b^2*c^3*d + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d
^2)*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 6.43584, size = 506, normalized size = 2.46

$$\frac{1}{6}d^{\frac{7}{2}}\left(\frac{3\left(5b^3c-6ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c-2\left(\sqrt{d}x-\sqrt{dx^2\right.\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4)*s
qrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c - 2*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d^4)*((s
qrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)
^3*a^3*d^3))
```

3.767 $\int \frac{x^4}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=130

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3\sqrt{a}c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*b*(b*c - a
*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.101606, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3\sqrt{a}c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*b*(b*c - a
*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5/2))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{ac-2bcx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{3abc^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2bc(bc-ad)^2}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3ac)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)^2}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3ac)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,\right.}{2(bc-ad)^2}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{3\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0212488, size = 54, normalized size = 0.42

$$\frac{cx^5\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{5a^2\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x^5*Hypergeometric2F1[2, 5/2, 7/2, ((-(b*c) + a*d)*x^2)/(a*(c + d*x^2))]
)/(5*a^2*(c + d*x^2)^(5/2))
```

Maple [B] time = 0.019, size = 1498, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b^2*x/c/(d*x^2+c)^(1/2)-1/4/b^2*a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
```

```
)-3/4/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b^2*a^2*d^2/(a*d-b*c)^
2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*x+3/4/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4/b^2*a/(a*d-b*c)/c/((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-
1/4/b^2*a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b^2*a*d*(-a*b)^(1/2)
/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+3/4/b^2*a^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b^2*a*
d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))-5/4/b^2*a/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-3/4/b*a/(-a*b)^(1/2)/(a*d-
b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/
4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] time = 3.82377, size = 1134, normalized size = 8.72

$$\Bigg[\frac{3\,(bcdx^4+ac^2+(bc^2+acd)x^2)\sqrt{-\frac{a}{bc-ad}}\log\Big(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^3-(abc^2-a^2cd)x}{b^2x^4+2\,abx^2+a^2}}{8\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
```

```
)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*s
qrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((2*b
*c + a*d)*x^3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*
c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*
d - a^2*b*c*d^2 + a^3*d^3)*x^2), 1/4*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d
)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((2*b*c + a*d)*x^3 + 3*a*c
*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d -
2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 5.24698, size = 402, normalized size = 3.09

$$\frac{3\,ac\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{cx}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{dx^2+c}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{\cdots}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/2*a*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + c*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - ((sqrt(
d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sq
rt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d +
b*c^2)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2))
```

3.768 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{a}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c - a*d)*(a +
 b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.119274, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c - a*d)*(a +
 b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4(bc-ad)^2}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2d(bc-ad)^2}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.029961, size = 91, normalized size = 0.68

$$\frac{\left(a+bx^2\right)(ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+a(bc-ad)}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2,
(b*(c + d*x^2))/(b*c - a*d)])/(2*b*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2
])
```

Maple [B] time = 0.012, size = 1456, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b*a*d/(a*
d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)-3/4/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4
/b*a*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))+1/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4/b^2*(-a*b)^(1/2)/(a*d
-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b*a*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b^
2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b*a*d/(a*d-b*c)^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/b^2*(-a*b)^(1/2)/(
a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d-1/2/b/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/b/(a*d-b*c)/((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)+1/2/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21015, size = 1492, normalized size = 11.13

$$\left[\frac{\left(\left(2\,b^2cd+abd^2\right)x^4+2\,abc^2+a^2cd+\left(2\,b^2c^2+3\,abcd+a^2d^2\right)x^2\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right.}{b^2x^4+2\,abx}\right.}{8\left(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^2)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a
*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d
)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*
```

```
a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x
^2 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (
b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4
 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2), -1/4*(((2*b^2*c*d + a
*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^2)*
sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*
d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*
b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^
5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4 -
 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.18451, size = 244, normalized size = 1.82

$$\frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{2\,(dx^2+c)bcd-2\,bc^2d+(dx^2+c)ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^2 + c)*b*c*d -
2*b*c^2*d + (d*x^2 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d)))/d
```

3.769 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}$$

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a + b*x^2)*S
qrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0909396, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a + b*x^2)*S
qrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{\int \frac{c-2dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2(bc-ad)}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{\int \frac{c(bc+2ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2c(bc-ad)^2}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)^2}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x}\right.}{2(bc-ad)^2}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.05945, size = 133, normalized size = 1.08

$$\frac{x^3\left(\frac{8x^2\left(c+dx^2\right)(bc-ad)\,{}_2F_1\left(2,3;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}+7c\left(5c+2dx^2\right){}_2F_1\left(1,2;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)\right)}{105c^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x^3*(7*c*(5*c + 2*d*x^2)*Hypergeometric2F1[1, 2, 7/2, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))] + (8*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 3, 9/2
, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(105*c^3*(a + b*x^2)^2*
Sqrt[c + d*x^2])
```

Maple [B] time = 0.01, size = 1453, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b*d*(-a*b)^(1/2)/(a*d-b*
c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-3/4/b*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b*d*(-a*b)^(1/2)/(
a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4
/b/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/4/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/(-a*b
)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))-1/4/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b/(a*d-b*c)/c/((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/
4/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))+1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d*(-a*b)^(1
/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d^2*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*d*(-a
*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] time = 4.56242, size = 1512, normalized size = 12.29

$$\left[-\frac{\left(\left(b^2cd+2\,abd^2\right)x^4+abc^2+2\,a^2cd+\left(b^2c^2+3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-\cdots\right.}{b^2x^4\cdots}\right.}{8\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(ab^4c^3d-3\,a^2b^3c^2d^2+3\,a^3b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*
c*d + 2*a^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
```

```
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2)*
x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*
c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x^4
 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2), 1/4*(((b^2
*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*c*d + 2*a^2*
d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
x)) - 2*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2
)*x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^
5*c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x
^4 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 5.32356, size = 404, normalized size = 3.28

$$-\frac{dx}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{dx^2+c}}-\frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\sqrt{d}x-\sqrt{dx^2\cdots\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - 1/2*(b*c*sqrt(d) +
 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^
2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^
4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*d + b*c^2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

3.770 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{3d}{2\sqrt{c+dx^2}(bc-ad)^2}-\frac{1}{2(a+bx^2)\sqrt{c+dx^2}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a + b*x^2)*Sqr
t[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/(2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0852439, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$-\frac{3d}{2\sqrt{c+dx^2}(bc-ad)^2}-\frac{1}{2(a+bx^2)\sqrt{c+dx^2}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a + b*x^2)*Sqr
t[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/(2*(b*c - a*d)^(5/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3d)\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{4(bc-ad)}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3bd)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x\right.}{4(bc-ad)^2}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{\right.}{2(bc-ad)^2}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0160044, size = 52, normalized size = 0.46

$$-\frac{d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{\sqrt{c+dx^2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^2))/(-(b*c) + a*d))])/((
-(b*c) + a*d)^2*Sqrt[c + d*x^2]))
```

Maple [B] time = 0.011, size = 989, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*d/(a*d-b*
c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-3/4*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-
b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2*(-a
*b)^(1/2)/a/b/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
```

```
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1
/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*(-a*b)^(1/2)/b*d
^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*(-a*b)^(1/2)/a/b/(a*d-b*c)/c/((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04625, size = 1110, normalized size = 9.82

$$\left[\frac{3\left(bd^2x^4+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2}}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((
b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*b*d*x^2 +
b*c + 2*a*d)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*x^2), -1/4*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt
(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b
/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*b*d*x^2 + b*c + 2*a*d)*sqrt(d*x^2 + c
))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^
2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.12848, size = 203, normalized size = 1.8

$$-\frac{1}{2}\,d\left(\frac{3\,b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\left(dx^2+c\right)b-2\,bc+2\,ad}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*d*(3*b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (3*(d*x^2 + c)*b - 2*b*c + 2*a*d)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c
 + sqrt(d*x^2 + c)*a*d)))
```

3.771 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)/(2*a*(b*c
 - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.102182, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```



```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)/(2*a*(b*c
 - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(5/2))
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-bc+2ad-2bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\int \frac{bc(bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ac(bc-ad)^2}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(b(bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{2a(bc-ad)^2}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(b(bc-4ad))\,\mathrm{Subst}\left(\int \frac{}{a-(-bc}\right.}{2a(bc-ad}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{b(bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}$$

Mathematica [C] time = 4.4101, size = 210, normalized size = 1.48

$$\frac{x\left(16x^2\left(c+dx^2\right)^2(bc-ad)\mathrm{HypergeometricPFQ}\left(\{2,2,3\},\left\{1,\frac{9}{2}\right\},\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)+16x^2\left(4c^2+7cdx^2+3d^2x^4\right)(bc-ad)\,{}_2F_1}{105c^4\left(a+bx^2\right)^3\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(7*c*(a + b*x^2)*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*Hypergeometric2F1[1,
2, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 16*(b*c - a*d)*x^2*(4*c^2 + 7*
c*d*x^2 + 3*d^2*x^4)*Hypergeometric2F1[2, 3, 9/2, ((b*c - a*d)*x^2)/(c*(a +
 b*x^2))] + 16*(b*c - a*d)*x^2*(c + d*x^2)^2*HypergeometricPFQ[{2, 2, 3}, {
1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(105*c^4*(a + b*x^2)^3*Sqrt[c
 + d*x^2])
```

Maple [B] time = 0.01, size = 1461, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a*d*(-a*b)^(1/2)/(a*d-b
*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)+3/4*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a*d*(-a*b)^(1/2)/(a*d
-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/
(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x*d-1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/
4/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-
3/4/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))-1/4/a/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/4/a/(-a*b)^(1/2)/(a*d-
b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] time = 5.1823, size = 1705, normalized size = 12.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c
^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
```

```
(a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c)
)/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4
*c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^
4*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2), 1/4*((a*b^2*c^
3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 3*a*b^2*c^
2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d
)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b
*c^2 - a^2*c*d)*x)) + 2*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
(a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c))
/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4*
c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^4
*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 5.26109, size = 429, normalized size = 3.02

$$\frac{d^2x}{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\sqrt{dx^2+c}} - \frac{\left(b^2c\sqrt{d} - 4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{2\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\sqrt{abcd - a^2d^2}} - \frac{\left(\sqrt{d}x - \sqrt{d\ldots\right.}{\left(\left(\sqrt{dx} - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}x\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*x/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)) - 1/2*(b^2*c*sq
rt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt
(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2))
```

3.772 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a*d
)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(3
/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*a^2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.244308, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a*d
)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(3
/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*a^2*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)^2-\frac{1}{4}bd(bc+2}{x(a+bx)\sqrt{c+dx}}\right.}{ac(bc-ad)^2}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2c}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+}\right.}{a^2cd}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}} + \frac{b^{3/2}(2bc}{}$$

Mathematica [C] time = 0.110675, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a}-\frac{2d}{c}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right) + \frac{b}{a+bx^2}}{2a\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b/(a + b*x^2) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c +
 d*x^2))/(b*c - a*d)])/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(2*a*(b*c - a*d)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.013, size = 1672, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/a^2/c/(d*x^2+c)^(1/2)-1/a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
+1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d
-b*c)^2*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)-3/4/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*
d/(a*d-b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)+1/2/(-a*b)^(1/2)/a/(a*d-b*c)*b/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/a^2/(a*d-b*c)*b/((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)+1/2/a^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a^2/(a*d-b*c)*b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/a^2/(a*d-b*c)*
b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)-1/2/a^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a^2/(a*d-b*c
)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/(-a*b)
^(1/2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d-b*c)^2*b/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+3/4/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*d/(a*d-b*c)
^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/(-a*b
)^(1/2)/a/(a*d-b*c)*b/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [B] time = 21.1527, size = 4030, normalized size = 23.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(b/(b*c - a*d))*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d
*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^
2*c^2*d + 2*a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d
 + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 +
(a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), 1/8*(8*(a
*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d
^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d
 - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c
*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)) + 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^2)
*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4
*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d
- a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (
2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c
^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d
*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*(a*b^2*c^3 - 2*a^2*b*c^
2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 -
a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^
2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*
a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d
^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 -
5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*
c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*
b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 4*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a
^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^
2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
- a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.13165, size = 319, normalized size = 1.88

$$-\frac{1}{2}\,d^2\left(\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^2c^2d^2-2\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{\left(dx^2+c\right)b^2c+2\left(dx^2+c\right)abd-2\,abcd+2\,a^2d^2}{\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+\dots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*d^2*((2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/((a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)) - ((
d*x^2 + c)*b^2*c + 2*(d*x^2 + c)*a*b*d - 2*a*b*c*d + 2*a^2*d^2)/((a*b^2*c^3
*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*
c + sqrt(d*x^2 + c)*a*d)) - 2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c
)*c*d^2))
```

3.773 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad}{2acx\sqrt{c+dx}}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a
*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*b*c*d + 4*a^2*d^2)*S
qrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```

Rubi [A] time = 0.268332, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad}{2acx\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a
*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*b*c*d + 4*a^2*d^2)*S
qrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-3bc+2ad-4bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-3b^2c^2+4abcd-4a^2d^2-2\cdots}{x^2\left(a+bx^2\right)\sqrt{c+\cdots}}}{2ac(bc-a\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd+4a^{\cdots}\right.}{2a^2c^2(bc-a\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd+4a^{\cdots}\right.}{2a^2c^2(bc-a\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd+4a^{\cdots}\right.}{2a^2c^2(bc-a\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd+4a^{\cdots}\right.}{2a^2c^2(bc-a\cdots}$$

Mathematica [A] time = 5.35102, size = 145, normalized size = 0.71

$$\sqrt{c+dx^2}\left(-\frac{\frac{b^3x}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{1}{c^2x}}{a^2} - \frac{d^3x}{c^2\left(c+dx^2\right)(bc-ad)^2}\right) - \frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-((d^3*x)/(c^2*(b*c - a*d)^2*(c + d*x^2))) - (1/(c^2*x) +
(b^3*x)/(2*(b*c - a*d)^2*(a + b*x^2)))/a^2) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```

Maple [B] time = 0.015, size = 1524, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a^2*d*(-a*b)^(1/2)/(
a*d-b*c)^2*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)-3/4/a*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a^2*d*(-a*
b)^(1/2)/(a*d-b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))-1/4/a^2/(a*d-b*c)*b/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a
*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)-3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a*d
^2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/a^2/(a*d-b*c)*b/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x*d-3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2/(-a
*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))+3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^2/(-a*b)^(1/2)
/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/a
^2/c/x/(d*x^2+c)^(1/2)-2/a^2*d/c^2*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [B] time = 6.08287, size = 2020, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)*sqrt(-a*b*c + a^2*d)*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*
c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 +
c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a
^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2
*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3
- 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*
a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a
^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 + (a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*
c^4*d^2 - a^7*c^3*d^3)*x), -1/4*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^
4*c^4 - a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d
)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2
- a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
 2*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*
b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b
^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3 - 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))
/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x
^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 +
(a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*c^4*d^2 - a^7*c^3*d^3)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 6.96892, size = 748, normalized size = 3.65

$$-\frac{d^3 x}{\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\sqrt{dx^2+c}} + \frac{3\left(b^3c\sqrt{d} - 2\,ab^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\sqrt{abcd-a^2d^2}} + \frac{3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d^3*x/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(d*x^2 + c)) + 3/2*(b^3*c
*sqrt(d) - 2*a*b^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^
2)*sqrt(a*b*c*d - a^2*d^2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^3*c^2*sq
rt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c*d^(3/2) + 2*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a^2*b*d^(5/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^
3*sqrt(d) + 18*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) - 20*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c*d^(5/2) + 8*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a^3*d^(7/2) + 3*b^3*c^4*sqrt(d) - 4*a*b^2*c^3*d^(3/2) + 2*a^2*b*c^2*d^(
5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*(a^2*b^2*c
^3 - 2*a^3*b*c^2*d + a^4*c*d^2))
```

3.774 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{b(2bc-}{2a^2c\left(a+bx^2\right)\sqrt{c}}$$

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2
]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.336123, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{b(2bc-}{2a^2c\left(a+bx^2\right)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2
]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)}{}\right.}{2}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

Mathematica [C] time = 0.115818, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^2\left(a+bx^2\right)(4bc-7ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right) - (ad-bc)\left(x^2\left(a+bx^2\right)\left(3a^2d^2+abcd-4b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}\right.\right.}{2a^3c^2x^2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^2*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, (b
*(c + d*x^2))/(b*c - a*d)] - (-(b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^2 + a*b
*(-c + d*x^2)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^2*(a + b*x^2)*Hyperge
ometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c]))/(2*a^3*c^2*(b*c - a*d)^2*x^2*(a +
 b*x^2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.016, size = 1778, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -2*b/a^3/c/(d*x^2+c)^(1/2)+2*b/a^3/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2
))/x)-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4
```

```
*b^2/a^2*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x+3/4*b^2/a^2*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))-1/2*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-b
^2/a^3/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+b^2
/a^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
-b^2/a^3/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)+b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+b
^2/a^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^
2/a^2*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*x+3/4*b^2/a^2*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))+1/2*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/
a^2/c/x^2/(d*x^2+c)^(1/2)-3/2/a^2*d/c^2/(d*x^2+c)^(1/2)+3/2/a^2*d/c^(5/2)*l
n((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [B] time = 41.5104, size = 5106, normalized size = 21.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b*c -
```

```
a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x
^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(
(4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^
4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4
*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(c)*
log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b^2*c^4 - 2*a^
3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^
3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 +
(a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*
c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), -1/8*(4*((4*b^4*c^3*d - 5*a*b^3*c^
2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a
^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^
3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x
^2 + c)) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*
d - 7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b
*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*
d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*
c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2
 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^
3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4
*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*
d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*
x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^
4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^
2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3
*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*
d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4
+ (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4*c^4*d - 7*
a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^4 +
(4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x
^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2
*((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*
b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 +
(4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^
3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/
((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a
^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^
5*d + a^6*c^4*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.18386, size = 510, normalized size = 2.12

$$\frac{1}{2}\,d^3\left(\frac{\left(4\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3b^2c^2d^3-2\,a^4bcd^4+a^5d^5\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^2+c\right)^2b^3c^2-2\left(dx^2+c\right)b^3c^3-2\left(dx^2+c\right)^2ab^2cd+3\left(dx^2+c\right)ab}{\left(a^2b^2c^4d^2-2\,a^3bc^3d^3+a^4c^2d^4\right)\left(\left(dx^2+c\right)^{\frac{5}{2}}b-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*((4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b^2*c^2*d^3 - 2*a^4*b*c*d^4 + a^5*d^5)*sqrt(-b^2*c + a*b*d)) - (2*
(d*x^2 + c)^2*b^3*c^2 - 2*(d*x^2 + c)*b^3*c^3 - 2*(d*x^2 + c)^2*a*b^2*c*d +
 3*(d*x^2 + c)*a*b^2*c^2*d + 3*(d*x^2 + c)^2*a^2*b*d^2 - 7*(d*x^2 + c)*a^2*
b*c*d^2 + 2*a^2*b*c^2*d^2 + 3*(d*x^2 + c)*a^3*d^3 - 2*a^3*c*d^3)/((a^2*b^2*
c^4*d^2 - 2*a^3*b*c^3*d^3 + a^4*c^2*d^4)*((d*x^2 + c)^(5/2)*b - 2*(d*x^2 +
c)^(3/2)*b*c + sqrt(d*x^2 + c)*b*c^2 + (d*x^2 + c)^(3/2)*a*d - sqrt(d*x^2 +
 c)*a*c*d)) - (4*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c
)*c^2*d^3))
```

3.775 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt{c+dx^2}\left(-8a^2bcd^2+16a^3d^3-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{b}}{\sqrt{a}\sqrt{}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(2*a*(b*c -
 a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 4*a*b*c*d + 8*a^2*d^
2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((15*b^3*c^3 - 14*a*b^2
*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d
)^2*x) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.396469, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(-8a^2bcd^2+16a^3d^3-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{b}}{\sqrt{a}\sqrt{}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(2*a*(b*c -
 a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 4*a*b*c*d + 8*a^2*d^
2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((15*b^3*c^3 - 14*a*b^2
*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d
)^2*x) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5bc+2ad-6bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5b^2c^2+4abcd-8a^2d^2-4b\cdots}{x^4\left(a+bx^2\right)\sqrt{c+d\cdots}}}{2ac(bc-ad\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4abcd+8a^2\cdots\right.}{6a^2c^2(bc-ad\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4abcd+8a^2\cdots\right.}{6a^2c^2(bc-ad\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4abcd+8a^2\cdots\right.}{6a^2c^2(bc-ad\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4abcd+8a^2\cdots\right.}{6a^2c^2(bc-ad\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4abcd+8a^2\cdots\right.}{6a^2c^2(bc-ad\cdots}$$

Mathematica [A] time = 5.43444, size = 167, normalized size = 0.6

$$\sqrt{c+dx^2}\left(\frac{\frac{b^4x}{2(a+bx^2)(bc-ad)^2}+\frac{2b}{c^2x}}{a^3}-\frac{c-5dx^2}{3a^2c^3x^3}+\frac{d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2}\right)+\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(c - 5*d*x^2)/(3*a^2*c^3*x^3) + (d^4*x)/(c^3*(b*c - a*d)^
2*(c + d*x^2)) + ((2*b)/(c^2*x) + (b^4*x)/(2*(b*c - a*d)^2*(a + b*x^2)))/a^
3) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.016, size = 1608, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^3*d*(-a*b)^
(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4*b
^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))+3/4*b^2/a^3/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/3/a^2/c/x^3/(d*
x^2+c)^(1/2)+4/3/a^2*d/c^2/x/(d*x^2+c)^(1/2)+8/3/a^2*d^2/c^3*x/(d*x^2+c)^(1
/2)-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^3*d*(-a*
b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/
4*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))+3/4*b^2/a^3/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+5/4*b^3/a^3/(-
a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4*b^3/a^3/(-a*b)^(1/2
)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+2*b/a^3/c/x/(d*x^2+c)^(1/2)+4*b/
a^3*d/c^2*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [B] time = 8.48678, size = 2504, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d
- 8*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sq
rt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b^3*c^5 - 6*a^4*b^2*
c^4*d + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*
d^2 + 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*
c^5 - 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b
*c*d^4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*
c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^
6*d - 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4
*c^7 - 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7
- 3*a^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3), 1/12*(3*((5*b^5*c^
4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2
)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d -
a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^3*c^5 - 6*a^4*b^2*c^4*d
 + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*d^2 +
 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*c^5 -
 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b*c*d^
4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d
^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^6*d -
 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4*c^7
- 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7 - 3*a
^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 11.8805, size = 656, normalized size = 2.37

$$\frac{d^4x}{\left(b^2c^5 - 2\,abc^4d + a^2c^3d^2\right)\sqrt{dx^2+c}} - \frac{\left(5\,b^4c\sqrt{d} - 8\,ab^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2\right)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}\right.}{\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^4*x/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(d*x^2 + c)) - 1/2*(5*b^4*
c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d
^2)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^4*c*sqrt(
d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^3*d^(3/2) - b^4*c^2*sqrt(d))/((a
^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
 + b*c^2)) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 6*b*c^3*sqrt(
d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^2)
```

3.776 $\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=174

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

```
[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)/(2*b*(b*c
 - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a*d)*x)/(6*(b*c - a*d
)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.211019, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)/(2*b*(b*c
 - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a*d)*x)/(6*(b*c - a*d
)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(7/2))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{ac-2(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{5abc^2-2bc(2bc+3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,d}{6bc(bc-ad)^2}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.962807, size = 133, normalized size = 0.76

$$\frac{x^5\left(\frac{8x^2\left(c+dx^2\right)(bc-ad)\,{}_2F_1\left(2,3;\frac{11}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}+9c\left(7c+2dx^2\right)\,{}_2F_1\left(1,2;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)\right)}{315c^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^5*(9*c*(7*c + 2*d*x^2)*Hypergeometric2F1[1, 2, 9/2, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))] + (8*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 3, 11/
2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(315*c^3*(a + b*x^2)^2
*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.023, size = 2463, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b*a/(-a*b)^(1/2)/(a*d-b
*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)-3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/b^2*a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)-1/4/b^2*a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-7/6/b^2*a*d/(a*d-
b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*x-7/12/b^2*a*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-7/6/b^2*a*d/(a*d-
b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*x-5/4/b*a^2*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b*a*d*(-a*
b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))-7/12/b^2*a*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)+3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))+5/4/b*a*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/1
2/b^2*a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4/b*a*d*(-a*b)^(1/2)/(a*d-b*
c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+5/12/b^2*a^2*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/12/b^2*a^2*d^2/
(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*x+5/6/b^2*a^2*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b
*a^2*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/b*a*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+5/6/b^2*a^2*d^2/(a*
d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+3/4/b*a/(a*d-b
*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*x*d+1/3/b^2*x/c/(d*x^2+c)^(3/2)+2/3/b^2/c^2*x/(d*x^2+c)^(
1/2)+3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*a/(-a*b)^(1/2)/(a*d-b*c)^
2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] time = 7.75318, size = 2048, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2
*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*
d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
 a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) - 4*((4*b^2*c*d + 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2
+ 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x^2 + c)
)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d
^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*
a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 -
 a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/1
2*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2*c^2*
d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*d^2)*
x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((4*b^2*c*d + 11*a*b*d^2)*x^
5 + 2*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2
*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
(2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)
*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4
*c*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 6.43751, size = 802, normalized size = 4.61

$$\frac{\left(\frac{2\left(b^4c^5d^2-ab^3c^4d^3-3\,a^2b^2c^3d^4+5\,a^3bc^2d^5-2\,a^4cd^6\right)x^2}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}+\frac{3\left(b^4c^6d-2\,ab^3c^5d^2+2\,a^3bc^3d^4-a^4c^2d^5\right)}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^4*c^5*d^2 - a*b^3*c^4*d^3 - 3*a^2*b^2*c^3*d^4 + 5*a^3*b*c^2*d^5 -
 2*a^4*c*d^6)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^
3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 3*(b^4*
c^6*d - 2*a*b^3*c^5*d^2 + 2*a^3*b*c^3*d^4 - a^4*c^2*d^5)/(b^6*c^7*d - 6*a*b
^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 -
 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) + 1/2*(3*a*b*c*sqrt(d) +
 2*a^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt
(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2))
```

3.777 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{a}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}(bc-ad)^3}+\frac{3ad+2bc}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{b}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c\cdots}}{\sqrt{bc\cdots}}\right.}{2(bc-ad)^{7/2}}$$

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b*c - a*d)*
(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.164626, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}(bc-ad)^3}+\frac{3ad+2bc}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{b}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c\cdots}}{\sqrt{bc\cdots}}\right.}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b*c - a*d)*
(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$=\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{(2bc+3ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx,x,x^2\right)}{4b(bc-ad)}$$

$$=\frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{(2bc+3ad)\operatorname{Subst}\left(\int \frac{}{(a+} \right.}{4(bc-a}$$

$$=\frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{2bc+3ad}{2(bc-ad)^3\sqrt{c+dx^2}}+$$

$$=\frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{2bc+3ad}{2(bc-ad)^3\sqrt{c+dx^2}}+$$

$$=\frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{2bc+3ad}{2(bc-ad)^3\sqrt{c+dx^2}}-$$

Mathematica [C] time = 0.0380696, size = 93, normalized size = 0.55

$$\frac{\left(a+bx^2\right)(3ad+2bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+3a(bc-ad)}{6b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a*(b*c - a*d) + (2*b*c + 3*a*d)*(a + b*x^2)*Hypergeometric2F1[-3/2, 1, -
1/2, (b*(c + d*x^2))/(b*c - a*d)])/(6*b*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)^(3/2))
```

Maple [B] time = 0.015, size = 2400, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -5/12/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/b^2*(-a*b)^(1/2)
*d^2*a/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/b*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^3/c/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x+5/6/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/12/b^2*
(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/2/(a*d-b*c)^2/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/
(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-1/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/b^
2*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/2/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)*x+1/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/b/(a*d-b*c)
^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x*d-5/4*a*d/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*a*d/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/6/b/(a*d-
b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)-1/2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))+1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)+5/12/b*a*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*a*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+5/4*a*d/(a*d-b*c)^3/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6/b/(a*d-b*c)/((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-
1/2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)-5/4/b*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b^2*(-a*b)^(1/2)/(
a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12/b*a*d/(a*d-b*c)^2/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75511, size = 2022, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2
*c^2*d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*
d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
+ (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
2*a*b*x^2 + a^2)) - 4*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2 + 4*a^2*c
*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5
 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3
*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^
2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d
 - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/12*(3*((2*b^2
*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2*c^2*d + 8*a*b*c*
d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*d^2)*x^2)*sqrt(-b
/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b
*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2
 + 4*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/
(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2
 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*
b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a
*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.18173, size = 351, normalized size = 2.06

$$\frac{\frac{3\sqrt{dx^2+c}abd^2}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\left(dx^2+c\right)b-bc+ad\right)}+\frac{3\left(2\,b^2cd+3\,abd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{2\left(3\left(dx^2+c\right)bcd+bc^2d+3\left(dx^2+c\right)ad^2-acd^2\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(dx^2+c\right)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*a*b*d^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 3*(2*b^2*c*d + 3*a*b*d^2)*arctan(sq
rt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*(3*(d*x^2 + c)*b*c*d + b*c^2*d +
3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*(d*x^2 + c)^(3/2)))/d
```

3.778 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5dx}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(a + b*x^2)
*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c - a*d)^3*Sqrt[c + d*
x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.160389, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5dx}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(a + b*x^2)
*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c - a*d)^3*Sqrt[c + d*
x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{c-4dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{c(3bc+2ad)-10bcdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{6c(bc-ad)^2}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}}$$

Mathematica [C] time = 2.03246, size = 211, normalized size = 1.29

$$\frac{x^3\left(16x^2\left(c+dx^2\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\{2,2,3\},\left\{1,\frac{11}{2}\right\},\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)+48x^2\left(2c^2+3cdx^2+d^2x^4\right)(bc-ad\right.}{945c^4\left(a+bx^2\right)^3\left(c+dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^3*(9*c*(a + b*x^2)*(35*c^2 + 28*c*d*x^2 + 8*d^2*x^4)*Hypergeometric2F1[1
, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 48*(b*c - a*d)*x^2*(2*c^2 +
3*c*d*x^2 + d^2*x^4)*Hypergeometric2F1[2, 3, 11/2, ((b*c - a*d)*x^2)/(c*(a
+ b*x^2))] + 16*(b*c - a*d)*x^2*(c + d*x^2)^2*HypergeometricPFQ[{2, 2, 3},
{1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(945*c^4*(a + b*x^2)^3*(c +
 d*x^2)^(3/2))
```

Maple [B] time = 0.014, size = 2369, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/4*d^2*a/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/12/b*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
3/2)+5/4*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))-1/4/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/4*d*(-a*b)^(1/2)/(a
*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/
(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2)))+1/4/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*d*(-a*b)^(1
/2)/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+5/4*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/(a
*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)-1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/12/(-a*b)^(1/2)/(a*d-b*c)/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)+1/12/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12/b*d/(a*d-b*c)/c/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*x+5/6/b*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*d^2*a/(a*d-b*c)^3/c/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
5/12/b*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6/b*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/(a*d
-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x*d-1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-5/12/b*d*(-a*b)^(1/2)/(a*d-b*c)^2/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)-5/12/b*d^2*a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/b*d^2*a/(a*d-b*c)^2/c^
2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x-5/12/b*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/b*d^2*a/(a*d-b*c)^
2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] time = 8.94826, size = 2588, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")

[Out] [1/24*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*d^3)*x^6 + (
2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 6*a*b^2*c^3
*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^5 + 2*(9*
a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*x^3 + 3*(a*b^3*c
^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^4*c^7 -
4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4 + (a*b^
5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4 - 4*a^4*b^2*c^2*d^5 + a^5
*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^2 + 8*a^3*b^3*c^4*d^3 - 2*
a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^4 + (a*b^5*c^7 - 2*a^2*b^4
*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 - 7*a^5*b*c^3*d^4 + 2*a^6*c^
2*d^5)*x^2), 1/12*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*
d^3)*x^6 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 +
 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(
a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^
2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3
- 2*a^3*b*d^4)*x^5 + 2*(9*a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 -
 a^4*d^4)*x^3 + 3*(a*b^3*c^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d
*x^2 + c))/(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4
*d^3 + a^6*c^3*d^4 + (a*b^5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4
 - 4*a^4*b^2*c^2*d^5 + a^5*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^
2 + 8*a^3*b^3*c^4*d^3 - 2*a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^
4 + (a*b^5*c^7 - 2*a^2*b^4*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 -
7*a^5*b*c^3*d^4 + 2*a^6*c^2*d^5)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 6.33806, size = 803, normalized size = 4.93

$$-\frac{\left(\frac{\left(5\,b^4c^4d^3-14\,ab^3c^3d^4+12\,a^2b^2c^2d^5-2\,a^3bcd^6-a^4d^7\right)x^2}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}+\frac{6\left(b^4c^5d^2-3\,ab^3c^4d^3+3\,a^2b^2c^3d^4-a^3bc^2d^5\right)}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((5*b^4*c^4*d^3 - 14*a*b^3*c^3*d^4 + 12*a^2*b^2*c^2*d^5 - 2*a^3*b*c*d^
6 - a^4*d^7)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3
*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 6*(b^4*c
^5*d^2 - 3*a*b^3*c^4*d^3 + 3*a^2*b^2*c^3*d^4 - a^3*b*c^2*d^5)/(b^6*c^7*d -
6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*
d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) - 1/2*(b^2*c*sqrt(d
) + 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*s
qrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^
2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*d + b*c^2))
```

3.779 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x^2)*(
c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (5*b^(3/2)*
d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.102821, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x^2)*(
c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (5*b^(3/2)*
d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{(5d)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx,x,x^2\right)}{4(bc-ad)}$$

$$= -\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{(5bd)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx}\right.}{4(bc-ad)^2}$$

$$= -\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{5bd}{2(bc-ad)^3\sqrt{c+dx^2}} - \frac{(5}{}$$

$$= -\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{5bd}{2(bc-ad)^3\sqrt{c+dx^2}} - \frac{(5}{}$$

$$= -\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{5bd}{2(bc-ad)^3\sqrt{c+dx^2}} + \frac{5b}{}$$

Mathematica [C] time = 0.0237449, size = 54, normalized size = 0.39

$$-\frac{d\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{3\left(c+dx^2\right)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(c + d*x^2))/(-(b*c) + a*d))])/(3
*(-(b*c) + a*d)^2*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.012, size = 1639, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12*d/(a*d-b
*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)-5/12*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6*(-a*b
```

```
)^(1/2)/b*d^2/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*b*d/(a*d-b*c)^3/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4
*(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/3*(-a*b)^(1/2)/a/b*d/(a
*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)*x+2/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4*
(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12*d/(a*d-b*c)^
2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(3/2)+5/12*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6*(-a*b)^(1
/2)/b*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*b*d/(a*d-b*c)^3/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*(-a
*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/3*(-a*b)^(1/2)/a/b*d/(a*d-b
*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)*x-2/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37066, size = 1816, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^
2*d + 2*a*b*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*
c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 14*a*b*c*
d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5 -
 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c
^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2
+ 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d -
```

```
 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), -1/12*(15*(b^2*d^
3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^2*d + 2*a*b*c*d^2)
*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(15*b^2*d^2*x^4 + 3*b^2*c^2 +
14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*
b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 -
3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3
*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^
3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.14011, size = 301, normalized size = 2.15

$$-\frac{1}{6}\left(\frac{15\,b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{3\,\sqrt{dx^2+c}\,b^2}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\left(dx^2+c\right)b-bc+ad\right)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 3*sqrt(d*x^2
+ c)*b^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x^2 + c)*
b - b*c + a*d)) + 2*(6*(d*x^2 + c)*b + b*c - a*d)/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^(3/2)))*d
```

3.780 $\int \frac{1}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=201

$$\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2ad}{6ac\left(c+dx^2\right)^{3}}$$

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 16*a*b*c*d - 4
*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (b^2*(b*c - 6*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d
)^(7/2))
```

Rubi [A] time = 0.221752, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2ad}{6ac\left(c+dx^2\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 16*a*b*c*d - 4
*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (b^2*(b*c - 6*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d
)^(7/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-bc+2ad-4bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3b^2c^2+12abcd-4a^2d^2-2bd\cdots}{\left(a+bx^2\right)\left(c+dx^2\right)^{3\cdots}}}{6ac(bc-ad)\cdots}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16abcd-4a^{2}\cdots\right.}{6ac^2(bc-ad)^3\sqrt{c+\cdots}}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16abcd-4a^{2}\cdots\right.}{6ac^2(bc-ad)^3\sqrt{c+\cdots}}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16abcd-4a^{2}\cdots\right.}{6ac^2(bc-ad)^3\sqrt{c+\cdots}}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16abcd-4a^{2}\cdots\right.}{6ac^2(bc-ad)^3\sqrt{c+\cdots}}$$

Mathematica [A] time = 5.37815, size = 170, normalized size = 0.85

$$\frac{1}{6}\left(\frac{3b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{7/2}} + x\sqrt{c+dx^2}\left(-\frac{3b^3}{a\left(a+bx^2\right)(ad-bc)^3} + \frac{4d^2(4bc-ad)}{c^2\left(c+dx^2\right)(bc-ad)^3} + \frac{2d^2}{c\left(c+dx^2\right)^2(bc-a\cdots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*((-3*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2)/(c*
(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^2*(4*b*c - a*d))/(c^2*(b*c - a*d)^3*(c
+ d*x^2))) + (3*b^2*(b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2]])])/(a^(3/2)*(b*c - a*d)^(7/2)))/6
```

Maple [B] time = 0.013, size = 2405, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a*b/(a*d-b*c)^2/c/((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x*d-1/4/a*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/12/(-a*b)^(1/2)/a/(a*d-b*c)*b/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)+1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/12/(-a*b)^(1/2)/a/
(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)+5/12*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/4/(-a*b)^(1/2)/a
*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))-5/4*b*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/a*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/2/
a*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b/a*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))-1/4/a*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/2/a*d/(a*d-b*c)/c^2/
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*x+5/4*b/a*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*b*d^2/(a*d-b*c
)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x-1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a/(a*d
-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/6*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/
12/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/6*d^2/(a*d-b*c)^2/c^2/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x+5/12*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/12/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)+5/4*b/a*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))-5/4*b/a*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] time = 11.2082, size = 2849, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^
6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4
*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*
((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - 4*((3*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2
*c*d^4 + 4*a^4*b*d^5)*x^5 + 2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^
2*c^2*d^3 - 7*a^4*b*c*d^4 + 2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d +
 6*a^3*b^2*c^3*d^2 - 8*a^4*b*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^
3*b^4*c^8 - 4*a^4*b^3*c^7*d + 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4
*d^4 + (a^2*b^5*c^6*d^2 - 4*a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2
*c^3*d^5 + a^6*b*c^2*d^6)*x^6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^
4*b^3*c^5*d^3 - 2*a^5*b^2*c^4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a
^2*b^5*c^8 - 2*a^3*b^4*c^7*d - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^
6*b*c^4*d^4 + 2*a^7*c^3*d^5)*x^2), 1/12*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (
b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^
2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*
a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2*c*d^4 + 4*a^4*b*d^5)*x^5 +
2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 - 7*a^4*b*c*d^4 +
2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - 8*a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^3*b^4*c^8 - 4*a^4*b^3*c^7*d
+ 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4*d^4 + (a^2*b^5*c^6*d^2 - 4*
a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2*c^3*d^5 + a^6*b*c^2*d^6)*x^
6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^4*b^3*c^5*d^3 - 2*a^5*b^2*c^
4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a^2*b^5*c^8 - 2*a^3*b^4*c^7*d
 - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^6*b*c^4*d^4 + 2*a^7*c^3*d^5)
*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 6.51488, size = 836, normalized size = 4.16

$$\frac{\left(\frac{2\left(4\,b^4c^4d^4-13\,ab^3c^3d^5+15\,a^2b^2c^2d^6-7\,a^3bcd^7+a^4d^8\right)x^2}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}+\frac{3\left(3\,b^4c^5d^3-10\,ab^3c^4d^4+12\,a^2b^2c^3d^5-6\,a^3bc^2d^6+a^4cd^7\right)}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(4*b^4*c^4*d^4 - 13*a*b^3*c^3*d^5 + 15*a^2*b^2*c^2*d^6 - 7*a^3*b*c*d
^7 + a^4*d^8)*x^2/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^
3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7) + 3*(3*
b^4*c^5*d^3 - 10*a*b^3*c^4*d^4 + 12*a^2*b^2*c^3*d^5 - 6*a^3*b*c^2*d^6 + a^4
*c*d^7)/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^5*
d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7))*x/(d*x^2 + c)^(3
/2) + 1/2*(b^3*c*sqrt(d) - 6*a*b^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^3*c^3 - 3*a^2
*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^3*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
b^2*d^(3/2) - b^3*c^2*sqrt(d))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^
2 - a^4*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

3.781 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*a*b*c*d - 2*a^2*d^2)
)/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.327889, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*a*b*c*d - 2*a^2*d^2)
)/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(7/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}(bc-ad)}{x(a+}\right.}{3ac(}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6abcd-\right.}{2ac^2(bc-ad)^3\sqrt{c}}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6abcd-\right.}{2ac^2(bc-ad)^3\sqrt{c}}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6abcd-\right.}{2ac^2(bc-ad)^3\sqrt{c}}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6abcd-\right.}{2ac^2(bc-ad)^3\sqrt{c}}$$

Mathematica [C] time = 0.094317, size = 114, normalized size = 0.51

$$\frac{-\frac{b(2bc-7ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(dx^2+c)}{bc-ad}\right)}{(bc-ad)^2}+\frac{3ab}{(a+bx^2)(bc-ad)}+\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)}{c}}{6a^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((3*a*b)/((b*c - a*d)*(a + b*x^2)) - (b*(2*b*c - 7*a*d)*Hypergeometric2F1[-
3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)])/(b*c - a*d)^2 + (2*Hypergeometr
ic2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/c)/(6*a^2*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.015, size = 2837, normalized size = 12.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -2/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+2/3/(-a*b)^(1/2)/a*d/(a*d
-b*c)*b/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x+1/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/2/
a^2*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/3/(-a*b)^(1/2)/a*d/(a*d-b*
c)*b/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)*x+1/6/a^2/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/a^2*b^2/(a*d-b*c)^2/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)+1/6/a^2/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/a^2*b^2/(a*d-b*c)^2/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/1
2/a*d/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*x+5/6/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/(-a*b)^(1/2)*d^
2*b^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/3/a^2/c/(d*x^2+c)^(3/2)+1/a^2/c^2/(d*x^2+
c)^(1/2)-1/a^2/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/4/(-a*b)^(1/
2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d-b*c)^3*b^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/
a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d-b*c)^3*b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4/a*d/(a*d-b*c)^3*
b^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
```

```
b*c)/b)^(1/2)+1/2/a^2*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))-1/3/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/
12/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/(-a*b)^(1/2)*d^2*b/(a*d
-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x+5/4/(-a*b)^(1/2)*d^2*b^2/(a*d-b*c)^3/c/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
1/3/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/6/a^2*d*(-a*b)^(1/2)/(a*
d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x-1/6/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/2/a^2
*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))+5/12/a*d/(a*d-b*c)^2*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4/a*d/(a*d-b*c)^3*b^2/((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a
^2*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [B] time = 47.739, size = 6770, normalized size = 30.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3)
*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c^
6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d
^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 12*(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(3*a*b^3*c^5 + 20*a^3*b*c^3*d^2 -
```

```
8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*d^4)*x^4 +
 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*c*d^4)*x^2
)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c
^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c
^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^
5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c
^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2), 1/24*(24*(a*b^3*c^5 - 3*a^2
*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3
 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^
2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2
*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + 3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a
*b^3*c^3*d^3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^
4 + (2*b^4*c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d
))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*
b^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c
^2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3
*b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^
7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
+ 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6
*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8
- a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2)
, -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^
3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*
c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 6*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b
^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^
2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7
*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 +
 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*
d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 -
 a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2),
 -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3
)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c
^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
b*c)) - 12*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(3*a*b^3*c^5 + 20*a^3*b*
c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*
d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*
c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d
^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4
 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^
5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3
*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.17279, size = 410, normalized size = 1.82

$$\frac{1}{6}\left(\frac{3\sqrt{dx^2+c}b^3}{\left(ab^3c^3d-3a^2b^2c^2d^2+3a^3bcd^3-a^4d^4\right)\left(\left(dx^2+c\right)b-bc+ad\right)}-\frac{3\left(2b^4c-7ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^3c^3d^2-3a^3b^2c^2d^3+3a^4bcd^4-a^5d^5\right)\sqrt{-b^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*b^3/((a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^
3 - a^4*d^4)*((d*x^2 + c)*b - b*c + a*d)) - 3*(2*b^4*c - 7*a*b^3*d)*arctan(
sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^3*c^3*d^2 - 3*a^3*b^2*c^2*d
^3 + 3*a^4*b*c*d^4 - a^5*d^5)*sqrt(-b^2*c + a*b*d)) + 2*(9*(d*x^2 + c)*b*c
+ b*c^2 - 3*(d*x^2 + c)*a*d - a*c*d)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^
3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^(3/2)) + 6*arctan(sqrt(d*x^2 + c)/sqrt(-c)
)/(a^2*sqrt(-c)*c^2*d^2))*d^2
```

3.782 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{\sqrt{c+dx^2}\left(40a^2bcd^2-16a^3d^3-18ab^2c^2d+9b^3c^3\right)}{6a^2c^3x(bc-ad)^3}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}(bc-ad)^3}-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b/(2*a*(b*c
 - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 20*a*b*c*d - 8*a
^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*x^2]) - ((9*b^3*c^3 - 18*a*b^2
*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*
d)^3*x) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.443202, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\left(40a^2bcd^2-16a^3d^3-18ab^2c^2d+9b^3c^3\right)}{6a^2c^3x(bc-ad)^3}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}(bc-ad)^3}-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b/(2*a*(b*c
 - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 20*a*b*c*d - 8*a
^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*x^2]) - ((9*b^3*c^3 - 18*a*b^2
*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*
d)^3*x) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}} \, dx = \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3bc+2ad-6bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}} \, dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-9b^2c^2+12abcd-}{x^2\left(a+b\right.}}{6ac(}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+20a\right.}{6ac^2(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+20a\right.}{6ac^2(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+20a\right.}{6ac^2(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+20a\right.}{6ac^2(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x \left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x \left(a+bx^2\right) \left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+20a\right.}{6ac^2(bc-ad)}$$

Mathematica [A] time = 5.53379, size = 188, normalized size = 0.67

$$\sqrt{c+dx^2}\left(\frac{b^4x}{2a^2\left(a+bx^2\right)(ad-bc)^3}-\frac{1}{a^2c^3x}+\frac{d^3x(5ad-11bc)}{3c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3x}{3c^2\left(c+dx^2\right)^2(bc-ad)^2}\right)-\frac{b^3(3bc-8ad)\tan^{-1}}{2a^{5/2}(bc-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*c^3*x)) + (b^4*x)/(2*a^2*(-(b*c) + a*d)^3*(a + b*
x^2)) - (d^3*x)/(3*c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-11*b*c + 5*a*d
)*x)/(3*c^3*(b*c - a*d)^3*(c + d*x^2))) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.018, size = 2513, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12/a^2*d/(a*d-b*c)*b/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^3*b^2
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)-3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-8/3/a^2*d/c^3*x/(d*x^2+c)^(1/2)+1/4*b^2/a
^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4*b^3/a^2/(-a*b)^(1/
2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+1/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
-1/a^2/c/x/(d*x^2+c)^(3/2)+3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-5/12/a^2*d*(-a*b)^(1/2)/(
a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+5/12/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4/
a^2*d*(-a*b)^(1/2)/(a*d-b*c)^3*b^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4/a^2*d*(-a*b)^(1/2)/(a*d-b*
c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/12*
b/a^2*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6*b/a^2*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*b
^2/a^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/12/a*d^2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5
```

```
/6/a*d^2*b/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/a*d^2*b^2/(a*d-b*c)^3/c/((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*x-5/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))+3/4*b^2/a^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/12/a
*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/a*d^2*b/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/
4/a*d^2*b^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/6/a^2*d/(a*d-b*c)*b/c^2/((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-4/3/a^2*d/c^2*x/(d*x^2+
c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [B] time = 10.4207, size = 3333, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*
c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*
c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(-a*b*c + a^2*d)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*a^
4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*a^
2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6 +
 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^
2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*d
- 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5)*
x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c^5
*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*b^
4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8*c
^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*b^
2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b^3
*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x), -1/12*(3*((
```

```
3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*c^4*d^2 - 8*
a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2)*x^3
 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a
*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2
)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*
a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*
a^2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6
 + 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*
c^2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*
d - 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5
)*x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c
^5*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*
b^4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8
*c^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*
b^2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b
^3*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 9.19209, size = 1266, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((11*b^4*c^6*d^5 - 38*a*b^3*c^5*d^6 + 48*a^2*b^2*c^4*d^7 - 26*a^3*b*c^
3*d^8 + 5*a^4*c^2*d^9)*x^2/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*
d^3 - 20*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d
^7) + 6*(2*b^4*c^7*d^4 - 7*a*b^3*c^6*d^5 + 9*a^2*b^2*c^5*d^6 - 5*a^3*b*c^4*
d^7 + a^4*c^3*d^8)/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*d^3 - 20
*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d^7))*x/(
d*x^2 + c)^(3/2) + 1/2*(3*b^4*c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2
*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b*c*d - a^2*d^
2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^4*c^3*sqrt(d) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*a*b^3*c^2*d^(3/2) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*b^2*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b*d^(7/2) - 6*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*b^4*c^4*sqrt(d) + 22*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a*b^3*c^3*d^(3/2) - 36*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^2*d^(5
/2) + 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(7/2) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a^4*d^(9/2) + 3*b^4*c^5*sqrt(d) - 6*a*b^3*c^4*d^(3/2) + 6
*a^2*b^2*c^3*d^(5/2) - 2*a^3*b*c^2*d^(7/2))/((a^2*b^3*c^5 - 3*a^3*b^2*c^4*d
 + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(s
qrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d +
```

```
 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*c*d - b*c^3))
```

3.783 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{2a^2c^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\,\text{ta}}{2a^3}$$

```
[Out] -(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(6*a^2*c^2*(b*c - a*d)^2*(c + d*x^
2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^
(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d)*(b^
2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (
(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7/2
)*(4*b*c - 9*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^
3*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.484342, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{2a^2c^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\,\text{ta}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```


```
[Out] -(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(6*a^2*c^2*(b*c - a*d)^2*(c + d*x^
2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^
(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d)*(b^
2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (
(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7/2
)*(4*b*c - 9*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^
3*(b*c - a*d)^(7/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+5ad)+\frac{7bdx}{2}}{x(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-}{\ldots}\right.}{\ldots}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)}$$

Mathematica [C] time = 0.11999, size = 190, normalized size = 0.62

$$\frac{b^2c^2x^2\left(a+bx^2\right)(4bc-9ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right) - (bc-ad)\left(x^2\left(a+bx^2\right)\left(-5a^2d^2+abcd+4b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}\right.\right.}{6a^3c^2x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 9*a*d)*x^2*(a + b*x^2)*Hypergeometric2F1[-3/2, 1, -1/2, (
b*(c + d*x^2))/(b*c - a*d)] - (b*c - a*d)*(-3*a*c*(a^2*d - 2*b^2*c*x^2 + a*
b*(-c + d*x^2)) + (4*b^2*c^2 + a*b*c*d - 5*a^2*d^2)*x^2*(a + b*x^2)*Hyperge
ometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c]))/(6*a^3*c^2*(b*c - a*d)^2*x^2*(a
+ b*x^2)*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.016, size = 2980, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/12*b^2/a/(-a*b)^(1
/2)*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c^
2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x+5/4*b^3/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/3*b^2/
a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+2/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-
b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*x-b^3/a^3/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))-5/12*b^2/a^2*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^2
*d/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)-b^3/a^3/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2/a^2/c/x^2/(d*x^2+c)^(3/2)-5/6/a^2*d/c^2/
(d*x^2+c)^(3/2)-5/2/a^2*d/c^3/(d*x^2+c)^(1/2)+5/2/a^2*d/c^(7/2)*ln((2*c+2*c
^(1/2)*(d*x^2+c)^(1/2))/x)-1/3*b^2/a^3/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+b^3/a^3/(a*d-b*c
)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)-2/3*b/a^3/c/(d*x^2+c)^(3/2)-2*b/a^3/c^2/(d*x^2+c)^(1/2)+2*b/a
^3/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/3*b^2/a^3/(a*d-b*c)/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)+b^3/a^3/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1
/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)+b^2/a^3/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
-5/12*b^2/a^2*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^2*d/(a*d-b*c)^3/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5
/4*b^3/a^2*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)-5/4*b^3/a^2*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))-b^2/a^3/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/3*b/a^3*d*
(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-2/3*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/c^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x+1/3*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/4*b^3/a/(-a
*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c
)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)*x-2/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/12*b
^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+2/3*b/a^3*d*(-a*b)^(1/2)/(a
*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [B] time = 69.8787, size = 8246, normalized size = 27.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c
^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5
*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(b*c - a*d))*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^
2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*((4*b^5*c^4*d^
2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x
^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*
c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b
^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2
*d^4)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*
(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a
*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6
+ (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*
d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^
2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*
d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b
^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*
d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d
^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2
- a^7*c^6*d^3)*x^2), -1/24*(12*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^
3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c
^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)
*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 +
17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a
^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - 3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*
c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*
d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*
```

```
(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*
x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*
c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^
4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*
c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a
^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*
c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c
^6*d^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*
d^2 - a^7*c^6*d^3)*x^2), 1/12*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (
8*b^5*c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^
4*c^6*d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sq
rt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 3*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*
a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a
*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^
5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*
d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(c)*log(-
(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a^2*b^3*c^6 - 9*a^3*b
^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a*b^4*c^4*d^2 - 3*a^2*b^3
*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6 + (12*a*b^4*c^5*d - 15*a
^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4
+ (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^2 + 41*a^4*b*c^3*d^3 - 2
0*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3
+ 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7
*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^6 + (a^3*b^4*c^9
- a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^4
+ (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^2), 1/1
2*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c^5*d
^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5*d^2
)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) - 6*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2
*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^
3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*
b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*
a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^
4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 - 2*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3
*(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)
*x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b
*c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c
^4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4
*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*
a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7
*c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*
c^6*d^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7
*d^2 - a^7*c^6*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.21261, size = 684, normalized size = 2.25

$$\frac{1}{6}d^3\left(\frac{3\left(4b^5c-9ab^4d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3b^3c^3d^3-3a^4b^2c^2d^4+3a^5bcd^5-a^6d^6\right)\sqrt{-b^2c+abd}}-\frac{3\left(2\left(dx^2+c\right)^{\frac{3}{2}}b^4c^3-2\sqrt{dx^2+c}b^4c^4-3\left(dx^2+c\right)^{\frac{3}{2}}ab^3c^2\right.}{\left(a^2b^3c^6d^2-3a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*d^3*(3*(4*b^5*c - 9*a*b^4*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/((a^3*b^3*c^3*d^3 - 3*a^4*b^2*c^2*d^4 + 3*a^5*b*c*d^5 - a^6*d^6)*sqrt(
-b^2*c + a*b*d)) - 3*(2*(d*x^2 + c)^(3/2)*b^4*c^3 - 2*sqrt(d*x^2 + c)*b^4*c
^4 - 3*(d*x^2 + c)^(3/2)*a*b^3*c^2*d + 4*sqrt(d*x^2 + c)*a*b^3*c^3*d + 3*(d
*x^2 + c)^(3/2)*a^2*b^2*c*d^2 - 6*sqrt(d*x^2 + c)*a^2*b^2*c^2*d^2 - (d*x^2
+ c)^(3/2)*a^3*b*d^3 + 4*sqrt(d*x^2 + c)*a^3*b*c*d^3 - sqrt(d*x^2 + c)*a^4*
d^4)/((a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)
*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) -
 2*(12*(d*x^2 + c)*b*c + b*c^2 - 6*(d*x^2 + c)*a*d - a*c*d)/((b^3*c^6 - 3*a
*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^(3/2)) - 3*(4*b*c +
 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^3*d^3))
```

3.784 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{\sqrt{c+dx^2}\left(-12a^2b^2c^2d^2+64a^3bcd^3-32a^4d^4-20ab^3c^3d+15b^4c^4\right)}{6a^3c^4x(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(32a^2bcd^2-16a^3d^3-6ab^2c^2d+5b^3c^3\right.}{6a^2c^3x^3(bc-ad)^3}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) + b/(2*a*(b
*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 8*a*b*c*d - 4*
a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c + d*x^2]) - ((5*b^3*c^3 - 6*a*b
^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c -
a*d)^3*x^3) + ((15*b^4*c^4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b
*c*d^3 - 32*a^4*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*
(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
7/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.596045, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(-12a^2b^2c^2d^2+64a^3bcd^3-32a^4d^4-20ab^3c^3d+15b^4c^4\right)}{6a^3c^4x(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(32a^2bcd^2-16a^3d^3-6ab^2c^2d+5b^3c^3\right.}{6a^2c^3x^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) + b/(2*a*(b
*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 8*a*b*c*d - 4*
a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c + d*x^2]) - ((5*b^3*c^3 - 6*a*b
^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c -
a*d)^3*x^3) + ((15*b^4*c^4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b
*c*d^3 - 32*a^4*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*
(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
7/2)*(b*c - a*d)^(7/2))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
```

```
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-5bc+2ad-8bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3\left(5b^2c^2-4ab\right.}{x^4(}}{6}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+8a\right.}{2ac^2(bc-ad}$$

Mathematica [A] time = 5.69681, size = 210, normalized size = 0.58

$$\frac{\sqrt{c+dx^2}\left(-\frac{3b^5x^4}{a^3\left(a+bx^2\right)(ad-bc)^3}+\frac{4x^2(4ad+3bc)}{a^3c^4}-\frac{2}{a^2c^3}+\frac{4d^4x^4(7bc-4ad)}{c^4\left(c+dx^2\right)(bc-ad)^3}+\frac{2d^4x^4}{c^3\left(c+dx^2\right)^2(bc-ad)^2}\right)}{6x^3} + \frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/(a^2*c^3) + (4*(3*b*c + 4*a*d)*x^2)/(a^3*c^4) - (3*b^5
*x^4)/(a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^4*x^4)/(c^3*(b*c - a*d)^2*(
c + d*x^2)^2) + (4*d^4*(7*b*c - 4*a*d)*x^4)/(c^4*(b*c - a*d)^3*(c + d*x^2))
))/(6*x^3) + (5*b^4*(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.018, size = 2623, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12*b^2/a^2*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/12*b^2/a^2*d^2/(a*d-b*c)^2/c/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)*x-5/4*b^3/a^2*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^3*d*(-a*b)^(
1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))+1/12*b^2/a^3*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6*b^2/a^3*d/(a*d-b*c)/c^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x-5/4*b^3/a^3/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+5/6*b^2/a^2*d^2/(a*d-b*c)^2/
c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*x+2/a^2*d/c^2/x/(d*x^2+c)^(3/2)+8/3/a^2*d^2/c^3*x/(d*x^2+c)^(
3/2)+1/12*b^2/a^3*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6*b^2/a^3*d/(a*d-b*c)/c^2/((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x+16/3/a^2*d^2/c^4*x/(d*x^2+c)^(1/2)-5/12*b^3/a^3/(-a*b)^(1/2)/(a*d-
b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b^2/a^3/(
a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+2*b/a^3/c/x/(d*x^2+c)^(3/2)+5/12*b^
3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/6*b^2/a^2
*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^2*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
5/4*b^3/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))-5/4*b^3/a^3/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/3/a^2/c/
x^3/(d*x^2+c)^(3/2)+5/4*b^3/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*b^
4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))-5/12*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^4/
a^3/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))+5/12*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^3/a^
3*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+8/3*b/a^3*d/c^2*x/(d*x^2+c)^(3/2)+16
/3*b/a^3*d/c^3*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [B] time = 9.49402, size = 3780, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c^
5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*b^5
*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a
*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 4*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2*c^5*d^2 - 8*a^6*
b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5*c^4*d^3 + 8*a^3*
b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a^6*b*d^7)*x^8 - 2
*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + 61*a^4*b^3*c^3*
d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^6 - 3*(5*a*b^6*c^
7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*d^3 + 26*a^5*b^2*
c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*b^5*c^7 - 14*a^3*
b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a
^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^5*b^4*c^7*d^3 + 6
*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9 + (2*a^4*b^5*c^9*
d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*d^4 - 2*a^8*b*c^5
*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d - 2*a^6*b^3*c^8*d
^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*x^5 + (a^5*b^4*c^
10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3 + a^9*c^6*d^4)*x
^3), 1/12*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5
*c^5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*
b^5*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c -
 a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2
*c^5*d^2 - 8*a^6*b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5
*c^4*d^3 + 8*a^3*b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a
^6*b*d^7)*x^8 - 2*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3
+ 61*a^4*b^3*c^3*d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^
6 - 3*(5*a*b^6*c^7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*
d^3 + 26*a^5*b^2*c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*
b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^
6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^
5*b^4*c^7*d^3 + 6*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9
+ (2*a^4*b^5*c^9*d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*
d^4 - 2*a^8*b*c^5*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d
- 2*a^6*b^3*c^8*d^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*
x^5 + (a^5*b^4*c^10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3
 + a^9*c^6*d^4)*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 20.5077, size = 1065, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(7*b^4*c^7*d^6 - 25*a*b^3*c^6*d^7 + 33*a^2*b^2*c^5*d^8 - 19*a^3*b*c^
4*d^9 + 4*a^4*c^3*d^10)*x^2/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^1
1*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^
7*d^7) + 3*(5*b^4*c^8*d^5 - 18*a*b^3*c^7*d^6 + 24*a^2*b^2*c^6*d^7 - 14*a^3*
b*c^5*d^8 + 3*a^4*c^4*d^9)/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^11
*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^7
*d^7))*x/(d*x^2 + c)^(3/2) - 5/2*(b^5*c*sqrt(d) - 2*a*b^4*d^(3/2))*arctan(1
/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2
))/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b*c*d
- a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^5*c*sqrt(d) - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b^4*d^(3/2) - b^5*c^2*sqrt(d))/((a^3*b^3*c^3 - 3*a^
4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b -
 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
a*d + b*c^2)) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c^2*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt
(d) + 4*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^3)
```

3.785 $\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=212

$$-\frac{2a^{7/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-5aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\sqrt{a+bx^2}(11Ab-5aB)}{231b^2}+2$$

```
[Out] (4*a*(11*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (2*(11*A*b -
 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2
)^(3/2))/(11*b*e) - (2*a^(7/4)*(11*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.159939, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 220}

$$-\frac{2a^{7/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\sqrt{a+bx^2}(11Ab-5aB)}{231b^2}+\frac{2(ex)^{5/2}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (4*a*(11*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (2*(11*A*b -
 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2
)^(3/2))/(11*b*e) - (2*a^(7/4)*(11*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be} - \frac{\left(2\left(-\frac{11Ab}{2}+\frac{5aB}{2}\right)\right)\int (ex)^{3/2}\sqrt{a+bx^2}\,dx}{11b}$$

$$= \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be} + \frac{(2a(11Ab-5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^2}}$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right.}{11be}$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right.}{11be}$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right.}{11be}$$

Mathematica [C] time = 0.142285, size = 110, normalized size = 0.52

$$\frac{2e\sqrt{ex}\sqrt{a+bx^2}\left(a(5aB-11Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\left(5aB-11Ab-7bBx^2\right)\right)}{77b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(-11*A*b
+ 5*a*B - 7*b*B*x^2)) + a*(-11*A*b + 5*a*B)*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.038, size = 276, normalized size = 1.3

$$-\frac{2e}{231\,xb^3}\sqrt{ex}\left(-21\,Bx^7b^4+11\,A\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-ab}a^2b-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] -2/231*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-21*B*x^7*b^4+11*A*2^(1/2)*((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-a*b)^(1/2)*a^2*b-33*A*x^5*b^4-5*B*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-a*b)^(1/2)*a^3-27*B*x^5*a*b^3-55*A*x^3*a*b^3+4*B*x^3*a^2*b^2-22*A*x*a^2*
b^2+10*B*x*a^3*b)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bex^3 + Aex\right)\sqrt{bx^2 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Sympy [C] time = 25.69, size = 97, normalized size = 0.46

$$\frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(9/4)) + B*sqrt(a)*e**(3/2)*x**(9/2)*gamma(9/4)*hy
per((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

3.786 $\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=337

$$\frac{2a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-}{15b^{7/4}\sqrt{a+bx}}$$

```
[Out] (2*(3*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*b*e) + (4*a*(3*A*b - a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) - (4*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]
) + (2*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.269853, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\right.}{15b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*(3*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*b*e) + (4*a*(3*A*b - a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) - (4*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]
) + (2*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{3aB}{2}\right)\right)\int \sqrt{ex}\sqrt{a+bx^2}\,dx}{9b}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{(2a(3Ab-aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{15b}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{(4a(3Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{b\ldots}{e}}}\ldots\right)}{15be}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{\left(4a^{3/2}(3Ab-aB)\right)\text{Subst}\left(\int \ldots\right)}{15b^{3/2}}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{4a(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} - \ldots$$

Mathematica [C] time = 0.0873176, size = 93, normalized size = 0.28

$$\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left((3Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+B\sqrt{\frac{bx^2}{a}+1}\left(a+bx^2\right)\right)}{9b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (3*A*b
- a*B)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x
^2)/a])
```

Maple [A] time = 0.029, size = 414, normalized size = 1.2

$$\frac{2}{45\,b^{2}x}\sqrt{ex}\left(5\,Bx^{6}b^{3}+18\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)a^{2}b-9\,A\sqrt{\frac{b}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] 2/45*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(5*B*x^6*b^3+18*A*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*b-9*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*a^3+3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+9*A*x^4*b^3+7*B*x^4*a*b^2+
9*A*x^2*a*b^2+2*B*x^2*a^2*b)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^{2}+A\right)\sqrt{bx^{2}+a}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bx^{2}+A\right)\sqrt{bx^{2}+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Sympy [C] time = 3.38528, size = 95, normalized size = 0.28

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{a}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_pol
ar(I*pi)/a)/(2*e*gamma(7/4)) + B*sqrt(a)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/
2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**3*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

3.787 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=176

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{21be}+\frac{2B\sqrt{ex}\left(a+b\right.}{7be}$$

```
[Out] (2*(7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b*e) + (2*B*Sqrt[e*x]*(a +
b*x^2)^(3/2))/(7*b*e) + (2*a^(3/4)*(7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.110792, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 220}

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{21be}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*(7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b*e) + (2*B*Sqrt[e*x]*(a +
b*x^2)^(3/2))/(7*b*e) + (2*a^(3/4)*(7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{\sqrt{ex}}dx = \frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{aB}{2}\right)\right)\int\frac{\sqrt{a+bx^2}}{\sqrt{ex}}dx}{7b}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be} + \frac{(2a(7Ab-aB))\int\frac{1}{\sqrt{ex}\sqrt{a+bx^2}}dx}{21b}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be} + \frac{(4a(7Ab-aB))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}dx, x,\right.}{21be}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be} + \frac{2a^{3/4}(7Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)}}}{21b^{5/4}\sqrt{e}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0604962, size = 93, normalized size = 0.53

$$\frac{2x\sqrt{a+bx^2}\left((7Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+B\sqrt{\frac{bx^2}{a}+1}\left(a+bx^2\right)\right)}{7b\sqrt{ex}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (7*A*b - a*B)*Hyp
ergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[e*x]*Sqrt[1 + (b*x
^2)/a])
```

Maple [A] time = 0.023, size = 246, normalized size = 1.4

$$\frac{2}{21\,b^2}\left(7\,A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-B\sqrt{-ab}\sqrt{\left(bx+\sqrt{-ab}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)*(7*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-B*(-
a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*x^5*b^3+7*A*x^3*b^3+5*B*x^3*
a*b^2+7*A*x*a*b^2+2*B*x*a^2*b)/(e*x)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] time = 3.42848, size = 97, normalized size = 0.55

$$\frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(e)*gamma(5/4)) + B*sqrt(a)*x**(5/2)*gamma(5/4)*hyper((-1/2,
5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

3.788 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)E\left(2\right.}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (2*(5*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^3) + (4*(5*A*b + a*B)*
Sqrt[e*x]*Sqrt[a + b*x^2])/(5*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
+ b*x^2)^(3/2))/(a*e*Sqrt[e*x]) - (4*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[
b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)
*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) +
 (2*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.261273, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)E\left(2\tan^{-1}\left(\right.\right.}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```


```
[Out] (2*(5*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^3) + (4*(5*A*b + a*B)*
Sqrt[e*x]*Sqrt[a + b*x^2])/(5*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
+ b*x^2)^(3/2))/(a*e*Sqrt[e*x]) - (4*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[
b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)
*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) +
 (2*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}+\frac{(5Ab+aB)\int \sqrt{ex}\sqrt{a+bx^2}\,dx}{ae^2}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}+\frac{(2(5Ab+aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5e^2}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}+\frac{(4(5Ab+aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\right.}{5e^3}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}+\frac{\left(4\sqrt{a}(5Ab+aB)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx\right.}{5\sqrt{b}e^2}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3}+\frac{4(5Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{5\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}-\frac{4\sqrt[4]{a}(5A}{}$$

Mathematica [C] time = 0.0511436, size = 96, normalized size = 0.29

$$\frac{2x\sqrt{a+bx^2}\left(x^2(aB+5Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-3A\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\right)}{3a(ex)^{3/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (5*A*b + a*B)*
x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sqrt
[1 + (b*x^2)/a])
```

Maple [A] time = 0.034, size = 391, normalized size = 1.2

$$\frac{2}{5\,be}\left(10\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)ab-5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x)
```

```
[Out] 2/5*(10*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
*b+2*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b^
2*B*x^4-5*A*x^2*b^2+B*x^2*a*b-5*A*a*b)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2+A)\sqrt{bx^2+a}\sqrt{ex}}{e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] time = 3.7647, size = 100, normalized size = 0.3

$$\frac{A\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{B\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] A*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*x**(3/2)*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^{2}+A\right)\sqrt{bx^{2}+a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

3.789 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{3ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

```
[Out] (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*a*e^3) - (2*A*(a + b*x^2)^(3/2
))/(3*a*e*(e*x)^(3/2)) + (2*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4
)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.11044, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 220}

$$\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{3ae^3}+\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*a*e^3) - (2*A*(a + b*x^2)^(3/2
))/(3*a*e*(e*x)^(3/2)) + (2*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4
)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(Ab+aB)\int \frac{\sqrt{a+bx^2}}{\sqrt{ex}}\,dx}{ae^2}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{3ae^3} - \frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2(Ab+aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{3e^2}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{3ae^3} - \frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(4(Ab+aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{3e^3}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{3ae^3} - \frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\right.}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0741247, size = 82, normalized size = 0.48

$$\frac{2x\sqrt{a+bx^2}\left(\frac{3x^2(aB+Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}} - A\left(a+bx^2\right)\right)}{3a(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-(A*(a + b*x^2)) + (3*(A*b + a*B)*x^2*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*a*(e*x)^(5/2))
```

Maple [A] time = 0.026, size = 234, normalized size = 1.4

$$\frac{2}{3\,xe^2b}\left(A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-abx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*b+B*((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a+b^2*B*x^4-A*x^2*b^2+B*x^2*a*b-A*a
*b)/e^2/(e*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^3*x^3), x)
```

Sympy [C] time = 13.6463, size = 100, normalized size = 0.58

$$\frac{A\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{B\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] A*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/
2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

3.790 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=338

$$\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)E}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*(A*b + 5*a*B)*Sqrt[a + b*x^2])/(5*a*e^3*Sqrt[e*x]) + (4*Sqrt[b]*(A*b +
5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
 + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) - (4*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^
(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^
2]) + (2*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.26434, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 277, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)E\left(2\tan^{-1}\right.}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```


```
[Out] (-2*(A*b + 5*a*B)*Sqrt[a + b*x^2])/(5*a*e^3*Sqrt[e*x]) + (4*Sqrt[b]*(A*b +
5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
 + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) - (4*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^
(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^
2]) + (2*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{7/2}}\,dx &= -\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(Ab+5aB)\int \frac{\sqrt{a+bx^2}}{(ex)^{3/2}}\,dx}{5ae^2}\\
&= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(2b(Ab+5aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5ae^4}\\
&= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(4b(Ab+5aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5ae^5}\\
&= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{\left(4\sqrt{b}(Ab+5aB)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right.}{5\sqrt{a}e^4}\\
&= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}}+\frac{4\sqrt{b}(Ab+5aB)\sqrt{ex}\sqrt{a+bx^2}}{5ae^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}-\frac{4\sqrt[4]{b}(Ab+5a}{}
\end{aligned}
$$

Mathematica [C] time = 0.0593729, size = 95, normalized size = 0.28

$$
-\frac{2x\sqrt{a+bx^2}\left(x^2(5aB+Ab)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)+A\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\right)}{5a(ex)^{7/2}\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*(A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (A*b + 5*a*B)*x^
2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[
1 + (b*x^2)/a])
```

Maple [A] time = 0.032, size = 417, normalized size = 1.2

$$\frac{2}{5x^2e^3a}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-A*((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a*b+10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-5*B*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*a^2-2*A*b^2*x^4-5*B*x^4*a*b-3*a*A*b*x^2-5*B*x^2*a^2-A*a^2)/(b
*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [C] time = 109.453, size = 107, normalized size = 0.32

$$\frac{A\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] A*sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1
/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

3.791 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}} dx$

Optimal. Leaf size=152

$$-\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(Ab-7aB)}{21ax^{3/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

```
[Out] (2*(A*b - 7*a*B)*Sqrt[a + b*x^2])/(21*a*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/
(7*a*x^(7/2)) - (2*b^(3/4)*(A*b - 7*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]
, 1/2])/(21*a^(5/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0930551, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 277, 329, 220}

$$-\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(Ab-7aB)}{21ax^{3/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```


```
[Out] (2*(A*b - 7*a*B)*Sqrt[a + b*x^2])/(21*a*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/
(7*a*x^(7/2)) - (2*b^(3/4)*(A*b - 7*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]
, 1/2])/(21*a^(5/4)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{9/2}}\,dx = -\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{7aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{x^{5/2}}\,dx}{7a}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}} - \frac{(2b(Ab-7aB))\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21a}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}} - \frac{(4b(Ab-7aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{21a}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}} - \frac{2b^{3/4}(Ab-7aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\,\mathrm{ta}\right.}{21a^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0942077, size = 79, normalized size = 0.52

$$\frac{2\sqrt{a+bx^2}\left(\frac{x^2(Ab-7aB)\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}-3A\left(a+bx^2\right)\right)}{21ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2) + ((A*b - 7*a*B)*x^2*Hypergeometric2F1
[-3/4, -1/2, 1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(21*a*x^(7/2))
```

Maple [A] time = 0.038, size = 242, normalized size = 1.6

$$-\frac{2}{21\,a}\left(A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/21/(b*x^2+a)^(1/2)*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^3*b-7*B
*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^3*a+2*A*b^2*x^4+7*B*x^4*a*b+5*a
*A*b*x^2+7*B*x^2*a^2+3*A*a^2)/x^(7/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Sympy [C] time = 139.503, size = 97, normalized size = 0.64

$$\frac{A\sqrt{a}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4}, -\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(9/2),x)
```

```
[Out] A*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*x**(7/2)*gamma(-3/4)) + B*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**2*exp_polar(I*pi)/a)/(2*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

3.792 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}-\frac{4b^{3/2}\sqrt{x}\sqrt{a+bx^2}(Ab-3aB)}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4b^{5/4}\left(\sqrt{a}+\right.}{}$$

```
[Out] (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/(15*a*x^(5/2)) + (4*b*(A*b - 3*a*B)*Sqrt[
a + b*x^2])/(15*a^2*Sqrt[x]) - (4*b^(3/2)*(A*b - 3*a*B)*Sqrt[x]*Sqrt[a + b*
x^2])/(15*a^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(9*a*x^(9/2)
) + (4*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*
a^(7/4)*Sqrt[a + b*x^2]) - (2*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.233339, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {453, 277, 325, 329, 305, 220, 1196}

$$-\frac{4b^{3/2}\sqrt{x}\sqrt{a+bx^2}(Ab-3aB)}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}+\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\left.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

```
[Out] (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/(15*a*x^(5/2)) + (4*b*(A*b - 3*a*B)*Sqrt[
a + b*x^2])/(15*a^2*Sqrt[x]) - (4*b^(3/2)*(A*b - 3*a*B)*Sqrt[x]*Sqrt[a + b*
x^2])/(15*a^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(9*a*x^(9/2)
) + (4*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*
a^(7/4)*Sqrt[a + b*x^2]) - (2*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11/2}}\,dx = -\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{9aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{x^{7/2}}dx}{9a}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}} - \frac{(2b(Ab-3aB))\int \frac{1}{x^{3/2}\sqrt{a+bx^2}}dx}{15a}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}} - \frac{\left(2b^2(Ab-3aB)\right)\int \frac{}{\sqrt{a}}}{15a^2}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}} - \frac{\left(4b^2(Ab-3aB)\right)\text{Sub}}{15}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}} - \frac{\left(4b^{3/2}(Ab-3aB)\right)\text{Su}}{15}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{4b^{3/2}(Ab-3aB)\sqrt{x}\sqrt{a+bx^2}}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A\left(a+\right.}{9ax}$$

Mathematica [C] time = 0.0944826, size = 80, normalized size = 0.24

$$\frac{2\sqrt{a+bx^2}\left(\frac{3x^2(Ab-3aB)\,_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}-5A\left(a+bx^2\right)\right)}{45ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-5*A*(a + b*x^2) + (3*(A*b - 3*a*B)*x^2*Hypergeometric2
F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(45*a*x^(9/2))
```

Maple [A] time = 0.041, size = 439, normalized size = 1.3

$$-\frac{2}{45\,a^2}\left(6\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^4ab^2-3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x)
```

```
[Out] -2/45*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^4*a*b^2-18*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*b+9*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
```

```
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*x^4*a^2*b-6*A*x^6*b^3+18*B*x^6*a*b^2-4*A*x^4*a*b^2+27*B*x^4
*a^2*b+7*A*x^2*a^2*b+9*B*x^2*a^3+5*A*a^3)/(b*x^2+a)^(1/2)/x^(9/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

3.793 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-11aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}+\frac{4b\sqrt{a+bx^2}(5Ab-11aB)}{231a^2x^{3/2}}+\frac{2\sqrt{a+bx^2}(5Ab-11aB)}{77ax^{7/2}}$$

```
[Out] (2*(5*A*b - 11*a*B)*Sqrt[a + b*x^2])/(77*a*x^(7/2)) + (4*b*(5*A*b - 11*a*B)
*Sqrt[a + b*x^2])/(231*a^2*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/(11*a*x^(11/2
)) + (2*b^(7/4)*(5*A*b - 11*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(231*a^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.112352, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {453, 277, 325, 329, 220}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-11aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}+\frac{4b\sqrt{a+bx^2}(5Ab-11aB)}{231a^2x^{3/2}}+\frac{2\sqrt{a+bx^2}(5Ab-11aB)}{77ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] (2*(5*A*b - 11*a*B)*Sqrt[a + b*x^2])/(77*a*x^(7/2)) + (4*b*(5*A*b - 11*a*B)
*Sqrt[a + b*x^2])/(231*a^2*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/(11*a*x^(11/2
)) + (2*b^(7/4)*(5*A*b - 11*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(231*a^(9/4)*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{13/2}}\,dx = -\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{11aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{x^{9/2}}\,dx}{11a}$$

$$= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}} - \frac{(2b(5Ab-11aB))\int \frac{1}{x^{5/2}\sqrt{a+bx^2}}\,dx}{77a}$$

$$= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}} + \frac{\left(2b^2(5Ab-1\right.}{}$$

$$= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}} + \frac{\left(4b^2(5Ab-1\right.}{}$$

$$= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}} + \frac{2b^{7/4}(5Ab-1}{}$$

Mathematica [C] time = 0.100848, size = 80, normalized size = 0.43

$$\frac{2\sqrt{a+bx^2}\left(\frac{x^2(5Ab-11aB)\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}-7A\left(a+bx^2\right)\right)}{77ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-7*A*(a + b*x^2) + ((5*A*b - 11*a*B)*x^2*Hypergeometric
2F1[-7/4, -1/2, -3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(77*a*x^(11/2))
```

Maple [A] time = 0.028, size = 270, normalized size = 1.4

$$\frac{2}{231\,a^2}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^5b^2-11\,B\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x)
```

```
[Out] 2/231/(b*x^2+a)^(1/2)*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^5*b^2
-11*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^5*a*b+10*A*x^6*b^3-22*B*x^
6*a*b^2+4*A*x^4*a*b^2-55*B*x^4*a^2*b-27*A*x^2*a^2*b-33*B*x^2*a^3-21*A*a^3)/
x^(11/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

3.794 $\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=252

$$-\frac{4a^{11/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{231b^2}+\frac{4a(ex)^{5}}{}$$

```
[Out] (8*a^2*(3*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*(3*A*b -
 a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*(3*A*b - a*B)*(e*x)^(5/2)*
(a + b*x^2)^(3/2))/(33*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2)^(5/2))/(15*b*e)
- (4*a^(11/4)*(3*A*b - a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.166786, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 220}

$$-\frac{4a^{11/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{231b^2}+\frac{4a(ex)^{5/2}\sqrt{a+b}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (8*a^2*(3*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*(3*A*b -
 a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*(3*A*b - a*B)*(e*x)^(5/2)*
(a + b*x^2)^(3/2))/(33*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2)^(5/2))/(15*b*e)
- (4*a^(11/4)*(3*A*b - a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx = \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be} - \frac{\left(2\left(-\frac{15Ab}{2}+\frac{5aB}{2}\right)\right)\int (ex)^{3/2}\left(a+bx^2\right)^{3/2}dx}{15b}$$

$$= \frac{2(3Ab-aB)(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{33be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be} + \frac{(2a(3Ab-aB))\int (ex}{11b}$$

$$= \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{33be} + \frac{2B(ex)^{5/2}(a}{15b}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(e}{3}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(e}{3}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(e}{3}$$

Mathematica [C] time = 0.159391, size = 114, normalized size = 0.45

$$\frac{2e\sqrt{ex}\sqrt{a+bx^2}\left(5a^2(aB-3Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}\left(5aB-15Ab-11bBx^2\right)\right)}{165b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*(-15*A*
b + 5*a*B - 11*b*B*x^2)) + 5*a^2*(-3*A*b + a*B)*Hypergeometric2F1[-3/2, 1/4
, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.023, size = 300, normalized size = 1.2

$$-\frac{2e}{1155\,xb^{3}}\sqrt{ex}\left(-77\,b^{5}Bx^{9}-105\,Ax^{7}b^{5}-196\,Bx^{7}ab^{4}+30\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\cdots}{\cdots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-77*b^5*B*x^9-105*A*x^7*b^5-196*B*
x^7*a*b^4+30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^3*b-300*A*x^5*a*b
^4-10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^4-131*B*x^5*a^2*b^3-255*
A*x^3*a^2*b^3+8*B*x^3*a^3*b^2-60*a^3*b^2*A*x+20*a^4*b*B*x)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^{2}+A\right)\left(bx^{2}+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bbex^{5}+(Ba+Ab)ex^{3}+Aaex\right)\sqrt{bx^{2}+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e*x^5 + (B*a + A*b)*e*x^3 + A*a*e*x)*sqrt(b*x^2 + a)*sqrt(e*x
), x)
```

Sympy [C] time = 84.8159, size = 199, normalized size = 0.79

$$\frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{A\sqrt{a}be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{B\sqrt{a}be^{\frac{3}{2}}x^{\frac{13}{2}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(9/4)) + A*sqrt(a)*b*e**(3/2)*x**(9/2)*gamma(9/4)
*hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4)) + B*
a**(3/2)*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4)) + B*sqrt(a)*b*e**(3/2)*x**(13/2)*gamma(13/
4)*hyper((-1/2, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(17/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\left(bx^2 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

3.795 $\int \sqrt{ex} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=377

$$\frac{4a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{ex}\sqrt{a+bx^2}(13Ab-3aB)}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{8a^{9/4}\sqrt{}}{}$$

```
[Out] (4*a*(13*A*b - 3*a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(195*b*e) + (8*a^2*(13*A
*b - 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(195*b^(3/2)*(Sqrt[a] + Sqrt[b]*x))
+ (2*(13*A*b - 3*a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(117*b*e) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(5/2))/(13*b*e) - (8*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*A
rcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b
*x^2]) + (4*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.289959, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 305, 220, 1196}

$$\frac{8a^2\sqrt{ex}\sqrt{a+bx^2}(13Ab-3aB)}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{e}\left(\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```


```
[Out] (4*a*(13*A*b - 3*a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(195*b*e) + (8*a^2*(13*A
*b - 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(195*b^(3/2)*(Sqrt[a] + Sqrt[b]*x))
+ (2*(13*A*b - 3*a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(117*b*e) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(5/2))/(13*b*e) - (8*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*A
rcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b
*x^2]) + (4*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^2])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx = \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be} - \frac{\left(2\left(-\frac{13Ab}{2}+\frac{3aB}{2}\right)\right)\int \sqrt{ex}\left(a+bx^2\right)^{3/2}dx}{13b}$$

$$= \frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be} + \frac{(2a(13Ab-3aB))\int}{39b}$$

$$= \frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be} + \frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be} + \frac{2B(ex)^{3/2}}{1}$$

$$= \frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be} + \frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be} + \frac{2B(ex)^{3/2}}{1}$$

$$= \frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be} + \frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be} + \frac{2B(ex)^{3/2}}{1}$$

$$= \frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be} + \frac{8a^2(13Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2(13Ab-3a}{}$$

Mathematica [C] time = 0.108196, size = 97, normalized size = 0.26

$$\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left(a(13Ab-3aB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+3B\sqrt{\frac{bx^2}{a}+1}\left(a+bx^2\right)^2\right)}{39b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(3*B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*(
13*A*b - 3*a*B)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(39*b*Sqr
t[1 + (b*x^2)/a])
```

Maple [A] time = 0.023, size = 438, normalized size = 1.2

$$\frac{2}{585\,b^2x}\sqrt{ex}\left(45\,Bx^8b^4+65\,Ax^6b^4+120\,Bx^6ab^3+156\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{}}{\sqrt{-a}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x)
```

```
[Out] 2/585/(b*x^2+a)^(1/2)*(e*x)^(1/2)/b^2*(45*B*x^8*b^4+65*A*x^6*b^4+120*B*x^6*
a*b^3+156*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b-78*A*((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*a^3*b-36*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4+18*B*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a^4+208*A*x^4*a*b^3+87*B*x^4*a^2*b^2+143*A*x^2*a^2*b^2+12*B*x^2*a^3*b)
/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Sympy [C] time = 12.5934, size = 197, normalized size = 0.52

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{A\sqrt{a}b(ex)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}+\frac{Ba^{\frac{3}{2}}(ex)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}+\frac{B\sqrt{a}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_po
lar(I*pi)/a)/(2*e*gamma(7/4)) + A*sqrt(a)*b*(e*x)**(7/2)*gamma(7/4)*hyper((
-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**3*gamma(11/4)) + B*a**
(3/2)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*e**3*gamma(11/4)) + B*sqrt(a)*b*(e*x)**(11/2)*gamma(11/4)*hyper
((-1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**5*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

3.796 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=214

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(11Ab-aB)}{77be}+\frac{4a\sqrt{ex}\sqrt{a}\cdots}{}$$

```
[Out] (4*a*(11*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(77*b*e) + (2*(11*A*b - a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(77*b*e) + (2*B*Sqrt[e*x]*(a + b*x^2)^(5/2))/
(11*b*e) + (4*a^(7/4)*(11*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(77*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.136428, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 220}

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(11Ab-aB)}{77be}+\frac{4a\sqrt{ex}\sqrt{a+bx^2}(11\cdots}{77be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```


```
[Out] (4*a*(11*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(77*b*e) + (2*(11*A*b - a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(77*b*e) + (2*B*Sqrt[e*x]*(a + b*x^2)^(5/2))/
(11*b*e) + (4*a^(7/4)*(11*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(77*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{\sqrt{ex}}\,dx = \frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be} - \frac{\left(2\left(-\frac{11Ab}{2}+\frac{aB}{2}\right)\right)\int\frac{\left(a+bx^2\right)^{3/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{2(11Ab-aB)\sqrt{ex}\left(a+bx^2\right)^{3/2}}{77be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be} + \frac{(6a(11Ab-aB))\int\frac{\sqrt{a+bx^2}}{\sqrt{ex}}\,dx}{77b}$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\left(a+bx^2\right)^{3/2}}{77be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be} +$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\left(a+bx^2\right)^{3/2}}{77be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be} +$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\left(a+bx^2\right)^{3/2}}{77be} + \frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be} +$$

Mathematica [C] time = 0.0754132, size = 96, normalized size = 0.45

$$\frac{2x\sqrt{a+bx^2}\left(a(11Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+B\sqrt{\frac{bx^2}{a}+1}\left(a+bx^2\right)^2\right)}{11b\sqrt{ex}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*(11*A*b - a*B
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[e*x]*Sqrt[1
+ (b*x^2)/a])
```

Maple [A] time = 0.013, size = 272, normalized size = 1.3

$$\frac{2}{77\,b^2}\left(7\,Bx^7b^4+22\,A\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-ab}a^2b+11\,Ax^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x)
```

```
[Out] 2/77/(b*x^2+a)^(1/2)*(7*B*x^7*b^4+22*A*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)
```

```
^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-a*b)^(
1/2)*a^2*b+11*A*x^5*b^4-2*B*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)*a^3+20
*B*x^5*a*b^3+44*A*x^3*a*b^3+17*B*x^3*a^2*b^2+33*A*x*a^2*b^2+4*B*x*a^3*b)/b^
2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bbx^4 + (Ba + Ab)x^2 + Aa\right)\sqrt{bx^2 + a}\sqrt{ex}}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e*x)/(e*x),
 x)
```

Sympy [C] time = 12.2051, size = 199, normalized size = 0.93

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1}{2\sqrt{e}\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(e)*gamma(5/4)) + A*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-1/
2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4)) + B*a**(3
/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a
)/(2*sqrt(e)*gamma(9/4)) + B*sqrt(a)*b*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4
), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

3.797 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{4a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{8a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)E\left(2\right.}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (4*(9*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*e^3) + (8*a*(9*A*b + a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) + (2*(9*
A*b + a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*a*e^3) - (2*A*(a + b*x^2)^(5/2
))/(a*e*Sqrt[e*x]) - (8*a^(5/4)*(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (4*a^(5/4)*
(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(
3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.294509, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 305, 220, 1196}

$$\frac{4a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{8a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)E\left(2\tan^{-1}\left(\right.\right.}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```


```
[Out] (4*(9*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*e^3) + (8*a*(9*A*b + a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) + (2*(9*
A*b + a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*a*e^3) - (2*A*(a + b*x^2)^(5/2
))/(a*e*Sqrt[e*x]) - (8*a^(5/4)*(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (4*a^(5/4)*
(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(
3/4)*e^(3/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx=-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}+\frac{(9Ab+aB)\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\,dx}{ae^2}$$

$$=\frac{2(9Ab+aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9ae^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}+\frac{(2(9Ab+aB))\int \sqrt{ex}\sqrt{a+bx^2}\,d}{3e^2}$$

$$=\frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}+\frac{2(9Ab+aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9ae^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}+$$

$$=\frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}+\frac{2(9Ab+aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9ae^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}+$$

$$=\frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}+\frac{2(9Ab+aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9ae^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}+$$

$$=\frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}+\frac{8a(9Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{15\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(9Ab+aB)(ex)^{3/2}\left(a+\right.}{9ae^3}$$

Mathematica [C] time = 0.0720828, size = 84, normalized size = 0.23

$$\frac{2x\sqrt{a+bx^2}\left(\frac{x^2(aB+9Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}-\frac{3A\left(a+bx^2\right)^2}{a}\right)}{3(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*((-3*A*(a + b*x^2)^2)/a + ((9*A*b + a*B)*x^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^(3/2
))
```

Maple [A] time = 0.017, size = 421, normalized size = 1.2

$$\frac{2}{45\,be}\left(5\,Bx^6b^3+108\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2b-54\,A\sqrt{\frac{bx+\sqrt{}}{\sqrt{-a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x)
```

```
[Out] 2/45*(5*B*x^6*b^3+108*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-54*A*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*b+12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
E(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-6*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a^3+9*A*x^4*b^3+16*B*x^4*a*b^2-36*A*x^2*a*b^2+11*B*x^2*a^2*
b-45*A*a^2*b)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}\sqrt{ex}}{e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^2*x
^2), x)
```

Sympy [C] time = 14.8636, size = 202, normalized size = 0.55

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{A\sqrt{a}bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2}{2e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + A*sqrt(a)*b*x**(3/2)*gamma(3/4)*hyper((
-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4)) + B*a
**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*e**(3/2)*gamma(7/4)) + B*sqrt(a)*b*x**(7/2)*gamma(7/4)*hyper((-1/2
, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

3.798 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=210

$$\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3aB+7Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(3aB+7Ab)}{21ae^3}+\frac{4\sqrt{ex}\sqrt{a}}{}$$

```
[Out] (4*(7*A*b + 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*e^3) + (2*(7*A*b + 3*a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(21*a*e^3) - (2*A*(a + b*x^2)^(5/2))/(3*a*e*(
e*x)^(3/2)) + (4*a^(3/4)*(7*A*b + 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(21*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.136615, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 220}

$$\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3aB+7Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(3aB+7Ab)}{21ae^3}+\frac{4\sqrt{ex}\sqrt{a+bx^2}(3a}{21e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (4*(7*A*b + 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*e^3) + (2*(7*A*b + 3*a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(21*a*e^3) - (2*A*(a + b*x^2)^(5/2))/(3*a*e*(
e*x)^(3/2)) + (4*a^(3/4)*(7*A*b + 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(21*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}dx = -\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(7Ab+3aB)\int \frac{(a+bx^2)^{3/2}}{\sqrt{ex}}dx}{3ae^2}$$

$$= \frac{2(7Ab+3aB)\sqrt{ex}(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(2(7Ab+3aB))\int \frac{\sqrt{a+bx^2}}{\sqrt{ex}}dx}{7e^2}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4a}{}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(8a}{}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{4a}{}$$

Mathematica [C] time = 0.0760289, size = 85, normalized size = 0.4

$$\frac{2x\sqrt{a+bx^2}\left(\frac{x^2(3aB+7Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}} - \frac{A(a+bx^2)^2}{a}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) + ((7*A*b + 3*a*B)*x^2*Hyperge
ometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^(5
/2))
```

Maple [A] time = 0.015, size = 255, normalized size = 1.2

$$\frac{2}{21\,bxe^2}\left(14\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}xab+6\,B\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)/x*(14*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a*b
+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a^2+3*B*x^6*b^3+7*A*x^4*b^3
+12*B*x^4*a*b^2+9*B*x^2*a^2*b-7*A*a^2*b)/b/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}\sqrt{ex}}{e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^3*x
^3), x)
```

Sympy [C] time = 25.1801, size = 202, normalized size = 0.96

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{A\sqrt{a}b\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1}{2e^{\frac{5}{2}}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(5/2),x)
```

```
[Out] A*a**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + A*sqrt(a)*b*sqrt(x)*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4)) + B*a
**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*e**(5/2)*gamma(5/4)) + B*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-1/2,
 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

3.799 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=365

$$\frac{12\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}}-\frac{2\left(a+bx^2\right)^{3/2}(aB+Ab)}{ae^3\sqrt{ex}}+\frac{12b(ex)^{3/2}\sqrt{a+}}{5a}$$

```
[Out] (12*b*(A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^5) + (24*Sqrt[b]*(A*b
 + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b
+ a*B)*(a + b*x^2)^(3/2))/(a*e^3*Sqrt[e*x]) - (2*A*(a + b*x^2)^(5/2))/(5*a*
e*(e*x)^(5/2)) - (24*a^(1/4)*b^(1/4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(1/
4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(
7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.288736, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {453, 277, 279, 329, 305, 220, 1196}

$$-\frac{2\left(a+bx^2\right)^{3/2}(aB+Ab)}{ae^3\sqrt{ex}}+\frac{12b(ex)^{3/2}\sqrt{a+bx^2}(aB+Ab)}{5ae^5}+\frac{24\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{5e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{12\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{}{(\sqrt{}}}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```


```
[Out] (12*b*(A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^5) + (24*Sqrt[b]*(A*b
 + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b
+ a*B)*(a + b*x^2)^(3/2))/(a*e^3*Sqrt[e*x]) - (2*A*(a + b*x^2)^(5/2))/(5*a*
e*(e*x)^(5/2)) - (24*a^(1/4)*b^(1/4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(1/
4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(
7/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(Ab+aB)\int \frac{(a+bx^2)^{3/2}}{(ex)^{3/2}}\,dx}{ae^2}$$

$$= -\frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(6b(Ab+aB))\int \sqrt{ex}\sqrt{a+bx^2}\,dx}{ae^4}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(12b(Ab+}{}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(24b(Ab+}{}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(24\sqrt{a}\sqrt{b}(}{}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} + \frac{24\sqrt{b}(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{5e^4\left(\sqrt{a}+\sqrt{bx}\right)} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}}$$

Mathematica [C] time = 0.0682737, size = 84, normalized size = 0.23

$$\frac{2x\sqrt{a+bx^2}\left(-\frac{5x^2(aB+Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}} - \frac{A(a+bx^2)^2}{a}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) - (5*(A*b + a*B)*x^2*Hypergeom
etric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(5*(e*x)^(7/
2))
```

Maple [A] time = 0.017, size = 422, normalized size = 1.2

$$\frac{2}{5x^2e^3}\left(12A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab - 6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(12*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-6*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
```

```
(1/2))*x^2*a*b+12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-6*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*x^2*a^2+B*b^2*x^6-7*A*b^2*x^4-4*B*x^4*a*b-8*a*A*b*x^2-5*B*x^2*
a^2-A*a^2)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bbx^4+(Ba+Ab)x^2+Aa)\sqrt{bx^2+a}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e*x)/(e^4*x
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

3.800 $\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=338

$$-\frac{a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}+\frac{2a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9A}{15b^{11/4}\sqrt{a}}$$

```
[Out] (2*(9*A*b - 7*a*B)*e*(e*x)^(3/2)*Sqrt[a + b*x^2])/(45*b^2) + (2*B*(e*x)^(7/
2)*Sqrt[a + b*x^2])/(9*b*e) - (2*a*(9*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b
*x^2])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*a^(5/4)*(9*A*b - 7*a*B)*e^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt
[a + b*x^2]) - (a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])
/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.257584, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 321, 329, 305, 220, 1196}

$$-\frac{a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}+\frac{2a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)}{15b^{11/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(9*A*b - 7*a*B)*e*(e*x)^(3/2)*Sqrt[a + b*x^2])/(45*b^2) + (2*B*(e*x)^(7/
2)*Sqrt[a + b*x^2])/(9*b*e) - (2*a*(9*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b
*x^2])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*a^(5/4)*(9*A*b - 7*a*B)*e^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt
[a + b*x^2]) - (a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])
/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^2}}\,dx}{9b}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{\left(a(9Ab-7aB)e^2\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{15b^2}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{(2a(9Ab-7aB)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,\right.}{15b^2}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{\left(2a^{3/2}(9Ab-7aB)e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{15b^{5/2}}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{2a(9Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2a^{5/4}(}{}$$

Mathematica [C] time = 0.111846, size = 96, normalized size = 0.28

$$\frac{2e(ex)^{3/2}\left(a\sqrt{\frac{bx^2}{a}+1}(7aB-9Ab)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\left(7aB-9Ab-5bBx^2\right)\right)}{45b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-((a + b*x^2)*(-9*A*b + 7*a*B - 5*b*B*x^2)) + a*(-9*A*b +
 7*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])
)/(45*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.025, size = 417, normalized size = 1.2

$$-\frac{e^2}{45\,xb^3}\sqrt{ex}\left(-10\,Bx^6b^3+54\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2b-27\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/45/x*e^2*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^3*(-10*B*x^6*b^3+54*A*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*b-27*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-18*A*x^4*b^3+4
*B*x^4*a*b^2-18*A*x^2*a*b^2+14*B*x^2*a^2*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)(ex)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Be^2x^4+Ae^2x^2\right)\sqrt{ex}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 98.2592, size = 94, normalized size = 0.28

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{11}{4}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*sqrt(a)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hyper((
1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

3.801 $\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=174

$$-\frac{a^{3/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{21b^2}+\frac{2B(ex$$

```
[Out] (2*(7*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*B*(e*x)^(5/2)
*Sqrt[a + b*x^2])/(7*b*e) - (a^(3/4)*(7*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.109648, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 321, 329, 220}

$$-\frac{a^{3/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{21b^2}+\frac{2B(ex)^{5/2}\sqrt{a+}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(7*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*B*(e*x)^(5/2)
*Sqrt[a + b*x^2])/(7*b*e) - (a^(3/4)*(7*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}dx = \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^2}}dx}{7b}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{\left(a(7Ab-5aB)e^2\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}dx}{21b^2}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{(2a(7Ab-5aB)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}dx, x,\right.}{21b^2}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{a^{3/4}(7Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx}{(\sqrt{a}+\sqrt{b}}}}{21b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.104504, size = 96, normalized size = 0.55

$$\frac{2e\sqrt{ex}\left(a\sqrt{\frac{bx^2}{a}+1}(5aB-7Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\left(5aB-7Ab-3bBx^2\right)\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*e*Sqrt[e*x]*(-((a + b*x^2)*(-7*A*b + 5*a*B - 3*b*B*x^2)) + a*(-7*A*b + 5
*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/
(21*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.024, size = 250, normalized size = 1.4

$$-\frac{e}{21\,xb^3}\sqrt{ex}\left(7\,A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-5\,B\sqrt{-ab}\sqrt{\frac{bx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/21*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(7*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*a*b-5*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*B*x^5*b^3-1
4*A*x^3*b^3+4*B*x^3*a*b^2-14*A*x*a*b^2+10*B*x*a^2*b)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bex^3 + Aex)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 14.6974, size = 94, normalized size = 0.54

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((1/2,
 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

3.802 $\int \frac{\sqrt{ex}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(5Ab-3aB)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\sqrt[4]{a}\sqrt{e}\left(\sqrt{a} \ldots\right.}{\ldots}$$

```
[Out] (2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*Sqrt[e*x]*Sq
rt[a + b*x^2])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*(5*A*b - 3*a*
B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2]) + (a^(1/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.214203, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 329, 305, 220, 1196}

$$\frac{2\sqrt{ex}\sqrt{a+bx^2}(5Ab-3aB)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right) \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*Sqrt[e*x]*Sq
rt[a + b*x^2])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*(5*A*b - 3*a*
B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2]) + (a^(1/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5b}$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{(2(5Ab-3aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5be}$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{\left(2\sqrt{a}(5Ab-3aB)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^{3/2}} - \frac{\left(2\sqrt{a}(5Ab-3aB)\right)\mathrm{S}}{}$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{2(5Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\sqrt[4]{a}(5Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx}{\left(\sqrt{a}+\sqrt{}\right.}}}{5b^{7/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0791491, size = 80, normalized size = 0.27

$$\frac{2x\sqrt{ex}\left(\sqrt{\frac{bx^2}{a}+1}(5Ab-3aB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+3B\left(a+bx^2\right)\right)}{15b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*B*(a + b*x^2) + (5*A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(15*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.013, size = 379, normalized size = 1.3

$$\frac{1}{5\,b^2x}\sqrt{ex}\left(10\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(10*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-5
*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^2+2*b^2*B*x^4+2*B*x^2*a*b)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [C] time = 3.00019, size = 92, normalized size = 0.31

$$\frac{A(ex)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e\Gamma\left(\frac{7}{4}\right)} + \frac{B(ex)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*sqrt(a)*e*gamma(7/4)) + B*(e*x)**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (
```

```
11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**3*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

3.803 $\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

```
[Out] (2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.0811609, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {459, 329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x
^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{3b}$$
$$= \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} + \frac{(2(3Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3be}$$
$$= \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} + \frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0554849, size = 79, normalized size = 0.57

$$\frac{2x\left(\sqrt{\frac{bx^2}{a}+1}(3Ab-aB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+B\left(a+bx^2\right)\right)}{3b\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*(B*(a + b*x^2) + (3*A*b - a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.016, size = 214, normalized size = 1.5

$$\frac{1}{3\,b^2}\left(3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}b-B\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3/(b*x^2+a)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b-B*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+2*b^2*B*x^3+2*B*x*a*b)/(e*x)^(1/2)/b^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{bx^2+a}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}}{bex^3+aex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b*e*x^3 + a*e*x), x)
```

Sympy [C] time = 2.37705, size = 94, normalized size = 0.68

$$\frac{A\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*sqrt(a)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*sqrt(e)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{bx^2+a}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

3.804 $\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=290

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a +
b*x^2])/(a*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a +
 b*x^2]) + ((A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2
])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.222748, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 329, 305, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right.}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```


```
[Out] (-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a +
b*x^2])/(a*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a +
 b*x^2]) + ((A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2
])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{ae^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(2(Ab+aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(2(Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{a}\sqrt{b}e^2} - \frac{(2(Ab+aB))\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}e}}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx\right.}{\sqrt{a}\sqrt{b}e^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{a\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0423765, size = 82, normalized size = 0.28

$$\frac{x\left(2x^2\sqrt{\frac{bx^2}{a}+1}(aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-6A\left(a+bx^2\right)\right)}{3a(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (x*(-6*A*(a + b*x^2) + 2*(A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric
2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.017, size = 378, normalized size = 1.3

$$\frac{1}{aeb}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b+2*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*E
llipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*A*x^2*b
^2-2*A*a*b)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{be^2x^4 + ae^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b*e^2*x^4 + a*e^2*x^2), x)
```

Sympy [C] time = 4.85839, size = 97, normalized size = 0.33

$$\frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (
```

```
7/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

3.805 $\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(5/4)*b^(1/4)*e^(5/2)*Sqrt[a +
b*x^2])
```

Rubi [A] time = 0.0864553, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 329, 220}

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(5/4)*b^(1/4)*e^(5/2)*Sqrt[a +
b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(Ab-3aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{3ae^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(2(Ab-3aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ae^3}$$

$$= -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0455389, size = 81, normalized size = 0.59

$$-\frac{2x\left(x^2\sqrt{\frac{bx^2}{a}+1}(Ab-3aB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+A\left(a+bx^2\right)\right)}{3a(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*(A*(a + b*x^2) + (A*b - 3*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.014, size = 223, normalized size = 1.6

$$-\frac{1}{3\,bxae^2}\left(A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*b-3*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a+2*A*x^2*b^2+2*A*a*b)/b/a/e^2/(
e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}}{be^3x^5+ae^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b*e^3*x^5 + a*e^3*x^3), x)
```

Sympy [C] time = 25.2779, size = 97, normalized size = 0.7

$$\frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (
5/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(5/2)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

3.806 $\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^3\sqrt{ex}}-\frac{2\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}}{5a^2e^4\left(\sqrt{a}+\right.}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) + (2*(3*A*b - 5*a*B)*Sqrt[a + b*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*Sqrt[b]*(3*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(5*a^2*e^4*(Sqrt[a] + Sqrt[b]*x)) + (2*b^(1/4)*(3*A*b - 5*a*B)*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[
a + b*x^2]) - (b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.251416, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 325, 329, 305, 220, 1196}

$$\frac{2\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^3\sqrt{ex}}-\frac{2\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\right|}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```


```
[Out] (-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) + (2*(3*A*b - 5*a*B)*Sqrt[a + b*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*Sqrt[b]*(3*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(5*a^2*e^4*(Sqrt[a] + Sqrt[b]*x)) + (2*b^(1/4)*(3*A*b - 5*a*B)*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[
a + b*x^2]) - (b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx &= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}-\frac{(3Ab-5aB)\int \frac{1}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx}{5ae^2}\\
&= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}}-\frac{(b(3Ab-5aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5a^2e^4}\\
&= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}}-\frac{(2b(3Ab-5aB))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5a^2e^5}\\
&= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}}-\frac{\left(2\sqrt{b}(3Ab-5aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5a^{3/2}e^4}\\
&= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}}-\frac{2\sqrt{b}(3Ab-5aB)\sqrt{ex}\sqrt{a+bx^2}}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2\sqrt[4]{b}(3Ab-5aB}{}
\end{aligned}$$

Mathematica [C] time = 0.0464111, size = 82, normalized size = 0.24

$$-\frac{2x\left(x^2\sqrt{\frac{bx^2}{a}+1}(5aB-3Ab)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)+A\left(a+bx^2\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*(A*(a + b*x^2) + (-3*A*b + 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 417, normalized size = 1.2

$$-\frac{1}{5\,x^2e^3a^2}\left(6\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)x^2ab-3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b-10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2+5*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*x^2*a^2-6*A*b^2*x^4+10*B*x^4*a*b-4*a*A*b*x^2+10*B*x^2*a^2+2*A*
a^2)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{be^4x^6 + ae^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b*e^4*x^6 + a*e^4*x^4), x)
```

Sympy [C] time = 136.821, size = 104, normalized size = 0.3

$$\frac{A\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt
(a)*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,
), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

3.807 $\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{5a^{3/4}e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-9aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\sqrt{a+bx^2}(7Ab-9aB)}{21b^3}-\frac{e(ex)^{5/}}{7b^2}$$

```
[Out] -((7*A*b - 9*a*B)*e*(e*x)^(5/2))/(7*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(9/2)
)/(7*b*e*Sqrt[a + b*x^2]) + (5*(7*A*b - 9*a*B)*e^3*Sqrt[e*x]*Sqrt[a + b*x^2
])/(21*b^3) - (5*a^(3/4)*(7*A*b - 9*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(42*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.137075, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 288, 321, 329, 220}

$$-\frac{5a^{3/4}e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-9aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\sqrt{a+bx^2}(7Ab-9aB)}{21b^3}-\frac{e(ex)^{5/2}(7Ab-}{7b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((7*A*b - 9*a*B)*e*(e*x)^(5/2))/(7*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(9/2)
)/(7*b*e*Sqrt[a + b*x^2]) + (5*(7*A*b - 9*a*B)*e^3*Sqrt[e*x]*Sqrt[a + b*x^2
])/(21*b^3) - (5*a^(3/4)*(7*A*b - 9*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(42*b^(13/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}dx = \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{9aB}{2}\right)\right)\int\frac{(ex)^{7/2}}{\left(a+bx^2\right)^{3/2}}dx}{7b}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{\left(5(7Ab-9aB)e^2\right)\int\frac{(ex)^{3/2}}{\sqrt{a+bx^2}}dx}{14b^2}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{\left(5a(7Ab-9aB)e^4\right.}{42b}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{\left(5a(7Ab-9aB)e^3\right.}{}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{5a^{3/4}(7Ab-9aB)e}{}$$

Mathematica [C] time = 0.143431, size = 111, normalized size = 0.53

$$\frac{e^3\sqrt{ex}\left(-45a^2B+5a\sqrt{\frac{bx^2}{a}+1}(9aB-7Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+ab\left(35A-18Bx^2\right)+2b^2x^2\left(7A+3Bx^2\right)\right)}{21b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-45*a^2*B + a*b*(35*A - 18*B*x^2) + 2*b^2*x^2*(7*A + 3*B*x^
2) + 5*a*(-7*A*b + 9*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5
/4, -((b*x^2)/a)]))/(21*b^3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.035, size = 252, normalized size = 1.2

$$-\frac{e^3}{42\,xb^4}\sqrt{ex}\left(35\,A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-45\,B\sqrt{-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/42*e^3/x*(e*x)^(1/2)*(35*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2)
)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-
45*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-12*B*x^5*b^3-28*A*x^3*b^3
+36*B*x^3*a*b^2-70*A*x*a*b^2+90*B*x*a^2*b)/(b*x^2+a)^(1/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)(ex)^{\frac{7}{2}}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^3x^5 + Ae^3x^3\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^3*x^5 + A*e^3*x^3)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

3.808 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=337

$$\frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}}+\frac{3e^2\sqrt{ex}\sqrt{a+bx^2}(5Ab-7aB)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{3\sqrt[4]{a}e^{5/2}\left(\right.}{}$$

```
[Out] -((5*A*b - 7*a*B)*e*(e*x)^(3/2))/(5*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(7/2)
)/(5*b*e*Sqrt[a + b*x^2]) + (3*(5*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^2
])/(5*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)*(
Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*
ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(11/4)*Sqrt[a + b
*x^2]) + (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
 b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.252037, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 288, 329, 305, 220, 1196}

$$\frac{3e^2\sqrt{ex}\sqrt{a+bx^2}(5Ab-7aB)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```


```
[Out] -((5*A*b - 7*a*B)*e*(e*x)^(3/2))/(5*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(7/2)
)/(5*b*e*Sqrt[a + b*x^2]) + (3*(5*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^2
])/(5*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)*(
Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*
ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(11/4)*Sqrt[a + b
*x^2]) + (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
 b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^2])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{5b}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{\left(3(5Ab-7aB)e^2\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{10b^2}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{(3(5Ab-7aB)e)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{\left(3\sqrt{a}(5Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^{5/2}}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{3(5Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{3\sqrt[4]{a}(5Ab-7aB)e}{}$$

Mathematica [C] time = 0.112121, size = 84, normalized size = 0.25

$$\frac{2e(ex)^{3/2}\left(\sqrt{\frac{bx^2}{a}+1}(7aB-5Ab)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-7aB+5Ab+bBx^2\right)}{5b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(5*A*b - 7*a*B + b*B*x^2 + (-5*A*b + 7*a*B)*Sqrt[1 + (b*x^
2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[a + b*x^
2])
```

Maple [A] time = 0.032, size = 391, normalized size = 1.2

$$\frac{e^2}{10\,xb^3}\sqrt{ex}\left(30\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-15\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/10/x*e^2*(e*x)^(1/2)*(30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-15*A*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))*a*b-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+21*B*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*a^2+4*b^2*B*x^4-10*A*x^2*b^2+14*B*x^2*a*b)/(b*x^2+a)^(1/2
)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Be^2x^4+Ae^2x^2\right)\sqrt{bx^2+a}\sqrt{ex}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

3.809 $\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(3Ab-5aB)}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

```
[Out] -((3*A*b - 5*a*B)*e*Sqrt[e*x])/(3*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(5/2))/
(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*
x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.111641, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 288, 329, 220}

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(3Ab-5aB)}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((3*A*b - 5*a*B)*e*Sqrt[e*x])/(3*b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(5/2))/
(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*
x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}dx = \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/2}}dx}{3b}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}} + \frac{\left((3Ab-5aB)e^2\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}dx}{6b^2}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}} + \frac{((3Ab-5aB)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{3b^2}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}} + \frac{(3Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.104776, size = 85, normalized size = 0.49

$$\frac{e\sqrt{ex}\left(\sqrt{\frac{bx^2}{a}+1}(3Ab-5aB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+5aB-3Ab+2bBx^2\right)}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (e*Sqrt[e*x]*(-3*A*b + 5*a*B + 2*b*B*x^2 + (3*A*b - 5*a*B)*Sqrt[1 + (b*x^2)
/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2]
)
```

Maple [A] time = 0.019, size = 225, normalized size = 1.3

$$\frac{e}{6\,x b^3}\sqrt{ex}\left(3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}b-5\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*e/x*(e*x)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b-5*B*((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+4*b^2*B*x^3-6*A*x*b^2+10*B*x*a*b)/(
```

```
b*x^2+a)^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bex^3 + Aex)\sqrt{bx^2 + a}\sqrt{ex}}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^2 +
 a^2), x)
```

Sympy [C] time = 61.2484, size = 94, normalized size = 0.54

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3/2
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.810 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\,\text{ta}\right.}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e*x
]*Sqrt[a + b*x^2])/(a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 3*a*B)*Sqrt[
e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(7/4)*S
qrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.224851, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 329, 305, 220, 1196}

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```


```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e*x
]*Sqrt[a + b*x^2])/(a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 3*a*B)*Sqrt[
e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(7/4)*S
qrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}}+\frac{\left(-\frac{Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{abe}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{\sqrt{a}b^{3/2}}+\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}e}}{\sqrt{a+\frac{bx^4}{e^2}}}\,d\right.}{\sqrt{a}b^{3/2}}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{(Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\,\mathrm{ta}\right.}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.103095, size = 76, normalized size = 0.25

$$\frac{2x\sqrt{ex}\left(\sqrt{\frac{bx^2}{a}+1}(Ab-3aB)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)+3aB\right)}{3ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*a*B + (A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1
[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.017, size = 382, normalized size = 1.3

$$-\frac{1}{2\,b^2xa}\sqrt{ex}\left(2\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(e*x)^(1/2)*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-A*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*a*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a^2-2*A*x^2*b^2+2*B*x^2*a*b)/(b*x^2+a)^(1/2)/b^2/x/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^2 + a^2),
 x)
```

Sympy [C] time = 12.4046, size = 94, normalized size = 0.31

$$\frac{A\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((3/2,
7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

3.811 $\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(5/4)*Sqrt[e]*Sqrt[a
 + b*x^2])
```

Rubi [A] time = 0.0899193, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {457, 329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(5/4)*Sqrt[e]*Sqrt[a
 + b*x^2])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx = \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}} + \frac{(Ab+aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{2ab}$$
$$= \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}} + \frac{(Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe}$$
$$= \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}} + \frac{(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0570964, size = 75, normalized size = 0.52

$$\frac{x\left(\sqrt{\frac{bx^2}{a}+1}(aB+Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-aB+Ab\right)}{ab\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(A*b - a*B + (A*b + a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^2)/a)]))/(a*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.021, size = 213, normalized size = 1.5

$$\frac{1}{2\,ab^2}\left(A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 1/2*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b+B*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*(-a*b)^(1/2)*a+2*A*x*b^2-2*B*x*a*b)/(b*x^2+a)^(1/2)/a/(e*x)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}}{b^2ex^5+2\,abex^3+a^2ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*e*x^5 + 2*a*b*e*x^3 + a
^2*e*x), x)
```

Sympy [C] time = 18.2242, size = 94, normalized size = 0.65

$$\frac{A\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/
4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*sqrt(e)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

3.812 $\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\right.\right.}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*(e*x)^(3/2))/(a^2*e
^3*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(a^2*Sqrt[b
]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(a^(7/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((3*A*
b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b
^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.257015, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 305, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\right.}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```


```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*(e*x)^(3/2))/(a^2*e
^3*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(a^2*Sqrt[b
]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(a^(7/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((3*A*
b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b
^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}}+\frac{(3Ab-aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{2a^2e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}}+\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}}+\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{a^{3/2}\sqrt{b}e^2}-\frac{(3Ab-\cdots}{\cdots}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}}+\frac{(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{a^2\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\cdots}{a^{7/4}\cdots}$$

Mathematica [C] time = 0.0417129, size = 77, normalized size = 0.23

$$\frac{x\left(2x^2\sqrt{\frac{bx^2}{a}+1}(aB-3Ab)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-6aA\right)}{3a^2(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(-6*a*A + 2*(-3*A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4
, 3/2, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.022, size = 386, normalized size = 1.2

$$\frac{1}{2\,be{a}^{2}}\left(6\,A\sqrt{{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}\sqrt{2}\sqrt{{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}}\sqrt{-{\frac{bx}{\sqrt{-ab}}}}{\it EllipticE} \left(\sqrt{{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}},1/2\,\sqrt{2} \right) ab-3\,A\sqrt{{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}\sqrt{2}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*
b-2*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*A
*x^2*b^2+2*B*x^2*a*b-4*A*a*b)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^{2} + A}{{\left(bx^{2} + a\right)}^{\frac{3}{2}} \left(ex\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(Bx^{2} + A\right)} \sqrt{bx^{2} + a} \sqrt{ex}}{b^{2} e^{2} x^{6} + 2 \, a b e^{2} x^{4} + a^{2} e^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*e^2*x^6 + 2*a*b*e^2*x^4
 + a^2*e^2*x^2), x)
```

Sympy [C] time = 49.5871, size = 97, normalized size = 0.29

$$\frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3
/2)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2),
(7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.813 $\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{\sqrt{ex}(5Ab-3aB)}{3a^2e^3\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*Sqrt[e*x])/(3
*a^2*e^3*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(6*a^(9/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.113333, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 220}

$$-\frac{\sqrt{ex}(5Ab-3aB)}{3a^2e^3\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*Sqrt[e*x])/(3
*a^2*e^3*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(6*a^(9/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx}{3ae^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{6a^2e^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3a^2e^3}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0549425, size = 91, normalized size = 0.52

$$\frac{x\left(x^2\sqrt{\frac{bx^2}{a}+1}(3aB-5Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-2aA+3aBx^2-5Abx^2\right)}{3a^2(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(-2*a*A - 5*A*b*x^2 + 3*a*B*x^2 + (-5*A*b + 3*a*B)*x^2*Sqrt[1 + (b*x^2)/
a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*Sqrt
[a + b*x^2])
```

Maple [A] time = 0.02, size = 232, normalized size = 1.3

$$-\frac{1}{6a^2bxe^2}\left(5A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}xb-3B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6/x*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*b-3*B*((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*x*a+10*A*x^2*b^2-6*B*x^2*a*b+4*A*a*b)/(b*x^2+a)^
```

```
(1/2)/b/a^2/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^2e^3x^7 + 2\,abe^3x^5 + a^2e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*e^3*x^7 + 2*a*b*e^3*x^5
 + a^2*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.814 $\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=379

$$-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{10a^{11/4}e^{7/2}\sqrt{a+bx^2}}+\frac{3\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^3\sqrt{ex}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx}}{5a^3e^4\left(\sqrt{a}\right.}$$

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - (7*A*b - 5*a*B)/(5*a^2*e^3*Sqr
t[e*x]*Sqrt[a + b*x^2]) + (3*(7*A*b - 5*a*B)*Sqrt[a + b*x^2])/(5*a^3*e^3*Sq
rt[e*x]) - (3*Sqrt[b]*(7*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a^3*e^4
*(Sqrt[a] + Sqrt[b]*x)) + (3*b^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2]) - (3*b
^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2]
)/(10*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.298067, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {453, 290, 325, 329, 305, 220, 1196}

$$\frac{3\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^3\sqrt{ex}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5a}{10a^{11/4}e^{7/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```


```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - (7*A*b - 5*a*B)/(5*a^2*e^3*Sqr
t[e*x]*Sqrt[a + b*x^2]) + (3*(7*A*b - 5*a*B)*Sqrt[a + b*x^2])/(5*a^3*e^3*Sq
rt[e*x]) - (3*Sqrt[b]*(7*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a^3*e^4
*(Sqrt[a] + Sqrt[b]*x)) + (3*b^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2]) - (3*b
^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2]
)/(10*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{(7Ab-5aB)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx}{5ae^2}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3(7Ab-5aB))\int \frac{1}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx}{10a^2e^2}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - \frac{(3b(7Ab-5aB))\int \cdots}{10a^3e^4}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - \frac{(3b(7Ab-5aB))\,\text{Sub}\cdots}{5\cdots}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - \frac{\left(3\sqrt{b}(7Ab-5aB)\right)\text{S}\cdots}{\cdots}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - \frac{3\sqrt{b}(7Ab-5aB)\sqrt{ex}\cdots}{5a^3e^4\left(\sqrt{a}+\sqrt{\cdots}\right.}$$

Mathematica [C] time = 0.0440894, size = 78, normalized size = 0.21

$$\frac{x\left(2x^2\sqrt{\frac{bx^2}{a}+1}(7Ab-5aB)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)-2aA\right)}{5a^2(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(-2*a*A + 2*(7*A*b - 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1
/4, 3/2, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.021, size = 417, normalized size = 1.1

$$-\frac{1}{10\,x^2e^3a^3}\left(42\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-21\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-21*A*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*a*b-30*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
```

```
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2+15*B*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a^2-42*A*b^2*x^4+30*B*x^4*a*b-28*a*A*b*x^2+20*B*x^2*a^
2+4*A*a^2)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^2e^4x^8 + 2\,abe^4x^6 + a^2e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^2*e^4*x^8 + 2*a*b*e^4*x^6
 + a^2*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.815 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{5e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}b^{13/4}\sqrt{a+bx^2}}-\frac{5e^3\sqrt{ex}(Ab-3aB)}{6b^3\sqrt{a+bx^2}}-\frac{e(ex)^{5/2}(Ab-3aB)}{3b^2\left(a+bx^2\right)^{3/2}}+$$

```
[Out] -((A*b - 3*a*B)*e*(e*x)^(5/2))/(3*b^2*(a + b*x^2)^(3/2)) + (2*B*(e*x)^(9/2)
)/(3*b*e*(a + b*x^2)^(3/2)) - (5*(A*b - 3*a*B)*e^3*Sqrt[e*x])/(6*b^3*Sqrt[a
 + b*x^2]) + (5*(A*b - 3*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(12*a^(1/4)*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.132842, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 288, 329, 220}

$$-\frac{5e^3\sqrt{ex}(Ab-3aB)}{6b^3\sqrt{a+bx^2}}+\frac{5e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{13/4}\sqrt{a+bx^2}}-\frac{e(ex)^{5/2}(Ab-3aB)}{3b^2\left(a+bx^2\right)^{3/2}}+\frac{2B}{3be\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -((A*b - 3*a*B)*e*(e*x)^(5/2))/(3*b^2*(a + b*x^2)^(3/2)) + (2*B*(e*x)^(9/2)
)/(3*b*e*(a + b*x^2)^(3/2)) - (5*(A*b - 3*a*B)*e^3*Sqrt[e*x])/(6*b^3*Sqrt[a
 + b*x^2]) + (5*(A*b - 3*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(12*a^(1/4)*b^(13/4)*Sqrt[a + b*x^2])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{5/2}}\,dx}{3b}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} + \frac{\left(5(Ab-3aB)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{6b^2}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{\left(5(Ab-3aB)e^4\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,d}{12b^3}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{\left(5(Ab-3aB)e^3\right)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{6b^3}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{5(Ab-3aB)e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)}{12\sqrt[4]{a}b^{13}}$$

Mathematica [C] time = 0.173096, size = 116, normalized size = 0.56

$$\frac{e^3\sqrt{ex}\left(15a^2B+5\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(Ab-3aB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a\left(21bBx^2-5Ab\right)+b^2x^2\left(4Bx^2-7A\right)\right)}{6b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(15*a^2*B + b^2*x^2*(-7*A + 4*B*x^2) + a*(-5*A*b + 21*b*B*x^
2) + 5*(A*b - 3*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4,
 1/2, 5/4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.034, size = 439, normalized size = 2.1

$$\frac{e^3}{12\,xb^4}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}x^2b^2-15\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b^2-15*B*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*a*b+5*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a*b-15*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+8*B*x^5*b^3-1
4*A*x^3*b^3+42*B*x^3*a*b^2-10*A*x*a*b^2+30*B*x*a^2*b)*e^3/x*(e*x)^(1/2)/b^4
/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)(ex)^{\frac{7}{2}}}{\left(bx^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^3x^5 + Ae^3x^3\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^3*x^5 + A*e^3*x^3)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*x^6 + 3*a*b
^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

3.816 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}+\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)E\left(\right.}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b - 7*a*B)*e*(e
*x)^(3/2))/(6*a*b^2*Sqrt[a + b*x^2]) - ((A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a
+ b*x^2])/(2*a*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 7*a*B)*e^(5/2)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(11/4)*Sqrt[
a + b*x^2]) - ((A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(4*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.260169, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 288, 329, 305, 220, 1196}

$$-\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}+\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)E\left(2\tan^{-1}\right.}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```


```
[Out] ((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b - 7*a*B)*e*(e
*x)^(3/2))/(6*a*b^2*Sqrt[a + b*x^2]) - ((A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a
+ b*x^2])/(2*a*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 7*a*B)*e^(5/2)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(11/4)*Sqrt[
a + b*x^2]) - ((A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(4*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{\left((Ab-7aB)e^2\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{4ab^2}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{((Ab-7aB)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{\left((Ab-7aB)e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2\sqrt{a}b^{5/2}} + \frac{((Ab}{}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{(Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{(Ab-7aB)e^{5/2}\left(\sqrt{a}+\right.}{2}$$

Mathematica [C] time = 0.127703, size = 97, normalized size = 0.28

$$-\frac{2e(ex)^{3/2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(7aB-Ab)\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)+a\left(-7aB+Ab-3bBx^2\right)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*(a*(A*b - 7*a*B - 3*b*B*x^2) + (-(A*b) + 7*a*B)*(a + b*x^
2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*
a*b^2*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.037, size = 767, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*a*b^2-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+21*B*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a^2*b+6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*a^3-6*A*x^4*b^3+18*B*x^4*a*b^2-2*A*x^2*a*b^2+14*B*x^
2*a^2*b)/x*e^2*(e*x)^(1/2)/b^3/a/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^2x^4 + Ae^2x^2\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*x^6 + 3*a*b
^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

3.817 $\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(5aB+Ab)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b + 5*a*B)*e*Sq
rt[e*x])/(6*a*b^2*Sqrt[a + b*x^2]) + ((A*b + 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(5/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.114255, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 288, 329, 220}

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(5aB+Ab)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```


```
[Out] ((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b + 5*a*B)*e*Sq
rt[e*x])/(6*a*b^2*Sqrt[a + b*x^2]) + ((A*b + 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(5/4)*b^(9/4)*Sqrt[a + b*x^2])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{\left((Ab+5aB)e^2\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{12ab^2}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{((Ab+5aB)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6ab^2}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{(Ab+5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.148075, size = 105, normalized size = 0.57

$$\frac{e\sqrt{ex}\left(-5a^2B+\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(5aB+Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-ab\left(A+7Bx^2\right)+Ab^2x^2\right)}{6ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[e*x]*(-5*a^2*B + A*b^2*x^2 - a*b*(A + 7*B*x^2) + (A*b + 5*a*B)*(a +
 b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])
)/(6*a*b^2*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.018, size = 429, normalized size = 2.3

$$\frac{e}{12\,axb^3}\left(A\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
```

```
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b^2+5*B*((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*x^2*a*b+A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*a*b+5*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+2*A*x^3*b^3-14*B*x^
3*a*b^2-2*A*x*a*b^2-10*B*x*a^2*b)*e/x*(e*x)^(1/2)/a/b^3/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bex^3 + Aex)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*x^6 + 3*a*b^2*x^4
 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.818 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)}{}$$

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a*B)*(e*x)^
(3/2))/(2*a^2*b*e*Sqrt[a + b*x^2]) - ((A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2]
)/(2*a^2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2
]) - ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(4*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.258791, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 290, 329, 305, 220, 1196}

$$-\frac{\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{bx}\right)}}}{2a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a*B)*(e*x)^
(3/2))/(2*a^2*b*e*Sqrt[a + b*x^2]) - ((A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2]
)/(2*a^2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2
]) - ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(4*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab+aB)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/2}}\,dx}{2ab}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{4a^2b}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2be}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^{3/2}b^{3/2}} + \frac{(Ab+aB)\,\text{Sul}}{}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{(Ab+aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{}{(}}}{2a^{7/4}b^{7/4}}$$

Mathematica [C] time = 0.104803, size = 84, normalized size = 0.24

$$\frac{2x\sqrt{ex}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(aB+Ab)\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-a^2B\right)}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(-(a^2*B) + (A*b + a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hype
rgeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.021, size = 764, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*a*b^2+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b-3*B*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*x^2*a^2*b+6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a^2*b+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-3*B*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))*a^3-6*A*x^4*b^3-6*B*x^4*a*b^2-10*A*x^2*a*b^2-2*B*x^2*a^2
*b)*(e*x)^(1/2)/b^2/a^2/x/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3), x)
```

Sympy [C] time = 162.735, size = 94, normalized size = 0.27

$$\frac{A\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4,
5/2), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\sqrt{ex}}{\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

3.819 $\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/2}} dx$

Optimal. Leaf size=187

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(aB+5Ab)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a*B)*Sqrt[e
*x])/(6*a^2*b*e*Sqrt[a + b*x^2]) + ((5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x
])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(9/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.111794, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 290, 329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(aB+5Ab)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```


```
[Out] ((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a*B)*Sqrt[e
*x])/(6*a^2*b*e*Sqrt[a + b*x^2]) + ((5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x
])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(9/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/2}}\,dx = \frac{(Ab-aB)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/2}}+\frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$=\frac{(Ab-aB)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/2}}+\frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}}+\frac{(5Ab+aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{12a^2b}$$

$$=\frac{(Ab-aB)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/2}}+\frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}}+\frac{(5Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{6a^2be}$$

$$=\frac{(Ab-aB)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/2}}+\frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}}+\frac{(5Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\right.}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0823972, size = 108, normalized size = 0.58

$$\frac{-a^2Bx+x\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(aB+5Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+abx\left(7A+Bx^2\right)+5Ab^2x^3}{6a^2b\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-(a^2*B*x) + 5*A*b^2*x^3 + a*b*x*(7*A + B*x^2) + (5*A*b + a*B)*x*(a + b*x^
2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(6*a
^2*b*Sqrt[e*x]*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.023, size = 425, normalized size = 2.3

$$\frac{1}{12\,b^2a^2}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}x^2b^2+B\sqrt{\left(bx+\sqrt{-ab}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] 1/12*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
```

```
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^2*b^2+B*((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*x^2*a*b+5*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a*b+B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+10*A*x^3*b^3+2*B*x^
3*a*b^2+14*A*x*a*b^2-2*B*x*a^2*b)/(e*x)^(1/2)/a^2/b^2/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3ex^7 + 3\,ab^2ex^5 + 3\,a^2bex^3 + a^3ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*e*x^7 + 3*a*b^2*e*x^5 +
 3*a^2*b*e*x^3 + a^3*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

3.820 $\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=377

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)E\left(2\tan^{-1}\left(\right.\right.}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(3*a
^2*e^3*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(2*a^3*e^3*Sqrt[a +
 b*x^2]) + ((7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(2*a^3*Sqrt[b]*e^2*(Sq
rt[a] + Sqrt[b]*x)) - ((7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(2*a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((7*A*b - a*
B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*a^(11/4)*b^(3/4
)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.292428, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 305, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\right.}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```


```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(3*a
^2*e^3*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(2*a^3*e^3*Sqrt[a +
 b*x^2]) + ((7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(2*a^3*Sqrt[b]*e^2*(Sq
rt[a] + Sqrt[b]*x)) - ((7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(2*a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((7*A*b - a*
B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*a^(11/4)*b^(3/4
)*e^(3/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/2}}\,dx}{2a^2e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{4a^3e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB)\,\mathrm{Subst}\left(\int \ldots\right)}{2a^3e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB)\,\mathrm{Subst}\left(\int \ldots\right)}{2a^{5/2}\sqrt{b}e^{2}\ldots}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{2a^3\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}$$

Mathematica [C] time = 0.062209, size = 86, normalized size = 0.23

$$\frac{x\left(2x^2\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(aB-7Ab)\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-6a^2A\right)}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x*(-6*a^2*A + 2*(-7*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hyperge
ometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2)^(3
/2))
```

Maple [A] time = 0.025, size = 771, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2-21*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
```

```
(1/2))*x^2*a*b^2-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+3*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*x^2*a^2*b+42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-21*A*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+3*B*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^3-42*A*x^4*b^3+6*B*x^4*a*b^2-70*A*x^2*a*b^2+10*B*x^
2*a^2*b-24*A*a^2*b)/b/a^3/e/(e*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3e^2x^8 + 3\,ab^2e^2x^6 + 3\,a^2be^2x^4 + a^3e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*e^2*x^8 + 3*a*b^2*e^2*x
^6 + 3*a^2*b*e^2*x^4 + a^3*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.821 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{ex}(3Ab-aB)}{6a^3e^3\sqrt{a+bx^2}}-\frac{\sqrt{ex}(3Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{}{3ae(ex)}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*Sqrt[e*x])/(3
*a^2*e^3*(a + b*x^2)^(3/2)) - (5*(3*A*b - a*B)*Sqrt[e*x])/(6*a^3*e^3*Sqrt[a
 + b*x^2]) - (5*(3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(12*a^(13/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.141613, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 220}

$$-\frac{5\sqrt{ex}(3Ab-aB)}{6a^3e^3\sqrt{a+bx^2}}-\frac{\sqrt{ex}(3Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```


```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*Sqrt[e*x])/(3
*a^2*e^3*(a + b*x^2)^(3/2)) - (5*(3*A*b - a*B)*Sqrt[e*x])/(6*a^3*e^3*Sqrt[a
 + b*x^2]) - (5*(3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(12*a^(13/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(5(3Ab-aB))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx}{6a^2e^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{(5(3Ab-aB))\int \frac{1}{\sqrt{ex}\sqrt{a}}}{12a^3e^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{(5(3Ab-aB))\,\mathrm{Subst}}{6a^3}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{5(3Ab-aB)\left(\sqrt{a}+\sqrt{b}\right.}{12a}$$

Mathematica [C] time = 0.0765028, size = 120, normalized size = 0.56

$$\frac{x\left(a^2\left(7Bx^2-4A\right)+5x^2\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}(aB-3Ab)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a\left(5bBx^4-21Abx^2\right)-15Ab^2x^4\right)}{6a^3(ex)^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x*(-15*A*b^2*x^4 + a^2*(-4*A + 7*B*x^2) + a*(-21*A*b*x^2 + 5*b*B*x^4) + 5*
(-3*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1
/2, 5/4, -((b*x^2)/a)]))/(6*a^3*(e*x)^(5/2)*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.025, size = 446, normalized size = 2.1

$$-\frac{1}{12\,xe^2a^3b}\left(15\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^3b^2-5\,B\sqrt{\frac{bx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^3*b^2-5*B*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^3*a*b+15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a
*b)^(1/2)*x*a*b-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a^2+30*A*x^4
*b^3-10*B*x^4*a*b^2+42*A*x^2*a*b^2-14*B*x^2*a^2*b+8*A*a^2*b)/x/e^2/(e*x)^(1
/2)/a^3/b/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2 + A\right)\sqrt{bx^2 + a}\sqrt{ex}}{b^3e^3x^9 + 3\,ab^2e^3x^7 + 3\,a^2be^3x^5 + a^3e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x)/(b^3*e^3*x^9 + 3*a*b^2*e^3*x
^7 + 3*a^2*b*e^3*x^5 + a^3*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(bx^2 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.822 $\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=288

$$-\frac{2c^{7/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(11a^2d^2+bc(3bc-10ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2(ex)^{5/2}\sqrt{c+dx^2}\left(11a^2d^2\right.}{77d^2e}$$

```
[Out] (4*c*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) + (2*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*(e*x)^(5/2)*Sqrt[c + d*x^2])/
(77*d^2*e) - (2*b*(3*b*c - 10*a*d)*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(55*d^2*e
) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(3/2))/(15*d*e^3) - (2*c^(7/4)*(11*a^2*d
^2 + b*c*(3*b*c - 10*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.291296, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {464, 459, 279, 321, 329, 220}

$$-\frac{2c^{7/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(11a^2d^2+bc(3bc-10ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2(ex)^{5/2}\sqrt{c+dx^2}\left(11a^2d^2+bc(3bc\right.}{77d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (4*c*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) + (2*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*(e*x)^(5/2)*Sqrt[c + d*x^2])/
(77*d^2*e) - (2*b*(3*b*c - 10*a*d)*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(55*d^2*e
) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(3/2))/(15*d*e^3) - (2*c^(7/4)*(11*a^2*d
^2 + b*c*(3*b*c - 10*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = \frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}+\frac{2\int (ex)^{3/2}\sqrt{c+dx^2}\left(\frac{15a^2d}{2}-\frac{3}{2}b(3bc-10ad)x^2\right)dx}{15d}$$

$$= -\frac{2b(3bc-10ad)(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{55d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}+\frac{1}{11}\left(11a^2+\frac{bc(3bc\cdots}{d\cdots}\right.\cdots$$

$$= \frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{77e}-\frac{2b(3bc-10ad)(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{55d^2e}+\frac{2b^2(e\cdots}{\cdots}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}+\frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{77e}-\frac{2\cdots}{\cdots}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}+\frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{77e}-\frac{2\cdots}{\cdots}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}+\frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{77e}-\frac{2\cdots}{\cdots}$$

Mathematica [C] time = 0.27916, size = 225, normalized size = 0.78

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(55a^2d^2(2c+3dx^2)+10abd(-10c^2+6cdx^2+21d^2x^4)+b^2(-18c^2dx^2+30c^3+14cd^2x^4+77d^3x^6)\right)}{5d^3}-\frac{4ic^2x\sqrt{\frac{c}{dx^2}+1}\left(11a^2d^2-10abcd+3b^2c^2\right)\text{Ellip}}{d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(55*a^2*d^2*(2*c + 3*d*x^2) + 10*a*b*d
*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4) + b^2*(30*c^3 - 18*c^2*d*x^2 + 14*c*d^2
*x^4 + 77*d^3*x^6)))/(5*d^3) - ((4*I)*c^2*(3*b^2*c^2 - 10*a*b*c*d + 11*a^2*
d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sq
rt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(3/2)*Sqrt[c + d*x^2]
)
```

Maple [A] time = 0.063, size = 448, normalized size = 1.6

$$-\frac{2e}{1155\,xd^4}\sqrt{ex}\left(-77\,x^9b^2d^5-210\,x^7abd^5-91\,x^7b^2cd^4+55\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+}{\sqrt{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-77*x^9*b^2*d^5-210*x^7*a*b*d^5-91
*x^7*b^2*c*d^4+55*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c^2*d^2-50*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^3*d+15*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*
d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*(-c*d)^(1/2)*b^2*c^4-165*x^5*a^2*d^5-270*x^5*a*b*c*d^4+4*x^5*b^2*c^2*
d^3-275*x^3*a^2*c*d^4+40*x^3*a*b*c^2*d^3-12*x^3*b^2*c^3*d^2-110*x*a^2*c^2*d
^3+100*x*a*b*c^3*d^2-30*x*b^2*c^4*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^2\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2ex^5 + 2\,abex^3 + a^2ex\right)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [C] time = 73.795, size = 150, normalized size = 0.52

$$\frac{a^2\sqrt{c}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{ab\sqrt{c}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{13}{4}\right)} + \frac{b^2\sqrt{c}e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2
*exp_polar(I*pi)/c)/(2*gamma(9/4)) + a*b*sqrt(c)*e**(3/2)*x**(9/2)*gamma(9/
4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/gamma(13/4) + b**2
*sqrt(c)*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((-1/2, 13/4), (17/4,), d*x**2
*exp_polar(I*pi)/c)/(2*gamma(17/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^2 \sqrt{dx^2 + c}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

3.823 $\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=425

$$\frac{2c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}}-\frac{4c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c\text{-}}{\left(\sqrt{c}\text{-}\right.}}}{}$$

```
[Out] (2*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*d^
2*e) + (4*c*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/
(195*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*b*(7*b*c - 26*a*d)*(e*x)^(3/2)*(c
+ d*x^2)^(3/2))/(117*d^2*e) + (2*b^2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e
^3) - (4*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqr
t[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2]) + (2
*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.414861, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 329, 305, 220, 1196}

$$\frac{2c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}}-\frac{4c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39\right.}{195}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```


```
[Out] (2*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*d^
2*e) + (4*c*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/
(195*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*b*(7*b*c - 26*a*d)*(e*x)^(3/2)*(c
+ d*x^2)^(3/2))/(117*d^2*e) + (2*b^2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e
^3) - (4*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqr
t[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2]) + (2
*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3} + \frac{2\int \sqrt{ex}\sqrt{c+dx^2}\left(\frac{13a^2d}{2}-\frac{1}{2}b(7bc-26ad)x^2\right)dx}{13d}$$

$$= -\frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{117d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3} + \frac{1}{39}\left(39a^2+\frac{bc(7bc-26\right.}{d^2}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e} - \frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{117d^2e} + \frac{2b^2(ex)^{7/2}}{1}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e} - \frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{117d^2e} + \frac{2b^2(ex)^{7/2}}{1}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e} - \frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{117d^2e} + \frac{2b^2(ex)^{7/2}}{1}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e} + \frac{4c\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{195\sqrt{d}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2b(7bc}{}$$

Mathematica [C] time = 0.132363, size = 145, normalized size = 0.34

$$\frac{2\sqrt{ex}\left(6cx\sqrt{\frac{c}{dx^2}+1}\left(39a^2d^2-26abcd+7b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-x\left(c+dx^2\right)\left(-117a^2d^2-26abd\left(2c+5dx^2\right)+b^2\left(14\right.\right.\right.}{585d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-117*a^2*d^2 - 26*a*b*d*(2*c + 5*d*x^2) + b^
2*(14*c^2 - 10*c*d*x^2 - 45*d^2*x^4))) + 6*c*(7*b^2*c^2 - 26*a*b*c*d + 39*a
^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2)
)]))/(585*d^2*Sqrt[c + d*x^2])
```

Maple [A] time = 0.04, size = 658, normalized size = 1.6

$$\frac{2}{585\,d^3x}\sqrt{ex}\left(45\,x^8b^2d^4+130\,x^6abd^4+55\,x^6b^2cd^3+234\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-c}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 2/585*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(45*x^8*b^2*d^4+130*x^6*a*b*d^4+55*x^
6*b^2*c*d^3+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-156*((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b*c^3*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
```

```
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-117*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a^2*c^2*d^2+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-21*
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4+117*x^4*a^2*d^4+182*x^4*a*b*c*d^3-4*x^4
*b^2*c^2*d^2+117*x^2*a^2*c*d^3+52*x^2*a*b*c^2*d^2-14*x^2*b^2*c^3*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2 \sqrt{dx^2+c}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [C] time = 6.68144, size = 148, normalized size = 0.35

$$\frac{a^2\sqrt{c}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{ab\sqrt{c}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^3\Gamma\left(\frac{11}{4}\right)}+\frac{b^2\sqrt{c}\,(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^5\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_
polar(I*pi)/c)/(2*e*gamma(7/4)) + a*b*sqrt(c)*(e*x)**(7/2)*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**3*gamma(11/4)) + b**2*
sqrt(c)*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**5*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2 \sqrt{dx^2+c}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

3.824 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

Optimal. Leaf size=244

$$\frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-22abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-\right.}{231d^2e}$$

```
[Out] (2*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^
2*e) - (2*b*(5*b*c - 22*a*d)*Sqrt[e*x]*(c + d*x^2)^(3/2))/(77*d^2*e) + (2*b
^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3) + (2*c^(3/4)*(5*b^2*c^2 - 22*a
*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt
[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(231*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.203183, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {464, 459, 279, 329, 220}

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-22abcd+5b^2c^2\right)}{231d^2e}+\frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-22abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```


```
[Out] (2*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^
2*e) - (2*b*(5*b*c - 22*a*d)*Sqrt[e*x]*(c + d*x^2)^(3/2))/(77*d^2*e) + (2*b
^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3) + (2*c^(3/4)*(5*b^2*c^2 - 22*a
*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt
[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(231*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}+\frac{2\int \frac{\sqrt{c+dx^2}\left(\frac{11a^2d}{2}-\frac{1}{2}b(5bc-22ad)x^2\right)}{\sqrt{ex}}\,dx}{11d}$$

$$= -\frac{2b(5bc-22ad)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{77d^2e}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}-\frac{1}{77}\left(-77a^2-\frac{bc(5bc-22ad)}{d^2}\right)$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e}-\frac{2b(5bc-22ad)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{77d^2e}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right.}{11de^3}$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e}-\frac{2b(5bc-22ad)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{77d^2e}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right.}{11de^3}$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e}-\frac{2b(5bc-22ad)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{77d^2e}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right.}{11de^3}$$

Mathematica [C] time = 0.196546, size = 189, normalized size = 0.77

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}\left(c+dx^2\right)\left(77a^2d^2+22abd\left(2c+3dx^2\right)+b^2\left(-10c^2+6cdx^2+21d^2x^4\right)\right)}{d^2}+\frac{4icx\sqrt{\frac{c}{dx^2}+1}\left(77a^2d^2-22abcd+5b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{d^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(2*c + 3*d*x^2) + b
^2*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4)))/d^2 + ((4*I)*c*(5*b^2*c^2 - 22*a*b*
```

```
c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]
)/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*
Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 401, normalized size = 1.6

$$\frac{2}{231\,d^3}\left(21\,x^7b^2d^4+77\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}a^2cd^2-22\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(21*x^7*b^2*d^4+77*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)
^(1/2)*a^2*c*d^2-22*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d+5*((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+66*x^5*a*b*d^4+27*x^5*b^2*c*
d^3+77*x^3*a^2*d^4+110*x^3*a*b*c*d^3-4*x^3*b^2*c^2*d^2+77*x*a^2*c*d^3+44*x*
a*b*c^2*d^2-10*x*b^2*c^3*d)/(e*x)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(e*x), x)
```

Sympy [C] time = 7.34902, size = 150, normalized size = 0.61

$$\frac{a^2\sqrt{c}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)}+\frac{ab\sqrt{c}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\Gamma\left(\frac{9}{4}\right)}+\frac{b^2\sqrt{c}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar
(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a*b*sqrt(c)*x**(5/2)*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(9/4)) + b**2*sq
rt(c)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi
)/c)/(2*sqrt(e)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

3.825 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}-\frac{4\sqrt{ex}\sqrt{c}}{}$$

```
[Out] (-2*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*d*
e^3) - (4*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*
d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(c*e*Sqrt[e*
x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*d*e^3) + (4*c^(1/4)*(b^2*c^2
 - 3*a*d*(2*b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (2*c^(1/4)*(b^2*c^2 - 3*a*d*(2*
b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(
7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.395773, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 459, 279, 329, 305, 220, 1196}

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-3ad(5ad+2bc)\right)}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)F}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```


```
[Out] (-2*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*d*
e^3) - (4*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*
d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(c*e*Sqrt[e*
x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*d*e^3) + (4*c^(1/4)*(b^2*c^2
 - 3*a*d*(2*b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (2*c^(1/4)*(b^2*c^2 - 3*a*d*(2*
b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(
7/4)*e^(3/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}} + \frac{2\int \sqrt{ex}\left(\frac{1}{2}a(2bc+5ad)+\frac{1}{2}b^2cx^2\right)\sqrt{c+dx^2}\,dx}{ce^2}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3} - \frac{\left(b^2c^2-3ad(2bc+5ad)\right)\int \sqrt{ex}\sqrt{c+dx^2}\,dx}{3cde^2}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3} - \frac{4\left(b^2c^2-3ad(2bc+5ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)} -$$

Mathematica [C] time = 0.107526, size = 129, normalized size = 0.31

$$\frac{x\left(12x^2\sqrt{\frac{c}{dx^2}+1}\left(15a^2d^2+6abcd-b^2c^2\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+2\left(c+dx^2\right)\left(-45a^2d+18abdx^2+b^2x^2\left(2c+5dx^2\right)\right)\right)}{45d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

```
[Out] (x*(2*(c + d*x^2)*(-45*a^2*d + 18*a*b*d*x^2 + b^2*x^2*(2*c + 5*d*x^2)) + 12
*(-(b^2*c^2) + 6*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 624, normalized size = 1.5

$$\frac{2}{45\,ed^2}\left(5\,x^6b^2d^3+90\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)a^2cd^2+36\,\sqrt{\frac{dx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x)
```

```
[Out] 2/45*(5*x^6*b^2*d^3+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+36*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
```

```
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-45*((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2*c*d^2-18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+18*x^4*a*b*d^3+7*x^4*b^2*c*d^2-45*x^2*a^
2*d^3+18*x^2*a*b*c*d^2+2*x^2*b^2*c^2*d-45*a^2*c*d^2)/(d*x^2+c)^(1/2)/d^2/e/
(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^2*x^2), x
)
```

Sympy [C] time = 8.01265, size = 153, normalized size = 0.36

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{ab\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2\sqrt{c}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)
/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*x**(3/2)*gamma(3/4)*hyper
((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(7/4)) + b**
2*sqrt(c)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(
```

```
I*pi)/c)/(2*e**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

3.826 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-7ad(ad+2bc)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(\ldots}{2\ldots}$$

```
[Out] (-2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d*e^3)
 - (2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*(c + d*
x^2)^(3/2))/(7*d*e^3) - (2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[
d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(5/4)*e^(5/2)*Sqrt[c +
d*x^2])
```

Rubi [A] time = 0.190562, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {462, 459, 279, 329, 220}

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}}-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-7ad(ad+2bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-7a\ldots\right.}{21cde^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```


```
[Out] (-2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d*e^3)
 - (2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*(c + d*
x^2)^(3/2))/(7*d*e^3) - (2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[
d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(5/4)*e^(5/2)*Sqrt[c +
d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}a(2bc+ad)+\frac{3}{2}b^2cx^2\right)\sqrt{c+dx^2}}{\sqrt{ex}}\,dx}{3ce^2}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3} - \frac{\left(b^2c^2-7ad(2bc+ad)\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{ex}}\,dx}{7cde^2}$$

$$= -\frac{2\left(b^2c^2-7ad(2bc+ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3} - \frac{(2(\cdots}{\cdots}$$

$$= -\frac{2\left(b^2c^2-7ad(2bc+ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3} - \frac{(4(\cdots}{\cdots}$$

$$= -\frac{2\left(b^2c^2-7ad(2bc+ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3} - \frac{2(b\cdots}{\cdots}$$

Mathematica [C] time = 0.184193, size = 171, normalized size = 0.73

$$\frac{x^{5/2}\left(\frac{2\left(c+dx^2\right)\left(-7a^2d+14abdx^2+b^2x^2\left(2c+3dx^2\right)\right)}{dx^{3/2}}+\frac{4ix\sqrt{\frac{c}{dx^2}+1}\left(7a^2d^2+14abcd-b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(-7*a^2*d + 14*a*b*d*x^2 + b^2*x^2*(2*c + 3*d*x^2)
))/(d*x^(3/2)) + ((4*I)*(-(b^2*c^2) + 14*a*b*c*d + 7*a^2*d^2)*Sqrt[1 + c/(d
```

```
*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt
[(I*Sqrt[c])/Sqrt[d]]*d)))/(21*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.028, size = 383, normalized size = 1.6

$$\frac{2}{21\,xe^2d^2}\left(7\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)xa^2d^2+14\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] 2/21/(d*x^2+c)^(1/2)/x*(7*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*d^
2+14*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b*c*d-(-c*d)^(1/2)*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*x*b^2*c^2+3*x^6*b^2*d^3+14*x^4*a*b*d^3+5*x^4*b^2*c*d^
2-7*x^2*a^2*d^3+14*x^2*a*b*c*d^2+2*x^2*b^2*c^2*d-7*a^2*c*d^2)/e^2/(e*x)^(1/
2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^3*x^3), x
)
```

Sympy [C] time = 17.4519, size = 153, normalized size = 0.65

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{ab\sqrt{c}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{b^2\sqrt{c}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi)
/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(c)*sqrt(x)*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(5/4)) + b**
2*sqrt(c)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I
*pi)/c)/(2*e**(5/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

3.827 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=421

$$\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}-\frac{4\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{}{}}}{}$$

```
[Out] (2*(b^2*c^2 + a*d*(10*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*c^2*e^5)
+ (4*(b^2*c^2 + a*d*(10*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(5*c*e*(e*x)^(5/2))
 - (2*a*(10*b*c + a*d)*(c + d*x^2)^(3/2))/(5*c^2*e^3*Sqrt[e*x]) - (4*(b^2*c
^2 + a*d*(10*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (2*(b^2*c^2 + a*d*(10*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.395567, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 279, 329, 305, 220, 1196}

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{4\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```



```
[Out] (2*(b^2*c^2 + a*d*(10*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*c^2*e^5)
+ (4*(b^2*c^2 + a*d*(10*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(5*c*e*(e*x)^(5/2))
 - (2*a*(10*b*c + a*d)*(c + d*x^2)^(3/2))/(5*c^2*e^3*Sqrt[e*x]) - (4*(b^2*c
^2 + a*d*(10*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (2*(b^2*c^2 + a*d*(10*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}dx = -\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(10bc+ad)+\frac{5}{2}b^2cx^2\right)\sqrt{c+dx^2}}{(ex)^{3/2}}dx}{5ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} + \frac{(b^2c^2+ad(10bc+ad))\int \sqrt{ex}\sqrt{c+dx^2}dx}{c^2e^4}$$

$$= \frac{2(b^2c^2+ad(10bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} +$$

$$= \frac{2(b^2c^2+ad(10bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} +$$

$$= \frac{2(b^2c^2+ad(10bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} +$$

$$= \frac{2(b^2c^2+ad(10bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} + \frac{4(b^2c^2+ad(10bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{5c\sqrt{d}e^4(\sqrt{c}+\sqrt{d}x)} - \frac{2a^2(c}{5ce}$$

Mathematica [C] time = 0.126577, size = 125, normalized size = 0.3

$$\frac{x\left(4x^4\sqrt{\frac{c}{dx^2}+1}(a^2d^2+10abcd+b^2c^2)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-2(c+dx^2)\left(a^2(c+2dx^2)+10abcx^2-b^2cx^4\right)\right)}{5c(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

```
[Out] (x*(-2*(c + d*x^2)*(10*a*b*c*x^2 - b^2*c*x^4 + a^2*(c + 2*d*x^2)) + 4*(b^2*
c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/4,
 1/2, 3/4, -(c/(d*x^2))]))/(5*c*(e*x)^(7/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 648, normalized size = 1.5

$$\frac{2}{5\,dx^2e^3c}\left(2\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2+20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+20*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*x^2*a*b*c^2*d+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(
```

```
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*
d-((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+x^6*b^2*c*d^2-2*x^4*a^2*d^3-10*x^
4*a*b*c*d^2+x^4*b^2*c^2*d-3*x^2*a^2*c*d^2-10*x^2*a*b*c^2*d-a^2*c^2*d)/(d*x^
2+c)^(1/2)/d/e^3/(e*x)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^4*x^4), x
)
```

Sympy [C] time = 147.089, size = 160, normalized size = 0.38

$$\frac{a^2\sqrt{c}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{ab\sqrt{c}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{b^2\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + a*b*sqrt(c)*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(e**(7/2)*sqrt(x)*gamma(3/4)) +
 b**2*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_pol
```

```
ar(I*pi)/c)/(2*e**(7/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

3.828 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx$

Optimal. Leaf size=213

$$\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(14bc-ad)+7b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}+\frac{2\sqrt{x}\sqrt{c+dx^2}}{}$$

```
[Out] (2*(7*b^2*c^2 + a*d*(14*b*c - a*d))*Sqrt[x]*Sqrt[c + d*x^2])/(21*c^2) - (2*
a^2*(c + d*x^2)^(3/2))/(7*c*x^(7/2)) - (2*a*(14*b*c - a*d)*(c + d*x^2)^(3/2
))/(21*c^2*x^(3/2)) + (2*(7*b^2*c^2 + a*d*(14*b*c - a*d))*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*
Sqrt[x])/c^(1/4)], 1/2])/(21*c^(5/4)*d^(1/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.170769, antiderivative size = 210, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {462, 453, 279, 329, 220}

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}+\frac{2}{21}\sqrt{x}\sqrt{c+dx^2}\left(\frac{ad(14bc-ad)}{c^2}+7b^2\right)+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(14bc-ad)+7b^2c^2\right)F\left(2\text{t}\right.}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*(7*b^2 + (a*d*(14*b*c - a*d))/c^2)*Sqrt[x]*Sqrt[c + d*x^2])/21 - (2*a^2*
(c + d*x^2)^(3/2))/(7*c*x^(7/2)) - (2*a*(14*b*c - a*d)*(c + d*x^2)^(3/2))/(
21*c^2*x^(3/2)) + (2*(7*b^2*c^2 + a*d*(14*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)
*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt
[x])/c^(1/4)], 1/2])/(21*c^(5/4)*d^(1/4)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}+\frac{2\int \frac{\left(\frac{1}{2}a(14bc-ad)+\frac{7}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{5/2}}\,dx}{7c}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}-\frac{2a(14bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^{3/2}}+\frac{1}{7}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{x}}\,dx$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}-\frac{2a(14bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^{3/2}}+\frac{1}{21}$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}-\frac{2a(14bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^{3/2}}+\frac{1}{21}$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}-\frac{2a(14bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^{3/2}}+\frac{2c}{}$$

Mathematica [C] time = 0.204546, size = 160, normalized size = 0.75

$$\frac{2\left(\left(c+dx^2\right)\left(-a^2\left(3c+2dx^2\right)-14abcx^2+7b^2cx^4\right)+\frac{2ix^{9/2}\sqrt{\frac{c}{dx^2}+1}\left(-a^2d^2+14abcd+7b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21cx^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*((c + d*x^2)*(-14*a*b*c*x^2 + 7*b^2*c*x^4 - a^2*(3*c + 2*d*x^2)) + ((2*I
)*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(9/2)*EllipticF[
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]
]))/(21*c*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.046, size = 385, normalized size = 1.8

$$-\frac{2}{21\,cd}\left(\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}x^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x)
```

```
[Out] -2/21/(d*x^2+c)^(1/2)*(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a^2*d^2
-14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a*b*c*d-7*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*(-c*d)^(1/2)*x^3*b^2*c^2-7*x^6*b^2*c*d^2+2*x^4*a^2*d^3+14*x^4*a
*b*c*d^2-7*x^4*b^2*c^2*d+5*x^2*a^2*c*d^2+14*x^2*a*b*c^2*d+3*a^2*c^2*d)/x^(7
/2)/d/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(9/2), x)
```

Sympy [C] time = 140.602, size = 144, normalized size = 0.68

$$\frac{a^2\sqrt{c}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}+\frac{ab\sqrt{c}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{b^2\sqrt{c}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(9/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*x**(7/2)*gamma(-3/4)) + a*b*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2),
 (1/4,), d*x**2*exp_polar(I*pi)/c)/(x**(3/2)*gamma(1/4)) + b**2*sqrt(c)*sqr
t(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2*gam
ma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2 \sqrt{dx^2 + c}}{x^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

3.829 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

Optimal. Leaf size=386

$$\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-ad)+15b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}-\frac{2\sqrt{c+dx^2}}{}$$

```
[Out] (-2*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[c + d*x^2])/(15*c^2*Sqrt[x]) + (4
*Sqrt[d]*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[x]*Sqrt[c + d*x^2])/(15*c^2*
(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(9*c*x^(9/2)) - (2*a*(6*
b*c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^(5/2)) - (4*d^(1/4)*(15*b^2*c^2 + a
*d*(6*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]
*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqr
t[c + d*x^2]) + (2*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c] + Sqrt
[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4
)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.323593, antiderivative size = 383, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {462, 453, 277, 329, 305, 220, 1196}

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}-\frac{2\sqrt{c+dx^2}\left(\frac{ad(6bc-ad)}{c^2}+15b^2\right)}{15\sqrt{x}}+\frac{4\sqrt{d}\sqrt{x}\sqrt{c+dx^2}\left(ad(6bc-ad)+15b^2c^2\right)}{15c^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (-2*(15*b^2 + (a*d*(6*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/(15*Sqrt[x]) + (4*S
qrt[d]*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[x]*Sqrt[c + d*x^2])/(15*c^2*(S
qrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(9*c*x^(9/2)) - (2*a*(6*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^(5/2)) - (4*d^(1/4)*(15*b^2*c^2 + a*d
*(6*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x
)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[
c + d*x^2]) + (2*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*
Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}dx = -\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} + \frac{2\int \frac{\left(\frac{3}{2}a(6bc-ad)+\frac{9}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{7/2}}dx}{9c}$$
$$= -\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)(c+dx^2)^{3/2}}{15c^2x^{5/2}} + \frac{1}{15}\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\int \frac{\sqrt{c+dx^2}}{x^{3/2}}c$$
$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)(c+dx^2)^{3/2}}{15c^2x^{5/2}} + \frac{1}{15}\Big(2d$$
$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)(c+dx^2)^{3/2}}{15c^2x^{5/2}} + \frac{1}{15}\Big(4d$$
$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)(c+dx^2)^{3/2}}{15c^2x^{5/2}} + \frac{1}{15}\Big(4\sqrt{}$$

$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} + \frac{4\sqrt{d}\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2}}{15\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2a^2(c+dx^2)^{3/}}{9cx^{9/2}}$$

Mathematica [C] time = 0.141972, size = 148, normalized size = 0.38

$$\frac{12dx^6\sqrt{\frac{c}{dx^2}+1}\left(-a^2d^2+6abcd+15b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-2\left(c+dx^2\right)\left(a^2\left(5c^2+2cdx^2-6d^2x^4\right)+18abcx^2\left(c+\right.\right.}{45c^2x^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (-2*(c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c + 2*d*x^2) + a^2*(5*c^2 +
 2*c*d*x^2 - 6*d^2*x^4)) + 12*d*(15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*Sqrt[1 +
 c/(d*x^2)]*x^6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))])/(45*c^2*x^
(9/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.05, size = 659, normalized size = 1.7

$$-\frac{2}{45c^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^4a^2cd^2-36\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x)
```

```
[Out] -2/45*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d^2-36*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^4*a*b*c^2*d-90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3-3*((d*x+(-c
```

```
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*x^4*a^2*c*d^2+18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b*c^2
*d+45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3-6*x^6*a^2*d^3+36*x^6*a*b*c*d^
2+45*x^6*b^2*c^2*d-4*x^4*a^2*c*d^2+54*x^4*a*b*c^2*d+45*x^4*b^2*c^3+7*x^2*a^
2*c^2*d+18*x^2*a*b*c^3+5*a^2*c^3)/(d*x^2+c)^(1/2)/x^(9/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

3.830 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{13/2}} \, dx$

Optimal. Leaf size=217

$$\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-22abcd+77b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(5a^2d^2-22abcd+\right.}{231c^2x^{3/2}}$$

```
[Out] (-2*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(231*c^2*x^(3/2)
) - (2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) - (2*a*(22*b*c - 5*a*d)*(c +
d*x^2)^(3/2))/(77*c^2*x^(7/2)) + (2*d^(3/4)*(77*b^2*c^2 - 22*a*b*c*d + 5*a^
2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.189566, antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {462, 453, 277, 329, 220}

$$\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-22abcd+77b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}-\frac{2\sqrt{c+dx^2}\left(77b^2-\right.}{231x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*(77*b^2 - (a*d*(22*b*c - 5*a*d))/c^2)*Sqrt[c + d*x^2])/(231*x^(3/2)) -
(2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) - (2*a*(22*b*c - 5*a*d)*(c + d*x^
2)^(3/2))/(77*c^2*x^(7/2)) + (2*d^(3/4)*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^
2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}+\frac{2\int\frac{\left(\frac{1}{2}a(22bc-5ad)+\frac{11}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{9/2}}\,dx}{11c}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}-\frac{2a(22bc-5ad)\left(c+dx^2\right)^{3/2}}{77c^2x^{7/2}}-\frac{1}{77}\left(-77b^2+\frac{ad(22bc-5ad)}{c^2}\right)\int\frac{\sqrt{c\cdots}}{x\cdots}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}-\frac{2a(22bc-5ad)\left(c+dx^2\right)^{3/2}}{77c^2x^{7/2}}+\frac{1}{23\cdots}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}-\frac{2a(22bc-5ad)\left(c+dx^2\right)^{3/2}}{77c^2x^{7/2}}+\frac{1}{23\cdots}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}-\frac{2a(22bc-5ad)\left(c+dx^2\right)^{3/2}}{77c^2x^{7/2}}+\frac{2d\cdots}{\cdots}$$

Mathematica [C] time = 0.156693, size = 187, normalized size = 0.86

$$-\frac{2\sqrt{c+dx^2}\left(a^2\left(21c^2+6cdx^2-10d^2x^4\right)+22abcx^2\left(3c+2dx^2\right)+77b^2c^2x^4\right)}{231c^2x^{11/2}}+\frac{4idx\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2-22abcd+77b^2\cdots\right.}{231c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[c + d*x^2]*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c + 2*d*x^2) + a^2*(2
1*c^2 + 6*c*d*x^2 - 10*d^2*x^4)))/(231*c^2*x^(11/2)) + (((4*I)/231)*d*(77*b
^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*Sqr
t[c + d*x^2])
```

Maple [A] time = 0.03, size = 403, normalized size = 1.9

$$\frac{2}{231\,c^2}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^5a^2d^2-22\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a^2*d
^2-22*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a*b*c*d+77*((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*b^2*c^2+10*x^6*a^2*d^3-44*x^6*a*b*c*d^2-77*
x^6*b^2*c^2*d+4*x^4*a^2*c*d^2-110*x^4*a*b*c^2*d-77*x^4*b^2*c^3-27*x^2*a^2*c
^2*d-66*x^2*a*b*c^3-21*a^2*c^3)/x^(11/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

3.831 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

Optimal. Leaf size=441

$$\frac{2d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-26abcd+39b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d\sqrt{c+dx^2}\left(7a^2d^2-26abcd\cdots\right.}{195c^3\sqrt{x}}$$

```
[Out] (-2*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(195*c^2*x^(5/2)
) - (4*d*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(195*c^3*Sq
rt[x]) + (4*d^(3/2)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[x]*Sqrt[c +
d*x^2])/(195*c^3*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(13*c*x
^(13/2)) - (2*a*(26*b*c - 7*a*d)*(c + d*x^2)^(3/2))/(117*c^2*x^(9/2)) - (4*
d^(5/4)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(
1/4)], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2]) + (2*d^(5/4)*(39*b^2*c^2 - 26*a
*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(195*c^(11/4)
*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.392979, antiderivative size = 437, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {462, 453, 277, 325, 329, 305, 220, 1196}

$$-\frac{4d\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\sqrt{x}}+\frac{4d^{3/2}\sqrt{x}\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (-2*(39*b^2 - (a*d*(26*b*c - 7*a*d))/c^2)*Sqrt[c + d*x^2])/(195*x^(5/2)) -
(4*d*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(195*c^3*Sqrt[x
]) + (4*d^(3/2)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[x]*Sqrt[c + d*x^
2])/(195*c^3*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(13*c*x^(13
/2)) - (2*a*(26*b*c - 7*a*d)*(c + d*x^2)^(3/2))/(117*c^2*x^(9/2)) - (4*d^(5
/4)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)
], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2]) + (2*d^(5/4)*(39*b^2*c^2 - 26*a*b*c
*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x
)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(195*c^(11/4)*Sqr
t[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}dx = -\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(26bc-7ad)+\frac{13}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{11/2}}dx}{13c}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} - \frac{2a(26bc-7ad)(c+dx^2)^{3/2}}{117c^2x^{9/2}} - \frac{1}{39}\left(-39b^2+\frac{ad(26bc-7ad)}{c^2}\right)\int \frac{\sqrt{c+d}}{x^{7/2}}$$

$$= -\frac{2\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} - \frac{2a(26bc-7ad)(c+dx^2)^{3/2}}{117c^2x^{9/2}} + \frac{1}{195}($$

$$= -\frac{2\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}} - \frac{2a^2(c+dx^2)}{13cx^{13/2}}$$

$$= -\frac{2\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}} - \frac{2a^2(c+dx^2)}{13cx^{13/2}}$$

$$= -\frac{2\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}} - \frac{2a^2(c+dx^2)}{13cx^{13/2}}$$

$$= -\frac{2\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}} + \frac{4d^{3/2}(39b^2c^2}{}$$

Mathematica [C] time = 0.173301, size = 182, normalized size = 0.41

$$\frac{4d^2x^8\sqrt{\frac{dx^2}{c}+1}(7a^2d^2-26abcd+39b^2c^2)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)-2(c+dx^2)\left(a^2(10c^2dx^2+45c^3-14cd^2x^4+42d^3x^6)+2\right.}{585c^3x^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (-2*(c + d*x^2)*(117*b^2*c^2*x^4*(c + 2*d*x^2) + 26*a*b*c*x^2*(5*c^2 + 2*c*
d*x^2 - 6*d^2*x^4) + a^2*(45*c^3 + 10*c^2*d*x^2 - 14*c*d^2*x^4 + 42*d^3*x^6
)) + 4*d^2*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*x^8*Sqrt[1 + (d*x^2)/c]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)])/(585*c^3*x^(13/2)*Sqrt[c + d*
x^2])
```

Maple [A] time = 0.034, size = 706, normalized size = 1.6

$$\frac{2}{585\,c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^6a^2cd^3-156\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x)
```

```
[Out] 2/585*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6*a^2*c*d^3-156*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
```

```
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*x^6*a*b*c^2*d^2+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6*b^2*c^3*d-21*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^6*a^2*c*d^3+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6
*a*b*c^2*d^2-117*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6*b^2*c^3*d-42*x^8*a^2*d^4+1
56*x^8*a*b*c*d^3-234*x^8*b^2*c^2*d^2-28*x^6*a^2*c*d^3+104*x^6*a*b*c^2*d^2-3
51*x^6*b^2*c^3*d+4*x^4*a^2*c^2*d^2-182*x^4*a*b*c^3*d-117*x^4*b^2*c^4-55*x^2
*a^2*c^3*d-130*x^2*a*b*c^4-45*a^2*c^4)/(d*x^2+c)^(1/2)/x^(13/2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 \sqrt{dx^2+c}}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

3.832 $\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=530

$$-\frac{4c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}-\frac{8c^3e^2\sqrt{ex}\sqrt{c+dx^2}\,(}{3315d^7}$$

```
[Out] (8*c^2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/
(9945*d^3) + (4*c*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*Sqrt[c +
 d*x^2])/(1989*d^2*e) - (8*c^3*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e^2*Sqr
t[e*x]*Sqrt[c + d*x^2])/(3315*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(51*a^2*d
^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(663*d^2*e) - (2
*b*(11*b*c - 42*a*d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(357*d^2*e) + (2*b^2*(e
*x)^(11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + (8*c^(13/4)*(51*a^2*d^2 + b*c*(1
1*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(3315*d^(15/4)*Sqrt[c + d*x^2]) - (4*c^(13/4)*(51*a^2*d^2 + b*c*(11*b*c
 - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
3315*d^(15/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.562325, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {464, 459, 279, 321, 329, 305, 220, 1196}

$$-\frac{8c^3e^2\sqrt{ex}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{3315d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{4c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)F\left(2\right.}{3315d^{15/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```


```
[Out] (8*c^2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/
(9945*d^3) + (4*c*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*Sqrt[c +
 d*x^2])/(1989*d^2*e) - (8*c^3*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e^2*Sqr
t[e*x]*Sqrt[c + d*x^2])/(3315*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(51*a^2*d
^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(663*d^2*e) - (2
*b*(11*b*c - 42*a*d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(357*d^2*e) + (2*b^2*(e
*x)^(11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + (8*c^(13/4)*(51*a^2*d^2 + b*c*(1
1*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(3315*d^(15/4)*Sqrt[c + d*x^2]) - (4*c^(13/4)*(51*a^2*d^2 + b*c*(11*b*c
 - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
3315*d^(15/4)*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
```

```
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx = \frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3} + \frac{2\int (ex)^{5/2}\left(c+dx^2\right)^{3/2}\left(\frac{21a^2d}{2} - \frac{1}{2}b(11bc-42ad)x^2\right) dx}{21d}$$

$$= -\frac{2b(11bc-42ad)(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{357d^2e} + \frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3} + \frac{1}{51}\left(51a^2 + \frac{bc}{\ }\right.$$

$$= \frac{2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{663e} - \frac{2b(11bc-42ad)(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{357d^2e}$$

$$= \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c+dx^2}}{1989e} + \frac{2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\left(c+dx\right.}{663e}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c-}}{1989e}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c-}}{1989e}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c-}}{1989e}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c-}}{1989e}$$

Mathematica [C] time = 0.167632, size = 210, normalized size = 0.4

$$\frac{2e(ex)^{3/2}\left(\left(c+dx^2\right)\left(357a^2d^2\left(4c^2+25cdx^2+15d^2x^4\right)+42abd\left(20c^2dx^2-28c^3+285cd^2x^4+195d^3x^6\right)+b^2\left(180c^2d^2\right.\right.\right.}{69615d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*((c + d*x^2)*(357*a^2*d^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4
) + 42*a*b*d*(-28*c^3 + 20*c^2*d*x^2 + 285*c*d^2*x^4 + 195*d^3*x^6) + b^2*(
308*c^4 - 220*c^3*d*x^2 + 180*c^2*d^2*x^4 + 4485*c*d^3*x^6 + 3315*d^4*x^8))
 - 84*c^3*(11*b^2*c^2 - 42*a*b*c*d + 51*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(69615*d^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 743, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -2/69615/x*e^2*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-3315*x^12*b^2*d^6-8190*x^1
0*a*b*d^6-7800*x^10*b^2*c*d^5-5355*x^8*a^2*d^6-20160*x^8*a*b*c*d^5-4665*x^8
```

```
*b^2*c^2*d^4-14280*x^6*a^2*c*d^5-12810*x^6*a*b*c^2*d^4+40*x^6*b^2*c^3*d^3+4
284*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^4*d^2-3528*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^5*d+924*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^6-2142*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*c^4*d^2+1764*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^5*d-462*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*b^2*c^6-10353*x^4*a^2*c^2*d^4+336*x^4*a*b*c^3*d^3-88*
x^4*b^2*c^4*d^2-1428*x^2*a^2*c^3*d^3+1176*x^2*a*b*c^4*d^2-308*x^2*b^2*c^5*d
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2de^2x^8+\left(b^2c+2\,abd\right)e^2x^6+a^2ce^2x^2+\left(2\,abc+a^2d\right)e^2x^4\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*e^2*x^8 + (b^2*c + 2*a*b*d)*e^2*x^6 + a^2*c*e^2*x^2 + (2*a*
b*c + a^2*d)*e^2*x^4)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2), x)
```

3.833 $\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=340

$$-\frac{4c^{11/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(57a^2d^2+bc(9bc-38ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}\left(57a^2\right.}{4389}$$

```
[Out] (8*c^2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(43
89*d^3) + (4*c*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*Sqrt[c + d*x
^2])/(1463*d^2*e) + (2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*(c +
 d*x^2)^(3/2))/(627*d^2*e) - (2*b*(9*b*c - 38*a*d)*(e*x)^(5/2)*(c + d*x^2)^
(5/2))/(285*d^2*e) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) - (4*
c^(11/4)*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4389*d^(13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.323289, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {464, 459, 279, 321, 329, 220}

$$-\frac{4c^{11/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(57a^2d^2+bc(9bc-38ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}\left(57a^2d^2+bc(9\right.}{4389d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (8*c^2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(43
89*d^3) + (4*c*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*Sqrt[c + d*x
^2])/(1463*d^2*e) + (2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*(c +
 d*x^2)^(3/2))/(627*d^2*e) - (2*b*(9*b*c - 38*a*d)*(e*x)^(5/2)*(c + d*x^2)^
(5/2))/(285*d^2*e) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) - (4*
c^(11/4)*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4389*d^(13/4)*Sqrt[c + d*x^2])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}+\frac{2\int (ex)^{3/2}\left(c+dx^2\right)^{3/2}\left(\frac{19a^2d}{2}-\frac{1}{2}b(9bc-38ad)x^2\right)dx}{19d}$$

$$= -\frac{2b(9bc-38ad)(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{285d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}+\frac{1}{57}\left(57a^2+\frac{bc(9\cdots\right.$$

$$= \frac{2\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{627e}-\frac{2b(9bc-38ad)(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{285d^2e}+\cdots$$

$$= \frac{4c\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{1463e}+\frac{2\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\left(c+dx^2\cdots\right.}{627e}$$

$$= \frac{8c^2\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{4389d}+\frac{4c\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx\cdots}}{1463e}$$

$$= \frac{8c^2\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{4389d}+\frac{4c\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx\cdots}}{1463e}$$

$$= \frac{8c^2\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{4389d}+\frac{4c\left(57a^2+\frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx\cdots}}{1463e}$$

Mathematica [C] time = 0.263317, size = 259, normalized size = 0.76

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(285a^2d^2(4c^2+13cdx^2+7d^2x^4)+38abd(12c^2dx^2-20c^3+119cd^2x^4+77d^3x^6)+3b^2(28c^2d^2x^4-36c^3dx^2+60c^4+539cd^3x^6+385d^4x^8)\right)}{5d^3}-\frac{8ic^3x\sqrt{\cdots}}{\cdots}\right)}{4389x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(285*a^2*d^2*(4*c^2 + 13*c*d*x^2 + 7*d
^2*x^4) + 38*a*b*d*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^4 + 77*d^3*x^6) +
3*b^2*(60*c^4 - 36*c^3*d*x^2 + 28*c^2*d^2*x^4 + 539*c*d^3*x^6 + 385*d^4*x^8
)))/(5*d^3) - ((8*I)*c^3*(9*b^2*c^2 - 38*a*b*c*d + 57*a^2*d^2)*Sqrt[1 + c/(
d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqr
t[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(4389*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.026, size = 489, normalized size = 1.4

$$-\frac{2e}{21945\,xd^4}\sqrt{ex}\left(-1155\,x^{11}b^2d^6-2926\,x^9abd^6-2772\,x^9b^2cd^5-1995\,x^7a^2d^6-7448\,x^7abcd^5-1701\,x^7b^2c^2d^4+570\,\sqrt{-c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -2/21945*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1155*x^11*b^2*d^6-2926*x^9*a*b*d
^6-2772*x^9*b^2*c*d^5-1995*x^7*a^2*d^6-7448*x^7*a*b*c*d^5-1701*x^7*b^2*c^2*
d^4+570*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^3*d^2-380*(-c*d)^(1/
2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^4*d+90*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*b^2*c^5-5700*x^5*a^2*c*d^5-4978*x^5*a*b*c^2*d^4+24*x^5*b^2*c^3*d^
3-4845*x^3*a^2*c^2*d^4+304*x^3*a*b*c^3*d^3-72*x^3*b^2*c^4*d^2-1140*x*a^2*c^
3*d^3+760*x*a*b*c^4*d^2-180*x*b^2*c^5*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dex^7 + \left(b^2c + 2\,abd\right)ex^5 + a^2cex + \left(2\,abc + a^2d\right)ex^3\right)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*e*x^7 + (b^2*c + 2*a*b*d)*e*x^5 + a^2*c*e*x + (2*a*b*c + a^
2*d)*e*x^3)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^2 \left(dx^2 + c\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2), x)
```

3.834 $\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=482

$$\frac{4c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}}+\frac{8c^2\sqrt{ex}\sqrt{c+dx^2}\left(221a^2\right.}{3315d^{5/2}\left(\sqrt{\right.}}$$

```
[Out] (4*c*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(3
315*d^2*e) + (8*c^2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*Sqrt[e*x]*Sqrt[c
 + d*x^2])/(3315*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(221*a^2*d^2 + 3*b*c*(
7*b*c - 34*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(1989*d^2*e) - (2*b*(7*b*c
- 34*a*d)*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(221*d^2*e) + (2*b^2*(e*x)^(7/2)*(
c + d*x^2)^(5/2))/(17*d*e^3) - (8*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*
a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d
^(11/4)*Sqrt[c + d*x^2]) + (4*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d)
)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(11
/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.468646, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 279, 329, 305, 220, 1196}

$$\frac{8c^2\sqrt{ex}\sqrt{c+dx^2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{4c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)F\left(2\tan^{-}\right.}{3315d^{11/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```


```
[Out] (4*c*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(3
315*d^2*e) + (8*c^2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*Sqrt[e*x]*Sqrt[c
 + d*x^2])/(3315*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(221*a^2*d^2 + 3*b*c*(
7*b*c - 34*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(1989*d^2*e) - (2*b*(7*b*c
- 34*a*d)*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(221*d^2*e) + (2*b^2*(e*x)^(7/2)*(
c + d*x^2)^(5/2))/(17*d*e^3) - (8*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*
a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d
^(11/4)*Sqrt[c + d*x^2]) + (4*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d)
)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(11
/4)*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}+\frac{2\int \sqrt{ex}\left(c+dx^2\right)^{3/2}\left(\frac{17a^2d}{2}-\frac{1}{2}b(7bc-34ad)x^2\right)dx}{17d}$$

$$= -\frac{2b(7bc-34ad)(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{221d^2e}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}-\frac{1}{221}\left(-221a^2-\frac{3bc(7 \ldots\right.$$

$$= \frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{1989e}-\frac{2b(7bc-34ad)(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{221d^2e}+ \ldots$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e}+\frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\left(c+dx^2\right) \ldots}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e}+\frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\left(c+dx^2\right) \ldots}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e}+\frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\left(c+dx^2\right) \ldots}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e}+\frac{8c^2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{3315\sqrt{d}\left(\sqrt{c}+\sqrt{d}x\right)}$$

Mathematica [C] time = 0.150643, size = 179, normalized size = 0.37

$$\frac{2\sqrt{ex}\left(12c^2x\sqrt{\frac{c}{dx^2}+1}\left(221a^2d^2-102abcd+21b^2c^2\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-x\left(c+dx^2\right)\left(-221a^2d^2\left(11c+5dx^2\right)-102abd \ldots\right.\right.}{9945d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-221*a^2*d^2*(11*c + 5*d*x^2) - 102*a*b*d*(4
*c^2 + 25*c*d*x^2 + 15*d^2*x^4) + b^2*(84*c^3 - 60*c^2*d*x^2 - 855*c*d^2*x^
4 - 585*d^3*x^6))) + 12*c^2*(21*b^2*c^2 - 102*a*b*c*d + 221*a^2*d^2)*Sqrt[1
 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(9945*d^2
*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 699, normalized size = 1.5

$$\frac{2}{9945\,d^3x}\sqrt{ex}\left(585\,x^{10}b^2d^5+1530\,x^8abd^5+1440\,x^8b^2cd^4+1105\,x^6a^2d^5+4080\,x^6abcd^4+915\,x^6b^2c^2d^3+2652\sqrt{\frac{dx+\ldots}{\sqrt{-c \ldots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x)
```

```
[Out] 2/9945/(d*x^2+c)^(1/2)*(e*x)^(1/2)/d^3*(585*x^10*b^2*d^5+1530*x^8*a*b*d^5+1
440*x^8*b^2*c*d^4+1105*x^6*a^2*d^5+4080*x^6*a*b*c*d^4+915*x^6*b^2*c^2*d^3+2
652*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
```

```
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^3*d^2-1224*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^4*d+252*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^5-1326*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*c^3*d^2+612*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^4*d-126*((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*b^2*c^5+3536*x^4*a^2*c*d^4+2958*x^4*a*b*c^2*d^3-24*x^4
*b^2*c^3*d^2+2431*x^2*a^2*c^2*d^3+408*x^2*a*b*c^3*d^2-84*x^2*b^2*c^4*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [C] time = 26.8029, size = 304, normalized size = 0.63

$$\frac{a^2c^{\frac{3}{2}}\left(ex\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{a^2\sqrt{c}d\left(ex\right)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}+\frac{abc^{\frac{3}{2}}\left(ex\right)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^3\Gamma\left(\frac{11}{4}\right)}+\frac{ab\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)*(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp
_polar(I*pi)/c)/(2*e*gamma(7/4)) + a**2*sqrt(c)*d*(e*x)**(7/2)*gamma(7/4)*h
```

```
yper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**3*gamma(11/4)) +
 a*b*c**(3/2)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*ex
p_polar(I*pi)/c)/(e**3*gamma(11/4)) + a*b*sqrt(c)*d*(e*x)**(11/2)*gamma(11/
4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(e**5*gamma(15/4)
) + b**2*c**(3/2)*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*
x**2*exp_polar(I*pi)/c)/(2*e**5*gamma(15/4)) + b**2*sqrt(c)*d*(e*x)**(15/2)
*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**7
*gamma(19/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} \sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x), x)
```

3.835 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{\sqrt{ex}}\,dx$

Optimal. Leaf size=286

$$\frac{4c^{7/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(33a^2d^2+bc(bc-6ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(33a^2d^2+\right.}{231d^2e}$$

```
[Out] (4*c*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^2*e
) + (2*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))/(231*d
^2*e) - (2*b*(b*c - 6*a*d)*Sqrt[e*x]*(c + d*x^2)^(5/2))/(33*d^2*e) + (2*b^2
*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15*d*e^3) + (4*c^(7/4)*(33*a^2*d^2 + b*c*(
b*c - 6*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^
(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.266818, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {464, 459, 279, 329, 220}

$$\frac{4c^{7/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(33a^2d^2+bc(bc-6ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(33a^2d^2+bc(bc-6\right.}{231d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (4*c*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^2*e
) + (2*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))/(231*d
^2*e) - (2*b*(b*c - 6*a*d)*Sqrt[e*x]*(c + d*x^2)^(5/2))/(33*d^2*e) + (2*b^2
*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15*d*e^3) + (4*c^(7/4)*(33*a^2*d^2 + b*c*(
b*c - 6*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^
(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{\sqrt{ex}}\,dx = \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3} + \frac{2\int \frac{\left(c+dx^2\right)^{3/2}\left(\frac{15a^2d}{2}-\frac{5}{2}b(bc-6ad)x^2\right)}{\sqrt{ex}}\,dx}{15d}$$

$$= -\frac{2b(bc-6ad)\sqrt{ex}\left(c+dx^2\right)^{5/2}}{33d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3} + \frac{1}{33}\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\int$$

$$= \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{231e} - \frac{2b(bc-6ad)\sqrt{ex}\left(c+dx^2\right)^{5/2}}{33d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right.}{15de^3}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{231e} - \frac{2b(bc-6a}{}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{231e} - \frac{2b(bc-6a}{}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{231e} - \frac{2b(bc-6a}{}$$

Mathematica [C] time = 0.229998, size = 223, normalized size = 0.78

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}\left(c+dx^2\right)\left(165a^2d^2\left(3c+dx^2\right)+30abd\left(4c^2+13cdx^2+7d^2x^4\right)+b^2\left(12c^2dx^2-20c^3+119cd^2x^4+77d^3x^6\right)\right)}{5d^2} + \frac{8ic^2x\sqrt{\frac{c}{dx^2}+1}\left(33a^2d^2-6abcd+b^2c^2\right)\text{EllipticF}\left(i\sin\right.}{d^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{231\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(165*a^2*d^2*(3*c + d*x^2) + 30*a*b*d*(4*c
^2 + 13*c*d*x^2 + 7*d^2*x^4) + b^2*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^4
+ 77*d^3*x^6)))/(5*d^2) + ((8*I)*c^2*(b^2*c^2 - 6*a*b*c*d + 33*a^2*d^2)*Sqr
t[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
-1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.016, size = 444, normalized size = 1.6

$$\frac{2}{1155\,d^3}\left(77\,x^9b^2d^5+210\,x^7abd^5+196\,x^7b^2cd^4+330\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-c}}{\sqrt{-c}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 2/1155/(d*x^2+c)^(1/2)*(77*x^9*b^2*d^5+210*x^7*a*b*d^5+196*x^7*b^2*c*d^4+33
0*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c^2*d^2-60*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*a*b*c^3*d+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/
2)*b^2*c^4+165*x^5*a^2*d^5+600*x^5*a*b*c*d^4+131*x^5*b^2*c^2*d^3+660*x^3*a^
2*c*d^4+510*x^3*a*b*c^2*d^3-8*x^3*b^2*c^3*d^2+495*x*a^2*c^2*d^3+120*x*a*b*c
^3*d^2-20*x*b^2*c^4*d)/d^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex}}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x)/(e*x), x)
```

Sympy [C] time = 30.0793, size = 306, normalized size = 1.07

$$\frac{a^2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)}+\frac{a^2\sqrt{c}dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)}+\frac{abc^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\Gamma\left(\frac{9}{4}\right)}+\frac{ab\sqrt{c}dx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F}{\sqrt{e}\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a**2*sqrt(c)*d*x**(5/2)*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(9/4)) + a
*b*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar
(I*pi)/c)/(sqrt(e)*gamma(9/4)) + a*b*sqrt(c)*d*x**(9/2)*gamma(9/4)*hyper((-
1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(13/4)) + b**2*
c**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I
*pi)/c)/(2*sqrt(e)*gamma(13/4)) + b**2*sqrt(c)*d*x**(13/2)*gamma(13/4)*hype
r((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(17/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

3.836 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=476

$$-\frac{4c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-13ad(9ad+2bc)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}-\frac{8c\sqrt{ex}}{}$$

```
[Out] (-4*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*
d*e^3) - (8*c*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2
])/(195*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*(3*b^2*c^2 - 13*a*d*(2*b*c
+ 9*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(117*c*d*e^3) - (2*a^2*(c + d*x^2)
^(5/2))/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e^3)
+ (8*c^(5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (4*c^(
5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.453706, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 459, 279, 329, 305, 220, 1196}

$$-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}-\frac{8c\sqrt{ex}\sqrt{c+dx^2}\left(3b^2c^2-13ad(9ad+2bc)\right)}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{4c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-13ad(9ad+2\right.}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (-4*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*
d*e^3) - (8*c*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2
])/(195*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*(3*b^2*c^2 - 13*a*d*(2*b*c
+ 9*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(117*c*d*e^3) - (2*a^2*(c + d*x^2)
^(5/2))/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e^3)
+ (8*c^(5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (4*c^(
5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}} dx = -\frac{2a^2 (c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2\int \sqrt{ex}\left(\frac{1}{2}a(2bc+9ad)+\frac{1}{2}b^2cx^2\right)(c+dx^2)^{3/2} dx}{ce^2}$$

$$= -\frac{2a^2 (c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}(c+dx^2)^{5/2}}{13de^3} - \frac{\left(4\left(\frac{3b^2c^2}{4}-\frac{13}{4}ad(2bc+9ad)\right)\right)\int \sqrt{ex}\,(c+}{13cde^2}$$

$$= -\frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c+dx^2)^{3/2}}{117cde^3} - \frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}(c+}{13de^3}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c}{117cde^3}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c}{117cde^3}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c}{117cde^3}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{8c(3b^2c^2-13ad(2bc+9ad))\sqrt{ex}\sqrt{c}}{195d^{3/2}e^2(\sqrt{c}+\sqrt{d}x)}$$

Mathematica [C] time = 0.145505, size = 161, normalized size = 0.34

$$\frac{x\left(24cx^2\sqrt{\frac{c}{dx^2}+1}\,(117a^2d^2+26abcd-3b^2c^2)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+2(c+dx^2)(117a^2d(dx^2-5c)+26abdx^2(11c+\right.}{585d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (x*(2*(c + d*x^2)*(117*a^2*d*(-5*c + d*x^2) + 26*a*b*d*x^2*(11*c + 5*d*x^2)
 + 3*b^2*x^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4)) + 24*c*(-3*b^2*c^2 + 26*a*b
*c*d + 117*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c/(d*x^2))]))/(585*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.02, size = 669, normalized size = 1.4

$$\frac{2}{585\,ed^2}\left(45\,x^8b^2d^4+130\,x^6abd^4+120\,x^6b^2cd^3+1404\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+}{\sqrt{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x)
```

```
[Out] 2/585*(45*x^8*b^2*d^4+130*x^6*a*b*d^4+120*x^6*b^2*c*d^3+1404*((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
```

```
,1/2*2^(1/2))*a^2*c^2*d^2+312*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-36*((d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2),1/2*2^(1/2))*b^2*c^4-702*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2
-156*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+18*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b
^2*c^4+117*x^4*a^2*d^4+416*x^4*a*b*c*d^3+87*x^4*b^2*c^2*d^2-468*x^2*a^2*c*d
^3+286*x^2*a*b*c^2*d^2+12*x^2*b^2*c^3*d-585*a^2*c^2*d^2)/(d*x^2+c)^(1/2)/d^
2/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] time = 35.2059, size = 309, normalized size = 0.65

$$\frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{a^2\sqrt{c}dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{abc^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{ab\sqrt{c}dx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F}{e^{\frac{3}{2}}\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a**2*sqrt(c)*d*x**(3/2)*gamma(3/4)*h
yper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma(7/4))
 + a*b*c**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_p
olar(I*pi)/c)/(e**(3/2)*gamma(7/4)) + a*b*sqrt(c)*d*x**(7/2)*gamma(7/4)*hyp
er((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(11/4)) +
 b**2*c**(3/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**(3/2)*gamma(11/4)) + b**2*sqrt(c)*d*x**(11/2)*gamma(11/
4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma
(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

3.837 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{4c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-11ad(7ad+6bc)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}-\frac{2\sqrt{ex}\left(c+\right.}{}$$

```
[Out] (-4*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d*
e^3) - (2*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))
/(231*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqr
t[e*x]*(c + d*x^2)^(5/2))/(11*d*e^3) - (4*c^(3/4)*(3*b^2*c^2 - 11*a*d*(6*b*
c + 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(5
/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.241653, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {462, 459, 279, 329, 220}

$$-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}-\frac{4c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-11ad(7ad+6bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```


```
[Out] (-4*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d*
e^3) - (2*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))
/(231*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqr
t[e*x]*(c + d*x^2)^(5/2))/(11*d*e^3) - (4*c^(3/4)*(3*b^2*c^2 - 11*a*d*(6*b*
c + 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(5
/4)*e^(5/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{5/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(6bc+7ad)+\frac{3}{2}b^2cx^2\right)\left(c+dx^2\right)^{3/2}}{\sqrt{ex}}\,dx}{3ce^2}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de^3} - \frac{\left(3b^2c^2-11ad(6bc+7ad)\right)\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{ex}}\,dx}{33cde^2}$$

$$= -\frac{2\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\left(c+dx^2\right)^{3/2}}{231cde^3} - \frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\left(c+dx^2\right)}{11de^3}$$

$$= -\frac{4\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\left(c+d\right.}{231cde^3}$$

$$= -\frac{4\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\left(c+d\right.}{231cde^3}$$

$$= -\frac{4\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2\left(3b^2c^2-11ad(6bc+7ad)\right)\sqrt{ex}\left(c+d\right.}{231cde^3}$$

Mathematica [C] time = 0.228935, size = 202, normalized size = 0.7

$$\frac{x^{5/2}\left(\frac{2\left(c+dx^2\right)\left(77a^2d\left(dx^2-c\right)+66abdx^2\left(3c+dx^2\right)+3b^2x^2\left(4c^2+13cdx^2+7d^2x^4\right)\right)}{dx^{3/2}} + \frac{8icx\sqrt{\frac{c}{dx^2}+1}\left(77a^2d^2+66abcd-3b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(77*a^2*d*(-c + d*x^2) + 66*a*b*d*x^2*(3*c + d*x^2
) + 3*b^2*x^2*(4*c^2 + 13*c*d*x^2 + 7*d^2*x^4)))/(d*x^(3/2)) + ((8*I)*c*(-3
*b^2*c^2 + 66*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))
/(231*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 415, normalized size = 1.4

$$\frac{2}{231\,xd^2e^2}\left(21\,x^8b^2d^4+154\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{-cd}xa^2cd^2+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)/x*(21*x^8*b^2*d^4+154*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c
*d)^(1/2)*x*a^2*c*d^2+132*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*a*b*c^
2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*b^2*c^3+66*x^6*a*b*d^4+60*
x^6*b^2*c*d^3+77*x^4*a^2*d^4+264*x^4*a*b*c*d^3+51*x^4*b^2*c^2*d^2+198*x^2*a
*b*c^2*d^2+12*x^2*b^2*c^3*d-77*a^2*c^2*d^2)/d^2/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^3*x^3), x)
```

Sympy [C] time = 55.4485, size = 309, normalized size = 1.07

$$\frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{a^2\sqrt{c}d\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{abc^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ab\sqrt{c}dx^{\frac{5}{2}}\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(5/2),x)
```

```
[Out] a**2*c**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a**2*sqrt(c)*d*sqrt(x)*gamma(1/4)*h
yper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma(5/4))
 + a*b*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_po
lar(I*pi)/c)/(e**(5/2)*gamma(5/4)) + a*b*sqrt(c)*d*x**(5/2)*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(9/4)) + b*
*2*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar
(I*pi)/c)/(2*e**(5/2)*gamma(9/4)) + b**2*sqrt(c)*d*x**(9/2)*gamma(9/4)*hype
r((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

3.838 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} dx$

Optimal. Leaf size=468

$$\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{5ce(ex)^{5/2}}-\frac{8\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\right.}{}$$

```
[Out] (4*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*e^5)
+ (8*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) + (2*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2
)*(c + d*x^2)^(3/2))/(9*c^2*e^5) - (2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(
5/2)) - (2*a*(2*b*c + a*d)*(c + d*x^2)^(5/2))/(c^2*e^3*Sqrt[e*x]) - (8*c^(1
/4)*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/
(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (4*c^(1/4)*(b^2*c^2 +
 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqr
t[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])
/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.444668, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 279, 329, 305, 220, 1196}

$$-\frac{2a^2\left(c+dx^2\right)^{5/2}}{5ce(ex)^{5/2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{8\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{}{(\sqrt{}}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```


```
[Out] (4*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*e^5)
+ (8*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) + (2*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2
)*(c + d*x^2)^(3/2))/(9*c^2*e^5) - (2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(
5/2)) - (2*a*(2*b*c + a*d)*(c + d*x^2)^(5/2))/(c^2*e^3*Sqrt[e*x]) - (8*c^(1
/4)*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/
(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (4*c^(1/4)*(b^2*c^2 +
 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqr
t[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])
/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} dx = -\frac{2a^2 (c+dx^2)^{5/2}}{5ce(ex)^{5/2}} + \frac{2\int \frac{\left(\frac{5}{2}a(2bc+ad)+\frac{5}{2}b^2cx^2\right)(c+dx^2)^{3/2}}{(ex)^{3/2}} dx}{5ce^2}$$

$$= -\frac{2a^2 (c+dx^2)^{5/2}}{5ce(ex)^{5/2}} - \frac{2a(2bc+ad)(c+dx^2)^{5/2}}{c^2e^3\sqrt{ex}} + \frac{(b^2c^2+9ad(2bc+ad))\int \sqrt{ex}(c+dx^2)^{3}}{c^2e^4}$$

$$= \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}} - \frac{2a(2bc+ad)(c+dx^2)^{5}}{c^2e^3\sqrt{ex}}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{8(b^2c^2+9ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{15\sqrt{d}e^4(\sqrt{c}+\sqrt{d}x)} + \frac{2(}{}$$

Mathematica [C] time = 0.162997, size = 141, normalized size = 0.3

$$\frac{x\left(24x^4\sqrt{\frac{c}{dx^2}+1}(9a^2d^2+18abcd+b^2c^2)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-2(c+dx^2)(9a^2(c+7dx^2)-18abx^2(dx^2-5c)-b^2x^4(1\right.}{45(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

```
[Out] (x*(-2*(c + d*x^2)*(-18*a*b*x^2*(-5*c + d*x^2) - b^2*x^4*(11*c + 5*d*x^2) +
 9*a^2*(c + 7*d*x^2)) + 24*(b^2*c^2 + 18*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + c/(d
*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*(e*x)^(7/2
)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 668, normalized size = 1.4

$$\frac{2}{45\,dx^2e^3}\left(5\,b^2d^3x^8+108\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2+216\sqrt{\frac{d}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x)
```

```
[Out] 2/45/x^2*(5*b^2*d^3*x^8+108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+216*(
```

```
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*b^2*c^3-54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-108*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+18*x^6*a
*b*d^3+16*x^6*b^2*c*d^2-63*x^4*a^2*d^3-72*x^4*a*b*c*d^2+11*x^4*b^2*c^2*d-72
*x^2*a^2*c*d^2-90*x^2*a*b*c^2*d-9*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

3.839 $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=430

$$-\frac{c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}+\frac{2c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right.}{\ldots}$$

```
[Out] (2*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
585*d^3) - (2*b*(11*b*c - 26*a*d)*(e*x)^(7/2)*Sqrt[c + d*x^2])/(117*d^2*e)
+ (2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*e^3) - (2*c*(117*a^2*d^2 + 7*b
*c*(11*b*c - 26*a*d))*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(195*d^(7/2)*(Sqrt[c]
+ Sqrt[d]*x)) + (2*c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c
+ d*x^2]) - (c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.405569, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {464, 459, 321, 329, 305, 220, 1196}

$$-\frac{c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}+\frac{2c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+\ldots}{(\sqrt{c}+\ldots}}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```


```
[Out] (2*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
585*d^3) - (2*b*(11*b*c - 26*a*d)*(e*x)^(7/2)*Sqrt[c + d*x^2])/(117*d^2*e)
+ (2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*e^3) - (2*c*(117*a^2*d^2 + 7*b
*c*(11*b*c - 26*a*d))*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(195*d^(7/2)*(Sqrt[c]
+ Sqrt[d]*x)) + (2*c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c
+ d*x^2]) - (c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c + d*x^
2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3} + \frac{2\int \frac{(ex)^{5/2}\left(\frac{13a^2d}{2}-\frac{1}{2}b(11bc-26ad)x^2\right)}{\sqrt{c+dx^2}}dx}{13d}$$

$$= -\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3} - \frac{1}{117}\left(-117a^2 - \frac{7bc(11bc-26ad}{d^2}\right.$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d} - \frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}}{13de}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d} - \frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}}{13de}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d} - \frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}}{13de}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d} - \frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}}{13de}$$

Mathematica [C] time = 0.137503, size = 143, normalized size = 0.33

$$\frac{2e(ex)^{3/2}\left(\left(c+dx^2\right)\left(117a^2d^2+26abd\left(5dx^2-7c\right)+b^2\left(77c^2-55cdx^2+45d^2x^4\right)\right)-3c\sqrt{\frac{c}{dx^2}+1}\left(117a^2d^2-182abcd+\right.\right.}{585d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*e*(e*x)^(3/2)*((c + d*x^2)*(117*a^2*d^2 + 26*a*b*d*(-7*c + 5*d*x^2) + b^
2*(77*c^2 - 55*c*d*x^2 + 45*d^2*x^4)) - 3*c*(77*b^2*c^2 - 182*a*b*c*d + 117
*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2)
)]))/(585*d^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.028, size = 661, normalized size = 1.5

$$-\frac{e^2}{585\,xd^4}\sqrt{ex}\left(-90\,x^8b^2d^4-260\,x^6abd^4+20\,x^6b^2cd^3+702\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/585/x*e^2*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-90*x^8*b^2*d^4-260*x^6*a*b*d
^4+20*x^6*b^2*c*d^3+702*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-1092*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
```

```
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-351
*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+546*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b*c^3*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-234*x^4*a^2*d^4+104*x^4*a*b
*c*d^3-44*x^4*b^2*c^2*d^2-234*x^2*a^2*c*d^3+364*x^2*a*b*c^2*d^2-154*x^2*b^2
*c^3*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2\right)\sqrt{ex}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(e*x)/sqrt(d*x^2 +
 c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

3.840 $\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=240

$$-\frac{c^{3/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2+5bc(9bc-22ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2\right.}{231d^3}$$

```
[Out] (2*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) - (2*b*(9*b*c - 22*a*d)*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*d^2*e) + (2*b
^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) - (c^(3/4)*(77*a^2*d^2 + 5*b*c*(
9*b*c - 22*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.219103, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {464, 459, 321, 329, 220}

$$-\frac{c^{3/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2+5bc(9bc-22ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2+5bc(9b\right.}{231d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```


```
[Out] (2*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) - (2*b*(9*b*c - 22*a*d)*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*d^2*e) + (2*b
^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) - (c^(3/4)*(77*a^2*d^2 + 5*b*c*(
9*b*c - 22*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}+\frac{2\int \frac{(ex)^{3/2}\left(\frac{11a^2d}{2}-\frac{1}{2}b(9bc-22ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{11d}$$

$$= -\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}-\frac{1}{77}\left(-77a^2-\frac{5bc(9bc-22ad)}{d^2}\right)\int$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}-\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+d}}{11de^3}$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}-\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+d}}{11de^3}$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d}-\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+d}}{11de^3}$$

Mathematica [C] time = 0.211704, size = 190, normalized size = 0.79

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}\left(c+dx^2\right)\left(77a^2d^2+22abd\left(3dx^2-5c\right)+3b^2\left(15c^2-9cdx^2+7d^2x^4\right)\right)}{d^3}-\frac{2icx\sqrt{\frac{c}{dx^2}+1}\left(77a^2d^2-110abcd+45b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(-5*c + 3*d*x^2
) + 3*b^2*(15*c^2 - 9*c*d*x^2 + 7*d^2*x^4)))/d^3 - ((2*I)*c*(45*b^2*c^2 - 1
10*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(
```

```
3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.026, size = 405, normalized size = 1.7

$$-\frac{e}{231\,xd^4}\sqrt{ex}\left(-42\,x^7b^2d^4+77\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{-cd}a^2cd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/231*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-42*x^7*b^2*d^4+77*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d^2-110*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(
1/2)*a*b*c^2*d+45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3-132*x^5*
a*b*d^4+12*x^5*b^2*c*d^3-154*x^3*a^2*d^4+88*x^3*a*b*c*d^3-36*x^3*b^2*c^2*d^
2-154*x*a^2*c*d^3+220*x*a*b*c^2*d^2-90*x*b^2*c^3*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2ex^5+2\,abex^3+a^2ex\right)\sqrt{ex}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

Sympy [C] time = 46.3341, size = 144, normalized size = 0.6

$$\frac{a^2e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{9}{4}\right)}+\frac{abe^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\Gamma\left(\frac{13}{4}\right)}+\frac{b^2e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*gamma(9/4)) + a*b*e**(3/2)*x**(9/2)*gamma(9/4)*hyper(
(1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(13/4)) + b**2
*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((1/2, 13/4), (17/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*gamma(17/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

3.841 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=375

$$\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2+bc(\right.}{15d^{5/2}\left(\sqrt{c}+\sqrt{dx}\right.}$$

```
[Out] (-2*b*(7*b*c - 18*a*d)*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*d^2*e) + (2*b^2*(e*
x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + (2*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d)
)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*c^(1/4
)*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2]) + (c^(1/4)*(15*a^2*d
^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.355625, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {464, 459, 329, 305, 220, 1196}

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2+bc(7bc-18ad)\right)}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{15d^{11/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*b*(7*b*c - 18*a*d)*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*d^2*e) + (2*b^2*(e*
x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + (2*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d)
)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*c^(1/4
)*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2]) + (c^(1/4)*(15*a^2*d
^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{2\int \frac{\sqrt{ex}\left(\frac{9a^2d}{2}-\frac{1}{2}b(7bc-18ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{9d}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{1}{15}\left(15a^2+\frac{bc(7bc-18ad)}{d^2}\right)\int \frac{\sqrt{}}{\sqrt{c+}}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{\left(2\left(15a^2+\frac{bc(7bc-18ad)}{d^2}\right)\right)\text{Subst}\left(\int -\right.}{15e}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{\left(2\sqrt{c}\left(15a^2+\frac{bc(7bc-18ad)}{d^2}\right)\right)\text{Subst}\left(\right.}{15\sqrt{d}}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{2\left(15a^2+\frac{bc(7bc-18ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{15\sqrt{d}\left(\sqrt{c}+\sqrt{d}x\right)}$$

Mathematica [C] time = 0.110666, size = 111, normalized size = 0.3

$$\frac{2\sqrt{ex}\left(3x\sqrt{\frac{c}{dx^2}+1}\left(15a^2d^2-18abcd+7b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+bx\left(c+dx^2\right)\left(18ad-7bc+5bdx^2\right)\right)}{45d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[e*x]*(b*x*(c + d*x^2)*(-7*b*c + 18*a*d + 5*b*d*x^2) + 3*(7*b^2*c^2
- 18*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/
2, 3/4, -(c/(d*x^2))]))/(45*d^2*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 604, normalized size = 1.6

$$\frac{1}{45\,d^3x}\sqrt{ex}\left(10\,x^6b^2d^3+90\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-108\sqrt{\frac{d}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/45*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(10*x^6*b^2*d^3+90*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*c*d^2-108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+42*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^3-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+54*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+36*x
^4*a*b*d^3-4*x^4*b^2*c*d^2+36*x^2*a*b*c*d^2-14*x^2*b^2*c^2*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ex}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

Sympy [C] time = 5.29669, size = 143, normalized size = 0.38

$$\frac{a^2 (ex)^{\frac{3}{2}} \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2\sqrt{c}e\Gamma\left(\frac{7}{4}\right)} + \frac{ab (ex)^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{\sqrt{c}e^3\Gamma\left(\frac{11}{4}\right)} + \frac{b^2 (ex)^{\frac{11}{2}} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2\sqrt{c}e^5\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_polar(I*p
i)/c)/(2*sqrt(c)*e*gamma(7/4)) + a*b*(e*x)**(7/2)*gamma(7/4)*hyper((1/2, 7/
4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**3*gamma(11/4)) + b**2*(e
*x)**(11/2)*gamma(11/4)*hyper((1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/
c)/(2*sqrt(c)*e**5*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 \sqrt{ex}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

3.842 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-14abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{21d^2e}+\frac{2b^2}{}$$

```
[Out] (-2*b*(5*b*c - 14*a*d)*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*d^2*e) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(9/4)*S
qrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.156124, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {464, 459, 329, 220}

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-14abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{}}{7de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*b*(5*b*c - 14*a*d)*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*d^2*e) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(9/4)*S
qrt[e]*Sqrt[c + d*x^2])
```


Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}dx = \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{2\int \frac{\frac{7a^2d}{2}-\frac{1}{2}b(5bc-14ad)x^2}{\sqrt{ex}\sqrt{c+dx^2}}dx}{7d}$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} - \frac{1}{21}\left(-21a^2 - \frac{bc(5bc-14ad)}{d^2}\right)\int \frac{1}{\sqrt{ex}\sqrt{c}$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{\left(2\left(21a^2+\frac{bc(5bc-14ad)}{d^2}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{d}{e}}}\right.}{21e}$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{\left(21a^2+\frac{bc(5bc-14ad)}{d^2}\right)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+}{(\sqrt{c}+}}}{21\sqrt[4]{c}\sqrt[4]{d}\sqrt{e}\sqrt{c+}}$$

Mathematica [C] time = 0.228814, size = 148, normalized size = 0.77

$$\frac{2x\left(-b\left(c+dx^2\right)\left(-14ad+5bc-3bdx^2\right)+\frac{i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(21a^2d^2-14abcd+5b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*x*(-(b*(c + d*x^2)*(5*b*c - 14*a*d - 3*b*d*x^2)) + (I*(5*b^2*c^2 - 14*a*
b*c*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(21*d^2*Sqrt[
e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 350, normalized size = 1.8

$$\frac{1}{21d^3}\left(21\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-cd}a^2d^2 - 14\sqrt{2}\sqrt{\frac{-dx+\sqrt{-}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)*(21*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),
1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*a^2*d^2-1
4*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*a*b*c*d+5*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*(-c*d)^(1/2)*b^2*c^2+6*x^5*b^2*d^3+28*x^3*a*b*d^3-4*x^3*b^2*c*d^2+28*
x*a*b*c*d^2-10*x*b^2*c^2*d)/(e*x)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{dex^3+cex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e*x^3 + c
*e*x), x)
```

Sympy [C] time = 5.21442, size = 144, normalized size = 0.75

$$\frac{a^2\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\Gamma\left(\frac{5}{4}\right)}+\frac{abx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}+\frac{b^2x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(c)*sqrt(e)*gamma(5/4)) + a*b*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4),
 (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*sqrt(e)*gamma(9/4)) + b**2*x**(
9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqr
t(c)*sqrt(e)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2}{\sqrt{dx^2 + c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

3.843 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-5ad(ad+2bc)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(3b^2\right.}{5cd^{3/2}e^2\left(\sqrt{\right.}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*Sqrt[c + d*x^
2])/(5*d*e^3) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x
^2])/(5*c*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) + (2*(3*b^2*c^2 - 5*a*d*(2*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d^(7/4)*e^(3/2)*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.340267, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {462, 459, 329, 305, 220, 1196}

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(3b^2c^2-5ad(ad+2bc)\right)}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-5ad(ad+2bc)\right)F\left(2\tan^{-1}\left(\right.\right.}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*Sqrt[c + d*x^
2])/(5*d*e^3) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x
^2])/(5*c*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) + (2*(3*b^2*c^2 - 5*a*d*(2*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d^(7/4)*e^(3/2)*
Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2\int \frac{\sqrt{ex}\left(\frac{1}{2}a(2bc+ad)+\frac{1}{2}b^2cx^2\right)}{\sqrt{c+dx^2}}\,dx}{ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}-\frac{\left(4\left(\frac{3b^2c^2}{4}-\frac{5}{4}ad(2bc+ad)\right)\right)\int \frac{\sqrt{ex}}{\sqrt{c+dx^2}}\,dx}{5cde^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}-\frac{\left(8\left(\frac{3b^2c^2}{4}-\frac{5}{4}ad(2bc+ad)\right)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx,x,\sqrt{\right.}{5cde^3}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}-\frac{\left(2\left(3b^2c^2-5ad(2bc+ad)\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx,x,\sqrt{\right.}{5\sqrt{c}d^{3/2}e^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}-\frac{2\left(3b^2c^2-5ad(2bc+ad)\right)\sqrt{ex}\sqrt{c+dx^2}}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\left(3b^2c^2-\right.}{}$$

Mathematica [C] time = 0.111314, size = 115, normalized size = 0.31

$$\frac{x\left(2x^2\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2+10abcd-3b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+2\left(c+dx^2\right)\left(b^2cx^2-5a^2d\right)\right)}{5cd(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(2*(-5*a^2*d + b^2*c*x^2)*(c + d*x^2) + 2*(-3*b^2*c^2 + 10*a*b*c*d + 5*a
^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^
2))]))/(5*c*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.02, size = 595, normalized size = 1.6

$$\frac{1}{5\,ed^2c}\left(10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2+20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-d}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/5*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+20*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-5*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a^2*c*d^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b^2*c^3+2*x^4*b^2*c*d^2-10*x^2*a^2*d^3+2*x^2*b^2*c^2*d-10*a^2*c*
d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^2x^4+ce^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^2*x^4 +
 c*e^2*x^2), x)
```

Sympy [C] time = 6.53332, size = 148, normalized size = 0.4

$$\frac{a^2\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{abx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*sq
rt(c)*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*x**(3/2)*gamma(3/4)*hyper((1/2, 3/
4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(3/2)*gamma(7/4)) + b**2*
x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2
*sqrt(c)*e**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

3.844 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(a^2d^2-6abcd+b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*Sqrt[c + d*
x^2])/(3*d*e^3) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sq
rt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^(5/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.138926, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {462, 459, 329, 220}

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(a^2d^2-6abcd+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*Sqrt[c + d*
x^2])/(3*d*e^3) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sq
rt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^(5/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\frac{1}{2}a(6bc-ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{3ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{(b^2c^2-6abcd+a^2d^2)\int \frac{1}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{3cde^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{\left(2(b^2c^2-6abcd+a^2d^2)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3cde^3}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{(b^2c^2-6abcd+a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}F\left(2\tan\right.}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.167646, size = 165, normalized size = 0.9

$$\frac{x\left(2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(c+dx^2)(b^2cx^2-a^2d) - 2ix^{5/2}\sqrt{\frac{c}{dx^2}+1}\,(a^2d^2-6abcd+b^2c^2)\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)\right)}{3cd\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-(a^2*d) + b^2*c*x^2)*(c + d*x^2) - (2*I)*
(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(5/2)*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3*c*Sqrt[(I*Sqrt[c])/Sqrt[d
]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 352, normalized size = 1.9

$$-\frac{1}{3\,cxe^2d^2}\left(\sqrt{-cd}\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3/(d*x^2+c)^(1/2)/x*((-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*d^2-
```

```
6*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b*c*d+(-c*d)^(1/2)*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*x*b^2*c^2-2*x^4*b^2*c*d^2+2*x^2*a^2*d^3-2*x^2*b^2*c^2*d+
2*a^2*c*d^2)/c/e^2/(e*x)^(1/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^3x^5+ce^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^3*x^5 +
 c*e^3*x^3), x)
```

Sympy [C] time = 34.4227, size = 148, normalized size = 0.8

$$\frac{a^2\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{ab\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{b^2x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), d*x**2*exp_polar(I*pi)/c)/(2*sq
rt(c)*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(x)*gamma(1/4)*hyper((1/4, 1/
2), (5/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(5/2)*gamma(5/4)) + b**2*
x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*
sqrt(c)*e**(5/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

3.845 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=387

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2+10abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(-3a^2d^2+10abcd\right.}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c - 3*a*d)*Sqrt[c
 + d*x^2])/(5*c^2*e^3*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*
Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^2*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*(
5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/
(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + ((5*b^2*c^2 +
10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.345841, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {462, 453, 329, 305, 220, 1196}

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(-3a^2d^2+10abcd+5b^2c^2\right)}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2+10abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c - 3*a*d)*Sqrt[c
 + d*x^2])/(5*c^2*e^3*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*
Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^2*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*(
5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/
(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + ((5*b^2*c^2 +
10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}+\frac{2\int \frac{\frac{1}{2}a(10bc-3ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}+\frac{\left(5b^2c^2+10abcd-3a^2d^2\right)\int \frac{\sqrt{ex}}{\sqrt{c+dx^2}}\,dx}{5c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}+\frac{\left(2\left(5b^2c^2+10abcd-3a^2d^2\right)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{c+\frac{dx^4}{e^2}}}\,d\right.}{5c^2e^5}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}+\frac{\left(2\left(5b^2c^2+10abcd-3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,d\right.}{5c^{3/2}\sqrt{d}e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}+\frac{2\left(5b^2c^2+10abcd-3a^2d^2\right)\sqrt{ex}\sqrt{c+dx^2}}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2\left(5\right.}{}$$

Mathematica [C] time = 0.124356, size = 116, normalized size = 0.3

$$\frac{x\left(2x^4\sqrt{\frac{c}{dx^2}+1}\left(-3a^2d^2+10abcd+5b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-2a\left(c+dx^2\right)\left(a\left(c-3dx^2\right)+10bcx^2\right)\right)}{5c^2(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(-2*a*(c + d*x^2)*(10*b*c*x^2 + a*(c - 3*d*x^2)) + 2*(5*b^2*c^2 + 10*a*b
*c*d - 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(c/(d*x^2))]))/(5*c^2*(e*x)^(7/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.021, size = 626, normalized size = 1.6

$$-\frac{1}{5\,dx^2e^3c^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2-20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-20*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*a*b*c^2*d-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
E(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-3*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-6*x^4*a^2*d^3+20*x^4*a*b*c*
d^2-4*x^2*a^2*c*d^2+20*x^2*a*b*c^2*d+2*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*
x)^(1/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^4x^6+ce^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^4*x^6 +
 c*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

3.846 $\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-14abcd+21b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a\sqrt{c+dx^2}(14bc-}{21c^2e^3(ex)^{3/2}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) - (2*a*(14*b*c - 5*a*d)*Sqrt[c
 + d*x^2])/(21*c^2*e^3*(e*x)^(3/2)) + ((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2
)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(9/4)*d^(1/4)
*e^(9/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.168304, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {462, 453, 329, 220}

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-14abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{21c^2e^3(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) - (2*a*(14*b*c - 5*a*d)*Sqrt[c
 + d*x^2])/(21*c^2*e^3*(e*x)^(3/2)) + ((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2
)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(9/4)*d^(1/4)
*e^(9/2)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} + \frac{2\int \frac{\frac{1}{2}a(14bc-5ad)+\frac{7}{2}b^2cx^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx}{7ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} - \frac{\left(4\left(-\frac{21}{4}b^2c^2+\frac{1}{4}ad(14bc-5ad)\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{21c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} - \frac{\left(8\left(-\frac{21}{4}b^2c^2+\frac{1}{4}ad(14bc-5ad)\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+}}\right.}{21c^2e^5}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} + \frac{\left(21b^2c^2-ad(14bc-5ad)\right)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}\ldots}}}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.18989, size = 159, normalized size = 0.82

$$\frac{x^{9/2}\left(\frac{2a(c+dx^2)(-3ac+5adx^2-14bcx^2)}{c^2x^{7/2}}+\frac{2ix\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2-14abcd+21b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21(ex)^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(9/2)*((2*a*(c + d*x^2)*(-3*a*c - 14*b*c*x^2 + 5*a*d*x^2))/(c^2*x^(7/2))
 + ((2*I)*(21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqrt[c
])/Sqrt[d]])))/(21*(e*x)^(9/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.03, size = 370, normalized size = 1.9

$$\frac{1}{21\,x^3dc^2e^4}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^3a^2d^2-14\sqrt{\frac{dx+\sqrt{\ldots}}{\sqrt{-\ldots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)/x^3*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a^
2*d^2-14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a*b*c*d+21*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*b^2*c^2+10*x^4*a^2*d^3-28*x^4*a*b*c*d^2+
4*x^2*a^2*c*d^2-28*x^2*a*b*c^2*d-6*a^2*c^2*d)/d/c^2/e^4/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^5x^7+ce^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^5*x^7 +
 c*e^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

3.847 $\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=438

$$\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-18abcd+15b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(7a^2d^2-18abcd+1\right.}{15c^3e^5\sqrt{ex}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) - (2*a*(18*b*c - 7*a*d)*Sqrt[c
 + d*x^2])/(45*c^2*e^3*(e*x)^(5/2)) - (2*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d
^2)*Sqrt[c + d*x^2])/(15*c^3*e^5*Sqrt[e*x]) + (2*Sqrt[d]*(15*b^2*c^2 - 18*a
*b*c*d + 7*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*c^3*e^6*(Sqrt[c] + Sqrt[
d]*x)) - (2*d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
 + (d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.423363, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 325, 329, 305, 220, 1196}

$$-\frac{2\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^5\sqrt{ex}}+\frac{2\sqrt{d}\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^6\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) - (2*a*(18*b*c - 7*a*d)*Sqrt[c
 + d*x^2])/(45*c^2*e^3*(e*x)^(5/2)) - (2*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d
^2)*Sqrt[c + d*x^2])/(15*c^3*e^5*Sqrt[e*x]) + (2*Sqrt[d]*(15*b^2*c^2 - 18*a
*b*c*d + 7*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*c^3*e^6*(Sqrt[c] + Sqrt[
d]*x)) - (2*d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
 + (d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} + \frac{2\int \frac{\frac{1}{2}a(18bc-7ad)+\frac{9}{2}b^2cx^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx}{9ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{\left(4\left(-\frac{45}{4}b^2c^2+\frac{3}{4}ad(18bc-7ad)\right)\right)\int \frac{1}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx}{45c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2\left(15b^2c^2-ad(18bc-7ad)\right)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}} - \frac{\left(4d\left(-\frac{45}{4}b\right.\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2\left(15b^2c^2-ad(18bc-7ad)\right)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}} - \frac{\left(8d\left(-\frac{45}{4}b\right.\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2\left(15b^2c^2-ad(18bc-7ad)\right)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}} - \frac{\left(8\sqrt{d}\left(-\frac{4}{4}\right.\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2\left(15b^2c^2-ad(18bc-7ad)\right)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}} + \frac{2\sqrt{d}\left(15b\right.}{}$$

Mathematica [C] time = 0.182344, size = 155, normalized size = 0.35

$$\frac{2\sqrt{ex}\left(dx^6\sqrt{\frac{dx^2}{c}+1}\left(7a^2d^2-18abcd+15b^2c^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)-\left(c+dx^2\right)\left(a^2\left(5c^2-7cdx^2+21d^2x^4\right)+18abcx^2\left(c-\right.\right.\right.}{45c^3e^6x^5\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*Sqrt[e*x]*(-((c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c - 3*d*x^2) +
a^2*(5*c^2 - 7*c*d*x^2 + 21*d^2*x^4))) + d*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2
*d^2)*x^6*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)
]))/(45*c^3*e^6*x^5*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 667, normalized size = 1.5

$$\frac{1}{45\,x^4e^5c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^4a^2cd^2-108\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/45/x^4*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d^2-108*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
```

```
/2*2^(1/2))*x^4*a*b*c^2*d+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3-21*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d^2+54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a
*b*c^2*d-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3-42*x^6*a^2*d^3+108*x^6*
a*b*c*d^2-90*x^6*b^2*c^2*d-28*x^4*a^2*c*d^2+72*x^4*a*b*c^2*d-90*x^4*b^2*c^3
+4*x^2*a^2*c^2*d-36*x^2*a*b*c^3-10*a^2*c^3)/(d*x^2+c)^(1/2)/e^5/(e*x)^(1/2)
/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^6x^8+ce^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^6*x^8 +
 c*e^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(11/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

3.848 $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77b^2c^2-5ad(22bc-9ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-2\sqrt{c+d}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) - (2*a*(22*b*c - 9*a*d)*Sqrt
[c + d*x^2])/(77*c^2*e^3*(e*x)^(7/2)) - (2*(77*b^2*c^2 - 5*a*d*(22*b*c - 9*
a*d))*Sqrt[c + d*x^2])/(231*c^3*e^5*(e*x)^(3/2)) - (d^(3/4)*(77*b^2*c^2 - 5
*a*d*(22*b*c - 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2]
)/(231*c^(13/4)*e^(13/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.2216, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {462, 453, 325, 329, 220}

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77b^2c^2-5ad(22bc-9ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(77b^2c\right.}{231c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) - (2*a*(22*b*c - 9*a*d)*Sqrt
[c + d*x^2])/(77*c^2*e^3*(e*x)^(7/2)) - (2*(77*b^2*c^2 - 5*a*d*(22*b*c - 9*
a*d))*Sqrt[c + d*x^2])/(231*c^3*e^5*(e*x)^(3/2)) - (d^(3/4)*(77*b^2*c^2 - 5
*a*d*(22*b*c - 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2]
)/(231*c^(13/4)*e^(13/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} + \frac{2\int \frac{\frac{1}{2}a(22bc-9ad)+\frac{11}{2}b^2cx^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx}{11ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} + \frac{\left(77b^2c^2-5ad(22bc-9ad)\right)\int \frac{1}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx}{77c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2\left(77b^2c^2-5ad(22bc-9ad)\right)\sqrt{c+dx^2}}{231c^3e^5(ex)^{3/2}} - \frac{\left(d\left(77b^2\right.\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2\left(77b^2c^2-5ad(22bc-9ad)\right)\sqrt{c+dx^2}}{231c^3e^5(ex)^{3/2}} - \frac{\left(2d\left(77b\right.\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2\left(77b^2c^2-5ad(22bc-9ad)\right)\sqrt{c+dx^2}}{231c^3e^5(ex)^{3/2}} - \frac{d^{3/4}\left(77b\right.}{}$$

Mathematica [C] time = 0.23611, size = 196, normalized size = 0.81

$$\frac{x^{13/2}\left(-\frac{2\left(c+dx^2\right)\left(3a^2\left(7c^2-9cdx^2+15d^2x^4\right)+22abcx^2\left(3c-5dx^2\right)+77b^2c^2x^4\right)}{c^3x^{11/2}}-\frac{2idx\sqrt{\frac{c}{dx^2}+1}\left(45a^2d^2-110abcd+77b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{c^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231(ex)^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(13/2)*((-2*(c + d*x^2)*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c - 5*d*x^2) +
 3*a^2*(7*c^2 - 9*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^(11/2)) - ((2*I)*d*(77*b^2
```

```
*c^2 - 110*a*b*c*d + 45*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]])))/
(231*(e*x)^(13/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.031, size = 411, normalized size = 1.7

$$-\frac{1}{231\,x^5c^3e^6}\left(45\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^5a^2d^2-110\sqrt{\frac{dx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/231/(d*x^2+c)^(1/2)/x^5*(45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5
*a^2*d^2-110*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a*b*c*d+77*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*b^2*c^2+90*x^6*a^2*d^3-220*x^6*a*b*c
*d^2+154*x^6*b^2*c^2*d+36*x^4*a^2*c*d^2-88*x^4*a*b*c^2*d+154*x^4*b^2*c^3-12
*x^2*a^2*c^2*d+132*x^2*a*b*c^3+42*a^2*c^3)/c^3/e^6/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{de^7x^9+ce^7x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d*e^7*x^9 +
 c*e^7*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(13/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

3.849 $\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{5c^{3/4}e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-198abcd+117b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{462d^{17/4}\sqrt{c+dx^2}}+\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\,(}{}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (5*(117*b^2*c^2 - 1
98*a*b*c*d + 77*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^4) - ((117*b
^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*c*d^3
) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d^2*e) - (5*c^(3/4)*(117*b^2*c^
2 - 198*a*b*c*d + 77*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(462*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.248713, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {463, 459, 321, 329, 220}

$$\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-198abcd+117b^2c^2\right)}{231d^4}-\frac{5c^{3/4}e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-198abcd+117b^2c^2\right)F\left(\right.}{462d^{17/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (5*(117*b^2*c^2 - 1
98*a*b*c*d + 77*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^4) - ((117*b
^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*c*d^3
) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d^2*e) - (5*c^(3/4)*(117*b^2*c^
2 - 198*a*b*c*d + 77*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(462*d^(17/4)*Sqrt[c + d*x^2])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx = \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{7/2}\left(\frac{1}{2}\left(-2a^2d^2+9(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e} - \frac{\left(117b^2c^2-198abcd+77a^2d^2\right)\int \frac{(ex)^{7/2}}{\sqrt{c+dx^2}}dx}{22cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(117b^2c^2-198abcd+77a^2d^2\right)e(ex)^{5/2}\sqrt{c+dx^2}}{77cd^3} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e} + \frac{(5\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5\left(117b^2c^2-198abcd+77a^2d^2\right)e^3\sqrt{ex}\sqrt{c+dx^2}}{231d^4} - \frac{\left(117b^2c^2-198abcd+7\cdots\right.}{77c\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5\left(117b^2c^2-198abcd+77a^2d^2\right)e^3\sqrt{ex}\sqrt{c+dx^2}}{231d^4} - \frac{\left(117b^2c^2-198abcd+7\cdots\right.}{77c\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5\left(117b^2c^2-198abcd+77a^2d^2\right)e^3\sqrt{ex}\sqrt{c+dx^2}}{231d^4} - \frac{\left(117b^2c^2-198abcd+7\cdots\right.}{77c\cdots}$$

Mathematica [C] time = 0.243443, size = 226, normalized size = 0.76

$$\frac{e^3\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(77a^2d^2\left(5c+2dx^2\right)+66abd\left(-15c^2-6cdx^2+2d^2x^4\right)+3b^2\left(78c^2dx^2+195c^3-26cd^2x^4+14d^3x^6\right)\right)-5ic\cdots\right.}{231d^4\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(77*a^2*d^2*(5*c + 2*d*x^2) + 66*
a*b*d*(-15*c^2 - 6*c*d*x^2 + 2*d^2*x^4) + 3*b^2*(195*c^3 + 78*c^2*d*x^2 - 2
6*c*d^2*x^4 + 14*d^3*x^6)) - (5*I)*c*(117*b^2*c^2 - 198*a*b*c*d + 77*a^2*d^
2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]
]/Sqrt[x]], -1]))/(231*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4*Sqrt[c + d*x^2])
```

Maple [A] time = 0.037, size = 407, normalized size = 1.4

$$-\frac{e^3}{462\,xd^5}\sqrt{ex}\left(-84\,x^7b^2d^4+385\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{-cd}a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/462*e^3/x*(e*x)^(1/2)*(-84*x^7*b^2*d^4+385*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(
-c*d)^(1/2)*a^2*c*d^2-990*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*
d+585*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3-264*x^5*a*b*d^4+156*
x^5*b^2*c*d^3-308*x^3*a^2*d^4+792*x^3*a*b*c*d^3-468*x^3*b^2*c^2*d^2-770*x*a
^2*c*d^3+1980*x*a*b*c^2*d^2-1170*x*b^2*c^3*d)/(d*x^2+c)^(1/2)/d^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{7}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2e^3x^7+2\,abe^3x^5+a^2e^3x^3\right)\sqrt{dx^2+c}\sqrt{ex}}{d^2x^4+2\,cdx^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^3*x^7 + 2*a*b*e^3*x^5 + a^2*e^3*x^3)*sqrt(d*x^2 + c)*sqrt(e
*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{7}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.850 $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=436

$$\frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{30d^{15/4}\sqrt{c+dx^2}}+\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(45a^2\right.}{15d^{7/2}\left(\right.}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 126*
a*b*c*d + 45*a^2*d^2)*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*c*d^3) + (2*b^2*(e
*x)^(7/2)*Sqrt[c + d*x^2])/(9*d^2*e) + ((77*b^2*c^2 - 126*a*b*c*d + 45*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) - (c
^(1/4)*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x
)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(15/4)*Sqrt[c + d*x^2]) + (c^(1/4)*
(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[
(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], 1/2])/(30*d^(15/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.378029, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {463, 459, 321, 329, 305, 220, 1196}

$$\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{15d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)F\left(2\tan\right.}{30d^{15/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 126*
a*b*c*d + 45*a^2*d^2)*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*c*d^3) + (2*b^2*(e
*x)^(7/2)*Sqrt[c + d*x^2])/(9*d^2*e) + ((77*b^2*c^2 - 126*a*b*c*d + 45*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) - (c
^(1/4)*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x
)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(15/4)*Sqrt[c + d*x^2]) + (c^(1/4)*
(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[
(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], 1/2])/(30*d^(15/4)*Sqrt[c + d*x^2])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{5/2}\left(\frac{1}{2}\left(-2a^2d^2+7(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)\int \frac{(ex)^{5/2}}{\sqrt{c+dx^2}}\,dx}{18cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} +$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} +$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} +$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} +$$

Mathematica [C] time = 0.138021, size = 133, normalized size = 0.31

$$\frac{e(ex)^{3/2}\left(3\sqrt{\frac{c}{dx^2}+1}\left(45a^2d^2-126abcd+77b^2c^2\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)-45a^2d^2+18abd\left(7c+2dx^2\right)+b^2\left(-77c^2-22\right.\right.}{45d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*(e*x)^(3/2)*(-45*a^2*d^2 + 18*a*b*d*(7*c + 2*d*x^2) + b^2*(-77*c^2 - 22*
c*d*x^2 + 10*d^2*x^4) + 3*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*Sqrt[1 +
c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d^3*Sqrt[c
 + d*x^2])
```

Maple [A] time = 0.036, size = 618, normalized size = 1.4

$$\frac{e^2}{90\,xd^4}\sqrt{ex}\left(20\,x^6b^2d^3+270\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)a^2cd^2-756\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/90/x*e^2*(e*x)^(1/2)*(20*x^6*b^2*d^3+270*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*
c*d^2-756*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
```

```
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+462*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*b^2*c^3-135*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+378*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*b*c^2*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+72*x^4*a*b*d^
3-44*x^4*b^2*c*d^2-90*x^2*a^2*d^3+252*x^2*a*b*c*d^2-154*x^2*b^2*c^2*d)/(d*x
^2+c)^(1/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2 (ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2e^2x^6 + 2\,abe^2x^4 + a^2e^2x^2)\sqrt{dx^2+c}\sqrt{ex}}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(d*x^2 + c)*sqrt(e
*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.851 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=245

$$\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-70abcd+45b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{42\sqrt[4]{c}d^{13/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\sqrt{c+dx^2}\left(21a^2d^2-70ab\right.}{21cd^3}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((45*b^2*c^2 - 70*a
*b*c*d + 21*a^2*d^2)*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d^3) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d^2*e) + ((45*b^2*c^2 - 70*a*b*c*d + 21*a^2*d^2)
*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(42*c^(1/4)*
d^(13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.197784, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {463, 459, 321, 329, 220}

$$\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{42\sqrt[4]{c}d^{13/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\sqrt{c+dx^2}\left(21a^2d^2-70abcd+45b\right.}{21cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((45*b^2*c^2 - 70*a
*b*c*d + 21*a^2*d^2)*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d^3) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d^2*e) + ((45*b^2*c^2 - 70*a*b*c*d + 21*a^2*d^2)
*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(42*c^(1/4)*
d^(13/4)*Sqrt[c + d*x^2])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{3/2}\left(\frac{1}{2}\left(-2a^2d^2+5(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e} - \frac{\left(45b^2c^2-70abcd+21a^2d^2\right)\int \frac{(ex)^{3/2}}{\sqrt{c+dx^2}}\,dx}{14cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(45b^2c^2-70abcd+21a^2d^2\right)e\sqrt{ex}\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e} + \frac{((4\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(45b^2c^2-70abcd+21a^2d^2\right)e\sqrt{ex}\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e} + \frac{((4\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(45b^2c^2-70abcd+21a^2d^2\right)e\sqrt{ex}\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e} + \frac{(45\cdots}{\cdots}$$

Mathematica [C] time = 0.196336, size = 191, normalized size = 0.78

$$\frac{e\sqrt{ex}\left(i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(21a^2d^2-70abcd+45b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(-21a^2d^2+14abd\left(5c+2dx^{\cdots}\right.\right.\right.}{21d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-21*a^2*d^2 + 14*a*b*d*(5*c + 2*d*
x^2) - 3*b^2*(15*c^2 + 6*c*d*x^2 - 2*d^2*x^4)) + I*(45*b^2*c^2 - 70*a*b*c*d
 + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[c])/Sqrt[d]]/Sqrt[x]], -1]))/(21*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3*Sqrt[c + d*
```

```
x^2])
```

Maple [A] time = 0.023, size = 363, normalized size = 1.5

$$\frac{e}{42\,xd^4}\sqrt{ex}\left(21\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-cd}a^2d^2-70\sqrt{2}\sqrt{\frac{-dx+}{\sqrt{-c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/42*e/x*(e*x)^(1/2)*(21*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),
1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*a^2*d^2-7
0*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*a*b*c*d+45*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-c*d)^(1/2)*b^2*c^2+12*x^5*b^2*d^3+56*x^3*a*b*d^3-36*x^3*b^2*c*d^2-
42*x*a^2*d^3+140*x*a*b*c*d^2-90*x*b^2*c^2*d)/(d*x^2+c)^(1/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2ex^5+2\,abex^3+a^2ex\right)\sqrt{dx^2+c}\sqrt{ex}}{d^2x^4+2\,cdx^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2
*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((e*x)**(3/2)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.852 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=384

$$-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-30abc\right.}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*(e*x)^(3/2)*
Sqrt[c + d*x^2])/(5*d^2*e) - ((21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*Sqrt[e*
x]*Sqrt[c + d*x^2])/(5*c*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + ((21*b^2*c^2 - 30
*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[
c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(5*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2]) - ((21*b^2*c^2 - 30*a*b*c*d +
 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
10*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.326508, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {463, 459, 329, 305, 220, 1196}

$$-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-30abcd+21b^2c^2\right)}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*(e*x)^(3/2)*
Sqrt[c + d*x^2])/(5*d^2*e) - ((21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*Sqrt[e*
x]*Sqrt[c + d*x^2])/(5*c*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + ((21*b^2*c^2 - 30
*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[
c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(5*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2]) - ((21*b^2*c^2 - 30*a*b*c*d +
 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
10*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}\left(\frac{1}{2}\left(-2a^2d^2+3(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$=\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}-\frac{\left(21b^2c^2-30abcd+5a^2d^2\right)\int \frac{\sqrt{ex}}{\sqrt{c+dx^2}}\,dx}{10cd^2}$$

$$=\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}-\frac{\left(21b^2c^2-30abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx,x,\sqrt{\right.}{5cd^2e}$$

$$=\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}-\frac{\left(21b^2c^2-30abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx,x,\sqrt{\right.}{5\sqrt{c}d^{5/2}}$$

$$=\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}-\frac{\left(21b^2c^2-30abcd+5a^2d^2\right)\sqrt{ex}\sqrt{c+dx^2}}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(21b^2c^2\right.}{}$$

Mathematica [C] time = 0.121057, size = 119, normalized size = 0.31

$$\frac{\sqrt{ex}\left(x\sqrt{\frac{c}{dx^2}+1}\left(-5a^2d^2+30abcd-21b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+x\left(5a^2d^2-10abcd+b^2c\left(7c+2dx^2\right)\right)\right)}{5cd^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[e*x]*(x*(-10*a*b*c*d + 5*a^2*d^2 + b^2*c*(7*c + 2*d*x^2)) + (-21*b^2*
c^2 + 30*a*b*c*d - 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4,
 1/2, 3/4, -(c/(d*x^2))]))/(5*c*d^2*Sqrt[c + d*x^2])
```

Maple [A] time = 0.021, size = 597, normalized size = 1.6

$$-\frac{1}{10\,d^3xc}\sqrt{ex}\left(10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-60\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/10*(e*x)^(1/2)*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-60*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a*b*c^2*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-5*((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2),1/2*2^(1/2))*a^2*c*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*El
lipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-4*x^4*b^2*c*d^2-10*x^2*a^2*d^3+20*x^2*a*
b*c*d^2-14*x^2*b^2*c^2*d)/(d*x^2+c)^(1/2)/d^3/x/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2\sqrt{ex}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2} x^{4} + 2\,abx^{2} + a^{2}\right)\sqrt{dx^{2} + c}\sqrt{ex}}{d^{2} x^{4} + 2\,cdx^{2} + c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2*x^4 + 2
*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}\left(a + bx^{2}\right)^{2}}{\left(c + dx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2} + a\right)^{2}\sqrt{ex}}{\left(dx^{2} + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

3.853 $\int \frac{\left(a+bx^2\right)^2}{\sqrt{ex}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2-6abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e}$$

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*Sqrt[e*x]*Sqrt
[c + d*x^2])/(3*d^2*e) - ((5*b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sq
rt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(5/4)*d^(9/4)*Sqrt[e]*Sqrt[c
+ d*x^2])
```

Rubi [A] time = 0.155666, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {463, 459, 329, 220}

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2-6abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*Sqrt[e*x]*Sqrt
[c + d*x^2])/(3*d^2*e) - ((5*b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sq
rt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(5/4)*d^(9/4)*Sqrt[e]*Sqrt[c
+ d*x^2])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{ex}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{\frac{1}{2}\left(-2a^2d^2+(bc-ad)^2\right)-b^2cdx^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e} - \frac{\left(5b^2c^2-6abcd-3a^2d^2\right)\int \frac{1}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{6cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e} - \frac{\left(5b^2c^2-6abcd-3a^2d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3cd^2e}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e} - \frac{\left(5b^2c^2-6abcd-3a^2d^2\right)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}F\left(2\,\text{t}\right.}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.15125, size = 174, normalized size = 0.9

$$\frac{ix^{3/2}\sqrt{\frac{c}{dx^2}+1}\left(3a^2d^2+6abcd-5b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)+x\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(3a^2d^2-6abcd+b^2c\left(5c+2dx^2\right)\right)}{3cd^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[d]]*x*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(5*c + 2*d*x^2
)) + I*(-5*b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(3*c*Sqrt[(I*Sqrt
[c])/Sqrt[d]]*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 341, normalized size = 1.8

$$\frac{1}{6\,cd^3}\left(3\,\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-cd}a^2d^2+6\,\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 1/6*(3*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*a^2*d^2+6*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-c*d)^(1/2)*a*b*c*d-5*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-c*d)^(1/2)*
b^2*c^2+4*x^3*b^2*c*d^2+6*x*a^2*d^3-12*x*a*b*c*d^2+10*x*b^2*c^2*d)/(d*x^2+c
)^(1/2)/c/(e*x)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^2ex^5+2\,cdex^3+c^2ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2*e*x^5 +
 2*c*d*e*x^3 + c^2*e*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{ex}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.854 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=393

$$\frac{(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}(3a^2d^2-2abcd+3b^2c^2)\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}(3a^2d^2-2abcd+3b^2c^2}{c^2d^{3/2}e^2(\sqrt{c}+\sqrt{d}x)}$$

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^
2)*(e*x)^(3/2))/(c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 - 2*a*b*c*d + 3*a
^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(c^2*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x))
- ((3*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^
2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) + ((3*b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(2*c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.361189, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {462, 457, 329, 305, 220, 1196}

$$\frac{\sqrt{ex}\sqrt{c+dx^2}(3a^2d^2-2abcd+3b^2c^2)}{c^2d^{3/2}e^2(\sqrt{c}+\sqrt{d}x)}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{c^2de^3\sqrt{c+dx^2}}+\frac{(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}(3a^2d^2-2abcd+3b^2}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^
2)*(e*x)^(3/2))/(c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 - 2*a*b*c*d + 3*a
^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(c^2*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x))
- ((3*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^
2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) + ((3*b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(2*c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx = -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} + \frac{2\int \frac{\sqrt{ex}\left(\frac{1}{2}a(2bc-3ad)+\frac{1}{2}b^2cx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} - \frac{\left(2ab-\frac{3b^2c}{d}-\frac{3a^2d}{c}\right)\int \frac{\sqrt{ex}}{\sqrt{c+dx^2}}\,dx}{2ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} - \frac{\left(2ab-\frac{3b^2c}{d}-\frac{3a^2d}{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{c+\frac{dx^4}{e^2}}}\right.}{ce^3}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} + \frac{\left(3b^2c^2-2abcd+3a^2d^2\right)\text{Subst}\left(\int -\right.}{c^{3/2}d^{3/2}e^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{\left(b^2c^2-2abcd+3a^2d^2\right)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} + \frac{\left(3b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}\sqrt{c+dx}}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}$$

Mathematica [C] time = 0.107433, size = 126, normalized size = 0.32

$$\frac{x\left(x^2\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-2abcd+3b^2c^2\right)\,_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)-3a^2d\left(2c+3dx^2\right)+6abcdx^2-3b^2c^2x^2\right)}{3c^2d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-3*b^2*c^2*x^2 + 6*a*b*c*d*x^2 - 3*a^2*d*(2*c + 3*d*x^2) + (3*b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*x^2*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4,
 7/4, -((d*x^2)/c)]))/(3*c^2*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 594, normalized size = 1.5

$$\frac{1}{2\,d^2ec^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-4\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx}{\ }}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-4*((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/
2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*c^2*d+6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-3*((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^
(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*c*d^2+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-3*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*b^2*c^3-6*x^2*a^2*d^3+4*x^2*a*b*c*d^2-2*x^2*b^2*c^2*d-4*a^2*c*d^2)/
(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{dx^{2}+c}\sqrt{ex}}{d^{2}e^{2}x^{6}+2\,cde^{2}x^{4}+c^{2}e^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2*e^2*x^6
 + 2*c*d*e^2*x^4 + c^2*e^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^{2}\right)^{2}}{\left(ex\right)^{\frac{3}{2}}\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((e*x)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{2}+a\right)^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.855 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=207

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-5ad)+3b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\left(5a^2d^2-6abcd+3b^2c^2\right)}{3c^2de^3\sqrt{c+dx^2}}-\frac{}{3ce(e}$$

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 6*a*b*c*d + 5*
a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 + a*d*(6*b*
c - 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(9/4
)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.181223, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {462, 457, 329, 220}

$$-\frac{\sqrt{ex}\left(5a^2d^2-6abcd+3b^2c^2\right)}{3c^2de^3\sqrt{c+dx^2}}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-5ad)+3b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 6*a*b*c*d + 5*
a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 + a*d*(6*b*
c - 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(9/4
)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} + \frac{2\int \frac{\frac{1}{2}a(6bc-5ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}\left(c+dx^2\right)^{3/2}}\,dx}{3ce^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-6abcd+5a^2d^2\right)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{\left(3b^2c^2+ad(6bc-5ad)\right)\int \frac{1}{\sqrt{ex}\sqrt{c+d}}}{6c^2de^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-6abcd+5a^2d^2\right)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{\left(3b^2c^2+ad(6bc-5ad)\right)\text{Subst}\left(\int\right.}{3c^2de^3}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-6abcd+5a^2d^2\right)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{\left(3b^2c^2+ad(6bc-5ad)\right)\left(\sqrt{c}+\sqrt{d}\right.}{6c^{9/4}d^{5/4}}$$

Mathematica [C] time = 0.180499, size = 181, normalized size = 0.87

$$\frac{x\left(-ix^{5/2}\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2-6abcd-3b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)-\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(a^2d\left(2c+5dx^2\right)-6abcdx^2+3b^2c\right.}{3c^2d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 + a^2*d*(2*c
 + 5*d*x^2))) - I*(-3*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*
x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3*c^
2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.024, size = 353, normalized size = 1.7

$$-\frac{1}{6xc^2e^2d^2}\left(5\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)xa^2d^2-6\sqrt{-cd}\sqrt{\frac{dx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/6/x*(5*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*d^2-6*(-c*d)^(1/2)
*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b*c*d-3*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x*b^2*c^2+10*x^2*a^2*d^3-12*x^2*a*b*c*d^2+6*x^2*b^2*c^2*d+4*a^2*c*d^
2)/(d*x^2+c)^(1/2)/c^2/e^2/(e*x)^(1/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^2e^3x^7+2\,cde^3x^5+c^2e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2*e^3*x^7
 + 2*c*d*e^3*x^5 + c^2*e^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\left(ex\right)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((e*x)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.856 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)}{}$$

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) - (2*a*(10*b*c - 7*a*d))/(5*c^
2*e^3*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(e
*x)^(3/2))/(5*c^3*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*
d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^3*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) +
((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(5*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2
 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
1/2])/(10*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.422497, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 290, 329, 305, 220, 1196}

$$-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+d}{(\sqrt{c}+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) - (2*a*(10*b*c - 7*a*d))/(5*c^
2*e^3*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(e
*x)^(3/2))/(5*c^3*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*
d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^3*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) +
((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(5*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2
 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
1/2])/(10*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} + \frac{2\int \frac{\frac{1}{2}a(10bc-7ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))\int \frac{\sqrt{ex}}{(c+dx^2)^{3/2}}\,dx}{5c^2e^4}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-3a}{}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-3a}{}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-3a}{}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-3a}{5c}$$

Mathematica [C] time = 0.12478, size = 141, normalized size = 0.32

$$\frac{x\left(x^4\sqrt{\frac{dx^2}{c}+1}\left(-21a^2d^2+30abcd-5b^2c^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)+a^2\left(-6c^2+42cdx^2+63d^2x^4\right)-30abcx^2\left(2c+3dx^2\right)+15\right.}{15c^3(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(15*b^2*c^2*x^4 - 30*a*b*c*x^2*(2*c + 3*d*x^2) + a^2*(-6*c^2 + 42*c*d*x^
2 + 63*d^2*x^4) + (-5*b^2*c^2 + 30*a*b*c*d - 21*a^2*d^2)*x^4*Sqrt[1 + (d*x^
2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(15*c^3*(e*x)^(7/2)*
Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 638, normalized size = 1.5

$$-\frac{1}{10\,dx^2e^3c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2-60\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-60*((d*x+(-c*d)^(1/
```

```
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^2*a*b*c^2*d+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-21*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b*c^2*d-5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-42*x^4*a^2*d^3+60*x^4*a*
b*c*d^2-10*x^4*b^2*c^2*d-28*x^2*a^2*c*d^2+40*x^2*a*b*c^2*d+4*a^2*c^2*d)/(d*
x^2+c)^(1/2)/d/e^3/(e*x)^(1/2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^2e^4x^8+2\,cde^4x^6+c^2e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^2*e^4*x^8
 + 2*c*d*e^4*x^6 + c^2*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

3.857 $\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{5e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-42abcd+39b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{84\sqrt[4]{c}d^{17/4}\sqrt{c+dx^2}}-\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-\right.}{42cd^4}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((39*b^2*c^2 -
42*a*b*c*d + 7*a^2*d^2)*e*(e*x)^(5/2))/(14*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(9/2))/(7*d^2*e*Sqrt[c + d*x^2]) - (5*(39*b^2*c^2 - 42*a*b*c*d + 7*a^
2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(42*c*d^4) + (5*(39*b^2*c^2 - 42*a*b*
c*d + 7*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(84*c^(1/4)*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.233711, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {463, 459, 288, 321, 329, 220}

$$-\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{42cd^4}+\frac{5e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-42abcd+39b^2c^2\right)F\left(2\tan^{-1}\left(\right.\right.}{84\sqrt[4]{c}d^{17/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((39*b^2*c^2 -
42*a*b*c*d + 7*a^2*d^2)*e*(e*x)^(5/2))/(14*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(9/2))/(7*d^2*e*Sqrt[c + d*x^2]) - (5*(39*b^2*c^2 - 42*a*b*c*d + 7*a^
2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(42*c*d^4) + (5*(39*b^2*c^2 - 42*a*b*
c*d + 7*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(84*c^(1/4)*d^(17/4)*Sqrt[c + d*x^2])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{7/2}\left(-\frac{3}{2}\left(2a^2d^2-3(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)\int \frac{(ex)^{7/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{14cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{\left(5\left(39b^2c^2-42abc\right.\right.}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5\left(39b^2c^2-42abcd\right.}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5\left(39b^2c^2-42abcd\right.}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5\left(39b^2c^2-42abcd\right.}{}$$

Mathematica [C] time = 0.280124, size = 222, normalized size = 0.74

$$\frac{(ex)^{7/2}\left(\frac{\sqrt{x}\left(-7a^2d^2\left(5c+7dx^2\right)+14abd\left(15c^2+21cdx^2+4d^2x^4\right)+b^2\left(-\left(273c^2dx^2+195c^3+52cd^2x^4-12d^3x^6\right)\right)\right)}{d^4\left(c+dx^2\right)}+\frac{5ix\sqrt{\frac{c}{dx^2}+1}\left(7a^2d^2-42abcd+39b^2c^2\right)\text{Elliptic}}{d^4\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{42x^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(7/2)*((Sqrt[x]*(-7*a^2*d^2*(5*c + 7*d*x^2) + 14*a*b*d*(15*c^2 + 21*
c*d*x^2 + 4*d^2*x^4) - b^2*(195*c^3 + 273*c^2*d*x^2 + 52*c*d^2*x^4 - 12*d^3
*x^6)))/(d^4*(c + d*x^2)) + ((5*I)*(39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*Sq
rt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4)))/(42*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.039, size = 696, normalized size = 2.3

$$\frac{e^3}{84\,xd^5}\left(35\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^2a^2d^3-210\sqrt{\frac{dx+\sqrt{-}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/84*(35*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a^2*d^3-210*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a*b*c*d^2+195*((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^
(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2
))*(-c*d)^(1/2)*x^2*b^2*c^2*d+24*x^7*b^2*d^4+35*((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*(-c*d)^(1/2)*a^2*c*d^2-210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^
2*d+195*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+112*x^5*a*b*d^4-10
4*x^5*b^2*c*d^3-98*x^3*a^2*d^4+588*x^3*a*b*c*d^3-546*x^3*b^2*c^2*d^2-70*x*a
^2*c*d^3+420*x*a*b*c^2*d^2-390*x*b^2*c^3*d)*e^3/x*(e*x)^(1/2)/d^5/(d*x^2+c)
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{7}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2e^3x^7 + 2\,abe^3x^5 + a^2e^3x^3\right)\sqrt{dx^2 + c}\sqrt{ex}}{d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^3*x^7 + 2*a*b*e^3*x^5 + a^2*e^3*x^3)*sqrt(d*x^2 + c)*sqrt(e
*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 (ex)^{\frac{7}{2}}}{\left(dx^2 + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

3.858 $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=442

$$-\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-70abcd+77b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-\right.}{10cd^{7/2}\left(\sqrt{c}+\right.}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((77*b^2*c^2 -
70*a*b*c*d + 5*a^2*d^2)*e*(e*x)^(3/2))/(30*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(7/2))/(5*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) +
((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2]) - ((77*b^2*
c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(20*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.383712, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {463, 459, 288, 329, 305, 220, 1196}

$$-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-70abcd+77b^2c^2\right)}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-70abcd+77b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((77*b^2*c^2 -
70*a*b*c*d + 5*a^2*d^2)*e*(e*x)^(3/2))/(30*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(7/2))/(5*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) +
((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2]) - ((77*b^2*
c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(20*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{5/2}\left(\frac{1}{2}\left(-6a^2d^2+7(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)\int \frac{(ex)^{5/2}}{\left(c+dx^2\right)^{3/2}}dx}{30cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(\left(77b^2c^2-70a\ldots\right.\right.}{\ldots}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(\left(77b^2c^2-70a\ldots\right.\right.}{\ldots}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(\left(77b^2c^2-70a\ldots\right.\right.}{\ldots}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-70ab\ldots\right.}{10c\ldots}$$

Mathematica [C] time = 0.184941, size = 153, normalized size = 0.35

$$\frac{e(ex)^{3/2}\left(-3\sqrt{\frac{c}{dx^2}+1}\left(c+dx^2\right)\left(5a^2d^2-70abcd+77b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+5a^2d^2\left(c+3dx^2\right)-10abcd\left(7c+9dx\ldots\right.\right.}{30cd^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (e*(e*x)^(3/2)*(5*a^2*d^2*(c + 3*d*x^2) - 10*a*b*c*d*(7*c + 9*d*x^2) + b^2*
c*(77*c^2 + 99*c*d*x^2 + 12*d^2*x^4) - 3*(77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d
^2)*Sqrt[1 + c/(d*x^2)]*(c + d*x^2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(
d*x^2))]))/(30*c*d^3*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.041, size = 1191, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/60*(30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3-420*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
```

```
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*x^2*a*b*c^2*d^2+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-15*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^
2*a*b*c^2*d^2-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-24*x^6*b^2*c*d^
3+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-420*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-15*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-231*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*b^2*c^4-30*x^4*a^2*d^4+180*x^4*a*b*c*d^3-198*x^4*b^2*c^2
*d^2-10*x^2*a^2*c*d^3+140*x^2*a*b*c^2*d^2-154*x^2*b^2*c^3*d)/x*e^2*(e*x)^(1
/2)/d^4/c/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(d*x^2 + c)*sqrt(e
*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2(ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)
```

3.859 $\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-10abcd+15b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}+\frac{e\sqrt{ex}\left(-a^2d^2-10abcd+15b\right.}{6cd^3\sqrt{c+dx^2}}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((15*b^2*c^2 -
10*a*b*c*d - a^2*d^2)*e*Sqrt[e*x])/(6*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*(e*x)
^(5/2))/(3*d^2*e*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^
(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(5/4)*d^(
13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.194239, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {463, 459, 288, 329, 220}

$$-\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-10abcd+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}+\frac{e\sqrt{ex}\left(-a^2d^2-10abcd+15b^2c^2\right)}{6cd^3\sqrt{c+dx^2}}+\frac{(\ldots}{3c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((15*b^2*c^2 -
10*a*b*c*d - a^2*d^2)*e*Sqrt[e*x])/(6*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*(e*x)
^(5/2))/(3*d^2*e*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^
(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(5/4)*d^(
13/4)*Sqrt[c + d*x^2])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{3/2}\left(\frac{1}{2}\left(-6a^2d^2+5(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-10abcd-a^2d^2\right)\int \frac{(ex)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{6cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{\left(\left(15b^2c^2-10abcd\right.\right.}{}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{\left(\left(15b^2c^2-10abcd\right.\right.}{}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-10abcd-\right.}{}$$

Mathematica [C] time = 0.253136, size = 204, normalized size = 0.82

$$\frac{(ex)^{3/2}\left(\frac{\sqrt{x}\left(a^2d^2\left(dx^2-c\right)-2abcd\left(5c+7dx^2\right)+b^2c\left(15c^2+21cdx^2+4d^2x^4\right)\right)}{cd^3\left(c+dx^2\right)}+\frac{ix\sqrt{\frac{c}{dx^2}+1}\left(a^2d^2+10abcd-15b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{cd^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{6x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(3/2)*((Sqrt[x]*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(5*c + 7*d*x^2) +
b^2*c*(15*c^2 + 21*c*d*x^2 + 4*d^2*x^4)))/(c*d^3*(c + d*x^2)) + (I*(-15*b^2
*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(6
*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.022, size = 674, normalized size = 2.7

$$\frac{e}{12\,cxd^4}\left(\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}x^2a^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a^2*d^3+10*((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a*b*c*d^2-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-
c*d)^(1/2)*x^2*b^2*c^2*d+((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d^2
+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d-15*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+8*x^5*b^2*c*d^3+2*x^3*a^2*d^4-28*x^3*a*b*c*
d^3+42*x^3*b^2*c^2*d^2-2*x*a^2*c*d^3-20*x*a*b*c^2*d^2+30*x*b^2*c^3*d)*e/x*(
e*x)^(1/2)/c/d^4/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2ex^5+2\,abex^3+a^2ex\right)\sqrt{dx^2+c}\sqrt{ex}}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3
*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.860 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}} dx$

Optimal. Leaf size=403

$$\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}\left(-a^2d^2-2abcd+7\right.}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{dx}\right)}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3
*b*c + a*d)*(e*x)^(3/2))/(2*c^2*d^2*e*Sqrt[c + d*x^2]) + ((7*b^2*c^2 - 2*a*
b*c*d - a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^2*d^(5/2)*(Sqrt[c] + Sqrt[
d]*x)) - ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*S
qrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2]) + ((7*b
^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(4*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.341972, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {463, 457, 329, 305, 220, 1196}

$$\frac{\sqrt{ex}\sqrt{c+dx^2}\left(-a^2d^2-2abcd+7b^2c^2\right)}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3
*b*c + a*d)*(e*x)^(3/2))/(2*c^2*d^2*e*Sqrt[c + d*x^2]) + ((7*b^2*c^2 - 2*a*
b*c*d - a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^2*d^(5/2)*(Sqrt[c] + Sqrt[
d]*x)) - ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*S
qrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2]) + ((7*b
^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(4*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{\sqrt{ex}\left(-\frac{3}{2}\left(2a^2d^2-(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{\left(7b^2c^2-2abcd-a^2d^2\right)\int \frac{\sqrt{ex}}{\sqrt{c+dx^2}}\,dx}{4c^2d^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{\left(7b^2c^2-2abcd-a^2d^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx\right.}{2c^2d^2e}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{\left(7b^2c^2-2abcd-a^2d^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,dx\right.}{2c^{3/2}d^{5/2}}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{\left(7b^2c^2-2abcd-a^2d^2\right)\sqrt{ex}\sqrt{c+dx^2}}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{(7b}{}$$

Mathematica [C] time = 0.162822, size = 147, normalized size = 0.36

$$\frac{\sqrt{ex}\left(3x\sqrt{\frac{c}{dx^2}+1}\left(c+dx^2\right)\left(-a^2d^2-2abcd+7b^2c^2\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)+x\left(a^2d^2\left(5c+3dx^2\right)+2abcd\left(c+3dx^2\right)-b^2c^2\right.\right.}{6c^2d^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e*x]*(x*(2*a*b*c*d*(c + 3*d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*
(7*c + 9*d*x^2)) + 3*(7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*
x*(c + d*x^2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(6*c^2*d^2*
(c + d*x^2)^(3/2))
```

Maple [B] time = 0.023, size = 1176, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/12*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3+12*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b*c^2*d^2-42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-3*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2+21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d+6*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-42*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^4-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-6*x^
4*a^2*d^4-12*x^4*a*b*c*d^3+18*x^4*b^2*c^2*d^2-10*x^2*a^2*c*d^3-4*x^2*a*b*c^
2*d^2+14*x^2*b^2*c^3*d)*(e*x)^(1/2)/d^3/c^2/x/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)\sqrt{dx^2+c}\sqrt{ex}}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3*x^6 + 3
*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

3.861 $\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2+2abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(5ad+7bc)(bc-ad)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc}{3cd^2e(c}$$

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(7*b
*c + 5*a*d)*Sqrt[e*x])/(6*c^2*d^2*e*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 2*a*b*
c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*
c^(9/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.162365, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {463, 457, 329, 220}

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2+2abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(5ad+7bc)(bc-ad)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(7*b
*c + 5*a*d)*Sqrt[e*x])/(6*c^2*d^2*e*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 2*a*b*
c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*
c^(9/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{\int \frac{\frac{1}{2}\left(-6a^2d^2+(bc-ad)^2\right)-3b^2cdx^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\int \frac{1}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{12c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\frac{dx^4}{e^2}}}\,d\right.}{6c^2d^2e}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c}{(\sqrt{c}}}}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+}}$$

Mathematica [C] time = 0.266587, size = 169, normalized size = 0.79

$$\frac{x\left(\frac{i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2+2abcd+5b^2c^2\right)\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}+5a^2d^2+\frac{2c(bc-ad)^2}{c+dx^2}+2abcd-7b^2c^2\right)}{6c^2d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(-7*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
(I*(5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[
d]]))/(6*c^2*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.025, size = 660, normalized size = 3.1

$$\frac{1}{12\,c^2d^3}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^2a^2d^3+2\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] 1/12*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a^2*d^3+2*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*(-c*d)^(1/2)*x^2*a*b*c*d^2+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-
c*d)^(1/2)*x^2*b^2*c^2*d+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d
^2+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d+5*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+10*x^3*a^2*d^4+4*x^3*a*b*c*d^3-14*x^3*b^2*c
^2*d^2+14*x*a^2*c*d^3-4*x*a*b*c^2*d^2-10*x*b^2*c^3*d)/(e*x)^(1/2)/c^2/d^3/(
d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^3ex^7+3\,cd^2ex^5+3\,c^2dex^3+c^3ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3*e*x^7 +
 3*c*d^2*e*x^5 + 3*c^2*d*e*x^3 + c^3*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.862 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=442

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(2bc-7ad)+b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{\ldots}{ce\sqrt{\ldots}}$$

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 7*a^2*
d^2)*(e*x)^(3/2))/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + a*d*(2*b*c
- 7*a*d))*(e*x)^(3/2))/(2*c^3*d*e^3*Sqrt[c + d*x^2]) - ((b^2*c^2 + a*d*(2*b
*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^3*d^(3/2)*e^2*(Sqrt[c] + Sqrt[
d]*x)) + ((b^2*c^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(2*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((b^2*c
^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(4*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.420859, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 457, 290, 329, 305, 220, 1196}

$$-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(ad(2bc-7ad)+b^2c^2\right)}{2c^3d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\ldots}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```


```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 7*a^2*
d^2)*(e*x)^(3/2))/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + a*d*(2*b*c
- 7*a*d))*(e*x)^(3/2))/(2*c^3*d*e^3*Sqrt[c + d*x^2]) - ((b^2*c^2 + a*d*(2*b
*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^3*d^(3/2)*e^2*(Sqrt[c] + Sqrt[
d]*x)) + ((b^2*c^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(2*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((b^2*c
^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(4*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeBQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} + \frac{2\int \frac{\sqrt{ex}\left(\frac{1}{2}a(2bc-7ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^{5/2}}dx}{ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)(ex)^{3/2}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7ad))\int \frac{\sqrt{ex}}{(c+dx^2)^{3/2}}dx}{2c^2de^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)(ex)^{3/2}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7ad))(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}} - \frac{(b^2c}{}$$

$$= -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)(ex)^{3/2}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7ad))(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}} - \frac{(b^2c}{}$$

$$= -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)(ex)^{3/2}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7ad))(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}} - \frac{(b^2c}{}$$

$$= -\frac{2a^2}{ce\sqrt{ex}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)(ex)^{3/2}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7ad))(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}} - \frac{(b^2c}{}$$

Mathematica [C] time = 0.146539, size = 161, normalized size = 0.36

$$\frac{x\left(-x^2(c+dx^2)\sqrt{\frac{dx^2}{c}+1}\,(-7a^2d^2+2abcd+b^2c^2)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)+a^2(-d)(12c^2+35cdx^2+21d^2x^4)+2abcdx^2(5c\right.}{6c^3d(ex)^{3/2}(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(b^2*c^2*x^2*(c + 3*d*x^2) + 2*a*b*c*d*x^2*(5*c + 3*d*x^2) - a^2*d*(12*c
^2 + 35*c*d*x^2 + 21*d^2*x^4) - (b^2*c^2 + 2*a*b*c*d - 7*a^2*d^2)*x^2*(c +
d*x^2)*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))
/(6*c^3*d*(e*x)^(3/2)*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.026, size = 1187, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3-12*((d*x+(-c*d)^(1/2))/(
```

```
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b*c^2*d^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-21*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3+6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d+42*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2-12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-6*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c^4-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-42*x^
4*a^2*d^4+12*x^4*a*b*c*d^3+6*x^4*b^2*c^2*d^2-70*x^2*a^2*c*d^3+20*x^2*a*b*c^
2*d^2+2*x^2*b^2*c^3*d-24*a^2*c^2*d^2)/d^2/c^3/e/(e*x)^(1/2)/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^3e^2x^8+3\,cd^2e^2x^6+3\,c^2de^2x^4+c^3e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3*e^2*x^8
 + 3*c*d^2*e^2*x^6 + 3*c^2*d*e^2*x^4 + c^3*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.863 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=258

$$\frac{(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}\left(5ad(2bc-3ad)+b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\left(3a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{}{3ce}$$

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 3*
a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + 5*a*d*(2*
b*c - 3*a*d))*Sqrt[e*x])/(6*c^3*d*e^3*Sqrt[c + d*x^2]) + ((b^2*c^2 + 5*a*d*
(2*b*c - 3*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*
c^(13/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.234409, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {462, 457, 290, 329, 220}

$$-\frac{\sqrt{ex}\left(3a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}+\frac{(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}\left(5ad(2bc-3ad)+b^2c^2\right)F\left(2\tan^{-1}\right.}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```


```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 3*
a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + 5*a*d*(2*
b*c - 3*a*d))*Sqrt[e*x])/(6*c^3*d*e^3*Sqrt[c + d*x^2]) + ((b^2*c^2 + 5*a*d*
(2*b*c - 3*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*
c^(13/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{5/2}\left(c+dx^2\right)^{5/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}+\frac{2\int \frac{\frac{3}{2}a(2bc-3ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}\left(c+dx^2\right)^{5/2}}\,dx}{3ce^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}-\frac{\left(b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}}{3c^2de^3\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2c^2+5ad(2bc-3ad)\right)\int \frac{1}{\sqrt{ex}\left(c+dx^2\right.}}{6c^2de^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}-\frac{\left(b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}}{3c^2de^3\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2c^2+5ad(2bc-3ad)\right)\sqrt{ex}}{6c^3de^3\sqrt{c+dx^2}}+\frac{\left(b^2c\right.}{}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}-\frac{\left(b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}}{3c^2de^3\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2c^2+5ad(2bc-3ad)\right)\sqrt{ex}}{6c^3de^3\sqrt{c+dx^2}}+\frac{\left(b^2c\right.}{}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}-\frac{\left(b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}}{3c^2de^3\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2c^2+5ad(2bc-3ad)\right)\sqrt{ex}}{6c^3de^3\sqrt{c+dx^2}}+\frac{\left(b^2c\right.}{}$$

Mathematica [C] time = 0.267107, size = 211, normalized size = 0.82

$$\frac{x^{5/2}\left(\frac{a^2(-d)\left(4c^2+21cdx^2+15d^2x^4\right)+2abcdx^2\left(7c+5dx^2\right)+b^2c^2x^2\left(dx^2-c\right)}{c^3dx^{3/2}\left(c+dx^2\right)}+\frac{ix\sqrt{\frac{c}{dx^2}+1}\left(-15a^2d^2+10abcd+b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{c^3d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{6(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^(5/2)*((b^2*c^2*x^2*(-c + d*x^2) + 2*a*b*c*d*x^2*(7*c + 5*d*x^2) - a^2*d
*(4*c^2 + 21*c*d*x^2 + 15*d^2*x^4))/(c^3*d*x^(3/2)*(c + d*x^2)) + (I*(b^2*c
^2 + 10*a*b*c*d - 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(
6*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.026, size = 686, normalized size = 2.7

$$-\frac{1}{12\,xe^2c^3d^2}\left(15\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)x^3a^2d^3-10\,\sqrt{-cd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/12*(15*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^3*a^2*d^3-10*(-c*d)^(1
/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^3*a*b*c*d^2-(-c*d)^(1/2)*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^3*b^2*c^2*d+15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*a^2
*c*d^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*a*b*c^2*d-((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*(-c*d)^(1/2)*x*b^2*c^3+30*x^4*a^2*d^4-20*x^4*a*b*c*d^3-2*x^
4*b^2*c^2*d^2+42*x^2*a^2*c*d^3-28*x^2*a*b*c^2*d^2+2*x^2*b^2*c^3*d+8*a^2*c^2
*d^2)/x/e^2/(e*x)^(1/2)/c^3/d^2/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{dx^2 + c}\sqrt{ex}}{d^3e^3x^9 + 3\,cd^2e^3x^7 + 3\,c^2de^3x^5 + c^3e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3*e^3*x^9
 + 3*c*d^2*e^3*x^7 + 3*c^2*d*e^3*x^5 + c^3*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\left(dx^2 + c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.864 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=489

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{20c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-70ab\right.}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}\right.}$$

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) - (2*a*(10*b*c - 11*a*d))/(5
*c^2*e^3*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d
^2)*(e*x)^(3/2))/(15*c^3*e^5*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d
+ 77*a^2*d^2)*(e*x)^(3/2))/(10*c^4*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*
a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c^4*Sqrt[d]*e^4*(Sqrt[
c] + Sqrt[d]*x)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c +
d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], 1/2])/(20*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.469583, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {462, 453, 290, 329, 305, 220, 1196}

$$-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4e^5\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{15c^3e^5\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```



```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) - (2*a*(10*b*c - 11*a*d))/(5
*c^2*e^3*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d
^2)*(e*x)^(3/2))/(15*c^3*e^5*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d
+ 77*a^2*d^2)*(e*x)^(3/2))/(10*c^4*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*
a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c^4*Sqrt[d]*e^4*(Sqrt[
c] + Sqrt[d]*x)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c +
d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], 1/2])/(20*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```


Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} + \frac{2\int \frac{\frac{1}{2}a(10bc-11ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}(c+dx^2)^{5/2}}dx}{5ce^2}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} - \frac{\left(4\left(-\frac{5}{4}b^2c^2+\frac{7}{4}ad(10bc-11ad)\right)\right)\int \cdots}{5c^2e^4}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11ad))(ex)^{3/2}}{15c^3e^5(c+dx^2)^{3/2}} - \cdots$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11ad))(ex)^{3/2}}{15c^3e^5(c+dx^2)^{3/2}} + \cdots$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11ad))(ex)^{3/2}}{15c^3e^5(c+dx^2)^{3/2}} + \cdots$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11ad))(ex)^{3/2}}{15c^3e^5(c+dx^2)^{3/2}} + \cdots$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11ad))(ex)^{3/2}}{15c^3e^5(c+dx^2)^{3/2}} + \cdots$$

Mathematica [C] time = 0.175168, size = 181, normalized size = 0.37

$$\frac{x\left(-x^4(c+dx^2)\sqrt{\frac{dx^2}{c}+1}\,(77a^2d^2-70abcd+5b^2c^2)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)+a^2(132c^2dx^2-12c^3+385cd^2x^4+231d^3x^6 \cdots\right.}{30c^4(ex)^{7/2}(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(5*b^2*c^2*x^4*(5*c + 3*d*x^2) - 10*a*b*c*x^2*(12*c^2 + 35*c*d*x^2 + 21*
d^2*x^4) + a^2*(-12*c^3 + 132*c^2*d*x^2 + 385*c*d^2*x^4 + 231*d^3*x^6) - (5
*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*x^4*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(30*c^4*(e*x)^(7/2)*(c + d*x^
2)^(3/2))
```

Maple [B] time = 0.03, size = 1231, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/60*(462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d^3-420*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^4*a*b*c^2*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3*d-231*
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d^3+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x
^4*a*b*c^2*d^2-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3*d+462*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a^2*c^2*d^2-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^
3*d+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^4-231*((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a^2*c^2*d^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^3*d-15*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*x^2*b^2*c^4-462*x^6*a^2*d^4+420*x^6*a*b*c*d^3-30*x^6*
b^2*c^2*d^2-770*x^4*a^2*c*d^3+700*x^4*a*b*c^2*d^2-50*x^4*b^2*c^3*d-264*x^2*
a^2*c^2*d^2+240*x^2*a*b*c^3*d+24*a^2*c^3*d)/x^2/d/c^4/e^3/(e*x)^(1/2)/(d*x^
2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex}}{d^3e^4x^{10}+3\,cd^2e^4x^8+3\,c^2de^4x^6+c^3e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x)/(d^3*e^4*x^1
0 + 3*c*d^2*e^4*x^8 + 3*c^2*d*e^4*x^6 + c^3*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

3.865 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{21b^2d}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{}}{}$$

```
[Out] (2*(2*b*c - 7*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2*d) - (2*e*(e*x)^(
5/2)*Sqrt[c - d*x^2])/(7*b) - (2*c^(1/4)*(2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d
^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d
)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.80242, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 478, 582, 523, 224, 221, 409, 1219, 1218}

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{21b^2d}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (2*(2*b*c - 7*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2*d) - (2*e*(e*x)^(
5/2)*Sqrt[c - d*x^2])/(7*b) - (2*c^(1/4)*(2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d
^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d
)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(2e)\,\text{Subst}\left(\int \frac{x^4\left(5ac+\frac{(2bc-7ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{7b}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{(2e^5)\,\text{Subst}\left(\int \frac{\frac{ac(2bc-7ad)}{e^2}-\frac{(2b^2c^2+14abcd-21a^2d^2)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,d\right.}{21b^2d}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(2a^2(bc-ad)e^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,\right.}{b^3}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(a(bc-ad)e^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{b^3}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{2\sqrt[4]{c}\left(2b^2c^2+14abcd-21a^2d^2\right)e^{7/2}\sqrt{1-\frac{dx^2}{c}}}{21b^3d^{5/4}\sqrt{c-dx^2}}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{2\sqrt[4]{c}\left(2b^2c^2+14abcd-21a^2d^2\right)e^{7/2}\sqrt{1-\frac{dx^2}{c}}}{21b^3d^{5/4}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.215411, size = 187, normalized size = 0.5

$$\frac{2e^3\sqrt{ex}\left(x^2\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ac\sqrt{1-\frac{dx^2}{c}}(7ad-2bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.}{105ab^2d\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-2*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-2
*b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*
x^2)/a] + (2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*d*Sqrt[c - d*x^2
])
```

Maple [B] time = 0.071, size = 1479, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/42*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2/d*(42*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)-70*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)+24*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*a*b^2*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*
b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^
(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+12*x^5*a*b^2*d^4*(a*b)^(1/2)-1
2*x^5*b^3*c*d^3*(a*b)^(1/2)+28*x^3*a^2*b*d^4*(a*b)^(1/2)-48*x^3*a*b^2*c*d^3
*(a*b)^(1/2)+20*x^3*b^3*c^2*d^2*(a*b)^(1/2)-28*x*a^2*b*c*d^3*(a*b)^(1/2)+36
*x*a*b^2*c^2*d^2*(a*b)^(1/2)-8*x*b^3*c^3*d*(a*b)^(1/2))/x/(d*x^2-c)/(a*b)^(
1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

3.866 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=414

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-$$

```
[Out] (-2*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) - (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)*e
^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqr
t[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.817372, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 478, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\sqrt{a}\sqrt[4]{c}e^{5/}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) - (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)*e
^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqr
t[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{(2e)\,\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(2bc-5ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{(2e)\,\text{Subst}\left(\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5\left(-abc+a^2d\right)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right)}{5b}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} - \frac{(2(2bc-5ad)e)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2} + \frac{(2a(bc-ad)e)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{\left(2\sqrt{c}(2bc-5ad)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2\sqrt{d}} - \frac{\left(2\sqrt{c}(2bc-5ad)e^2\cdots\right.}{\cdots}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{\left(2\sqrt{c}(2bc-5ad)e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2\sqrt{d}\sqrt{c-dx^2}} - \frac{\left(2\sqrt{c}(2bc\cdots\right.}{\cdots}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}} - \frac{\sqrt{a}\sqrt[4]{c}(bc-ad)e\cdots}{\cdots}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} - \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}} + \frac{2c^{3/4}(2bc-5ad\cdots}{\cdots}$$

Mathematica [C] time = 0.134432, size = 143, normalized size = 0.35

$$\frac{2e(ex)^{3/2}\left(x^2\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\left(c-dx^2\right)\right)}{35ab\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^2)
/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(35*a*b*Sqrt[c - d*
x^2])
```

Maple [B] time = 0.043, size = 1491, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/10*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-5*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-5
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*(c*d)^(1/2)*a*b*c*d-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+28*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b^2*c^2*d-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+10*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a^2*b*c*d^2-14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d+4*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3
*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^
2*b*c*d^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d+4*x^4*a*b^
2*d^3-4*x^4*b^3*c*d^2-4*x^2*a*b^2*c*d^2+4*x^2*b^3*c^2*d)/x/b^2/(d*x^2-c)/((
a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}{bx^2 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}{bx^2 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.867 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+$$

```
[Out] (-2*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) - (2*c^(1/4)*(2*b*c - 3*a*d)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(3*b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.508994, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 478, 523, 224, 221, 409, 1219, 1218}

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (-2*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) - (2*c^(1/4)*(2*b*c - 3*a*d)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(3*b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(b^2*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} + \frac{(2e)\,\text{Subst}\left(\int \frac{ac+\frac{(2bc-3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b}$$

$$= -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} - \frac{(2(2bc-3ad)e)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b^2} + \frac{(2a(bc-ad)e)\,\text{Subst}\left(\int \frac{}{\left(a-\frac{b}{}\right.}\right.}{b^2}$$

$$= -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{}{(1}\right.}{b^2}$$

$$= -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} - \frac{2\sqrt[4]{c}(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\left((bc-ad)e\sqrt{1-\frac{dx^2}{c}}\right)}{}$$

$$= -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} - \frac{2\sqrt[4]{c}(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{1-}}{b}$$

Mathematica [C] time = 0.117163, size = 143, normalized size = 0.45

$$\frac{2e\sqrt{ex}\left(x^2\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5a\left(c-dx^2\right)\right)}{15ab\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c
]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[c - d*x^2
])
```

Maple [B] time = 0.017, size = 1286, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] -1/6*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
```

```
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-3*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*(c*d)^(1/2)*a*b*c*d-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a
*b*c*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-4*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d-3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*(c*d)^(1/2)*a*b*c*d-4*x^3*a*b*d^3*(a*b)^(1/2)+4*x^3*b^2*c*d^2*(a*b)^(1/2
)+4*x*a*b*c*d^2*(a*b)^(1/2)-4*x*b^2*c^2*d*(a*b)^(1/2))/b/x/(d*x^2-c)/(a*b)^
(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.868 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=365

$$-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{\ldots}}{\ldots}$$

```
[Out] (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*
x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.535333, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {466, 491, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*
x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= -\frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{((bc-ad)e \cdots}{\cdots}$$

$$= -\frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{c-dx^2}} + \frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \cdots\right)}{b\sqrt{c-dx^2}}$$

$$= -\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}} \cdots\right.\right.}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}$$

$$= \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} - \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(\cdots}{\cdots}$$

Mathematica [C] time = 0.0416688, size = 69, normalized size = 0.19

$$\frac{2x\sqrt{ex}\sqrt{c-dx^2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{1-\frac{dx^2}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[c - d*x^2]*AppellF1[3/4, -1/2, 1, 7/4, (d*x^2)/c, (b*x^
2)/a])/(3*a*Sqrt[1 - (d*x^2)/c])
```

Maple [B] time = 0.017, size = 701, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*d*(Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*a*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
```

```
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+(c*d)^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c+EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*
b*c*d+(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*b*c-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d
-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2)/b/
x/(d*x^2-c)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

3.869 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\left(a-bx^2\right)}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc$$

```
[Out] (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.370585, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {466, 406, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\left(a-bx^2\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be} + \frac{(2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe}$$

$$= \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}} + \frac{\left((bc-ad)\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x,\right.}{abe\sqrt{c-dx^2}}$$

$$= \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right.}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.0376311, size = 67, normalized size = 0.24

$$\frac{2x\sqrt{c-dx^2}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\sqrt{ex}\sqrt{1-\frac{dx^2}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*x*Sqrt[c - d*x^2]*AppellF1[1/4, -1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(a*
Sqrt[e*x]*Sqrt[1 - (d*x^2)/c])
```

Maple [B] time = 0.031, size = 651, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x)
```

```
[Out] -1/2*(-d*x^2+c)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
a*d^2*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)-Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*b*c*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b
*d*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*d^2*(a*b)^(1/2)+EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c*d*
(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a*d^2*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*b*c*d*(a*b)^(1/2))/(e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(
c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.870 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx$

Optimal. Leaf size=392

$$\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc}{a}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/
c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)
*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) -
 (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3
/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt
[c - d*x^2])
```

Rubi [A] time = 0.717934, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 475, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+2c^{3/4}\sqrt[4]{d}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/
c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)
*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) -
 (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3
/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt
[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
```

```
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^2\left(a-\frac{bx^4}{e^2}\right)}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(\frac{bc-2ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}}+\frac{(bc-ad)x^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx,x,\sqrt{ex}\right)}{ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae^3} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{\right.}}{ae^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae^2} - \frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae^2}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\sqrt{ex}\right)}{ae^2\sqrt{c-dx^2}} - \frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int\right.}{ae^2\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\right.}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}} + \frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-\right.}{ae^{3/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.119749, size = 143, normalized size = 0.36

$$\frac{x\left(6bdx^4\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+14x^2\sqrt{1-\frac{dx^2}{c}}(bc-2ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-42a\left(c-dx^2\right)\right)}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-42*a*(c - d*x^2) + 14*(b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3
/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e*x)^(3/2)*Sqrt[c
- d*x^2])
```

Maple [B] time = 0.038, size = 1274, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x)
```

```
[Out] 1/2*(-d*x^2+c)^(1/2)*d*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-4*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*b^2*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*(c*d)^(1/2)*a*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d-((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b*c+(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2+4*a*b
*d^2*x^2-4*b^2*c*d*x^2-4*c*a*b*d+4*b^2*c^2)/e/(e*x)^(1/2)/(d*x^2-c)/a/((a*b
)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-dx^2}}{-a(ex)^{\frac{3}{2}}+bx^2(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.871 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx$

Optimal. Leaf size=308

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*e
^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c -
 a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqr
t[c - d*x^2])
```

Rubi [A] time = 0.495098, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 475, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*e
^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c -
 a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqr
t[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^4\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{3bc-2ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ae^3} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{ae^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a^2e^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\right.}\right.}{a^2e^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}} + \frac{\left((bc-ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{a^2e^3\sqrt{c-dx}}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{si}\right.}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx}}$$

Mathematica [C] time = 0.13117, size = 146, normalized size = 0.47

$$\frac{x\left(10x^2\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(bdx^4\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(c-dx^2\right)\right)\right)}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (x*(10*(3*b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (
d*x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*(e*x)^(5/2)*Sqrt[c
 - d*x^2])
```

Maple [B] time = 0.034, size = 1167, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x)
```

```
[Out] -1/6*(-d*x^2+c)^(1/2)*b*d*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b*c
*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
```

```
1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*
d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
x*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*x*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)-4*x^2*a*d^2*(a*b)^(1/2)+4*x^2*b*c*d*(a*b)^(1/2)+4*a*c*d*(a*b)^(
1/2)-4*b*c^2*(a*b)^(1/2))/x/a/e^2/(e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a*b)^
(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.872 $\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx$

Optimal. Leaf size=457

$$\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-2ad)}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 2*a*d)*Sqrt[c - d*x^
2])/(5*a^2*c*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/c
]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(1
/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/
c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(
1/4)*e^(7/2)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
 + (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.983984, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 475, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{2\sqrt{c-dx^2}(5bc-2ad)}{5a^2ce^3\sqrt{ex}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 2*a*d)*Sqrt[c - d*x^
2])/(5*a^2*c*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/c
]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(1
/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/
c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(
1/4)*e^(7/2)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
 + (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
```

```
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^6\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{5bc-2ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{5b^2c^2-10abcd+2a^2d^2}{e^4}-\frac{bd(5bc-2ad)x^4}{e^6}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{5a^2ce}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\,\text{Subst}\left(\int \left(\frac{d(5bc-2ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5\left(b^2c^2-abcd\right)x^2}{e^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right.}{5a^2ce}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{(2d(5bc-2ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5a^2ce^5} + \frac{(2b(b\cdots}{\cdots}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{\left(2\sqrt{d}(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5a^2\sqrt{c}e^4} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{\left(2\sqrt{d}(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \cdots\right.}{5a^2\sqrt{c}e^4\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{2\sqrt[4]{d}(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}} - \cdots$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\sqrt[4]{d}(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}} + \cdots$$

Mathematica [C] time = 0.218595, size = 190, normalized size = 0.42

$$\frac{x\left(14x^4\sqrt{1-\frac{dx^2}{c}}\left(2a^2d^2-10abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(bdx^6\sqrt{1-\frac{dx^2}{c}}(2ad-5bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\cdots\right.\right.}{105a^3c(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (x*(14*(5*b^2*c^2 - 10*a*b*c*d + 2*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF
1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*b*c
*x^2 - 2*a*d*x^2) + b*d*(-5*b*c + 2*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*c*(e*x)^(7/2)*Sqrt[c -
```

d*x^2])

Maple [B] time = 0.041, size = 1553, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x)
```

```
[Out] 1/10*(-d*x^2+c)^(1/2)*d*b*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*x^2*a*b*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*c*d-5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^2*c^3-5*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d
)^(1/2)*x^2*b*c^2-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+28*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a*b*c^2*d-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+4*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*a^2*c*d^2-14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d+10*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*x^2*a*b*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*c*d-5*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^2*c^3+5*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
b*c^2-8*x^4*a^2*d^3+28*x^4*a*b*c*d^2-20*x^4*b^2*c^2*d+12*x^2*a^2*c*d^2-32*x
^2*a*b*c^2*d+20*x^2*b^2*c^3-4*a^2*c^2*d+4*a*b*c^3)/x^2/e^3/(e*x)^(1/2)/(d*x
^2-c)/a^2/c/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.873 $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=485

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}$$

```
[Out] (-2*(11*b*c - 9*a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(45*b^2) + (2*d*(e*x)^(
7/2)*Sqrt[c - d*x^2])/(9*b*e) - (2*c^(3/4)*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2
*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(4*b^2*c
^2 - 21*a*b*c*d + 15*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2
]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c
- a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1
/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.10645, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 477, 582, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)E\left(\sin\right.}{15b^3d^{3/4}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```


```
[Out] (-2*(11*b*c - 9*a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(45*b^2) + (2*d*(e*x)^(
7/2)*Sqrt[c - d*x^2])/(9*b*e) - (2*c^(3/4)*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2
*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(4*b^2*c
^2 - 21*a*b*c*d + 15*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2
]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c
- a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1
/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} - \frac{(2e)\,\text{Subst}\left(\int \frac{x^6\left(-\frac{c(9bc-7ad)}{e^2}+\frac{d(11bc-9ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{9b}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2e^5\right)\text{Subst}\left(\int \frac{x^2\left(\frac{3acd(11bc-9ad)}{e^4}+\frac{3d(4b\cdots}{\cdots}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{\cdots}}\cdots\right)}{45b^2d\cdots}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2e^5\right)\text{Subst}\left(\int\left(-\frac{3d\left(4b^2c^2-21abcd+1\cdots\right.}{be^4\sqrt{c-\frac{dx^4}{e^2}}}\cdots\right)\cdots\right)}{\cdots}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2a(bc-ad)^2e\right)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{\cdots}}\cdots\right)}{b^3}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2\sqrt{c}\left(4b^2c^2-21abcd+15a^2d^2\right)e^{\cdots}\right.}{15b^3\cdots}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2\sqrt{c}\left(4b^2c^2-21abcd+15a^2d^2\right)e^{\cdots}\right.}{15b^3\cdots}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{2c^{3/4}\left(4b^2c^2-21abcd+15a^2d^2\right)e^{5}\cdots}{15b^3d^{3/4}\cdots}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} - \frac{2c^{3/4}\left(4b^2c^2-21abcd+15a^2d^2\right)e^{5}\cdots}{15b^3d^{3/4}\cdots}$$

Mathematica [C] time = 0.202092, size = 183, normalized size = 0.38

$$-\frac{2e(ex)^{3/2}\left(-3x^2\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7ac\sqrt{1-\frac{dx^2}{c}}(9ad-11bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cdots\right.\right.}{315ab^2\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2)*(-11*b*c + 9*a*d + 5*b*d*x^2) + 7*a*c*(
-11*b*c + 9*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c,
(b*x^2)/a] - 3*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c
]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(315*a*b^2*Sqrt[c - d
*x^2])
```

Maple [B] time = 0.033, size = 2183, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x)
```

```
[Out] -1/90*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(36*x^4*a^2*b^2*d^4+64*x^4*b^4*c^2*d
^2-44*x^2*b^4*c^3*d+45*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*a*b^3*c^3*d+45*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*a*b^3*c^3*d+90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^3+90*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)-100*x^4*a*b^3*c*d^3-45*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d+45
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*(c*d)^(1/2)*a*b^2*c^2*d-216*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d^2+150*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a*b^3*c^3*d+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^
3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-90*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+45
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)-90*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2
*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+20*x^6*a*b^3*d^4-20*x^6*b^4*c*d^3-1
80*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^3+432*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d^
```

```
2-300*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3*d+48*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4-36*x
^2*a^2*b^2*c*d^3+80*x^2*a*b^3*c^2*d^2-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4-90*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2))/x/b^3/(d*x^2-c)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c
*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.874 $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{21b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\right.}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] (-2*(9*b*c - 7*a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2) + (2*d*(e*x)^(5/2
)*Sqrt[c - d*x^2])/(7*b*e) - (2*c^(1/4)*(12*b^2*c^2 - 35*a*b*c*d + 21*a^2*d
^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^
2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.778367, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 477, 582, 523, 224, 221, 409, 1219, 1218}

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\right.\right.}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*(9*b*c - 7*a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2) + (2*d*(e*x)^(5/2
)*Sqrt[c - d*x^2])/(7*b*e) - (2*c^(1/4)*(12*b^2*c^2 - 35*a*b*c*d + 21*a^2*d
^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^
2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
```

```
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{(2e)\,\text{Subst}\left(\int \frac{x^4\left(-\frac{c(7bc-5ad)}{e^2}+\frac{d(9bc-7ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{7b}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{\left(2e^5\right)\text{Subst}\left(\int \frac{\frac{acd(9bc-7ad)}{e^4}+\frac{d(12b^2c^2-35abcd}{e^6}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{21b^2d}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{\left(2a(bc-ad)^2e\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{d}{e}}}\right.}{b^3}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{\left((bc-ad)^2e\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{b^3}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{2\sqrt[4]{c}\left(12b^2c^2-35abcd+21a^2d^2\right)e^{3/2}}{21b^3\sqrt[4]{d}\sqrt{c}}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{2\sqrt[4]{c}\left(12b^2c^2-35abcd+21a^2d^2\right)e^{3/2}}{21b^3\sqrt[4]{d}\sqrt{c}}$$

Mathematica [C] time = 0.189403, size = 182, normalized size = 0.49

$$-\frac{2e\sqrt{ex}\left(x^2\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+35abcd-12b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ac\sqrt{1-\frac{dx^2}{c}}(7ad-9bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\right.\right.}{105ab^2\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*e*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-9*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-9*
b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x
^2)/a] + (-12*b^2*c^2 + 35*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*Sqrt[c - d*x^2]
)
```

Maple [B] time = 0.024, size = 1920, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x)
```

```
[Out] 1/42*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2*(21*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-42
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)-42*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d+21*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d-2
1*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+42*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^
2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-42*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3*
d+21*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d-42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)+112*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-9
4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^
2*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+24*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*(a*b)^(1/2)
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-12*x^5*a*b^2*d^4*(a*b)^(1/2)+12*x^5
*b^3*c*d^3*(a*b)^(1/2)-28*x^3*a^2*b*d^4*(a*b)^(1/2)+76*x^3*a*b^2*c*d^3*(a*b
)^(1/2)-48*x^3*b^3*c^2*d^2*(a*b)^(1/2)+28*x*a^2*b*c*d^3*(a*b)^(1/2)-64*x*a*
b^2*c^2*d^2*(a*b)^(1/2)+36*x*b^3*c^3*d*(a*b)^(1/2))/x/(d*x^2-c)/(a*b)^(1/2)
/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int -\frac{dx^2\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x
**2*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.875 $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=421

$$-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}$$

```
[Out] (2*d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e) + (2*c^(3/4)*d^(1/4)*(7*b*c - 5*a
*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*(7*b*c - 5*
a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.823461, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 477, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e) + (2*c^(3/4)*d^(1/4)*(7*b*c - 5*a
*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*(7*b*c - 5*
a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
```

```
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{(2e)\,\text{Subst}\left(\int \frac{x^2\left(-\frac{c(5bc-3ad)}{e^2}+\frac{d(7bc-5ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{(2e)\,\text{Subst}\left(\int \left(-\frac{d(7bc-5ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5\left(b^2c^2-2abcd+a^2d^2\right)x^2}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right)}{5b}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} + \frac{(2d(7bc-5ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2e} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{\ldots}{\left(a-\ldots\right.}\right.}{b^2e}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{\left(2\sqrt{c}\sqrt{d}(7bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2} + \frac{\left(2\sqrt{c}\sqrt{d}(7bc-5ad)\right)\ldots}{\ldots}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{\left(2\sqrt{c}\sqrt{d}(7bc-5ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2\sqrt{c-dx^2}} + \frac{\left(2\sqrt{c}\sqrt{d}(7b\ldots\right.}{\ldots}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{2c^{3/4}\sqrt[4]{d}(7bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{e}\ldots}{\ldots}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} + \frac{2c^{3/4}\sqrt[4]{d}(7bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}} - \frac{2c^{3/4}\sqrt[4]{d}(7bc-5a\ldots}{\ldots}$$

Mathematica [C] time = 0.205163, size = 155, normalized size = 0.37

$$\frac{2x\sqrt{ex}\left(7c\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d\left(x^2\sqrt{1-\frac{dx^2}{c}}(5ad-7bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7a\left(c-dx^2\right)\right)\ldots}{105ab\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(7*c*(5*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2,
1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*(7*a*(c - d*x^2) + (-7*b*c + 5*a*d)*x^2
*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(
105*a*b*Sqrt[c - d*x^2])
```

Maple [B] time = 0.021, size = 1927, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/10*(-d*x^2+c)^(1/2)*(e*x)^(1/2)*d*(20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-48
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-10*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*b*c*d^2+24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d-14*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*b^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d+5*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*(c*d)^(1/2)*b^2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d-5*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*(c*d)^(1/2)*b^2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*a^2*b*c*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d-5*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(
```

```
c*d)^(1/2))^(1/2)*2^(1/2)*b^3*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-5*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*b^3*c^3-4*x^4*a*b^2*d^3+4*x^4*b^3*c*d
^2+4*x^2*a*b^2*c*d^2-4*x^2*b^3*c^2*d)/b^2/x/(d*x^2-c)/((a*b)^(1/2)*d+(c*d)^
(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int -\frac{dx^2\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)*(e*x)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x**2
*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

3.876 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)}\,dx$

Optimal. Leaf size=328

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\ldots$$

```
[Out] (2*d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e) + (2*c^(1/4)*d^(3/4)*(5*b*c - 3*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^
2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.562045, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 416, 523, 224, 221, 409, 1219, 1218}

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```


```
[Out] (2*d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e) + (2*c^(1/4)*d^(3/4)*(5*b*c - 3*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^
2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c-dx^2\right)^{3/2}}{\sqrt{ex}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} - \frac{(2e)\,\text{Subst}\left(\int \frac{-\frac{c(3bc-ad)}{e^2}+\frac{d(5bc-3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{3b}$$

$$= \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} + \frac{(2d(5bc-3ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{3b^2e} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\cdots}\right.}{b^2e}$$

$$= \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{ab^2e} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\cdots}\right.}{ab^2e}$$

$$= \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} + \frac{2\sqrt[4]{c}d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}} + \frac{\left((bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Su}\cdots}{ab\cdots}$$

$$= \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} + \frac{2\sqrt[4]{c}d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\Pi\cdots}{ab^2\sqrt[4]{d}\cdots}$$

Mathematica [C] time = 0.160598, size = 153, normalized size = 0.47

$$\frac{2x\left(dx^2\sqrt{1-\frac{dx^2}{c}}(3ad-5bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5c\sqrt{1-\frac{dx^2}{c}}(3bc-ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ad\left(c-dx^2\right)\right)}{15ab\sqrt{ex}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*x*(5*a*d*(c - d*x^2) + 5*c*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/
4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(-5*b*c + 3*a*d)*x^2*Sqrt[1 - (d*
x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[e*x
]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.023, size = 1721, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x)
```

```
[Out] 1/6*(-d*x^2+c)^(1/2)*d*(3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
```

```
*d),1/2*2^(1/2))*a^2*b*c*d^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-6*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-6*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*a*b*c*d+3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*b^3*c^3+3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*(c*d)^(1/2)*b^2*c^2-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)+16*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*a*b*c*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-10*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2+3*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a
^2*d^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
a*b^2*c^2*d-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d-3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*b^3*c^3+3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(c*d)^(1/2)*b^2*c^2-4*x^3*a*b*d^
3*(a*b)^(1/2)+4*x^3*b^2*c*d^2*(a*b)^(1/2)+4*x*a*b*c*d^2*(a*b)^(1/2)-4*x*b^2
*c^2*d*(a*b)^(1/2))/b/(e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d
)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx-\int -\frac{dx^2\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x) - Integr
al(-d*x**2*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.877 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

Optimal. Leaf size=417

$$\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\ldots$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*b*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]
) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^
(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.788696, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 474, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```


```
[Out] (-2*c*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*b*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]
) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^
(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
```

```
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^2\left(a-\frac{bx^4}{e^2}\right)}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(\frac{c(bc-3ad)}{e^2}+\frac{d(bc+ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{d(bc+ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^2}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx,x,\sqrt{ex}\right)}{ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{abe^3} - \frac{(2d(bc+ad))\,\text{Subst}\left(\int \cdots\right.}{abe^3}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{abe^2} - \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad)\right)\text{Subst}\cdots}{abe^2}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\sqrt{ex}\right)}{abe^2\sqrt{c-dx^2}} - \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad\cdots\right.}{\cdots}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\Pi\cdots}{a^{3/2}b^{3/2}\sqrt[4]{\cdots}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}} + \frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\cdots}}{abe^{3/2}\sqrt{\cdots}}$$

Mathematica [C] time = 0.1428, size = 151, normalized size = 0.36

$$\frac{x\left(6dx^4\sqrt{1-\frac{dx^2}{c}}(ad+bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+14cx^2\sqrt{1-\frac{dx^2}{c}}(bc-3ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-42ac\left(c-dx^2\right)\right)}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-42*a*c*(c - d*x^2) + 14*c*(b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*d*(b*c + a*d)*x^4*Sqrt[1 - (
d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e*x)
^(3/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.027, size = 1754, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x)
```

```
[Out] -1/2*(-d*x^2+c)^(1/2)*d*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a^2*b*c*d^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*
b*c*d+((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*b^3
*c^3+((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*(c*d)^(1/2)*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+4*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b^3*c^3+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*b^3*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d+((-d*x+(c
```

```
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*b^3*c^3-((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(c*d)^(
1/2)*b^2*c^2-4*x^2*a*b^2*c*d^2+4*x^2*b^3*c^2*d+4*a*c^2*d*b^2-4*c^3*b^3)/b/e
/(e*x)^(1/2)/(d*x^2-c)/a/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)
^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c-dx^2}}{-a\,(ex)^{\frac{3}{2}}+bx^2\,(ex)^{\frac{3}{2}}}\,dx-\int -\frac{dx^2\sqrt{c-dx^2}}{-a\,(ex)^{\frac{3}{2}}+bx^2\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x) -
Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.878 $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx$

Optimal. Leaf size=330

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\cdots$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*(b*c - 3*a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(3*a*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.591964, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 474, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```


```
[Out] (-2*c*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*(b*c - 3*a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(3*a*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
```

```
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^4\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{c(3bc-5ad)}{e^2}-\frac{d(bc-3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{(2d(bc-3ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3abe^3} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\left(a-\frac{bx^4}{e^2}\right.}\right.}{abe^3}$$

$$= -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a^2be^3} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{}{\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right.}\right.}{a^2be^3}$$

$$= -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}} + \frac{\left((bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\right.\text{S}}{a}$$

$$= -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\text{I}}{a^2b\sqrt[4]{}}$$

Mathematica [C] time = 0.152822, size = 153, normalized size = 0.46

$$\frac{x\left(-2dx^4\sqrt{1-\frac{dx^2}{c}}(bc-3ad)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+10cx^2\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-10ac\left(c-dx^2\right)\right.}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-10*a*c*(c - d*x^2) + 10*c*(3*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appe
llF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - 2*d*(b*c - 3*a*d)*x^4*Sqrt[1
 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2*(e
*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.025, size = 1740, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x)
```

```
[Out] 1/6*(-d*x^2+c)^(1/2)*d*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*
d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
```

```
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a
^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-6*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-6*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*(c*d)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^3*c^3*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*(c*d)^(1/2)+8*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(
c*d)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2
^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+
3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*(c*d)^(1/2)+6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^2
*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*2^(1/2)*x*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*(c*d)^(1/2)+4*x^2*a*b*c*d^2*(a*b)^(1/2)-4*x^2*b^2*c^2*d*(a*b)^(1/2)-4*a
*b*c^2*d*(a*b)^(1/2)+4*b^2*c^3*(a*b)^(1/2))/x/a/e^2/(e*x)^(1/2)/(d*x^2-c)/(
a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\,dx-\int -\frac{dx^2\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x) -
Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.879 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx$

Optimal. Leaf size=459

$$\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-2\sqrt{c}$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 7*a*d)*Sqrt[c - d*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*
a^2*e^(7/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5
*a^2*e^(7/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a
^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.05931, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 474, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5}{5a^2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 7*a*d)*Sqrt[c - d*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*
a^2*e^(7/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5
*a^2*e^(7/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a
^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{7/2}\left(a-bx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^6\left(a-\frac{bx^4}{e^2}\right)}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{c(5bc-7ad)}{e^2}-\frac{d(3bc-5ad)x^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{c\left(5b^2c^2-15abcd+12a^2d^2\right)}{e^4}-\frac{bcd(5bc-7ad)x^4}{e^6}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\right.}{5a^2ce}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\,\text{Subst}\left(\int \left(\frac{cd(5bc-7ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}}-\frac{5\left(b^2c^3-2abc^2d+a^2cd^2\right)x^2}{e^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx,\right.}{5a^2ce}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{(2d(5bc-7ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5a^2e^5} + \frac{(2(b}{}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{5a^2e^4} -$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,\right.}{5a^2e^4\sqrt{c-dx^2}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.22234, size = 187, normalized size = 0.41

$$\frac{x\left(14x^4\sqrt{1-\frac{dx^2}{c}}\left(12a^2d^2-15abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(bdx^6\sqrt{1-\frac{dx^2}{c}}(7ad-5bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right.\right.\right.}{105a^3(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (x*(14*(5*b^2*c^2 - 15*a*b*c*d + 12*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*b*
c*x^2 - 7*a*d*x^2) + b*d*(-5*b*c + 7*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[
7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*(e*x)^(7/2)*Sqrt[c - d
```

*x^2])

Maple [B] time = 0.027, size = 2028, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x)
```

```
[Out] -1/10*(-d*x^2+c)^(1/2)*d*(-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a^2*b*c*d^2-48*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c^2+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*c
*d^2-48*x^4*a*b^2*c*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3+5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3+5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^3+24*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*d^2+20*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*x^2*b^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*x^2*a^2*b*c*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*d^2-5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*(c*d)^(1/2)*x^2*b^2*c^2-14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(
```

```
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2-10*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c
*d)^(1/2)*x^2*a*b*c*d-4*a*b^2*c^3+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*c*d-32*x^2*a^2*
b*c*d^2+52*x^2*a*b^2*c^2*d-20*x^2*b^3*c^3+28*x^4*a^2*b*d^3+20*x^4*b^3*c^2*d
+4*a^2*b*c^2*d)/x^2/e^3/(e*x)^(1/2)/(d*x^2-c)/a^2/((a*b)^(1/2)*d+(c*d)^(1/2
)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.880 $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=305

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\cdots}{\cdots}$$

```
[Out] (2*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (2*c^(1/4)*(b*c + 3*a*d)*e^(7/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (a*c^(1/
4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c
 - d*x^2])
```

Rubi [A] time = 0.489501, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 479, 523, 224, 221, 409, 1219, 1218}

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```


```
[Out] (2*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (2*c^(1/4)*(b*c + 3*a*d)*e^(7/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (a*c^(1/
4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c
 - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{ac-\frac{(bc+3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3bd}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} + \frac{(2a^2e^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} - \frac{(2(bc+3ad)e^3)\,\text{Subst}\left(\right.}{3b^2 \ldots}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\ldots}\right.}{b^2}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{2\sqrt[4]{c}(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}} + \frac{\left(ae^3\sqrt{1-\frac{dx^2}{c}}\right)\text{Su}\ldots}{\ldots}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{2\sqrt[4]{c}(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}} + \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\ldots}{\ldots}$$

Mathematica [C] time = 0.128182, size = 147, normalized size = 0.48

$$\frac{2e^3\sqrt{ex}\left(x^2\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(c-dx^2\right)\right)}{15abd\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(5*a*(c - d*x^2) - 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (b*c + 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*d*Sqrt[c - d*x^
2])
```

Maple [B] time = 0.035, size = 853, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^
```

```
2*b*c*d^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a*b*c*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+2*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2+3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2
*d^2-4*x^3*a*b*d^3*(a*b)^(1/2)+4*x^3*b^2*c*d^2*(a*b)^(1/2)+4*x*a*b*c*d^2*(a
*b)^(1/2)-4*x*b^2*c^2*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/b/d/x
/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d
*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.881 $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}} \, dx$

Optimal. Leaf size=349

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\cdots}{b\cdots}$$

```
[Out] (-2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sq
rt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)
*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.491857, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {466, 483, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\cdots}}{b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sq
rt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)
*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{(2e)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{(2ae)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b}$$

$$= \frac{\left(2\sqrt{c}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{d}} - \frac{\left(2\sqrt{c}e^2\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{d}} + \frac{\left(ae^3\right)\text{Subst}}{}$$

$$= \frac{\left(2\sqrt{c}e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{d}\sqrt{c-dx^2}} - \frac{\left(2\sqrt{c}e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{}\right.}{b\sqrt{d}\sqrt{c-dx^2}}$$

$$= \frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} - \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}$$

$$= -\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} + \frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} - \frac{\sqrt{a}\sqrt[4]{}}{}$$

Mathematica [C] time = 0.0499359, size = 70, normalized size = 0.2

$$\frac{2x(ex)^{5/2}\sqrt{\frac{c-dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{7a\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*(e*x)^(5/2)*Sqrt[(c - d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a])/(7*a*Sqrt[c - d*x^2])
```

Maple [A] time = 0.032, size = 470, normalized size = 1.4

$$-\frac{\sqrt{2}e^2}{2x(dx^2-c)b}\left(\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}}, b\sqrt{cd}\left(\sqrt{cd}b-\sqrt{abd}\right)^{-1}, \frac{\sqrt{2}}{2}\right)abcd + \sqrt{cd}\sqrt{ab}\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+(c*d)^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*a*d-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
```

```
/2),1/2*2^(1/2))*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*b^2*c^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*b^2*c^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-(c*d)^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*d)*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*
d+(c*d)^(1/2)*b)/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.882 $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}}{}$$

```
[Out] (-2*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])
+ (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*
Sqrt[c - d*x^2])
```

Rubi [A] time = 0.356394, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {466, 483, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F(}{b\sqrt[4]{d}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])
+ (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*
Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{e\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} - \frac{\left(2e\sqrt{1-\frac{dx}{c}}\right.}{}$$

$$= -\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right.}{b\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} +$$

Mathematica [C] time = 0.041824, size = 70, normalized size = 0.27

$$\frac{2x(ex)^{3/2}\sqrt{\frac{c-dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{5a\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*(e*x)^(3/2)*Sqrt[(c - d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c,
(b*x^2)/a])/(5*a*Sqrt[c - d*x^2])
```

Maple [B] time = 0.023, size = 415, normalized size = 1.6

$$\frac{\sqrt{2}e}{2x\left(dx^2-c\right)}\left(\mathrm{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},b\sqrt{cd}\left(\sqrt{cd}b-\sqrt{abd}\right)^{-1},\frac{\sqrt{2}}{2}\right)ab\sqrt{cd}+\mathrm{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*a*d*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*d*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*b*c*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*(c*d)^(1
/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*d*(a*b)^(1/2))*(c*d)^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x^2+c)^(1/2)*e*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*
b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral((e*x)**(3/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.883 $\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}} dx$

Optimal. Leaf size=203

$$\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] -((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]
*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])) + (c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.260219, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {466, 490, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] -((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]
*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])) + (c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}x^2\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{b}} - \frac{e\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}e+\sqrt{b}x^2\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{b}}$$

$$= \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}x^2\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{b}\sqrt{c-dx^2}} - \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}e+\sqrt{b}x^2\right)\sqrt{1-\frac{dx^4}{ce^2}}}\right.}{\sqrt{b}\sqrt{c-dx^2}}$$

$$= -\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle| -1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle| -\right.}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.041193, size = 70, normalized size = 0.34

$$\frac{2x\sqrt{ex}\sqrt{\frac{c-dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[(c - d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b
*x^2)/a])/(3*a*Sqrt[c - d*x^2])
```

Maple [B] time = 0.019, size = 337, normalized size = 1.7

$$-\frac{d\sqrt{2}}{2\,x\left(dx^2-c\right)}\left(\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}}, b\sqrt{cd}\left(\sqrt{abd}+\sqrt{cdb}\right)^{-1}, \frac{\sqrt{2}}{2}\right)bc-\sqrt{ab}\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}}, b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c-(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c+(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2))*d*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
```

```
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x^2+c)^(1/2)*(e*x)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c
*d)^(1/2)*b)/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}}{\left(bx^2-a\right)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(sqrt(e*x)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{ex}}{\left(bx^2-a\right)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.884 $\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

```
[Out] (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*Sqrt[e]
*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.251575, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {466, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*Sqrt[e]
*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae}$$

$$= \frac{\sqrt{1-\frac{dx^2}{c}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{ae\sqrt{c-dx^2}} + \frac{\sqrt{1-\frac{dx^2}{c}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,d\right.}{ae\sqrt{c-dx^2}}$$

$$= \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-\right.}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.0443237, size = 68, normalized size = 0.36

$$\frac{2x\sqrt{\frac{c-dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\sqrt{ex}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*Sqrt[(c - d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/
(a*Sqrt[e*x]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.025, size = 344, normalized size = 1.8

$$\frac{\sqrt{2}b}{2\,dx^2-2\,c}\left(\sqrt{cd}\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}}, b\sqrt{cd}\left(\sqrt{cd}b-\sqrt{abd}\right)^{-1}, \frac{\sqrt{2}}{2}\right)b+\sqrt{ab}\text{EllipticPi}\left(\sqrt{\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*((c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b+(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*d-(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b+(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*d)*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
```

```
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d
)^(1/2)*b*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*
d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\sqrt{ex}\sqrt{c-dx^2}+bx^2\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)/(e*x)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(1/(-a*sqrt(e*x)*sqrt(c - d*x**2) + b*x**2*sqrt(e*x)*sqrt(c - d*x*
*2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.885 $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=379

$$\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{a^{3/2}\sqrt[4]{d}e^{3}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(a*c*e*Sqrt[e*x]) - (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*Ell
ipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/
2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
 - (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2
)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.683064, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 480, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin\right.}{a\sqrt[4]{c}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(a*c*e*Sqrt[e*x]) - (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*Ell
ipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/
2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
 - (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2
)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(\frac{bc-ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}}+\frac{bcx^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx,x,\sqrt{ex}\right)}{ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{ae^3} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,\right.}{ace^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{\left(2\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{a\sqrt{c}e^2} - \frac{\left(2\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x\right.}{a\sqrt{c}e^2}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{\left(2\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\sqrt{ex}\right)}{a\sqrt{c}e^2\sqrt{c-dx^2}} - \frac{\left(2\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Sub}}{a\sqrt{ce}}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{si}\right.}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-}}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} - \frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}} + \frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.116515, size = 146, normalized size = 0.39

$$\frac{x\left(6bdx^4\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+14x^2\sqrt{1-\frac{dx^2}{c}}(bc-ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-42a\left(c-dx^2\right)\right)}{21a^2c(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-42*a*(c - d*x^2) + 14*(b*c - a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appell
F1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c*(e*x)^(3/2)*Sqrt[c
- d*x^2])
```

Maple [B] time = 0.03, size = 835, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c
^2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*b*c^2+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*c-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d+4*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b*c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2-
4*x^2*a*d^2+4*x^2*b*c*d+4*a*c*d-4*b*c^2)*d*b*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*
b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/a/c/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2 - a\right)\sqrt{-dx^2 + c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}+bx^2\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(1/(-a*(e*x)**(3/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(3/2)*sqrt(c
- d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.886 $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=297

$$\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin\ldots\right.}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-\ldots}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*c*e*(e*x)^(3/2)) + (2*d^(3/4)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*c^(3/4)
*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.46489, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 480, 523, 224, 221, 409, 1219, 1218}

$$\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{c}\ldots}\right.\right.}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*c*e*(e*x)^(3/2)) + (2*d^(3/4)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*c^(3/4)
*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{3bc+ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x,\right.}{3ace^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a^2e^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx\right.}{a^2e^3}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}} + \frac{\left(b\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)}\right.}{a^2e^3\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\right.}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.131744, size = 148, normalized size = 0.5

$$\frac{x\left(10x^2\sqrt{1-\frac{dx^2}{c}}(ad+3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(bdx^4\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(c-dx^2\right)\right)\right)}{15a^2c(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(10*(3*b*c + a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*
x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appel
lF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*c*(e*x)^(5/2)*Sqrt[c
 - d*x^2])
```

Maple [B] time = 0.026, size = 740, normalized size = 2.5

$$\frac{bd}{6\,cxa\left(dx^2-c\right)e^2}\left(2\,\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},1/2\sqrt{2}\right)\sqrt{2}xad\sqrt{ab}\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}-2\,\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*x*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
```

```
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-2*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^
(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)+4*x^2*a*d^2*(a*b)^(1/2)-4*x^2*b*c*d*(a*b)^(1/2)-4*a*c*d*(a
*b)^(1/2)+4*b*c^2*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/x/c/a/((c*d)^(1/2)*b-(a*b)^
(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)/e^2/(e*x)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}+bx^2\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(1/(-a*(e*x)**(5/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(5/2)*sqrt(c
- d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.887 $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=444

$$\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{\cdot}}{}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*c*e*(e*x)^(5/2)) - (2*(5*b*c + 3*a*d)*Sqrt[c - d*
x^2])/(5*a^2*c^2*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*
c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2
*c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2]) + (b^(3
/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e
^(7/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.920595, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 480, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(3ad+}{5a^2c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*c*e*(e*x)^(5/2)) - (2*(5*b*c + 3*a*d)*Sqrt[c - d*
x^2])/(5*a^2*c^2*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*
c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2
*c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2]) + (b^(3
/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e
^(7/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
+ 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^6\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{5bc+3ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2\left(-\frac{5b^2c^2-5abcd-3a^2d^2}{e^4}-\frac{bd(5bc+3ad)x^4}{e^6}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{5a^2c^2e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\,\mathrm{Subst}\left(\int \left(\frac{d(5bc+3ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5b^2c^2x^2}{e^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx\right.}{5a^2c^2e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a^2e^5} - \frac{(2}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{d}(5bc+3ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{5a^2c^{3/2}e^4}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{d}(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\right.}{5a^2c^{3/2}e^4\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{2\sqrt[4]{d}(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right| -}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\sqrt[4]{d}(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right| -}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.216993, size = 188, normalized size = 0.42

$$\frac{x\left(14x^4\sqrt{1-\frac{dx^2}{c}}\left(-3a^2d^2-5abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+6bdx^6\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-4\right.}{105a^3c^2(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-42*a*(c - d*x^2)*(5*b*c*x^2 + a*(c + 3*d*x^2)) + 14*(5*b^2*c^2 - 5*a*b
*c*d - 3*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a] + 6*b*d*(5*b*c + 3*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(105*a^3*c^2*(e*x)^(7/2)*Sqrt[c -
```

```
d*x^2])
```

Maple [B] time = 0.029, size = 1109, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/10*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
x^2*b^2*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b*c^2-12*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d+20*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+4*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a*b*c^2*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3-12*x^4*a
^2*d^3-8*x^4*a*b*c*d^2+20*x^4*b^2*c^2*d+8*x^2*a^2*c*d^2+12*x^2*a*b*c^2*d-20
*x^2*b^2*c^3+4*a^2*c^2*d-4*a*b*c^3)*b*d*(-d*x^2+c)^(1/2)/x^2/((c*d)^(1/2)*b
-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/a^2/c^2/(d*x^2-c)/e^3/(e*x)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2 - a\right)\sqrt{-dx^2 + c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

3.888 $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{7/4}\sqrt{c-dx^2}(bc-ad)}-\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\ldots$$

```
[Out] -((c*e^3*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*(3*b*c -
2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*(3
*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (a^(
3/2)*c^(1/4)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/
2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*e^(9/2)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c -
d*x^2])
```

Rubi [A] time = 0.790681, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 470, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -((c*e^3*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*(3*b*c -
2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*(3
*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (a^(
3/2)*c^(1/4)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/
2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*e^(9/2)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c -
d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\,\mathrm{Subst}\left(\int \frac{x^2\left(3ac-\frac{(3bc-2ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{d(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\,\mathrm{Subst}\left(\int \left(\frac{(3bc-2ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{2a^2dx^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{d(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{\left(2a^2e^3\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)} + \frac{\left((3bc-2ad)e^3\right)\mathrm{S}}{}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}(3bc-2ad)e^4\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bd^{3/2}(bc-ad)} + \frac{\left(\sqrt{c}(3bc-2a\right.}{}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}(3bc-2ad)e^4\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{bd^{3/2}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(\sqrt{\right.}{}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}} - \frac{a^{3/2}\sqrt[4]{c}e^{9/2}}{}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}(3bc-}{}$$

Mathematica [C] time = 0.145338, size = 148, normalized size = 0.33

$$-\frac{e^3(ex)^{3/2}\left(x^2\sqrt{1-\frac{dx^2}{c}}(2ad-3bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7ac\right)}{7ad\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(e^3*(e*x)^(3/2)*(-7*a*c + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1,
 7/4, (d*x^2)/c, (b*x^2)/a] + (-3*b*c + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(7*a*d*(-(b*c) + a*d)*Sqrt[
c - d*x^2])
```

Maple [B] time = 0.044, size = 1039, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*
b*c*d^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+10*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b^2*c^2*d-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a^2*b*c*d^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d+3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*b^3*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+2*x^2*a*b^2*c*d^2-2*x^2*b^3*c^2*d)*(-d*x
^2+c)^(1/2)*e^4*(e*x)^(1/2)/x/d/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*
d+(c*d)^(1/2)*b)/(a*d-b*c)/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2 - a\right)\left(-dx^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.889 $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=338

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{5/4}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3\sqrt{ex}}{d\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((c*e^3*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(1/4)*(b*c - 2*a*
d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(b*d^(5/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.520474, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 470, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{bd^{5/4}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3\sqrt{ex}}{d\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -((c*e^3*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(1/4)*(b*c - 2*a*
d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(b*d^(5/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

p, q, x]

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\,\text{Subst}\left(\int \frac{ac-\frac{(bc-2ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{d(bc-ad)}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{(2a^2e^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)} + \frac{((bc-2ad)e^3)\,\text{Subs}}{bd(b}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{}{\left(1+\frac{\sqrt{b}}{\sqrt{}}\right.}\right.}{b(bc-}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(ae^3\sqrt{1-\frac{dx^2}{c}}\right)}{}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}} + \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{d}{}}}{b}$$

Mathematica [C] time = 0.139369, size = 148, normalized size = 0.44

$$-\frac{e^3\sqrt{ex}\left(x^2\sqrt{1-\frac{dx^2}{c}}(2ad-bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5ac\right)}{5ad\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(e^3*Sqrt[e*x]*(-5*a*c + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5
/4, (d*x^2)/c, (b*x^2)/a] + (-(b*c) + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appell
F1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(5*a*d*(-(b*c) + a*d)*Sqrt[c -
 d*x^2])
```

Maple [B] time = 0.039, size = 825, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2
*b*c*d^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)+3*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*a*b*c*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2+((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-2*x*
a*b*c*d^2*(a*b)^(1/2)+2*x*b^2*c^2*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^3*(e*x)
^(1/2)/d/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b
)^(1/2)/(a*d-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.890 $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=414

$$-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\cdots\right.}{\sqrt{b}\sqrt[4]{d}\sqrt{c-\cdots}}$$

```
[Out] -((e*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*e^(5/2)*Sqrt[1
- (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c
]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*(b
*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) +
(Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt
[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.691038, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 471, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\cdots\right.}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((e*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*e^(5/2)*Sqrt[1
- (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c
]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*(b
*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) +
(Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt
[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{x^2\left(3a-\frac{bx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}} + \frac{2ax^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right)}{bc-ad}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{(2ae)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,d\right.}{bc-ad}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{d}(bc-ad)} + \frac{\left(\sqrt{c}e^2\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{\sqrt{d}(bc-ad)}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{d}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}e^2\sqrt{1-\frac{dx^2}{c}}\right.}{\sqrt{d}(}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\right.}{\sqrt{b}\sqrt[4]{d}(bc}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin\right.}{d^{3/4}(bc-ad)\sqrt{}}$$

Mathematica [C] time = 0.0983634, size = 133, normalized size = 0.32

$$-\frac{e(ex)^{3/2}\left(bx^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7a\right)}{7a\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(e*(e*x)^(3/2)*(7*a - 7*a*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (
d*x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4
, (d*x^2)/c, (b*x^2)/a]))/(7*a*(b*c - a*d)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.025, size = 839, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*d+((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*
d)^(1/2)*a*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*a*b*c*d+2*a*b*d^2*x^2-2*b^2*c*d*x^2)*(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/
((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*d-b*c)/(d*x^
2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{5}{2}}}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{5}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral((e*x)**(5/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.891 $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt[4]{d}\sqrt{c-}}$$

```
[Out] -((e*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) - (c^(1/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d
^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)
*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.426559, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 471, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin\right.}{\sqrt[4]{d}\sqrt{c-dx^2}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -((e*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) - (c^(1/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d
^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)
*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{a+\frac{bx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{bc-ad}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{e\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{bc-ad}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{(bc-ad)\sqrt{}}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\right.}{\sqrt[4]{d}(bc-ad)\sqrt{}}$$

Mathematica [C] time = 0.0974128, size = 133, normalized size = 0.42

$$\frac{e\sqrt{ex}\left(bx^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5a\right)}{5a\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*Sqrt[e*x]*(-5*a + 5*a*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*
x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (
d*x^2)/c, (b*x^2)/a]))/(5*a*(b*c - a*d)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.026, size = 702, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/2*b*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
```

```
*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*(c*d)^(1/2)*a*d-EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b*c*(a
*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*d-2*x*a*d^
2*(a*b)^(1/2)+2*x*b*c*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e*(e*x)^(1/2)/x/((c*d
)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c
)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral((e*x)**(3/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{3}{2}}}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.892 $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=420

$$-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{d(ex)^{3/2}}{ce\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}}{}$$

```
[Out] -((d*(e*x)^(3/2))/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) + (d^(1/4)*Sqrt[e]*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^
2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(1/4
)*(b*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/
c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) + (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.716668, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 472, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{d(ex)^{3/2}}{ce\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((d*(e*x)^(3/2))/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) + (d^(1/4)*Sqrt[e]*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^
2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(1/4
)*(b*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/
c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) + (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{x^2\left(-\frac{2bc+ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{2bcx^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{(bc-ad)e} + \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}} \cdots\right.}{c(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{c}(bc-ad)} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{ce}}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \cdots\right.}{\sqrt{c}(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\left(\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{c}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\cdots}{\sqrt{c}(\cdots}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\cdots\right.}{\sqrt{a}\sqrt[4]{d}(bc\cdots}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin\cdots\right.}{\sqrt[4]{c}(bc-ad)\cdots}$$

Mathematica [C] time = 0.186754, size = 148, normalized size = 0.35

$$\frac{\sqrt{ex}\left(7x\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right) - 3dx\left(bx^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7a\right)\right)}{21ac\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[e*x]*(7*(2*b*c + a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4
, (d*x^2)/c, (b*x^2)/a] - 3*d*x*(7*a + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(21*a*c*(b*c - a*d)*Sqrt[c - d*x
^2])
```

Maple [B] time = 0.026, size = 830, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c^
2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*b*c^2+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*c+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d-2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*b*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2+2*x^2
*a*d^2-2*x^2*b*c*d)*d*b*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/((c*d)^(1/2)*b-(a*b)^(
1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/c/(a*d-b*c)/(d*x^2-c)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{ex}}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.893 $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{c^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}-\frac{d\sqrt{ex}}{ce\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\ldots$$

```
[Out] -((d*Sqrt[e*x])/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) - (d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(3/
4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) +
 (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c -
a*d)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.48782, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 414, 523, 224, 221, 409, 1219, 1218}

$$-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}-\frac{d\sqrt{ex}}{ce\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((d*Sqrt[e*x])/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) - (d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(3/
4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) +
 (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c -
a*d)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{-\frac{2bc-ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{(bc-ad)e} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{c(bc-ad}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{a(bc-ad)e} + \frac{b\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)}\right.}{a(bc-}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}(bc-ad)\sqrt{e}\sqrt{c-dx^2}} + \frac{\left(b\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int\right.}{a(bc-a}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}(bc-ad)\sqrt{e}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{}}{\sqrt{a}\sqrt{}}\right.}{a\sqrt[4]{d}(bc-ad}$$

Mathematica [C] time = 0.126273, size = 147, normalized size = 0.45

$$\frac{bdx^3\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5x\sqrt{1-\frac{dx^2}{c}}(2bc-ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5adx}{5ac\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (-5*a*d*x + 5*(2*b*c - a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4
, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1,
 9/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*(b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.03, size = 706, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] -1/2*b*d*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
```

```
*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*b*c-EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*2^(1/2)*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)+EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b*c*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*b*c-2*x*a*
d^2*(a*b)^(1/2)+2*x*b*c*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/c/((c*d)^(1/2)*b-(a
*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c)/(d*x^2-c)/
(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\sqrt{ex}\sqrt{c-dx^2}+adx^2\sqrt{ex}\sqrt{c-dx^2}+bcx^2\sqrt{ex}\sqrt{c-dx^2}-bdx^4\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)/(-d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] -Integral(1/(-a*c*sqrt(e*x)*sqrt(c - d*x**2) + a*d*x**2*sqrt(e*x)*sqrt(c -
d*x**2) + b*c*x**2*sqrt(e*x)*sqrt(c - d*x**2) - b*d*x**4*sqrt(e*x)*sqrt(c -
```

```
d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.894 $\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=493

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1\cdots}}{a^{3}\cdots}$$

```
[Out] -(d/(c*(b*c - a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])) - ((2*b*c - 3*a*d)*Sqrt[c
- d*x^2])/(a*c^2*(b*c - a*d)*e*Sqrt[e*x]) - (d^(1/4)*(2*b*c - 3*a*d)*Sqrt[1
 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(2*b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d
)*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.984656, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 472, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc\cdots}{ac^{5/4}e^{3/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -(d/(c*(b*c - a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])) - ((2*b*c - 3*a*d)*Sqrt[c
- d*x^2])/(a*c^2*(b*c - a*d)*e*Sqrt[e*x]) - (d^(1/4)*(2*b*c - 3*a*d)*Sqrt[1
 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(2*b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d
)*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{e\,\mathrm{Subst}\left(\int \frac{-\frac{2bc-3ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{e\,\mathrm{Subst}\left(\int \frac{x^2\left(\frac{2b^2c^2-2abcd+3a^2d^2}{e^4}+\frac{bd(}{}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx}{e^2}}}\right.}{ac^2(bc-ad}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{e\,\mathrm{Subst}\left(\int\left(-\frac{d(2bc-3ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}}+\frac{}{e^4(a}\right.\right.}{ac^2(bc-a}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,d\right.}{a(bc-ad)e^3}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\left(\sqrt{d}(2bc-3ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-}}\right.}{ac^{3/2}(bc-ad)e^2}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\left(\sqrt{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subs}}{ac^{3/2}(bc-ad)e^2}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\sqrt[4]{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-}\right.}{ac^{5/4}(bc-ad)e^{3/2}\sqrt{c}}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} - \frac{\sqrt[4]{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-}\right.}{ac^{5/4}(bc-ad)e^{3/2}\sqrt{c}}$$

Mathematica [C] time = 0.229079, size = 198, normalized size = 0.4

$$\frac{x\left(7x^2\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-2abcd+2b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bdx^4\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+21a\right.}{21a^2c^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(21*a*(a*d*(2*c - 3*d*x^2) - 2*b*c*(c - d*x^2)) + 7*(2*b^2*c^2 - 2*a*b*c
*d + 3*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/
c, (b*x^2)/a] + 3*b*d*(2*b*c - 3*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4,
 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c^2*(b*c - a*d)*(e*x)^(3/2)*
```

```
Sqrt[c - d*x^2])
```

Maple [B] time = 0.034, size = 1058, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*
c^3-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*b*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*b^2*c^3+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d
^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-2*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*b^2*c^3+6*x^2*a^2*d^3-10*x^2*a*b*c*d^2+4*x^2*b^2*c^2*d-4*a^2*c*d^2+8
*a*b*c^2*d-4*b^2*c^3)*b*d*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((
a*b)^(1/2)*d+(c*d)^(1/2)*b)/a/(a*d-b*c)/c^2/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}+adx^2\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}+bcx^2\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}-bdx^4\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral(1/(-a*c*(e*x)**(3/2)*sqrt(c - d*x**2) + a*d*x**2*(e*x)**(3/2)*sqr
t(c - d*x**2) + b*c*x**2*(e*x)**(3/2)*sqrt(c - d*x**2) - b*d*x**4*(e*x)**(3
/2)*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.895 $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=397

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ac^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-}}{a^2}$$

```
[Out] -(d/(c*(b*c - a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])) - ((2*b*c - 5*a*d)*Sqrt[
c - d*x^2])/(3*a*c^2*(b*c - a*d)*e*(e*x)^(3/2)) + (d^(3/4)*(2*b*c - 5*a*d)*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(3*a*c^(7/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/
2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.789251, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 472, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2bc-5a}{3ac^{7/4}e^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -(d/(c*(b*c - a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])) - ((2*b*c - 5*a*d)*Sqrt[
c - d*x^2])/(3*a*c^2*(b*c - a*d)*e*(e*x)^(3/2)) + (d^(3/4)*(2*b*c - 5*a*d)*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(3*a*c^(7/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/
2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{-\frac{2bc-5ad}{e^2}-\frac{5bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{6b^2c^2+2abcd-5a^2d^2}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{\cdots}}\cdots\right)}{3ac^2(bc\cdots}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c\cdots}}\cdots\right)}{a(bc-ad)e\cdots}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\cdots}}\cdots\right)}{a^2(bc-ad)e\cdots}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{d^{3/4}(2bc-5ad)\sqrt{1-\frac{dx^2}{c}}F\cdots}{3ac^{7/4}(bc-ad)e\cdots}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{d^{3/4}(2bc-5ad)\sqrt{1-\frac{dx^2}{c}}F\cdots}{3ac^{7/4}(bc-ad)e\cdots}$$

Mathematica [C] time = 0.247626, size = 197, normalized size = 0.5

$$\frac{x\left(5x^2\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+2abcd+6b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bdx^4\sqrt{1-\frac{dx^2}{c}}(5ad-2bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5\cdots\right.}{15a^2c^2(ex)^{5/2}\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(5*a*(a*d*(2*c - 5*d*x^2) - 2*b*c*(c - d*x^2)) + 5*(6*b^2*c^2 + 2*a*b*c*
d - 5*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a] + b*d*(-2*b*c + 5*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1
/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2*c^2*(b*c - a*d)*(e*x)^(5/2)*Sqr
t[c - d*x^2])
```

Maple [B] time = 0.033, size = 896, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/6*b*d*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^3*c^3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-5*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*(c*d)^(1/2)+7*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*2^(1/2)*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1
/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*2^(1/2)*x*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*(c*d)^(1/2)-10*x^2*a^2*d^3*(a*b)^(1/2)+14*x^2*a*b*c*d^2*(a*b)^(1/2)-4*x
^2*b^2*c^2*d*(a*b)^(1/2)+4*a^2*c*d^2*(a*b)^(1/2)-8*a*b*c^2*d*(a*b)^(1/2)+4*
b^2*c^3*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/x/c^2/a/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c)/(d*x^2-c)/e^2/(e*x)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-a\right)\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.896 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=362

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}}-$$

```
[Out] (7*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*(e*x)^(5/2)*Sqrt[c - d*x^2])
/(2*b*(a - b*x^2)) + (c^(1/4)*(8*b*c - 21*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*
EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*b^3*d^(1/4
)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1
/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.697441, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 467, 582, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (7*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*(e*x)^(5/2)*Sqrt[c - d*x^2])
/(2*b*(a - b*x^2)) + (c^(1/4)*(8*b*c - 21*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*
EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*b^3*d^(1/4
)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1
/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{x^4\left(5c-\frac{7dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{e^5\,\text{Subst}\left(\int \frac{-\frac{7acd}{e^2}-\frac{d(8bc-21ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6b^2d}$$

$$= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{\left((8bc-21ad)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6b^3} - \frac{\left(a(5bc - \right.}{}$$

$$= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{\left((5bc-7ad)e^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4b^3} - \frac{((}{}$$

$$= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{\sqrt[4]{c}(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{((5}{}$$

$$= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{\sqrt[4]{c}(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}}{}$$

Mathematica [C] time = 0.178961, size = 184, normalized size = 0.51

$$\frac{e^3\sqrt{ex}\left(x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(-(21ad-8bc))F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+35ac\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\right.}{30ab^2\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e^3*Sqrt[e*x]*(5*a*(7*a - 4*b*x^2)*(-c + d*x^2) + 35*a*c*(a - b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-8*b*c +
 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*
x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.048, size = 2561, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)

[Out] 1/24*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+16*x^5*b^3*c*d^2*(a*b)^(1/2)+28*x^3*a^2*b*d^3*(a*b)^(1/2)-16*x^3*b^3*c^2*d*(a*b)^(1/2)-16*x^5*a*b^2*d^3*(a*b)^(1/2)-42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-58*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-12*x^3*a*b^2*c*d^2*(a*b)^(1/2)-28*x*a^2*b*c*d^2*(a*b)^(1/2)+28*x*a*b^2*c^2*d*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+58*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*

```
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+16*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-16*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)+42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2))/b^2/x/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b
)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\,(ex)^{\frac{7}{2}}}{\left(bx^2 - a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.897 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=413

$$\frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{5/2}\cdots}{\cdots}$$

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (5*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b
^2*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) - (
c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.759654, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 467, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (5*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b
^2*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) - (
c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^6\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{e\,\mathrm{Subst}\left(\int \frac{x^2\left(3c-\frac{5dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{e\,\mathrm{Subst}\left(\int \left(\frac{5dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(3bc-5ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{(5de)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} - \frac{((3bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,d\right.}{2b^2}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{\left(5\sqrt{c}\sqrt{d}e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} - \frac{\left(5\sqrt{c}\sqrt{d}e^2\right)\mathrm{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,\right.}{2b^2}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{\left(5\sqrt{c}\sqrt{d}e^2\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2\sqrt{c-dx^2}} - \frac{\left(5\sqrt{c}\sqrt{d}e^2\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Su}}{2b^2\sqrt{}}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\right.}{4\sqrt{a}b^{5/2}\sqrt[4]{d}}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} + \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\right.\right.}{2b^2\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.127682, size = 163, normalized size = 0.39

$$\frac{e(ex)^{3/2}\left(5dx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7c\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\left(c-dx^2\right)\right)}{14ab\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 5*d*x^2*(-a + b*x^2)*Sqrt[1
- (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*b*(-
a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.046, size = 2542, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(4*x^2*a*b^2*c*d-5*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2
)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*a^2*b*c*d-4*x^4*a*b^2*d^2+4*x^4*b^3*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c
*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+5*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d+3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*
c-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c
*d-4*x^2*b^3*c^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+5*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+2
0*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
```

```
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*
d)^(1/2)*a*b*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*a*b^2*c^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-10*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*x^2*b^3*c^2-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+10*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*a*b^2*c^2)/x/b^2/(d*x^2-c)/(b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*
b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.898 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=328

$$-\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/}}{}$$

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (3*c^(1/4)*d^(3/4)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.485284, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 467, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (3*c^(1/4)*d^(3/4)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
```

ialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}dx,x,\sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{c-\frac{3dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx,x,\sqrt{ex}\right)}{2b}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{(3de)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx,x,\sqrt{ex}\right)}{2b^2} - \frac{((bc-3ad)e)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx,x\right.}{2b^2}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{((bc-3ad)e)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx,x,\sqrt{ex}\right)}{4ab^2} - \frac{((bc-3ad)e)\,\text{Subst}\left(\int \frac{}{(1+}\right.}{4ab}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - \frac{\left((bc-3ad)e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int\right.}{4ab^2\sqrt{c-}}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} - \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{}}{\sqrt{}}\right.}{4ab^2\sqrt[4]{d}\sqrt{c-}}$$

Mathematica [C] time = 0.126045, size = 163, normalized size = 0.5

$$\frac{e\sqrt{ex}\left(3dx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5c\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5a\left(c-dx^2\right)\right)}{10ab\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^2*(-a + b*x^2)*Sqrt[1 -
(d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(10*a*b*(-a +
 b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.023, size = 2255, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b*d*(6*2^(1/2)*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1
```

/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(

```
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-4*x^3*a*b*d^2*(a*b)^(1/2)+4*x^3*b^2*c*d*(a*b
)^(1/2)+4*x*a*b*c*d*(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^(1/2))/x/(d*x^2-c)/(b*x^2
-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{\left(-a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.899 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=417

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\ldots}{\ldots}$$

```
[Out] ((e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) - (c^(3/4)*d^(1/4)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*a*b*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*S
qrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*
(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.72038, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 469, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```


```
[Out] ((e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) - (c^(3/4)*d^(1/4)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*a*b*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*S
qrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*
(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
```

```
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-c-\frac{dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2ae}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(bc+ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx,x,\sqrt{ex}\right)}{2ae}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2abe} + \frac{(bc+ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right.}{2abe}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2ab} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}e}}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right.}{2ab}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\sqrt{ex}\right)}{2ab\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int\right.}{2ab\sqrt{c-d}}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc+ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}}{\sqrt{a}}\right.}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c}}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}} + \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{2ab\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.119936, size = 163, normalized size = 0.39

$$\frac{\sqrt{ex}\left(3dx^3\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7cx\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+21ax\left(dx^2-c\right)\right)}{42a^2\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (Sqrt[e*x]*(21*a*x*(-c + d*x^2) + 7*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^3*(-a + b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^2*
(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.018, size = 2534, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(-4*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d
+4*x^4*a*b^2*d^2-4*x^4*b^3*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
a*b*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+4*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*a*b^2*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+4*x^2*b^3*c^2+2*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-4*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2
*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
```

```
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)
^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-4*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^3*c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+4*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a*b^2*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)/b/x/(d*x^2-c)/a/(b*x^2
-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{\left(-a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.900 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}}{}$$

```
[Out] (Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) + (c^(1/4)*d^(3/4)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
2*a*b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c
^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*
d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.451297, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 412, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) + (c^(1/4)*d^(3/4)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
2*a*b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c
^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*
d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \frac{-3c+\frac{dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2abe} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{e\right.}$$

$$= \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4a^2be} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{}{\left(1+\frac{\sqrt{bx^2}}{\sqrt{ae}}\right.}\right.}{4a^2be}$$

$$= \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}} + \frac{\left((3bc-ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{4a^2be\sqrt{c-dx}}$$

$$= \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{si}\right.}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx}}$$

Mathematica [C] time = 0.113822, size = 161, normalized size = 0.48

$$\frac{dx^3\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+15cx\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ax\left(dx^2-c\right)}{10a^2\sqrt{ex}\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (5*a*x*(-c + d*x^2) + 15*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*Sqrt[e*x]*(-a +
b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.029, size = 2251, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*(-d*x^2+c)^(1/2)*d*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
```

```
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2
)*x^2*b^2*c-2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*x^2*a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+2*2^(1
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
a*b*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x
^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(
a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(
1/2)*a*b*c+2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-2*2^(1/2)*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*
d)^(1/2)*a*b*c-4*x^3*a*b*d^2*(a*b)^(1/2)+4*x^3*b^2*c*d*(a*b)^(1/2)+4*x*a*b*
c*d*(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^(1/2))/a/(e*x)^(1/2)/(d*x^2-c)/(b*x^2-a)/
(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\left(-a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c - d*x**2)/(sqrt(e*x)*(-a + b*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.901 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=444

$$\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(\cdots}{\cdots}$$

```
[Out] (-5*Sqrt[c - d*x^2])/(2*a^2*e*Sqrt[e*x]) + Sqrt[c - d*x^2]/(2*a*e*Sqrt[e*x]
*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) +
 (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[
b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c
- d*x^2])
```

Rubi [A] time = 0.911519, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 469, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{5c^{3/4}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c - d*x^2])/(2*a^2*e*Sqrt[e*x]) + Sqrt[c - d*x^2]/(2*a*e*Sqrt[e*x]
*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) +
 (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[
b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c
- d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^2\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} - \frac{\mathrm{Subst}\left(\int \frac{-5c+\frac{3dx^4}{e^2}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(\frac{c(5bc-8ad)}{e^2}+\frac{5bcdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2ce}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} + \frac{\mathrm{Subst}\left(\int \left(-\frac{5cdx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{\left(5bc^2-3acd\right)x^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right)}{2a^2ce}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2e^3} + \frac{(5bc-3ad)\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} + \frac{\left(5\sqrt{c}\sqrt{d}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2e^2} - \frac{\left(5\sqrt{c}\sqrt{d}\right)\mathrm{Sub}}{}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} + \frac{\left(5\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2e^2\sqrt{c-dx^2}} - \frac{\left(5\sqrt{c}\right.}{}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} + \frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc-3a}{}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\left(a-bx^2\right)} - \frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}} + \frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-}}{2}$$

Mathematica [C] time = 0.177885, size = 182, normalized size = 0.41

$$\frac{x\left(15bdx^4\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(8ad-5bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+21a\left(\right.\right.}{42a^3(ex)^{3/2}\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(21*a*(4*a - 5*b*x^2)*(c - d*x^2) + 7*(-5*b*c + 8*a*d)*x^2*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 15*b*
d*x^4*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/
```

```
c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.033, size = 2568, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)*d*(-4*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+20*x^
4*a*b^2*d^2-20*x^4*b^3*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
a*b*d-16*a*b^2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d+5*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c
+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*
d+20*x^2*b^3*c^2-16*x^2*a^2*b*d^2+16*a^2*b*c*d+10*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*
b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*(c*d)^(1/2)*a^2*d-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
E(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+5*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(
1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
```

```
^(1/2))*x^2*a*b^2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+5*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-20*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*b^3*c^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-5*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+20*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a*b^2*c^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)/e/(e*x)^(1/2)/(d*x^2-c)
/a^2/(b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2 (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2 (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.902 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=355

$$\frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\cdots}}{\cdots}$$

```
[Out] (-7*Sqrt[c - d*x^2])/(6*a^2*e*(e*x)^(3/2)) + Sqrt[c - d*x^2]/(2*a*e*(e*x)^(
3/2)*(a - b*x^2)) + (7*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*e^(5/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2])
```

Rubi [A] time = 0.651905, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 469, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{7\sqrt[4]{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```


```
[Out] (-7*Sqrt[c - d*x^2])/(6*a^2*e*(e*x)^(3/2)) + Sqrt[c - d*x^2]/(2*a*e*(e*x)^(
3/2)*(a - b*x^2)) + (7*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*e^(5/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
```

```
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^4\left(a-\frac{bx^4}{e^2}\right)^2}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \frac{-7c+\frac{5dx^4}{e^2}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2ae}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{c(21bc-8ad)}{e^2}-\frac{7bcdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{6a^2ce}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{6a^2e^3} + \frac{(7bc-5ad)\,\text{Subs}}{}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{(7bc-5ad)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{4a^3e^3} + \frac{(7}{}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}} + \frac{\left((7bc-5a\right.}{}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(7bc-}{}$$

Mathematica [C] time = 0.17856, size = 181, normalized size = 0.51

$$\frac{x\left(7bdx^4\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(8ad-21bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(4\right.\right.}{30a^3(ex)^{5/2}\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(5*a*(4*a - 7*b*x^2)*(c - d*x^2) + 5*(-21*b*c + 8*a*d)*x^2*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 7*b*d
*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c,
(b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.03, size = 2316, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/24*(-d*x^2+c)^(1/2)*b*d*(-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*
a*b^2*c*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*x^3*b^2*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+14*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*b^2*c*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c
*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*x^3*a*b^2*c*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*x^3*b^2*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*d*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+14*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2
*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^3*a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)-28*x^4*a*b*d^2*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x^3*a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-14*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a*b*d*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*2^(1/2)*x^3*b^3*c^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+28*x^4*b^2*c*d*(a*b)^(1/
2)+16*x^2*a^2*d^2*(a*b)^(1/2)-28*x^2*b^2*c^2*(a*b)^(1/2)-16*a^2*c*d*(a*b)^(
1/2)+16*a*b*c^2*(a*b)^(1/2)+12*x^2*a*b*c*d*(a*b)^(1/2)-14*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*b*c*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*x*a*b*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-21*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+21*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
```

```
c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^2*
c^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2))/x/a^2/e^2/(e*x)^(1/2)/(d*x^2-c)/(b*x^2-a)/(
a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2 (ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2 (ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.903 $\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2} dx$

Optimal. Leaf size=429

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{42b^4\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e^3\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{42b^3}-\frac{\sqrt[4]{c}e^{7}}{}$$

```
[Out] ((57*b*c - 77*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(42*b^3) - (11*d*e*(e*x)^
(5/2)*Sqrt[c - d*x^2])/(14*b^2) + (e*(e*x)^(5/2)*(c - d*x^2)^(3/2))/(2*b*(a
 - b*x^2)) + (c^(1/4)*(48*b^2*c^2 - 259*a*b*c*d + 231*a^2*d^2)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(42*b^4*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)
*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqr
t[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.97485, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 467, 581, 582, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{42b^4\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e^3\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{42b^3}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```


```
[Out] ((57*b*c - 77*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(42*b^3) - (11*d*e*(e*x)^
(5/2)*Sqrt[c - d*x^2])/(14*b^2) + (e*(e*x)^(5/2)*(c - d*x^2)^(3/2))/(2*b*(a
 - b*x^2)) + (c^(1/4)*(48*b^2*c^2 - 259*a*b*c*d + 231*a^2*d^2)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(42*b^4*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)
*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqr
t[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{x^4\left(5c-\frac{11dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= -\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{x^4\left(-\frac{5c(7bc-11ad)}{e^2}+\frac{d(57bc-77ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{14b^2}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e^7\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\left(a(5bc-11a\right.}{}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\left((5bc-11ad\right.}{}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\sqrt[4]{c}\left(48b^2c^2\right.}{}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\sqrt[4]{c}\left(48b^2c^2\right.}{}$$

Mathematica [C] time = 0.2768, size = 233, normalized size = 0.54

$$\frac{e^3\sqrt{ex}\left(x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(c-dx^2\right)\left(77a^2d-ab\left(57c+\right.\right.\right.}{210ab^3\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e^3*Sqrt[e*x]*(5*a*(c - d*x^2)*(77*a^2*d - 12*b^2*x^2*(-3*c + d*x^2) - a*b
*(57*c + 44*d*x^2)) - 5*a*c*(-57*b*c + 77*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)
/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (48*b^2*c^2 - 259*a*
b*c*d + 231*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(210*a*b^3*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.036, size = 3790, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/168*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(-105*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*2^(1/2)*a^2*b^2*c^2*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-
980*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^
3*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+231*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)-462*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*(a*b)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-48*x^7*a*b^3*d^4*(a*b)^(1/2)+536*x*
a^2*b^2*c^2*d^2*(a*b)^(1/2)-228*x*a*b^3*c^3*d*(a*b)^(1/2)+48*x^7*b^4*c*d^3*
(a*b)^(1/2)-176*x^5*a^2*b^2*d^4*(a*b)^(1/2)-192*x^5*b^4*c^2*d^2*(a*b)^(1/2)
+308*x^3*a^3*b*d^4*(a*b)^(1/2)+144*x^3*b^4*c^3*d*(a*b)^(1/2)-360*x^3*a^2*b^
2*c*d^3*(a*b)^(1/2)-92*x^3*a*b^3*c^2*d^2*(a*b)^(1/2)+105*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^2*a*b^4*c^3*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-96*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^3*c^3*(a*b
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-308*x*a^3*b*c*d^3*(a*b)^(1/2
)+336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*(a*b)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+368*x^5*a*b^3*c*d^3*(a*b)^(1/2)+614*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*
d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+336*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*(a*b)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*x^2*a^3*b^2*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
```

```
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+336*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-105*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^4*c^3*d*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)+96*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*x^2*b^4*c^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-336*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^2*d^2*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b^2*c*d^3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^4*d^3*(a*b)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-231*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*2^(1/2)*a^4*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+33
6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b^2*c^2*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^3*d*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^4*b*c*d^3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^4*b*c
*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+462*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*a^4*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)-336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b^2*c^2*d^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^3
*c^3*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-614*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*(a*b)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)-336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*
b^2*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+105*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)-336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2
*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+105*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
```

```
)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(c*d)^(1/2)+980*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2))/b^3/x/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)
*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.904 $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=485

$$\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(11bc-15ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{10b^3\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}\right.}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] (-9*d*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(10*b^2) + (e*(e*x)^(3/2)*(c - d*x^2)^
(3/2))/(2*b*(a - b*x^2)) - (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*
d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(
7/2)*d^(1/4)*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2
)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(7/2)*d
^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.01205, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 467, 581, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```


```
[Out] (-9*d*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(10*b^2) + (e*(e*x)^(3/2)*(c - d*x^2)^
(3/2))/(2*b*(a - b*x^2)) - (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*
d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(
7/2)*d^(1/4)*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2
)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(7/2)*d
^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^6\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\,\mathrm{Subst}\left(\int \frac{x^2\left(3c-\frac{9dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\,\mathrm{Subst}\left(\int \frac{x^2\left(-\frac{3c(5bc-9ad)}{e^2}+\frac{3d(11bc-15ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{e\ldots}\right.}{10b^2}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\,\mathrm{Subst}\left(\int \left(-\frac{3d(11bc-15ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{15\left(b^2c^2-4abcd+3a^2\ldots\right)}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\ldots}}\ldots\right.\right.}{10b^2}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{(3d(11bc-15ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\ldots\right.}{10b^3}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\left(3\sqrt{c}\sqrt{d}(11bc-15ad)e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, \ldots\right.}{10b^3}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\left(3\sqrt{c}\sqrt{d}(11bc-15ad)e^2\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \ldots\right.}{10b^3\sqrt{c-dx^2}}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{c}\ldots}\right.\right.}{10b^3\sqrt{c-dx^2}}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{c}\ldots}\right.\right.}{10b^3\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.231637, size = 196, normalized size = 0.4

$$\frac{e(ex)^{3/2}\left(3dx^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(15ad-11bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right) - 7c\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(9ad-5bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c}\ldots\right.\right.}{70ab^2\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*(7*a*(c - d*x^2)*(-5*b*c + 9*a*d - 4*b*d*x^2) - 7*c*(-5*b*c
+ 9*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)
```

```
/c, (b*x^2)/a] + 3*d*(-11*b*c + 15*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(70*a*b^2*(-a + b*x^2)
*Sqrt[c - d*x^2])
```

Maple [B] time = 0.031, size = 3886, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/40*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(45*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-60*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+180*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a^3*b*c*d^2-312*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-90*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a^3*b*c*d^2+156*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+40*x^4*a*b^
3*c*d^2+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2
^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+60*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(c*d)^(1/2)*(a*b)^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*
d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-15*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3+36*x^2*a^2*b^2*c*d
^2-56*x^2*a*b^3*c^2*d+66*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3+15*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3+132*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a*b^3*c^3+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
```

)*d),1/2*2^(1/2))*x^2*b^4*c^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-180*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+312*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-156*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-132*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+20*x^2*b^4*c^3-36*x^4*a^2*b^2*d^3-4*x^4*b^4*c^2*d+16*x^6*a*b^3*d^3-16*x^6*b^4*c*d^2+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))

```
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-66*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*a*b^3*c^3+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2
*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))/x/b^3/(d*x^2-c)/(b*x^2-a)/((a*b)^(1/
2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")

[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.905 $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2} dx$

Optimal. Leaf size=381

$$-\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6b^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\right.}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] (-7*d*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*Sqrt[e*x]*(c - d*x^2)^(3/2)
)/(2*b*(a - b*x^2)) - (c^(1/4)*d^(3/4)*(17*b*c - 21*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*
b^3*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.728981, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 467, 528, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```


```
[Out] (-7*d*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*Sqrt[e*x]*(c - d*x^2)^(3/2)
)/(2*b*(a - b*x^2)) - (c^(1/4)*d^(3/4)*(17*b*c - 21*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*
b^3*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{\left(c-\frac{7dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{-\frac{c(3bc-7ad)}{e^2}+\frac{d(17bc-21ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6b^2}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{(d(17bc-21ad)e)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6b^3} -$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{((bc-7ad)(bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,d\right.}{4ab^3}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\sqrt[4]{c}d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{6b^3\sqrt{c-dx^2}}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\sqrt[4]{c}d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{6b^3\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.214911, size = 195, normalized size = 0.51

$$\frac{e\sqrt{ex}\left(dx^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(21ad-17bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5c\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(7ad-3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\right.\right.}{30ab^2\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*(5*a*(c - d*x^2)*(-3*b*c + 7*a*d - 4*b*d*x^2) - 5*c*(-3*b*c +
7*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a] + d*(-17*b*c + 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*Sqrt
[c - d*x^2])
```

Maple [B] time = 0.03, size = 3466, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/24*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2*d*(21*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-24*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+16*x^5*b^
3*c*d^2*(a*b)^(1/2)+28*x^3*a^2*b*d^3*(a*b)^(1/2)-4*x^3*b^3*c^2*d*(a*b)^(1/2
)-16*x^5*a*b^2*d^3*(a*b)^(1/2)-42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
^(1/2)-76*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1
/2)*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2+3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
(c*d)^(1/2)*x^2*b^3*c^2-24*x^3*a*b^2*c*d^2*(a*b)^(1/2)-28*x*a^2*b*c*d^2*(a*
b)^(1/2)+40*x*a*b^2*c^2*d*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+76*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2)*(a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c
*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-24*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*
c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+34*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1
/2)*(a*b)^(1/2)-34*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)
-12*x*b^3*c^3*(a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2
*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/
2)*(a*b)^(1/2))/x/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2
)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.906 $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=474

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab^2\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin\right.}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] ((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b
^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b^2*c^
2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.839543, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 468, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b
^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b^2*c^
2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \frac{x^2\left(\frac{c(bc+3ad)}{e^2}+\frac{d(bc-5ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \left(-\frac{d(bc-5ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{\left(b^2c^2+4abcd-5a^2d^2\right)x^2}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{ex}\right)}{2ab}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} - \frac{(d(bc-5ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2e} + \frac{((bc-ad)(bc+5ad}{}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}(bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2} - \frac{\left(\sqrt{c}\sqrt{d}(bc-5\right.}{}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}(bc-5ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\right.}{}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{c^{3/4}\sqrt[4]{d}(bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-a}{}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} - \frac{c^{3/4}\sqrt[4]{d}(bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}} + \frac{c^{3/4}\sqrt[4]{d}(bc}{}$$

Mathematica [C] time = 0.218393, size = 189, normalized size = 0.4

$$\frac{\sqrt{ex}\left(3dx^3\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(bc-5ad)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7cx\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.}{42a^2b\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (Sqrt[e*x]*(21*a*(-(b*c) + a*d)*x*(c - d*x^2) + 7*c*(b*c + 3*a*d)*x*(-a + b
*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a]
+ 3*d*(b*c - 5*a*d)*x^3*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2,
 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^2*b*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.029, size = 3858, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+20*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a^3*b*c*d^2-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-10*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a^3*b*c*d^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+8*x^4*a*b^3*c*d^2+5*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^
2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
```

+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3+4*x^2*a^2*b^2*c*d^2-8*x^2*a*b^3*c^2*
d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3
+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^
3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*(c*d)^(1/2)*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*
c^2-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+24*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b^3*c^2*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-12*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*a*b^3*c^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4
*c^3-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1
/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d

```
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*
b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*x^2*b^4*c^3-4*
x^4*a^2*b^2*d^3-4*x^4*b^4*c^2*d+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^
2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-2*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a*b^3*c^3+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2))/b^2/x/(d*x^2-c)/a/(b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.907 $\int \frac{\left(c-dx^2\right)^{3/2}}{\sqrt{ex}\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=366

$$\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

```
[Out] ((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*(b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(
b*c - a*d)*(b*c + a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^(1/4)*Sqrt[e]*S
qrt[c - d*x^2])
```

Rubi [A] time = 0.597167, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 413, 523, 224, 221, 409, 1219, 1218}

$$\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```


```
[Out] ((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*(b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(
b*c - a*d)*(b*c + a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^(1/4)*Sqrt[e]*S
qrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c-dx^2\right)^{3/2}}{\sqrt{ex}\left(a-bx^2\right)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{-\frac{c(3bc+ad)}{e^2}+\frac{d(bc+3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{(3(bc-ad)(bc+ad))\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2e} + \frac{(d(bc+3ad}{}$$

$$= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{(3(bc-ad)(bc+ad))\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4a^2b^2e} + \frac{(3(bc-ad}{}$$

$$= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}} + \frac{\left(3(bc-ad)(bc+ad\right.}{}$$

$$= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}} + \frac{3\sqrt[4]{c}(bc-ad)(bc+}{}$$

Mathematica [C] time = 0.177567, size = 187, normalized size = 0.51

$$\frac{dx^3\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5cx\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(ad+3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ax\left(\right.}{10a^2b\sqrt{ex}\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (5*a*(-(b*c) + a*d)*x*(c - d*x^2) + 5*c*(3*b*c + a*d)*x*(-a + b*x^2)*Sqrt[1
 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(b*c + 3
*a*d)*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2
)/c, (b*x^2)/a])/(10*a^2*b*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.031, size = 2531, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*(-d*x^2+c)^(1/2)*d*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b
```

^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*x^3*a^2*b*d^3*(a*b)^(1/2)+4*x^3*b^3*c
^2*d*(a*b)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/
2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3+3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3-3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+3*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(
1/2)*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*(c*d)^(1/2)*x^2*b^3*c^2-8*x^3*a*b^2*c*d^2*(a*b)^(1/2)-4*x*a^2*b*c*d^2*(a*b
)^(1/2)+8*x*a*b^2*c^2*d*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2
^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2)+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*

```
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+
2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*
b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x*b^3*c^3*(a*b
)^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))/b/a/(e*x)^(
1/2)/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(
1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.908 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=519

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -((5*b*c - a*d)*Sqrt[c - d*x^2])/(2*a^2*b*e*Sqrt[e*x]) + ((b*c - a*d)*Sqrt[
c - d*x^2])/(2*a*b*e*Sqrt[e*x]*(a - b*x^2)) - (c^(3/4)*d^(1/4)*(5*b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d
 - a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3
/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d - a^
2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3/2)*d^(1
/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.1202, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 468, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\right.}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] -((5*b*c - a*d)*Sqrt[c - d*x^2])/(2*a^2*b*e*Sqrt[e*x]) + ((b*c - a*d)*Sqrt[
c - d*x^2])/(2*a*b*e*Sqrt[e*x]*(a - b*x^2)) - (c^(3/4)*d^(1/4)*(5*b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d
 - a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3
/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d - a^
2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3/2)*d^(1
/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^2\left(a-\frac{bx^4}{e^2}\right)^2}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{e\,\mathrm{Subst}\left(\int \frac{\frac{c(5bc-ad)}{e^2}-\frac{d(3bc+ad)x^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2ab}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{e\,\mathrm{Subst}\left(\int \frac{x^2\left(-\frac{bc^2(5bc-9ad)}{e^4}-\frac{bcd(5bc-ad)x^4}{e^6}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2a^2bc}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{e\,\mathrm{Subst}\left(\int \left(\frac{cd(5bc-ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(5b^2c^3-4abc^2d-a^2cd^2)x^2}{e^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\right.}{2a^2bc}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{(d(5bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2a^2be^3} + \cdots$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{c}\sqrt{d}(5bc-ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right.}{2a^2be^2}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{c}\sqrt{d}(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\right.}{2a^2be^2\sqrt{c-dx^2}}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{c^{3/4}\sqrt[4]{d}(5bc-ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right| -}{2a^2be^{3/2}\sqrt{c-dx^2}}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}(5bc-ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right| -}{2a^2be^{3/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.227345, size = 197, normalized size = 0.38

$$\frac{x\left(3dx^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}(ad-5bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7cx^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}(9ad-5bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right.}{42a^3(ex)^{3/2}(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(21*a*(c - d*x^2)*(4*a*c - 5*b*c*x^2 + a*d*x^2) + 7*c*(-5*b*c + 9*a*d)*x
^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b
```

```
*x^2)/a] + 3*d*(-5*b*c + a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[
7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*x^2)
*Sqrt[c - d*x^2])
```

Maple [B] time = 0.037, size = 3879, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)*d*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*
c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x
^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^
2-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+
12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+24*x^4*a*b^3*c*d^2+EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^
(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)*(a*b)^(1/2)+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*a*b^3*c^3-12*x^2*a^2*b^2*c*d^2-8*x^2*a*b^3*c^2*d+10*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^4*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*x^2*b^4*c^3+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+5*((d*x+(
```

c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-16*a*b
^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*x^2*b^3*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+24*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*a*b^3*c^2*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-12*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a*b^3*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-20*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*b^4*c^3-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)
+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^
2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+20*x^2*b^4*c
^3-4*x^4*a^2*b^2*d^3-20*x^4*b^4*c^2*d+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2
)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1
/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-10*((d*x+(c*d)^(1/2

```
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a*b^3*c^3+16*a^2*b^2*c^2*d-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^
2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))/b/e/(e*x)^(1/2)/(d*x^2-c)/a^2/(b*x^
2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.909 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2} dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

```
[Out] -((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(6*a^2*b*e*(e*x)^(3/2)) + ((b*c - a*d)*S
qrt[c - d*x^2])/(2*a*b*e*(e*x)^(3/2)*(a - b*x^2)) + (c^(1/4)*d^(3/4)*(7*b*c
 - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(6*a^2*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(
7*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/
4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]
)
```

Rubi [A] time = 0.85135, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 468, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```


```
[Out] -((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(6*a^2*b*e*(e*x)^(3/2)) + ((b*c - a*d)*S
qrt[c - d*x^2])/(2*a*b*e*(e*x)^(3/2)*(a - b*x^2)) + (c^(1/4)*d^(3/4)*(7*b*c
 - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(6*a^2*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(
7*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/
4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]
)
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
```

```
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}dx = \frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^4\left(a-\frac{bx^4}{e^2}\right)^2}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{e\,\text{Subst}\left(\int \frac{\frac{c(7bc-3ad)}{e^2}-\frac{d(5bc-ad)x^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2ab}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} - \frac{e\,\text{Subst}\left(\int \frac{-\frac{bc^2(21bc-17ad)}{e^4}+\frac{bcd(7bc-3ad)x^4}{e^6}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{\right.}}{6a^2bc}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{(d(7bc-3ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right.}{6a^2be^3}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{((bc-ad)(7bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-}}\right.}{4a^3be^3}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{\sqrt[4]{c}d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{6a^2be^{5/2}\sqrt{c-dx^2}}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{\sqrt[4]{c}d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{6a^2be^{5/2}\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.222114, size = 199, normalized size = 0.48

$$\frac{x\left(-dx^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}(3ad-7bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5cx^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}(17ad-21bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\right.\right.}{30a^3(ex)^{5/2}(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(5*a*(c - d*x^2)*(4*a*c - 7*b*c*x^2 + 3*a*d*x^2) + 5*c*(-21*b*c + 17*a*d
)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a] - d*(-7*b*c + 3*a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a + b*x^
2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.033, size = 3484, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/24*(-d*x^2+c)^(1/2)*d*(-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*x^3*a^2*b*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*a^2*b*d^2*(a*b)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*x^3*a^2*b*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+1
4*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*
b^2*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-21*2^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*x*a*b^2*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a*b^2*c^2*(a*b)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-12*x^4*a^2*b*d^3*(a*b)^(1
/2)+16*a^2*b*c^2*d*(a*b)^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*
a*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*x*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*x*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*x*a^3*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)+24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a^2*b^2*c^2*d*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)-14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
-3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*a^2*b^2*c*d^2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)+24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*a*b^3*c^2*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)+28*x^2*b^3*c^3*(a*b)^(1/2)-16*a*b^2*c^3*(a*b)^(1/2)+2
1*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(c*d)^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^3*a
^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-24*2^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*x^3*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*2^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)+6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*x*a^3*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+3*2^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*x*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*2^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*a^3*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)+40*x^4*a*b^2*c*d^2*(a*b)^(1/2)-28*x^4*b^3*c^2*d*(a*b)^
(1/2)-4*x^2*a^2*b*c*d^2*(a*b)^(1/2)-24*x^2*a*b^2*c^2*d*(a*b)^(1/2)-21*2^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21
*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)-20*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*x*a^2*b*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+24*2^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*a^2*b*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)+24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a^2*b*c*d*(a
*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-24*2^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*x^3*a*b^2*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)+20*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*x^3*a*b^2*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-24*2^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*a*b^2*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2))/x/a^2/e^2/(e*x)^(1/2)/(d*x^2-c)/(b*x^2-a)/(a*b)^(
1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2 (ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")

[Out] Timed out
```

3.910 $\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=484

$$-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\ldots$$

```
[Out] (a*e^3*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(3/4
)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt
[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.873992, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 470, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] (a*e^3*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(3/4
)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt
[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc-5ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{e^3\,\text{Subst}\left(\int \left(-\frac{(4bc-5ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{\left(-7abc+5a^2d\right)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} + \frac{\left((4bc-5ad)e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2(bc-ad)} - \frac{\left(a(7bc-5ad)e^3\right)\cdots}{\cdots}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{\left(\sqrt{c}(4bc-5ad)e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2\sqrt{d}(bc-ad)} + \frac{\left(\sqrt{c}(4bc-5a\cdots\right.}{\cdots}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{\left(\sqrt{c}(4bc-5ad)e^4\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2\sqrt{d}(bc-ad)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}\cdots\right.}{\cdots}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt{a}\sqrt[4]{c}(7bc\cdots}{\cdots}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} + \frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}(4bc-5\cdots}{\cdots}$$

Mathematica [C] time = 0.197152, size = 184, normalized size = 0.38

$$\frac{e^3(ex)^{3/2}\left(-7a^2\left(c-dx^2\right)+7ac\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(5ad-4bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\right.\right.}{14ab\left(a-bx^2\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(-7*a^2*(c - d*x^2) + 7*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 5*a*d)*x^2*(
a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a]))/(14*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.031, size = 2956, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-36*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*b^2*c^2*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+18*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*b^2*c^2*d+4*x^4*a*b^3*c*d^2+5*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2
^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*
c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-7*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
^(1/2)+4*x^2*a^2*b^2*c*d^2-4*x^2*a*b^3*c^2*d+8*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4
```

```
*c^3+16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*
c*d^2+36*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+10*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
^2*b^2*c*d^2-18*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-16*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*b^4*c^3-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/
2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^4*a
^2*b^2*d^3+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^
2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+7*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))*(-d
*x^2+c)^(1/2)*e^4*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*
d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)^2 \sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.911 $\int \frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=376

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] (a*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*
(4*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
- (c^(1/4)*(5*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(4*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a
*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^2*d^(1/4)*(b*
c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.545467, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 470, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (a*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*
(4*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
- (c^(1/4)*(5*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(4*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a
*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^2*d^(1/4)*(b*
c - a*d)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{e^3\,\text{Subst}\left(\int \frac{ac+\frac{(4bc-3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} + \frac{\left((4bc-3ad)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2(bc-ad)} - \frac{\left(a(5bc-3ad)e^3\right)}{}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} - \frac{\left((5bc-3ad)e^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{ae}}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4b^2(bc-ad)} - \frac{\left((5bc-3\right.}{}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} + \frac{\sqrt[4]{c}(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} - \frac{\left((5bc-3ad\right.}{}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)} + \frac{\sqrt[4]{c}(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc-3}{}$$

Mathematica [C] time = 0.190136, size = 184, normalized size = 0.49

$$\frac{e^3\sqrt{ex}\left(-5a^2\left(c-dx^2\right)+5ac\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(3ad-4bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4}\right.\right.}{10ab\left(a-bx^2\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-5*a^2*(c - d*x^2) + 5*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*
AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 3*a*d)*x^2*(a
- b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/
a]))/(10*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.03, size = 2520, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d
```

)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)+4*x^3*a^2*b*d^3*(a*b)^(1/2)-6*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^
(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)*(a*b)^(1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a
*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/
2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^3*a*b^2
*c*d^2*(a*b)^(1/2)-4*x*a^2*b*c*d^2*(a*b)^(1/2)+4*x*a*b^2*c^2*d*(a*b)^(1/2)-
3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*
b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/
2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+14*Ellip
ticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c
d((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a
*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
d)^(1/2))^(1/2)(c*d)^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2
^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+8*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/

```
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-8*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2
*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2
)*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/b/x/((c*d)^(1/2)*b-(a*b)^(1
/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d*x^2
-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.912 $\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=460

$$\frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{5}\cdots}{\cdots}$$

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) - (c^(3/4)*d^(1
/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1
/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^(5/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2])
```

Rubi [A] time = 0.779607, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 471, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) - (c^(3/4)*d^(1
/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1
/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^(5/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{x^2\left(3c-\frac{dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(3bc-ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{(de)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{((3bc-ad)e)\,\text{Subst}\left(\int \frac{x}{\left(a-\frac{bx^4}{e^2}\right)\cdots}\right.}{2b(bc-ad)}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{d}e^2\right)\text{Subst}\left(\int \frac{1+\cdots}{\sqrt{c\cdots}}\right.}{2b(bc-a\cdots}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\sqrt{d}e^2\sqrt{1-\cdots}\right.}{\cdots}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-ad)e^{5/2}\sqrt{1\cdots}}{4\sqrt{ab^3\cdots}}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\cdots}{2b(bc-a\cdots}$$

Mathematica [C] time = 0.150799, size = 168, normalized size = 0.37

$$\frac{e(ex)^{3/2}\left(dx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7c\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\left(c-dx^2\right)\right)}{14a\left(a-bx^2\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqrt[1 -
(d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*(-(b*c
) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.028, size = 2548, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(4*x^2*a*b^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-4*x^4*a*b^2*d^2+4*x^4*b^3*c*d+(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c
+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^
2*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*(c*d)^(1/2)*a*b*c-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*x^2*a*b^2*c*d-4*x^2*b^3*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
```

```
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*(c*d)^(1/2)*a^2*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
b^2*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2
*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^
2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+
2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)*d*(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b
*c)/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.913 $\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=363

$$\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}}{}$$

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*d^(3/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e
^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c
- a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.511338, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 471, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{d}{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*d^(3/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e
^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c
- a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{c+\frac{dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{((bc+ad)e)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\cdots}\right.}{2b(bc-ad)}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} - \frac{((bc+ad)e)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4ab(bc-ad)} - \frac{((bc+ad)e)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} - \frac{\left((bc+ad)e\sqrt{1-\frac{dx^2}{c}}\cdots\right.}{4a\cdots}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc+ad)e^{3/2}\sqrt{1-\cdots}}{4ab\sqrt[4]{\cdots}}$$

Mathematica [C] time = 0.136031, size = 169, normalized size = 0.47

$$-\frac{e\sqrt{ex}\left(dx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5c\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(c-dx^2\right)\right)}{10a\left(a-bx^2\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -(e*Sqrt[e*x]*(5*a*(c - d*x^2) + 5*c*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Appel
lF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqrt[1 - (
d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(10*a*(-(b*c)
+ a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.028, size = 2258, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*d*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
```

```
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^
2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-2*2^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*d*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+2*2^(1/2)*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*(c*d)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2
)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+2*2^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
(c*d)^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
```

```
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-4*x^3*a*b*d^2*(a*b)
^(1/2)+4*x^3*b^2*c*d*(a*b)^(1/2)+4*x*a*b*c*d*(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^
(1/2))*(-d*x^2+c)^(1/2)*e*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b
)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.914 $\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=464

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1}\cdots}{\cdots}$$

```
[Out] (b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)
*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
)), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*
d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d)*Sqrt[e]*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*d^(1/4)*(b*c - a
*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.786553, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {466, 472, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] (b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)
*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
)), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*
d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d)*Sqrt[e]*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*d^(1/4)*(b*c - a
*d)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(bc-3ad)x^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} - \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)e} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{2a(bc-ad)e}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{ce}}}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{2a(bc-ad)}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)}{2a(b}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-3ad)\sqrt{e}\sqrt{1-}}{4a^{3/2}\sqrt{b}}$$

$$= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\right.}{2a(bc-ad)}$$

Mathematica [C] time = 0.233247, size = 181, normalized size = 0.39

$$\frac{x\sqrt{ex}\left(7\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3b\left(dx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\left(\right.\right.\right.}{42a^2\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*Sqrt[e*x]*(7*(b*c - 4*a*d)*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*(-7*a*(c - d*x^2) + d*x^2*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a
])))/(42*a^2*(b*c - a*d)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.027, size = 2545, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(-4*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+4*x^4*a*b^2*d^2-4*x^4*b
^3*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d
-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+4*x^2*b^3*c^2+2*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
```

```
^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^
(1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*
b^3*c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*
c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)*d*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/((c*d
)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/a/(a*d-b*c
)/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.915 $\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=367

$$\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3}{4}$$

```
[Out] (b*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*a*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c -
5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c
- a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*
x^2])
```

Rubi [A] time = 0.518724, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {466, 414, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}d^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (b*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*a*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c -
5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c
- a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*
x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\sqrt{c-dx^2}}dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2a(bc-ad)e} + \frac{(3bc-5ad)\,\text{Subst}\left(\int \frac{\ldots}{\left(a-\frac{bx^4}{e^2}\right)\ldots}\ldots\right)}{2a(bc-ad)\ldots}$$

$$= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{(3bc-5ad)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{4a^2(bc-ad)e} + \frac{(3bc-5ad\ldots}{\ldots}$$

$$= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{e}\sqrt{c-dx^2}} + \frac{\left((3bc-5ad)\sqrt{1-\frac{dx^2}{c}}\ldots\right.}{4a\ldots}$$

$$= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-5ad)\sqrt{1-\ldots}}{4a^2\sqrt[4]{d}\ldots}$$

Mathematica [C] time = 0.172312, size = 180, normalized size = 0.49

$$\frac{bdx^3\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5x\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(4ad-3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5abx\left(dx^2-c\right)}{10a^2\sqrt{ex}\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (5*a*b*x*(-c + d*x^2) + 5*(-3*b*c + 4*a*d)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*(a - b*x^2)*Sq
rt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2
*(b*c - a*d)*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.031, size = 2266, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*b*d*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
```

/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*x^2*a*b^2*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-
2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*
a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+2*2^(1/2)*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c
^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*(c*d)^(1/2)*x^2*b^2*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*a*b*c+2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-2*2^(1/2)*Ellip
ticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d

```
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*
d)^(1/2)*a*b*c-4*x^3*a*b*d^2*(a*b)^(1/2)+4*x^3*b^2*c*d*(a*b)^(1/2)+4*x*a*b*
c*d*(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/a/((c*d)^(1/2)*b-
(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c
)/(d*x^2-c)/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.916 $\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=535

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-}}{}$$

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(2*a^2*c*(b*c - a*d)*e*Sqrt[e*x]) + (b*S
qrt[c - d*x^2])/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)) - (d^(1/4)*(5*b*c
 - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(
1/4)*(5*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x
^2]) - (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqr
t[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.07725, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 472, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5b}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(2*a^2*c*(b*c - a*d)*e*Sqrt[e*x]) + (b*S
qrt[c - d*x^2])/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)) - (d^(1/4)*(5*b*c
 - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(
1/4)*(5*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x
^2]) - (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqr
t[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} + \frac{e\,\text{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int \frac{x^2\left(-\frac{(bc-2ad)(5bc-2ad)}{e^4}-\frac{bd\cdots}{}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{\cdots}}}\cdots\right)}{2a^2c(bc-a\cdots}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} - \frac{e\,\text{Subst}\left(\int\left(\frac{d(5bc-4ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(5b^2\cdots}{e^4\left(a-\cdots\right.}\cdots\right.\right.}{2a^2c(bc-\cdots}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} + \frac{(b(5bc-7ad))\,\text{Subst}\left(\int \frac{\cdots}{\left(a-\frac{bx^4}{e^2}\right)\cdots}\cdots\right.}{2a^2(bc-ad)\cdots}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} + \frac{\left(\sqrt{d}(5bc-4ad)\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{c-\cdots}}\cdots\right.}{2a^2\sqrt{c}(bc-ad)e\cdots}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} + \frac{\left(\sqrt{d}(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Sub}\cdots}{2a^2\sqrt{c}(bc-ad)e\cdots}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} + \frac{\sqrt[4]{d}(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}F\left(\sin\cdots\right.}{2a^2\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{\cdots}}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)} - \frac{\sqrt[4]{d}(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin\cdots\right.}{2a^2\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{\cdots}}$$

Mathematica [C] time = 0.276013, size = 235, normalized size = 0.44

$$\frac{x\left(7x^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-21a\left(c-dx^2\right)\left(4a^2d-4ab\left(c+dx^2\right)+5b^2cx^2\cdots\right.\right.}{42a^3c(ex)^{3/2}\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-21*a*(c - d*x^2)*(4*a^2*d + 5*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 7*(5*b^
2*c^2 - 12*a*b*c*d + 4*a^2*d^2)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(-5*b*c + 4*a*d)*x^4*(a
```

```
- b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)
/a]))/(42*a^3*c*(b*c - a*d)*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 2982, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b^2*c^3+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2
-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^3+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d+36*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*
a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c^2-16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*
c*d^2+36*x^4*a*b^2*c*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3-5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3-5*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^3-18*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3+16*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^3*c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
```

```
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c^2+8*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d-5*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*
a*b*c^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*c*d-16*a*b^2*c^3-7*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*
c*d-16*x^2*a^2*b*c*d^2-20*x^2*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2+16*x^2*a
^3*d^3+20*x^2*b^3*c^3-16*x^4*a^2*b*d^3-20*x^4*b^3*c^2*d-16*a^3*c*d^2+32*a^2
*b*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c^2*d-36*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c^2*d+
18*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^2*b*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d)*b*d*(-d*x^2+c)^(1/2)/c/((c*d)^(1/
2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)/a^2/(
d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.917 $\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=429

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{3/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}}{}$$

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(6*a^2*c*(b*c - a*d)*e*(e*x)^(3/2)) + (b
*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)) + (d^(3/4)*(7
*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(6*a^2*c^(3/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) +
(b*c^(1/4)*(7*b*c - 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*c - 9
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*
d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.81428, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 472, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{3/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] -((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(6*a^2*c*(b*c - a*d)*e*(e*x)^(3/2)) + (b
*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)) + (d^(3/4)*(7
*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(6*a^2*c^(3/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) +
(b*c^(1/4)*(7*b*c - 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*c - 9
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*
d)*e^(5/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{e\,\mathrm{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{5bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} - \frac{e\,\mathrm{Subst}\left(\int \frac{-\frac{21b^2c^2-20abcd-4a^2d^2}{e^4}\ldots}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c\ldots}}\ldots\right.}{6a^2c(bc-\ldots}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{(b(7bc-9ad))\,\mathrm{Subst}\left(\int \frac{\ldots}{\left(a-\ldots\right.}\ldots\right.}{2a^2(bc-a\ldots}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{(b(7bc-9ad))\,\mathrm{Subst}\left(\int \frac{\ldots}{\left(1-\ldots\right.}\ldots\right.}{4a^3(bc-\ldots}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{d^{3/4}(7bc-4ad)\sqrt{1-\frac{dx^2}{c}}F\left(\ldots\right.}{6a^2c^{3/4}(bc-ad)e^{5/\ldots}}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{d^{3/4}(7bc-4ad)\sqrt{1-\frac{dx^2}{c}}F\left(\ldots\right.}{6a^2c^{3/4}(bc-ad)e^{5/\ldots}}$$

Mathematica [C] time = 0.289774, size = 234, normalized size = 0.55

$$\frac{x\left(5x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(4a^2d^2+20abcd-21b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5a\left(c-dx^2\right)\left(4a^2d-4ab\left(c+dx^2\right)+7b^2cx^2\ldots\right.\right.}{30a^3c(ex)^{5/2}\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-5*a*(c - d*x^2)*(4*a^2*d + 7*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 5*(-21*b
^2*c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - b*d*(-7*b*c + 4*a*d)*x^4*(a -
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]
))/(30*a^3*c*(b*c - a*d)*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.035, size = 2622, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*b*d*(-16*x^2*a^3*d^3*(a*b)^(1/2)+16*a^3*c*d^2*(a*b)^(1/2)+8*2^(1/2)*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^3*a^2*b*d^2*(a
*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-14*2^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b^2*c^2*(a*b)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*x*a*b^2*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+21*2^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a*b^2*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*b^4*c^3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)+16*x^4*a^2*b*d^3*(a*b)^(1/2)-32*a^2*b*c^2*d*(a*b)^
(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*a*b^3*c^3*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*a^2*b^2*c^2*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a^2*b^2*c
^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(c*d)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*a*b^3*c^2*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)-28*x^2*b^3*c^3*(a*b)^(1/2)+16*a*b^2*c^3*(a*b)^(1
/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*x^3*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*x^3*b^3*c^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-8
*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^3
*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-44*x^4*a*b^2*c*d^2*
(a*b)^(1/2)+28*x^4*b^3*c^2*d*(a*b)^(1/2)+16*x^2*a^2*b*c*d^2*(a*b)^(1/2)+28*
x^2*a*b^2*c^2*d*(a*b)^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x*a*b
^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+22*2^(1/2)*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*b*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1
```

```
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x*a^2*b*
c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-27*2^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*x*a^2*b*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^3*a*b^2*c*d*(a*b)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-22*2^(1/2)*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^3*a*b^2*c*d*(a*b)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^3*
a*b^2*c*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2))*(-d*x^2+c)^(1
/2)/x/c/a^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*
b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.918 $\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=529

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2bd^{3/4}\sqrt{c-dx^2}(bc-ad)^2}+\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] ((2*b*c + a*d)*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^
3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (c^(3/4)*(2*
b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (c^
(3/4)*(2*b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) + (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (Sqrt
[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.07994, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 470, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^
3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (c^(3/4)*(2*
b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (c^
(3/4)*(2*b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) + (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (Sqrt
[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc-ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^5\,\text{Subst}\left(\int \frac{x^2\left(-\frac{18abc^2}{e^2}+\frac{2bc(2bc\cdots}{e^{\cdots}}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{d\cdots}{\cdots}}}\right.}{4bc(bc-ad\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^5\,\text{Subst}\left(\int\left(-\frac{2c(2bc+ad)x^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}}+\cdots\right.\right.}{4bc(bc\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(a(7bc-ad)e^3\right)\text{Subst}\left(\int \cdots\right.}{2b(bc-\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}(2bc+ad)e^4\right)\text{Subst}\left(\int\cdots\right.}{2b\sqrt{d}(bc-\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}(2bc+ad)e^4\sqrt{1-\frac{dx^2}{c}}\right)\cdots}{2b\sqrt{d}(bc-\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{c^{3/4}(2bc+ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}F\cdots}{2bd^{3/4}(bc-ad)^{2\cdots}}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{c^{3/4}(2bc+ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}E\cdots}{2bd^{3/4}(bc-ad)^{2\cdots}}$$

Mathematica [C] time = 0.221764, size = 189, normalized size = 0.36

$$\frac{e^3(ex)^{3/2}\left(x^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+21ac\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7a\cdots\right.}{14a\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(7*a*(-3*a*c + 2*b*c*x^2 + a*d*x^2) + 21*a*c*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*b*
```

```
c + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (
d*x^2)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.036, size = 2964, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/8*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+4*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a^2*b^2*c^2*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a^2*b^2*c^2*d-4*x^4*a*b^3*c*d^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+12
*x^2*a^2*b^2*c*d^2-12*x^2*a*b^3*c^2*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-8*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a*b^3*c^3-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-4*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^
2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
```

```
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3
-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2
*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^
3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-7*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1
/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^4*a^2*b^2*d^3+8*x^4*b^4*
c^2*d+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+7*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))*(-d*x^2+c)
^(1/2)*e^4*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)
^(1/2)*b)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.919 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx$

Optimal. Leaf size=420

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{e^3\sqrt{ex}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}-$$

```
[Out] ((2*b*c + a*d)*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^3*
Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (c^(1/4)*(2*b*c
+ a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(2*b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4
)*(5*b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*
b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d^(1/4)*(b*c - a*d)^2*Sqrt
[c - d*x^2])
```

Rubi [A] time = 0.692244, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 470, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{e^3\sqrt{ex}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{7/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^3*
Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (c^(1/4)*(2*b*c
+ a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(2*b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4
)*(5*b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*
b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d^(1/4)*(b*c - a*d)^2*Sqrt
[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}\left(\int \frac{ac+\frac{(4bc+ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^5\,\text{Subst}\left(\int \frac{-\frac{6abc^2}{e^2}-\frac{2bc(2bc+ad)x}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{4bc(bc-ad)^2}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left((2bc+ad)e^3\right)\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{2b(bc-ad)^2}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left((5bc+ad)e^3\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{4b(bc-}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(2bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F}{2b\sqrt[4]{d}(bc-ad)^2}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(2bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}F}{2b\sqrt[4]{d}(bc-ad)^2}$$

Mathematica [C] time = 0.202534, size = 191, normalized size = 0.45

$$-\frac{e^3\sqrt{ex}\left(x^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+15ac\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(\right.\right.}{10a\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(e^3*Sqrt[e*x]*(5*a*(3*a*c - 2*b*c*x^2 - a*d*x^2) + 15*a*c*(-a + b*x^2)*Sq
rt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c
 + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*
x^2)/c, (b*x^2)/a]))/(10*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.035, size = 2530, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)

[Out] -1/8*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*x^3*a^2*b*d^3*(a*b)^(1/2)+8*x^3*b^3*c^2*d*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^3*a*b^2*c*d^2*(a*b)^(1/2)+12*x*a^2*b*c*d^2*(a*b)^(1/2)-12*x*a*b^2*c^2*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^

```
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*
c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(
1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/x/((c*d)^(1/2)*
b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b
*c)^2/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.920 $\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=485

$$\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(}{4\sqrt{}}$$

```
[Out] (3*d*e*(e*x)^(3/2))/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*(e*x)^(3/2))/(2*
(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[
1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)
^2*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*
x^2])
```

Rubi [A] time = 0.957945, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 471, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (3*d*e*(e*x)^(3/2))/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*(e*x)^(3/2))/(2*
(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[
1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)
^2*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*
x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{x^2\left(3c+\frac{3dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(-\frac{6c(bc+2ad)}{e^2}+\frac{6bcdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx \ldots\right.}{4c(bc-ad)^2}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^3\,\text{Subst}\left(\int\left(-\frac{6cdx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{6\left(bc^2 \ldots\right.}{e^2\left(a-\frac{bx^4}{e^2}\ldots\right.}\right.\right.}{4c(bc-ad)^2}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{(3de)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex} \ldots\right.}{2(bc-ad)^2}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(3\sqrt{c}\sqrt{d}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, \ldots\right.}{2(bc-ad)^2}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(3\sqrt{c}\sqrt{d}e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \ldots\right.}{2(bc-ad)^2\sqrt{c-d\ldots}}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\ldots\right.\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\ldots\right.\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.211889, size = 185, normalized size = 0.38

$$\frac{e(ex)^{3/2}\left(3bdx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7a\left(2a\ldots\right.\right.}{14a\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(2*a*d + b*(c - 3*d*x^2)) + 7*(b*c + 2*a*d)*(a - b*x^2
)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*
```

```
b*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2
)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.032, size = 2561, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(4*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+12*x^4*a*b^2*d^2-12*x^4*b
^3*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+3*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^
2*c*d-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+4*x^2*b^3*c^2-8*x^2*a^2*b*d^2+6*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-12*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
```

```
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c
^2-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*
c^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)*
d*(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(
1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.921 $\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\ldots}{\ldots}$$

```
[Out] (3*d*e*Sqrt[e*x])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*Sqrt[e*x])/(2*(b*c
 - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (3*c^(1/4)*d^(3/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d
*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.645462, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 471, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (3*d*e*Sqrt[e*x])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*Sqrt[e*x])/(2*(b*c
 - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (3*c^(1/4)*d^(3/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d
*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e\,\text{Subst}\left(\int \frac{c+\frac{5dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e^3\,\text{Subst}\left(\int \frac{-\frac{2c(bc+2ad)}{e^2}-\frac{6bcdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \right.}{4c(bc-ad)^2}$$

$$= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{(3de)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{2(bc-ad)^2}$$

$$= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{((bc+5ad)e)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{\cdots}}\right.}{4a(bc-ad)^2}$$

$$= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

$$= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

Mathematica [C] time = 0.19959, size = 186, normalized size = 0.48

$$-\frac{e\sqrt{ex}\left(3bdx^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(2ad\cdots\right.\right.}{10a\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(e*Sqrt[e*x]*(5*a*(2*a*d + b*(c - 3*d*x^2)) + 5*(b*c + 2*a*d)*(-a + b*x^2)
*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 3*b
*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/
c, (b*x^2)/a]))/(10*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.033, size = 2277, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/8*b*d*(-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-5*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*(c*d)^(1/2)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a^2*b*c*d+6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-
12*x^3*a*b*d^2*(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^(1/2)+5*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b
*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*(c*d)^(1/2)*x^2*a*b*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+5*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2
)*a*b*c+8*x*a^2*d^2*(a*b)^(1/2)+12*x^3*b^2*c*d*(a*b)^(1/2)-4*x*a*b*c*d*(a*b
)^(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-5*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/
2)*a*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
x^2*b^3*c^2-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a*b*c*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+6*2^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
```

```
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2)*(-d*x^2+c)^(1/2)*e*(e*
x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b
)^(1/2)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.922 $\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}} dx$

Optimal. Leaf size=531

$$\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+$$

```
[Out] (d*(b*c + 2*a*d)*(e*x)^(3/2))/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*
(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) - (d^(1/4)*(b*
c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (d^
(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) - (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (Sqrt
[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.0903, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 472, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d)*(e*x)^(3/2))/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*
(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) - (d^(1/4)*(b*
c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (d^
(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) - (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (Sqrt
[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}-\frac{3bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(-\frac{2(b^2c^2-8abc}{e^4}\right.}{\left(a-\right.}\right.}{4ac(}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}\left(\int\left(\frac{2d(bc+2ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}}\right.\right.}{4ac(}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{(b(bc-7ad))\,\text{Subst}\left(\int\right.}{2a(bc-}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(\sqrt{d}(bc+2ad)\right)\text{Subst}\left(\right.}{2a\sqrt{c}(bc-}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\left(\sqrt{d}(bc+2ad)\sqrt{1-\frac{dx^2}{c}}\right)}{2a\sqrt{c}(bc-}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{\sqrt[4]{d}(bc+2ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}}{2a\sqrt[4]{c}(bc-a}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}(bc+2ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}}{2a\sqrt[4]{c}(bc-a}$$

Mathematica [C] time = 0.273861, size = 230, normalized size = 0.43

$$\frac{\sqrt{ex}\left(7x\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(2a^2d^2+8abcd-b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+21ax\left(-2a^2d^2+2abd^2x^2+b^2c\left(dx^2-c\right)\right)\right.}{42a^2c\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[e*x]*(21*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 7*(-
(b^2*c^2) + 8*a*b*c*d + 2*a^2*d^2)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
```

```
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(b*c + 2*a*d)*x^3*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a
]))/(42*a^2*c*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.034, size = 2950, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^
2*c^3+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2-2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*a*b^2*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*a*b^2*c^3+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^
2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*(c*d)^(1/2)*x^2*b^2*c^2+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2-4*x^4
*a*b^2*c*d^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3-((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3-((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^3+2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*a*b^2*c^2*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3-8*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a^3*c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*x^2*a*b^2*c^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
```

```
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)
^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
^2*b*c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2+7*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*x^2*a*b*c*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*c*d+8*x^2*a^2*b*c*d^2-4*x^2*a*b^2*c^2
*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*(c*d)^(1/2)*a*b*c^2-8*x^2*a^3*d^3+4*x^2*b^3*c^3+8*x^4*a^2*b*d^3-4*x^4
*b^3*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c^2*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c^2*d
-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^2*b*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d)*d*b*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/
c/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/a/(
a*d-b*c)^2/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.923 $\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=426

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ac^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\cdots$$

```
[Out] (d*(b*c + 2*a*d)*Sqrt[e*x])/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*Sq
rt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) + (d^(3/4)*(b*c +
2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(2*a*c^(3/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*
d^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^2*
Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.774581, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[e*x])/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*Sq
rt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) + (d^(3/4)*(b*c +
2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(2*a*c^(3/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*
d^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^2*
Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e\,\mathrm{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{5bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\mathrm{Subst}\left(\int -\frac{2(3b^2c^2-}{}\right.}{4}$$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{(3b(bc-3ad))\,\mathrm{Subs}}{2a}$$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{(3b(bc-3ad))\,\mathrm{Subs}}{4a}$$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{d^{3/4}(bc+2ad)\sqrt{1-}}{2ac^{3/4}(bc-}$$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{d^{3/4}(bc+2ad)\sqrt{1-}}{2ac^{3/4}(bc-}$$

Mathematica [C] time = 0.261117, size = 229, normalized size = 0.54

$$\frac{5x\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}\left(2a^2d^2-8abcd+3b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5ax\left(-2a^2d^2+2abd^2x^2+b^2c\left(dx^2-c\right)\right)+bdx}{10a^2c\sqrt{ex}\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (5*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 5*(3*b^2*c^2 - 8
*a*b*c*d + 2*a^2*d^2)*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(b*c + 2*a*d)*x^3*(a - b*x^2)*Sqrt[1 -
 (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*c*(b*
c - a*d)^2*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 2554, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/8*b*d*(9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)-8*x^3*a^2*b*d^3*(a*b)^(1/2)+4*x^3*b^3*c^2*d*(a*b)^(1/2)+
8*x*a^3*d^3*(a*b)^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a
*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-9*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3+3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3-3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1
/2)*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^2-3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
(c*d)^(1/2)*x^2*b^3*c^2+4*x^3*a*b^2*c*d^2*(a*b)^(1/2)-8*x*a^2*b*c*d^2*(a*b)
^(1/2)+4*x*a*b^2*c^2*d*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*
b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
```

```
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/
2)*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x*b^3*c^
3*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)
)*(-d*x^2+c)^(1/2)/c/a/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(
1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.924 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=628

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)E\left(\sin^{-1}\right.}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*Sqrt[c - d*x^2]) + b/(2*
a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]) - ((5*b^2*c^2 - 8*a*
b*c*d + 6*a^2*d^2)*Sqrt[c - d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*e*Sqrt[e*x]) -
 (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c -
a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d
^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)
*c^(1/4)*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
4*a^(5/2)*d^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)
*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d
^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.40339, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {466, 472, 579, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*Sqrt[c - d*x^2]) + b/(2*
a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]) - ((5*b^2*c^2 - 8*a*
b*c*d + 6*a^2*d^2)*Sqrt[c - d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*e*Sqrt[e*x]) -
 (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c -
a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d
^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)
*c^(1/4)*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
4*a^(5/2)*d^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)
*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d
^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{7bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8ab\cdots\right.}{2a^2c^2(\cdots}$$

Mathematica [C] time = 0.467891, size = 319, normalized size = 0.51

$$\frac{x\left(3bdx^4\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(-8a^2bcd^2+6a^3d^3\cdots\right.\right.}{42a^3c\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(21*a*(2*a^3*d^2*(2*c - 3*d*x^2) - 5*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*c
*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 3*d^2*x^4))
+ 7*(-5*b^3*c^3 + 16*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 6*a^3*d^3)*x^2*(a - b*x^
2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3
*b*d*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*x^4*(-a + b*x^2)*Sqrt[1 - (d*x^2)/
c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*c^2*(b*c - a
*d)^2*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.04, size = 3385, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(-11*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d+11*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5
2*x^4*a*b^3*c^2*d^2+40*x^2*a^3*b*c*d^3-36*x^2*a*b^3*c^3*d-56*x^4*a^2*b^2*c*
d^3-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a^4*c*d^3+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4+12*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^4*c*d^3-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4+5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^4+5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^4-5*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^4
-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^4+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^4-20
*x^4*b^4*c^3*d+24*x^4*a^3*b*d^4-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^4-24*x^2*a^4*d^4+20*x^2*b^4*c^4+16*a^4*c
*d^3-16*a*b^3*c^4+56*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c^2*d^2-52*((d*x+(c*d)^(1/2
```

))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^3*d-28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c^2*d^2+26*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^3*d-11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b^2*c^3*d-11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^3*d-11*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c^2*d+11*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d-48*a^3*b*c^2*d^2+48*a^2*b^2*c^3*d+11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^3*c^3*d+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^3-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^3-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^3+52*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^3*d-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^3*b*c*d^3+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2-26*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^3*d+11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^3*c^3*d+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3+24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^3*b*c*d^3-56*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2)*d*b*(-d*x^2+c)^(1/2)/c^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)^2/a^2/(d*x^2-c)/e/(e*x)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.925 $\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=512

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(10a^2d^2-8abcd+7b^2c^2\right)}{6a^2c^2e(ex)^{3/2}(bc-ad)^2}+\frac{b^2\sqrt[4]{c}\sqrt{1-}}{}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*Sqrt[c - d*x^2]) + b/(
2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]) - ((7*b^2*c^2 -
8*a*b*c*d + 10*a^2*d^2)*Sqrt[c - d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*e*(e*x)^(
3/2)) + (d^(3/4)*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*E
llipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(7/4)
*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e
^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x
^2])
```

Rubi [A] time = 1.07839, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 472, 579, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(10a^2d^2-8abcd+7b^2c^2\right)}{6a^2c^2e(ex)^{3/2}(bc-ad)^2}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7b}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*Sqrt[c - d*x^2]) + b/(
2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]) - ((7*b^2*c^2 -
8*a*b*c*d + 10*a^2*d^2)*Sqrt[c - d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*e*(e*x)^(
3/2)) + (d^(3/4)*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*E
llipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(7/4)
*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e
^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x
^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{9bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{e^3\,\text{Subst}}{\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-\ldots\right.}{6a\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-\ldots\right.}{6a\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-\ldots\right.}{6a\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-\ldots\right.}{6a\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-\ldots\right.}{6a\ldots}$$

Mathematica [C] time = 0.46396, size = 318, normalized size = 0.62

$$\frac{x\left(bdx^4\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5x^2\left(bx^2-a\right)\sqrt{1-\frac{dx^2}{c}}\left(-8a^2bcd^2+10a^3d^3\ldots\right.\right.}{30a^3c\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(5*a*(2*a^3*d^2*(2*c - 5*d*x^2) - 7*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*c*
(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 5*d^2*x^4)) +
 5*(21*b^3*c^3 - 32*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 10*a^3*d^3)*x^2*(-a + b*x
^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] +
b*d*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*c^2*(b*c - a*d
)^2*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.045, size = 2871, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/24*b*d*(-40*x^4*a^3*b*d^4*(a*b)^(1/2)+48*a^3*b*c^2*d^2*(a*b)^(1/2)-48*a^
2*b^2*c^3*d*(a*b)^(1/2)-20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*2^(1/2)*x^3*a^3*b*d^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*x*a*b^3*c^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^3*c^3*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^3
*c^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+20*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^4*d^3*(a*b)^(1/2)*
(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+28*x^4*b^4*c^3*d*(a*b)^(1/2)+16*a*b^
3*c^4*(a*b)^(1/2)+40*x^2*a^4*d^4*(a*b)^(1/2)-28*x^2*b^4*c^4*(a*b)^(1/2)-16*
a^4*c*d^3*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b^4*c
^3*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-30*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a*b^3*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b^3*c^
2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*2^(1/2)*x^3*a*b^3*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)-36*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*x*a^3*b*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
+30*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*
a^2*b^2*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-39*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d
```

```
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2
*c^2*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-39*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b^3*c^3*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+39*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^3*c^3*d*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)+14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
x^3*b^4*c^3*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+72*x^4*a^2*b
^2*c*d^3*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^4*c^3*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*2^(1/2)*x^3*b^5*c^4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-60*x^4*a*b^3*c^2*d
^2*(a*b)^(1/2)-56*x^2*a^3*b*c*d^3*(a*b)^(1/2)+44*x^2*a*b^3*c^3*d*(a*b)^(1/2
)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*b^5*c^4*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)+36*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*x^3*a^2*b^2*c*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2
1*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^4*c^4*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^4*c^4*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b^4*c^3*d*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*b^4*c^3*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))*(-d*x^2+c)^(1/2)/x/c^2/
a^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)
/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 - a\right)^2 \left(-dx^2 + c\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.926 $\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=568

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3(ex)^{3/2}}{2\sqrt{c-d}}$$

```
[Out] ((2*b*c + 3*a*d)*e^3*(e*x)^(3/2))/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (
a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + ((b*c
+ 4*a*d)*e^3*(e*x)^(3/2))/(2*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (c^(3/4)*(b*c
 + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (c^(3/
4)*(b*c + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) +
 (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[a]
*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.36229, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 470, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3(ex)^{3/2}(4ad+}{2\sqrt{c-dx^2}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] ((2*b*c + 3*a*d)*e^3*(e*x)^(3/2))/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (
a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + ((b*c
+ 4*a*d)*e^3*(e*x)^(3/2))/(2*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (c^(3/4)*(b*c
 + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (c^(3/
4)*(b*c + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) +
 (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[a]
*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc+3ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e^5\,\text{Subst}\left(\int \frac{x^2\left(-\frac{30abc^2}{e^2}-\right.}{\left(a-\frac{bx^4}{e^2}\right)}\right.}{12bc(b}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} -$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} -$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} -$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} +$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} +$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} +$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} -$$

Mathematica [C] time = 0.404213, size = 256, normalized size = 0.45

$$-\frac{e^3(ex)^{3/2}\left(7a\left(a^2d\left(7c-9dx^2\right)+4ab\left(2c^2-4cdx^2+3d^2x^4\right)+b^2cx^2\left(3dx^2-5c\right)\right)+3bx^2\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\right.(}{42a\left(bx^2-a\right)\left(c-dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -(e^3*(e*x)^(3/2)*(7*a*(a^2*d*(7*c - 9*d*x^2) + b^2*c*x^2*(-5*c + 3*d*x^2)
+ 4*a*b*(2*c^2 - 4*c*d*x^2 + 3*d^2*x^4)) + 7*a*(8*b*c + 7*a*d)*(-a + b*x^2)
*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x
^2)/a] + 3*b*(b*c + 4*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*
AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a*(b*c - a*d)^3*(-a
 + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] time = 0.063, size = 5126, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{\left(bx^2 - a\right)^2 \left(-dx^2 + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.927 $\int \frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=454

$$\frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{5e^3\sqrt{ex}(2ad+bc)}{6\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3\sqrt{ex}(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{}{2b\left(a-b\right.}$$

```
[Out] ((2*b*c + 3*a*d)*e^3*Sqrt[e*x])/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (a*
e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (5*(b*c +
2*a*d)*e^3*Sqrt[e*x])/(6*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (5*c^(1/4)*(b*c +
 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(6*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/
4)*(b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d
^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*d)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c -
d*x^2])
```

Rubi [A] time = 0.86722, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 470, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{5e^3\sqrt{ex}(2ad+bc)}{6\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3\sqrt{ex}(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin\right.}{6\sqrt[4]{d}\sqrt{c-dx^2}(bc-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] ((2*b*c + 3*a*d)*e^3*Sqrt[e*x])/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (a*
e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (5*(b*c +
2*a*d)*e^3*Sqrt[e*x])/(6*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (5*c^(1/4)*(b*c +
 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(6*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/
4)*(b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d
^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*d)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c -
d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e^3\,\text{Subst}\left(\int \frac{ac+\frac{(4bc+5ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e^5\,\text{Subst}\left(\int \frac{-\frac{10abc^2}{e^2}-\frac{10bc(2bc\cdots}{e\cdots}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx\cdots}{e\cdots}\right)}\cdots\right)}{12bc(bc-a\cdots}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} - \cdots$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} - \cdots$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} - \cdots$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} + \cdots$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} + \cdots$$

Mathematica [C] time = 0.38687, size = 252, normalized size = 0.56

$$\frac{e^3\sqrt{ex}\left(a\left(a^2d\left(7dx^2-5c\right)-2ab\left(5c^2-8cdx^2+5d^2x^4\right)+b^2cx^2\left(7c-5dx^2\right)\right)+bx^2\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\cdots\right.}{6a\left(bx^2-a\right)\left(c-dx^2\right)^{3/2}(bc-ad\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (e^3*Sqrt[e*x]*(a*(b^2*c*x^2*(7*c - 5*d*x^2) + a^2*d*(-5*c + 7*d*x^2) - 2*a
*b*(5*c^2 - 8*c*d*x^2 + 5*d^2*x^4)) + 5*a*(2*b*c + a*d)*(a - b*x^2)*(c - d*
x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] +
 b*(b*c + 2*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5
/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(6*a*(b*c - a*d)^3*(-a + b*x^2)*(c
```

```
- d*x^2)^(3/2))
```

Maple [B] time = 0.061, size = 4403, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] 1/24*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*b*(15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a^2*b*d^3+15
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*(c*d)^(1/2)*x^4*a^2*b*d^3-15*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d-15*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*x^2*a*b^2*c^2*d-40*x^5*a^2*b*d^4*(a*b)^(1/2)+15*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a
*b^2*c*d^2-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*c*d^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a*b^2*c*
d^2-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*c*d^2+20*x^5*a*b^2*c*d^3*(a*b)^(1/2)+20*x^5
*b^3*c^2*d^2*(a*b)^(1/2)-28*x^3*b^3*c^3*d*(a*b)^(1/2)-20*x*a^3*c*d^3*(a*b)^
(1/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*x^4*a*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+10*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*
b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-20*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c^2*d*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+15*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^4*a^2*b^2*c*d^3-15*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b^2*c^3*d+15*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
```

```
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^3*d
+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^4*a
*b^3*c^2*d^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*x^2*a^3*b*c*d^3+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^3*c*d^2-15*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*a^2*b^2*c*d^
3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*
a*b^3*c^2*d^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^3*d^3+15*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c^2*d+1
5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*a^2*b*c^2*d+28*x^3*a^3*d^4*(a*b)^(1/2)-15*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^3*c^3*d+15*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^3*b*c^2*d^2-1
5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^3*b*c
^2*d^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*x^2*a*b^3*c^3*d-20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*2^(1/2)*x^4*a^2*b*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1
/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*x^4*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+10*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c^2*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-20*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*c*d^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)*(a*b)^(1/2)-10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*2^(1/2)*x^2*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
```

```
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2
)+20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x
^2*a^3*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+36*x^3*a^2*b*
c*d^3*(a*b)^(1/2)-36*x^3*a*b^2*c^2*d^2*(a*b)^(1/2)-20*x*a^2*b*c^2*d^2*(a*b)
^(1/2)+40*x*a*b^2*c^3*d*(a*b)^(1/2)-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2+15*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^3*c*d
^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*(c*d)^(1/2)*x^2*a^3*d^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a^3*b*c*d^3)/(d*x^2-c)^2/((c*d)^(1/2)*b-(a*b
)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)^3/
x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.928 $\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=551

$$\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt{b}\sqrt[4]{c}\cdots}{\cdots}$$

```
[Out] (5*d*e*(e*x)^(3/2))/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*(e*x)^(3/2))/(
2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(4*b*c + a*d)*e*(e*x)^(3/
2))/(2*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (d^(1/4)*(4*b*c + a*d)*e^(5/2)*Sq
rt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(4*b*c + a*d)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(3
*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sq
rt[a]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(3*b*c + 7*
a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4
)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.29642, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 471, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt{b}\sqrt[4]{c}e^{5/2}\sqrt{1-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (5*d*e*(e*x)^(3/2))/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*(e*x)^(3/2))/(
2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(4*b*c + a*d)*e*(e*x)^(3/
2))/(2*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (d^(1/4)*(4*b*c + a*d)*e^(5/2)*Sq
rt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(4*b*c + a*d)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(3
*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sq
rt[a]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(3*b*c + 7*
a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4
)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}dx,x,\sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e\,\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}dx,x,\sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(-\frac{6c(3bc+2ad)}{e^2}-30\ldots\right.}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)\ldots}\ldots\right)}{12c(bc-ad)\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} - \frac{e^5\,\text{S}\ldots}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} - \frac{e^5\,\text{S}\ldots}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} - \frac{(d(\ldots}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} + \frac{(\sqrt{\ldots}}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} + \frac{\left(\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} + \frac{\sqrt[4]{d}\ldots}{\ldots}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}\ldots}{\ldots}$$

Mathematica [C] time = 0.455667, size = 278, normalized size = 0.5

$$-\frac{e(ex)^{3/2}\left(7\left(bx^2-a\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(a^2d^2+11abcd+3b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7a\left(a^2d^2\left(c-3dx^2\right)+abd\left(11c^2\ldots\right.\right.\right.}{42ac\left(bx^2-a\right)\left(c\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -(e*(e*x)^(3/2)*(7*a*(a^2*d^2*(c - 3*d*x^2) + a*b*d*(11*c^2 - 10*c*d*x^2 +
3*d^2*x^4) + b^2*c*(3*c^2 - 17*c*d*x^2 + 12*d^2*x^4)) + 7*(3*b^2*c^2 + 11*a
*b*c*d + a^2*d^2)*(-a + b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(4*b*c + a*d)*x^2*(a - b*x^2)*
(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x
^2)/a]))/(42*a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] time = 0.046, size = 5078, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{\left(bx^2 - a\right)^2 \left(-dx^2 + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.929 $\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=447

$$\frac{d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+14bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6c^{3/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right.}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}$$

```
[Out] (5*d*e*Sqrt[e*x])/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*Sqrt[e*x])/(2*(b
*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(14*b*c + a*d)*e*Sqrt[e*x])/(
6*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(3/4)*(14*b*c + a*d)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(6*c^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)
^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.916765, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 471, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+14bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6c^{3/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}-\frac{b\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (5*d*e*Sqrt[e*x])/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*Sqrt[e*x])/(2*(b
*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(14*b*c + a*d)*e*Sqrt[e*x])/(
6*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(3/4)*(14*b*c + a*d)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(6*c^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)
^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e\,\mathrm{Subst}\left(\int \frac{c+\frac{9dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e^3\,\mathrm{Subst}\left(\int \frac{-\frac{2c(3bc+2ad)}{e^2}-5\ldots}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx\ldots}{e^2}\right.}\right.}{12c(bc-a\ldots}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} - \ldots$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} + \ldots$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} - \ldots$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} + \ldots$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} + \ldots$$

Mathematica [C] time = 0.439187, size = 275, normalized size = 0.62

$$\frac{e\sqrt{ex}\left(-5\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(a^2d^2-13abcd-3b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5a\left(a^2d^2\left(c+dx^2\right)-abd\left(13c^2\ldots\right.\right.\right.}{30ac\left(bx^2-a\right)\left(\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (e*Sqrt[e*x]*(5*a*(a^2*d^2*(c + d*x^2) + b^2*c*(-3*c^2 + 19*c*d*x^2 - 14*d^
2*x^4) - a*b*d*(13*c^2 - 10*c*d*x^2 + d^2*x^4)) - 5*(-3*b^2*c^2 - 13*a*b*c*
d + a^2*d^2)*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(14*b*c + a*d)*x^2*(a - b*x^2)*(c - d*
x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))
```

```
/(30*a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] time = 0.043, size = 4403, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] 1/24*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*b*d*(-30*(c*d)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d
-30*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/
2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x^4*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*x^4*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-4*x^5*a^2*b*d^4*(a*b)^(1/2
)-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3
*c^4+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*
b^3*c^4-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*x^2*b^4*c^4+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a*b^2*c*d^2-27*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*
c*d^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*x^4*a*b^2*c*d^2-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*c*d^2+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*
c^4-52*x^5*a*b^2*c*d^3*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x^4*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+3*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
```

```
2))^(1/2)*(a*b)^(1/2)+56*x^5*b^3*c^2*d^2*(a*b)^(1/2)-76*x^3*b^3*c^3*d*(a*b)
^(1/2)+4*x*a^3*c*d^3*(a*b)^(1/2)-26*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2)*(a*b)^(1/2)+28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*2^(1/2)*x^2*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2
^(1/2)*x^2*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+1
2*x*b^3*c^4*(a*b)^(1/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*a^2*b^2*c^3*d+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^3*d+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^4*a*b^3*c^2*d^2+27*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2-27*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*a*b^3*c^2*d
^2+27*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*a^2*b*c^2*d+27*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d+4*x^3*a^3*d^
4*(a*b)^(1/2)-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*x^2*a*b^3*c^3*d+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^3-3*(c*d)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3
*c^3+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*a*b^2*c^3+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^3*c^3*d-3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3-2*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a^2*
b*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+28*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b^3*c^2*d*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-26*EllipticF(((d*x+(c*d)^(1/2))
```

```
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c^2*d*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(c*d)^(1/2)*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*a^3*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)+28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*a*b^2*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-28*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c
^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^3*d^3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+36*x^3*a^2*b*c*d^3*(a*b)^(1/2)+36*x^3
*a*b^2*c^2*d^2*(a*b)^(1/2)-56*x*a^2*b*c^2*d^2*(a*b)^(1/2)+40*x*a*b^2*c^3*d*
(a*b)^(1/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*x^2*a^2*b^2*c^2*d^2)/x/(a*d-b*c)^3/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)
*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(d*x^2-c)^2/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.930 $\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=625

$$\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{2ac^2e\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\ldots}}{\ldots}$$

```
[Out] (d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) +
 (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^
2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(2*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
 d*x^2]) - (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(
5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d
^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b^(3/2)*c
^(1/4)*(b*c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(b*
c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.43087, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {466, 472, 579, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{2ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2\ldots\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```



```
[Out] (d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) +
 (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^
2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(2*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
 d*x^2]) - (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(
5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d
^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b^(3/2)*c
^(1/4)*(b*c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(b*
c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
```

```
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}-\frac{7bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2\left(-\frac{6(b^2\cdots}{}\right)}{}\cdots\right)}{\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+5abcd-a\cdots\right)}{2ac^2(bc-ad)^3e\cdots}$$

Mathematica [C] time = 0.546268, size = 327, normalized size = 0.52

$$\frac{\sqrt{ex}\left(3bdx^3\left(bx^2-a\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7x\left(bx^2-a\right)\left(c-dx^2\right)\sqrt{1-\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[e*x]*(7*a*x*(a*b^2*c*d^2*x^2*(17*c - 15*d*x^2) + a^3*d^3*(5*c - 3*d*x
^2) - 3*b^3*c^2*(c - d*x^2)^2 + a^2*b*d^2*(-17*c^2 + 10*c*d*x^2 + 3*d^2*x^4
)) + 7*(b^3*c^3 - 12*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3)*x*(-a + b*x^2)*
(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^
2)/a] + 3*b*d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*x^3*(-a + b*x^2)*(c - d*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42
*a^2*c^2*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] time = 0.05, size = 5689, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.931 $\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=514

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6ac^{7/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{6ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-}}{}$$

```
[Out] (d*(3*b*c + 2*a*d)*Sqrt[e*x])/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) + (
b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*
c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
d*x^2]) + (d^(3/4)*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a*c^(7/4)
*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*S
qrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x
^2])
```

Rubi [A] time = 0.975388, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {466, 414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{6ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6ac^{7/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3b}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d)*Sqrt[e*x])/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) + (
b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*
c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
d*x^2]) + (d^(3/4)*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a*c^(7/4)
*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*S
qrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x
^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{9bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e^3\,\text{Subst}\left(\int \frac{-\frac{2(9b^2}{}}{}\right.}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+17abcd\right.}{6ac^2(bc-ad)^3}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+17abcd\right.}{6ac^2(bc-ad)^3}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+17abcd\right.}{6ac^2(bc-ad)^3}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+17abcd\right.}{6ac^2(bc-ad)^3}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+17abcd\right.}{6ac^2(bc-ad)^3}$$

Mathematica [C] time = 0.509803, size = 328, normalized size = 0.64

$$-\frac{bdx^3\left(bx^2-a\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-5x\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -(5*a*x*(3*b^3*c^2*(c - d*x^2)^2 + a^3*d^3*(-7*c + 5*d*x^2) + a*b^2*c*d^2*x
^2*(-19*c + 17*d*x^2) + a^2*b*d^2*(19*c^2 - 10*c*d*x^2 - 5*d^2*x^4)) - 5*(-
9*b^3*c^3 + 36*a*b^2*c^2*d - 17*a^2*b*c*d^2 + 5*a^3*d^3)*x*(a - b*x^2)*(c -
 d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + b*d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*x^3*(-a + b*x^2)*(c - d*x^2)*S
```

```
qrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(30*a^
2*c^2*(b*c - a*d)^3*Sqrt[e*x]*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] time = 0.05, size = 4776, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] 1/24*(-d*x^2+c)^(1/2)*b*d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*a*b^4*c^3*d^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*a*b^4*c^
3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*a^2*b^3*c^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+30*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*2^(1/2)*x^2*a*b^4*c^4*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+10*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^4*c*d^3*(a*b)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^
(1/2)*x^2*b^4*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-39*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^3*d^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)-30*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^4*c^4*d*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*b^4*c^4*(a*b
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-10*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^4*d^4*(a*b)^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^4*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(c*d)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*b^5*c^5*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a*b^4*c^5*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
```

)*(-x*d/(c*d)^(1/2))^(1/2)+20*x^5*a^3*b*d^5*(a*b)^(1/2)+12*x^5*b^4*c^3*d^2*
(a*b)^(1/2)-24*x^3*b^4*c^4*d*(a*b)^(1/2)+28*x*a^4*c*d^4*(a*b)^(1/2)-88*x^5*
a^2*b^2*c*d^4*(a*b)^(1/2)-20*x^3*a^4*d^5*(a*b)^(1/2)+12*x*b^4*c^5*(a*b)^(1/
2)+56*x^5*a*b^3*c^2*d^3*(a*b)^(1/2)+60*x^3*a^3*b*c*d^4*(a*b)^(1/2)+36*x^3*a
^2*b^2*c^2*d^3*(a*b)^(1/2)-52*x^3*a*b^3*c^3*d^2*(a*b)^(1/2)-104*x*a^3*b*c^2
d^3(a*b)^(1/2)+76*x*a^2*b^2*c^3*d^2*(a*b)^(1/2)-12*x*a*b^3*c^4*d*(a*b)^(1
/2)-34*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^3*c^3*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+28*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a*b^3*
c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-39*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)-30*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c
^3*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-39*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)-30*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^3*
d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+34*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b*c*d^3*(a*b)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+16*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*(a*b)^(1/2)*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*x^4*a*b^3*c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-44*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a^2
*b^2*c*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+39*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*a*b^3*c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*b^5*c^5
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)-44*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*a^3*b*c^2*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)+28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*a^2*b^2*c^3*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2
)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*b^4*c^3*d*(a*b)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x
^4*b^4*c^3*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+10*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a^3*b*d^4*
(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)

```
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+6*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b^4*c^3*d*(a*b)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*a^2*b^2*c^3*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+39*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^3*d*(a*b)^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a*b^3*c^
4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*a*b^3*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*b^5*c^4*d*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*b^5*c^4*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^4*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^
3*c^4*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))/(e*x)^(1/2)/(a*d-b*c)^3/(b*x^2-a)/(a*b)
^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(d*x^2-c
)^2/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.932 $\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=735

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(19a^2bcd^2-7a^3d^3-12ab^2c^2d+5b^3c^3\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(19a^2bcd^2-7a^3d^3\right.}{2a^2c^{9/4}e^3}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*(c - d*x^2)^(3/2)) + b
/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*c^
2 + 19*a*b*c*d - 7*a^2*d^2))/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[e*x]*Sqrt[c - d*
x^2]) - ((5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[c -
 d*x^2])/(2*a^2*c^3*(b*c - a*d)^3*e*Sqrt[e*x]) - (d^(1/4)*(5*b^3*c^3 - 12*a
*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^
3*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*
b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d
*x^2]) - (5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) +
(5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.84293, antiderivative size = 735, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {466, 472, 579, 583, 584, 307, 224, 221, 1200, 1199, 424, 490, 1219, 1218}

$$\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(19a^2bcd^2-7a^3d^3-12ab^2c^2d+5b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(19a^2bcd^2-7a^3d^3-12ab^2\right.}{2a^2c^{9/4}e^{3/2}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*(c - d*x^2)^(3/2)) + b
/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*c^
2 + 19*a*b*c*d - 7*a^2*d^2))/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[e*x]*Sqrt[c - d*
x^2]) - ((5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[c -
 d*x^2])/(2*a^2*c^3*(b*c - a*d)^3*e*Sqrt[e*x]) - (d^(1/4)*(5*b^3*c^3 - 12*a
*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^
3*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*
b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d
*x^2]) - (5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) +
(5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\,\mathrm{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{11bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e^3\,\mathrm{Subs}}{}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d\left(3b^2c\right.}{6ac^2(bc}$$

Mathematica [C] time = 1.28527, size = 407, normalized size = 0.55

$$x\left(3bdx^4\sqrt{1-\frac{dx^2}{c}}\left(-19a^2bcd^2+7a^3d^3+12ab^2c^2d-5b^3c^3\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-7x^2\sqrt{1-\frac{dx^2}{c}}\left(12a^2b^2c^2d^2-19a^3b\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-7*a*(15*b^4*c^3*x^2*(c - d*x^2)^2 - 12*a*b^3*c^2*(c - d*x^2)^2*(c + 3
*d*x^2) + a^4*d^3*(12*c^2 - 35*c*d*x^2 + 21*d^2*x^4) - a^3*b*d^2*(36*c^3 -
83*c^2*d*x^2 + 22*c*d^2*x^4 + 21*d^3*x^6) + a^2*b^2*c*d*(36*c^3 - 36*c^2*d*
x^2 - 59*c*d^2*x^4 + 57*d^3*x^6)))/((a - b*x^2)*(c - d*x^2)) - 7*(5*b^4*c^4
 - 20*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 - 19*a^3*b*c*d^3 + 7*a^4*d^4)*x^2*Sq
rt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*
(-5*b^3*c^3 + 12*a*b^2*c^2*d - 19*a^2*b*c*d^2 + 7*a^3*d^3)*x^4*Sqrt[1 - (d*
x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*c^3*(-(
b*c) + a*d)^3*(e*x)^(3/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.059, size = 6334, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.933 $\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=606

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(35a^2bcd^2-15a^3d^3-12ab^2c^2d+7b^3c^3\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{11/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(35a^2bcd^2-15a^3d^3\cdots\right.}{6a^2c^3e(ex)^{3/2}(bc\cdots}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*(c - d*x^2)^(3/2)) +
 b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^2*
c^2 + 7*a*b*c*d - 3*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*e*(e*x)^(3/2)*Sqrt[c -
 d*x^2]) - ((7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt
[c - d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^3*c^3
- 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(11/4)*(b*c
 - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2
)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.46119, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 472, 579, 583, 523, 224, 221, 409, 1219, 1218}

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(35a^2bcd^2-15a^3d^3-12ab^2c^2d+7b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{11/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(35a^2bcd^2-15a^3d^3-12ab^2\cdots\right.}{6a^2c^3e(ex)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*(c - d*x^2)^(3/2)) +
 b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^2*
c^2 + 7*a*b*c*d - 3*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*e*(e*x)^(3/2)*Sqrt[c -
 d*x^2]) - ((7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt
[c - d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^3*c^3
- 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(11/4)*(b*c
 - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2
)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2])
```


Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{13bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{}\right.}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} -$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} +$$

Mathematica [C] time = 1.36853, size = 427, normalized size = 0.7

$$\frac{x\left(-\frac{bdx^4\sqrt{1-\frac{dx^2}{c}}\left(35a^2bcd^2-15a^3d^3-12ab^2c^2d+7b^3c^3\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bc-ad)^3}+\frac{5x^2\sqrt{1-\frac{dx^2}{c}}\left(-12a^2b^2c^2d^2+35a^3bcd^3-15a^4d^4-44ab^3c^3d+21b^4c^4\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\right.}{(bc-ad)^3}\right.}{30a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-5*a*(7*b^4*c^3*x^2*(c - d*x^2)^2 - 4*a*b^3*c^2*(c - d*x^2)^2*(c + 3*d
*x^2) + a^4*d^3*(4*c^2 - 21*c*d*x^2 + 15*d^2*x^4) - a^3*b*d^2*(12*c^3 - 45*
c^2*d*x^2 + 14*c*d^2*x^4 + 15*d^3*x^6) + a^2*b^2*c*d*(12*c^3 - 12*c^2*d*x^2
 - 37*c*d^2*x^4 + 35*d^3*x^6)))/((-(b*c) + a*d)^3*(a - b*x^2)*(c - d*x^2))
+ (5*(21*b^4*c^4 - 44*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 - 1
5*a^4*d^4)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b
*x^2)/a])/(b*c - a*d)^3 - (b*d*(7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2
 - 15*a^3*d^3)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c
, (b*x^2)/a])/(b*c - a*d)^3))/(30*a^3*c^3*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.063, size = 5248, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.934 $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx}}{24b^2d^2}$$

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b^2
*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b^2*d^2) +
(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)*(5*b^2*c^2 +
 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(16*b^(5/2)*d^(7/2))
```

Rubi [A] time = 0.248724, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx}}{24b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b^2
*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b^2*d^2) +
(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)*(5*b^2*c^2 +
 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(16*b^(5/2)*d^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{6bd}$$

$$= -\frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\text{Subst}\left(\right.}{16b^2d^2}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}}{6bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}}{6bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}}{6bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}}{6bd}$$

Mathematica [A] time = 0.415513, size = 187, normalized size = 0.89

$$\frac{-b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(3a^2d^2-2abd\left(dx^2-2c\right)+b^2\left(-15c^2+10cdx^2-8d^2x^4\right)\right)-3\left(a^2d^2+2abcd+5b^2c^2\right)(bc-a}{48b^3d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 - 2*a*b*d*(-2*c + d*x^2
) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - 3*(b*c - a*d)^(3/2)*(5*b^2*c
^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d
]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^3*d^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.051, size = 532, normalized size = 2.6

$$\frac{1}{96\,d^3b^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(16\,x^4b^2d^2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+4\,\sqrt{bdx^4+adx^2+bcx^2+ac}x^2abd^2\sqrt{bd}-20\,\sqrt{bdx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*x^4*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^2
*(b*d)^(1/2)-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)
+3*d^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+9*ln(1/2*(2*d*x^2*b+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2
*a*b^2*d-15*b^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^
2*d^2*(b*d)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)
+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/d^3/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/b^2/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23173, size = 960, normalized size = 4.59

$$\left[-\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+\right.\right.}{192\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2
 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4
*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d
^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4), 1/96*(3*(5
*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^
4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^2*d -
 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/(b^3*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.21419, size = 304, normalized size = 1.45

$$\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5b^3cd^3+7ab^2d^4}{b^3d^5}\right)+\frac{3\left(5b^4c^2d^2+2ab^3cd^3+a^2b^2d^4\right)}{b^3d^5}\right)+\frac{3\left(5b^3c^3-3ab^2c^2d-\right.}{}}{48b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b*d) - (5*b^3*c*d^3 + 7*a*b^2*d^4)/(b^3*d^5)) + 3*(5*b^4*c^
2*d^2 + 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(b^3*d^5)) + 3*(5*b^3*c^3 - 3*a*b^2*c^
2*d - a^2*b*c*d^2 - a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3))/(b*abs(b))
```

3.935 $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b*d^2) + ((a + b*x^2)^(
3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

Rubi [A] time = 0.1272, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b*d^2) + ((a + b*x^2)^(
3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{8bd}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{\ldots}}\right.}{16bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\ldots}}\right.}{8b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,d\ldots\right.}{8b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.299721, size = 138, normalized size = 1.01

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(ad-3bc+2bdx^2\right)+(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^2d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.013, size = 339, normalized size = 2.5

$$-\frac{1}{16\,d^2b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-4\,\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}x^2bd+d^2\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*b*d+d^2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2+2*ln(1/2*(2*d*x^2*b+2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*b*d-3*b^
2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b
*c)/(b*d)^(1/2))*c^2-2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d+
6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)/d^2/b/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93929, size = 752, normalized size = 5.49

$$\left[-\frac{\left(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2}\right)\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{2}-4\left(2\,bdx^{2}+bc+ad\right)\sqrt{bx^{2}+a}\cdots}{32\,b^{2}d^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2
*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c
 + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(2*b^2*d^2*x^2 - 3*b
^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3), -1/16*((3*b^2
*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/(b^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\sqrt{a+bx^{2}}}{\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.21667, size = 207, normalized size = 1.51

$$\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(\frac{2\left(bx^2+a\right)}{bd}-\frac{3b^2cd+abd^2}{b^2d^3}\right)-\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}d^2}}{8|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b*d) - (3*b^2*c*d + a*b*d^2)/(b^2*d^3)) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)
*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
))/(sqrt(b*d)*d^2))/abs(b)
```

3.936 $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

Rubi [A] time = 0.0745375, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}\\ &= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2bd}\\ &= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2bd}\\ &= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.266458, size = 116, normalized size = 1.35

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)-(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{2bd^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2) - (b*c - a*d)^(3/2)*Sqrt[(b*(c + d*x
^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b*
d^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.01, size = 198, normalized size = 2.3

$$\frac{1}{4d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(a\ln\left(\frac{1}{2}\left(2dx^2b+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d-b\ln\left(\frac{1}{2}\left(2dx^2b+2\sqrt{b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(a*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d-b*ln(1/2*(2*d*x^2*b
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/d/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91502, size = 590, normalized size = 6.86

$$\left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}bd-(bc-ad)\sqrt{bd}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bdx^2+bc+ad\right)\sqrt{}\right.}{8bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b*d^2)
, 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b
^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.59166, size = 143, normalized size = 1.66

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}}{bd}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*((b*c - a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*
sqrt(b*x^2 + a)/(b*d))/abs(b)
```

3.937 $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2
])])/Sqrt[d]
```

Rubi [A] time = 0.0953195, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2
])])/Sqrt[d]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= a\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)+\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.135443, size = 129, normalized size = 1.4

$$
\frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{c+dx^2}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c]
```

Maple [B] time = 0.026, size = 177, normalized size = 1.9

$$
\frac{1}{2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(b\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{ac}-a\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(b*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)-a*ln((a*d
*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*
(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9764, size = 1708, normalized size = 18.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b/d)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^
4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), -1/2*sqrt(-b/d)*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2
*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
 (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), 1/2*
sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 1/4*sqrt(b/d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x
^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b
/d)), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 1/2*
sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{x\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.20248, size = 208, normalized size = 2.26

$$-\frac{b^2\left(\frac{2\sqrt{bd}a\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b}+\frac{\sqrt{bd}\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{bd}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*(2*sqrt(b*d)*a*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(
-a*b*c*d)*b) + sqrt(b*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2)/(b*d))/abs(b)
```

3.938 $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - ((b*c - a*d)*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

Rubi [A] time = 0.0719323, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 94, 93, 208}

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - ((b*c - a*d)*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2c}$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}$$

Mathematica [A] time = 0.0609534, size = 89, normalized size = 1.

$$\frac{1}{2}\left(-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}c^{3/2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2)))/2
```

Maple [B] time = 0.023, size = 207, normalized size = 2.3

$$\frac{1}{4\,cx^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2\,ac\right)\right)x^2ad - \ln\left(\frac{1}{x^2}\left(adx^2+bcx^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a*d-ln((a*d*x^2+b*c*x^2+2
*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*b*c-2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)/(a*c)^(1/2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 2.43489, size = 629, normalized size = 7.07

$$\left[-\frac{\sqrt{ac}(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)+4\,\sqrt{bx^2+a}\sqrt{dx^2+c}ac}{8\,ac^2x^2},\ -\frac{\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*a*c)/(a*c^2*x^2), 1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c + a*d
)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^
2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)/
(a*c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.939 $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=143

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - ((a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Rubi [A] time = 0.116061, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 96, 94, 93, 208}

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - ((a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{\left(\frac{bc}{2}+\frac{3ad}{2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac}\\ &= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{((bc-ad)(bc+3ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\right.}{16ac^2}\\ &= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{((bc-ad)(bc+3ad))\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x,\right.}{8ac^2}\\ &= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}+\frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0740333, size = 125, normalized size = 0.87

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-bcx^2\right)}{8ac^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2))/(8*a*c^2*x
^4) + ((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/
(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Maple [B] time = 0.026, size = 355, normalized size = 2.5

$$-\frac{1}{16\,ac^2x^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\,\ln\left(\frac{adx^2+bcx^2+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2\,ac}{x^2}\right)x^4a^2d^2-2\,\ln\left(\frac{adx^2+bcx^2+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2*(3*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*d^2-2*ln((a*d
```

```
*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*
x^4*a*b*c*d-ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+2*a*c)/x^2)*x^4*b^2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^
2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/(a*c)^(1/2)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.4225, size = 795, normalized size = 5.56

$$\left[-\frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\sqrt{ac}x^4 \log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right) + 4\left(2\,a^2c^2\right.}{32\,a^2c^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*(2*a^2
*c^2 + (a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3
*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*(
(b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*
d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*(2*a^2*c^2 + (a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{x^5\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.940 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=343

$$\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{15b^2d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2\cdots\right.}{15\cdots}$$

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*d^2*Sqrt[c
+ d*x^2]) - ((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b*d^2) +
(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c
*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.305972, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {478, 582, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{15b^2d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}\cdots}{15\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*d^2*Sqrt[c
+ d*x^2]) - ((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b*d^2) +
(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c
*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
```

```
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} - \frac{\int \frac{x^2\left(3ac+(4bc-ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5d}$$

$$= -\frac{(4bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{ac(4bc-ad)+\left(8b^2c^2-3abcd-2a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^2}$$

$$= -\frac{(4bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} + \frac{(ac(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^2} + \frac{\left(8b^2c^{\cdots}\right.}{\cdots}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}} - \frac{(4bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} + \frac{c^{3/2}\left(\cdots\right.}{\cdots}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}} - \frac{(4bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} - \frac{\sqrt{c}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.425054, size = 246, normalized size = 0.72

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+7abcd-8b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2+3abcd-\right.}{15bd^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8
*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d
 + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A] time = 0.025, size = 526, normalized size = 1.5

$$\frac{1}{15\,d^3\left(bdx^4+adx^2+bcx^2+ac\right)b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+4\sqrt{-\frac{b}{a}}x^5abd^3-\sqrt{-\frac{b}{a}}x^5b^2cd^2+\sqrt{-\frac{b}{a}}x^3a^2d^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+4*(-b/a)^(
1/2)*x^5*a*b*d^3-(-b/a)^(1/2)*x^5*b^2*c*d^2+(-b/a)^(1/2)*x^3*a^2*d^3-4*(-b/
a)^(1/2)*x^3*b^2*c^2*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3
-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+(-b/a)^(1/2)*x*a^2*
c*d^2-4*(-b/a)^(1/2)*x*a*b*c^2*d)/d^3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+a}\,x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

3.941 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=259

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

```
[Out] -((2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.163547, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {478, 531, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{x\sqrt{a}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -((2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{\int \frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}$$

$$= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{(ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} - \frac{(2bc-ad)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}$$

$$= -\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(c(2bc-ad)\ldots}{3\ldots}$$

$$= -\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{\sqrt{c}(2bc-ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/\ldots}}{\ldots}$$

Mathematica [C] time = 0.253636, size = 199, normalized size = 0.77

$$\frac{2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-2b\ldots}{3d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.016, size = 335, normalized size = 1.3

$$-\frac{1}{\left(3\,bd{x}^{4}+3\,ad{x}^{2}+3\,bc{x}^{2}+3\,ac\right){d}^{2}}\sqrt{b{x}^{2}+a}\sqrt{d{x}^{2}+c}\left(-\sqrt{-{\frac{b}{a}}}{x}^{5}b{d}^{2}-\sqrt{-{\frac{b}{a}}}{x}^{3}a{d}^{2}-\sqrt{-{\frac{b}{a}}}{x}^{3}bcd+2\,ac\sqrt{{\frac{b{x}^{2}+a}{a}}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^5*b*d^2-(-b/a)^(1/2)*
x^3*a*d^2-(-b/a)^(1/2)*x^3*b*c*d+2*a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-2*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d
+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*b*c^2-(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\,x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,x^2}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}x^2}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

3.942 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=232

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rubi [A] time = 0.148864, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+b\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx+\frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c}\\
&= \frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-d\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx\\
&= \frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\right.\right.}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.265678, size = 111, normalized size = 0.48

$$\frac{\frac{bcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}-\left(a+bx^2\right)\left(c+dx^2\right)}{cx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a + b*x^2)*(c + d*x^2)) + (b*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 168, normalized size = 0.7

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)cx}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-\sqrt{-\frac{b}{a}}x^4bd+bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}x\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^4*b*d+b*c*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-(-b/a)^
(1/2)*x^2*a*d-(-b/a)^(1/2)*x^2*b*c-(-b/a)^(1/2)*a*c)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)/c/(-b/a)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{dx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

3.943 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$-\frac{b\sqrt{d}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3ac^2x}+\frac{dx\sqrt{a+bx^2}(bc-2ad)}{3ac^2\sqrt{c+dx^2}}-\frac{\sqrt{d}\sqrt{a+b\ldots}}{}$$

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*a*c^2*Sqrt[c + d*x^2]) - (Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.262819, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {475, 583, 531, 418, 492, 411}

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3ac^2x}+\frac{dx\sqrt{a+bx^2}(bc-2ad)}{3ac^2\sqrt{c+dx^2}}-\frac{\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\ldots}{3c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```


```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*a*c^2*Sqrt[c + d*x^2]) - (Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} + \frac{\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x} - \frac{\int \frac{abcd-bd(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac^2}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x} - \frac{(bd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c} + \frac{(bd(bc-2ad))\int \frac{}{\sqrt{a}}}{3ac^2}$$

$$= \frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x} - \frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-}\right.}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= \frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x} - \frac{\sqrt{d}(bc-2ad)\sqrt{a+bx}}{3ac^{3/2}\sqrt{\frac{c(a}{a(c}}}$$

Mathematica [C] time = 0.75721, size = 228, normalized size = 0.74

$$\frac{icx^3\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(bc-ad)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-\frac{(a+bx^2)(c+dx^2)(ac-2adx^2+bcx^2)}{a}+icx^3\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+}}{3c^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((a + b*x^2)*(c + d*x^2)*(a*c + b*c*x^2 - 2*a*d*x^2))/a) + I*Sqrt[b/a]*c
*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*Sqrt[b/a]*c*(b*c - a*d)*x^3*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.026, size = 417, normalized size = 1.4

$$-\frac{1}{\left(3\,bd x^4+3\,ad x^2+3\,bc x^2+3\,ac\right)c^2x^3a}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-2\sqrt{-\frac{b}{a}}x^6abd^2+\sqrt{-\frac{b}{a}}x^6b^2cd+2\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{I}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-2*(-b/a)^(1/2)*x^6*a*b*d^2+(-b/a)^(1
/2)*x^6*b^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2-b*d*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c+
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*x^3*b^2*c^2-2*(-b/a)^(1/2)*x^4*a^2*d^2+(-b/a)^(1/2)*x^4*b^2*c^2-(-b/
a)^(1/2)*x^2*a^2*c*d+2*(-b/a)^(1/2)*x^2*a*b*c^2+(-b/a)^(1/2)*a^2*c^2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(1/2)/x^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{dx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

3.944 $\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=276

$$\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(3a^2d^2+10abcd+35b^2c^2)}{192b^2d^3}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)\,(3a^2d^2+10abcd+35b^2c^2)}{128b^2d^4}+\frac{(bc-ad}{}$$

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(128*b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*b*c + 3*a*d)*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(8
*b*d) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```

Rubi [A] time = 0.32544, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(3a^2d^2+10abcd+35b^2c^2)}{192b^2d^3}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)\,(3a^2d^2+10abcd+35b^2c^2)}{128b^2d^4}+\frac{(bc-ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```


```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(128*b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*b*c + 3*a*d)*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(8
*b*d) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8bd}$$

$$= -\frac{(7bc+3ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} + \frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\text{Sub}}{96b^2d^2}$$

$$= \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192b^2d^3} - \frac{(7bc+3ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} + \frac{x^2\left(a+b\right.}{}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)}{192b^2d^3}$$

Mathematica [A] time = 0.574768, size = 231, normalized size = 0.84

$$\frac{3(bc-ad)^{5/2}\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(3a^2bd^2\left(5c-2dx^2\right)+\right.}{384b^3d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^2) + a*b^2*d*(-145*c^2 + 92*c*d*x^2 - 72*d^2*x^4) + b^3*(105*c^3 - 70*c
^2*d*x^2 + 56*c*d^2*x^4 - 48*d^3*x^6))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^2 +
 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(384*b^3*d^(9/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.03, size = 770, normalized size = 2.8

$$\frac{1}{768\,b^2d^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(96\,x^6b^3d^3\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}+144\,x^4ab^2d^3\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(96*x^6*b^3*d^3*(b*d)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)+144*x^4*a*b^2*d^3*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)-112*x^4*b^3*c*d^2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)-1
84*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*c*b^2*d^2*(b*d)^(1/2)+140*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c^2*b^3*d*(b*d)^(1/2)+9*d^4*ln(1/2*(2*d
*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/
2))*a^4+12*a^3*c*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*d^3+54*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^2*b^2*d^2-180
*a*c^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*b^3*d+105*b^4*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-18*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*a^3*d^3*(b*d)^(1/2)-30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*a^2*c*b*d^2*(b*d)^(1/2)+290*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c^2*b^
2*d*(b*d)^(1/2)-210*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^3*b^3*(b*d)^(1/2)
)/b^2/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27163, size = 1265, normalized size = 4.58

$$\left[\frac{3\left(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(b*d)) + 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*b^3*c^2
*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^4 +
 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c))/(b^3*d^5), -1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*
b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4
)*x^4 + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c))/(b^3*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.23056, size = 410, normalized size = 1.49

$$\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(4\left(bx^2+a\right)\left(\frac{6\left(bx^2+a\right)}{bd}-\frac{7\,b^3cd^5+9\,ab^2d^6}{b^3d^7}\right)+\frac{35\,b^4c^2d^4+10\,ab^3cd^5+3\,a^2b^2d^6}{b^3d^7}\right)-\frac{3\left(35\,b^5\right.}{}\right.}{384\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b*d) - (7*b^3*c*d^5 + 9*a*b^2*d^6)/(b^3*d^7
)) + (35*b^4*c^2*d^4 + 10*a*b^3*c*d^5 + 3*a^2*b^2*d^6)/(b^3*d^7)) - 3*(35*b
^5*c^3*d^3 - 25*a*b^4*c^2*d^4 - 7*a^2*b^3*c*d^5 - 3*a^3*b^2*d^6)/(b^3*d^7))
 - 3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*
```

```
a^4*d^4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d)))/(sqrt(b*d)*d^4))/(b*abs(b))
```

3.945 $\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(ad+5bc)}{24bd^2}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)(ad+5bc)}{16bd^3}+$$

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b*d^3) - ((
5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d^2) + ((a + b*x^2)^(
5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rubi [A] time = 0.171749, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(ad+5bc)}{24bd^2}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)(ad+5bc)}{16bd^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b*d^3) - ((
5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d^2) + ((a + b*x^2)^(
5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd} - \frac{(5bc+ad)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{12bd}$$

$$= -\frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd} + \frac{((bc-ad)(5bc+ad))\operatorname{Subst}\left(\int\right.}{16bd^2}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+}}{6bd}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+}}{6bd}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+}}{6bd}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+}}{6bd}$$

Mathematica [A] time = 0.422248, size = 173, normalized size = 0.93

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(3a^2d^2+2abd\left(7dx^2-11c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}{48b^2d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - 3*(b*c - a*d)^(5/2)*(5*b*c +
a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sq
```

```
rt[b*c - a*d]])/(48*b^2*d^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.017, size = 532, normalized size = 2.8

$$-\frac{1}{96\,bd^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-16\,x^4b^2d^2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}-28\,\sqrt{bdx^4+adx^2+bcx^2+ac}x^2abd^2\sqrt{bd}+20\,\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-16*x^4*b^2*d^2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2)-28*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*
d^2*(b*d)^(1/2)+20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1
/2)+3*d^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2)+a*d+b*c)/(b*d)^(1/2))*a^3+9*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2-27*ln(1/2*(2*d*x^
2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*c^2*a*b^2*d+15*b^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*a^2*d^2*(b*d)^(1/2)+44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^
(1/2)-30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/b/d^3/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06232, size = 972, normalized size = 5.2

$$\left[\frac{3\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,bdx^2+bc+\right.\right.}{192\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) +
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c
*d^2 - 7*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4), 1/96*(
3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b*d)*arctan(1
```

```
/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*
d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c
^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.25854, size = 293, normalized size = 1.57

$$\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5b^2cd^3+abd^4}{b^2d^5}\right)+\frac{3\left(5b^3c^2d^2-4ab^2cd^3-a^2bd^4\right)}{b^2d^5}\right)+\frac{3\left(5b^3c^3-9ab^2c^2d+3a^2\right.}{}}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b*d) - (5*b^2*c*d^3 + a*b*d^4)/(b^2*d^5)) + 3*(5*b^3*c^2*d^
2 - 4*a*b^2*c*d^3 - a^2*b*d^4)/(b^2*d^5)) + 3*(5*b^3*c^3 - 9*a*b^2*c^2*d +
3*a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3))/abs(b)
```

3.946 $\int \frac{x\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a + b*x^2)^(3/
2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^(5/2))
```

Rubi [A] time = 0.104095, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a + b*x^2)^(3/
2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^(5/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{8d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,d\right.}{16d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx\right.}{8bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\right.}{8bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}$$

Mathematica [A] time = 0.361895, size = 131, normalized size = 1.05

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(5ad-3bc+2bdx^2\right)+\frac{3(bc-ad)^{5/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{b}}{8d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c + 5*a*d + 2*b*d*x^2) + (3*(b*c
 - a*d)^(5/2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b
*x^2])/Sqrt[b*c - a*d]])/b)/(8*d^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.013, size = 337, normalized size = 2.7

$$\frac{1}{16\,d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(4\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}x^2bd+3\,d^2\ln\left(1/2\,\frac{2\,dx^2b+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{ba}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*x^2*b*d+3*d^2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2-6*ln(1/2*(2*d*x^2*b+2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*b*d+3*b^
2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b
*c)/(b*d)^(1/2))*c^2+10*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d
-6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04739, size = 751, normalized size = 6.01

$$\left[\frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,bdx^2 + bc + ad\right)\sqrt{bx^2 + a}\sqrt{\right.}{32\,bd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*b^
2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3), -1/16*(3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + bx^2\right)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.23552, size = 201, normalized size = 1.61

$$\frac{\left(\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(\frac{2\left(bx^2+a\right)}{bd}-\frac{3\left(bcd-ad^2\right)}{bd^3}\right)-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}d^2}\right)b}{8\left|b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b*d) - 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(a
bs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sq
rt(b*d)*d^2))*b/abs(b)
```

3.947 $\int \frac{\left(a+bx^2\right)^{3/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=133

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rubi [A] time = 0.139794, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {446, 102, 157, 63, 217, 206, 93, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x\sqrt{c+dx^2}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$=\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}+\frac{\text{Subst}\left(\int \frac{a^2d-\frac{1}{2}b(bc-3ad)x}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{2d}$$

$$=\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)-\frac{(b(bc-3ad))\,\text{Subst}\left(\int \frac{}{\sqrt{a+b}}\right.}{4d}$$

$$=\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}+a^2\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)-\frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{2d}$$

$$=\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2d}$$

$$=\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

Mathematica [A] time = 0.749248, size = 195, normalized size = 1.47

$$\frac{\sqrt{d}\left(b\sqrt{a+bx^2}\left(c+dx^2\right)-\frac{2a^{3/2}d\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}\right)-\frac{\left(3a^2d^2-4abcd+b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}}{2d^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*Arc
Sinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]) + Sqrt[d]
*(b*Sqrt[a + b*x^2]*(c + d*x^2) - (2*a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c]))/(2*d^(3/2)*Sqrt
[c + d*x^2])
```

Maple [B] time = 0.014, size = 287, normalized size = 2.2

$$\frac{1}{4d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)\sqrt{ac}abd-\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d-ln(
1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(
b*d)^(1/2))*(a*c)^(1/2)*b^2*c-2*(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*d+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*b)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.18854, size = 2017, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/4*(a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b
```

```
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
(b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (b*c
 - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 2*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/8*(4*a*d*sqrt(-a/c)*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a
^2*c + (a*b*c + a^2*d)*x^2)) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^2*x^4 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2
+ b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*b)/d, 1/4*(2*a*d*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b
^2*c + a*b*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.3083, size = 284, normalized size = 2.14

$$-\frac{\frac{4\sqrt{bd}a^2\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b}-\frac{2\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}}{bd}-\frac{\left(\sqrt{bd}bc-3\sqrt{bd}ad\right)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{bd^2}}{4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*sqrt(b*d)*a^2*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a
*b*c*d)*b) - 2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)/(b*d)
- (sqrt(b*d)*b*c - 3*sqrt(b*d)*a*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b
^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d^2))*b^2/abs(b)
```

3.948 $\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - (Sqrt[a]*(3*b*c - a*d)*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) + (b
^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d
]
```

Rubi [A] time = 0.140248, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - (Sqrt[a]*(3*b*c - a*d)*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) + (b
^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)-b^2cx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right) + \frac{(a(3bc-ad))\,\text{Subst}\left(\int \frac{}{x\sqrt{a\cdots}}\right.}{4c}$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + b\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^2}\right) + \frac{(a(3bc-ad))\,\text{Subst}\left(\int \frac{}{-a\cdots}\right.}{2c}$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} + b\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Mathematica [A] time = 0.940658, size = 172, normalized size = 1.26

$$\frac{\sqrt{a}(ad-3bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}+\frac{(bc-ad)^{3/2}\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) + ((b*c - a*d)^(3/2)*((b*(c
+ d*x^2))/(b*c - a*d))^(3/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a
*d]])/(Sqrt[d]*(c + d*x^2)^(3/2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2))
```

Maple [B] time = 0.016, size = 298, normalized size = 2.2

$$\frac{1}{4\,cx^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\,\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^2b^2c\sqrt{ac}+\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^2*c*(a*c)^(1/
2)+ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*
a*c)/x^2)*x^2*a^2*d*(b*d)^(1/2)-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a*b*c*(b*d)^(1/2)-2*a*(b*d)^(1
/2)*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.52722, size = 2095, normalized size = 15.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d)*x^2*sqrt(a/c)*log(((b^2*c^2
+ 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2*a
*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4)
 - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/8*(4*b*c*x^2*sqrt(-b/d)
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b
/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + (3*b*c - a*d)*x^2*sqrt(a/c
)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c
*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(a/c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), 1/4*(b*c*x
^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d
 + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*sqrt(b/d)) + (3*b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/4*
(2*b*c*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (3*b*
c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 2
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.96656, size = 663, normalized size = 4.88

$$-\frac{b^3\left(\frac{\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{\sqrt{bd}}+\frac{\left(3\sqrt{bd}abc-\sqrt{bd}a^2d\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b^2c}+\frac{2\left(\sqrt{bd}ab^3c^2 \cdots\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2 \cdots\right.}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*(log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2)/sqrt(b*d) + (3*sqrt(b*d)*a*b*c - sqrt(b*d)*a^2*d)*arctan(-1/2*(b^
2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b^2*c) + 2*(sqrt(b*d)*a*b^3*c
^2 - 2*sqrt(b*d)*a^2*b^2*c*d + sqrt(b*d)*a^3*b*d^2 - sqrt(b*d)*(sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*c - sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*
d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - s
```

```
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d
) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*b*c))/abs(b)
```

3.949 $\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8c^2x^2}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Rubi [A] time = 0.106027, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 94, 93, 208}

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8c^2x^2}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{8c}$$
$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\right.}{16c^2}$$
$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right.}{8c^2}$$
$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}$$

Mathematica [A] time = 0.0707517, size = 110, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-5bcx^2\right)}{8c^2x^4}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 5*b*c*x^2 + 3*a*d*x^2))/(8*c^2*x
^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Maple [B] time = 0.014, size = 352, normalized size = 2.7

$$-\frac{1}{16\,c^2x^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\,\ln\left(\frac{adx^2+bcx^2+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2\,ac}{x^2}\right)x^4a^2d^2-6\,\ln\left(\frac{adx^2+bcx^2+2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1
/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*d^2-6*ln((a*d*x
^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^
4*a*b*c*d+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+2*a*c)/x^2)*x^4*b^2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^
2*(a*c)^(1/2)+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/(a*c)^(1/2)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.64661, size = 792, normalized size = 6.05

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ac}x^4\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)-4\left(2\,a^2c^2+\right.}{32\,ac^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*
d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^2*
c^2 + (5*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*
x^4), 1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*
x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 + (5*a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.950 $\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=429

$$-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}-$$

```
[Out] (-2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x*Sqrt[a + b*x^2])/(35*
b^2*d^3*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 11*a*b*c*d + a^2*d^2)*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(35*b*d^3) - (2*(3*b*c - 4*a*d)*x^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(35*d^2) + (b*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) +
 (2*Sqrt[c]*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(8*b^2*c^2
 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(35*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.518254, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {477, 582, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}-\frac{2x\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)}{35b^2d^3\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11\right.}{35bd^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```


```
[Out] (-2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x*Sqrt[a + b*x^2])/(35*
b^2*d^3*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 11*a*b*c*d + a^2*d^2)*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(35*b*d^3) - (2*(3*b*c - 4*a*d)*x^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(35*d^2) + (b*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) +
 (2*Sqrt[c]*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(8*b^2*c^2
 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(35*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^4\left(-a(5bc-7ad)-2b(3bc-4ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{7d}$$

$$= -\frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d} - \frac{\int \frac{x^2\left(-6abc(3bc-4ad)-3b\left(8b^2c^2-11abcd+a^2\right.\right.}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{35bd^2}$$

$$= \frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3} - \frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^5\sqrt{a+bx^2}}{7d}$$

$$= \frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3} - \frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^5\sqrt{a+bx^2}}{7d}$$

$$= -\frac{2(2bc-ad)\left(4b^2c^2-4abcd-a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}} + \frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3} -$$

$$= -\frac{2(2bc-ad)\left(4b^2c^2-4abcd-a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}} + \frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3} -$$

Mathematica [C] time = 0.607434, size = 305, normalized size = 0.71

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(15a^2bcd^2+a^3d^3-32ab^2c^2d+16b^3c^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right.}{35bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(-11*c + 8*d*x^2) +
 b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(8*b^3*c^3 - 12*a*b^2*c^2*d
 + 2*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(16*b^3*c^3 - 32*a*b^2*c^2*d
+ 15*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^4*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 782, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*x^9*b^3*d^4+13*(-b/a)^
(1/2)*x^7*a*b^2*d^4-(-b/a)^(1/2)*x^7*b^3*c*d^3+9*(-b/a)^(1/2)*x^5*a^2*b*d^4
-4*(-b/a)^(1/2)*x^5*a*b^2*c*d^3+2*(-b/a)^(1/2)*x^5*b^3*c^2*d^2+(-b/a)^(1/2)
*x^3*a^3*d^4-2*(-b/a)^(1/2)*x^3*a^2*b*c*d^3-9*(-b/a)^(1/2)*x^3*a*b^2*c^2*d^
2+8*(-b/a)^(1/2)*x^3*b^3*c^3*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
```

```
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+15*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2-32*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a*b^2*c^3*d+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3-4*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*
d^2+24*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*a*b^2*c^3*d-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1/2)*x*a^3*c*d^3-11*(-b
/a)^(1/2)*x*a^2*b*c^2*d^2+8*(-b/a)^(1/2)*x*a*b^2*c^3*d)/b/d^4/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx^6+ax^4)\sqrt{bx^2+a}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6 + a*x^4)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

3.951 $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=335

$$\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((13*a*c - (8*b*c^2)/d - (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*d*Sqrt[c + d*
x^2]) - (2*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) + (b
*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 13*a*b*
c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) + (2*c^(3/2)*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.340289, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {477, 582, 531, 418, 492, 411}

$$-\frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-\frac{3a^2d}{b}+13ac-\frac{8bc^2}{d}\right)}{15d\sqrt{c+dx^2}}+\frac{2c^{3/2}\sqrt{a+bx^2}(}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```


```
[Out] -((13*a*c - (8*b*c^2)/d - (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*d*Sqrt[c + d*
x^2]) - (2*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) + (b
*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 13*a*b*
c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) + (2*c^(3/2)*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
```

```
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}} \, dx = \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{x^2\left(-a(3bc-5ad)-2b(2bc-3ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}} \, dx}{5d}$$

$$= -\frac{2(2bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} - \frac{\int \frac{-2abc(2bc-3ad)-b\left(8b^2c^2-13abcd+3a^2d^2\right)x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15bd^2}$$

$$= -\frac{2(2bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} + \frac{(2ac(2bc-3ad)) \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}} \, dx}{15d^2}$$

$$= \frac{\left(8b^2c^2-13abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}} - \frac{2(2bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$= \frac{\left(8b^2c^2-13abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}} - \frac{2(2bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

Mathematica [C] time = 0.424786, size = 245, normalized size = 0.73

$$\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(9a^2d^2-17abcd+8b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-13abcd\right.}{15d^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 6*a*d + 3*b*d*x^2) - I*c*(
8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 - 17*a*b*c
*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

Maple [A] time = 0.018, size = 544, normalized size = 1.6

$$-\frac{1}{15\,d^3\left(bdx^4+adx^2+bcx^2+ac\right)}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-3\sqrt{-\frac{b}{a}}x^7b^2d^3-9\sqrt{-\frac{b}{a}}x^5abd^3+\sqrt{-\frac{b}{a}}x^5b^2cd^2-6\sqrt{-\frac{b}{a}}x^3a^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*(-b/a)^(1/2)*x^7*b^2*d^3-9*(-b/a)
^(1/2)*x^5*a*b*d^3+(-b/a)^(1/2)*x^5*b^2*c*d^2-6*(-b/a)^(1/2)*x^3*a^2*d^3-5*
(-b/a)^(1/2)*x^3*a*b*c*d^2+4*(-b/a)^(1/2)*x^3*b^2*c^2*d+9*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-
17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+13*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d
-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*b^2*c^3-6*(-b/a)^(1/2)*x*a^2*c*d^2+4*(-b/a)^(1/2)*x*a*b*c^2*d)/d^
3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{3}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + ax^2)\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x^2)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + bx^2\right)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}} x^2}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

3.952 $\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{2b\sqrt{c}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{x\sqrt{a+bx^2}(ad+bc)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\ldots\right.\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\ldots}$$

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (a*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(c*x) - ((b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.160478, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {474, 531, 418, 492, 411}

$$-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{x\sqrt{a+bx^2}(ad+bc)}{c\sqrt{c+dx^2}}+\frac{2b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\ldots\right.\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+\ldots}{a(c+\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```


```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (a*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(c*x) - ((b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx &= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{\int \frac{2abc+b(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c}\\
&= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+(2ab)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx+\frac{(b(bc+ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c}\\
&= \frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{2b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}+(-bc-ad)\int\\
&= \frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}-\frac{(bc+ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}+\frac{2b\sqrt{c}\sqrt{a}}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.271095, size = 206, normalized size = 0.84

$$
\frac{-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ad\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad\cdots}{cdx\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)) - I*b*c*(b*c + a*d)*x*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*b*c*(-(b*c) + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Maple [A] time = 0.018, size = 352, normalized size = 1.4

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)cdx}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-\sqrt{-\frac{b}{a}}x^4abd^2+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)xab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^4*a*b*d^2+((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*a*b*c*d
-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*x*b^2*c^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x*a*b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*b^2*c^2-(-b/a)^(1/2)*x^2*a^2*d^
2-(-b/a)^(1/2)*x^2*a*b*c*d-(-b/a)^(1/2)*a^2*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)/c/(-b/a)^(1/2)/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{dx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(d*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx^2 + c}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.953 $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=311

$$\frac{b\sqrt{a+bx^2}(3bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{3c^2x}+\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3c^2\sqrt{c+dx^2}}-\frac{2\sqrt{}}{}$$

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x^2]) - (a*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - (2*(2*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/(3*c^2*x) - (2*Sqrt[d]*(2*b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(3*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.292151, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {474, 583, 531, 418, 492, 411}

$$-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{3c^2x}+\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3c^2\sqrt{c+dx^2}}-\frac{2\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```


```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x^2]) - (a*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - (2*(2*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/(3*c^2*x) - (2*Sqrt[d]*(2*b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(3*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{2a(2bc-ad)+b(3bc-ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c}$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{2(2bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{\int \frac{-abc(3bc-ad)-2abd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac^2}$$

$$= -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{2(2bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}+\frac{(2bd(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c^2}+\frac{(b(3b}{}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{2(2bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}+\frac{b(3bc-ad)\sqrt{a+}}{3a\sqrt{c}\sqrt{d}}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{2(2bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{2\sqrt{d}(2bc-ad)\sqrt{a}}{3c^{3/2}}$$

Mathematica [C] time = 0.353747, size = 227, normalized size = 0.73

$$\frac{-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(-ac+2adx^2-4bcx^2\right)+2ibc}{3c^2x^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 4*b*c*x^2 + 2*a*d*x^2) + (2*I)
*b*c*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
/(3*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.018, size = 433, normalized size = 1.4

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)c^2x^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\sqrt{-\frac{b}{a}}x^6abd^2-4\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*x^6*a*b*d^2-4*(-b/a)^(1
/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2-2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d+
4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*x^3*b^2*c^2+2*(-b/a)^(1/2)*x^4*a^2*d^2-3*(-b/a)^(1/2)*x^4*a*b*c*d-
4*(-b/a)^(1/2)*x^4*b^2*c^2+(-b/a)^(1/2)*x^2*a^2*c*d-5*(-b/a)^(1/2)*x^2*a*b*
c^2-(-b/a)^(1/2)*a^2*c^2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(1/2)/x^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{dx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(d*x^6 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**4*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.954 $\int \frac{x^5\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\sqrt{a+}}{}$$

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^
2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^2*d^4) + ((63*b^2*c^2 + 14*a*b
*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*
b*c + a*d)*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^
2)^(7/2)*Sqrt[c + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*
d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]
)/(256*b^(5/2)*d^(11/2))
```

Rubi [A] time = 0.417532, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^
2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^2*d^4) + ((63*b^2*c^2 + 14*a*b
*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*
b*c + a*d)*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^
2)^(7/2)*Sqrt[c + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*
d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]
)/(256*b^(5/2)*d^(11/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{10bd}$$

$$= -\frac{3(3bc+ad)\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} + \frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right) \text{S}}{160b}$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{480b^2d^3} - \frac{3(3bc+ad)\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} + \frac{x^2\left(a\right.}{}$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{384b^2d^4} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a\right.}{480b^2d^3}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d\right.}{384b^2d^4}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d\right.}{384b^2d^4}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d\right.}{384b^2d^4}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d\right.}{384b^2d^4}$$

Mathematica [A] time = 1.15535, size = 271, normalized size = 0.8

$$\frac{\sqrt{c+dx^2}\left(\frac{5(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{16d^3\left(a+bx^2\right)^3}{15(bc-ad)^3}-\frac{4d^2\left(a+bx^2\right)^2}{3(bc-ad)^2}+\frac{2d\left(a+bx^2\right)}{bc-ad}-\frac{2\sqrt{d}\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}\right)}{4bd^5}-\frac{24\left(a+bx^2\right)^4(ad+3bc)}{bd}+64x^2\left(\right.\right.}{640bd\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*((-24*(3*b*c + a*d)*(a + b*x^2)^4)/(b*d) + 64*x^2*(a + b*x
^2)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((2*d*(a + b
*x^2))/(b*c - a*d) - (4*d^2*(a + b*x^2)^2)/(3*(b*c - a*d)^2) + (16*d^3*(a +
 b*x^2)^3)/(15*(b*c - a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(
b*c - a*d)]))))/(4*b*d^5)))/(640*b*d*Sqrt[a + b*x^2])
```

Maple [B] time = 0.031, size = 1054, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/7680*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(768*x^8*b^4*d^4*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)+2016*x^6*a*b^3*d^4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)-864*x^6*b^4*c*d^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(b*d)^(1/2)+1488*x^4*a^2*b^2*d^4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)-2368*x^4*a*b^3*c*d^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
1008*x^4*b^4*c^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+60*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^3*b*d^4*(b*d)^(1/2)-1924*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*c*b^2*d^3*(b*d)^(1/2)+2996*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*x^2*a*c^2*b^3*d^2*(b*d)^(1/2)-1260*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^2*c^3*b^4*d*(b*d)^(1/2)+45*d^5*ln(1/2*(2*d*x^2*b+2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^5+75*ln(
1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(
b*d)^(1/2))*a^4*c*b*d^4+450*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*c^2*b^2*d^3-2250*ln(1/2*(2*d
*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/
2))*a^2*c^3*b^3*d^2+2625*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4*a*b^4*d-945*b^5*ln(1/2*(2*d*x^2
*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*
c^5-90*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^4*d^4*(b*d)^(1/2)-180*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^3*c*b*d^3*(b*d)^(1/2)+3128*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*a^2*c^2*b^2*d^2*(b*d)^(1/2)-4620*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*a*c^3*b^3*d*(b*d)^(1/2)+1890*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*c^4*b^4*(b*d)^(1/2))/b^2/d^5/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73585, size = 1638, normalized size = 4.82

$$\left[-\frac{15\left(63\,b^5c^5 - 175\,ab^4c^4d + 150\,a^2b^3c^3d^2 - 30\,a^3b^2c^2d^3 - 5\,a^4bcd^4 - 3\,a^5d^5\right)\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15360*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(384*b^5*d^5*x^8 + 94
```

```
5*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4
- 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 -
148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*
d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(b^3*d^6), 1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*
d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^
2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(384*b^5*d^5*x^8 + 945*b^5*
c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a
^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 - 148*a*
b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 +
481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
b^3*d^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.32319, size = 536, normalized size = 1.58

$$\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(4\left(bx^2+a\right)\left(6\left(bx^2+a\right)\left(\frac{8\left(bx^2+a\right)}{bd}-\frac{9\,b^3cd^7+11\,ab^2d^8}{b^3d^9}\right)+\frac{63\,b^4c^2d^6+14\,ab^3cd^7+}{b^3d^9}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a
)*(4*(b*x^2 + a)*(6*(b*x^2 + a)*(8*(b*x^2 + a)/(b*d) - (9*b^3*c*d^7 + 11*a*
b^2*d^8)/(b^3*d^9)) + (63*b^4*c^2*d^6 + 14*a*b^3*c*d^7 + 3*a^2*b^2*d^8)/(b^
3*d^9)) - 5*(63*b^5*c^3*d^5 - 49*a*b^4*c^2*d^6 - 11*a^2*b^3*c*d^7 - 3*a^3*b
^2*d^8)/(b^3*d^9)) + 15*(63*b^6*c^4*d^4 - 112*a*b^5*c^3*d^5 + 38*a^2*b^4*c^
2*d^6 + 8*a^3*b^3*c*d^7 + 3*a^4*b^2*d^8)/(b^3*d^9)) + 15*(63*b^5*c^5 - 175*
a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*
a^5*d^5)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d)))/(sqrt(b*d)*d^5))/(b*abs(b))
```

3.955 $\int \frac{x^3\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=237

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}$$

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(128*b*d^4
) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b*
d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b*d^2) + ((a +
 b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(
9/2))
```

Rubi [A] time = 0.229626, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```


```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(128*b*d^4
) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b*
d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b*d^2) + ((a +
 b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(
9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd} - \frac{(7bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{16bd}$$

$$= -\frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd} + \frac{(5(bc-ad)(7bc+ad))\,\text{Subst}\left(\int\right.}{96bd^2}$$

$$= \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{\left(a+bx^2\right)^{7/2}}{8bd}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7b}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7b}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7b}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7b}{}$$

Mathematica [A] time = 0.574439, size = 214, normalized size = 0.9

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(a^2bd^2\left(118dx^2-191c\right)+15a^3d^3+ab^2d\left(265c^2-172cdx^2+136d^2x^4\right)+b^3\left(70c^2dx^2-105c^3\right.\right.}{384b^2d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^2) + a*b^2*d*(265*c^2 - 172*c*d*x^2 + 136*d^2*x^4) + b^3*(-105*c^3 +
70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)) + 15*(b*c - a*d)^(7/2)*(7*b*c +
a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sq
rt[b*c - a*d]])/(384*b^2*d^(9/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.019, size = 770, normalized size = 3.3

$$-\frac{1}{768\,bd^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-96\,x^6b^3d^3\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}-272\,x^4ab^2d^3\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}+11\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-96*x^6*b^3*d^3*(b*d)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)-272*x^4*a*b^2*d^3*(b*d)^(1/2)*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)+112*x^4*b^3*c*d^2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)-236*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2
)+344*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*c*b^2*d^2*(b*d)^(1/2)-140*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c^2*b^3*d*(b*d)^(1/2)+15*d^4*ln(1/2*
(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)
^(1/2))*a^4+60*a^3*c*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*d^3-270*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^2*b^2*d^
2+300*a*c^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*d-105*b^4*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-30*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*a^3*d^3*(b*d)^(1/2)+382*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*a^2*c*b*d^2*(b*d)^(1/2)-530*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*
c^2*b^2*d*(b*d)^(1/2)+210*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^3*b^3*(b*d)
^(1/2))/b/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27101, size = 1272, normalized size = 5.37

$$\left[-\frac{15\left(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c)*sqrt(b*d)) - 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a*b^3*c^2*
d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^4
 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b^2*d^5), -1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a
*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3
*d^4)*x^4 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + bx^2\right)^{\frac{5}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.23741, size = 397, normalized size = 1.68

$$\frac{\sqrt{b^2c + \left(bx^2 + a\right)bd - abd}\sqrt{bx^2 + a}\left(2\left(bx^2 + a\right)\left(4\left(bx^2 + a\right)\left(\frac{6\left(bx^2+a\right)}{bd} - \frac{7b^2cd^5+abd^6}{b^2d^7}\right) + \frac{5\left(7b^3c^2d^4-6ab^2cd^5-a^2bd^6\right)}{b^2d^7}\right) - \frac{15\left(7b^4\right.}{}\right.}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b*d) - (7*b^2*c*d^5 + a*b*d^6)/(b^2*d^7)) +
 5*(7*b^3*c^2*d^4 - 6*a*b^2*c*d^5 - a^2*b*d^6)/(b^2*d^7)) - 15*(7*b^4*c^3*d
^3 - 13*a*b^3*c^2*d^4 + 5*a^2*b^2*c*d^5 + a^3*b*d^6)/(b^2*d^7)) - 15*(7*b^4
*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*log(a
bs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sq
rt(b*d)*d^4))/abs(b)
```

3.956 $\int \frac{x\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=164

$$\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}{24d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}d^{7/2}}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(b*c - a*d)
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*x^2)^(5/2)*Sqrt[c +
d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]
*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(7/2))
```

Rubi [A] time = 0.140962, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}{24d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}d^{7/2}}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(b*c - a*d)
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*x^2)^(5/2)*Sqrt[c +
d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]
*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(7/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5(bc-ad))\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} + \frac{\left(5(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,\right.}{16d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{5(}{}$$

Mathematica [A] time = 0.514228, size = 164, normalized size = 1.

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(33a^2d^2+2abd\left(13dx^2-20c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right) - \frac{15(bc-ad)^{7/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right.}{b}}{48d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x^
2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - (15*(b*c - a*d)^(7/2)*Sqrt[(b
*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d
]])/b)/(48*d^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.014, size = 529, normalized size = 3.2

$$\frac{1}{96\,d^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(16\,x^4b^2d^2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+52\,\sqrt{bdx^4+adx^2+bcx^2+ac}x^2abd^2\sqrt{bd}-20\,\sqrt{bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*x^4*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+52*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^
2*(b*d)^(1/2)-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2
)+15*d^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)+a*d+b*c)/(b*d)^(1/2))*a^3-45*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+45*ln(1/2*(2*d*x^
2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*c^2*a*b^2*d-15*b^3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*a^2*d^2*(b*d)^(1/2)-80*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)
^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/d^3/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98681, size = 971, normalized size = 5.92

$$\left[-\frac{15\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,bdx^2 + bc + \right.\right.}{19}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) -
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*
c*d^2 - 13*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4), 1/96*(
15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/
2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d
^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^
2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + bx^2\right)^{\frac{5}{2}}}{\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [A] time = 1.23526, size = 284, normalized size = 1.73

$$\frac{\left(\sqrt{b^2c + \left(bx^2 + a\right)bd - abd}\sqrt{bx^2 + a}\left(2\left(bx^2 + a\right)\left(\frac{4\left(bx^2+a\right)}{bd} - \frac{5\left(bcd^3-ad^4\right)}{bd^5}\right) + \frac{15\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)}{bd^5}\right) + \frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bc\cdots\right.}{\cdots}\right.}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d)))/(sqrt(b*d)*d^3))*b/abs(b)
```

3.957 $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{8d^2}+\frac{b\left(a+b\right.}{}$$

```
[Out] -(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + (b*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d +
15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(
8*d^(5/2))
```

Rubi [A] time = 0.225928, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 102, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{8d^2}+\frac{b\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + (b*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d +
15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(
8*d^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(2a^2d-\frac{1}{2}b(3bc-7ad)x\right)}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{2a^3d^2+\frac{1}{4}b\left(3b^2c^2-10abcd+15a^2d^2\right)x}{x\sqrt{a+bx}\sqrt{c+dx}}\right.}{4d^2}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + a^3\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{\left(3b^2c^2-10\right.}{}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\left(3b^2c^2-\right.}{}$$

Mathematica [A] time = 0.637364, size = 213, normalized size = 1.14

$$\frac{1}{8}\left(\frac{\left(25a^2bcd^2-15a^3d^3-13ab^2c^2d+3b^3c^3\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{d^{5/2}\sqrt{c+dx^2}\sqrt{bc-ad}} - \frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{b\sqrt{a+bx^2}\sqrt{c+d}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 9*a*d + 2*b*d*x^2))/d^2 + ((3
*b^3*c^3 - 13*a*b^2*c^2*d + 25*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[(b*(c + d*x^2
))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(d^(5/2
)*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]) - (8*a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b
*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c])/8
```

Maple [B] time = 0.017, size = 446, normalized size = 2.4

$$\frac{1}{16\,d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(4\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}\sqrt{ac}x^2b^2d+15\ln\left(1/2\,\frac{2\,dx^2b+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{b}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)*(a*c)^(1/2)*x^2*b^2*d+15*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+
```

```
b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*b*d^2-
10*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+
b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b^2*c*d+3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^3*c^
2-8*(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+2*a*c)/x^2)*a^3*d^2+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)*(a*c)^(1/2)*a*b*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*
(a*c)^(1/2)*b^2*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)/(a*c
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.0371, size = 2361, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2
*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^
2*d^2)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/d^2, 1/16*(4*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^
2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (3*b^2*c^
2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*
b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/d^2, 1/32*(16*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c +
a^2*d)*x^2)) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/d)*log(8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(2*
b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/d^2, 1/16*(
8*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - (3*b
^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c
 + a*b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.23519, size = 354, normalized size = 1.89

$$-\frac{\left(\frac{16\sqrt{bd}a^3\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b}-2\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(\frac{2(bx^2+a)}{bd}-\frac{3b^2cd-7abd^2}{b^2d^3}\right)+\frac{(3\sqrt{b\cdots}}{\cdots}\right.}{16|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(16*sqrt(b*d)*a^3*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(
-a*b*c*d)*b) - 2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(
b*x^2 + a)/(b*d) - (3*b^2*c*d - 7*a*b*d^2)/(b^2*d^3)) + (3*sqrt(b*d)*b^2*c^
2 - 10*sqrt(b*d)*a*b*c*d + 15*sqrt(b*d)*a^2*d^2)*log((sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d^3))*b^2/abs(b)
```

3.958 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\sqrt{c+d}}{2cd}$$

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a + b*x^2)^(3
/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) - (b^(3/2)*(b*c - 5*
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/
2))
```

Rubi [A] time = 0.229505, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 98, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\sqrt{c+d}}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a + b*x^2)^(3
/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) - (b^(3/2)*(b*c - 5*
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/
2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-ad)-b(bc+ad)x\right)}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a^2d(5bc-ad)+\frac{1}{2}b^2c(bc-5ad)x}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,\right.}{2cd}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\left(b^2(bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx\right.}{4d}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{(b(bc-5ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right.}{2d}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} - \frac{(b}{}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} - \frac{b^3}{}$$

Mathematica [A] time = 0.965392, size = 196, normalized size = 1.05

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(b^2cx^2-a^2d\right)}{cdx^2} + \frac{a^{3/2}(ad-5bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{c^{3/2}} - \frac{(bc-5ad)(bc-ad)^{3/2}\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\right.}{d^{3/2}\left(c+dx^2\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-(a^2*d) + b^2*c*x^2)*Sqrt[c + d*x^2])/(c*d*x^2) - ((b*c
 - 5*a*d)*(b*c - a*d)^(3/2)*((b*(c + d*x^2))/(b*c - a*d))^(3/2)*ArcSinh[(Sq
rt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(d^(3/2)*(c + d*x^2)^(3/2)) + (a^(
3/2)*(-5*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])])/c^(3/2))/2
```

Maple [B] time = 0.017, size = 423, normalized size = 2.3

$$\frac{1}{4\,cdx^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(5\,\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^2ab^2cd\sqrt{ac}-\ln\left(\frac{1}{2}\left(2\,dx^2b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(5*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b^2*c*d*(a*c)
^(1/2)-ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
```

```
a*d+b*c)/(b*d)^(1/2))*x^2*b^3*c^2*(a*c)^(1/2)+ln((a*d*x^2+b*c*x^2+2*(a*c)^(
1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^3*d^2*(b*d)^(1/2
)-5*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2
*a*c)/x^2)*x^2*a^2*b*c*d*(b*d)^(1/2)+2*x^2*b^2*c*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)-2*a^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d/(b*d)^(1/
2)/(a*c)^(1/2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 14.4561, size = 2340, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [-1/8*((b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*
d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (5*a*b*c*d - a^2*d^2)*x^2
*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^
2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c))/(c*d*x^2), 1/8*(2*(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^
4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (5*a*b*c*d - a^2*d^2)*x^2*sqrt(a/c)*log
(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x
^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt
(a/c))/x^4) + 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x
^2), 1/8*(2*(5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x
^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2
*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(b^2*c
*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2), 1/4*((5*a*b*c*d -
 a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d
)*x^2)) + 2*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [B] time = 2.02664, size = 752, normalized size = 4.02

$$b^3\left(\frac{2\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}}{bd}+\frac{\left(\sqrt{bd}bc-5\sqrt{bd}ad\right)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{bd^2}-\frac{2\left(5\sqrt{bd}a^2bc-\sqrt{bd}a^3d\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+\right.}{}\right)}{\sqrt{-abcd}b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*(2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)/(b*d) + (s
qrt(b*d)*b*c - 5*sqrt(b*d)*a*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d^2) - 2*(5*sqrt(b*d)*a^2*b*c - sqrt(b*d
)*a^3*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b^2*c
) - 4*(sqrt(b*d)*a^2*b^3*c^2 - 2*sqrt(b*d)*a^3*b^2*c*d + sqrt(b*d)*a^4*b*d^
2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2*a^2*b*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2*a^3*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c -
 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*
b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4
)*b*c))/abs(b)
```

3.959 $\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{8c^2x^2}-\frac{a\left(a+\right.}{}$$

```
[Out] -(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - (a*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*b^2*c^2 - 10*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])
/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c +
 d*x^2])])/Sqrt[d]
```

Rubi [A] time = 0.2008, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 98, 149, 157, 63, 217, 206, 93, 208}

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{8c^2x^2}-\frac{a\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - (a*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*b^2*c^2 - 10*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])
/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c +
 d*x^2])])/Sqrt[d]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
```

```
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(7bc-3ad)-2b^2cx\right)}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{4}a\left(15b^2c^2-10abcd+3a^2d^2\right)-2b^3c^2}{x\sqrt{a+bx}\sqrt{c+dx}}\right.}{4c^2}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{1}{2}b^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a}\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\left(15b^2c^2-10abcd+3a^2d^2\right)\tanh^{-1}}{8c^{5/2}}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\left(15b^2c^2-10abcd+3a^2d^2\right)\tanh^{-1}}{8c^{5/2}}$$

Mathematica [A] time = 1.09908, size = 206, normalized size = 1.07

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}} + \frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-9bcx^2\right)}{8c^2x^4} + \frac{(bc-ad)^{5/2}\left(\frac{b(c+dx^2}{bc-ad}\right.}{\sqrt{d}\,(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 9*b*c*x^2 + 3*a*d*x^2))/(8*c^2
*x^4) + ((b*c - a*d)^(5/2)*((b*(c + d*x^2))/(b*c - a*d))^(5/2)*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*(c + d*x^2)^(5/2)) - (Sqrt
[a]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])
/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2))
```

Maple [B] time = 0.015, size = 464, normalized size = 2.4

$$\frac{1}{16\,c^2x^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(8\,\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^4b^3c^2\sqrt{ac}-3\,\ln\left(\frac{adx^2+bcx^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(8*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^4*b^3*c^2*(a*c
)^(1/2)-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
```

```
1/2)+2*a*c)/x^2)*x^4*a^3*d^2*(b*d)^(1/2)+10*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*b*c*d*(b*d)^(1/2
)-15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+
2*a*c)/x^2)*x^4*a*b^2*c^2*(b*d)^(1/2)+6*x^2*a^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)-18*x^2*a*b*c*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)-4*a^2*c*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2)*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)/
(a*c)^(1/2)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.4588, size = 2450, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^
2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)
*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/32*(16*b^2*c^2*x^4*sqrt(-b/d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/
(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d + 3*a
^2*d^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/16*(4*b^2*c^2*x^4*sqrt(b/d)
*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^
4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/16*(8*b^2*c^2*x^4*sqrt(-b/d)*
arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/
d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d +
3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^
2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.960 $\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}} dx$

Optimal. Leaf size=553

$$\frac{c^{3/2}\sqrt{a+bx^2}\left(105a^2bcd^2-5a^3d^3-156ab^2c^2d+64b^3c^3\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(75a}{315}$$

```
[Out] ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*c*d^3 - 10
*a^4*d^4)*x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^2]) - ((64*b^3*c^3 -
 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(315*b*d^4) + ((48*b^2*c^2 - 115*a*b*c*d + 75*a^2*d^2)*x^3*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(315*d^3) - (4*b*(2*b*c - 3*a*d)*x^5*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(63*d^2) + (b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9
*d) - (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^
3*b*c*d^3 - 10*a^4*d^4)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(315*b^2*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) + (c^(3/2)*(64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d
^2 - 5*a^3*d^3)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(315*b*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rubi [A] time = 0.718035, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {477, 581, 582, 531, 418, 492, 411}

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{315d^3}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(105a^2bcd^2-5a^3d^3-156ab^2c^2d+64b^3c^3\right)}{315bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```


```
[Out] ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*c*d^3 - 10
*a^4*d^4)*x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^2]) - ((64*b^3*c^3 -
 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(315*b*d^4) + ((48*b^2*c^2 - 115*a*b*c*d + 75*a^2*d^2)*x^3*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(315*d^3) - (4*b*(2*b*c - 3*a*d)*x^5*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(63*d^2) + (b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9
*d) - (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^
3*b*c*d^3 - 10*a^4*d^4)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(315*b^2*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) + (c^(3/2)*(64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d
^2 - 5*a^3*d^3)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(315*b*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
```

```
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d} + \frac{\int \frac{x^4\sqrt{a+bx^2}\left(-a(5bc-9ad)-4b(2bc-3ad)x^2\right)}{\sqrt{c+dx^2}}dx}{9d}$$

$$= -\frac{4b(2bc-3ad)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d^2} + \frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d} + \frac{\int \frac{x^4\left(a\left(40b^2c^2-95abcd+63a^2d^2\right)+\right.}{\sqrt{a+bx^2}}}{63}$$

$$= \frac{\left(48b^2c^2-115abcd+75a^2d^2\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315d^3} - \frac{4b(2bc-3ad)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d^2} + \frac{bx}{}$$

$$= -\frac{\left(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^4} + \frac{\left(48b^2c^2-115abcd+75a^2\right.}{315}$$

$$= -\frac{\left(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^4} + \frac{\left(48b^2c^2-115abcd+75a^2\right.}{315}$$

$$= \frac{\left(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4\right)x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}} - \frac{\left(64b^3c^3-156ab^2c^2d\right.}{}$$

$$= \frac{\left(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4\right)x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}} - \frac{\left(64b^3c^3-156ab^2c^2d\right.}{}$$

Mathematica [C] time = 1.66892, size = 379, normalized size = 0.69

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(-399a^2b^2c^2d^2+130a^3bcd^3+5a^4d^4+392ab^3c^3d-128b^4c^4\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3 + 15*a^2*b*d^2*(-7*c + 5*
d*x^2) + a*b^2*d*(156*c^2 - 115*c*d*x^2 + 95*d^2*x^4) + b^3*(-64*c^3 + 48*c
^2*d*x^2 - 40*c*d^2*x^4 + 35*d^3*x^6)) + I*c*(-128*b^4*c^4 + 328*a*b^3*c^3*
d - 243*a^2*b^2*c^2*d^2 + 25*a^3*b*c*d^3 + 10*a^4*d^4)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-
128*b^4*c^4 + 392*a*b^3*c^3*d - 399*a^2*b^2*c^2*d^2 + 130*a^3*b*c*d^3 + 5*a
^4*d^4)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.028, size = 1047, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-25*(-b/a)^(1/2)*x^7*a*b^3*c*d^4-328
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
```

```
^(1/2))*a*b^3*c^4*d+392*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^4*d+130*(-b/a)^(1/2)*x^9*a*b^3*d^5-6
4*(-b/a)^(1/2)*x*a*b^3*c^4*d-25*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d^3+243*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3
*d^2+130*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^3*b*c^2*d^3-399*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3*d^2+35*(-b/a)^(1/2)*x^
11*b^4*d^5+5*(-b/a)^(1/2)*x^3*a^4*d^5-50*(-b/a)^(1/2)*x^5*a^2*b^2*c*d^4+5*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^4*c*d^4-10*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^4+49*(-b/a)^(1/2)*x^5*a*b^3*c^2*d^3-25*(-
b/a)^(1/2)*x^3*a^3*b*c*d^4-64*(-b/a)^(1/2)*x^3*a^2*b^2*c^2*d^3+140*(-b/a)^(
1/2)*x^3*a*b^3*c^3*d^2-105*(-b/a)^(1/2)*x*a^3*b*c^2*d^3+156*(-b/a)^(1/2)*x*
a^2*b^2*c^3*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^5+128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^5+80*(-b/a)^(1/2)*x^5*a
^3*b*d^5-16*(-b/a)^(1/2)*x^5*b^4*c^3*d^2+5*(-b/a)^(1/2)*x*a^4*c*d^4-64*(-b/
a)^(1/2)*x^3*b^4*c^4*d-5*(-b/a)^(1/2)*x^9*b^4*c*d^4+170*(-b/a)^(1/2)*x^7*a^
2*b^2*d^5+8*(-b/a)^(1/2)*x^7*b^4*c^2*d^3)/b/d^5/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^8+2\,abx^6+a^2x^4\right)\sqrt{bx^2+a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^6 + a^2*x^4)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

3.961 $\int \frac{x^2\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=436

$$-\frac{c^{3/2}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24\right.}{105d^3}$$

```
[Out] -((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a +
b*x^2])/(105*b*d^3*Sqrt[c + d*x^2]) + ((24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^
2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (2*b*(3*b*c - 5*a*d)*x^3*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b*x^3*(a + b*x^2)^(3/2)*Sqrt[c
 + d*x^2])/(7*d) + (Sqrt[c]*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2]) - (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.489241, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {477, 581, 582, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{105d^3}-\frac{x\sqrt{a+bx^2}\left(103a^2bcd^2-15a^3d^3-128ab^2c^2d+48b^3c^3\right)}{105bd^3\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```


```
[Out] -((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a +
b*x^2])/(105*b*d^3*Sqrt[c + d*x^2]) + ((24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^
2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (2*b*(3*b*c - 5*a*d)*x^3*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b*x^3*(a + b*x^2)^(3/2)*Sqrt[c
 + d*x^2])/(7*d) + (Sqrt[c]*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2]) - (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^2\sqrt{a+bx^2}\left(-a(3bc-7ad)-2b(3bc-5ad)x^2\right)}{\sqrt{c+dx^2}}dx}{7d}$$

$$= -\frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^2\left(a\left(18b^2c^2-45abcd+35a^2d^2\right)+b(2\cdots}{\sqrt{a+bx^2}\sqrt{c+\cdots}}}{35d^2}$$

$$= \frac{\left(24b^2c^2-61abcd+45a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^3\left(a+\cdots\right.}{\cdots}$$

$$= \frac{\left(24b^2c^2-61abcd+45a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^3\left(a+\cdots\right.}{\cdots}$$

$$= -\frac{\left(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3\right)x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}} + \frac{\left(24b^2c^2-61abcd+45a^2d^2\right)x\sqrt{a+b\cdots}}{105d^3}$$

$$= -\frac{\left(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3\right)x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}} + \frac{\left(24b^2c^2-61abcd+45a^2d^2\right)x\sqrt{a+b\cdots}}{105d^3}$$

Mathematica [C] time = 0.5944, size = 306, normalized size = 0.7

$$\frac{4ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(-41a^2bcd^2+15a^3d^3+38ab^2c^2d-12b^3c^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(45*a^2*d^2 + a*b*d*(-61*c + 45*d*x^
2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - I*c*(-48*b^3*c^3 + 128*a*b^2*
c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*c*(-12*b^3*c^3 +
38*a*b^2*c^2*d - 41*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[b/a]*d
^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.019, size = 782, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-15*(-b/a)^(1/2)*x^9*b^3*d^4-60*(-b
/a)^(1/2)*x^7*a*b^2*d^4+3*(-b/a)^(1/2)*x^7*b^3*c*d^3-90*(-b/a)^(1/2)*x^5*a^
2*b*d^4+19*(-b/a)^(1/2)*x^5*a*b^2*c*d^3-6*(-b/a)^(1/2)*x^5*b^3*c^2*d^2-45*(
-b/a)^(1/2)*x^3*a^3*d^4-29*(-b/a)^(1/2)*x^3*a^2*b*c*d^3+55*(-b/a)^(1/2)*x^3
*a*b^2*c^2*d^2-24*(-b/a)^(1/2)*x^3*b^3*c^3*d+60*((b*x^2+a)/a)^(1/2)*((d*x^2
```

```
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3-164*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a^2*b*c^2*d^2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-15*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3
+103*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b*c^2*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-45*(-b/a
)^(1/2)*x*a^3*c*d^3+61*(-b/a)^(1/2)*x*a^2*b*c^2*d^2-24*(-b/a)^(1/2)*x*a*b^2
*c^3*d)/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^4+a^2x^2\right)\sqrt{bx^2+a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^4 + a^2*x^2)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

3.962 $\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=330

$$-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{b}{a}\right.}{3\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] ((7*a*b - (2*b^2*c)/d + (3*a^2*d)/c)*x*Sqrt[a + b*x^2])/(3*Sqrt[c + d*x^2])
 + (b*(b*c + 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d) - (a*(a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x) + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*S
qrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*
Sqrt[c]*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rubi [A] time = 0.290934, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {474, 528, 531, 418, 492, 411}

$$\frac{\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{c}+7ab-\frac{2b^2c}{d}\right)}{3\sqrt{c+dx^2}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9a}{3d^{3/2}\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```


```
[Out] ((7*a*b - (2*b^2*c)/d + (3*a^2*d)/c)*x*Sqrt[a + b*x^2])/(3*Sqrt[c + d*x^2])
 + (b*(b*c + 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d) - (a*(a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x) + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*S
qrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*
Sqrt[c]*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```


Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = -\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{\int \frac{\sqrt{a+bx^2}\left(4abc+b(bc+3ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{c}$$

$$=\frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{\int \frac{-abc(bc-9ad)-b\left(2b^2c^2-7abcd-3a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3cd}$$

$$=\frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}-\frac{(ab(bc-9ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}-\ (b(\ldots$$

$$=\frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}}+\frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}-\ b\sqrt{c}(\ldots$$

$$=\frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}}+\frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}+\ (2b^2c\ldots$$

Mathematica [C] time = 0.43935, size = 254, normalized size = 0.77

$$\frac{-2ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2+7a\right.}{3cd^2x\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*(a + b*x^2)*(3*a^2*d - b^2*c*x^2)*(c + d*x^2)) - I*b*c*(-2*b
^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b*c*(b^2*c^2 - 4*a*b*
c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A] time = 0.018, size = 568, normalized size = 1.7

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)d^2cx}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^6b^3cd^2-3\sqrt{-\frac{b}{a}}x^4a^2bd^3+\sqrt{-\frac{b}{a}}x^4ab^2cd^2+\sqrt{-\frac{b}{a}}x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^6*b^3*c*d^2-3*(-b/a)^(1
/2)*x^4*a^2*b*d^3+(-b/a)^(1/2)*x^4*a*b^2*c*d^2+(-b/a)^(1/2)*x^4*b^3*c^2*d+3
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*x*a^2*b*c*d^2+7*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*a*b^2*c^2*d-2*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*b^3*c^3+6*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*
a^2*b*c*d^2-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*x*a*b^2*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*b^3*c^3-3*(-b/a)^(1/2)*x^2*a^
3*d^3-3*(-b/a)^(1/2)*x^2*a^2*b*c*d^2+(-b/a)^(1/2)*x^2*a*b^2*c^2*d-3*(-b/a)^
(1/2)*a^3*c*d^2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{dx^4 + cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^4
 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{x^2\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\sqrt{dx^2 + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.963 $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=336

$$\frac{b\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{3c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\left(-2a^2d^2+7\right.}{3c^{3/2}}$$

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x
^2]) - (2*a*(3*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c^2*x) - (a*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*c*x^3) - ((3*b^2*c^2 + 7*a*b*c*d - 2*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(3*c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) + (b*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.291161, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {474, 580, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{3c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2a\sqrt{a+bx^2}\sqrt{c}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```


```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x
^2]) - (2*a*(3*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c^2*x) - (a*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*c*x^3) - ((3*b^2*c^2 + 7*a*b*c*d - 2*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(3*c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) + (b*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])
```


Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
```

```
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = -\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{\sqrt{a+bx^2}\left(2a(3bc-ad)+b(3bc+ad)x^2\right)}{x^2\sqrt{c+dx^2}}\,dx}{3c}$$

$$= -\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{abc(9bc-ad)+b\left(3b^2c^2+7abcd-2a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c^2}$$

$$= -\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}+\frac{(ab(9bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c}+ \ldots$$

$$= \frac{\left(3b^2c^2+7abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}+ \ldots$$

$$= \frac{\left(3b^2c^2+7abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}- \ldots$$

Mathematica [C] time = 0.454699, size = 261, normalized size = 0.78

$$\frac{-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+2abcd-3b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-7\right.}{3c^2dx^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 7*b*c*x^2 + 2*a*d*x^2) + I
*b*c*(-3*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-3*b^2*c
^2 + 2*a*b*c*d + a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*x^3*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.019, size = 583, normalized size = 1.7

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)c^2dx^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\sqrt{-\frac{b}{a}}x^6a^2bd^3-7\sqrt{-\frac{b}{a}}x^6ab^2cd^2+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*x^6*a^2*b*d^3-7*(-b/a)^
(1/2)*x^6*a*b^2*c*d^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a^2*b*c*d^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b^2*c^2*d-3*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*x^3*b^3*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x^3*a^2*b*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b^2*c^2*d+3*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
x^3*b^3*c^3+2*(-b/a)^(1/2)*x^4*a^3*d^3-6*(-b/a)^(1/2)*x^4*a^2*b*c*d^2-7*(-b
/a)^(1/2)*x^4*a*b^2*c^2*d+(-b/a)^(1/2)*x^2*a^3*c*d^2-8*(-b/a)^(1/2)*x^2*a^2
*b*c^2*d-(-b/a)^(1/2)*a^3*c^2*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(
1/2)/d/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{dx^6 + cx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^6
 + c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{x^4\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**4*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\sqrt{dx^2 + c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.964 $\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{27\sqrt{3}}-\frac{1}{15}\sqrt{2-3x^2}\sqrt{3x^2-1}x^3-\frac{7}{135}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}$$

```
[Out] (-7*x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/135 - (x^3*Sqrt[2 - 3*x^2]*Sqrt[-1
+ 3*x^2])/15 - (8*EllipticE[ArcCos[Sqrt[3/2]*x], 2])/(45*Sqrt[3]) - (2*Elli
pticF[ArcCos[Sqrt[3/2]*x], 2])/(27*Sqrt[3])
```

Rubi [A] time = 0.0934089, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {478, 582, 524, 425, 420}

$$-\frac{1}{15}\sqrt{2-3x^2}\sqrt{3x^2-1}x^3-\frac{7}{135}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{2F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/135 - (x^3*Sqrt[2 - 3*x^2]*Sqrt[-1
+ 3*x^2])/15 - (8*EllipticE[ArcCos[Sqrt[3/2]*x], 2])/(45*Sqrt[3]) - (2*Elli
pticF[ArcCos[Sqrt[3/2]*x], 2])/(27*Sqrt[3])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 425

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> -Simp
[(Sqrt[a - (b*c)/d]*EllipticE[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)])/
(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c
, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= -\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{-1+3x^2}+\frac{1}{15}\int \frac{x^2\left(-6+21x^2\right)}{\sqrt{2-3x^2}\sqrt{-1+3x^2}}\,dx \\
&= -\frac{7}{135}x\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{-1+3x^2}+\frac{1}{405}\int \frac{-42+216x^2}{\sqrt{2-3x^2}\sqrt{-1+3x^2}}\,dx \\
&= -\frac{7}{135}x\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{-1+3x^2}+\frac{2}{27}\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+3x^2}}\,dx+\frac{8}{45}\int \frac{\sqrt{}}{\sqrt{}} \\
&= -\frac{7}{135}x\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}-\frac{2F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right.\right.}{27\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0918039, size = 92, normalized size = 0.93

$$\frac{10\sqrt{3-9x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)-3x\sqrt{2-3x^2}\left(27x^4+12x^2-7\right)-24\sqrt{3-9x^2}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{405\sqrt{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-3*x*Sqrt[2 - 3*x^2]*(-7 + 12*x^2 + 27*x^4) - 24*Sqrt[3 - 9*x^2]*EllipticE
[ArcSin[Sqrt[3/2]*x], 2] + 10*Sqrt[3 - 9*x^2]*EllipticF[ArcSin[Sqrt[3/2]*x]
, 2])/(405*Sqrt[-1 + 3*x^2])
```

Maple [A] time = 0.02, size = 135, normalized size = 1.4

$$-\frac{\sqrt{2}}{7290\,x^4-7290\,x^2+1620}\sqrt{3\,x^2-1}\sqrt{-6\,x^2+4}\left(243\,x^7-54\,x^5+5\,\sqrt{2}\sqrt{3}\sqrt{-6\,x^2+4}\sqrt{-3\,x^2+1}\text{EllipticF}\left(1/2\,x\sqrt{2}\sqrt{3},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/810*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(243*x^7-54*x^5+5*2^(1/2)*3
^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),2^
(1/2))-12*2^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/2*x
*2^(1/2)*3^(1/2),2^(1/2))-135*x^3+42*x)/(9*x^4-9*x^2+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-1}x^4}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{3x^2-1}\sqrt{-3x^2+2}x^4}{3x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)*x^4/(3*x^2 - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-1}x^4}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

3.965 $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{36}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 + 3*x^2)^(3
/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rubi [A] time = 0.0492191, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 53, 619, 216}

$$-\frac{1}{36}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 + 3*x^2)^(3
/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)\\
&= -\frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} + \frac{7}{24}\,\text{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)\\
&= -\frac{7}{72}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} + \frac{7}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\sqrt{-1+3x}}\,dx, x,\right.\\
&= -\frac{7}{72}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} + \frac{7}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+9x-9x^2}}\,dx, x, x^2\right.\\
&= -\frac{7}{72}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} - \frac{7}{432}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 9\left(1-2x\right.\right.\\
&= -\frac{7}{72}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} - \frac{7}{144}\sin^{-1}\left(3-6x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0270139, size = 44, normalized size = 0.68

$$\frac{1}{72}\left(-\sqrt{-9x^4+9x^2-2}\left(6x^2+5\right)-7\sin^{-1}\left(\sqrt{2-3x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-((5 + 6*x^2)*Sqrt[-2 + 9*x^2 - 9*x^4]) - 7*ArcSin[Sqrt[2 - 3*x^2]])/72
```

Maple [A] time = 0.023, size = 81, normalized size = 1.3

$$\frac{1}{144}\sqrt{-3x^2+2}\sqrt{3x^2-1}\left(-12x^2\sqrt{-9x^4+9x^2-2}+7\arcsin\left(6x^2-3\right)-10\sqrt{-9x^4+9x^2-2}\right)\frac{1}{\sqrt{-9x^4+9x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/144*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-12*x^2*(-9*x^4+9*x^2-2)^(1/2)+7*ar
csin(6*x^2-3)-10*(-9*x^4+9*x^2-2)^(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

Maxima [A] time = 1.45833, size = 62, normalized size = 0.95

$$-\frac{1}{12}\sqrt{-9x^4+9x^2-2}x^2-\frac{5}{72}\sqrt{-9x^4+9x^2-2}+\frac{7}{144}\arcsin\left(6x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-9*x^4 + 9*x^2 - 2)*x^2 - 5/72*sqrt(-9*x^4 + 9*x^2 - 2) + 7/144*
arcsin(6*x^2 - 3)
```

Fricas [A] time = 1.78888, size = 185, normalized size = 2.85

$$-\frac{1}{72}\left(6x^2+5\right)\sqrt{3x^2-1}\sqrt{-3x^2+2}-\frac{7}{144}\arctan\left(\frac{3\sqrt{3x^2-1}\left(2x^2-1\right)\sqrt{-3x^2+2}}{2\left(9x^4-9x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 7/144*arctan(3/2*sqrt(
3*x^2 - 1)*(2*x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [A] time = 1.1517, size = 54, normalized size = 0.83

$$-\frac{1}{72}\left(6x^2+5\right)\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{7}{72}\arcsin\left(\sqrt{3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/72*arcsin(sqrt(3*x^2
 - 1))
```

3.966 $\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=70

$$-\frac{\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{1}{9}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

```
[Out] -(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/9 - EllipticE[ArcCos[Sqrt[3/2]*x], 2]
/(3*Sqrt[3]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/(9*Sqrt[3])
```

Rubi [A] time = 0.0455551, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {478, 524, 425, 420}

$$-\frac{1}{9}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/9 - EllipticE[ArcCos[Sqrt[3/2]*x], 2]
/(3*Sqrt[3]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/(9*Sqrt[3])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 425

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> -Simp
[(Sqrt[a - (b*c)/d]*EllipticE[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)])/
(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c
, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
```

```
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= -\frac{1}{9}x\sqrt{2-3x^2}\sqrt{-1+3x^2}+\frac{1}{9}\int \frac{-2+9x^2}{\sqrt{2-3x^2}\sqrt{-1+3x^2}}\,dx\\ &= -\frac{1}{9}x\sqrt{2-3x^2}\sqrt{-1+3x^2}+\frac{1}{9}\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+3x^2}}\,dx+\frac{1}{3}\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx\\ &= -\frac{1}{9}x\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0710557, size = 86, normalized size = 1.23

$$\frac{\sqrt{3-9x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)+3x\sqrt{2-3x^2}\left(1-3x^2\right)-3\sqrt{3-9x^2}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (3*x*(1 - 3*x^2)*Sqrt[2 - 3*x^2] - 3*Sqrt[3 - 9*x^2]*EllipticE[ArcSin[Sqrt[
3/2]*x], 2] + Sqrt[3 - 9*x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2])/(27*Sqrt[-
1 + 3*x^2])
```

Maple [A] time = 0.012, size = 129, normalized size = 1.8

$$-\frac{\sqrt{2}}{972\,x^4-972\,x^2+216}\sqrt{3\,x^2-1}\sqrt{-6\,x^2+4}\left(54\,x^5+\sqrt{2}\sqrt{3}\sqrt{-6\,x^2+4}\sqrt{-3\,x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{3}}{2},\sqrt{2}\right)-3\,\sqrt{2}\sqrt{3}\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/108*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(54*x^5+2^(1/2)*3^(1/2)*(-6
*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),2^(1/2))-3*2
^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/2*x*2^(1/2)*3^
(1/2),2^(1/2))-54*x^3+12*x)/(9*x^4-9*x^2+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2}x^2}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)*x^2/(3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

3.967 $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] -(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/6 - ArcSin[3 - 6*x^2]/12
```

Rubi [A] time = 0.0310672, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 53, 619, 216}

$$-\frac{1}{6}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/6 - ArcSin[3 - 6*x^2]/12
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)$$
$$= -\frac{1}{6}\sqrt{2-3x^2}\sqrt{-1+3x^2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\sqrt{-1+3x}}\,dx, x, x^2\right)$$
$$= -\frac{1}{6}\sqrt{2-3x^2}\sqrt{-1+3x^2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+9x-9x^2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{6}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 9\left(1-2x^2\right)\right)$$
$$= -\frac{1}{6}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

Mathematica [A] time = 0.0123984, size = 37, normalized size = 0.95

$$\frac{1}{6}\left(-\sqrt{-9x^4+9x^2-2} - \sin^{-1}\left(\sqrt{2-3x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-Sqrt[-2 + 9*x^2 - 9*x^4] - ArcSin[Sqrt[2 - 3*x^2]])/6
```

Maple [A] time = 0.009, size = 60, normalized size = 1.5

$$\frac{1}{12}\sqrt{-3\,x^2+2}\sqrt{3\,x^2-1}\left(-2\sqrt{-9\,x^4+9\,x^2-2} + \arcsin\left(6\,x^2-3\right)\right)\frac{1}{\sqrt{-9\,x^4+9\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/12*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-2*(-9*x^4+9*x^2-2)^(1/2)+arcsin(6*x
^2-3))/(-9*x^4+9*x^2-2)^(1/2)
```

Maxima [A] time = 1.47783, size = 36, normalized size = 0.92

$$-\frac{1}{6}\sqrt{-9\,x^4+9\,x^2-2} + \frac{1}{12}\arcsin\left(6\,x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-9*x^4 + 9*x^2 - 2) + 1/12*arcsin(6*x^2 - 3)
```

Fricas [B] time = 1.80314, size = 166, normalized size = 4.26

$$-\frac{1}{6}\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2} - \frac{1}{12}\arctan\left(\frac{3\sqrt{3\,x^2-1}\left(2\,x^2-1\right)\sqrt{-3\,x^2+2}}{2\left(9\,x^4-9\,x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 1/12*arctan(3/2*sqrt(3*x^2 - 1)*(2*
x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

Sympy [A] time = 6.1469, size = 66, normalized size = 1.69

$$\frac{\begin{cases} -\frac{\sqrt{2-3x^2}\sqrt{3x^2-1}}{2} + \frac{\operatorname{asin}\left(\sqrt{3x^2-1}\right)}{2} & \text{for}\ \left(x \geq \frac{\sqrt{3}}{3} \wedge x < \frac{\sqrt{6}}{3}\right) \vee \left(x \leq -\frac{\sqrt{3}}{3} \wedge x > -\frac{\sqrt{6}}{3}\right) \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2 - 3*x**2)*sqrt(3*x**2 - 1)/2 + asin(sqrt(3*x**2 - 1))/2,
 ((x >= sqrt(3)/3) & (x < sqrt(6)/3)) | ((x <= -sqrt(3)/3) & (x > -sqrt(6)/
3))))/3
```

Giac [A] time = 1.16384, size = 45, normalized size = 1.15

$$-\frac{1}{6}\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2}+\frac{1}{6}\,\arcsin\left(\sqrt{3\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/6*arcsin(sqrt(3*x^2 - 1))
```

3.968 $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{2}\sqrt{bx^2+2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{2\sqrt{2}(3b-d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}$$

```
[Out] (-2*(3*b - d)*x*Sqrt[2 + b*x^2])/(3*b*d*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^
2]*Sqrt[3 + d*x^2])/(3*d) + (2*Sqrt[2]*(3*b - d)*Sqrt[2 + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*d^(3/2)*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x
^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.149874, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {478, 531, 418, 492, 411}

$$-\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{2\sqrt{2}(3b-d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2x(3b}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```


```
[Out] (-2*(3*b - d)*x*Sqrt[2 + b*x^2])/(3*b*d*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^
2]*Sqrt[3 + d*x^2])/(3*d) + (2*Sqrt[2]*(3*b - d)*Sqrt[2 + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*d^(3/2)*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x
^2)]*Sqrt[3 + d*x^2])
```


Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d} - \frac{\int \frac{6+2(3b-d)x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx}{3d}$$

$$= \frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d} - \frac{2\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx}{d} - \frac{(2(3b-d))\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx}{3d}$$

$$= -\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}} + \frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d} - \frac{\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{(2(3b-d))\int}{bd}$$

$$= -\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}} + \frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d} + \frac{2\sqrt{2}(3b-d)\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} - \sqrt{2}$$

Mathematica [C] time = 0.112274, size = 127, normalized size = 0.53

$$\frac{-2i\sqrt{3}(3b-2d)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)+\sqrt{b}dx\sqrt{bx^2+2}\sqrt{dx^2+3}+2i\sqrt{3}(3b-d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] + (2*I)*Sqrt[3]*(3*b - d)*Elli
pticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - (2*I)*Sqrt[3]*(3*b - 2
*d)*EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d^2)
```

Maple [A] time = 0.022, size = 306, normalized size = 1.3

$$\frac{1}{\left(3\,bdx^4+9\,bx^2+6\,dx^2+18\right)db}\sqrt{bx^2+2}\sqrt{dx^2+3}\left(x^5b^2d\sqrt{-d}+3\,x^3b^2\sqrt{-d}+2\,x^3bd\sqrt{-d}+3\,\sqrt{2}\text{EllipticF}\left(1/3\,x\sqrt{3}\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(x^5*b^2*d*(-d)^(1/2)+3*x^3*b^2*(-d)^(1
/2)+2*x^3*b*d*(-d)^(1/2)+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2
^(1/2)*3^(1/2)*(1/d*b)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)-2*2^(1/2)*E
llipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*d*(b*x
^2+2)^(1/2)*(d*x^2+3)^(1/2)-6*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/
2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+2*2^(1/2
)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*d*(
b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+6*x*b*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*d*x^2+6
)/d/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8b^2d^2}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}$$

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x^2*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rubi [A] time = 0.158631, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8b^2d^2}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x^2*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} + \frac{\operatorname{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\operatorname{Subst}\left(\int \ldots\right.}{16b^2d^2}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\operatorname{Subst}\left(\int \ldots\right.}{8b^3d^2}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{1-\ldots}\right.}{8b^3d^2}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\ldots}}{\sqrt{b}\sqrt{\ldots}}\right.}{8b^{5/2}d^{5/2}}$$

Mathematica [A] time = 0.252941, size = 154, normalized size = 1.09

$$\frac{\sqrt{bc-ad}\left(3a^2d^2+2abcd+3b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(-3ad-3bc+2bdx^2\right)}{8b^3d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c - 3*a*d + 2*b*d*x^2) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*d^(5/2)*Sq
rt[c + d*x^2])
```

Maple [B] time = 0.029, size = 340, normalized size = 2.4

$$\frac{1}{16\,b^2d^2}\left(4\,\sqrt{bd}\sqrt{bdx^4+adx^2+bcx^2+ac}x^2bd+3\,d^2\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)a^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*b*d+3*d^2*ln(1/
2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*
d)^(1/2))*a^2+2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*b*d+3*b^2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2-6*(b*d)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d-6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/d^2/b^2/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05447, size = 761, normalized size = 5.4

$$\left[\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2}\right.}{32\,b^3d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*
b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3), -1/16*((3*
b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^
2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.23209, size = 212, normalized size = 1.5

$$\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}\left(\frac{2\left(bx^2+a\right)}{bd}-\frac{3\,b^2cd+5\,abd^2}{b^2d^3}\right)-\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}d^2}}{8\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b*d) - (3*b^2*c*d + 5*a*b*d^2)/(b^2*d^3)) - (3*b^2*c^2 + 2*a*b*c*d + 3*a^2*
d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d)))/(sqrt(b*d)*d^2))/(b*abs(b))
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0934393, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.166655, size = 123, normalized size = 1.4

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)-\sqrt{bc-ad}(ad+bc)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{2b^2d^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*
d]])/(2*b^2*d^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.016, size = 200, normalized size = 2.3

$$-\frac{1}{4\,bd}\left(a\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d+b\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(a*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*d+b*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/(b*d)^(1/2)/d/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17643, size = 595, normalized size = 6.76

$$\left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}bd + (bc+ad)\sqrt{bd}\log\left(8b^2d^2x^4 + b^2c^2 + 6abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 - 4\left(2bdx^2 + bc + a\right.\right.}{8b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b^2*d^
2), 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/
(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.22905, size = 140, normalized size = 1.59

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}d} + \frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}}{bd}}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*c + a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sq
rt(b*x^2 + a)/(b*d))/abs(b)
```

3.971 $\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt[
d])
```

Rubi [A] time = 0.0545664, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt[
d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.0655062, size = 82, normalized size = 1.82

$$\frac{\sqrt{c+dx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [B] time = 0.014, size = 103, normalized size = 2.3

$$\frac{1}{2}\ln\left(\frac{1}{2}\left(2\,dx^2b+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{bdx^4+adx^2+bcx^2+ac}}\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d
+b*c)/(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13194, size = 440, normalized size = 9.78

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}\right)}{4\,bd}, -\frac{\sqrt{-bd}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.22082, size = 73, normalized size = 1.62

$$-\frac{b\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -b*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

3.972 $\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Rubi [A] time = 0.0468969, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0118912, size = 46, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Maple [B] time = 0.019, size = 103, normalized size = 2.2

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac\right)\right)\sqrt{dx^2+c}\sqrt{bx^2+a}\frac{1}{\sqrt{ac}}\frac{1}{\sqrt{bdx^4+adx^2+bcx^2+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+
2*a*c)/x^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25657, size = 455, normalized size = 9.89

$$\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)}{4\,ac}, \frac{\sqrt{-ac}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2}\right.}{2\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c)*sqrt(a*c))/x^4)/(a*c), 1/2*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^2 +
 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^2))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.21049, size = 120, normalized size = 2.61

$$-\frac{\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a
bs(b))
```

3.973 $\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0785099, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 96, 93, 208}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac}$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2ac}$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2} + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.0550414, size = 91, normalized size = 1.

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Maple [B] time = 0.021, size = 209, normalized size = 2.3

$$\frac{1}{4\,ac{x}^{2}}\left(\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2\,ac\right)\right)x^2ad+\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\,\sqrt{ac}\sqrt{bdx^4+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a/c*(ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+2*a*c)/x^2)*x^2*a*d+ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*b*c-2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/x^2/(a*c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49727, size = 635, normalized size = 6.98

$$\left[\frac{\sqrt{ac}(bc+ad)x^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2+4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)-4\sqrt{bx^2+a}\sqrt{dx^2+c}ac}{8\,a^2c^2x^2},-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c)*(b*c + a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*a*c)/(a^2*c^2*x^2), -1/4*(sqrt(-a*c)*(b*c + a*d)*x^2*arctan(1/2*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 +
a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c
)/(a^2*c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.3526, size = 558, normalized size = 6.13

$$\frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd}-\frac{2\left(b^3c^2-2\,ab^2cd+a^2bd^2-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-\cdots}\right.\right.}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\sqrt{b\cdots}\right.\right.}\right.}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

3.974 $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8a^2c^2x^2}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*a*c*x^4) + (3*(b*c + a*d)*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2
)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*
c^(5/2))
```

Rubi [A] time = 0.13681, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 103, 151, 12, 93, 208}

$$-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8a^2c^2x^2}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*a*c*x^4) + (3*(b*c + a*d)*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2
)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*
c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

erQ[m]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+ad)+bdx}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\text{Subst}\left(\int \frac{3b^2c^2+2abcd+3a^2d^2}{4x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2c^2}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\text{Subst}\left(\int \cdots\right.}{16a^2c^2}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\text{Subst}\left(\int \cdots\right.}{8a^2c^2}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} - \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tanh^{-1}\left(\cdots\right.}{8a^{5/2}c^{5/2}}$$

Mathematica [A] time = 0.0800869, size = 126, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2+3bcx^2\right)}{8a^2c^2x^4} - \frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c + 3*b*c*x^2 + 3*a*d*x^2))/(8*a^2*c
^2*x^4) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

Maple [B] time = 0.022, size = 355, normalized size = 2.4

$$-\frac{1}{16 a^2c^2x^4}\left(3 \ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^4a^2d^2+2\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+\cdots}}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16/a^2/c^2*(3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*d^2+2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a*b*c*d+3*ln((a*d*x^2+b*c*
x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*b^2*c
^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-6*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*a*c*(a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/x^4/(a*c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.58152, size = 798, normalized size = 5.36

$$\left[\frac{\left(3 b^2c^2+2abcd+3a^2d^2\right)\sqrt{ac}x^4 \log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left((bc+ad)x^2+2ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)-4\left(2a^2c^2\cdots\right.}{32a^3c^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^
2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^
3*x^4), 1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*
c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 - 3*(a*b*c^2 +
 a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.975 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=342

$$\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)}{15b^3d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(8a^2d^2 \right.}{15b^}$$

```
[Out] ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^3*d^2*Sqrt[c
+ d*x^2]) - (4*(b*c + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b^2*d^2)
+ (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (Sqrt[c]*(8*b^2*c^2 + 7*a
*b*c*d + 8*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*b^3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]) + (4*c^(3/2)*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.31085, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {479, 582, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)}{15b^3d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4c^{3/2}\sqrt{a+bx^2}(}{15b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^3*d^2*Sqrt[c
+ d*x^2]) - (4*(b*c + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b^2*d^2)
+ (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (Sqrt[c]*(8*b^2*c^2 + 7*a
*b*c*d + 8*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*b^3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]) + (4*c^(3/2)*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
```

```
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} - \frac{\int \frac{x^2\left(3ac+4(bc+ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5bd}$$

$$= -\frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} + \frac{\int \frac{4ac(bc+ad)+\left(8b^2c^2+7abcd+8a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d^2}$$

$$= -\frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} + \frac{(4ac(bc+ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d^2} +$$

$$= \frac{\left(8b^2c^2+7abcd+8a^2d^2\right)x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}} - \frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}$$

$$= \frac{\left(8b^2c^2+7abcd+8a^2d^2\right)x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}} - \frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}$$

Mathematica [C] time = 0.458683, size = 249, normalized size = 0.73

$$\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2+3abcd+8b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2+7abcd+\right.}{15a^2d^3\left(\frac{b}{a}\right)^{5/2}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*b*c + 4*a*d - 3*b*d*x^2)) - I*c
*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 + 3*a*b*c
*d + 4*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A] time = 0.026, size = 546, normalized size = 1.6

$$-\frac{1}{15\,b^2d^3\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-3\sqrt{-\frac{b}{a}}x^7b^2d^3+\sqrt{-\frac{b}{a}}x^5abd^3+\sqrt{-\frac{b}{a}}x^5b^2cd^2+4\sqrt{-\frac{b}{a}}x^3a^2d^3+5\sqrt{-\frac{b}{a}}x^3abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(-3*(-b/a)^(1/2)*x^7*b^2*d^3+(-b/a)^(1/2)*x^5*a*b*d^3+(-b/a)^(1/2)*x^
5*b^2*c*d^2+4*(-b/a)^(1/2)*x^3*a^2*d^3+5*(-b/a)^(1/2)*x^3*a*b*c*d^2+4*(-b/a
)^(1/2)*x^3*b^2*c^2*d+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^
3-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a^2*c*d^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+4*(-b/a)^(1/2)*x*a
^2*c*d^2+4*(-b/a)^(1/2)*x*a*b*c^2*d)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3/b^
2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(b*d*x^4 + (b*c + a*d)*x^2 + a
*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{c^{3/2}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}(ad+b\cdots}{3b^2d\sqrt{c+dx^2}}$$

```
[Out] (-2*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*b*d) + (2*Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.162298, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {479, 531, 418, 492, 411}

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}(ad+bc)}{3b^2d\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*b*d) + (2*Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{\int \frac{ac+2(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3bd} \\
&= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{(ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3bd} - \frac{(2(bc+ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3bd} \\
&= -\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(2c(b}{} \\
&= -\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} + \frac{2\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right.}{3b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] time = 0.277404, size = 201, normalized size = 0.77

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+}{3bd^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(b*c + a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
I*c*(2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Maple [A] time = 0.02, size = 333, normalized size = 1.3

$$\frac{1}{3\,d^2b\left(bdx^4+adx^2+bcx^2+ac\right)}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d+a
*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)*x*a*c
*d)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/d^2/b/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(b*d*x^4 + (b*c + a*d)*x^2 + a
*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0492409, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {492, 411}

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx&=\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{b}\\&=\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [C] time = 0.0724129, size = 122, normalized size = 1.05

$$-\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right)}{d\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sq
rt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 129, normalized size = 1.1

$$\frac{c}{d\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{bx^2+a}\sqrt{d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2)))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*c*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(b*d*x^4 + (b*c + a*d)*x^2 + a
*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.978 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=153

$$\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(a*c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(a*c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2])
```

Rubi [A] time = 0.0948289, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {480, 12, 492, 411}

$$\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(a*c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(a*c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}+\frac{\int \frac{bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ac}$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}+\frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ac}$$
$$= \frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{d\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{a}$$
$$= \frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.324959, size = 146, normalized size = 0.95

$$\frac{-\frac{\left(a+bx^2\right)\left(c+dx^2\right)}{cx}-ia\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((a + b*x^2)*(c + d*x^2))/(c*x)) - I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - Elliptic
F[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
)
```

Maple [A] time = 0.019, size = 224, normalized size = 1.5

$$\frac{1}{axc\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-\sqrt{-\frac{b}{a}}x^4bd-bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}x\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-(-b/a)^(1/2)*x^4*b*d-b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-(-b/a)^(1/2)*x^2*a*d-(-b/a
)^(1/2)*x^2*b*c-(-b/a)^(1/2)*a*c)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/x/a/(-b/a
)^(1/2)/c/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{bdx^6+(bc+ad)x^4+acx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*x^6 + (b*c + a*d)*x^4 + a*c*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

3.979 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$-\frac{b\sqrt{d}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3a^2c^2x}-\frac{2dx\sqrt{a+bx^2}(ad+bc)}{3a^2c^2\sqrt{c+dx^2}}+\frac{2\sqrt{d}\sqrt{a+}}{}$$

```
[Out] (-2*d*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*a^2*c^2*Sqrt[c + d*x^2]) - (Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(3*a*c*x^3) + (2*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(3*a^2*c^2*x) + (2*Sqrt[d]*(b*c + a*d)*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*c^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.2505, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {480, 583, 531, 418, 492, 411}

$$\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3a^2c^2x}-\frac{2dx\sqrt{a+bx^2}(ad+bc)}{3a^2c^2\sqrt{c+dx^2}}+\frac{2\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{d}\sqrt{a+b}}{3a^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*d*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*a^2*c^2*Sqrt[c + d*x^2]) - (Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(3*a*c*x^3) + (2*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(3*a^2*c^2*x) + (2*Sqrt[d]*(b*c + a*d)*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*c^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{\int \frac{-2(bc+ad)-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}-\frac{\int \frac{abcd+2bd(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2c^2}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}-\frac{(bd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac}-\frac{(2bd(bc+\cdots}{\cdots}\\
&= -\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}-\frac{b\sqrt{d}\sqrt{a+\cdots}}{3a^2\cdots}\\
&= -\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}+\frac{2\sqrt{d}(bc+\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.34465, size = 229, normalized size = 0.75

$$
\frac{-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(-ac+2adx^2+2bcx^2\right)+2i\cdots}{3a^2c^2x^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) + 2*b*c*x^2 + 2*a*d*x^2) + (2*I)
*b*c*(b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*
a^2*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 435, normalized size = 1.4

$$\frac{1}{3\,a^2c^2x^3\left(bdx^4+adx^2+bcx^2+ac\right)}\left(2\sqrt{-\frac{b}{a}}x^6abd^2+2\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(2*(-b/a)^(1/2)*x^6*a*b*d^2+2*(-b/a)^(1/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a
*c+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*x^3*b^2*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2+2*(-b/a)^
(1/2)*x^4*a^2*d^2+3*(-b/a)^(1/2)*x^4*a*b*c*d+2*(-b/a)^(1/2)*x^4*b^2*c^2+(-b
/a)^(1/2)*x^2*a^2*c*d+(-b/a)^(1/2)*x^2*a*b*c^2-(-b/a)^(1/2)*a^2*c^2)*(d*x^2
+c)^(1/2)*(b*x^2+a)^(1/2)/x^3/a^2/(-b/a)^(1/2)/c^2/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{bdx^8+(bc+ad)x^6+acx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*x^8 + (b*c + a*d)*x^6 + a*c*x
^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

3.980 $\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}$$

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*d^(3/2))
```

Rubi [A] time = 0.164462, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*d^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{b^2(bc-ad)}\\
&= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}-\frac{(bc+3ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{4b^2d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}-\frac{(bc+3ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+\right.}{2b^3d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}-\frac{(bc+3ad)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2b^3d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}-\frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.345041, size = 185, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\sqrt{bc-ad}\left(-3a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\left(c+dx^2\right)\left(-3a^2d+ab\left(c-dx^2\right)+b^2cx^2\right)}{2b^3d^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^2)*(-3*a^2*d + b^2*c*x^2 + a*b*(c - d*x^2))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*
x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b
^3*d^(3/2)*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.038, size = 553, normalized size = 4.3

$$-\frac{1}{4\,b^{2}d\,(ad-bc)}\left(3\,\ln\left(1/2\,\frac{2\,dx^{2}b+2\,\sqrt{bdx^{4}+adx^{2}+bcx^{2}+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^{2}a^{2}bd^{2}-2\,\ln\left(1/2\,\frac{2\,dx^{2}b+2\,\sqrt{bdx^{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b^2*c*d-ln(1/2*
(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)
^(1/2))*x^2*b^3*c^2-2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*b*d+2*(
b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*b^2*c+3*ln(1/2*(2*d*x^2*b+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*d^2-2
*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*a^2*b*c*d-ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c^2-6*(b*d)^(1/2)*((b*x^2+a)
*(d*x^2+c))^(1/2)*a^2*d+2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a*b*c)/b^
2*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(b*d)^(1/2)/d/(a*d-b*c)/((b*x^2+a)*(d*x^2
+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69498, size = 1049, normalized size = 8.13

$$\left[\frac{\left(ab^{2}c^{2}+2\,a^{2}bcd-3\,a^{3}d^{2}+\left(b^{3}c^{2}+2\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{2}\right)\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)\right.}{8\left(ab^{4}cd^{2}-a^{2}b^{3}d^{3}+\left(b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2
*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b*d)) + 4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*
x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d
^2 - a*b^4*d^3)*x^2), 1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c +
```

```
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x^2)) + 2*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c
*d^2 - a*b^4*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.24991, size = 254, normalized size = 1.97

$$-\frac{\frac{8\sqrt{bd}a^2}{b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}-\frac{2\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\sqrt{bx^2+a}}{bd}-\frac{\left(\sqrt{bd}bc+3\sqrt{bd}ad\right)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{bd^2}}{4\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8*sqrt(b*d)*a^2/(b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2) - 2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
)*sqrt(b*x^2 + a)/(b*d) - (sqrt(b*d)*b*c + 3*sqrt(b*d)*a*d)*log((sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d^2))/(b*abs
(b))
```

3.981 $\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}$$

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt
[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d])
```

Rubi [A] time = 0.0885643, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 78, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt
[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.494791, size = 118, normalized size = 1.42

$$\frac{\frac{ab\left(c+dx^2\right)}{\sqrt{a+bx^2}(bc-ad)}+\frac{\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}}{b^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*(c + d*x^2))/((b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[b*c - a*d]*Sqrt[(b
*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d
]])/Sqrt[d])/(b^2*Sqrt[c + d*x^2])
```

Maple [B] time = 0.02, size = 320, normalized size = 3.9

$$\frac{1}{2\,(ad-bc)\,b}\left(\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)x^2abd-\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4+adx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*x^2*a*b*d-ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^2*c+ln(1/2*(2*d*x^2*b+2*
```

```
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*d
-ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*a*b*c-2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a)/b*(d*x^2
+c)^(1/2)/(b*x^2+a)^(1/2)/(b*d)^(1/2)/(a*d-b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46672, size = 790, normalized size = 9.52

$$\left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}abd + \left(abc - a^2d + \left(b^2c - abd\right)x^2\right)\sqrt{bd}\log\left(8b^2d^2x^4 + b^2c^2 + 6abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2\right.}{4\left(ab^3cd - a^2b^2d^2 + \left(b^4cd - ab^3d^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d + (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b*d)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2),
 1/2*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2))
)/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + bx^2\right)^{\frac{3}{2}}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.22551, size = 177, normalized size = 2.13

$$\frac{\frac{4\sqrt{bd}ab}{b^2c - abd - \left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c + \left(bx^2+a\right)bd - abd}\right)^2} - \frac{\sqrt{bd}\log\left(\left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c + \left(bx^2+a\right)bd - abd}\right)^2\right)}{d}}{2b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(b*d)*a*b/(b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2) - sqrt(b*d)*log((sqrt(b*x^2 + a)*sqrt(b*d
) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/d)/(b*abs(b))
```

3.982 $\int \frac{x}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(bc-ad)}$$

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rubi [A] time = 0.0268717, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {444, 37}

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0099777, size = 33, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/((-(b*c) + a*d)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$\frac{1}{ad-bc}\sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8887, size = 97, normalized size = 2.85

$$-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{abc-a^2d+\left(b^2c-abd\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.18841, size = 95, normalized size = 2.79

$$-\frac{2\sqrt{bd}b}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^2)*abs(b))
```

3.983 $\int \frac{x^5}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(3bc-2ad)}{3b^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*a*(3*b*c
- 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.141449, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(3bc-2ad)}{3b^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*a*(3*b*c
- 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(5/2)*Sqrt[d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{3b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a -}\right.}{b^3}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b^3}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

Mathematica [A] time = 0.483143, size = 214, normalized size = 1.56

$$\frac{\sqrt{c+dx^2}\left(\frac{\left(a+bx^2\right)\left(3b^2c^2-a^2d^2\right)}{d(bc-ad)^2}+\frac{a^2}{ad-bc}-\frac{3\left(a+bx^2\right)\left(\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{d\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}\right)}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^2))/
(d*(b*c - a*d)^2) - (3*(a + b*x^2)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b
*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt
[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])))/(3*b
^2*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.035, size = 651, normalized size = 4.8

$$\frac{1}{6\,b^2\,(ad-bc)^2\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-8\,\sqrt{bd}x^4a^2bd^2+12\,x^4ab^2cd\sqrt{bd}+3\,\ln\left(1/2\,\frac{2\,dx^2b+2\,\sqrt{bdx^4+adx^2+bcx^2}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-8*(b*d)^(1/2)*x^4*a^2*b*d^2+12*x^4*a*b^2*c*d*(b*d)^(1/2)+3*ln(1/2*(2*
d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1
/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a^2*b*d^2-6*ln(1/2*(2*d*x^2*b+2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*
(d*x^2+c))^(1/2)*x^2*a*b^2*c*d+3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)
*x^2*b^3*c^2-6*x^2*a^3*d^2*(b*d)^(1/2)+2*(b*d)^(1/2)*x^2*a^2*b*c*d+12*x^2*a
*b^2*c^2*(b*d)^(1/2)+3*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a^3*d^2-6
*ln(1/2*(2*d*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a^2*b*c*d+3*ln(1/2*(2*d*x^2*b+2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*
x^2+a)*(d*x^2+c))^(1/2)*a*b^2*c^2-6*a^3*c*d*(b*d)^(1/2)+10*(b*d)^(1/2)*a^2*
b*c^2)*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/(b*d)^(1/2)/(a*d-b*c)^2/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.48415, size = 1473, normalized size = 10.75

$$\left[\frac{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\right.}{12\left(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) +
4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3
+ (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d - 2*a^2*b^
5*c*d^2 + a^3*b^4*d^3)*x^2), -1/6*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 +
 (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x^2)) - 2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4
*b^3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d -
 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.35565, size = 443, normalized size = 3.23

$$-\frac{\frac{3\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{\sqrt{bd}}-\frac{8\left(3\sqrt{bd}ab^4c^2-5\sqrt{bd}a^2b^3cd+2\sqrt{bd}a^3b^2d^2-6\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2ab^2c+3\sqrt{bd}\left(\sqrt{\cdots}\right.\right.}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\cdots}\right.\right.}}{6\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b
*d))^2)/sqrt(b*d) - 8*(3*sqrt(b*d)*a*b^4*c^2 - 5*sqrt(b*d)*a^2*b^3*c*d + 2*
sqrt(b*d)*a^3*b^2*d^2 - 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b*d + 3*sqrt(b*d)*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a)/(b^2*c
- a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)^3)/(b*abs(b))
```

3.984 $\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{a\sqrt{c+dx^2}}{3b\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{c+dx^2}(3bc-ad)}{3b\sqrt{a+bx^2}(bc-ad)^2}$$

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c - a*d)*Sq
rt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0664083, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 78, 37}

$$\frac{a\sqrt{c+dx^2}}{3b\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{c+dx^2}(3bc-ad)}{3b\sqrt{a+bx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c - a*d)*Sq
rt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)(a+bx^2)^{3/2}} + \frac{(3bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{6b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)(a+bx^2)^{3/2}} - \frac{(3bc-ad)\sqrt{c+dx^2}}{3b(bc-ad)^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0260997, size = 54, normalized size = 0.61

$$\frac{\sqrt{c+dx^2}\left(-2ac+adx^2-3bcx^2\right)}{3\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^
2)^(3/2))
```

Maple [A] time = 0.006, size = 63, normalized size = 0.7

$$-\frac{-adx^2+3\,bcx^2+2\,ac}{3\,a^2d^2-6\,cabd+3\,b^2c^2}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)*(-a*d*x^2+3*b*c*x^2+2*a*c)/(b*x^2+a)^(3/2)/(a^2*d^2-2*
a*b*c*d+b^2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45018, size = 265, normalized size = 2.98

$$-\frac{\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c
^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*
(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.25334, size = 289, normalized size = 3.25

$$-\frac{2\left(3\sqrt{bd}b^5c^2-4\sqrt{bd}ab^4cd+\sqrt{bd}a^2b^3d^2-6\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^3c+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{\right.\right.}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)^3b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*d)*b^5*c^2 - 4*sqrt(b*d)*a*b^4*c*d + sqrt(b*d)*a^2*b^3*d^2 -
 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^2*b^3*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^4*b)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*b*abs(b))
```

3.985 $\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c + d*x^2])/
(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0438094, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 45, 37}

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c + d*x^2])/
(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{3(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2d\sqrt{c+dx^2}}{3(bc-ad)^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0161268, size = 52, normalized size = 0.7

$$\frac{\sqrt{c+dx^2}\left(3ad-bc+2bdx^2\right)}{3\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^2)
^(3/2))
```

Maple [A] time = 0.006, size = 60, normalized size = 0.8

$$\frac{2\,dx^2b+3\,ad-bc}{3\,a^2d^2-6\,cabd+3\,b^2c^2}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(2*b*d*x^2+3*a*d-b*c)/(b*x^2+a)^(3/2)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43684, size = 261, normalized size = 3.53

$$\frac{\left(2\,bdx^2-bc+3\,ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c^2
- 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*
b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.23678, size = 174, normalized size = 2.35

$$\frac{4\left(b^2c-abd-3\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)\sqrt{bd}b^2d}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(b^2*c - a*b*d - 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*abs(b))
```

3.986 $\int \frac{x^5}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2*a*(5*b*c
- 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - ((15*b
^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^3*Sqr
t[a + b*x^2])
```

Rubi [A] time = 0.180243, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 89, 78, 37}

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2*a*(5*b*c
- 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - ((15*b
^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^3*Sqr
t[a + b*x^2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(5bc-ad)+\frac{5}{2}b(bc-ad)x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{5b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd+3a^2d^2\right)}{30b^2}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{\left(15b^2c^2-10abcd+3a^2d^2\right)}{15b^2(bc-ad)^3\sqrt{a+b}}$$

Mathematica [A] time = 0.0536155, size = 91, normalized size = 0.59

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(8c^2-4cdx^2+3d^2x^4\right)+10abcx^2\left(2c-dx^2\right)+15b^2c^2x^4\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(2*c - d*x^2) + a^2*(8*c^2
 - 4*c*d*x^2 + 3*d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A] time = 0.007, size = 119, normalized size = 0.8

$$\frac{3\,a^2d^2x^4-10\,abcdx^4+15\,b^2c^2x^4-4\,a^2cdx^2+20\,ac^2bx^2+8\,a^2c^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*a^2*d^2*x^4-10*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*d
*x^2+20*a*b*c^2*x^2+8*a^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.06474, size = 522, normalized size = 3.39

$$-\frac{\left(\left(15\,b^2c^2-10\,abcd+3\,a^2d^2\right)x^4+8\,a^2c^2+4\left(5\,abc^2-a^2cd\right)x^2\right)\sqrt{bx^2+}}{15\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^6+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 8*a^2*c^2 + 4*(5*a*b*c^2
 - a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2
 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^
4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.39943, size = 806, normalized size = 5.23

$$-\frac{2\left(15\sqrt{bd}b^8c^4-40\sqrt{bd}ab^7c^3d+38\sqrt{bd}a^2b^6c^2d^2-16\sqrt{bd}a^3b^5cd^3+3\sqrt{bd}a^4b^4d^4-60\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(b\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(b*d)*b^8*c^4 - 40*sqrt(b*d)*a*b^7*c^3*d + 38*sqrt(b*d)*a^2*b
^6*c^2*d^2 - 16*sqrt(b*d)*a^3*b^5*c*d^3 + 3*sqrt(b*d)*a^4*b^4*d^4 - 60*sqrt
```

```
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2
*b^6*c^3 + 80*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^2*a*b^5*c^2*d - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 90*sqrt(b*d)*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c^2 -
40*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^4*a*b^3*c*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 60*sqrt(b*d)*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c + 15*sqrt(b*d)*(sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8)/((b^2*c
- a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)^5*b*abs(b))
```

3.987 $\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=138

$$\frac{a\sqrt{c+dx^2}}{5b\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2d\sqrt{c+dx^2}(5bc-ad)}{15b\sqrt{a+bx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}(5bc-ad)}{15b\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c - a*d)*Sq
rt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (2*d*(5*b*c - a*d)*
Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.100783, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 78, 45, 37}

$$\frac{a\sqrt{c+dx^2}}{5b\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2d\sqrt{c+dx^2}(5bc-ad)}{15b\sqrt{a+bx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}(5bc-ad)}{15b\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c - a*d)*Sq
rt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (2*d*(5*b*c - a*d)*
Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{(5bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{10b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}} - \frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{(d(5bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\cdots}\right.}{15b(bc-ad)^2}$$
$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}} - \frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{2d(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0353247, size = 91, normalized size = 0.66

$$\frac{\sqrt{c+dx^2}\left(-5a^2d\left(dx^2-2c\right)-2ab\left(c^2-13cdx^2+d^2x^4\right)-5b^2cx^2\left(c-2dx^2\right)\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-5*b^2*c*x^2*(c - 2*d*x^2) - 5*a^2*d*(-2*c + d*x^2) - 2*a
*b*(c^2 - 13*c*d*x^2 + d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A] time = 0.007, size = 125, normalized size = 0.9

$$-\frac{-2\,abd^2x^4+10\,b^2cdx^4-5\,a^2d^2x^2+26\,abcdx^2-5\,b^2c^2x^2+10\,a^2cd-2\,abc^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(-2*a*b*d^2*x^4+10*b^2*c*d*x^4-5*a^2*d^2*x^2+26*a*b*c
*d*x^2-5*b^2*c^2*x^2+10*a^2*c*d-2*a*b*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.32376, size = 537, normalized size = 3.89

$$\frac{\left(2\left(5b^2cd - abd^2\right)x^4 - 2abc^2 + 10a^2cd - \left(5b^2c^2 - 26abcd + 5a^2d^2\right)x^2\right)\sqrt{bx}}{15\left(a^3b^3c^3 - 3a^4b^2c^2d + 3a^5bcd^2 - a^6d^3 + \left(b^6c^3 - 3ab^5c^2d + 3a^2b^4cd^2 - a^3b^3d^3\right)x^6 + 3\left(ab^5c^3 - 3a^2b^4c^2d + 3a^3b^3cd^2 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*b^2*c*d - a*b*d^2)*x^4 - 2*a*b*c^2 + 10*a^2*c*d - (5*b^2*c^2 - 2
6*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 -
3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^
2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3
*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*
d^2 - a^5*b*d^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + bx^2\right)^{\frac{7}{2}} \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.31265, size = 637, normalized size = 4.62

$$\frac{4\left(5\sqrt{bd}b^8c^3d - 11\sqrt{bd}ab^7c^2d^2 + 7\sqrt{bd}a^2b^6cd^3 - \sqrt{bd}a^3b^5d^4 - 25\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c + \left(bx^2+a\right)bd - abd}\right)^2 b^6c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*sqrt(b*d)*b^8*c^3*d - 11*sqrt(b*d)*a*b^7*c^2*d^2 + 7*sqrt(b*d)*a^2*
b^6*c*d^3 - sqrt(b*d)*a^3*b^5*d^4 - 25*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^2*d + 30*sqrt(b*d)*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c*d^2
 - 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*a^2*b^4*d^3 + 35*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c*d + 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*d^2 - 15*sqrt(b*d)*(
```

```
sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*d)
/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d))^2)^5*b*abs(b))
```

3.988 $\int \frac{x}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=113

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c + d*x^2])/
(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c + d*x^2])/(15*(b*c - a
*d)^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0636799, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 45, 37}

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c + d*x^2])/
(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c + d*x^2])/(15*(b*c - a
*d)^3*Sqrt[a + b*x^2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}} - \frac{(2d)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{5(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\left(4d^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx\right.}{15(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{8d^2\sqrt{c+dx^2}}{15(bc-ad)^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.028407, size = 83, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\left(15a^2d^2-10abd\left(c-2dx^2\right)+b^2\left(3c^2-4cdx^2+8d^2x^4\right)\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x^2) + b^2*(3*c^2 - 4*c*d
*x^2 + 8*d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A] time = 0.007, size = 113, normalized size = 1.

$$\frac{8\,b^2d^2x^4+20\,abd^2x^2-4\,b^2cdx^2+15\,a^2d^2-10\,cabd+3\,b^2c^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4+20*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^2
-10*a*b*c*d+3*b^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d
-b^3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.14213, size = 520, normalized size = 4.6

$$-\frac{\left(8\,b^2d^2x^4+3\,b^2c^2-10\,abcd+15\,a^2d^2-4\,\left(b^2cd-5\,abd^2\right)x^2\right)\sqrt{bx^2+}}{15\,\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^6+3\,\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^2*d^2*x^4 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
 a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*
b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] time = 1.24168, size = 328, normalized size = 2.9

$$-\frac{16\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-5\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c+5\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right.}\right.\right.}{15\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -16/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((
b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2)^5*abs(b))
```

3.989 $\int \frac{x^5}{\left(a+bx^2\right)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=217

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c}}{35b^2\left(a\right.}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2*a*(7*b*c
- 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*x^2)^(5/2)) - ((35*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^3*(a
 + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^
2])/(105*b^2*(b*c - a*d)^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.268655, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 89, 78, 45, 37}

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c}}{35b^2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] -(a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2*a*(7*b*c
- 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*x^2)^(5/2)) - ((35*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^3*(a
 + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^
2])/(105*b^2*(b*c - a*d)^4*Sqrt[a + b*x^2])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{9/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(7bc-ad)+\frac{7}{2}b(bc-ad)x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{7b^2(bc-ad)}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}} + \frac{\left(35b^2c^2-14abcd+3a^2d^2\right)\text{Su}}{70b^2(bc}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}} - \frac{\left(35b^2c^2-14abcd+3a^2d^2\right)\sqrt{c}}{105b^2(bc-ad)^3\left(a+bx^2\right)^3}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}} - \frac{\left(35b^2c^2-14abcd+3a^2d^2\right)\sqrt{c}}{105b^2(bc-ad)^3\left(a+bx^2\right)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0763455, size = 151, normalized size = 0.7

$$
\frac{\sqrt{c+dx^2}\left(a^2b\left(200c^2dx^2-8c^3-101cd^2x^4+6d^3x^6\right)+7a^3d\left(8c^2-4cdx^2+3d^2x^4\right)-7ab^2cx^2\left(4c^2-37cdx^2+4d^2x^4\right)-35\right.}{105\left(a+bx^2\right)^{7/2}(bc-ad)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-35*b^3*c^2*x^4*(c - 2*d*x^2) + 7*a^3*d*(8*c^2 - 4*c*d*x^
2 + 3*d^2*x^4) - 7*a*b^2*c*x^2*(4*c^2 - 37*c*d*x^2 + 4*d^2*x^4) + a^2*b*(-8
*c^3 + 200*c^2*d*x^2 - 101*c*d^2*x^4 + 6*d^3*x^6)))/(105*(b*c - a*d)^4*(a +
 b*x^2)^(7/2))
```

Maple [A] time = 0.01, size = 213, normalized size = 1.

$$\frac{6\,a^{2}bd^{3}x^{6}-28\,ab^{2}cd^{2}x^{6}+70\,b^{3}c^{2}dx^{6}+21\,a^{3}d^{3}x^{4}-101\,a^{2}bcd^{2}x^{4}+259\,ab^{2}c^{2}dx^{4}-35\,b^{3}c^{3}x^{4}-28\,a^{3}cd^{2}x^{2}+200\,a^{2}bc}{105\,a^{4}d^{4}-420\,a^{3}bcd^{3}+630\,a^{2}c^{2}d^{2}b^{2}-420\,ac^{3}db^{3}+105\,c^{4}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(6*a^2*b*d^3*x^6-28*a*b^2*c*d^2*x^6+70*b^3*c^2*d*x^6+
21*a^3*d^3*x^4-101*a^2*b*c*d^2*x^4+259*a*b^2*c^2*d*x^4-35*b^3*c^3*x^4-28*a^
3*c*d^2*x^2+200*a^2*b*c^2*d*x^2-28*a*b^2*c^3*x^2+56*a^3*c^2*d-8*a^2*b*c^3)/
(b*x^2+a)^(7/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*
c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.54495, size = 914, normalized size = 4.21

$$\frac{\left(2\left(35\,b^{3}c^{2}d-14\,ab^{2}cd^{2}+3\,a^{2}bd^{3}\right)x^{6}-8\,a^{2}bc^{3}+56\,a}{105\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{8}+4\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*b^3*c^2*d - 14*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^6 - 8*a^2*b*c^3 +
56*a^3*c^2*d - (35*b^3*c^3 - 259*a*b^2*c^2*d + 101*a^2*b*c*d^2 - 21*a^3*d^3
)*x^4 - 4*(7*a*b^2*c^3 - 50*a^2*b*c^2*d + 7*a^3*c*d^2)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7
*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b
^5*c*d^3 + a^4*b^4*d^4)*x^8 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^
2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^6 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3
*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^4 + 4*(a^3*b^5*c^
4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

[Out] Timed out

Giac [B] time = 1.48882, size = 1407, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/105*(35*sqrt(b*d)*b^11*c^5*d - 119*sqrt(b*d)*a*b^10*c^4*d^2 + 150*sqrt(b*
d)*a^2*b^9*c^3*d^3 - 86*sqrt(b*d)*a^3*b^8*c^2*d^4 + 23*sqrt(b*d)*a^4*b^7*c*
d^5 - 3*sqrt(b*d)*a^5*b^6*d^6 - 245*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^9*c^4*d + 588*sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^8*c^3*d^2
 - 462*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d))^2*a^2*b^7*c^2*d^3 + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^6*c*d^4 - 21*sqrt(b*d)*(sqrt(b
*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^5*d^5
+ 630*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^4*b^7*c^3*d - 714*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^6*c^2*d^2 + 42*sqrt(b*d)*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^5*c*d^3 + 42*
sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^4*a^3*b^4*d^4 - 770*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^6*b^5*c^2*d + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b^4*c*d^2 - 210*sqrt(b*d
)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2
*b^3*d^3 + 455*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^8*b^3*c*d + 105*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8*a*b^2*d^2 - 105*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^10*b*d)/((b^2*c -
 a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
)^2)^7*b*abs(b))
```

3.990 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d]))
```

Rubi [A] time = 0.0570364, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {444, 63, 217, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx^2}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\right)}{b}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [B] time = 0.088637, size = 108, normalized size = 2.3

$$\frac{\sqrt{-b}\sqrt{-ad-bc}\sqrt{\frac{b(c+dx^2)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{-ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[-b]*Sqrt[-(b*c) - a*d]*Sqrt[(b*(c + d*x^2))/(b*c + a*d)]*ArcSin[(Sqrt
[-b]*Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[-(b*c) - a*d])])/(b^(3/2)*Sqrt[
d]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.037, size = 108, normalized size = 2.3

$$\frac{1}{2}\arctan\left(\frac{2\,dx^2b-ad+bc}{2\,bd}\sqrt{bd}\frac{1}{\sqrt{-bdx^4+adx^2-bcx^2+ac}}\right)\sqrt{-bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{bd}}\frac{1}{\sqrt{-bdx^4+adx^2-bcx^2+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(b*d)^(1/2)*(2*b*d*x^2-a*d+b*c)/b/d/(-b*d*x^4+a*d*x^2-b*c*x^
2+a*c)^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9173, size = 444, normalized size = 9.45

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^4+b^2c^2-6\,abcd+a^2d^2+8\left(b^2cd-abd^2\right)x^2-4\left(2\,bdx^2+bc-ad\right)\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}\right)}{4\,bd},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d - a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(d*x^2
 + c)*sqrt(-b*d))/(b*d), -1/2*sqrt(b*d)*arctan(1/2*(2*b*d*x^2 + b*c - a*d)*
sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)/(b^2*d^2*x^4 - a*b*c*d + (b^2*c*
d - a*b*d^2)*x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.21223, size = 77, normalized size = 1.64

$$\frac{b\log\left(\left|-\sqrt{-bx^2+a}\sqrt{-bd}+\sqrt{b^2c+\left(bx^2-a\right)bd+abd}\right|\right)}{\sqrt{-bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(-b*d) + sqrt(b^2*c + (b*x^2 - a)*b*d + a*b
*d)))/(sqrt(-b*d)*abs(b))
```

3.991 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

Rubi [A] time = 0.0580089, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 63, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx}\sqrt{c-dx}}\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx^2}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\right)}{b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [B] time = 0.0897496, size = 109, normalized size = 2.27

$$\frac{\sqrt{-b}\sqrt{ad-bc}\sqrt{\frac{b(c-dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[-b]*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c - d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[-b]*Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[-(b*c) + a*d])])/(b^(3/2)*Sqrt
[d]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.04, size = 111, normalized size = 2.3

$$\frac{1}{2}\ln\left(\frac{1}{2}\left(2\,dx^2b+2\,\sqrt{bdx^4-adx^2-bcx^2+ac}\sqrt{bd}-ad-bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{-bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{bdx^4-adx^2-bcx^2+ac}}\frac{1}{\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*d*x^2*b+2*(b*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-a*d
-b*c)/(b*d)^(1/2))*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(b*d)^(1/2)/(b*d*x^4-a
*d*x^2-b*c*x^2+a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89428, size = 446, normalized size = 9.29

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2-8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2-bc-ad\right)\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{bd}\right)}{4\,bd}, -\frac{\sqrt{-bd}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 - b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(-d*x^2
+ c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 - b*c - a*d)*s
qrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d - (b^2*c
*d + a*b*d^2)*x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [A] time = 1.20376, size = 77, normalized size = 1.6

$$\frac{b\log\left(\left|-\sqrt{-bx^2+a}\sqrt{bd}+\sqrt{b^2c-\left(bx^2-a\right)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c - (b*x^2 - a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

3.992 $\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.0451851, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {492, 411}

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx &= \frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}}-\frac{3\int \frac{\sqrt{2+bx^2}}{\left(3+dx^2\right)^{3/2}}\,dx}{b}\\ &= \frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}}-\frac{\sqrt{2}\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}\end{aligned}$$

Mathematica [C] time = 0.0477002, size = 72, normalized size = 0.65

$$-\frac{i\sqrt{3}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[3]*(EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - Ell
ipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)]))/(Sqrt[b]*d)
```

Maple [A] time = 0.021, size = 70, normalized size = 0.6

$$\frac{\sqrt{2}}{b}\left(-\text{EllipticF}\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{b}{d}}\right)+\text{EllipticE}\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{b}{d}}\right)\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] (-EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))+Ell
ipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2)))*2^(1/2)
/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}\sqrt{dx^2+3}x^2}{bdx^4+(3b+2d)x^2+6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)*x^2/(b*d*x^4 + (3*b + 2*d)*x^2 + 6
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.993 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c])
- (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.0697967, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {493, 426, 424, 421, 419}

$$\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c])
- (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[c + d*x^
2])
```


Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx &= \frac{\int \frac{\sqrt{c+dx^2}}{\sqrt{4-x^2}}\,dx}{d} - \frac{c\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx}{d} \\ &= \frac{\sqrt{c+dx^2}\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{4-x^2}}\,dx}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{\left(c\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}} \\ &= \frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{c\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0524682, size = 59, normalized size = 0.68

$$\frac{c\sqrt{\frac{dx^2}{c}+1}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)-\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (c*Sqrt[1 + (d*x^2)/c]*(EllipticE[ArcSin[x/2], (-4*d)/c] - EllipticF[ArcSin
[x/2], (-4*d)/c]))/(d*Sqrt[c + d*x^2])
```

Maple [A] time = 0.02, size = 59, normalized size = 0.7

$$\frac{c}{d}\left(-\text{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+\text{EllipticE}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{dx^2+c}{c}}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-EllipticF(1/2*x,2*(-d/c)^(1/2))+EllipticE(1/2*x,2*(-d/c)^(1/2)))/(d*x^2+c
)^(1/2)*c*((d*x^2+c)/c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{dx^2+c}\sqrt{-x^2+4}x^2}{dx^4+(c-4\,d)x^2-4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^2 + c)*sqrt(-x^2 + 4)*x^2/(d*x^4 + (c - 4*d)*x^2 - 4*c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.994 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi [A] time = 0.0361522, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {492, 411}

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx&=\frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{4\int \frac{\sqrt{c+dx^2}}{\left(4+x^2\right)^{3/2}}\,dx}{d}\\&=\frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c\left(4+x^2\right)}}}\end{aligned}$$

Mathematica [C] time = 0.0489789, size = 70, normalized size = 0.8

$$-\frac{ic\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{2}\right),\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[x/2], (4*d)/c] - EllipticF
[I*ArcSinh[x/2], (4*d)/c]))/(d*Sqrt[c + d*x^2])
```

Maple [A] time = 0.016, size = 76, normalized size = 0.9

$$-2\frac{1}{\sqrt{dx^2+c}}\sqrt{\frac{dx^2+c}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*(EllipticF(x*(-d/c)^(1/2),1/2*(c/d)^
(1/2))-EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2)))/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}\sqrt{x^2+4}x^2}{dx^4+(c+4\,d)x^2+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*sqrt(x^2 + 4)*x^2/(d*x^4 + (c + 4*d)*x^2 + 4*c), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.995 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)$$

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSin[x], -3/2
])/3
```

Rubi [A] time = 0.0296919, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSin[x], -3/2
])/3
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx &= \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-x^2}}\,dx-\frac{2}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx\\ &= \frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0305914, size = 24, normalized size = 0.77

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}(x)\left|-\frac{3}{2}\right.\right)-\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*(EllipticE[ArcSin[x], -3/2] - EllipticF[ArcSin[x], -3/2]))/3
```

Maple [A] time = 0.018, size = 25, normalized size = 0.8

$$-\frac{\left(\text{EllipticF}\left(x,\frac{i}{2}\sqrt{6}\right)-\text{EllipticE}\left(x,\frac{i}{2}\sqrt{6}\right)\right)\sqrt{2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*(EllipticF(x,1/2*I*6^(1/2))-EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2+2}\sqrt{-x^2+1}x^2}{3\,x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)*x^2/(3*x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.996 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sqrt{2}\text{EllipticF}\left(\sin^{-1}(x),\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], 3/2])/3 + (Sqrt[2]*EllipticF[ArcSin[x], 3/2]
)/3
```

Rubi [A] time = 0.0297203, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], 3/2])/3 + (Sqrt[2]*EllipticF[ArcSin[x], 3/2]
)/3
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx &= -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-x^2}}\,dx\right)+\frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|\frac{3}{2}\right)+\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|\frac{3}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0363519, size = 37, normalized size = 1.19

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)-E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] (-EllipticE[ArcSin[Sqrt[3/2]*x], 2/3] + EllipticF[ArcSin[Sqrt[3/2]*x], 2/3]
)/Sqrt[3]
```

Maple [A] time = 0.017, size = 23, normalized size = 0.7

$$\frac{\sqrt{2}}{3}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/3*2^(1/2)*(EllipticF(x,1/2*6^(1/2))-EllipticE(x,1/2*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}x^2}{3\,x^4-5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)*x^2/(3*x^4 - 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - 3*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.997 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-6\right)$$

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSin[x/2], -6
])/3
```

Rubi [A] time = 0.0314943, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSin[x/2], -6
])/3
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx &= \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{4-x^2}}\,dx-\frac{2}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx\\ &= \frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)\end{aligned}$$

Mathematica [A] time = 0.0300229, size = 28, normalized size = 0.8

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*(EllipticE[ArcSin[x/2], -6] - EllipticF[ArcSin[x/2], -6]))/3
```

Maple [A] time = 0.02, size = 29, normalized size = 0.8

$$-\frac{\sqrt{2}}{3}\left(\text{EllipticF}\left(\frac{x}{2},i\sqrt{6}\right)-\text{EllipticE}\left(\frac{x}{2},i\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*(EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2+2}\sqrt{-x^2+4}x^2}{3\,x^4-10\,x^2-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)*x^2/(3*x^4 - 10*x^2 - 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

3.998 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}\sqrt{2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], 6])/3 + (Sqrt[2]*EllipticF[ArcSin[x/2], 6]
)/3
```

Rubi [A] time = 0.0316317, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], 6])/3 + (Sqrt[2]*EllipticF[ArcSin[x/2], 6]
)/3
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx &= -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{4-x^2}}\,dx\right)+\frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)+\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)\end{aligned}$$

Mathematica [A] time = 0.0327616, size = 38, normalized size = 1.09

$$-\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)-\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] (-2*(EllipticE[ArcSin[Sqrt[3/2]*x], 1/6] - EllipticF[ArcSin[Sqrt[3/2]*x], 1
/6]))/Sqrt[3]
```

Maple [A] time = 0.018, size = 33, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x)
```

```
[Out] 2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*6^(1/2))-EllipticE(1/2*x*6^(1/2),1
/6*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}x^2}{3\,x^4-14\,x^2+8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)*x^2/(3*x^4 - 14*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+4)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(2 - 3*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.999 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{\text{EllipticF}\left(\sin^{-1}(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x], -3/8]/(3*
Sqrt[2])
```

Rubi [A] time = 0.0301921, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x], -3/8]/(3*
Sqrt[2])
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx &= \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-4x^2}}\,dx-\frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx\\ &= \frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.031602, size = 28, normalized size = 0.8

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)-\text{EllipticF}\left(\sin^{-1}(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (EllipticE[ArcSin[2*x], -3/8] - EllipticF[ArcSin[2*x], -3/8])/(3*Sqrt[2])
```

Maple [A] time = 0.023, size = 29, normalized size = 0.8

$$-\frac{\left(\text{EllipticF}\left(2x,\frac{i}{4}\sqrt{6}\right)-\text{EllipticE}\left(2x,\frac{i}{4}\sqrt{6}\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/6*(EllipticF(2*x,1/4*I*6^(1/2))-EllipticE(2*x,1/4*I*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{-4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3x^2+2}\sqrt{-4x^2+1}x^2}{12x^4+5x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)*x^2/(12*x^4 + 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{-4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1000 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\text{EllipticF}\left(\sin^{-1}(2x),\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] -EllipticE[ArcSin[2*x], 3/8]/(3*Sqrt[2]) + EllipticF[ArcSin[2*x], 3/8]/(3*S
qrt[2])
```

Rubi [A] time = 0.0319154, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{F\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

```
[Out] -EllipticE[ArcSin[2*x], 3/8]/(3*Sqrt[2]) + EllipticF[ArcSin[2*x], 3/8]/(3*S
qrt[2])
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx &= -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-4x^2}}\,dx\right)+\frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx\\ &= -\frac{E\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}+\frac{F\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0310396, size = 28, normalized size = 0.8

$$\frac{\text{EllipticF}\left(\sin^{-1}(2x),\frac{3}{8}\right)-E\left(\sin^{-1}(2x)|\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

```
[Out] (-EllipticE[ArcSin[2*x], 3/8] + EllipticF[ArcSin[2*x], 3/8])/(3*Sqrt[2])
```

Maple [A] time = 0.021, size = 27, normalized size = 0.8

$$\frac{\sqrt{2}}{6}\left(\text{EllipticF}\left(2x,\frac{\sqrt{6}}{4}\right)-\text{EllipticE}\left(2x,\frac{\sqrt{6}}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(EllipticF(2*x,1/4*6^(1/2))-EllipticE(2*x,1/4*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3x^2+2}\sqrt{-4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-3x^2+2}\sqrt{-4x^2+1}x^2}{12x^4-11x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)*x^2/(12*x^4 - 11*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(2 - 3*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1001 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[Sqrt[3/2]*x
], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0282783, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {493, 424, 419}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[Sqrt[3/2]*x
], -2/3]/Sqrt[3]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx &= -\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx+\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx\\ &= \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.029042, size = 37, normalized size = 0.88

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)-\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -2/3] - EllipticF[ArcSin[Sqrt[3/2]*x], -2/3
])/Sqrt[3]
```

Maple [A] time = 0.017, size = 35, normalized size = 0.8

$$-\frac{\sqrt{3}}{3}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/2
),1/3*I*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-3\,x^2+2}x^2}{3\,x^4+x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)*x^2/(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1002 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[ArcSin[Sqrt
[3/2]*x], -1/6])/Sqrt[3]
```

Rubi [A] time = 0.0289706, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {493, 424, 419}

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[ArcSin[Sqrt
[3/2]*x], -1/6])/Sqrt[3]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx &= -\left(4\int \frac{1}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx\right)+\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx\\ &= \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0305339, size = 38, normalized size = 0.88

$$\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)-\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*(EllipticE[ArcSin[Sqrt[3/2]*x], -1/6] - EllipticF[ArcSin[Sqrt[3/2]*x], -
1/6]))/Sqrt[3]
```

Maple [A] time = 0.017, size = 35, normalized size = 0.8

$$-\frac{2\sqrt{3}}{3}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i}{6}\sqrt{6}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i}{6}\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x)
```

```
[Out] -2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*I*6^(1/2))-EllipticE(1/2*x*6^(1/2
),1/6*I*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+4}\sqrt{-3\,x^2+2}x^2}{3\,x^4+10\,x^2-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)*x^2/(3*x^4 + 10*x^2 - 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+4)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.1003 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{8}{3}\right)}{4\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcSin[Sqrt[3/
2]*x], -8/3]/(4*Sqrt[3])
```

Rubi [A] time = 0.0312434, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 424, 419}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcSin[Sqrt[3/
2]*x], -8/3]/(4*Sqrt[3])
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx &= -\left(\frac{1}{4}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx\right)+\frac{1}{4}\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx\\ &= \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0328761, size = 40, normalized size = 0.85

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)-\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -8/3] - EllipticF[ArcSin[Sqrt[3/2]*x], -8/3
])/(4*Sqrt[3])
```

Maple [A] time = 0.017, size = 35, normalized size = 0.7

$$-\frac{\sqrt{3}}{12}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{2i}{3}\sqrt{6}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{2i}{3}\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x)
```

```
[Out] -1/12*3^(1/2)*(EllipticF(1/2*x*6^(1/2),2/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/
2),2/3*I*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4x^2+1}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4x^2+1}\sqrt{-3x^2+2}x^2}{12x^4-5x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)*x^2/(12*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(4*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1004 $\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0275457, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {492, 411}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx &= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\left(1+x^2\right)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}\end{aligned}$$

Mathematica [C] time = 0.0279188, size = 34, normalized size = 0.42

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}(x)|\frac{3}{2}\right)-\text{EllipticF}\left(i\sinh^{-1}(x),\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (-I/3)*Sqrt[2]*(EllipticE[I*ArcSinh[x], 3/2] - EllipticF[I*ArcSinh[x], 3/2]
)
```

Maple [A] time = 0.014, size = 30, normalized size = 0.4

$$\frac{i}{3}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{3\,x^2+2}\sqrt{x^2+1}x^2}{3\,x^4+5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)*x^2/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.1005 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```

Rubi [A] time = 0.0291869, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {492, 411}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```


```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```


Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx &= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\left(4+x^2\right)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}\end{aligned}$$

Mathematica [C] time = 0.0280067, size = 38, normalized size = 0.46

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)\middle|6\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{2}\right),6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (-I/3)*Sqrt[2]*(EllipticE[I*ArcSinh[x/2], 6] - EllipticF[I*ArcSinh[x/2], 6]
)
```

Maple [A] time = 0.015, size = 26, normalized size = 0.3

$$\frac{i}{3}\left(\text{EllipticF}\left(\frac{i}{2}x,\sqrt{6}\right)-\text{EllipticE}\left(\frac{i}{2}x,\sqrt{6}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*(EllipticF(1/2*I*x,6^(1/2))-EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3x^2+2}\sqrt{x^2+4}x^2}{3x^4+14x^2+8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)*x^2/(3*x^4 + 14*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 4)*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

3.1006 $\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\tan^{-1}(2x)\middle|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*EllipticE[ArcTan
[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rubi [A] time = 0.0307442, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {492, 411}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\tan^{-1}(2x)\middle|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*EllipticE[ArcTan
[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx&=\frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}-\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\left(1+4x^2\right)^{3/2}}\,dx\\&=\frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}-\frac{\sqrt{2+3x^2}E\left(\tan^{-1}(2x)\middle|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}\end{aligned}$$

Mathematica [C] time = 0.0325587, size = 50, normalized size = 0.57

$$-\frac{i\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{8}{3}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{8}{3}\right)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ((-I/4)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3] - EllipticF[I*ArcSinh[Sqrt[
3/2]*x], 8/3]))/Sqrt[3]
```

Maple [C] time = 0.017, size = 36, normalized size = 0.4

$$\frac{i}{12}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{6},\frac{2\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{i}{2}x\sqrt{6},\frac{2\sqrt{6}}{3}\right)\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x)
```

```
[Out] 1/12*I*(EllipticF(1/2*I*x*6^(1/2),2/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),2/
3*6^(1/2)))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}x^2}{12\,x^4+11\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)*x^2/(12*x^4 + 11*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(3*x**2 + 2)*sqrt(4*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

3.1007 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}\text{EllipticF}\left(\cos^{-1}(x),2\right)-\frac{1}{2}E\left(\cos^{-1}(x)\middle|2\right)$$

```
[Out] -EllipticE[ArcCos[x], 2]/2 - EllipticF[ArcCos[x], 2]/2
```

Rubi [A] time = 0.0278453, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {493, 425, 420}

$$-\frac{1}{2}F\left(\cos^{-1}(x)\middle|2\right)-\frac{1}{2}E\left(\cos^{-1}(x)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] -EllipticE[ArcCos[x], 2]/2 - EllipticF[ArcCos[x], 2]/2
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 425

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> -Simp
[(Sqrt[a - (b*c)/d]*EllipticE[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)])/
(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c
, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx+\frac{1}{2}\int \frac{\sqrt{-1+2x^2}}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}E\left(\cos^{-1}(x)\middle|2\right)-\frac{1}{2}F\left(\cos^{-1}(x)\middle|2\right)\end{aligned}$$

Mathematica [B] time = 0.0348798, size = 37, normalized size = 2.18

$$\frac{\sqrt{1-2x^2}\left(\text{EllipticF}\left(\sin^{-1}(x),2\right)-E\left(\sin^{-1}(x)\middle|2\right)\right)}{2\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*(-EllipticE[ArcSin[x], 2] + EllipticF[ArcSin[x], 2]))/(2*S
qrt[-1 + 2*x^2])
```

Maple [A] time = 0.018, size = 34, normalized size = 2.

$$\frac{\text{EllipticF}\left(x,\sqrt{2}\right)-\text{EllipticE}\left(x,\sqrt{2}\right)}{2}\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(x,2^(1/2))-EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{2\,x^2-1}\sqrt{-x^2+1}x^2}{2\,x^4-3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)*x^2/(2*x^4 - 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)\,(x+1)}\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.1008 $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=109

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTan[(1 + (2
- 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2])/(4*2^(2/3)) + (27*
Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.0877867, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTan[(1 + (2
- 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2])/(4*2^(2/3)) + (27*
Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}(3+x)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2}{\sqrt[3]{1-x}} - (1-x)^{2/3} + \frac{9}{\sqrt[3]{1-x}(3+x)}\right) dx, x, x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3} + \frac{3}{10}\left(1-x^2\right)^{5/3} + \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx, x, x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3} + \frac{3}{10}\left(1-x^2\right)^{5/3} - \frac{9\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{27}{4}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right.\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3} + \frac{3}{10}\left(1-x^2\right)^{5/3} - \frac{9\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}} - \frac{27\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{2\,2}\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3} + \frac{3}{10}\left(1-x^2\right)^{5/3} + \frac{9\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} - \frac{9\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right.}{4\,2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0754938, size = 106, normalized size = 0.97

$$-\frac{3}{40}\left(4\left(1-x^2\right)^{2/3}x^2-24\left(1-x^2\right)^{2/3}+15\sqrt[3]{2}\log\left(x^2+3\right)-45\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-30\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(-24*(1 - x^2)^(2/3) + 4*x^2*(1 - x^2)^(2/3) - 30*2^(1/3)*Sqrt[3]*ArcTa
n[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 15*2^(1/3)*Log[3 + x^2] - 45*2^(1/3)*L
og[2^(2/3) - (1 - x^2)^(1/3)]))/40
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^5}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.48205, size = 146, normalized size = 1.34

$$\frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
+ 3/2*(-x^2 + 1)^(2/3)
```

Fricas [A] time = 1.51807, size = 319, normalized size = 2.93

$$-\frac{3}{10}\left(x^2-6\right)\left(-x^2+1\right)^{\frac{2}{3}}+\frac{9}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/10*(x^2 - 6)*(-x^2 + 1)^(2/3) + 9/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*s
qrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)
*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 +
1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1009 $\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=94

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2^(2/3) - (1 - x^2)^(
1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.0653427, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2^(2/3) - (1 - x^2)^(
1/3)])/(4*2^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}}-\frac{9}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)+\frac{9\,\text{Subst}\,(\ldots}{\ldots}$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2-\ldots}\right)}{2\ 2^{2/3}}$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Mathematica [A] time = 0.0293063, size = 90, normalized size = 0.96

$$-\frac{3}{8}\left(2\left(1-x^2\right)^{2/3}-\sqrt[3]{2}\log\left(x^2+3\right)+3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/S
qrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)]))
/8
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.47669, size = 131, normalized size = 1.39

$$-\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) - 3/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Fricas [A] time = 1.52992, size = 408, normalized size = 4.34

$$-\frac{3}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)-\frac{3}{16}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/4*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*(-1)^(1/3)*(-
x^2 + 1)^(1/3) - 4^(1/3))) - 3/16*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(-1)^(2/3)
*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 3/8*4^(2/3)*(-
1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1010 $\int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=79

$$-\frac{\log(x^2+3)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - Log[3 + x^2
]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.052399, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log(x^2+3)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - Log[3 + x^2
]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)-\frac{3\operatorname{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx,x,\sqrt[3]{1-\right.}{4\ 2^{2/3}}$$
$$= -\frac{\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2-2x^2}\right)}{2\ 2^{2/3}}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Mathematica [A] time = 0.0166833, size = 67, normalized size = 0.85

$$\frac{-\log\left(x^2+3\right)+3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - Log[3 + x^2] + 3*Log[2
^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{x}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.47863, size = 116, normalized size = 1.47

$$\frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/
3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

Fricas [A] time = 1.5138, size = 271, normalized size = 3.43

$$\frac{1}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3
))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3
)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1011 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=136

$$\frac{\log(x^2+3)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

```
[Out] -ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(1 +
2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Log[3 + x^2]/(12*2^(2/
3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(4*2^(2/3
))
```

Rubi [A] time = 0.0990665, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log(x^2+3)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(1 +
2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Log[3 + x^2]/(12*2^(2/
3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(4*2^(2/3
))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(3+x)}\,dx,x,x^2\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx,x,x^2\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\log(x)}{6}+\frac{\log\left(3+x^2\right)}{12\ 2^{2/3}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^2}\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,\right.\\
&= -\frac{\log(x)}{6}+\frac{\log\left(3+x^2\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-}\right.\\
&= -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}+\frac{\log\left(3+x^2\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-
\end{aligned}$$

Mathematica [A] time = 0.0359399, size = 127, normalized size = 0.93

$$\frac{1}{24}\left(\sqrt[3]{2}\log\left(x^2+3\right)+6\log\left(1-\sqrt[3]{1-x^2}\right)-3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\right($$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 4*Sqrt[3]*Arc
Tan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 4*Log[x] + 2^(1/3)*Log[3 + x^2] + 6*
Log[1 - (1 - x^2)^(1/3)] - 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/24
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A] time = 1.5695, size = 591, normalized size = 4.35

$$-\frac{1}{12}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\,(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{48}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/12*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)^(1/3)*(
-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) - 1/48*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(
-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 1/24*
4^(2/3)*(-1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) + 1/6*sqrt(3
)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/12*log((-x^2 + 1)^
(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/6*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1012 $\int \frac{1}{x^3 \sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=97

$$-\frac{(1-x^2)^{2/3}}{6x^2}-\frac{\log(x^2+3)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^2)^(2/3)/(6*x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(6*2^(2/
3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/3)]/(1
2*2^(2/3))
```

Rubi [A] time = 0.0744409, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 103, 12, 55, 617, 204, 31}

$$-\frac{(1-x^2)^{2/3}}{6x^2}-\frac{\log(x^2+3)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] -(1 - x^2)^(2/3)/(6*x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(6*2^(2/
3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/3)]/(1
2*2^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^2(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2}-\frac{1}{6}\text{Subst}\left(\int -\frac{1}{3\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2}-\frac{\log\left(3+x^2\right)}{36\ 2^{2/3}}+\frac{1}{12}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)-\frac{\text{Subst}\left(\int \right.}{}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2}-\frac{\log\left(3+x^2\right)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2-2x^2}\right)}{6\ 2^{2/3}}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\log\left(3+x^2\right)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0447154, size = 93, normalized size = 0.96

$$\frac{1}{72}\left(-\frac{12\left(1-x^2\right)^{2/3}}{x^2}-\sqrt[3]{2}\log\left(x^2+3\right)+3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ((-12*(1 - x^2)^(2/3))/x^2 + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3
)])/72
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(x^{2}+3\right)} \frac{1}{\sqrt[3]{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}+3\right)\left(-x^{2}+1\right)^{\frac{1}{3}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A] time = 1.52997, size = 332, normalized size = 3.42

$$\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3} x^{2} \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{1}{3}} \sqrt{3} + 2 \sqrt{3} \left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right) - 4^{\frac{2}{3}} x^{2} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}} + \left(-x^{2}+1\right)^{\frac{2}{3}}\right) + 2 \cdot 4^{\frac{2}{3}} x^{2} \log\left(-4^{\frac{1}{3}}\right.}{144 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/144*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*sqrt(3
)*(-x^2 + 1)^(1/3))) - 4^(2/3)*x^2*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) +
 (-x^2 + 1)^(2/3)) + 2*4^(2/3)*x^2*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 24*(-
x^2 + 1)^(2/3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \sqrt[3]{-\left(x-1\right)\left(x+1\right)} \left(x^{2}+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1013 $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)} dx$

Optimal. Leaf size=172

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\left(1-x^2\right)^{2/3}}{12x^4}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2}{\ldots}\right)}{9\sqrt{\ldots}}$$

```
[Out] -(1 - x^2)^(2/3)/(12*x^4) - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/S
qrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2]/(108*2^(2/3)) + Log[1 - (1 -
 x^2)^(1/3)]/18 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(36*2^(2/3))
```

Rubi [A] time = 0.129859, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\left(1-x^2\right)^{2/3}}{12x^4}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2}{\ldots}\right)}{9\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] -(1 - x^2)^(2/3)/(12*x^4) - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/S
qrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2]/(108*2^(2/3)) + Log[1 - (1 -
 x^2)^(1/3)]/18 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(36*2^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^3(3+x)}\,dx,x,x^2\right)$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{1}{12}\,\text{Subst}\left(\int \frac{-2-\frac{4x}{3}}{\sqrt[3]{1-x}x^2(3+x)}\,dx,x,x^2\right)$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{\left(1-x^2\right)^{2/3}}{18x^2} + \frac{1}{36}\,\text{Subst}\left(\int \frac{4+\frac{2x}{3}}{\sqrt[3]{1-x}x(3+x)}\,dx,x,x^2\right)$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{\left(1-x^2\right)^{2/3}}{18x^2} - \frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right) + \frac{1}{27}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{\left(1-x^2\right)^{2/3}}{18x^2} - \frac{\log(x)}{27} + \frac{\log\left(3+x^2\right)}{108\ 2^{2/3}} - \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{\left(1-x^2\right)^{2/3}}{18x^2} - \frac{\log(x)}{27} + \frac{\log\left(3+x^2\right)}{108\ 2^{2/3}} + \frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{\log\left(2^{2/3}-\sqrt[3]{1-}\right.}{36\ 2^{2/3}}$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4} - \frac{\left(1-x^2\right)^{2/3}}{18x^2} - \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{9\sqrt{3}} - \frac{\log(x)}{27} + \frac{\log\left(3+x^2\right)}{108\ 2^{2/3}}$$

Mathematica [A] time = 0.0699829, size = 178, normalized size = 1.03

$$-\frac{12\left(1-x^2\right)^{2/3}x^2+18\left(1-x^2\right)^{2/3}+8x^4\log(x)-\sqrt[3]{2}x^4\log\left(x^2+3\right)-12x^4\log\left(1-\sqrt[3]{1-x^2}\right)+3\sqrt[3]{2}x^4\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right.}{216x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(18*(1 - x^2)^(2/3) + 12*x^2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*x^4*ArcTa
n[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 8*Sqrt[3]*x^4*ArcTan[(1 + 2*(1 - x^2)^
(1/3))/Sqrt[3]] + 8*x^4*Log[x] - 2^(1/3)*x^4*Log[3 + x^2] - 12*x^4*Log[1 -
(1 - x^2)^(1/3)] + 3*2^(1/3)*x^4*Log[2^(2/3) - (1 - x^2)^(1/3)])/(216*x^4)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A] time = 1.54489, size = 664, normalized size = 3.86

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}x^4\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^4\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/432*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)
^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 4^(2/3)*(-1)^(1/3)*x^4*log(4^
(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3))
- 2*4^(2/3)*(-1)^(1/3)*x^4*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 16
*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 8*x^4*log
((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 16*x^4*log((-x^2 + 1)^(1/3) - 1
) + 12*(2*x^2 + 3)*(-x^2 + 1)^(2/3))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1014 $\int \frac{x^4}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx$

Optimal. Leaf size=536

$$-\frac{18\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{3}{7}\left(1-x^2\right)^{2/3}x+\frac{54x}{7\left(-\sqrt[3]{1-x^2}-\right.}$$

```
[Out] (-3*x*(1 - x^2)^(2/3))/7 + (54*x)/(7*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (3*
Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) + (3*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^
(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) - (3*ArcTanh[x])/(2*2^(2/3)) + (9*A
rcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) + (27*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((
1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (18*Sqrt[2]*3^(
3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((1 - (
1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.278392, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {479, 530, 235, 304, 219, 1879, 393}

$$-\frac{3}{7}\left(1-x^2\right)^{2/3}x+\frac{54x}{7\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}+\frac{9\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{18\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] (-3*x*(1 - x^2)^(2/3))/7 + (54*x)/(7*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (3*
Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) + (3*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^
(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) - (3*ArcTanh[x])/(2*2^(2/3)) + (9*A
rcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) + (27*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((
1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (18*Sqrt[2]*3^(
3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((1 - (
1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```


Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3}{7}\int \frac{3-6x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$
$$= -\frac{3}{7}x\left(1-x^2\right)^{2/3}-\frac{18}{7}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx+9\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$
$$= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{3\tanh^{-1}(x)}{2\ 2^{2/3}}+\frac{9\tanh^{-1}\left(\right.}{2\ 2}$$
$$= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{3\tanh^{-1}(x)}{2\ 2^{2/3}}+\frac{9\tanh^{-1}\left(\right.}{2\ 2}$$
$$= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{54x}{7\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{3}{}$$

Mathematica [C] time = 0.1128, size = 156, normalized size = 0.29

$$\frac{1}{7}x\left(\frac{3\left(-\frac{27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}+x^2-1\right)}{\sqrt[3]{1-x^2}}-2x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x*(-2*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + (3*(-1 + x^2 - (27*App
ellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((3 + x^2)*(-9*AppellF1[1/2, 1/3, 1,
3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - Appell
F1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))))/(1 - x^2)^(1/3)))/7
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^4}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^2+1\right)^{\frac{2}{3}}x^4}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^4/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1015 $\int \frac{x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)} dx$

Optimal. Leaf size=515

$$\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{3x}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}-\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{2\cdots}$$

```
[Out] (-3*x)/(1 - Sqrt[3] - (1 - x^2)^(1/3)) - (Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(
2/3)) - (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2
/3)) + ArcTanh[x]/(2*2^(2/3)) - (3*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]
)/(2*2^(2/3)) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1
+ (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(2*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x
^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))^2)])
```

Rubi [A] time = 0.167402, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {483, 235, 304, 219, 1879, 393}

$$-\frac{3x}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\cdots}{\sqrt{-\frac{1-\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-x^2}-\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*x)/(1 - Sqrt[3] - (1 - x^2)^(1/3)) - (Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(
2/3)) - (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2
/3)) + ArcTanh[x]/(2*2^(2/3)) - (3*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]
)/(2*2^(2/3)) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1
+ (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(2*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x
^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))^2)])
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = -\left(3\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx\right) + \int \frac{1}{\sqrt[3]{1-x^2}}\,dx$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}(x)}{2\ 2^{2/3}} - \frac{3\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \left(3\sqrt{-x \cdots\right.$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}(x)}{2\ 2^{2/3}} - \frac{3\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} + \left(3\sqrt{-x \cdots\right.$$

$$= -\frac{3x}{1-\sqrt{3}-\sqrt[3]{1-x^2}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}(x)}{2\ 2^{2/3}} - \frac{3\tanh^{-1}\left(\cdots\right.}{2\ 2 \cdots}$$

Mathematica [C] time = 0.0184728, size = 28, normalized size = 0.05

$$\frac{1}{9}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/9
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^2+1\right)^{\frac{2}{3}}x^2}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^2/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1016 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.0130406, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.03098, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(
-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 7.79614, size = 5544, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
```

```
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1017 $\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)} dx$

Optimal. Leaf size=538

$$-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{x}{3\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{(1-x^2)^{2/3}}{3x}$$

```
[Out] -(1 - x^2)^(2/3)/(3*x) + x/(3*(1 - Sqrt[3] - (1 - x^2)^(1/3))) - ArcTan[Sqr
t[3]/x]/(6*2^(2/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3))
)/x]/(6*2^(2/3)*Sqrt[3]) + ArcTanh[x]/(18*2^(2/3)) - ArcTanh[x/(1 + 2^(1/3)
*(1 - x^2)^(1/3))]/(6*2^(2/3)) + (Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 +
 (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3]
 - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.23598, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {480, 530, 235, 304, 219, 1879, 393}

$$\frac{x}{3\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{(1-x^2)^{2/3}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{6\ 2^{2/3}}-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1}\cdots}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}\cdots\right.}}}{3\sqrt[4]{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] -(1 - x^2)^(2/3)/(3*x) + x/(3*(1 - Sqrt[3] - (1 - x^2)^(1/3))) - ArcTan[Sqr
t[3]/x]/(6*2^(2/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3))
)/x]/(6*2^(2/3)*Sqrt[3]) + ArcTanh[x]/(18*2^(2/3)) - ArcTanh[x/(1 + 2^(1/3)
*(1 - x^2)^(1/3))]/(6*2^(2/3)) + (Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 +
 (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3]
 - (1 - x^2)^(1/3))^2)])
```


Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{3x} + \frac{1}{3}\int \frac{-2-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{1}{9}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}(x)}{18\ 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{6\ 2^{2/3}} +$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}(x)}{18\ 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{6\ 2^{2/3}} -$$

$$= -\frac{(1-x^2)^{2/3}}{3x} + \frac{x}{3\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}(x)}{18\ 2^{2/3}} -$$

Mathematica [C] time = 0.0942054, size = 161, normalized size = 0.3

$$\frac{\frac{18x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}+x^2-1}{3x\sqrt[3]{1-x^2}} - \frac{1}{81}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/81 + (-1 + x^2 + (18*x^2*App
ellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((3 + x^2)*(-9*AppellF1[1/2, 1/3, 1,
3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - Appell
F1[3/2, 4/3, 1, 5/2, x^2, -x^2/3]))))/(3*x*(1 - x^2)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(x^2+3)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{x^6+2\,x^4-3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^6 + 2*x^4 - 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1018 $\int \frac{1}{x^4\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=556

$$-\frac{2\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{27\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{2x}{27\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{2(1-x^2)}{27x}$$

```
[Out] -(1 - x^2)^(2/3)/(9*x^3) - (2*(1 - x^2)^(2/3))/(27*x) + (2*x)/(27*(1 - Sqrt
[3] - (1 - x^2)^(1/3))) + ArcTan[Sqrt[3]/x]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(
Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(18*2^(2/3)*Sqrt[3]) - ArcTanh[x]
/(54*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(18*2^(2/3)) + (Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3
/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (
2*Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*x
*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.309975, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 583, 530, 235, 304, 219, 1879, 393}

$$\frac{2x}{27\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{2(1-x^2)^{2/3}}{27x}-\frac{(1-x^2)^{2/3}}{9x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{18\,2^{2/3}}-\frac{2\sqrt{2}\left(1-\sqrt[3]{1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] -(1 - x^2)^(2/3)/(9*x^3) - (2*(1 - x^2)^(2/3))/(27*x) + (2*x)/(27*(1 - Sqrt
[3] - (1 - x^2)^(1/3))) + ArcTan[Sqrt[3]/x]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(
Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(18*2^(2/3)*Sqrt[3]) - ArcTanh[x]
/(54*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(18*2^(2/3)) + (Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3
/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (
2*Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*x
*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```


Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
```

```
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\left(3+x^2\right)}dx = -\frac{\left(1-x^2\right)^{2/3}}{9x^3}+\frac{1}{9}\int \frac{2+\frac{5x^2}{3}}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)}dx$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{9x^3}-\frac{2\left(1-x^2\right)^{2/3}}{27x}-\frac{1}{27}\int \frac{-1+\frac{2x^2}{3}}{\sqrt[3]{1-x^2}\left(3+x^2\right)}dx$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{9x^3}-\frac{2\left(1-x^2\right)^{2/3}}{27x}-\frac{2}{81}\int \frac{1}{\sqrt[3]{1-x^2}}dx+\frac{1}{9}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}dx$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{9x^3}-\frac{2\left(1-x^2\right)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{54\ 2^{2/3}}+\frac{\tanh^{-1}}{1}$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{9x^3}-\frac{2\left(1-x^2\right)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{54\ 2^{2/3}}+\frac{\tanh^{-1}}{1}$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{9x^3}-\frac{2\left(1-x^2\right)^{2/3}}{27x}+\frac{2x}{27\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.115841, size = 166, normalized size = 0.3

$$\frac{-\frac{9x^4F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}+2x^4+x^2-3}{27x^3\sqrt[3]{1-x^2}}-\frac{2}{729}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/729 + (-3 + x^2 + 2*x^4 -
(9*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((3 + x^2)*(-9*AppellF1[1/2
, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3
] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3]))))/(27*x^3*(1 - x^2)^(1/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(-x^2+1)^{\frac{2}{3}}}{x^8+2\,x^6-3\,x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^8 + 2*x^6 - 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1019 $\int \frac{x^7}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2} \, dx$

Optimal. Leaf size=133

$$-\frac{3\left(1-x^2\right)^{2/3} x^4}{10\left(x^2+3\right)}+\frac{9\left(1-x^2\right)^{2/3}\left(14 x^2+69\right)}{40\left(x^2+3\right)}-\frac{99 \log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297 \log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69 + 14*x^2))
/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2
^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (297*Log[2^(2/3) - (1 - x^2)^(1/
3)])/(16*2^(2/3))
```

Rubi [A] time = 0.0857265, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 100, 146, 55, 617, 204, 31}

$$-\frac{3\left(1-x^2\right)^{2/3} x^4}{10\left(x^2+3\right)}+\frac{9\left(1-x^2\right)^{2/3}\left(14 x^2+69\right)}{40\left(x^2+3\right)}-\frac{99 \log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297 \log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69 + 14*x^2))
/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2
^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (297*Log[2^(2/3) - (1 - x^2)^(1/
3)])/(16*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
```

```
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}(3+x)^2}\,dx, x, x^2\right)$$

$$= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} - \frac{3}{10}\operatorname{Subst}\left(\int \frac{x(-6+7x)}{\sqrt[3]{1-x}(3+x)^2}\,dx, x, x^2\right)$$

$$= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} + \frac{99}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}} + \frac{297}{16}\operatorname{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/}}\right.$$

$$= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}} + \frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

$$= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} + \frac{99\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}} +$$

Mathematica [A] time = 0.168505, size = 120, normalized size = 0.9

$$\frac{3}{80}\left(-\frac{8\left(1-x^2\right)^{2/3}x^4}{x^2+3}+\frac{6\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{x^2+3}+\frac{165\left(-\log\left(x^2+3\right)+3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)}{2^{2/3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((-8*x^4*(1 - x^2)^(2/3))/(3 + x^2) + (6*(1 - x^2)^(2/3)*(69 + 14*x^2))/
(3 + x^2) + (165*(2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - Log[3
 + x^2] + 3*Log[2^(2/3) - (1 - x^2)^(1/3)]))/2^(2/3)))/80
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.49337, size = 170, normalized size = 1.28

$$\frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1
/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A] time = 1.56803, size = 394, normalized size = 2.96

$$\frac{3\left(660\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-165\cdot 4^{\frac{2}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)\right.}{320\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/320*(660*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) +
2*(-x^2 + 1)^(1/3))) - 165*4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 +
1)^(1/3) + (-x^2 + 1)^(2/3)) + 330*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 +
 1)^(1/3)) - 8*(4*x^4 - 42*x^2 - 207)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1020 $\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*Sqrt[3]*Ar
cTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (21*Log[3 + x^2])/(16*
2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi [A] time = 0.0774002, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 89, 80, 55, 617, 204, 31}

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*Sqrt[3]*Ar
cTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (21*Log[3 + x^2])/(16*
2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}(3+x)^2}\,dx,x,x^2\right)$$

$$= -\frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{1}{8}\,\text{Subst}\left(\int \frac{-9+4x}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{21}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - \frac{63}{16}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{\cdots}\right.$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - \frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}} + \frac{63\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{21\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - \frac{63\log\left(2^{2/3}-\cdots\right.}{16\ 2^{2/\cdots}}$$

Mathematica [A] time = 0.147833, size = 110, normalized size = 0.95

$$\frac{3}{32}\left(-8\left(1-x^2\right)^{2/3} - \frac{12\left(1-x^2\right)^{2/3}}{x^2+3} + 7\sqrt[3]{2}\log\left(x^2+3\right) - 21\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) - 14\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(-8*(1 - x^2)^(2/3) - (12*(1 - x^2)^(2/3))/(3 + x^2) - 14*2^(1/3)*Sqrt[3
]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 7*2^(1/3)*Log[3 + x^2] - 21*2^(
1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)]))/32
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.46894, size = 155, normalized size = 1.34

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 21/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(
2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A] time = 1.54901, size = 478, normalized size = 4.12

$$-\frac{3\left(28\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)+7\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^2+1\right.\right.\right.}{64\left(x^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -3/64*(28*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(
2*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3))) + 7*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*
log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(
2/3)) - 14*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1
)^(1/3)) + 8*(2*x^2 + 9)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1021 $\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)) + (9*Log[2^(2/3) -
 (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi [A] time = 0.0659721, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)) + (9*Log[2^(2/3) -
 (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}(3+x)^2}\,dx,x,x^2\right)\\ &= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\ &= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{3\log\left(3+x^2\right)}{16\ 2^{2/3}}+\frac{9}{16}\operatorname{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)-\frac{9\operatorname{Subst}}{}\\ &= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{3\log\left(3+x^2\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{9\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2-}\right.}{8\ 2^{2/3}}\\ &= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\log\left(3+x^2\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0906556, size = 97, normalized size = 0.96

$$\frac{3}{32}\left(\frac{4\left(1-x^2\right)^{2/3}}{x^2+3}-\sqrt[3]{2}\log\left(x^2+3\right)+3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((4*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^
2)^(1/3))/Sqrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^
2)^(1/3)]))/32
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.50316, size = 140, normalized size = 1.39

$$\frac{3}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{3}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{3}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 3/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/3)
/(x^2 + 3)
```

Fricas [A] time = 1.53422, size = 351, normalized size = 3.48

$$\frac{3\left(4\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+2\cdot 4^{\frac{2}{3}}\right.}{64\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/64*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(
-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3
) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)
) + 8*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1022 $\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2} dx$

Optimal. Leaf size=101

$$-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{\log\left(x^2+3\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^2)^(2/3)/(8*(3 + x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8
*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/
3)]/(16*2^(2/3))
```

Rubi [A] time = 0.0630688, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {444, 51, 55, 617, 204, 31}

$$-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{\log\left(x^2+3\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(1 - x^2)^(2/3)/(8*(3 + x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8
*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/
3)]/(16*2^(2/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{1}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{\log\left(3+x^2\right)}{48\ 2^{2/3}}+\frac{1}{16}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)-\frac{\text{Subst}\left(\int\right.}{}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{\log\left(3+x^2\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2-2x^2}\right)}{8\ 2^{2/3}}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\log\left(3+x^2\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0075964, size = 34, normalized size = 0.34

$$-\frac{3}{64}\left(1-x^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{1}{4}\left(1-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, (1 - x^2)/4])/64
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

[Out] int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x)

Maxima [A] time = 1.48261, size = 140, normalized size = 1.39

$$\frac{1}{96} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3}\left(4^{\frac{1}{3}} + 2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{192} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{96} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")

[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 1/96*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2/3)/(x^2 + 3)

Fricas [A] time = 1.51495, size = 365, normalized size = 3.61

$$\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3}\left(x^2+3\right) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}} \sqrt{3} + 2\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - 4^{\frac{2}{3}}\left(x^2+3\right) \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + 2 \cdot 4^{\frac{2}{3}}}{192\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")

[Out] 1/192*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*sqrt(3)*(-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)

[Out] Exception raised: ValueError

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.1023 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=158

$$\frac{(1-x^2)^{2/3}}{24(x^2+3)}+\frac{5\log(x^2+3)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+}{\sqrt{3}}\right)}{6\sqrt{3}}$$

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(6*Sqrt[3]
) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/1
2 - (5*Log[2^(2/3) - (1 - x^2)^(1/3)])/(48*2^(2/3))
```

Rubi [A] time = 0.110738, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$\frac{(1-x^2)^{2/3}}{24(x^2+3)}+\frac{5\log(x^2+3)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(6*Sqrt[3]
) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/1
2 - (5*Log[2^(2/3) - (1 - x^2)^(1/3)])/(48*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(3+x)^2}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}+\frac{1}{24}\,\text{Subst}\left(\int \frac{4-\frac{x}{3}}{\sqrt[3]{1-x}x(3+x)}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx,x,x^2\right)-\frac{5}{72}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\ 2^{2/3}}-\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^2}\right)+\frac{1}{12}\,\text{Subst}\left(\int\right.\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\ 2^{2/3}}-\frac{1}{6}\,\text{S}\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{5\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1\right.
\end{aligned}$$

Mathematica [A] time = 0.130308, size = 148, normalized size = 0.94

$$\frac{1}{288}\left(\frac{12\left(1-x^2\right)^{2/3}}{x^2+3}+5\sqrt[3]{2}\log\left(x^2+3\right)+24\log\left(1-\sqrt[3]{1-x^2}\right)-15\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-10\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3))/(3 + x^2) - 10*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2
)^(1/3))/Sqrt[3]] + 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 16
*Log[x] + 5*2^(1/3)*Log[3 + x^2] + 24*Log[1 - (1 - x^2)^(1/3)] - 15*2^(1/3)
*Log[2^(2/3) - (1 - x^2)^(1/3)])/288
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A] time = 1.59731, size = 711, normalized size = 4.5

$$-\frac{20\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+5\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/576*(20*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(2*sqrt(
3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 5*4^(2/3)*(-1)^(1/3)*(
x^2 + 3)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x
^2 + 1)^(2/3)) - 10*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) +
(-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^2 + 3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3
) + 1/3*sqrt(3)) + 16*(x^2 + 3)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1
) - 32*(x^2 + 3)*log((-x^2 + 1)^(1/3) - 1) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)

[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1024 $\int \frac{1}{x^3 \sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{(1-x^2)^{2/3}}{6x^2(x^2+3)}-\frac{5(1-x^2)^{2/3}}{72(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\ \text{ta}$$

```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 + x^2)) + A
rcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2*
(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x]/54 - Log[3 + x^2]/(48*2^(2/
3)) - Log[1 - (1 - x^2)^(1/3)]/36 + Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2
/3))
```

Rubi [A] time = 0.130042, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{(1-x^2)^{2/3}}{6x^2(x^2+3)}-\frac{5(1-x^2)^{2/3}}{72(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\ \text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 + x^2)) + A
rcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2*
(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x]/54 - Log[3 + x^2]/(48*2^(2/
3)) - Log[1 - (1 - x^2)^(1/3)]/36 + Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2
/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\left(3+x^2\right)^2}dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^2(3+x)^2}dx, x, x^2\right)$$
$$= -\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} - \frac{1}{6}\text{Subst}\left(\int \frac{1-\frac{4x}{3}}{\sqrt[3]{1-x}x(3+x)^2}dx, x, x^2\right)$$
$$= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} - \frac{1}{72}\text{Subst}\left(\int \frac{4-\frac{5x}{3}}{\sqrt[3]{1-x}x(3+x)}dx, x, x^2\right)$$
$$= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} - \frac{1}{54}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}dx, x, x^2\right) + \frac{1}{24}\text{Subst}\left(\int \frac{}{\sqrt[3]{1-x}}\right.$$
$$= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} + \frac{\log(x)}{54} - \frac{\log\left(3+x^2\right)}{48\ 2^{2/3}} + \frac{1}{36}\text{Subst}\left(\int \frac{1}{1-x}dx, x, \sqrt[3]{1-}\right.$$
$$= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} + \frac{\log(x)}{54} - \frac{\log\left(3+x^2\right)}{48\ 2^{2/3}} - \frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right) + \frac{\log\left(2^{2/}\right.}{1}$$
$$= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)} + \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}} + \frac{\log(x)}{54} - \frac{\log(}{48}$$

<u>**Mathematica**</u> **[A]** time = 0.149302, size = 171, normalized size = 0.93

$$\frac{1}{864}\left(-\frac{144\left(1-x^2\right)^{2/3}}{x^2\left(x^2+3\right)} - \frac{60\left(1-x^2\right)^{2/3}}{x^2+3} - 9\sqrt[3]{2}\log\left(x^2+3\right) - 24\log\left(1-\sqrt[3]{1-x^2}\right) + 27\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) + 18\sqrt[3]{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((-60*(1 - x^2)^(2/3))/(3 + x^2) - (144*(1 - x^2)^(2/3))/(x^2*(3 + x^2)) +
18*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 16*Sqrt[3]*Arc
Tan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 16*Log[x] - 9*2^(1/3)*Log[3 + x^2] -
 24*Log[1 - (1 - x^2)^(1/3)] + 27*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/8
64
```

<u>**Maple**</u> **[F]** time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2 \left(-x^2+1\right)^{\frac{1}{3}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A] time = 1.59575, size = 663, normalized size = 3.62

$$36 \cdot 4^{\frac{1}{6}} \sqrt{3} \left(x^4+3 x^2\right) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{1}{3}} \sqrt{3}+2 \sqrt{3} \left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - 9 \cdot 4^{\frac{2}{3}} \left(x^4+3 x^2\right) \log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}} \left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1728*(36*4^(1/6)*sqrt(3)*(x^4 + 3*x^2)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3
) + 2*sqrt(3)*(-x^2 + 1)^(1/3))) - 9*4^(2/3)*(x^4 + 3*x^2)*log(4^(2/3) + 4^
(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 18*4^(2/3)*(x^4 + 3*x^2)*log(-
4^(1/3) + (-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^4 + 3*x^2)*arctan(2/3*sqrt(3)*(
-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 16*(x^4 + 3*x^2)*log((-x^2 + 1)^(2/3) + (-
x^2 + 1)^(1/3) + 1) - 32*(x^4 + 3*x^2)*log((-x^2 + 1)^(1/3) - 1) - 24*(5*x^
2 + 12)*(-x^2 + 1)^(2/3))/(x^4 + 3*x^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1025 $\int \frac{1}{x^5 \sqrt[3]{1-x^2}\left(3+x^2\right)^2} dx$

Optimal. Leaf size=208

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-$$

```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^2)) - (1 -
 x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt
[3]) - Log[x]/54 + (13*Log[3 + x^2])/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/
3)]/36 - (13*Log[2^(2/3) - (1 - x^2)^(1/3)])/(432*2^(2/3))
```

Rubi [A] time = 0.146718, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^2)) - (1 -
 x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt
[3]) - Log[x]/54 + (13*Log[3 + x^2])/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/
3)]/36 - (13*Log[2^(2/3) - (1 - x^2)^(1/3)])/(432*2^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^3(3+x)^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{1}{12}\text{Subst}\left(\int \frac{-1-\frac{7x}{3}}{\sqrt[3]{1-x}x^2(3+x)^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{36}\text{Subst}\left(\int \frac{6+\frac{4x}{3}}{\sqrt[3]{1-x}x(3+x)^2}\,dx,x,x^2\right)$$

$$= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{432}\text{Subst}\left(\int \frac{24-\frac{2x}{3}}{\sqrt[3]{1-x}x(3+x)}\,dx,x,x^2\right.$$

$$= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{54}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx,x,x^2\right) - \frac{13}{648}$$

$$= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{\log(x)}{54} + \frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}} - \frac{1}{36}\text{Subst}\Big(\int$$

$$= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{\log(x)}{54} + \frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}} + \frac{1}{36}\log\Big(1-$$

$$= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{13\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}}$$

Mathematica [A] time = 0.166932, size = 194, normalized size = 0.93

$$\frac{-\frac{72\left(1-x^2\right)^{2/3}}{x^2\left(x^2+3\right)} - \frac{216\left(1-x^2\right)^{2/3}}{x^4\left(x^2+3\right)} + \frac{12\left(1-x^2\right)^{2/3}}{x^2+3} + 13\sqrt[3]{2}\log\left(x^2+3\right) + 72\log\left(1-\sqrt[3]{1-x^2}\right) - 39\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) - 26\sqrt[3]{2}}{2592}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3))/(3 + x^2) - (216*(1 - x^2)^(2/3))/(x^4*(3 + x^2)) - (
72*(1 - x^2)^(2/3))/(x^2*(3 + x^2)) - 26*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]] + 48*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]
- 48*Log[x] + 13*2^(1/3)*Log[3 + x^2] + 72*Log[1 - (1 - x^2)^(1/3)] - 39*2^
(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/2592
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A] time = 1.63802, size = 778, normalized size = 3.74

$$-\frac{52\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\left(x^6+3x^4\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+13\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^6+3x^4\right)\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(\cdot\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/5184*(52*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*arctan(1/6*4^(1/6)*(2*
sqrt(3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 13*4^(2/3)*(-1)^(
1/3)*(x^6 + 3*x^4)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(
1/3) + (-x^2 + 1)^(2/3)) - 26*4^(2/3)*(-1)^(1/3)*(x^6 + 3*x^4)*log(-4^(1/3)
*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 96*sqrt(3)*(x^6 + 3*x^4)*arctan(2/3*sqrt(
3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 48*(x^6 + 3*x^4)*log((-x^2 + 1)^(2/3)
+ (-x^2 + 1)^(1/3) + 1) - 96*(x^6 + 3*x^4)*log((-x^2 + 1)^(1/3) - 1) - 24*(
x^4 - 6*x^2 - 18)*(-x^2 + 1)^(2/3))/(x^6 + 3*x^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1026 $\int \frac{x^4}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=543

$$\frac{9\,3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4\sqrt{2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{3\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}-\frac{27x}{8\left(-\sqrt[3]{1-x^2}-\right.}$$

```
[Out] (3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (27*x)/(8*(1 - Sqrt[3] - (1 - x^2)^(1
/3))) - (5*Sqrt[3]*ArcTan[Sqrt[3]/x])/(8*2^(2/3)) - (5*Sqrt[3]*ArcTan[(Sqrt
[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(8*2^(2/3)) + (5*ArcTanh[x])/(8*2^(2
/3)) - (15*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(8*2^(2/3)) - (27*3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1
- x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt
[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + (9
*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*x*S
qrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.238137, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {470, 530, 235, 304, 219, 1879, 393}

$$\frac{3\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}-\frac{27x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}}-\frac{15\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}+\frac{9\,3^{3/4}\left(1-\sqrt[3]{1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] (3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (27*x)/(8*(1 - Sqrt[3] - (1 - x^2)^(1
/3))) - (5*Sqrt[3]*ArcTan[Sqrt[3]/x])/(8*2^(2/3)) - (5*Sqrt[3]*ArcTan[(Sqrt
[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(8*2^(2/3)) + (5*ArcTanh[x])/(8*2^(2
/3)) - (15*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(8*2^(2/3)) - (27*3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1
- x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt
[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + (9
*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*x*S
qrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```


Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{3x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{1}{8}\int \frac{3-9x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{3x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{9}{8}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{15}{4}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{3x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}} + \frac{5\tanh^{-1}(x)}{8\ 2^{2/3}} - \frac{15\tanh^{-1}}{}$$

$$= \frac{3x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}} + \frac{5\tanh^{-1}(x)}{8\ 2^{2/3}} - \frac{15\tanh^{-1}}{}$$

$$= \frac{3x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{27x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}} +$$

Mathematica [C] time = 0.11431, size = 157, normalized size = 0.29

$$\frac{1}{8}x\left(x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{3\left(\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}-x^2+1\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + (3*(1 - x^2 + (9*AppellF1
[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3
] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1,
5/2, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2))))/8
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(-x^2+1)^{\frac{2}{3}}x^4}{x^6+5x^4+3x^2-9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^4/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1027 $\int \frac{x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=543

$$-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}$$

```
[Out] -(x*(1 - x^2)^(2/3))/(8*(3 + x^2)) + x/(8*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
+ ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
 x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x/
(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3
] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1
 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2
)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt
[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.236746, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {471, 530, 235, 304, 219, 1879, 393}

$$-\frac{\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}\ldots}{\left(-\sqrt[3]{1-x^2}\ldots\right.}}}{4\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] -(x*(1 - x^2)^(2/3))/(8*(3 + x^2)) + x/(8*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
+ ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
 x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x/
(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3
] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1
 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2
)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt
[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```


Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{1}{8}\int \frac{1-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{1}{24}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx+\frac{1}{4}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right.}{8\,2^{2/3}}$$

$$= -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right.}{8\,2^{2/3}}$$

$$= -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(}{24\,2^{2/3}}$$

Mathematica [C] time = 0.0941147, size = 156, normalized size = 0.29

$$\frac{x\left(\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+x^2-1\right)}{8\sqrt[3]{1-x^2}\left(x^2+3\right)}-\frac{1}{216}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3]])/216 + (x*(-1 + x^2 + (9*Appe
llF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3]])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^
2/3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] + AppellF1[3/2, 4/3,
 1, 5/2, x^2, -x^2/3]]))))/(8*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^2+1\right)^{\frac{2}{3}}x^2}{x^6+5\,x^4+3\,x^2-9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^2/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1028 $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2} dx$

Optimal. Leaf size=543

$$\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{12\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}-\frac{x}{24\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right.}$$

```
[Out] (x*(1 - x^2)^(2/3))/(24*(3 + x^2)) - x/(24*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
 + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1
- x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x
/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) - (Sqrt[2 + Sqrt[3]]*(1 - (1 -
x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1
- x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt
[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*x*Sqrt[-((1 - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + ((1 - (1 - x^2)^(1/3))*Sqr
t[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.229481, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {414, 530, 235, 304, 219, 1879, 393}

$$\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}-\frac{x}{24\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\ 2^{2/3}}+\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\dots}{\left(-\sqrt[3]{1-x^2}\dots\right.}}}{12\sqrt{\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(1 - x^2)^(2/3))/(24*(3 + x^2)) - x/(24*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
 + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1
- x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x
/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) - (Sqrt[2 + Sqrt[3]]*(1 - (1 -
x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1
- x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt
[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*x*Sqrt[-((1 - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + ((1 - (1 - x^2)^(1/3))*Sqr
t[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{x\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)} - \frac{1}{24}\int \frac{-7-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{x\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)} + \frac{1}{72}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx + \frac{1}{4}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{x\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{24\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{8\ 2^{2/3}}$$

$$= \frac{x\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{24\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{8\ 2^{2/3}}$$

$$= \frac{x\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)} - \frac{x}{24\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x}{24\ 2^{2/3}}$$

Mathematica [C] time = 0.102374, size = 157, normalized size = 0.29

$$\frac{1}{648}x\left(x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{27\left(\frac{63F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}-x^2+1\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + (27*(1 - x^2 + (63*Appell
F1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/
3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] + AppellF1[3/2, 4/3, 1
, 5/2, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2))))/648
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{x^6+5\,x^4+3\,x^2-9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1029 $\int \frac{1}{x^2 \sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{8x}$$

```
[Out] -(1 - x^2)^(2/3)/(8*x) + (1 - x^2)^(2/3)/(24*x*(3 + x^2)) + x/(8*(1 - Sqrt[
3] - (1 - x^2)^(1/3))) - (7*ArcTan[Sqrt[3]/x])/(72*2^(2/3)*Sqrt[3]) - (7*Ar
cTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(72*2^(2/3)*Sqrt[3]) + (7*
ArcTanh[x])/(216*2^(2/3)) - (7*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(7
2*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elliptic
E[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2
)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2
)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqr
t[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2)])
```

Rubi [A] time = 0.309029, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {472, 583, 530, 235, 304, 219, 1879, 393}

$$\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{8x}+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}}-\frac{7\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{72\ 2^{2/3}}-\frac{\left(1-\sqrt[3]{1}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] -(1 - x^2)^(2/3)/(8*x) + (1 - x^2)^(2/3)/(24*x*(3 + x^2)) + x/(8*(1 - Sqrt[
3] - (1 - x^2)^(1/3))) - (7*ArcTan[Sqrt[3]/x])/(72*2^(2/3)*Sqrt[3]) - (7*Ar
cTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(72*2^(2/3)*Sqrt[3]) + (7*
ArcTanh[x])/(216*2^(2/3)) - (7*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(7
2*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elliptic
E[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2
)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2
)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqr
t[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2)])
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} - \frac{1}{24}\int \frac{-9+\frac{5x^2}{3}}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{8x} + \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} + \frac{1}{72}\int \frac{-23-3x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{8x} + \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} - \frac{1}{24}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{7}{36}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{8x} + \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}} + \frac{7\tanh^{-1}(x)}{216\ 2^{2/3}} -$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{8x} + \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}} + \frac{7\tanh^{-1}(x)}{216\ 2^{2/3}} -$$

$$= -\frac{\left(1-x^2\right)^{2/3}}{8x} + \frac{\left(1-x^2\right)^{2/3}}{24x\left(3+x^2\right)} + \frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\right.}{x}\right.}{72\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.109034, size = 168, normalized size = 0.3

$$\frac{\frac{9\left(\frac{69x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+3x^4+5x^2-8\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)} - x^4F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{216x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-(x^4*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3]) + (9*(-8 + 5*x^2 + 3*x^4 +
(69*x^2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(-9*AppellF1[1/2, 1/3, 1,
3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - Appell
F1[3/2, 4/3, 1, 5/2, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2)))/(216*x)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(-x^2+1)^{\frac{2}{3}}}{x^8+5x^6+3x^4-9x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^8 + 5*x^6 + 3*x^4 - 9*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1030 $\int \frac{1}{x^4 \sqrt[3]{1-x^2} \left(3+x^2\right)^2} \, dx$

Optimal. Leaf size=581

$$\frac{11 \left(1-\sqrt[3]{1-x^2}\right) \sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}} \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{324\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x} - \frac{11x}{648\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)} + \frac{11 (1-}{648}$$

```
[Out] (-11*(1 - x^2)^(2/3))/(216*x^3) + (11*(1 - x^2)^(2/3))/(648*x) + (1 - x^2)^
(2/3)/(24*x^3*(3 + x^2)) - (11*x)/(648*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (
11*ArcTan[Sqrt[3]/x])/(216*2^(2/3)*Sqrt[3]) + (11*ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x])/(216*2^(2/3)*Sqrt[3]) - (11*ArcTanh[x])/(648*2^(2
/3)) + (11*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(216*2^(2/3)) - (11*Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(432*3^
(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) +
 (11*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/
4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.36951, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {472, 583, 530, 235, 304, 219, 1879, 393}

$$-\frac{11x}{648\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)} + \frac{11\left(1-x^2\right)^{2/3}}{648x} - \frac{11\left(1-x^2\right)^{2/3}}{216x^3} + \frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x^3} + \frac{11\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{216\ 2^{2/3}\sqrt{3}} + \frac{11\tanh^{-}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] (-11*(1 - x^2)^(2/3))/(216*x^3) + (11*(1 - x^2)^(2/3))/(648*x) + (1 - x^2)^
(2/3)/(24*x^3*(3 + x^2)) - (11*x)/(648*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (
11*ArcTan[Sqrt[3]/x])/(216*2^(2/3)*Sqrt[3]) + (11*ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x])/(216*2^(2/3)*Sqrt[3]) - (11*ArcTanh[x])/(648*2^(2
/3)) + (11*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(216*2^(2/3)) - (11*Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(432*3^
(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) +
 (11*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/
4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```


Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}-\frac{1}{24}\int \frac{-11+\frac{11x^2}{3}}{x^4\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}+\frac{1}{216}\int \frac{-11+\frac{55x^2}{3}}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}-\frac{1}{648}\int \frac{-77-\frac{11x^2}{3}}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}+\frac{11\int \frac{1}{\sqrt[3]{1-x^2}}\,dx}{1944}+\frac{11}{108}\int \frac{1}{\sqrt[3]{1-x^2}\left(3+\right.}$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{\right.}}{x}\right.}{216\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{\right.}}{x}\right.}{216\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(3+x^2\right)}-\frac{11x}{648\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{11\tan^{-1}\left(\right.}{216\ 2^{2/3}}$$

Mathematica [C] time = 0.142986, size = 173, normalized size = 0.3

$$\frac{11x^6F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{27\left(\frac{693x^4F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}-11x^6+11x^4+72x^2-72\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)}}{17496x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (11*x^6*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + (27*(-72 + 72*x^2 + 11*x^
4 - 11*x^6 + (693*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*AppellF1[
1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -x
^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x
^2)))/(17496*x^3)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(x^{2}+3\right)^{2}} \frac{1}{\sqrt[3]{-x^{2}+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}+3\right)^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(-x^{2}+1\right)^{\frac{2}{3}}}{x^{10}+5\,x^{8}+3\,x^{6}-9\,x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^10 + 5*x^8 + 3*x^6 - 9*x^4), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}+3\right)^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1031 $\int \frac{x^7}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=136

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(1
1/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)
*(2 - 3*x^2)^(1/4))])/81
```

Rubi [A] time = 0.0878525, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {440, 261, 266, 43, 439}

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(1
1/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)
*(2 - 3*x^2)^(1/4))])/81
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}dx = \int \left(-\frac{16x}{27\sqrt[4]{2-3x^2}}-\frac{4x^3}{9\sqrt[4]{2-3x^2}}-\frac{x^5}{3\sqrt[4]{2-3x^2}}+\frac{64x}{27\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^5}{\sqrt[4]{2-3x^2}}dx\right)-\frac{4}{9}\int \frac{x^3}{\sqrt[4]{2-3x^2}}dx-\frac{16}{27}\int \frac{x}{\sqrt[4]{2-3x^2}}dx+\frac{64}{27}\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3\right.}$$

$$= \frac{32}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{1}{6}$$

$$= \frac{32}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{1}{6}$$

$$= \frac{56}{243}\left(2-3x^2\right)^{3/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{2}{891}\left(2-3x^2\right)^{11/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Mathematica [C] time = 0.0296407, size = 46, normalized size = 0.34

$$\frac{2\left(2-3x^2\right)^{3/4}\left(-2464\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)+189x^4+540x^2+1712\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(1712 + 540*x^2 + 189*x^4 - 2464*Hypergeometric2F1[3/4
, 1, 7/4, -1 + (3*x^2)/2]))/18711
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{x^7}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.51897, size = 204, normalized size = 1.5

$$\frac{2}{891}\left(-3x^2+2\right)^{\frac{11}{4}}-\frac{16}{567}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/891*(-3*x^2 + 2)^(11/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*2^(1/4)*arcta
n(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4)*arctan(-1/2
*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log(2^(3/4)*(-3*
x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*log(-2^(3/4)*(
-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^(3/4)
```

Fricas [B] time = 1.3434, size = 814, normalized size = 5.99

$$\frac{2}{18711}\left(189\,x^{4}+540\,x^{2}+1712\right)\left(-3\,x^{2}+2\right)^{\frac{3}{4}}+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3\,x^{2}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 540*x^2 + 1712)*(-3*x^2 + 2)^(3/4) + 32/81*8^(1/4)*sqrt(
2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*s
qrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1)
+ 32/81*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + 1) + 8/81*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*
x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8/81*8^(1/4)*sqrt(2)*l
og(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{7}}{3x^{2}\sqrt[4]{2-3x^{2}}-4\sqrt[4]{2-3x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.2818, size = 216, normalized size = 1.59

$$\frac{2}{891}\left(3\,x^{2}-2\right)^{2}\left(-3\,x^{2}+2\right)^{\frac{3}{4}}-\frac{16}{567}\left(-3\,x^{2}+2\right)^{\frac{7}{4}}-\frac{8}{81}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{81}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 2)^2*(-3*x^2 + 2)^(3/4) - 16/567*(-3*x^2 + 2)^(7/4) - 8/81*8
^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 8/81*8^(3/4)*
```

```
arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log(2
^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*log
(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2
 + 2)^(3/4)
```

3.1032 $\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=121

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)*ArcTan[(S
qrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27 + (8*2^(1/4)*Arc
Tanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27
```

Rubi [A] time = 0.0645026, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {440, 261, 266, 43, 439}

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)*ArcTan[(S
qrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27 + (8*2^(1/4)*Arc
Tanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx &= \int \left(-\frac{4x}{9\sqrt[4]{2-3x^2}}-\frac{x^3}{3\sqrt[4]{2-3x^2}}+\frac{16x}{9\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\right)dx \\
&= -\left(\frac{1}{3}\int \frac{x^3}{\sqrt[4]{2-3x^2}}\,dx\right)-\frac{4}{9}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx+\frac{16}{9}\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx \\
&= \frac{8}{81}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{1}{6}\,S \\
&= \frac{8}{81}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{1}{6}\,S \\
&= \frac{4}{27}\left(2-3x^2\right)^{3/4}-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}}{2^{3}}\right.
\end{aligned}$$

Mathematica [C] time = 0.0215337, size = 42, normalized size = 0.35

$$\frac{2}{567}\left(2-3x^2\right)^{3/4}\left(9\left(x^2+4\right)-56\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(9*(4 + x^2) - 56*Hypergeometric2F1[3/4, 1, 7/4, -1 +
(3*x^2)/2]))/567
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{x^5}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.52376, size = 189, normalized size = 1.56

$$-\frac{2}{189}\left(-3\,x^2+2\right)^{\frac{7}{4}}-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
+ 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

Fricas [B] time = 1.34583, size = 784, normalized size = 6.48

$$\frac{2}{63}\left(x^2+4\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/63*(x^2 + 4)*(-3*x^2 + 2)^(3/4) + 8/27*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)
*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^
2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*8^(1/4)*sqrt(2
)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1
6*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) +
 1) + 2/27*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sq
rt(2) + 16*sqrt(-3*x^2 + 2)) - 2/27*8^(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.72053, size = 189, normalized size = 1.56

$$-\frac{2}{189}\left(-3x^2+2\right)^{\frac{7}{4}}+\frac{1}{27}\cdot 8^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{27}\cdot 8^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) + 1/27*8^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) - 1/27*8^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2
*(-3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x
^2 + 2)^(1/4))) + 4/27*(-3*x^2 + 2)^(3/4)
```

3.1033 $\int \frac{x^3}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=106

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2
^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

Rubi [A] time = 0.0461909, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {440, 261, 439}

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```



```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2
^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```


Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx = \int \left(-\frac{x}{3\sqrt[4]{2-3x^2}} + \frac{4x}{3\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx\right) + \frac{4}{3}\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$$
$$= \frac{2}{27}\left(2-3x^2\right)^{3/4} + \frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Mathematica [C] time = 0.0114363, size = 36, normalized size = 0.34

$$-\frac{2}{27}\left(2-3x^2\right)^{3/4}\left(2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(2 - 3*x^2)^(3/4)*(-1 + 2*Hypergeometric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2
]))/27
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^3}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.57022, size = 174, normalized size = 1.64

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

Fricas [B] time = 1.38666, size = 768, normalized size = 7.25

$$\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) - 1) + 2/9*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8
^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2
*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) + 1/18*8^(1/4)*sqrt(2)*log(4*8^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/18*8^
(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*s
qrt(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.18734, size = 174, normalized size = 1.64

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

3.1034 $\int \frac{x}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3/4))
 + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3
/4))
```

Rubi [A] time = 0.0134464, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {439}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3/4))
 + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3
/4))
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Mathematica [C] time = 0.0074835, size = 34, normalized size = 0.37

$$-\frac{1}{9}\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1}{2}\left(3x^2-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -((2 - 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, (-2 + 3*x^2)/2])/9
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.55128, size = 159, normalized size = 1.75

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}(-3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [B] time = 1.37552, size = 574, normalized size = 6.31

$$\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}(-3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*2^(1/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 1/3*2^(1/4)*arctan(1/2*2^
(1/4)*sqrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
- 2^(1/4)*(-3*x^2 + 2)^(1/4) + 1) + 1/12*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)
^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*log(-4*2^(3/4)*(-3*
x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.1499, size = 159, normalized size = 1.75

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1035 $\int \frac{1}{x\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=145

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1/4
)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 -
3*x^2)^(1/4))]/(4*2^(3/4))
```

Rubi [A] time = 0.0833905, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {440, 266, 63, 298, 203, 206, 439}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1/4
)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 -
3*x^2)^(1/4))]/(4*2^(3/4))
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x\sqrt[4]{2-3x^2}}-\frac{3x}{4\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx\\
&= \frac{1}{4}\int \frac{1}{x\sqrt[4]{2-3x^2}}\,dx-\frac{3}{4}\int \frac{x}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\,dx\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-3x}x}\,dx,x,x^2\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)+\frac{1}{4}\,\text{Su}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0407167, size = 73, normalized size = 0.5

$$
\frac{1}{24}\left(3\,2^{3/4}\left(\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\right)-2\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (3*2^(3/4)*(ArcTan[(1 - (3*x^2)/2)^(1/4)] - ArcTanh[(1 - (3*x^2)/2)^(1/4)])
 - 2*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2])/24
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-3\,x^{2}+4\right)}\frac{1}{\sqrt[4]{-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^{2}-4\right)\left(-3\,x^{2}+2\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x), x)
```

Fricas [B] time = 1.54202, size = 844, normalized size = 5.82

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-3\,x^{2}+2}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*2^(3
/4)*(-3*x^2 + 2)^(1/4)) - 1/16*2^(3/4)*log(2^(1/4) + (-3*x^2 + 2)^(1/4)) +
1/16*2^(3/4)*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) + 1/4*2^(1/4)*arctan(2^(1/4
)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*(
-3*x^2 + 2)^(1/4) - 1) + 1/4*2^(1/4)*arctan(1/2*2^(1/4)*sqrt(-4*2^(3/4)*(-3
*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1
/4) + 1) + 1/16*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sq
rt(-3*x^2 + 2)) - 1/16*2^(1/4)*log(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2
) + 4*sqrt(-3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^{3}\sqrt[4]{2-3x^{2}}-4x\sqrt[4]{2-3x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(1/4) - 4*x*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.26835, size = 292, normalized size = 2.01

$$-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/16*4^(3/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(3/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) + 1/32*4^(3/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/32*4^(3/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/8*
4^(1/8)*sqrt(2)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) + 1/16*4^(1/8)*sqrt(
2)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8)) - 1/16*4^(3/8)*log((-3*x^2 + 2)^(1/4)
 + 4^(1/8))
```

3.1036 $\int \frac{1}{x^3 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=163

$$-\frac{(2-3x^2)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(16*x^2) + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(
1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3*Ar
cTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4)
)
```

Rubi [A] time = 0.132789, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {440, 266, 51, 63, 298, 203, 206, 439}

$$-\frac{(2-3x^2)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(16*x^2) + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(
1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3*Ar
cTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4)
)
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\left(4-3x^2\right)}dx = \int \left(\frac{1}{4x^3\sqrt[4]{2-3x^2}}+\frac{3}{16x\sqrt[4]{2-3x^2}}-\frac{9x}{16\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx$$
$$= \frac{3}{16}\int \frac{1}{x\sqrt[4]{2-3x^2}}dx+\frac{1}{4}\int \frac{1}{x^3\sqrt[4]{2-3x^2}}dx-\frac{9}{16}\int \frac{x}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}dx$$
$$= \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3}{32}\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-3x}x}dx,x,x^2\right)+\frac{1}{8}$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3}{64}\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-3}}\right.$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{1}{16}\text{Subst}\left(\int \frac{x^2}{\frac{2}{3}-\frac{x^4}{3}}\right.$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\sqrt[4]{2}}+\frac{3\tan}{}$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan}{}$$

Mathematica [C] time = 0.0506956, size = 102, normalized size = 0.63

$$-\frac{4\left(2-3x^2\right)^{3/4}x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)+4\left(2-3x^2\right)^{3/4}-9\ 2^{3/4}x^2\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+9\ 2^{3/4}x^2\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(4*(2 - 3*x^2)^(3/4) - 9*2^(3/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] + 9*2^(
3/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 4*x^2*(2 - 3*x^2)^(3/4)*Hypergeom
etric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2])/(64*x^2)
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-3x^2+4\right)}\frac{1}{\sqrt[4]{-3x^2+2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{1}{4}}x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3), x)
```

Fricas [B] time = 1.45525, size = 909, normalized size = 5.58

$$-\frac{36 \cdot 2^{\frac{3}{4}} x^2 \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \sqrt{\sqrt{2} + \sqrt{-3\,x^2 + 2}} - \frac{1}{2} \cdot 2^{\frac{3}{4}} \left(-3\,x^2 + 2\right)^{\frac{1}{4}}\right) + 9 \cdot 2^{\frac{3}{4}} x^2 \log\left(2^{\frac{1}{4}} + \left(-3\,x^2 + 2\right)^{\frac{1}{4}}\right) - 9 \cdot 2^{\frac{3}{4}} x^2 \log\left(-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/128*(36*2^(3/4)*x^2*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2))
- 1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) + 9*2^(3/4)*x^2*log(2^(1/4) + (-3*x^2 + 2
)^(1/4)) - 9*2^(3/4)*x^2*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) - 24*2^(1/4)*x^
2*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 +
2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) - 24*2^(1/4)*x^2*arctan(1/2*2^(1/4)*s
qrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/
4)*(-3*x^2 + 2)^(1/4) + 1) - 6*2^(1/4)*x^2*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4)
 + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 6*2^(1/4)*x^2*log(-4*2^(3/4)*(-3*x^2 +
 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 8*(-3*x^2 + 2)^(3/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^5\sqrt[4]{2-3x^2} - 4x^3\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(1/4) - 4*x**3*(2 - 3*x**2)**(1/4)), x)
```

Giac [A] time = 1.33623, size = 259, normalized size = 1.59

$$\frac{9}{64} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \left(-3\,x^2 + 2\right)^{\frac{1}{4}}\right) - \frac{9}{128} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{1}{4}} + \left(-3\,x^2 + 2\right)^{\frac{1}{4}}\right) + \frac{9}{128} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{1}{4}} - \left(-3\,x^2 + 2\right)^{\frac{1}{4}}\right) - \frac{3}{32} \cdot 2^{\frac{1}{4}} \mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 9/64*2^(3/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 9/128*2^(3/4)*log(2^(
1/4) + (-3*x^2 + 2)^(1/4)) + 9/128*2^(3/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)
) - 3/32*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/3
2*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 3/64*2^(1
```

```
/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 3/64*2^(
1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/16*(
-3*x^2 + 2)^(3/4)/x^2
```

3.1037 $\int \frac{x^4}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=164

$$\frac{2}{45}\left(2-3x^2\right)^{3/4}x+\frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{16\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] (2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (4*2^(1/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (16*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rubi [A] time = 0.072696, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {440, 228, 321, 397}

$$\frac{2}{45}\left(2-3x^2\right)^{3/4}x+\frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{16\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (4*2^(1/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (16*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
```

```
b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx = \int \left(-\frac{4}{9\sqrt[4]{2-3x^2}} - \frac{x^2}{3\sqrt[4]{2-3x^2}} + \frac{16}{9\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\right) - \frac{4}{9}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx + \frac{16}{9}\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$$

$$= \frac{2}{45}x\left(2-3x^2\right)^{3/4} + \frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} + \frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{8\sqrt[4]{2}E\left(\frac{1}{2}\right.}{}$$

$$= \frac{2}{45}x\left(2-3x^2\right)^{3/4} + \frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} + \frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{16\sqrt[4]{2}E\left(\right.}{}$$

Mathematica [C] time = 0.128451, size = 184, normalized size = 1.12

$$\frac{1}{45}x\left(3\,2^{3/4}x^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right) + \frac{2\left(\frac{32F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)} - 3x^2 + \right.}{\sqrt[4]{2-3x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (x*(3*2^(3/4)*x^2*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2*(2
- 3*x^2 + (32*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^
2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/
2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/
2, (3*x^2)/4])))))/(2 - 3*x^2)^(1/4)))/45
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{x^4}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(-3\,x^2+2)^{\frac{3}{4}}x^4}{9\,x^4-18\,x^2+8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(3/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**4/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1038 $\int \frac{x^2}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

```
[Out] (2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^
(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])
/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3]) - (2*2^(1/4)*EllipticE[ArcSin[
Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```

Rubi [A] time = 0.0555915, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {440, 228, 397}

$$\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```


```
[Out] (2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^
(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])
/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3]) - (2*2^(1/4)*EllipticE[ArcSin[
Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```


Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int\left(-\frac{1}{3\sqrt[4]{2-3x^2}} + \frac{4}{3\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\right) + \frac{4}{3}\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$= \frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} + \frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} - \frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

Mathematica [C] time = 0.0175562, size = 37, normalized size = 0.25

$$\frac{x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\sqrt[4]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(1/4))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-3x^2+2)^{\frac{3}{4}}x^2}{9x^4-18x^2+8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(3/4)*x^2/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1039 $\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)} dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi [A] time = 0.0153221, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)} dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.0265612, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x^2)^(1/4)*
(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*A
ppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2
, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [B] time = 20.6601, size = 1553, normalized size = 12.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1040 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt[3]*ArcTanh[(2^(3/4)
 + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) - (
Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rubi [A] time = 0.0640898, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {440, 325, 228, 397}

$$-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```


```
[Out] -(2 - 3*x^2)^(3/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt[3]*ArcTanh[(2^(3/4)
 + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) - (
Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```


Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
```

```
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx=\int\left(\frac{1}{4x^2\sqrt[4]{2-3x^2}}-\frac{3}{4\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx$$
$$=\frac{1}{4}\int\frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx-\frac{3}{4}\int\frac{1}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\,dx$$
$$=-\frac{\left(2-3x^2\right)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}-\frac{3}{16}\int\frac{1}{\sqrt[4]{2-3x^2}}$$
$$=-\frac{\left(2-3x^2\right)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\right.}{4\,2^{3/4}}$$

Mathematica [C] time = 0.0363516, size = 56, normalized size = 0.34

$$\frac{3x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{64\sqrt[4]{2}}-\frac{\left(2-3x^2\right)^{3/4}}{8x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(8*x) + (3*x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*
x^2)/4])/(64*2^(1/4))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-3x^2+4\right)}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{9\,x^6-18\,x^4+8\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(3/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^4\sqrt[4]{2-3x^2}-4x^2\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**4*(2 - 3*x**2)**(1/4) - 4*x**2*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

3.1041 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{3\left(2-3x^2\right)^{3/4}}{16x}-\frac{\left(2-3x^2\right)^{3/4}}{24x^3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(24*x^3) - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*Sqrt[3]*Arc
Tan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(32
*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]
*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2
]*x]/2, 2])/(8*2^(3/4))
```

Rubi [A] time = 0.0870461, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {440, 325, 228, 397}

$$-\frac{3\left(2-3x^2\right)^{3/4}}{16x}-\frac{\left(2-3x^2\right)^{3/4}}{24x^3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(24*x^3) - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*Sqrt[3]*Arc
Tan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(32
*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]
*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2
]*x]/2, 2])/(8*2^(3/4))
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx = \int \left(\frac{1}{4x^4\sqrt[4]{2-3x^2}}+\frac{3}{16x^2\sqrt[4]{2-3x^2}}-\frac{9}{16\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx$$
$$= \frac{3}{16}\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx+\frac{1}{4}\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx-\frac{9}{16}\int \frac{1}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\,dx$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{24x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{32x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-}}{\sqrt{3}x\sqrt[4]{2-3}}\right.}{32\ 2^{3/4}}$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{24x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-}}{\sqrt{3}x\sqrt[4]{2-3}}\right.}{32\ 2^{3/4}}$$
$$= -\frac{\left(2-3x^2\right)^{3/4}}{24x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-}}{\sqrt{3}x\sqrt[4]{2-3}}\right.}{32\ 2^{3/4}}$$

Mathematica [C] time = 0.154913, size = 156, normalized size = 0.85

$$\frac{1}{8}\left(2-3x^2\right)^{3/4}\left(\frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}-\frac{9x^2}{6x}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ((2 - 3*x^2)^(3/4)*(-(2 + 9*x^2)/(6*x^3) + (9*x*AppellF1[1/2, -3/4, 1, 3/2,
 (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, -3/4, 1, 3/2, (3*x^2
)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4]
- 3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])))))/8
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-3x^2+4\right)}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{9\,x^8-18\,x^6+8\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(3/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^6\sqrt[4]{2-3x^2}-4x^4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**6*(2 - 3*x**2)**(1/4) - 4*x**4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

3.1042 $\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1 + 3*x^2)^
(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(
1/4)])/81
```

Rubi [A] time = 0.0532299, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 63, 298, 203, 206}

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1 + 3*x^2)^
(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(
1/4)])/81
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{7}{27\sqrt[4]{-1+3x}}+\frac{8}{27(-2+3x)\sqrt[4]{-1+3x}}+\frac{4}{27}(-1+3x)^{3/4}+\frac{1}{27}(-1+3x)^{7/4}\right)\right.\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{4}{27}\,\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{}}\right.\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{16}{81}\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,\right.\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}-\frac{8}{81}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\right.\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-
\end{aligned}$$

Mathematica [A] time = 0.0413076, size = 57, normalized size = 0.73

$$\frac{2\left(\left(3x^2-1\right)^{3/4}\left(189x^4+270x^2+428\right)+924\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-924\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4)*(428 + 270*x^2 + 189*x^4) + 924*ArcTan[(-1 + 3*x^2)^
(1/4)] - 924*ArcTanh[(-1 + 3*x^2)^(1/4)]))/18711
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{x^7}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.44358, size = 100, normalized size = 1.28

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.28734, size = 208, normalized size = 2.67

$$\frac{2}{18711}\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 270*x^2 + 428)*(3*x^2 - 1)^(3/4) + 8/81*arctan((3*x^2 -
1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) -
1)
```

Sympy [A] time = 18.0649, size = 88, normalized size = 1.13

$$\frac{2\left(3x^2-1\right)^{\frac{11}{4}}}{891}+\frac{8\left(3x^2-1\right)^{\frac{7}{4}}}{567}+\frac{14\left(3x^2-1\right)^{\frac{3}{4}}}{243}+\frac{4\log\left(\sqrt[4]{3x^2-1}-1\right)}{81}-\frac{4\log\left(\sqrt[4]{3x^2-1}+1\right)}{81}+\frac{8\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(11/4)/891 + 8*(3*x**2 - 1)**(7/4)/567 + 14*(3*x**2 - 1)**(
3/4)/243 + 4*log((3*x**2 - 1)**(1/4) - 1)/81 - 4*log((3*x**2 - 1)**(1/4) +
1)/81 + 8*atan((3*x**2 - 1)**(1/4))/81
```

Giac [A] time = 1.22234, size = 101, normalized size = 1.29

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1043 $\int \frac{x^5}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTan[(-1 + 3*
x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi [A] time = 0.0480356, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 63, 298, 203, 206}

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTan[(-1 + 3*
x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{3\sqrt[4]{-1+3x}}+\frac{4}{9(-2+3x)\sqrt[4]{-1+3x}}+\frac{1}{9}(-1+3x)^{3/4}\right)dx, x, x^2\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{8}{27}\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}-\frac{4}{27}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)+\frac{4}{27}\,\mathrm{S}\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{-1+3\right.\end{aligned}$$

Mathematica [A] time = 0.0382267, size = 51, normalized size = 0.81

$$\frac{2}{189}\left(3\left(3x^2-1\right)^{3/4}\left(x^2+2\right)+14\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-14\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(3*(2 + x^2)*(-1 + 3*x^2)^(3/4) + 14*ArcTan[(-1 + 3*x^2)^(1/4)] - 14*Arc
Tanh[(-1 + 3*x^2)^(1/4)]))/189
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^5}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.44819, size = 85, normalized size = 1.35

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.33612, size = 182, normalized size = 2.89

$$\frac{2}{63}\left(3x^2-1\right)^{\frac{3}{4}}\left(x^2+2\right)+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/63*(3*x^2 - 1)^(3/4)*(x^2 + 2) + 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/27*lo
g((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [A] time = 13.9766, size = 75, normalized size = 1.19

$$\frac{2\left(3x^2-1\right)^{\frac{7}{4}}}{189}+\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}+\frac{2\log\left(\sqrt[4]{3x^2-1}-1\right)}{27}-\frac{2\log\left(\sqrt[4]{3x^2-1}+1\right)}{27}+\frac{4\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(7/4)/189 + 2*(3*x**2 - 1)**(3/4)/27 + 2*log((3*x**2 - 1)**
(1/4) - 1)/27 - 2*log((3*x**2 - 1)**(1/4) + 1)/27 + 4*atan((3*x**2 - 1)**(1
/4))/27
```

Giac [A] time = 1.22778, size = 86, normalized size = 1.37

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
 1))
```

3.1044 $\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(
-1 + 3*x^2)^(1/4)])/9
```

Rubi [A] time = 0.032934, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 63, 298, 203, 206}

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(
-1 + 3*x^2)^(1/4)])/9
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{4}{9}\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt[4]{-1+3x^2}\right)\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}-\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{-1+3x^2}\right)+\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{-\right.\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0101469, size = 44, normalized size = 0.92

$$\frac{2}{27}\left(\left(3x^2-1\right)^{3/4}+3\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-3\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 + 3*x
^2)^(1/4)]))/27
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{x^3}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.51309, size = 70, normalized size = 1.46

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.32912, size = 165, normalized size = 3.44

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [A] time = 10.2749, size = 58, normalized size = 1.21

$$\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}+\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{9}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{9}+\frac{2\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(3/4)/27 + log((3*x**2 - 1)**(1/4) - 1)/9 - log((3*x**2 - 1
)**(1/4) + 1)/9 + 2*atan((3*x**2 - 1)**(1/4))/9
```

Giac [A] time = 1.23998, size = 72, normalized size = 1.5

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1045 $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi [A] time = 0.0224928, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 63, 298, 203, 206}

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\ &= \frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.005736, size = 33, normalized size = 1.

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.46885, size = 55, normalized size = 1.67

$$\frac{1}{3}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.34098, size = 131, normalized size = 3.97

$$\frac{1}{3}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

Sympy [A] time = 5.485, size = 42, normalized size = 1.27

$$\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}+\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 + atan((3*x
**2 - 1)**(1/4))/3
```

Giac [A] time = 1.17008, size = 57, normalized size = 1.73

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(ab
s((3*x^2 - 1)^(1/4) - 1))
```

3.1046 $\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2\right.}}{2\sqrt{2}}$$

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 +
 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(
4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt
[2])
```

Rubi [A] time = 0.131985, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {446, 86, 63, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2\right.}}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 +
 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(
4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt
[2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-2+3x)\sqrt[4]{-1+3x}}dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}dx, x, x^2\right)\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}dx, x, x^2\right)$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}dx, x, \sqrt[4]{-1+3x^2}\right)\right) + \text{Subst}\left(\int \frac{x^2}{-1+x^4}dx, x, \sqrt[4]{-1+3x^2}\right)$$
$$= \frac{1}{6}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\text{S}$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \sqrt[4]{}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x}\right.}{4\sqrt{2}}$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{1}{2}$$

Mathematica [C] time = 0.0158875, size = 63, normalized size = 0.36

$$-\frac{1}{3}\left(3x^2-1\right)^{3/4} {}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-3x^2\right) + \frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 - ((-1 + 3*x^2
)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - 3*x^2])/3
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(3x^2-2\right)}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x), x)
```

Fricas [A] time = 1.39738, size = 649, normalized size = 3.75

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) - 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) + 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)

[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [A] time = 1.22451, size = 209, normalized size = 1.21

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")

[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1047 $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{(3x^2-1)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9}{}$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(4*x^2) + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (9*ArcTan[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTan[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 - (9*Log[1 -
 Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.147334, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {446, 103, 156, 63, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{(3x^2-1)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(3/4)/(4*x^2) + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (9*ArcTan[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTan[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 - (9*Log[1 -
 Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2}+\frac{9x}{4}}{x(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{9}{16}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}\,dx, x, x^2\right) + \frac{9}{8}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1}}\right.$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx\right.$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{8}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{3}{8}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,\right.$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{9}{16}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{9\log\left(1-\sqrt{2}\sqrt[4]{}\right.}{}$$
$$= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} - \frac{9\tan^{-1}\left(1\right.}{}$$

Mathematica [C] time = 0.0308546, size = 77, normalized size = 0.4

$$\frac{1}{4}\left(-3\left(3x^2-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-3x^2\right) - \frac{\left(3x^2-1\right)^{3/4}}{x^2} + 3\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - 3\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (-((-1 + 3*x^2)^(3/4)/x^2) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 +
 3*x^2)^(1/4)] - 3*(-1 + 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - 3*
x^2])/4
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3x^2-2\right)}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A] time = 1.36684, size = 720, normalized size = 3.77

$$36\,\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1}-\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)+36\,\sqrt{2}x^2\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{-4\,\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/32*(36*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 36*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) + 9*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) +
 4*sqrt(3*x^2 - 1) + 4) - 9*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) +
4*sqrt(3*x^2 - 1) + 4) + 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x
^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(3/4
))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [A] time = 1.23334, size = 228, normalized size = 1.19

$$-\frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{9}{16}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{9}{32}\,\sqrt{2}\log\left(\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] -9/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 9/16*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 9/32*sqrt(2)*l
og(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 9/32*sqrt(2)*log(-sqr
t(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(3/4)/x^2 +
 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

3.1048 $\int \frac{x^4}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=244

$$\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{15\sqrt{3}x}+\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}x}{15\left(\sqrt{3x^2-1}+1\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}$$

```
[Out] (2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + Sqrt[-1 + 3
*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (Sqrt[2/
3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (8*Sqrt[x^2/(1 + Sqrt[-1
+ 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)],
 1/2])/(15*Sqrt[3]*x) + (4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x)
```

Rubi [A] time = 0.185831, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {440, 230, 305, 220, 1196, 321, 398}

$$\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}x}{15\left(\sqrt{3x^2-1}+1\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + Sqrt[-1 + 3
*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (Sqrt[2/
3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (8*Sqrt[x^2/(1 + Sqrt[-1
+ 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)],
 1/2])/(15*Sqrt[3]*x) + (4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x)
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^4}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=\int \left(\frac{2}{9\sqrt[4]{-1+3x^2}}+\frac{x^2}{3\sqrt[4]{-1+3x^2}}+\frac{4}{9\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\right)dx$$

$$=\frac{2}{9}\int \frac{1}{\sqrt[4]{-1+3x^2}}\,dx+\frac{1}{3}\int \frac{x^2}{\sqrt[4]{-1+3x^2}}\,dx+\frac{4}{9}\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$$

$$=\frac{2}{45}x\left(-1+3x^2\right)^{3/4}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{2}{45}\int$$

$$=\frac{2}{45}x\left(-1+3x^2\right)^{3/4}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{\left(4\sqrt{x^2}\right.}{}$$

$$=\frac{2}{45}x\left(-1+3x^2\right)^{3/4}+\frac{4x\sqrt[4]{-1+3x^2}}{9\left(1+\sqrt{-1+3x^2}\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\right.$$

$$=\frac{2}{45}x\left(-1+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-1+3x^2}}{15\left(1+\sqrt{-1+3x^2}\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}$$

Mathematica [C] time = 0.0929506, size = 177, normalized size = 0.73

$$\frac{2x\left(-3\sqrt[4]{1-3x^2}x^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-\frac{4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}+3x^2-1\right)}{45\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - 3*x^2*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2
, (3*x^2)/2] - (4*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^
2)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2, 1
/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*x^2)/2
])))))/(45*(-1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^4}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^2-1\right)^{\frac{3}{4}}x^4}{9x^4-9x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(3/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049 $\int \frac{x^2}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{3\sqrt{3}x}+\frac{2\sqrt[4]{3x^2-1}x}{3\left(\sqrt{3x^2-1}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqrt[3/2]*x)/
(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]
/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])
*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^2/(1
 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x
^2)^(1/4)], 1/2])/(3*Sqrt[3]*x)
```

Rubi [A] time = 0.104839, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {440, 230, 305, 220, 1196, 398}

$$\frac{2\sqrt[4]{3x^2-1}x}{3\left(\sqrt{3x^2-1}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}-\frac{2\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqrt[3/2]*x)/
(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]
/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])
*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^2/(1
 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x
^2)^(1/4)], 1/2])/(3*Sqrt[3]*x)
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \left(\frac{1}{3\sqrt[4]{-1+3x^2}}+\frac{2}{3\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\right)dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt[4]{-1+3x^2}}\,dx+\frac{2}{3}\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}+\frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx,x,\sqrt[4]{-1+3x^2}\right)}{3\sqrt{3}x}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}+\frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt[4]{-1+3x^2}\right)}{3\sqrt{3}x}-$$

$$= \frac{2x\sqrt[4]{-1+3x^2}}{3\left(1+\sqrt{-1+3x^2}\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{2\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{\right.}}{}$$

Mathematica [C] time = 0.0203964, size = 52, normalized size = 0.23

$$-\frac{x^3\sqrt[4]{1-3x^2}F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(x^3*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(6*(-
1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^2}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^2-1\right)^{\frac{3}{4}}x^2}{9x^4-9x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(3/4)*x^2/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050 $\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0089125, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.0260957, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(-1 + 3*x^
2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[
3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*
x^2)/2])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3 x^2 - 2} \frac{1}{\sqrt[4]{3 x^2 - 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3 x^2 - 1\right)^{\frac{1}{4}} \left(3 x^2 - 2\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [B] time = 24.0225, size = 281, normalized size = 4.61

$$\frac{1}{12} \sqrt{6} \arctan\left(\frac{\sqrt{6} \left(3 x^2 - 1\right)^{\frac{1}{4}}}{3 x}\right) + \frac{1}{24} \sqrt{6} \log\left(-\frac{9 x^4 - 6 \sqrt{6} \left(3 x^2 - 1\right)^{\frac{1}{4}} x^3 + 12 \sqrt{3 x^2 - 1} x^2 - 4 \sqrt{6} \left(3 x^2 - 1\right)^{\frac{3}{4}} x + 12 x^2}{9 x^4 - 12 x^2 + 4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2 - 2\right) \sqrt[4]{3x^2 - 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1051 $\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{4x}+\frac{3\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{\left(3x^2-1\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(2*x) + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^
2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3/2]*
ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[
-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4
)], 1/2])/(2*x) + (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rubi [A] time = 0.122261, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {440, 325, 230, 305, 220, 1196, 398}

$$\frac{3\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{\left(3x^2-1\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(3/4)/(2*x) + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^
2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3/2]*
ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[
-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4
)], 1/2])/(2*x) + (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \left(-\frac{1}{2x^2\sqrt[4]{-1+3x^2}}+\frac{3}{2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^2\sqrt[4]{-1+3x^2}}\,dx\right)+\frac{3}{2}\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{3}{4}\int \frac{}{\sqrt[4]{-}}$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{\left(\sqrt{3}\sqrt{x^2}\right.}{}$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{\left(\sqrt{3}\sqrt{x^2}\right.}{}$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{2x}+\frac{3x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{}}\right.$$

Mathematica [C] time = 0.0276539, size = 64, normalized size = 0.26

$$\frac{-3\sqrt[4]{1-3x^2}x^4F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-12x^2+4}{8x\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (4 - 12*x^2 - 3*x^4*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*
x^2)/2])/(8*x*(-1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(3 x^{2}-2\right)} \frac{1}{\sqrt[4]{3 x^{2}-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3 x^{2}-1\right)^{\frac{1}{4}} \left(3 x^{2}-2\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3 x^{2}-1\right)^{\frac{3}{4}}}{9 x^{6}-9 x^{4}+2 x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(3/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(3x^{2}-2\right) \sqrt[4]{3x^{2}-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

3.1052 $\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=264

$$\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{4x}+\frac{9\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3\left(3x^2-1\right)^{3/4}}{2x}-\frac{\left(3x^2-1\right)^{3/4}}{6x^3}$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(6*x^3) - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x*(-1 + 3*x
^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(
-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/
4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2
])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 +
 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rubi [A] time = 0.214906, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {440, 325, 230, 305, 220, 1196, 398}

$$\frac{9\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3\left(3x^2-1\right)^{3/4}}{2x}-\frac{\left(3x^2-1\right)^{3/4}}{6x^3}-\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt{3}\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(3/4)/(6*x^3) - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x*(-1 + 3*x
^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(
-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/
4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2
])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 +
 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{x^4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \left(-\frac{1}{2x^4\sqrt[4]{-1+3x^2}} - \frac{3}{4x^2\sqrt[4]{-1+3x^2}} + \frac{9}{4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\right) dx$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^4\sqrt[4]{-1+3x^2}}\,dx\right) - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-1+3x^2}}\,dx + \frac{9}{4}\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{4x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{2x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{2x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{2x} + \frac{9x\sqrt[4]{-1+3x^2}}{4\left(1+\sqrt{-1+3x^2}\right)} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right.$$

$$= -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{2x} + \frac{9x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right.$$

Mathematica [C] time = 0.144385, size = 148, normalized size = 0.56

$$\frac{1}{2}\left(3x^2-1\right)^{3/4}\left(\frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)} - \frac{9x^2}{3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ((-1 + 3*x^2)^(3/4)*(-(1 + 9*x^2)/(3*x^3) + (9*x*AppellF1[1/2, -3/4, 1, 3/2
, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(2*AppellF1[1/2, -3/4, 1, 3/2, 3*x^2, (3
*x^2)/2] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, 3*x^2, (3*x^2)/2] - 3*AppellF
1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])))))/2
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(3x^2-2\right)}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,x^2-1\right)^{\frac{3}{4}}}{9\,x^8-9\,x^6+2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(3/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

3.1053 $\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(3*2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.0226888, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(3*2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2+3x^2\right)^{3/4}\left(4+3x^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [C] time = 0.0380499, size = 37, normalized size = 0.29

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/2, (-3*x^2)/4])/(12*2^(3/4))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{x^2}{3x^2+4}\left(3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x)
```

```
[Out] int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2+4\right)\left(3x^2+2\right)^{\frac{3}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

Fricas [B] time = 1.86533, size = 857, normalized size = 6.64

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2)
- 12*72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt(2)
*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(3*x^2 + 2
)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2) - 12*72^(1/4)*sqrt(2)
*(3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/4)*sqr
t(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2) + 1/86
4*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x - 18*sqrt(
2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2+2\right)^{\frac{3}{4}}\left(3x^2+4\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

```
[Out] Integral(x**2/((3*x**2 + 2)**(3/4)*(3*x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

3.1054 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.0226413, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [C] time = 0.0365984, size = 37, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(3/4))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [B] time = 2.00185, size = 871, normalized size = 7.26

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1055 $\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

```
[Out] -(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(
1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rubi [A] time = 0.030975, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(
1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2+bx^2\right)^{3/4}\left(4+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Mathematica [C] time = 0.045506, size = 39, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^2)/2, -(b*x^2)/4])/(12*2^(3/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2+4}\left(bx^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

```
[Out] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+4\right)\left(bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

Fricas [B] time = 1.93609, size = 1249, normalized size = 10.07

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{x^2}}\,\frac{1}{b^6}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}-\cdots}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2
)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x - 8
*sqrt(2)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) + sqrt(2)
*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*x*s
qrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/
4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4) - 8*sqrt(2
)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) - 1/4*sqrt(2)*(1
/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2
)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x
^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^
(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x +
2*sqrt(b*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+2)^{\frac{3}{4}}(bx^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
```

```
[Out] Integral(x**2/((b*x**2 + 2)**(3/4)*(b*x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

3.1056 $\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]
*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rubi [A] time = 0.0317682, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]
*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-bx^2\right)^{3/4}\left(4-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Mathematica [C] time = 0.0485828, size = 39, normalized size = 0.33

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/2, (b*x^2)/4])/(12*2^(3/4))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2+4}\left(-bx^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

```
[Out] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

Fricas [B] time = 1.72266, size = 1262, normalized size = 10.61

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{x^2}}\,\frac{1}{b^6}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(-bx^2+2\right)^{\frac{1}{4}}b^4}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 +
2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x -
 8*sqrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) + sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*
x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)
^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)*(b^(-6))^(3/4) - 8*s
qrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) - 1/4*sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*
sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2
 + 2))/x^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4
*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(
1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{bx^2\left(-bx^2+2\right)^{\frac{3}{4}}-4\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
```

```
[Out] -Integral(x**2/(b*x**2*(-b*x**2 + 2)**(3/4) - 4*(-b*x**2 + 2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

3.1057 $\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(3*Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rubi [A] time = 0.0282203, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(3*Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))]])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))]])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a+3x^2\right)^{3/4}\left(2a+3x^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Mathematica [C] time = 0.0497414, size = 65, normalized size = 0.54

$$\frac{x^3\left(\frac{a+3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a\left(a+3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] (x^3*((a + 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/
(2*a)])/(6*a*(a + 3*x^2)^(3/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{3x^{2}+2a}\left(3x^{2}+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

```
[Out] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(3x^{2}+2a\right)\left(3x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 1.63863, size = 495, normalized size = 4.12

$$-\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}ax\left(-\frac{1}{a}\right)^{\frac{3}{4}}\sqrt{\frac{3x^{2}\sqrt{-\frac{1}{a}}+2\sqrt{3x^{2}+a}}{x^{2}}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(3x^{2}+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(3*x^2 + a))/x^2) - (1/36)^(3/4)*(3*x
^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/
36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/a)^
(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (3*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+3x^2\right)^{\frac{3}{4}}\left(2a+3x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + 3*x**2)**(3/4)*(2*a + 3*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

3.1058 $\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rubi [A] time = 0.026276, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Mathematica [C] time = 0.0522317, size = 65, normalized size = 0.54

$$\frac{x^3\left(\frac{a-3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a\left(a-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] (x^3*((a - 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2
*a)])/(6*a*(a - 3*x^2)^(3/4))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{-3 x^{2}+2 a} \left(-3 x^{2}+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

```
[Out] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{2}}{\left(3 x^{2}-2 a\right) \left(-3 x^{2}+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 1.61002, size = 501, normalized size = 4.18

$$-\frac{2}{3} \left(\frac{1}{36}\right)^{\frac{1}{4}} \left(-\frac{1}{a}\right)^{\frac{1}{4}} \arctan\left(\frac{12 \left(\sqrt{\frac{1}{2}} \left(\frac{1}{36}\right)^{\frac{3}{4}} a x \left(-\frac{1}{a}\right)^{\frac{3}{4}} \sqrt{\frac{3 x^{2} \sqrt{-\frac{1}{a}}+2 \sqrt{-3 x^{2}+a}}{x^{2}}} - \left(\frac{1}{36}\right)^{\frac{3}{4}} \left(-3 x^{2}+a\right)^{\frac{1}{4}} a \left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right) - \frac{1}{6} \left(\frac{1}{36}\right)^{\frac{1}{4}} \left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(-3*x^2 + a))/x^2) - (1/36)^(3/4)*(-3
*x^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(
1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/
a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (-3*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-2a\left(a-3x^2\right)^{\frac{3}{4}}+3x^2\left(a-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - 3*x**2)**(3/4) + 3*x**2*(a - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(3\,x^2-2\,a\right)\left(-3\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

3.1059 $\int \frac{x^2}{\left(a+bx^2\right)^{3/4}\left(2a+bx^2\right)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/
4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.0346536, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/
4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{3/4}\left(2a+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Mathematica [C] time = 0.0566761, size = 67, normalized size = 0.58

$$\frac{x^3\left(\frac{a+bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] (x^3*((a + b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -(b*x^2
)/(2*a)])/(6*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2+2a}\left(bx^2+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

```
[Out] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2+2a\right)\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

Fricas [B] time = 1.653, size = 566, normalized size = 4.92

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
- (1/4)^(3/4)*(b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1
/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^2 +
 a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*
(-1/(a*b^6))^(1/4) - (b*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)^{\frac{3}{4}}(2a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**(3/4)*(2*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+2a)(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

3.1060 $\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(Sqr
t[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.0371274, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(Sqr
t[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^{3/4}\left(2a-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Mathematica [C] time = 0.0603899, size = 68, normalized size = 0.57

$$\frac{x^3\left(\frac{a-bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a\left(a-bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] (x^3*((a - b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2
*a)])/(6*a*(a - b*x^2)^(3/4))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2+2a}\left(-bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(bx^2-2a\right)\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

Fricas [B] time = 1.63471, size = 571, normalized size = 4.8

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{-bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(-bx^2+a\right)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(-b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
 - (1/4)^(3/4)*(-b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^
(1/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b*x^
2 + a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2
*x*(-1/(a*b^6))^(1/4) - (-b*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-2a\left(a - bx^2\right)^{\frac{3}{4}} + bx^2\left(a - bx^2\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - b*x**2)**(3/4) + b*x**2*(a - b*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(bx^2 - 2\,a\right)\left(-bx^2 + a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

3.1061 $\int \frac{x^7}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=188

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-}\right.$$

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/
4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81 + (16*2^(3/4)*ArcTa
n[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2
- 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81
```

Rubi [A] time = 0.211457, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {443, 261, 266, 43, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/
4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81 + (16*2^(3/4)*ArcTa
n[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2
- 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx = \int \left(-\frac{16x}{27\left(2-3x^2\right)^{3/4}} - \frac{4x^3}{9\left(2-3x^2\right)^{3/4}} - \frac{x^5}{3\left(2-3x^2\right)^{3/4}} + \frac{64x}{27\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right) dx$$

$$= -\left(\frac{1}{3}\int \frac{x^5}{\left(2-3x^2\right)^{3/4}}\,dx\right) - \frac{4}{9}\int \frac{x^3}{\left(2-3x^2\right)^{3/4}}\,dx - \frac{16}{27}\int \frac{x}{\left(2-3x^2\right)^{3/4}}\,dx + \frac{64}{27}\int \frac{}{\left(2-3\right.}$$

$$= \frac{32}{81}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(2-3x)^{3/4}}\,dx, x, x^2\right) - \frac{2}{9}\text{Subst}\left(\int \frac{x}{(2-3x)^{3/4}}\,dx, x, x^2\right) +$$

$$= \frac{32}{81}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \left(\frac{4}{9(2-3x)^{3/4}} - \frac{4}{9}\sqrt[4]{2-3x} + \frac{1}{9}(2-3x)^{5/4}\right) dx, x, x^2\right) - \frac{2}{9}\text{S}$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{1}{81}\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{1}{81}\left(16\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x}\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} + \frac{8}{81}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{16}{81}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right) + \frac{16}{81}$$

Mathematica [A] time = 0.0968532, size = 179, normalized size = 0.95

$$\frac{2\left(45\sqrt[4]{2-3x^2}x^4 + 156\sqrt[4]{2-3x^2}x^2 + 1136\sqrt[4]{2-3x^2} + 180\ 2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right) - 180\ 2^{3/4}\log\left(\sqrt{\right.}\right.}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(1136*(2 - 3*x^2)^(1/4) + 156*x^2*(2 - 3*x^2)^(1/4) + 45*x^4*(2 - 3*x^2)
^(1/4) + 360*2^(3/4)*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 360*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)] + 180*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) +
 Sqrt[2 - 3*x^2]] - 180*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + S
qrt[2 - 3*x^2]]))/3645
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^7}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.48939, size = 204, normalized size = 1.09

$$\frac{2}{729}\left(-3x^2+2\right)^{\frac{9}{4}} - \frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/729*(-3*x^2 + 2)^(9/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4)
```

Fricas [A] time = 1.70798, size = 622, normalized size = 3.31

$$\frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}-2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}-1\right)+\frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 32/81*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 32/81*2^(3/4)*arctan(2^
(1/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1
/4)*(-3*x^2 + 2)^(1/4) + 1) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/3645*(45*x^4 + 156*x^2 + 1136)*(-3*x^2 +
 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^{7}}{3x^{2}\left(2-3x^{2}\right)^{\frac{3}{4}}-4\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.20685, size = 216, normalized size = 1.15

$$\frac{2}{729}\left(3x^{2}-2\right)^{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 2)^2*(-3*x^2 + 2)^(1/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(
2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/
4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
```

```
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2
+ 2)^(1/4)
```

3.1062 $\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=173

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\right.$$

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])
/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]
)/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
])/27
```

Rubi [A] time = 0.180285, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {443, 261, 266, 43, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```


```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])
/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]
)/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
])/27
```


Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}dx = \int \left(-\frac{4x}{9\left(2-3x^2\right)^{3/4}} - \frac{x^3}{3\left(2-3x^2\right)^{3/4}} + \frac{16x}{9\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{x^3}{\left(2-3x^2\right)^{3/4}}dx\right) - \frac{4}{9}\int \frac{x}{\left(2-3x^2\right)^{3/4}}dx + \frac{16}{9}\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}dx$$
$$= \frac{8}{27}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{x}{(2-3x)^{3/4}}dx, x, x^2\right) + \frac{8}{9}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}dx, x,\right.$$
$$= \frac{8}{27}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \left(\frac{2}{3(2-3x)^{3/4}} - \frac{1}{3}\sqrt[4]{2-3x}\right)dx, x, x^2\right) - \frac{32}{27}\text{Subst}\left(\int \frac{1}{2+x^4}\right.$$
$$= \frac{4}{9}\sqrt[4]{2-3x^2} - \frac{2}{135}\left(2-3x^2\right)^{5/4} - \frac{1}{27}\left(8\sqrt{2}\right)\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}dx, x, \sqrt[4]{2-3x^2}\right) - \frac{1}{27}\left(8\sqrt{2}\right.$$
$$= \frac{4}{9}\sqrt[4]{2-3x^2} - \frac{2}{135}\left(2-3x^2\right)^{5/4} - \frac{1}{27}\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}dx, x, \sqrt[4]{2-3x^2}\right) -$$
$$= \frac{4}{9}\sqrt[4]{2-3x^2} - \frac{2}{135}\left(2-3x^2\right)^{5/4} + \frac{2}{27}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right) - \frac{2}{27}2^{3/4}\log$$
$$= \frac{4}{9}\sqrt[4]{2-3x^2} - \frac{2}{135}\left(2-3x^2\right)^{5/4} - \frac{4}{27}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right) + \frac{4}{27}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-\right.$$

Mathematica [A] time = 0.0639823, size = 163, normalized size = 0.94

$$\frac{2}{135}\left(3\sqrt[4]{2-3x^2}x^2 + 28\sqrt[4]{2-3x^2} + 5\,2^{3/4}\log\left(\sqrt{2-3x^2} - 2^{3/4}\sqrt[4]{2-3x^2} + \sqrt{2}\right) - 5\,2^{3/4}\log\left(\sqrt{2-3x^2} + 2^{3/4}\sqrt[4]{2-3x^2} + \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(28*(2 - 3*x^2)^(1/4) + 3*x^2*(2 - 3*x^2)^(1/4) + 10*2^(3/4)*ArcTan[1 -
(4 - 6*x^2)^(1/4)] - 10*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)] + 5*2^(3/4)*L
og[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - 5*2^(3/4)*Log[S
qrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]))/135
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{x^5}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.52908, size = 189, normalized size = 1.09

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

Fricas [A] time = 1.7383, size = 601, normalized size = 3.47

$$\frac{8}{27} \cdot 2^{\frac{3}{4}} \arctan\left(2^{\frac{1}{4}} \sqrt{2^{\frac{3}{4}} \left(-3 x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3 x^{2}+2}}-2^{\frac{1}{4}} \left(-3 x^{2}+2\right)^{\frac{1}{4}}-1\right)+\frac{8}{27} \cdot 2^{\frac{3}{4}} \arctan\left(2^{\frac{1}{4}} \sqrt{-2^{\frac{3}{4}} \left(-3 x^{2}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 8/27*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*2^(3/4)*arctan(2^(1
/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4
)*(-3*x^2 + 2)^(1/4) + 1) - 2/27*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 2/135*(3*x^2 + 28)*(-3*x^2 + 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{5}}{3x^{2}\left(2-3x^{2}\right)^{\frac{3}{4}}-4\left(2-3x^{2}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.20761, size = 189, normalized size = 1.09

$$-\frac{4}{27} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}}+2 \left(-3 x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27} \cdot 2^{\frac{3}{4}} \arctan\left(-\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}}-2 \left(-3 x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{3}{4}}\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

3.1063 $\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=158

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)$$

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/9 + (2^(3
/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x
^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4))
```

Rubi [A] time = 0.152481, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {443, 261, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```


```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/9 + (2^(3
/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x
^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4))
```


Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(-\frac{x}{3(2-3x^2)^{3/4}}+\frac{4x}{3(2-3x^2)^{3/4}(4-3x^2)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{x}{(2-3x^2)^{3/4}}\,dx\right)+\frac{4}{3}\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx,x,x^2\right)$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{8}{9}\text{Subst}\left(\int \frac{1}{2+x^4}\,dx,x,\sqrt[4]{2-3x^2}\right)$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx,x,\sqrt[4]{2-3x^2}\right)-\frac{1}{9}\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2+x^4}\right.$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\text{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)}{9\sqrt[4]{2}}+\frac{\text{Subst}\left(\int \frac{2^{3/4}-2x}{-\sqrt{2}+2^{3/4}x-x^2}\,dx,x,\sqrt[4]{2-\right.}{9\sqrt[4]{2}}$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{9\sqrt[4]{2}}$$
$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{\log\left(\sqrt{2}-2^{\ldots}\right.}{}$$

Mathematica [A] time = 0.0382085, size = 146, normalized size = 0.92

$$\frac{1}{18}\left(4\sqrt[4]{2-3x^2}+2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+2\,2^{3/4}\tan^{-1}\left(1-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(1/4) + 2*2^(3/4)*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*2^(3/4)*
ArcTan[1 + (4 - 6*x^2)^(1/4)] + 2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(
1/4) + Sqrt[2 - 3*x^2]] - 2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) +
 Sqrt[2 - 3*x^2]])/18
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^3}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.49483, size = 174, normalized size = 1.1

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

Fricas [A] time = 1.70154, size = 578, normalized size = 3.66

$$\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3\,x^2+2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 2/9*2^(3/4)*arctan(2^(1/4
)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*
(-3*x^2 + 2)^(1/4) + 1) - 1/18*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.18944, size = 174, normalized size = 1.1

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

3.1064 $\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{\ldots}\right)}{6\sqrt[4]{2}}$$

```
[Out] -ArcTan[1 + (4 - 6*x^2)^(1/4)]/(6*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)
^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*
x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x
^2]]/(12*2^(1/4))
```

Rubi [A] time = 0.117628, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{\ldots}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[1 + (4 - 6*x^2)^(1/4)]/(6*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)
^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*
x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x
^2]]/(12*2^(1/4))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx, x, x^2\right)\\
&= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)}{3\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)}{3\sqrt{2}}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{6\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{6\sqrt{2}} + \frac{\text{Sub}}{}\\
&= \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}} - \frac{\text{Subst}}{}\\
&= -\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{6\sqrt[4]{2}} + \frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-\right.}{12\sqrt[4]{2}}
\end{aligned}
$$

Mathematica [A] time = 0.02572, size = 117, normalized size = 0.82

$$
\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\tan^{-1}\left(\sqrt[4]{4\right.}{12\sqrt[4]{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)] + Log[Sq
rt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - Log[Sqrt[2] + 2^(3/4
)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(12*2^(1/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.55516, size = 159, normalized size = 1.11

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [B] time = 1.68018, size = 741, normalized size = 5.18

$$\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+4\,\sqrt{2}+4\,\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/24*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) - 1) + 1/24*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/96*8^(3/4)*sqrt(2)*log(16*
8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1/
96*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2)
+ 64*sqrt(-3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.21157, size = 159, normalized size = 1.11

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1065 $\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=197

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}$$

```
[Out] -ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) - ArcTan[1 + (4 - 6*x^2)^(1/
4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - ArcTa
nh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^
2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2
)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4))
```

Rubi [A] time = 0.189395, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {443, 266, 63, 212, 206, 203, 444, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) - ArcTan[1 + (4 - 6*x^2)^(1/
4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - ArcTa
nh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^
2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2
)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4))
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx = \int \left(\frac{1}{4x\left(2-3x^2\right)^{3/4}} - \frac{3x}{4\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right) dx$$
$$= \frac{1}{4}\int \frac{1}{x\left(2-3x^2\right)^{3/4}}\,dx - \frac{3}{4}\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx$$
$$= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}x}\,dx, x, x^2\right) - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(-4+3x)}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx, x, \sqrt[4]{2-3x^2}\right)\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-2-x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{-2-x^4}\right.}{4}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{8\sqrt{2}} - \text{Subst}\Big($$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{16\sqrt[4]{2}} - \log\Big(\sqrt{2}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{8\sqrt[4]{2}} + \frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

Mathematica [A] time = 0.0567033, size = 156, normalized size = 0.79

$$\frac{-4\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) - 4\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + \sqrt{2}\left(\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right) - \log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}\right.\right.}{16\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*ArcTan[(1 - (3*x^2)/2)^(1/4)] - 4*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + Sqrt
[2]*(2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)] + Lo
g[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - Log[Sqrt[2] + 2^
(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]))/(16*2^(3/4))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-3x^2+4\right)}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x), x)
```

Fricas [B] time = 1.73771, size = 1019, normalized size = 5.17

$$\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+4\,\sqrt{2}+4\,\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/32*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) - 1) + 1/32*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/128*8^(3/4)*sqrt(2)*log(16
*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1
/128*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2
) + 64*sqrt(-3*x^2 + 2)) + 1/16*8^(3/4)*arctan(1/2*8^(1/4)*sqrt(sqrt(2) + s
qrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 1/64*8^(3/4)*log(8^(3/
4) + 4*(-3*x^2 + 2)^(1/4)) + 1/64*8^(3/4)*log(-8^(3/4) + 4*(-3*x^2 + 2)^(1/
4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^3\left(2-3x^2\right)^{\frac{3}{4}}-4x\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(3/4) - 4*x*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.25155, size = 284, normalized size = 1.44

$$-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/16*4^(1/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(1/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) - 1/32*4^(1/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/32*4^(1/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/8*
4^(1/8)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) - 1/16*4^(1/8)*log((-3*x^2 +
 2)^(1/4) + 4^(1/8)) + 1/16*4^(1/8)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8))
```

3.1066 $\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-$$

```
[Out] -(2 - 3*x^2)^(1/4)/(16*x^2) - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4)) + (3*ArcTan[1 - 2^(
1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/
4)])/(32*2^(3/4)) + (3*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3
*x^2]])/(64*2^(1/4)) - (3*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/(64*2^(1/4))
```

Rubi [A] time = 0.240897, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {443, 266, 51, 63, 212, 206, 203, 444, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(16*x^2) - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4)) + (3*ArcTan[1 - 2^(
1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/
4)])/(32*2^(3/4)) + (3*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3
*x^2]])/(64*2^(1/4)) - (3*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/(64*2^(1/4))
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=\int\left(\frac{1}{4x^3\left(2-3x^2\right)^{3/4}}+\frac{3}{16x\left(2-3x^2\right)^{3/4}}-\frac{9x}{16\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right)dx$$

$$=\frac{3}{16}\int\frac{1}{x\left(2-3x^2\right)^{3/4}}\,dx+\frac{1}{4}\int\frac{1}{x^3\left(2-3x^2\right)^{3/4}}\,dx-\frac{9}{16}\int\frac{x}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx$$

$$=\frac{3}{32}\text{Subst}\left(\int\frac{1}{(2-3x)^{3/4}x}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int\frac{1}{(2-3x)^{3/4}x^2}\,dx,x,x^2\right)-\frac{9}{32}\text{Su}$$

$$=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{1}{8}\text{Subst}\left(\int\frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)+\frac{9}{64}\text{Subst}\left(\int\frac{1}{(2-3x)^{3/4}x}\,dx,\right.$$

$$=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{3}{16}\text{Subst}\left(\int\frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)-\frac{3\,\text{Subst}\left(\int\frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{2-\right.}{16\sqrt{2}}$$

$$=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\,2^{3/4}}-\frac{3\,\text{Subst}\left(\int\frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx,x,\right.}{32\sqrt{2}}$$

$$=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}+\frac{3\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}\right.}{64\sqrt[4]{2}}$$

$$=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{3\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-\right.}{32\sqrt[4]{2}}$$

Mathematica [A] time = 0.0780583, size = 210, normalized size = 0.98

$$-\frac{8\sqrt[4]{2-3x^2}-3\,2^{3/4}x^2\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+3\,2^{3/4}x^2\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+30\sqrt[4]{2}x}{128}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(8*(2 - 3*x^2)^(1/4) + 30*2^(1/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] - 6*2^
(3/4)*x^2*ArcTan[1 - (4 - 6*x^2)^(1/4)] + 6*2^(3/4)*x^2*ArcTan[1 + (4 - 6*x
^2)^(1/4)] + 30*2^(1/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] - 3*2^(3/4)*x^2*
Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] + 3*2^(3/4)*x^2*
```

```
Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(128*x^2)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-3x^2+4\right)}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3), x)
```

Fricas [B] time = 1.81365, size = 1081, normalized size = 5.03

$$12\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+12\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/512*(12*8^(3/4)*sqrt(2)*x^2*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(
2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(
2)*(-3*x^2 + 2)^(1/4) - 1) + 12*8^(3/4)*sqrt(2)*x^2*arctan(1/16*8^(1/4)*sqr
t(2)*sqrt(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*
x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 3*8^(3/4)*sqrt(2)
*x^2*log(16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^
2 + 2)) + 3*8^(3/4)*sqrt(2)*x^2*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)
+ 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 60*8^(3/4)*x^2*arctan(1/2*8^(1/4)*sqr
t(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 15*8^(3/4
)*x^2*log(8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 15*8^(3/4)*x^2*log(-8^(3/4) + 4
*(-3*x^2 + 2)^(1/4)) - 32*(-3*x^2 + 2)^(1/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^5\left(2-3x^2\right)^{\frac{3}{4}}-4x^3\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(3/4) - 4*x**3*(2 - 3*x**2)**(3/4)), x)
```

Giac [A] time = 1.26565, size = 259, normalized size = 1.2

$$-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -3/64*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/64*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 3/128*2^(3/4
)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 3/128*2^(3
/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 15/64*2
^(1/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 15/128*2^(1/4)*log(2^(1/4)
+ (-3*x^2 + 2)^(1/4)) + 15/128*2^(1/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)) -
1/16*(-3*x^2 + 2)^(1/4)/x^2
```

3.1067 $\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=182

$$-\frac{160\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{567\sqrt{3}}+\frac{2}{63}\sqrt[4]{2-3x^2}x^3+\frac{80}{567}\sqrt[4]{2-3x^2}x+\frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\ 2^{3/4}\tanh^{-1}\cdots}{2\cdots}$$

```
[Out] (80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*2^(3/4)*Ar
cTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(2
7*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3
]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3]) - (160*2^(3/4)*EllipticF[ArcSin[Sqrt[
3/2]*x]/2, 2])/(567*Sqrt[3])
```

Rubi [A] time = 0.118129, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {443, 232, 321, 400, 441}

$$\frac{2}{63}\sqrt[4]{2-3x^2}x^3+\frac{80}{567}\sqrt[4]{2-3x^2}x+\frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\ 2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{160\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{567\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*2^(3/4)*Ar
cTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(2
7*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3
]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3]) - (160*2^(3/4)*EllipticF[ArcSin[Sqrt[
3/2]*x]/2, 2])/(567*Sqrt[3])
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(-\frac{16}{27(2-3x^2)^{3/4}} - \frac{4x^2}{9(2-3x^2)^{3/4}} - \frac{x^4}{3(2-3x^2)^{3/4}} + \frac{64}{27(2-3x^2)^{3/4}(4-3x^2)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx\right) - \frac{4}{9}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx - \frac{16}{27}\int \frac{1}{(2-3x^2)^{3/4}}\,dx + \frac{64}{27}\int \frac{}{(2-3}$$

$$= \frac{8}{81}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} - \frac{16\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}} - \frac{4}{21}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$$

$$= \frac{80}{567}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} + \frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} - \frac{8\ 2^{3/4}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{}}{\sqrt{3}x}\right.}{27\sqrt{3}}$$

$$= \frac{80}{567}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} + \frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} - \frac{8\ 2^{3/4}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{}}{\sqrt{3}x}\right.}{27\sqrt{3}}$$

Mathematica [C] time = 0.229756, size = 190, normalized size = 1.04

$$\frac{2}{567}x\left(31\sqrt[4]{2}x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right) + \frac{\frac{1280F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)} - 27x^4 -}{(2-3x^2)^{3/4}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(31*2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (80
 - 102*x^2 - 27*x^4 + (1280*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4
])/((-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*
(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4,
1, 5/2, (3*x^2)/2, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/567
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x^6}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^6}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-3x^2+2\right)^{\frac{1}{4}}x^6}{9x^4-18x^2+8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)*x^6/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^6}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**6/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^6}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1068 $\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=164

$$-\frac{4\,2^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{27\sqrt{3}}+\frac{2}{27}\sqrt[4]{2-3x^2}x+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}$$

```
[Out] (2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (2*2^(3/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

Rubi [A] time = 0.0984093, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {443, 232, 321, 400, 441}

$$\frac{2}{27}\sqrt[4]{2-3x^2}x+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (2*2^(3/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
```

```
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \left(-\frac{4}{9\left(2-3x^2\right)^{3/4}}-\frac{x^2}{3\left(2-3x^2\right)^{3/4}}+\frac{16}{9\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right)dx\\
&= -\left(\frac{1}{3}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx\right)-\frac{4}{9}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx+\frac{16}{9}\int \frac{1}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx\\
&= \frac{2}{27}x\sqrt[4]{2-3x^2}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}-\frac{4}{27}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx+\frac{4}{9}\int \frac{1}{\left(2-3x^2\right)^{3/}}\\
&= \frac{2}{27}x\sqrt[4]{2-3x^2}+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{4\,2^{3/4}F}{}
\end{aligned}$$

Mathematica [C] time = 0.120838, size = 184, normalized size = 1.12

$$\frac{2}{27}x\left(\sqrt[4]{2}x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+\frac{\frac{32F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}-3x^2+2}{\left(2-3x^2\right)^{3/4}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2 - 3
*x^2 + (32*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*
(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2,
3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^2)/2
, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/27
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^4}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-3x^2+2)^{\frac{1}{4}}x^4}{9x^4-18x^2+8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{3x^2(2-3x^2)^{\frac{3}{4}}-4(2-3x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**4/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1069 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.0225193, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [C] time = 0.0304062, size = 37, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(3/4))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [B] time = 1.71199, size = 871, normalized size = 7.26

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1070 $\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=148

$$\frac{\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(
4*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x
*(2 - 3*x^2)^(1/4))]/(4*2^(1/4)*Sqrt[3]) + EllipticF[ArcSin[Sqrt[3/2]*x]/2,
 2]/(2*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.0419964, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {400, 232, 441}

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}+\frac{F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(
4*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x
*(2 - 3*x^2)^(1/4))]/(4*2^(1/4)*Sqrt[3]) + EllipticF[ArcSin[Sqrt[3/2]*x]/2,
 2]/(2*2^(1/4)*Sqrt[3])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{1}{4}\int \frac{1}{(2-3x^2)^{3/4}}\,dx + \frac{3}{4}\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}} + \frac{F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

Mathematica [C] time = 0.0192556, size = 67, normalized size = 0.45

$$\frac{\sqrt{x^2}\left(\Pi\left(-i;-\sin^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\middle|-1\right)+\Pi\left(i;-\sin^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\middle|-1\right)\right)}{2\sqrt[4]{2}\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (Sqrt[x^2]*(EllipticPi[-I, -ArcSin[(1 - (3*x^2)/2)^(1/4)], -1] + EllipticPi
[I, -ArcSin[(1 - (3*x^2)/2)^(1/4)], -1]))/(2*2^(1/4)*Sqrt[3]*x)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{9\,x^4-18\,x^2+8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1071 $\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)} dx$

Optimal. Leaf size=166

$$\frac{\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{4\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqrt[3]*ArcTanh[(2^(3/4
) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) +
 (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(1/4))
```

Rubi [A] time = 0.090179, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {443, 325, 232, 400, 441}

$$-\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqrt[3]*ArcTanh[(2^(3/4
) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) +
 (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(1/4))
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
```

```
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int\left(\frac{1}{4x^2\left(2-3x^2\right)^{3/4}}-\frac{3}{4\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right)dx\\
&= \frac{1}{4}\int\frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx-\frac{3}{4}\int\frac{1}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{8x}+2\left(\frac{3}{16}\int\frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\right)-\frac{9}{16}\int\frac{x^2}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\right.\right.}{4\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.0499904, size = 37, normalized size = 0.22

$$-\frac{F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{4\ 2^{3/4}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -AppellF1[-1/2, 3/4, 1, 1/2, (3*x^2)/2, (3*x^2)/4]/(4*2^(3/4)*x)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-3\,x^2+4\right)}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-3\,x^{2}+2\right)^{\frac{1}{4}}}{9\,x^{6}-18\,x^{4}+8\,x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{3x^{4}\left(2-3x^{2}\right)^{\frac{3}{4}}-4x^{2}\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**4*(2 - 3*x**2)**(3/4) - 4*x**2*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{\left(3\,x^{2}-4\right)\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

3.1072 $\int \frac{1}{x^4(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=184

$$\frac{11\sqrt{3}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{4x}-\frac{\sqrt[4]{2-3x^2}}{24x^3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(24*x^3) - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3]*ArcTan[(
2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^(1
/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]
/2, 2])/(32*2^(1/4))
```

Rubi [A] time = 0.11609, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {443, 325, 232, 400, 441}

$$-\frac{\sqrt[4]{2-3x^2}}{4x}-\frac{\sqrt[4]{2-3x^2}}{24x^3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}+\frac{11\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```


```
[Out] -(2 - 3*x^2)^(1/4)/(24*x^3) - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3]*ArcTan[(
2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^(1
/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]
/2, 2])/(32*2^(1/4))
```


Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x^4\left(2-3x^2\right)^{3/4}}+\frac{3}{16x^2\left(2-3x^2\right)^{3/4}}-\frac{9}{16\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right)dx\\
&= \frac{3}{16}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx+\frac{1}{4}\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx-\frac{9}{16}\int \frac{1}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,d\\
&= -\frac{\sqrt[4]{2-3x^2}}{24x^3}-\frac{3\sqrt[4]{2-3x^2}}{32x}+2\left(\frac{9}{64}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\right)+\frac{5}{16}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx-\frac{2}{6}\\
&= -\frac{\sqrt[4]{2-3x^2}}{24x^3}-\frac{\sqrt[4]{2-3x^2}}{4x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}\\
&= -\frac{\sqrt[4]{2-3x^2}}{24x^3}-\frac{\sqrt[4]{2-3x^2}}{4x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.0507399, size = 37, normalized size = 0.2

$$-\frac{F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4}x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -AppellF1[-3/2, 3/4, 1, -1/2, (3*x^2)/2, (3*x^2)/4]/(12*2^(3/4)*x^3)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-3\,x^2+4\right)}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(-3x^2+2)^{\frac{1}{4}}}{9x^8-18x^6+8x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^6(2-3x^2)^{\frac{3}{4}} - 4x^4(2-3x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**6*(2 - 3*x**2)**(3/4) - 4*x**4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

3.1073 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.0163922, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.0394659, size = 52, normalized size = 0.85

$$-\frac{x^3\left(1-3x^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(6*(-
1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{x^2}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [B] time = 1.53749, size = 282, normalized size = 4.62

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-}{9x^4-12x^2+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1074 $\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.016959, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2-3x^2)\left(-1-3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.0417319, size = 52, normalized size = 0.85

$$-\frac{x^3\left(3x^2+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)}{6\left(-3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 + 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -3*x^2, (-3*x^2)/2])/(6*
(-1 - 3*x^2)^(3/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2-2}\left(-3x^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(3x^2+2\right)\left(-3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

Fricas [C] time = 1.66068, size = 343, normalized size = 5.62

$$-\frac{1}{36}\sqrt{6}\log\left(\frac{\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{\sqrt{6}x-2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)-\frac{1}{36}i\sqrt{6}\log\left(\frac{i\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*log(1/2*(sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*sqrt(6)*
log(-1/2*(sqrt(6)*x - 2*(-3*x^2 - 1)^(1/4))/x) - 1/36*I*sqrt(6)*log(1/2*(I*
sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*I*sqrt(6)*log(1/2*(-I*sqrt(6)*x
 + 2*(-3*x^2 - 1)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 - 1)**(3/4) + 2*(-3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3x^2+2)(-3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

3.1075 $\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rubi [A] time = 0.0242826, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+bx^2\right)\left(-1+bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

Mathematica [C] time = 0.0500228, size = 54, normalized size = 0.75

$$-\frac{x^3\left(1-bx^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)}{6\left(bx^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 - b*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, b*x^2, (b*x^2)/2])/(6*(-
1 + b*x^2)^(3/4))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2-2}\left(bx^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2-1\right)^{\frac{3}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

Fricas [B] time = 1.64654, size = 707, normalized size = 9.82

$$\left[-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}x}\right)-\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{bx^2-1}\,bx^2+4\,bx^2-4\sqrt{2}\left(bx^2-1\right)^{\frac{3}{4}}\sqrt{b}x-4}{b^2x^4-4\,bx^2+4}\right)}{4\,b^2}, \frac{2\sqrt{2}\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - sq
rt(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*s
qrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)
/(b^2*x^4 - 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x
^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 2*sqrt(2)*
(b*x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 + 4*sq
rt(2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2-2\right)\left(bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((b*x**2 - 2)*(b*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2-1\right)^{\frac{3}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1076 $\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rubi [A] time = 0.025874, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2-bx^2\right)\left(-1-bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

Mathematica [C] time = 0.0517411, size = 55, normalized size = 0.74

$$-\frac{x^3\left(bx^2+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)}{6\left(-bx^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 + b*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^2), -(b*x^2)/2])/(
6*(-1 - b*x^2)^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2-2}\left(-bx^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

Fricas [B] time = 1.70306, size = 682, normalized size = 9.22

$$\left[-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}x}\right)-\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4bx^2-2\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3+2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4bx^2+4}\right)}{4b^2}, \frac{2\sqrt{2}\sqrt{-b}\,\mathrm{arc}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - s
qrt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(
2)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^
4 + 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)
^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1
)*b*x^2 - 4*b*x^2 - 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3
/4)*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{bx^2\left(-bx^2-1\right)^{\frac{3}{4}}+2\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
```

```
[Out] -Integral(x**2/(b*x**2*(-b*x**2 - 1)**(3/4) + 2*(-b*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

3.1077 $\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rubi [A] time = 0.026389, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a+3x^2\right)\left(-a+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Mathematica [C] time = 0.0527904, size = 66, normalized size = 0.78

$$-\frac{x^3\left(1-\frac{3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a\left(3x^2-a\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 - (3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(
2*a)])/(6*a*(-a + 3*x^2)^(3/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^2}{3x^2-2a}\left(3x^2-a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-a\right)^{\frac{3}{4}}\left(3x^2-2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

Fricas [B] time = 1.67865, size = 437, normalized size = 5.14

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{3x^2-a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+\left(3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-(3x}{x}\right.}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(3*x^2 - a))/x^2) - (1/36)^(3/4)*(3*x^2 - a)^(1/4)*a^(1/4))/x
)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (3*x^2 - a)^(1
/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (3*x^2
- a)^(1/4))/x)/a^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-2a+3x^2\right)\left(-a+3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + 3*x**2)*(-a + 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-a\right)^{\frac{3}{4}}\left(3\,x^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

3.1078 $\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rubi [A] time = 0.0235324, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a-3x^2\right)\left(-a-3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Mathematica [C] time = 0.0560152, size = 67, normalized size = 0.79

$$-\frac{x^3\left(\frac{a+3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a\left(-a-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] -(x^3*((a + 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)
/(2*a)])/(6*a*(-a - 3*x^2)^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2 - 2a} \left(-3x^2 - a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(3x^2 + 2a\right)\left(-3x^2 - a\right)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

Fricas [B] time = 1.64697, size = 443, normalized size = 5.21

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}} \arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}} a^{\frac{1}{4}} x \sqrt{\frac{\frac{3x^2}{\sqrt{a}} + 2\sqrt{-3x^2 - a}}{x^2}} - \left(\frac{1}{36}\right)^{\frac{3}{4}} \left(-3x^2 - a\right)^{\frac{1}{4}} a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}} - \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}} \log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}} x}{a^{\frac{1}{4}}} + \left(-3x^2 - a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} + \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}} \log\left(-\frac{3\left(\frac{1}{36}\right)}{a^{\frac{1}{4}}} \cdots\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(-3*x^2 - a))/x^2) - (1/36)^(3/4)*(-3*x^2 - a)^(1/4)*a^(1/4))
/x)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (-3*x^2 - a)
^(1/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (-3*
x^2 - a)^(1/4))/x)/a^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2a\left(-a - 3x^2\right)^{\frac{3}{4}} + 3x^2\left(-a - 3x^2\right)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - 3*x**2)**(3/4) + 3*x**2*(-a - 3*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(3\,x^2+2\,a\right)\left(-3\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

3.1079 $\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.0330777, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a+bx^2\right)\left(-a+bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Mathematica [C] time = 0.0607403, size = 68, normalized size = 0.71

$$-\frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a\left(bx^2-a\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 - (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(
2*a)])/(6*a*(-a + b*x^2)^(3/4))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2 - 2a}\left(bx^2 - a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2 - a\right)^{\frac{3}{4}}\left(bx^2 - 2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

Fricas [B] time = 1.67302, size = 554, normalized size = 5.77

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}} - \left(\frac{1}{4}\right)^{\frac{3}{4}}\left(bx^2-a\right)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right) - \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (1
/4)^(3/4)*(b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)*(
1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (b*x^2 - a)^(1/
4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a*b^
6))^(1/4) - (b*x^2 - a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-2a + bx^2\right)\left(-a + bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + b*x**2)*(-a + b*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^2-a\right)^{\frac{3}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

3.1080 $\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.0332707, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a-bx^2\right)\left(-a-bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Mathematica [C] time = 0.0589936, size = 70, normalized size = 0.71

$$-\frac{x^3\left(\frac{a+bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a\left(-a-bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] -(x^3*((a + b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -(b*x^
2)/(2*a)])/(6*a*(-a - b*x^2)^(3/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2-2\,a}\left(-bx^2-a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

Fricas [B] time = 1.67294, size = 559, normalized size = 5.7

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{-bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(-bx^2-a\right)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(-b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (
1/4)^(3/4)*(-b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)
*(1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (-b*x^2 - a)^
(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a
*b^6))^(1/4) - (-b*x^2 - a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2a\left(-a-bx^2\right)^{\frac{3}{4}}+bx^2\left(-a-bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - b*x**2)**(3/4) + b*x**2*(-a - b*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(bx^2 + 2a)(-bx^2 - a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

3.1081 $\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1 + 3*x^2)^(
9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/
4)])/81
```

Rubi [A] time = 0.0509349, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 63, 212, 206, 203}

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1 + 3*x^2)^(
9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/
4)])/81
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{7}{27(-1+3x)^{3/4}}+\frac{8}{27(-2+3x)(-1+3x)^{3/4}}+\frac{4}{27}\sqrt[4]{-1+3x}+\frac{1}{27}(-1+3x\right.\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4}+\frac{2}{729}\left(-1+3x^2\right)^{9/4}+\frac{4}{27}\operatorname{Subst}\left(\int \frac{1}{(-2+3x)(-1\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4}+\frac{2}{729}\left(-1+3x^2\right)^{9/4}+\frac{16}{81}\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4}+\frac{2}{729}\left(-1+3x^2\right)^{9/4}-\frac{8}{81}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{}\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4}+\frac{2}{729}\left(-1+3x^2\right)^{9/4}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{8}{81}\end{aligned}$$

Mathematica [A] time = 0.0346687, size = 58, normalized size = 0.74

$$\frac{2\sqrt[4]{3x^2-1}\left(45x^4+78x^2+284\right)-360\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-360\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(284 + 78*x^2 + 45*x^4) - 360*ArcTan[(-1 + 3*x^2)^(1/
4)] - 360*ArcTanh[(-1 + 3*x^2)^(1/4)])/3645
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{x^7}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.5393, size = 100, normalized size = 1.28

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left((3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.53744, size = 204, normalized size = 2.62

$$\frac{2}{3645}\left(45\,x^4+78\,x^2+284\right)\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3645*(45*x^4 + 78*x^2 + 284)*(3*x^2 - 1)^(1/4) - 8/81*arctan((3*x^2 - 1)^
(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**7/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A] time = 1.2302, size = 101, normalized size = 1.29

$$\frac{2}{729}\left(3\,x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1082 $\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan[(-1 + 3*x
^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi [A] time = 0.0460011, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 63, 212, 206, 203}

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan[(-1 + 3*x
^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^2}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{4}{9(-2+3x)(-1+3x)^{3/4}}+\frac{1}{9}\sqrt[4]{-1+3x}\right)dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}(-1+3x^2)^{5/4}+\frac{2}{9}\mathrm{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}(-1+3x^2)^{5/4}+\frac{8}{27}\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}(-1+3x^2)^{5/4}-\frac{4}{27}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\mathrm{Su}\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}(-1+3x^2)^{5/4}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{-1+3x\right.
\end{aligned}$$

Mathematica [A] time = 0.0262596, size = 53, normalized size = 0.84

$$\frac{1}{135}\left(2\sqrt[4]{3x^2-1}\,(3x^2+14)-20\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-20\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(14 + 3*x^2) - 20*ArcTan[(-1 + 3*x^2)^(1/4)] - 20*Arc
Tanh[(-1 + 3*x^2)^(1/4)])/135
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{x^5}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.46538, size = 85, normalized size = 1.35

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(
1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.55292, size = 188, normalized size = 2.98

$$\frac{2}{135}\left(3x^2+14\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/135*(3*x^2 + 14)*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/2
7*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**5/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A] time = 1.21353, size = 86, normalized size = 1.37

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(
1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
1))
```

3.1083 $\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(-
1 + 3*x^2)^(1/4)])/9
```

Rubi [A] time = 0.0323884, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 63, 212, 206, 203}

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(-
1 + 3*x^2)^(1/4)])/9
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{4}{9}\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1\right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0098354, size = 44, normalized size = 0.92

$$\frac{2}{9}\left(\sqrt[4]{3x^2-1}-\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(1/4) - ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^
(1/4)]))/9
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^3}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.56127, size = 70, normalized size = 1.46

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.50087, size = 163, normalized size = 3.4

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**3/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A] time = 1.26419, size = 72, normalized size = 1.5

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1084 $\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi [A] time = 0.0212163, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 63, 212, 206, 203}

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)$$
$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)$$
$$= -\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

Mathematica [A] time = 0.0057361, size = 33, normalized size = 1.

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{x}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.57283, size = 55, normalized size = 1.67

$$-\frac{1}{3}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 1.57698, size = 132, normalized size = 4.

$$-\frac{1}{3}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

Sympy [A] time = 4.23762, size = 42, normalized size = 1.27

$$\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}-\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 - atan((3*x
**2 - 1)**(1/4))/3
```

Giac [A] time = 1.18501, size = 57, normalized size = 1.73

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

3.1085 $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-\right.}{2\sqrt{2}}$$

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*S
qrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1
+ 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/
(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqr
t[2])
```

Rubi [A] time = 0.125718, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {446, 86, 63, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*S
qrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1
+ 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/
(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqr
t[2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx, x, x^2\right)\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx,\right.$$
$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) + \text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right.$$
$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,\right.$$
$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1\right.}}{4\sqrt{2}}$$
$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0486646, size = 150, normalized size = 0.87

$$\frac{1}{8}\left(-4\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - 4\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right) + \sqrt{2}\left(\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right) - \log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-4*ArcTan[(-1 + 3*x^2)^(1/4)] - 4*ArcTanh[(-1 + 3*x^2)^(1/4)] + Sqrt[2]*(2
*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)] - 2*ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)
^(1/4)] + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]] - Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]))/8
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(3x^2-2\right)}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x), x)
```

Fricas [A] time = 1.68814, size = 649, normalized size = 3.75

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) - 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) + 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) - 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A] time = 1.19595, size = 209, normalized size = 1.21

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1086 $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{15}{}$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(4*x^2) - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (15*ArcTan
[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*ArcTan[1 + Sqrt[2]*(-1
+ 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 + (15*Log[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[
1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.146251, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {446, 103, 156, 63, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{15}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```


```
[Out] -(-1 + 3*x^2)^(1/4)/(4*x^2) - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (15*ArcTan
[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*ArcTan[1 + Sqrt[2]*(-1
+ 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 + (15*Log[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[
1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{15}{2}+\frac{27x}{4}}{x(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{15}{16}\text{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx,x,x^2\right) + \frac{9}{8}\text{Subst}\left(\int \frac{1}{(-2+3x)(-$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx\right.$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{8}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right) - \frac{5}{8}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x\right.$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{15}{16}\text{Subst}\left(\int -\right.$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\log\left(1-\sqrt{2}\right.}{}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} - \frac{15\tan^{-1}}{}$$

Mathematica [A] time = 0.0518076, size = 181, normalized size = 0.95

$$\frac{1}{32}\left(-\frac{8\sqrt[4]{3x^2-1}}{x^2}+15\sqrt{2}\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-15\sqrt{2}\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-24\tan^{-1}\left(\sqrt[4]{3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((-8*(-1 + 3*x^2)^(1/4))/x^2 - 24*ArcTan[(-1 + 3*x^2)^(1/4)] + 30*Sqrt[2]*A
rcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)] - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4)] - 24*ArcTanh[(-1 + 3*x^2)^(1/4)] + 15*Sqrt[2]*Log[1 - Sqrt[2
]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/32
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3x^2-2\right)}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A] time = 1.60238, size = 722, normalized size = 3.78

$$60\,\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1}-\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)+60\,\sqrt{2}x^2\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{-4\,\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/32*(60*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 60*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) - 15*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) + 15*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) - 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3
*x^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(1
/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A] time = 1.22364, size = 228, normalized size = 1.19

$$-\frac{15}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{15}{16}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{15}{32}\,\sqrt{2}\log\left(\sqrt{2}\left(3\,x^2-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -15/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 15/16*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 15/32*sqrt(2
)*log(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 15/32*sqrt(2)*log(
-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(1/4)/x
^2 - 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*l
og(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1087 $\int \frac{x^6}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=165

$$\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{567\sqrt{3}x}+\frac{2}{63}\sqrt[4]{3x^2-1}x^3+\frac{40}{567}\sqrt[4]{3x^2-1}x+\frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\vphantom{\frac{1}{1}}\right.$$

```
[Out] (40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (2*Sqrt[2/3
]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqrt[2/3]*ArcTanh[(Sqrt
[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(
1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(567*Sq
rt[3]*x)
```

Rubi [A] time = 0.181692, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {443, 234, 220, 321, 400, 442}

$$\frac{2}{63}\sqrt[4]{3x^2-1}x^3+\frac{40}{567}\sqrt[4]{3x^2-1}x+\frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}\right.}{56}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (2*Sqrt[2/3
]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqrt[2/3]*ArcTanh[(Sqrt
[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(
1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(567*Sq
rt[3]*x)
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^6}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int \left(\frac{4}{27\left(-1+3x^2\right)^{3/4}} + \frac{2x^2}{9\left(-1+3x^2\right)^{3/4}} + \frac{x^4}{3\left(-1+3x^2\right)^{3/4}} + \frac{8}{27\left(-2+3x^2\right)\left(-1+3x\right.}\right.$$

$$= \frac{4}{27}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx + \frac{2}{9}\int \frac{x^2}{\left(-1+3x^2\right)^{3/4}}\,dx + \frac{8}{27}\int \frac{1}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,d$$

$$= \frac{4}{81}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{4}{81}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx + \frac{2}{21}\int \frac{x^2}{\left(-1+3x^2\right)^{3/4}}\,d$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}$$

Mathematica [C] time = 0.204073, size = 184, normalized size = 1.12

$$\frac{2x\left(-31\left(1-3x^2\right)^{3/4}x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right) - \frac{80F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)} + 27\right.}{567\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-20 + 51*x^2 + 27*x^4 - 31*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1
, 5/2, 3*x^2, (3*x^2)/2] - (80*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2]
)/((-2 + 3*x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*Ap
pellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/2, 3
*x^2, (3*x^2)/2])))))/(567*(-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{x^{6}}{3x^{2}-2}\left(3x^{2}-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(3x^{2}-1\right)^{\frac{3}{4}}\left(3x^{2}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^{2}-1\right)^{\frac{1}{4}}x^{6}}{9x^{4}-9x^{2}+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)*x^6/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(3x^{2}-2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**6/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2-1)^{\frac{3}{4}}(3x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088 $\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{27\sqrt{3}x}+\frac{2}{27}\sqrt[4]{3x^2-1}x+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tan$$

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 + (2*Sq
rt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[
(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt[3]*x)
```

Rubi [A] time = 0.13704, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {443, 234, 220, 321, 400, 442}

$$\frac{2}{27}\sqrt[4]{3x^2-1}x+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2\right.\right.}}{27\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 + (2*Sq
rt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[
(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt[3]*x)
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^4}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int \left(\frac{2}{9\left(-1+3x^2\right)^{3/4}}+\frac{x^2}{3\left(-1+3x^2\right)^{3/4}}+\frac{4}{9\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\right)dx$$

$$= \frac{2}{9}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx+\frac{1}{3}\int \frac{x^2}{\left(-1+3x^2\right)^{3/4}}\,dx+\frac{4}{9}\int \frac{1}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2}+\frac{2}{27}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx-\frac{2}{9}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx+\frac{2}{3}\int \frac{}{\left(-2+3x^2\right.}$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{2\sqrt{(1}}{}$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)+\frac{2\sqrt{(1}}{}$$

Mathematica [C] time = 0.0820678, size = 179, normalized size = 1.22

$$\frac{2x\left(-2\left(1-3x^2\right)^{3/4}x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-\frac{4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}+3x^2}{27\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - 2*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2
, (3*x^2)/2] - (4*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^
2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2, 3
/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/2, 3*x^2, (3*x^2)
/2])))))/(27*(-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^{4}}{3 x^{2}-2} \left(3 x^{2}-1\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(3 x^{2}-1\right)^{\frac{3}{4}} \left(3 x^{2}-2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3 x^{2}-1\right)^{\frac{1}{4}} x^{4}}{9 x^{4}-9 x^{2}+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(3x^{2}-2\right) \left(3x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.0164688, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.0301524, size = 52, normalized size = 0.85

$$-\frac{x^3\left(1-3x^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(x^3*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(6*(-
1 + 3*x^2)^(3/4))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [B] time = 1.5339, size = 282, normalized size = 4.62

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x}{9x^4-12x^2+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090 $\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=127

$$-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{3}x}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - (Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1
+ Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*Sqrt[3
]*x)
```

Rubi [A] time = 0.0481679, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {400, 234, 220, 442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - (Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1
+ Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*Sqrt[3
]*x)
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
```

```
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx\right)+\frac{3}{2}\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt[4]{-1+3x^2}\right)}{\sqrt{3}x}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)F\left(2\tan^{-1}\left(\sqrt[4]{\right.\right.}{2\sqrt{3}x}$$

Mathematica [C] time = 0.0153465, size = 68, normalized size = 0.54

$$\frac{\sqrt[4]{-1}\sqrt{x^2}\left(\Pi\left(-i;\sin^{-1}\left((-1)^{3/4}\sqrt[4]{3x^2-1}\right)\middle|-1\right)+\Pi\left(i;\sin^{-1}\left((-1)^{3/4}\sqrt[4]{3x^2-1}\right)\middle|-1\right)\right)}{\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[x^2]*(EllipticPi[-I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)]
, -1] + EllipticPi[I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)], -1]))/(Sqrt[3]
*x)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,x^2-1\right)^{\frac{1}{4}}}{9\,x^4-9\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1091 $\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}} dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{2x}-\frac{\sqrt[4]{3x^2-1}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tan$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1
/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3
]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*Arc
Tan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x)
```

Rubi [A] time = 0.129288, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {443, 325, 234, 220, 400, 442}

$$-\frac{\sqrt[4]{3x^2-1}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-\right.\right.}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1
/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3
]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*Arc
Tan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x)
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \int \left(-\frac{1}{2x^2\left(-1+3x^2\right)^{3/4}}+\frac{3}{2\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{1}{x^2\left(-1+3x^2\right)^{3/4}}\,dx\right)+\frac{3}{2}\int \frac{1}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{-1+3x^2}}{2x}-2\left(\frac{3}{4}\int \frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx\right)+\frac{9}{4}\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{-1+3x^2}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-2\frac{\left(\sqrt{3}\right.}{}\\
&= -\frac{\sqrt[4]{-1+3x^2}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{\sqrt{3}\sqrt{}}{}
\end{aligned}$$

Mathematica [C] time = 0.0481973, size = 52, normalized size = 0.35

$$\frac{\left(1-3x^2\right)^{3/4}F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{2x\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((1 - 3*x^2)^(3/4)*AppellF1[-1/2, 3/4, 1, 1/2, 3*x^2, (3*x^2)/2])/(2*x*(-1
+ 3*x^2)^(3/4))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(3x^2-2\right)}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{3}{4}}(3x^2-2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3x^2-1)^{\frac{1}{4}}}{9x^6-9x^4+2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{3}{4}}(3x^2-2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")

[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

3.1092 $\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=165

$$-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{8x}-\frac{2\sqrt[4]{3x^2-1}}{x}-\frac{\sqrt[4]{3x^2-1}}{6x^3}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-\cdots}}\right.$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(6*x^3) - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[3/2]*ArcTa
n[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1
 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x)
```

Rubi [A] time = 0.166966, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {443, 325, 234, 220, 400, 442}

$$-\frac{2\sqrt[4]{3x^2-1}}{x}-\frac{\sqrt[4]{3x^2-1}}{6x^3}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(\cdots\right.}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(6*x^3) - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[3/2]*ArcTa
n[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1
 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x)
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{x^4\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int\left(-\frac{1}{2x^4\left(-1+3x^2\right)^{3/4}}-\frac{3}{4x^2\left(-1+3x^2\right)^{3/4}}+\frac{9}{4\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\right)dx$$

$$= -\left(\frac{1}{2}\int\frac{1}{x^4\left(-1+3x^2\right)^{3/4}}\,dx\right)-\frac{3}{4}\int\frac{1}{x^2\left(-1+3x^2\right)^{3/4}}\,dx+\frac{9}{4}\int\frac{1}{\left(-2+3x^2\right)\left(-1+\right.}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{3\sqrt[4]{-1+3x^2}}{4x}-2\left(\frac{9}{8}\int\frac{1}{\left(-1+3x^2\right)^{3/4}}\,dx\right)-\frac{5}{4}\int\frac{1}{x^2\left(-1+3x^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{2\sqrt[4]{-1+3x^2}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-1}}\right.$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{2\sqrt[4]{-1+3x^2}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-1}}\right.$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{2\sqrt[4]{-1+3x^2}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-1}}\right.$$

Mathematica [C] time = 0.045282, size = 52, normalized size = 0.32

$$\frac{\left(1-3x^2\right)^{3/4}F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{6x^3\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((1 - 3*x^2)^(3/4)*AppellF1[-3/2, 3/4, 1, -1/2, 3*x^2, (3*x^2)/2])/(6*x^3*(
-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(3 x^{2}-2\right)} \left(3 x^{2}-1\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3 x^{2}-1\right)^{\frac{3}{4}} \left(3 x^{2}-2\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3 x^{2}-1\right)^{\frac{1}{4}}}{9 x^{8}-9 x^{6}+2 x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(3x^{2}-2\right) \left(3x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

3.1093 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a}}{4be}$$

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(e*x)^(7/2)
*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*
Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4)) - (3*a*(8*b*c - 7*a*
d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^
(11/4))
```

Rubi [A] time = 0.126232, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 321, 329, 331, 298, 205, 208}

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a}}{4be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(e*x)^(7/2)
*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*
Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4)) - (3*a*(8*b*c - 7*a*
d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^
(11/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(-4bc+\frac{7ad}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(3a(8bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{32b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x\right.}{16b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \right.}{16b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(3a(8bc-7ad)e^3\right)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx, x,\right.}{32b^{5/2}}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} + \frac{3a(8bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}} - \frac{3a}{}$$

Mathematica [A] time = 0.165653, size = 131, normalized size = 0.76

$$\frac{(ex)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}\left(-7ad+8bc+4bdx^2\right)-3a(7ad-8bc)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+3a(7ad-8bc)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{32b^{11/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((e*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(8*b*c - 7*a*d + 4*b*d*x^
2) - 3*a*(-8*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*a
*(-8*b*c + 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(32*b^(11/
4)*x^(5/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c \right) \left(e x \right)^{\frac{5}{2}} \left(b \,x^{2}+a \right)^{-\frac{3}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{5}{2}}}{\left(b x^{2}+a\right)^{\frac{3}{4}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 142.568, size = 94, normalized size = 0.54

$$\frac{c e^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{2 a^{\frac{3}{4}} \Gamma\left(\frac{11}{4}\right)} + \frac{d e^{\frac{5}{2}} x^{\frac{11}{2}} \Gamma\left(\frac{11}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{2 a^{\frac{3}{4}} \Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(3/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

3.1094 $\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) + ((4*b*c -
3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4
*b^(7/4))
```

Rubi [A] time = 0.0939556, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 329, 331, 298, 205, 208}

$$-\frac{\sqrt{e}(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```


```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) + ((4*b*c -
3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4
*b^(7/4))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{\left(-2bc+\frac{3ad}{2}\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{2b}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{2be}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{2be}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{((4bc-3ad)e)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} - \frac{((4bc-3ad)e)\,\text{Subst}\left(\int \frac{}{e-}\right.}{4b^{3/2}}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{(4bc-3ad)\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} + \frac{(4bc-3ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

Mathematica [A] time = 0.0973939, size = 112, normalized size = 0.82

$$\frac{\sqrt{ex}\left(2b^{3/4}dx^{3/2}\sqrt[4]{a+bx^2}+(3ad-4bc)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[e*x]*(2*b^(3/4)*d*x^(3/2)*(a + b*x^2)^(1/4) + (-4*b*c + 3*a*d)*ArcTan
[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c - 3*a*d)*ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(4*b^(7/4)*Sqrt[x])
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)

[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] Timed out
```

Sympy [C] time = 3.84092, size = 92, normalized size = 0.68

$$\frac{c(ex)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e\Gamma\left(\frac{7}{4}\right)} + \frac{d(ex)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)

[Out] c*(e*x)**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(3/4)*e*gamma(7/4)) + d*(e*x)**(7/2)*gamma(7/4)*hyper((3/4, 7/4),
(11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*e**3*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1095 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=113

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2))
```

Rubi [A] time = 0.0800657, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {451, 329, 331, 298, 205, 208}

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2))
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{e^2}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{ex}\right)}{e^3}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{e^3}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}e}-\frac{d\,\text{Subst}\left(\int \frac{1}{e+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}e}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0303367, size = 100, normalized size = 0.88

$$\frac{x\left(-2b^{3/4}c\sqrt[4]{a+bx^2}-ad\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+ad\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{ab^{3/4}(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*b^(3/4)*c*(a + b*x^2)^(1/4) - a*d*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/(
a + b*x^2)^(1/4)] + a*d*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)
]))/(a*b^(3/4)*(e*x)^(3/2))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{3}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 13.9976, size = 85, normalized size = 0.75

$$\frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(3/2)*gamma(3/4)) +
d*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(3/4)*e**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

3.1096 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt[4]{a+bx^2}(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)*(a + b*x^
2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rubi [A] time = 0.0371541, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$\frac{2\sqrt[4]{a+bx^2}(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)*(a + b*x^
2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}-\frac{(4bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx}{5ae^2}\\ &= -\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(4bc-5ad)\sqrt[4]{a+bx^2}}{5a^2e^3\sqrt{ex}}\end{aligned}$$

Mathematica [A] time = 0.0196833, size = 44, normalized size = 0.66

$$-\frac{2x\sqrt[4]{a+bx^2}\left(a\left(c+5dx^2\right)-4bcx^2\right)}{5a^2(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(-4*b*c*x^2 + a*(c + 5*d*x^2)))/(5*a^2*(e*x)^(7/2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.6

$$-\frac{2\,x\left(5\,adx^{2}-4\,bcx^{2}+ac\right)}{5\,a^{2}}\sqrt[4]{bx^{2}+a}\,(ex)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(b*x^2+a)^(1/4)*(5*a*d*x^2-4*b*c*x^2+a*c)/a^2/(e*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{3}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

Fricas [A] time = 2.07496, size = 101, normalized size = 1.51

$$\frac{2\left((4\,bc-5\,ad)x^{2}-ac\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{ex}}{5\,a^{2}e^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c - 5*a*d)*x^2 - a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*e^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}} (ex)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

3.1097 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)*(a + b*x^
2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*(a + b*x^2)^(5/4))/(
45*a^3*e^3*(e*x)^(5/2))
```

Rubi [A] time = 0.0508128, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```


```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)*(a + b*x^
2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*(a + b*x^2)^(5/4))/(
45*a^3*e^3*(e*x)^(5/2))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} - \frac{(8bc-9ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx}{9ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} + \frac{(4(8bc-9ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{7/2}}\,dx}{9a^2e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} - \frac{8(8bc-9ad)\left(a+bx^2\right)^{5/4}}{45a^3e^3(ex)^{5/2}}$$

Mathematica [A] time = 0.0397203, size = 72, normalized size = 0.69

$$-\frac{2\sqrt{ex}\sqrt[4]{a+bx^2}\left(a^2\left(5c+9dx^2\right)-4abx^2\left(2c+9dx^2\right)+32b^2cx^4\right)}{45a^3e^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(32*b^2*c*x^4 - 4*a*b*x^2*(2*c + 9*d*x^2) +
 a^2*(5*c + 9*d*x^2)))/(45*a^3*e^6*x^5)
```

Maple [A] time = 0.005, size = 62, normalized size = 0.6

$$-\frac{2x\left(-36\,abdx^4+32\,b^2cx^4+9\,a^2dx^2-8\,abcx^2+5\,a^2c\right)}{45\,a^3}\sqrt[4]{bx^2+a}\,(ex)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(1/4)*(-36*a*b*d*x^4+32*b^2*c*x^4+9*a^2*d*x^2-8*a*b*c*x^2
+5*a^2*c)/a^3/(e*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

Fricas [A] time = 2.63642, size = 153, normalized size = 1.47

$$-\frac{2\left(4\left(8\,b^2c-9\,abd\right)x^4+5\,a^2c-\left(8\,abc-9\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\,a^3e^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(4*(8*b^2*c - 9*a*b*d)*x^4 + 5*a^2*c - (8*a*b*c - 9*a^2*d)*x^2)*(b*x^
2 + a)^(1/4)*sqrt(e*x)/(a^3*e^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

3.1098 $\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*a*d)*(a +
b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c - 13*a*d)*(a + b*x^2)^
(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c - 13*a*d)*(a + b*x^2)^(9/4))/
(585*a^4*e^3*(e*x)^(9/2))
```

Rubi [A] time = 0.0696162, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*a*d)*(a +
b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c - 13*a*d)*(a + b*x^2)^
(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c - 13*a*d)*(a + b*x^2)^(9/4))/
(585*a^4*e^3*(e*x)^(9/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} - \frac{(12bc-13ad)\int \frac{1}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx}{13ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} + \frac{(8(12bc-13ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{11/2}}\,dx}{13a^2e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} - \frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}} - \frac{(32(12bc-13\ldots}{6\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} - \frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}} + \frac{64(12bc-13\ldots}{585a^4\ldots}$$

Mathematica [A] time = 0.0477906, size = 94, normalized size = 0.67

$$-\frac{2\sqrt{ex}\sqrt[4]{a+bx^2}\left(-4a^2bx^2\left(15c+26dx^2\right)+5a^3\left(9c+13dx^2\right)+32ab^2x^4\left(3c+13dx^2\right)-384b^3cx^6\right)}{585a^4e^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(-384*b^3*c*x^6 + 32*a*b^2*x^4*(3*c + 13*d*
x^2) + 5*a^3*(9*c + 13*d*x^2) - 4*a^2*b*x^2*(15*c + 26*d*x^2)))/(585*a^4*e^
8*x^7)
```

Maple [A] time = 0.005, size = 86, normalized size = 0.6

$$-\frac{2\,x\left(416\,ab^2dx^6-384\,b^3cx^6-104\,a^2bdx^4+96\,ab^2cx^4+65\,a^3dx^2-60\,a^2bcx^2+45\,ca^3\right)}{585\,a^4}\sqrt[4]{bx^2+a}\,(ex)^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/585*x*(b*x^2+a)^(1/4)*(416*a*b^2*d*x^6-384*b^3*c*x^6-104*a^2*b*d*x^4+96*
a*b^2*c*x^4+65*a^3*d*x^2-60*a^2*b*c*x^2+45*a^3*c)/a^4/(e*x)^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

Fricas [A] time = 1.91521, size = 215, normalized size = 1.52

$$\frac{2\left(32\left(12\,b^{3}c-13\,ab^{2}d\right)x^{6}-8\left(12\,ab^{2}c-13\,a^{2}bd\right)x^{4}-45\,a^{3}c+5\left(12\,a^{2}bc-13\,a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{ex}}{585\,a^{4}e^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/585*(32*(12*b^3*c - 13*a*b^2*d)*x^6 - 8*(12*a*b^2*c - 13*a^2*b*d)*x^4 - 4
5*a^3*c + 5*(12*a^2*b*c - 13*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4*e
^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{3}{4}}\left(ex\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

3.1099 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=180

$$-\frac{a^{3/2}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-9ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{ae^3\sqrt{ex}\sqrt[4]{a+bx^2}(10bc-9ad)}{12b^3}+\frac{e(ex)^{5/2}\sqrt[4]{a+bx}}{30}$$

```
[Out] -(a*(10*b*c - 9*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(12*b^3) + ((10*b*c -
 9*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(30*b^2) + (d*(e*x)^(9/2)*(a + b*x
^2)^(1/4))/(5*b*e) - (a^(3/2)*(10*b*c - 9*a*d)*e^2*(1 + a/(b*x^2))^(3/4)*(e
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*b^(5/2)*(a + b*x
^2)^(3/4))
```

Rubi [A] time = 0.145938, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 321, 329, 237, 335, 275, 231}

$$-\frac{a^{3/2}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-9ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{ae^3\sqrt{ex}\sqrt[4]{a+bx^2}(10bc-9ad)}{12b^3}+\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(10bc-}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] -(a*(10*b*c - 9*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(12*b^3) + ((10*b*c -
 9*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(30*b^2) + (d*(e*x)^(9/2)*(a + b*x
^2)^(1/4))/(5*b*e) - (a^(3/2)*(10*b*c - 9*a*d)*e^2*(1 + a/(b*x^2))^(3/4)*(e
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*b^(5/2)*(a + b*x
^2)^(3/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}dx = \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{\left(-5bc+\frac{9ad}{2}\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{3/4}}dx}{5b}$$

$$= \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{\left(a(10bc-9ad)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/4}}dx}{12b^2}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} + \frac{\left(a^2(1\ldots\right.}{\ldots}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} + \frac{\left(a^2(1\ldots\right.}{\ldots}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} + \frac{\left(a^2(1\ldots\right.}{\ldots}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{\left(a^2(1\ldots\right.}{\ldots}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{\left(a^2(1\ldots\right.}{\ldots}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{a^{3/2}(1\ldots}{\ldots}$$

Mathematica [C] time = 0.131519, size = 123, normalized size = 0.68

$$\frac{e^3\sqrt{ex}\left(\left(a+bx^2\right)\left(45a^2d-2ab\left(25c+9dx^2\right)+4b^2x^2\left(5c+3dx^2\right)\right)+5a^2\left(\frac{bx^2}{a}+1\right)^{3/4}(10bc-9ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{60b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*((a + b*x^2)*(45*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*(
25*c + 9*d*x^2)) + 5*a^2*(10*b*c - 9*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeomet
ric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{7}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(de^3x^5+ce^3x^3)\sqrt{ex}}{(bx^2+a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^3*x^5 + c*e^3*x^3)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

3.1100 $\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-5ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(6bc-5ad)}{6b^2}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

```
[Out] ((6*b*c - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^2) + (d*(e*x)^(5/2)*(a
 + b*x^2)^(1/4))/(3*b*e) + (Sqrt[a]*(6*b*c - 5*a*d)*(1 + a/(b*x^2))^(3/4)*(
e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(3/2)*(a + b*x
^2)^(3/4))
```

Rubi [A] time = 0.108603, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 321, 329, 237, 335, 275, 231}

$$\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(6bc-5ad)}{6b^2}+\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((6*b*c - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^2) + (d*(e*x)^(5/2)*(a
 + b*x^2)^(1/4))/(3*b*e) + (Sqrt[a]*(6*b*c - 5*a*d)*(1 + a/(b*x^2))^(3/4)*(
e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(3/2)*(a + b*x
^2)^(3/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{\left(-3bc+\frac{5ad}{2}\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{3b}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{\left(a(6bc-5ad)e^2\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{12b^2}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{(a(6bc-5ad)e)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{\right.}{6b^2}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\right.}{6b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\right.}{6b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\right.}{12b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\sqrt{a}(6bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-}\right.}{6b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.103529, size = 97, normalized size = 0.7

$$\frac{e\sqrt{ex}\left(a\left(\frac{bx^2}{a}+1\right)^{3/4}(5ad-6bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\left(5ad-2b\left(3c+dx^2\right)\right)\right)}{6b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-((a + b*x^2)*(5*a*d - 2*b*(3*c + d*x^2))) + a*(-6*b*c + 5*a*
d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(
6*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dex^3+cex\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*e*x^3 + c*e*x)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

Sympy [C] time = 17.6159, size = 94, normalized size = 0.68

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

3.1101 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] (d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*x^2))^(3/4
)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]
*e^2*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0879124, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 329, 237, 335, 275, 231}

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*x^2))^(3/4
)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]
*e^2*(a + b*x^2)^(3/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx = \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} - \frac{\left(-bc+\frac{ad}{2}\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{b}$$

$$= \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} + \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} + \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{ex}\right)}{be\left(a+bx^2\right)^{3/4}}$$

$$= \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} - \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{be\left(a+bx^2\right)^{3/4}}$$

$$= \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} - \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{ex}\right)}{2be\left(a+bx^2\right)^{3/4}}$$

$$= \frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be} - \frac{(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0616701, size = 77, normalized size = 0.75

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/4}(2bc-ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+dx\left(a+bx^2\right)}{b\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*x*(a + b*x^2) + (2*b*c - a*d)*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[
1/4, 3/4, 5/4, -((b*x^2)/a)])/(b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right)\frac{1}{\sqrt{e x}}\left(b x^{2}+a\right)^{-\frac{3}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2}+c}{\left(b x^{2}+a\right)^{\frac{3}{4}}\sqrt{e x}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^{2}+a\right)^{\frac{1}{4}}\left(d x^{2}+c\right)\sqrt{e x}}{b e x^{3}+a e x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e*x^3 + a*e*x), x)
```

Sympy [C] time = 5.36119, size = 78, normalized size = 0.76

$$-\frac{c\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{a e^{i\pi}}{b x^{2}}\right)}{b^{\frac{3}{4}}\sqrt{e x}}+\frac{d x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{b x^{2} e^{i\pi}}{a}\right)}{2 a^{\frac{3}{4}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -c*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(e)*
x) + d*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)
```

```
/a)/(2*a**(3/4)*sqrt(e)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

3.1102 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c - 3*a*d)*(
1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0972325, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 329, 237, 335, 275, 231}

$$\frac{2\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c - 3*a*d)*(
1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}-\frac{(2bc-3ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{3ae^2}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}-\frac{(2(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{3ae^3}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}-\frac{\left(2(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{ex}\right)}{3ae^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}+\frac{\left(2(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{3ae^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}+\frac{\left((2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{ex}\right)}{3ae^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt{b}(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0441215, size = 84, normalized size = 0.79

$$\frac{x\left(2x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-2c\left(a+bx^2\right)\right)}{3a(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*c*(a + b*x^2) + 2*(-2*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hyperge
ometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*(a + b*x^2)^(3/4
```

```
))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c \right) \left(e x \right)^{-\frac{5}{2}} \left(b \,x^{2}+a \right)^{-\frac{3}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2} + c}{{\left(b x^{2} + a\right)}^{\frac{3}{4}} \left(e x\right)^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + a\right)}^{\frac{1}{4}} {\left(d x^{2} + c\right)} \sqrt{e x}}{b e^{3} x^{5} + a e^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^3*x^5 + a*e^3*x^3), x
)
```

Sympy [C] time = 111.809, size = 82, normalized size = 0.77

$$- \frac{d {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{a e^{i \pi}}{b x^{2}}} \right)}}{b^{\frac{3}{4}} e^{\frac{5}{2}} x} + \frac{c \Gamma\left(- \frac{3}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{2 a^{\frac{3}{4}} e^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -d*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*e**(5/2)
*x) + c*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/4)*e**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

3.1103 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-7ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{a+bx^2}(6bc-7ad)}{21a^2e^3(ex)^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) + (2*(6*b*c - 7*a*d)*(a + b*x^
2)^(1/4))/(21*a^2*e^3*(e*x)^(3/2)) - (4*b^(3/2)*(6*b*c - 7*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(
5/2)*e^6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.117044, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {453, 325, 329, 237, 335, 275, 231}

$$-\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{a+bx^2}(6bc-7ad)}{21a^2e^3(ex)^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```


```
[Out] (-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) + (2*(6*b*c - 7*a*d)*(a + b*x^
2)^(1/4))/(21*a^2*e^3*(e*x)^(3/2)) - (4*b^(3/2)*(6*b*c - 7*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(
5/2)*e^6*(a + b*x^2)^(3/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} - \frac{(6bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{(2b(6bc-7ad))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{21a^2e^4}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{(4b(6bc-7ad))\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{21a^2e^5}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{\left(4b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+}\right.}{21a^2e^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{\left(4b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+}\right.}{21a^2e^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{\left(2b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+}\right.}{21a^2e^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{4b^{3/2}(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\right.\right.}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0705322, size = 88, normalized size = 0.61

$$-\frac{2\sqrt{ex}\left(x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(7ad-6bc)\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)+3c\left(a+bx^2\right)\right)}{21ae^5x^4\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(3*c*(a + b*x^2) + (-6*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)
*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]))/(21*a*e^5*x^4*(a + b*x^2
)^(3/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{9}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)\sqrt{ex}}{be^5x^7+ae^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^5*x^7 + a*e^5*x^5), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

3.1104 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=182

$$\frac{8b^{5/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-11ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^3e^5(ex)^{3/2}}+\frac{2\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^2e^3(ex)^{7/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) + (2*(10*b*c - 11*a*d)*(a +
b*x^2)^(1/4))/(77*a^2*e^3*(e*x)^(7/2)) - (4*b*(10*b*c - 11*a*d)*(a + b*x^2)
^(1/4))/(77*a^3*e^5*(e*x)^(3/2)) + (8*b^(5/2)*(10*b*c - 11*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^(
7/2)*e^8*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.140598, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {453, 325, 329, 237, 335, 275, 231}

$$\frac{8b^{5/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^3e^5(ex)^{3/2}}+\frac{2\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^2e^3(ex)^{7/2}}-\frac{2c\sqrt[4]{\cdots}}{11ae\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) + (2*(10*b*c - 11*a*d)*(a +
b*x^2)^(1/4))/(77*a^2*e^3*(e*x)^(7/2)) - (4*b*(10*b*c - 11*a*d)*(a + b*x^2)
^(1/4))/(77*a^3*e^5*(e*x)^(3/2)) + (8*b^(5/2)*(10*b*c - 11*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^(
7/2)*e^8*(a + b*x^2)^(3/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} - \frac{(10bc-11ad)\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx}{11ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} + \frac{(6b(10bc-11ad))\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{77a^2e^4}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{\left(4b^2(10bc-11ad)\right)\ldots}{77a^3e\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{\left(8b^2(10bc-11ad)\right)\ldots}{\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{\left(8b^2(10bc-11ad)\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{\left(8b^2(10bc-11ad)\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{\left(4b^2(10bc-11ad)\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{8b^{5/2}(10bc-11ad)\left(\ldots\right.}{7\ldots}$$

Mathematica [C] time = 0.0684601, size = 88, normalized size = 0.48

$$-\frac{2\sqrt{ex}\left(x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(11ad-10bc)\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)+7c\left(a+bx^2\right)\right)}{77ae^7x^6\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(7*c*(a + b*x^2) + (-10*b*c + 11*a*d)*x^2*(1 + (b*x^2)/a)^(3/
4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)]))/(77*a*e^7*x^6*(a + b*
x^2)^(3/4))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{13}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)^{\frac{1}{4}}\left(dx^2 + c\right)\sqrt{ex}}{be^7x^9 + ae^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^7*x^9 + a*e^7*x^7), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

3.1105 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=171

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

```
[Out] -((4*b*c - 5*a*d)*e*Sqrt[e*x])/(2*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(5/2))/
(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e
*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)) + ((4*b*c - 5*a*d)*e^(3/2)*A
rcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rubi [A] time = 0.107683, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 288, 329, 240, 212, 208, 205}

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] -((4*b*c - 5*a*d)*e*Sqrt[e*x])/(2*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(5/2))/
(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e
*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)) + ((4*b*c - 5*a*d)*e^(3/2)*A
rcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} - \frac{\left(-2bc+\frac{5ad}{2}\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{2b}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\int \frac{1}{\sqrt{ex}\sqrt[4]{a+bx^2}}\,dx}{4b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{2b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} + \frac{((4bc-5}{}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{(4bc-5ad)e^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{(4bc-5ad)e^{3/2}\tanh^{-1}}{4b^{9/4}}$$

Mathematica [C] time = 0.114269, size = 77, normalized size = 0.45

$$\frac{x(ex)^{3/2}\left(\sqrt[4]{\frac{bx^2}{a}+1}(4bc-5ad)\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)+5ad\right)}{10ab\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (x*(e*x)^(3/2)*(5*a*d + (4*b*c - 5*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometri
c2F1[5/4, 5/4, 9/4, -((b*x^2)/a)]))/(10*a*b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [B] time = 2.06611, size = 2084, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*d*e*x^2 - (4*b*c - 5*a*d)*e)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 4*(b^3
*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 200
0*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*arctan(((4*b^8*c - 5*a*b^7*d)*(
b*x^2 + a)^(3/4)*sqrt(e*x)*e*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^
2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(3/4) + (b^8*x^2 + a*b
^7)*sqrt(((16*b^2*c^2 - 40*a*b*c*d + 25*a^2*d^2)*sqrt(b*x^2 + a)*e^3*x + (b
^5*x^2 + a*b^4)*sqrt((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2
 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9))/(b*x^2 + a))*((256*b^4*c^4 - 1
280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^
6/b^9)^(3/4))/((256*b^5*c^4 - 1280*a*b^4*c^3*d + 2400*a^2*b^3*c^2*d^2 - 200
0*a^3*b^2*c*d^3 + 625*a^4*b*d^4)*e^6*x^2 + (256*a*b^4*c^4 - 1280*a^2*b^3*c^
3*d + 2400*a^3*b^2*c^2*d^2 - 2000*a^4*b*c*d^3 + 625*a^5*d^4)*e^6)) + (b^3*x
^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*
a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c -
5*a*d)*sqrt(e*x)*e + (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2
400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/(b*x^
2 + a)) - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2
*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)
```

```
^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280
*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b
^9)^(1/4))/(b*x^2 + a)))/(b^3*x^2 + a*b^2)
```

Sympy [C] time = 60.2405, size = 94, normalized size = 0.55

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((5/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

3.1106 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=122

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(b^(1/4)*Sq
rt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]) + (d*ArcTanh[(b^(1
/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e])
```

Rubi [A] time = 0.0717827, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 329, 240, 212, 208, 205}

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(b^(1/4)*Sq
rt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]) + (d*ArcTanh[(b^(1
/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e])
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\int \frac{1}{\sqrt{ex}\sqrt[4]{a+bx^2}}\,dx}{b}\\ &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{be}\\ &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}\\ &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b}\\ &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}\end{aligned}$$

Mathematica [C] time = 0.0623474, size = 68, normalized size = 0.56

$$\frac{2\left(dx^3\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)+5cx\right)}{5a\sqrt{ex}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(5*c*x + d*x^3*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -(
(b*x^2)/a)]))/(5*a*Sqrt[e*x]*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (dx^2+c)\frac{1}{\sqrt{ex}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{5}{4}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

Fricas [B] time = 1.82888, size = 853, normalized size = 6.99

$$\frac{4\left(bx^2 + a\right)^{\frac{3}{4}}(bc - ad)\sqrt{ex} - 4\left(ab^2ex^2 + a^2be\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}b^4de\left(\frac{d^4}{b^5e^2}\right)^{\frac{3}{4}} - \left(b^5ex^2+ab^4e\right)\sqrt{\frac{\sqrt{bx^2+a}d^2ex+\left(b^3e^2x^2+ab^2e^2\right)\sqrt{\ldots}}{bx^2+a}}\ldots}{bd^4x^2+ad^4}\ldots\right)\ldots}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(b*x^2 + a)^(3/4)*(b*c - a*d)*sqrt(e*x) - 4*(a*b^2*e*x^2 + a^2*b*e)*
(d^4/(b^5*e^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(e*x)*b^4*d*e*(d^4/(b^
5*e^2))^(3/4) - (b^5*e*x^2 + a*b^4*e)*sqrt((sqrt(b*x^2 + a)*d^2*e*x + (b^3*
e^2*x^2 + a*b^2*e^2)*sqrt(d^4/(b^5*e^2)))/(b*x^2 + a))*(d^4/(b^5*e^2))^(3/4
))/(b*d^4*x^2 + a*d^4)) + (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^5*e^2))^(1/4)*log
(((b*x^2 + a)^(3/4)*sqrt(e*x)*d + (b^2*e*x^2 + a*b*e)*(d^4/(b^5*e^2))^(1/4)
)/(b*x^2 + a)) - (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^5*e^2))^(1/4)*log(((b*x^2
+ a)^(3/4)*sqrt(e*x)*d - (b^2*e*x^2 + a*b*e)*(d^4/(b^5*e^2))^(1/4))/(b*x^2
+ a)))/(a*b^2*e*x^2 + a^2*b*e)
```

Sympy [C] time = 26.9673, size = 83, normalized size = 0.68

$$\frac{c\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2} + 1}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*gamma(1/4)/(2*a*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)) + d*
x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*
```

```
a**(5/4)*sqrt(e)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

3.1107 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\sqrt{ex}(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)*Sqrt[e*x]
)/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0305918, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2\sqrt{ex}(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)*Sqrt[e*x]
)/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{(4bc-3ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{3ae^2}\\ &= -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(4bc-3ad)\sqrt{ex}}{3a^2e^3\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0213481, size = 45, normalized size = 0.67

$$\frac{x\left(-2ac+6adx^2-8bcx^2\right)}{3a^2(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-2*a*c - 8*b*c*x^2 + 6*a*d*x^2))/(3*a^2*(e*x)^(5/2)*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.6

$$-\frac{2\,x\left(-3\,adx^{2}+4\,bcx^{2}+ac\right)}{3\,a^{2}}\frac{1}{\sqrt[4]{bx^{2}+a}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/3*x*(-3*a*d*x^2+4*b*c*x^2+a*c)/(b*x^2+a)^(1/4)/a^2/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

Fricas [A] time = 1.87557, size = 124, normalized size = 1.85

$$-\frac{2\left((4\,bc-3\,ad)x^{2}+ac\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{3\left(a^{2}be^{3}x^{4}+a^{3}e^{3}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - 3*a*d)*x^2 + a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b*e^3*x^4
 + a^3*e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{5}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

3.1108 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d))/(7*a^2*e
^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(
21*a^3*e^3*(e*x)^(3/2))
```

Rubi [A] time = 0.047096, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d))/(7*a^2*e
^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(
21*a^3*e^3*(e*x)^(3/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{(8bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx}{7ae^2}$$
$$= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{(4(8bc-7ad))\int \frac{1}{(ex)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{7a^2e^2}$$
$$= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} + \frac{8(8bc-7ad)\left(a+bx^2\right)^{3/4}}{21a^3e^3(ex)^{3/2}}$$

Mathematica [A] time = 0.0295544, size = 71, normalized size = 0.68

$$-\frac{2\sqrt{ex}\left(a^2\left(3c+7dx^2\right)+ab\left(28dx^4-8cx^2\right)-32b^2cx^4\right)}{21a^3e^5x^4\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(-32*b^2*c*x^4 + a^2*(3*c + 7*d*x^2) + a*b*(-8*c*x^2 + 28*d*x
^4)))/(21*a^3*e^5*x^4*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.6

$$-\frac{2x\left(28abdx^4-32b^2cx^4+7a^2dx^2-8abcx^2+3a^2c\right)}{21a^3}\frac{1}{\sqrt[4]{bx^2+a}}(ex)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(28*a*b*d*x^4-32*b^2*c*x^4+7*a^2*d*x^2-8*a*b*c*x^2+3*a^2*c)/(b*x^2+
a)^(1/4)/a^3/(e*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

Fricas [A] time = 1.60484, size = 173, normalized size = 1.66

$$\frac{2\left(4\left(8\,b^2c-7\,abd\right)x^4-3\,a^2c+\left(8\,abc-7\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{21\left(a^3be^5x^6+a^4e^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(8*b^2*c - 7*a*b*d)*x^4 - 3*a^2*c + (8*a*b*c - 7*a^2*d)*x^2)*(b*x^2
 + a)^(3/4)*sqrt(e*x)/(a^3*b*e^5*x^6 + a^4*e^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

3.1109 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*a*d))/(11*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c - 11*a*d)*(a + b*x^2)^
(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c - 11*a*d)*(a + b*x^2)^(7/4))/
(231*a^4*e^3*(e*x)^(7/2))
```

Rubi [A] time = 0.0645813, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```


```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*a*d))/(11*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c - 11*a*d)*(a + b*x^2)^
(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c - 11*a*d)*(a + b*x^2)^(7/4))/
(231*a^4*e^3*(e*x)^(7/2))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{(12bc-11ad)\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{(8(12bc-11ad))\int \frac{1}{(ex)^{9/2}\sqrt[4]{a+bx^2}}\,dx}{11a^2e^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)\left(a+bx^2\right)^{3/4}}{33a^3e^3(ex)^{7/2}} + \frac{(32(12bc\ \cdots}{\cdots}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)\left(a+bx^2\right)^{3/4}}{33a^3e^3(ex)^{7/2}} - \frac{64(12bc-\cdots}{23\cdots}$$

Mathematica [A] time = 0.0397234, size = 68, normalized size = 0.48

$$\frac{2x\left(-x^2\left(-3a^2+8abx^2+32b^2x^4\right)(12bc-11ad)-21a^3c\right)}{231a^4(ex)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x*(-21*a^3*c - (12*b*c - 11*a*d)*x^2*(-3*a^2 + 8*a*b*x^2 + 32*b^2*x^4)))
/(231*a^4*(e*x)^(13/2)*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.006, size = 86, normalized size = 0.6

$$-\frac{2\,x\left(-352\,ab^2dx^6+384\,b^3cx^6-88\,a^2bdx^4+96\,ab^2cx^4+33\,a^3dx^2-36\,a^2bcx^2+21\,ca^3\right)}{231\,a^4}\frac{1}{\sqrt[4]{bx^2+a}}(ex)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/231*x*(-352*a*b^2*d*x^6+384*b^3*c*x^6-88*a^2*b*d*x^4+96*a*b^2*c*x^4+33*a
^3*d*x^2-36*a^2*b*c*x^2+21*a^3*c)/(b*x^2+a)^(1/4)/a^4/(e*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

Fricas [A] time = 1.6039, size = 238, normalized size = 1.69

$$-\frac{2\left(32\left(12\,b^{3}c-11\,ab^{2}d\right)x^{6}+8\left(12\,ab^{2}c-11\,a^{2}bd\right)x^{4}+21\,a^{3}c-3\left(12\,a^{2}bc-11\,a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{231\left(a^{4}be^{7}x^{8}+a^{5}e^{7}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*(12*b^3*c - 11*a*b^2*d)*x^6 + 8*(12*a*b^2*c - 11*a^2*b*d)*x^4 +
21*a^3*c - 3*(12*a^2*b*c - 11*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^4*
b*e^7*x^8 + a^5*e^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

3.1110 $\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=180

$$-\frac{7a^{3/2}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{7ae^3(ex)^{3/2}(10bc-11ad)}{60b^3\sqrt[4]{a+bx^2}}+\frac{e(ex)^{7/2}(10bc-11ad)}{30b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+b}}$$

```
[Out] (-7*a*(10*b*c - 11*a*d)*e^3*(e*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) + ((10*
b*c - 11*a*d)*e*(e*x)^(7/2))/(30*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(11/2))/
(5*b*e*(a + b*x^2)^(1/4)) - (7*a^(3/2)*(10*b*c - 11*a*d)*e^4*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(7/2)*
(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0984225, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 285, 284, 335, 196}

$$-\frac{7a^{3/2}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{7ae^3(ex)^{3/2}(10bc-11ad)}{60b^3\sqrt[4]{a+bx^2}}+\frac{e(ex)^{7/2}(10bc-11ad)}{30b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*(10*b*c - 11*a*d)*e^3*(e*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) + ((10*
b*c - 11*a*d)*e*(e*x)^(7/2))/(30*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(11/2))/
(5*b*e*(a + b*x^2)^(1/4)) - (7*a^(3/2)*(10*b*c - 11*a*d)*e^4*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(7/2)*
(a + b*x^2)^(1/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(-5bc+\frac{11ad}{2}\right)\int \frac{(ex)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5b}$$

$$= \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(7a(10bc-11ad)e^2\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{60b^2}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} + \frac{\left(7a^2(10bc-11ad)e^4\right)\int}{40b^3}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} + \frac{\left(7a^2(10bc-11ad)e^4\sqrt[4]{1}\right.}{40b^4}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(7a^2(10bc-11ad)e^4\sqrt[4]{1}\right.}{}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}(10bc-11ad)e^4\sqrt[4]{1}}{20b}$$

Mathematica [C] time = 0.138711, size = 112, normalized size = 0.62

$$\frac{e^3(ex)^{3/2}\left(77a^2d+7a\sqrt[4]{\frac{bx^2}{a}+1}(10bc-11ad)\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-2ab\left(35c+11dx^2\right)+4b^2x^2\left(5c+3dx^2\right)\right)}{60b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(77*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*(35*c + 11*d
*x^2) + 7*a*(10*b*c - 11*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4,
5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{9}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(de^4x^6+ce^4x^4)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^4*x^6 + c*e^4*x^4)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^2*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

3.1111 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}(6bc-7ad)}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

```
[Out] ((6*b*c - 7*a*d)*e*(e*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(7/2))
/(3*b*e*(a + b*x^2)^(1/4)) + (Sqrt[a]*(6*b*c - 7*a*d)*e^2*(1 + a/(b*x^2))^(
1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a +
 b*x^2)^(1/4))
```

Rubi [A] time = 0.0697724, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 285, 284, 335, 196}

$$\frac{\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}(6bc-7ad)}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] ((6*b*c - 7*a*d)*e*(e*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(7/2))
/(3*b*e*(a + b*x^2)^(1/4)) + (Sqrt[a]*(6*b*c - 7*a*d)*e^2*(1 + a/(b*x^2))^(
1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a +
 b*x^2)^(1/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(-3bc+\frac{7ad}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{3b}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(a(6bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b^2}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(a(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} + \frac{\left(a(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x\right.}{4b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.11123, size = 85, normalized size = 0.6

$$\frac{e(ex)^{3/2}\left(\sqrt[4]{\frac{bx^2}{a}+1}(7ad-6bc)\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-7ad+6bc+2bdx^2\right)}{6b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(6*b*c - 7*a*d + 2*b*d*x^2 + (-6*b*c + 7*a*d)*(1 + (b*x^2)/a
)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)
^(1/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(de^2x^4 + ce^2x^2)(bx^2 + a)^{\frac{3}{4}}\sqrt{ex}}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^2*x^4 + c*e^2*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^2*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

3.1112 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=99

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a/(b*x^2))^
(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*b^(3/
2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0482421, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 284, 335, 196}

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a/(b*x^2))^
(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*b^(3/
2)*(a + b*x^2)^(1/4))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{\left(-bc+\frac{3ad}{2}\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{b}$$

$$= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{\left(\left(-bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} + \frac{\left(\left(-bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.100897, size = 77, normalized size = 0.78

$$\frac{x\sqrt{ex}\left(\sqrt[4]{\frac{bx^2}{a}+1}(2bc-3ad)\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+3ad\right)}{3ab\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (x*Sqrt[e*x]*(3*a*d + (2*b*c - 3*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2
F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^2 + a^2
), x)
```

Sympy [C] time = 16.9099, size = 94, normalized size = 0.95

$$\frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((5/4,
7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

3.1113 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 + a/(b*x^2))
^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*Sqrt
[b]*e^2*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0592124, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 284, 335, 196}

$$\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 + a/(b*x^2))
^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*Sqrt
[b]*e^2*(a + b*x^2)^(1/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{(2bc-ad)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{ae^2}$$
$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left((2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{abe^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{\left((2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{abe^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{2(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0412828, size = 77, normalized size = 0.75

$$\frac{x\left(2x^2\sqrt[4]{\frac{bx^2}{a}+1}(ad-2bc)\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-6ac\right)}{3a^2(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-6*a*c + 2*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3
/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^2*x^6 + 2*a*b*e^2*x
^4 + a^2*e^2*x^2), x)
```

Sympy [C] time = 104.662, size = 82, normalized size = 0.8

$$-\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{5}{4}}e^{\frac{3}{2}}x}+\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -d*hyper((1/2, 5/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(5/4)*e**(3/2)
*x) + c*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(5/4)*e**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

3.1114 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=144

$$\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-5ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) + (2*(6*b*c - 5*a*d))/(5*a^2*e
^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (4*Sqrt[b]*(6*b*c - 5*a*d)*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*e
^4*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0748738, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 286, 284, 335, 196}

$$\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-5ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```


```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) + (2*(6*b*c - 5*a*d))/(5*a^2*e
^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (4*Sqrt[b]*(6*b*c - 5*a*d)*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*e
^4*(a + b*x^2)^(1/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{(6bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{5ae^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{(2b(6bc-5ad))\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{5a^2e^4}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{\left(2(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5a^2e^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left(2(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\right.}{5a^2e^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{4\sqrt{b}(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right|}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.0443797, size = 78, normalized size = 0.54

$$\frac{x\left(2x^2\sqrt[4]{\frac{bx^2}{a}+1}(6bc-5ad)\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^2}{a}\right)-2ac\right)}{5a^2(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-2*a*c + 2*(6*b*c - 5*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
-1/4, 5/4, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{7}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2e^4x^8+2\,abe^4x^6+a^2e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^4*x^8 + 2*a*b*e^4*x
^6 + a^2*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

3.1115 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=182

$$\frac{8b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) + (2*(10*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (4*b*(10*b*c - 9*a*d))/(15*a^3*e^5*Sq
rt[e*x]*(a + b*x^2)^(1/4)) + (8*b^(3/2)*(10*b*c - 9*a*d)*(1 + a/(b*x^2))^(1
/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(7/2)*e^6*
(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0996455, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 286, 284, 335, 196}

$$\frac{8b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```


```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) + (2*(10*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (4*b*(10*b*c - 9*a*d))/(15*a^3*e^5*Sq
rt[e*x]*(a + b*x^2)^(1/4)) + (8*b^(3/2)*(10*b*c - 9*a*d)*(1 + a/(b*x^2))^(1
/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(7/2)*e^6*
(a + b*x^2)^(1/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} - \frac{(10bc-9ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx}{9ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{(2b(10bc-9ad))\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{15a^2e^4}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left(4b^2(10bc-9ad\right.}{15a}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left(4b(10bc-9ad)\right.}{1}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{\left(4b(10bc-9ad)\right.}{}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{8b^{3/2}(10bc-9ac}{}$$

Mathematica [C] time = 0.0686539, size = 82, normalized size = 0.45

$$-\frac{2\sqrt{ex}\left(x^2\sqrt[4]{\frac{bx^2}{a}+1}(9ad-10bc)\,_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)+5ac\right)}{45a^2e^6x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(5*a*c + (-10*b*c + 9*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)]))/(45*a^2*e^6*x^5*(a + b*x^2)^(1/4
))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{11}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2e^6x^{10}+2\,abe^6x^8+a^2e^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^6*x^10 + 2*a*b*e^6*
x^8 + a^2*e^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

3.1116 $\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=184

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-a}{3abe\left(a+bx^2\right)}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c - 7*a*d)*
e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c - 7*a*d)*e^(5/2)*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4)) + ((4*b*c
- 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(4*b^(11/4))
```

Rubi [A] time = 0.119237, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {457, 321, 329, 331, 298, 205, 208}

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-a}{3abe\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c - 7*a*d)*
e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c - 7*a*d)*e^(5/2)*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4)) + ((4*b*c
- 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(4*b^(11/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{\left(2\left(-2bc+\frac{7ad}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{\left((4bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{\cdots}\right.}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+b\cdots}}\right.}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{\left((4bc-7ad)e^3\right)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx, x, \cdots\right.}{4b^{5/2}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} - \frac{(4bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}} + \frac{(4bc\cdots}{\cdots}$$

Mathematica [C] time = 0.127221, size = 77, normalized size = 0.42

$$\frac{x(ex)^{5/2}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(4bc-7ad)\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\frac{bx^2}{a}\right)+7ad\right)}{14ab\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (x*(e*x)^(5/2)*(7*a*d + (4*b*c - 7*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometri
c2F1[7/4, 7/4, 11/4, -((b*x^2)/a)]))/(14*a*b*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

3.1117 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=125

$$-\frac{d\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4) + (d*Sqrt[e]*Arc
Tanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4)
```

Rubi [A] time = 0.0764689, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 329, 331, 298, 205, 208}

$$-\frac{d\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4) + (d*Sqrt[e]*Arc
Tanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4)
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx &= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{d\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{b}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^{3/2}}-\frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{b}x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^{3/2}}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{d\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.059424, size = 69, normalized size = 0.55

$$\frac{2\sqrt{ex}\left(3dx^3\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\frac{bx^2}{a}\right)+7cx\right)}{21a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*Sqrt[e*x]*(7*c*x + 3*d*x^3*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/4,
7/4, 11/4, -((b*x^2)/a)]))/(21*a*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)\sqrt{ex}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 75.2104, size = 87, normalized size = 0.7

$$\frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{2a^{\frac{7}{4}}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)/(2*a**(7/4)*(1 + b*x**2/a)**(3/4)*gamma(7/4))
 + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(7/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

3.1118 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)^(3/2))/(3
*a^2*e^3*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0311342, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)^(3/2))/(3
*a^2*e^3*(a + b*x^2)^(3/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx &= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{(4bc-ad)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{7/4}}\,dx}{ae^2}\\ &= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{2(4bc-ad)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0185852, size = 44, normalized size = 0.68

$$\frac{2x\left(-3ac+adx^2-4bcx^2\right)}{3a^2(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*x*(-3*a*c - 4*b*c*x^2 + a*d*x^2))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(3/4))
```

Maple [A] time = 0.005, size = 40, normalized size = 0.6

$$-\frac{2x\left(-adx^2+4bcx^2+3ac\right)}{3a^2}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/3*x*(-a*d*x^2+4*b*c*x^2+3*a*c)/(b*x^2+a)^(3/4)/a^2/(e*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

Fricas [A] time = 2.16218, size = 122, normalized size = 1.88

$$-\frac{2\left((4\,bc-ad)x^2+3\,ac\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{3\left(a^2be^2x^3+a^3e^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - a*d)*x^2 + 3*a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*b*e^2*x^3
 + a^3*e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

3.1119 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8\sqrt[4]{a+bx^2}(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d))/(15*a^2*
e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(a + b*x^2)^(1/4))/(1
5*a^3*e^3*Sqrt[e*x])
```

Rubi [A] time = 0.0518137, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{8\sqrt[4]{a+bx^2}(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d))/(15*a^2*
e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(a + b*x^2)^(1/4))/(1
5*a^3*e^3*Sqrt[e*x])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{(8bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx}{5ae^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}} - \frac{(4(8bc-5ad))\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx}{15a^2e^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}} + \frac{8(8bc-5ad)\sqrt[4]{a+bx^2}}{15a^3e^3\sqrt{ex}}$$

Mathematica [A] time = 0.0262971, size = 66, normalized size = 0.63

$$\frac{x\left(-6a^2\left(c+5dx^2\right)+8abx^2\left(6c-5dx^2\right)+64b^2cx^4\right)}{15a^3(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(64*b^2*c*x^4 + 8*a*b*x^2*(6*c - 5*d*x^2) - 6*a^2*(c + 5*d*x^2)))/(15*a^
3*(e*x)^(7/2)*(a + b*x^2)^(3/4))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(20\,abdx^4-32\,b^2cx^4+15\,a^2dx^2-24\,abcx^2+3\,a^2c\right)}{15\,a^3}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/15*x*(20*a*b*d*x^4-32*b^2*c*x^4+15*a^2*d*x^2-24*a*b*c*x^2+3*a^2*c)/(b*x^
2+a)^(3/4)/a^3/(e*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

Fricas [A] time = 2.05389, size = 176, normalized size = 1.69

$$\frac{2\left(4\left(8b^2c-5abd\right)x^4-3a^2c+3\left(8abc-5a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{15\left(a^3be^4x^5+a^4e^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/15*(4*(8*b^2*c - 5*a*b*d)*x^4 - 3*a^2*c + 3*(8*a*b*c - 5*a^2*d)*x^2)*(b*x
^2 + a)^(1/4)*sqrt(e*x)/(a^3*b*e^4*x^5 + a^4*e^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

3.1120 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d))/(9*a^2*e
^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)*(a + b*x^2)^(1/4))/
(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)*(a + b*x^2)^(5/4))/(45*a^4*e^
3*(e*x)^(5/2))
```

Rubi [A] time = 0.0656395, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d))/(9*a^2*e
^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)*(a + b*x^2)^(1/4))/
(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)*(a + b*x^2)^(5/4))/(45*a^4*e^
3*(e*x)^(5/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{(4bc-3ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx}{3ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{(8(4bc-3ad))\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx}{9a^2e^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}} + \frac{(32(4bc}{}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}} - \frac{64(4bc-}{45}$$

Mathematica [A] time = 0.0401438, size = 88, normalized size = 0.62

$$\frac{4x^3\left(3a^2-24abx^2-32b^2x^4\right)\left(6bc-\frac{9ad}{2}\right)}{135a^4(ex)^{11/2}\left(a+bx^2\right)^{3/4}} - \frac{2cx}{9a(ex)^{11/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c*x)/(9*a*(e*x)^(11/2)*(a + b*x^2)^(3/4)) + (4*(6*b*c - (9*a*d)/2)*x^3*
(3*a^2 - 24*a*b*x^2 - 32*b^2*x^4))/(135*a^4*(e*x)^(11/2)*(a + b*x^2)^(3/4))
```

Maple [A] time = 0.005, size = 86, normalized size = 0.6

$$-\frac{2x\left(-96ab^2dx^6+128b^3cx^6-72a^2bdx^4+96ab^2cx^4+9a^3dx^2-12a^2bcx^2+5ca^3\right)}{45a^4}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/45*x*(-96*a*b^2*d*x^6+128*b^3*c*x^6-72*a^2*b*d*x^4+96*a*b^2*c*x^4+9*a^3*
d*x^2-12*a^2*b*c*x^2+5*a^3*c)/(b*x^2+a)^(3/4)/a^4/(e*x)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

Fricas [A] time = 1.6078, size = 228, normalized size = 1.62

$$-\frac{2\left(32\left(4b^3c-3ab^2d\right)x^6+24\left(4ab^2c-3a^2bd\right)x^4+5a^3c-3\left(4a^2bc-3a^3d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\left(a^4be^6x^7+a^5e^6x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*(4*b^3*c - 3*a*b^2*d)*x^6 + 24*(4*a*b^2*c - 3*a^2*b*d)*x^4 + 5*a^
3*c - 3*(4*a^2*b*c - 3*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4*b*e^6*x
^7 + a^5*e^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

3.1121 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=192

$$\frac{5\sqrt{a}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{5e^3\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-3ad)}{6b^3}-\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(2}{3ab^2}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (5*(2*b*c - 3*a*d
)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^3) - ((2*b*c - 3*a*d)*e*(e*x)^(5/2)
*(a + b*x^2)^(1/4))/(3*a*b^2) + (5*Sqrt[a]*(2*b*c - 3*a*d)*e^2*(1 + a/(b*x^
2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(5/
2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.134827, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {457, 321, 329, 237, 335, 275, 231}

$$\frac{5e^3\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-3ad)}{6b^3}+\frac{5\sqrt{a}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(2bc-3a}{3ab^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```


```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (5*(2*b*c - 3*a*d
)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^3) - ((2*b*c - 3*a*d)*e*(e*x)^(5/2)
*(a + b*x^2)^(1/4))/(3*a*b^2) + (5*Sqrt[a]*(2*b*c - 3*a*d)*e^2*(1 + a/(b*x^
2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(5/
2)*(a + b*x^2)^(3/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{\left(2\left(-3bc+\frac{9ad}{2}\right)\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left(5(2bc-3ad)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{6b^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left(5a(2bc-\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left(5a(2bc-\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left(5a(2bc-\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left(5a(2bc-\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left(5a(2bc-\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{5\sqrt{a}(2bc}{}$$

Mathematica [C] time = 0.138387, size = 110, normalized size = 0.57

$$\frac{e^3\sqrt{ex}\left(-15a^2d+5a\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+ab\left(10c-9dx^2\right)+2b^2x^2\left(3c+dx^2\right)\right)}{6b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-15*a^2*d + a*b*(10*c - 9*d*x^2) + 2*b^2*x^2*(3*c + d*x^2)
+ 5*a*(-2*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/
4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{7}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(de^3x^5+ce^3x^3)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^3*x^5 + c*e^3*x^3)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(b^2*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

3.1122 $\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-5ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-5ad)}{3ab^2}+\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c - 5*a*d)*
e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(3*a*b^2) - ((2*b*c - 5*a*d)*(1 + a/(b*x^2))
^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*
b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.111519, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {457, 321, 329, 237, 335, 275, 231}

$$-\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-5ad)}{3ab^2}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c - 5*a*d)*
e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(3*a*b^2) - ((2*b*c - 5*a*d)*(1 + a/(b*x^2))
^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*
b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{\left(2\left(-bc+\frac{5ad}{2}\right)\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left((2bc-5ad)e^2\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{6b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{((2bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{e\cdots}\right.}{3b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{3b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{3b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int \cdots\right.}{6b^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{(2bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\cdots\right.\right.}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.112639, size = 85, normalized size = 0.56

$$\frac{e\sqrt{ex}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(2bc-5ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+5ad-2bc+3bdx^2\right)}{3b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-2*b*c + 5*a*d + 3*b*d*x^2 + (2*b*c - 5*a*d)*(1 + (b*x^2)/a)^
(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2)^(
3/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dex^3+cex\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((d*e*x^3 + c*e*x)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^2
 + a^2), x)
```

Sympy [C] time = 160.785, size = 94, normalized size = 0.62

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(7/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((7/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{2}+c\right)\left(ex\right)^{\frac{3}{2}}}{\left(bx^{2}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

3.1123 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c + a*d)*(1
 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.0934183, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 329, 237, 335, 275, 231}

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c + a*d)*(1
 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{\left(2\left(bc+\frac{ad}{2}\right)\right)\int\frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{3ab}\\
&= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{(2(2bc+ad))\,\mathrm{Subst}\left(\int\frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{ex}\right)}{3abe}\\
&= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{\left(2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{ex}\right)}{3abe\left(a+bx^2\right)^{3/4}}\\
&= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{\left(2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int\frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{3abe\left(a+bx^2\right)^{3/4}}\\
&= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{\left((2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{ex}\right)}{3abe\left(a+bx^2\right)^{3/4}}\\
&= \frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0564527, size = 79, normalized size = 0.68

$$\frac{2x\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-ad+bc\right)}{3ab\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4
, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (dx^2+c)\frac{1}{\sqrt{ex}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2ex^5+2\,abex^3+a^2ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e*x^5 + 2*a*b*e*x^3 +
 a^2*e*x), x)
```

Sympy [C] time = 141.547, size = 78, normalized size = 0.67

$$-\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{7}{4}}\sqrt{ex}}+\frac{c\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -d*hyper((1/2, 7/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(7/4)*sqrt(e)*
x) + c*sqrt(x)*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(7/4)*sqrt(e)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

3.1124 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=144

$$\frac{4\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{ex}(2bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - (2*(2*b*c - a*d)*Sqrt[e*x])/
(3*a^2*e^3*(a + b*x^2)^(3/4)) + (4*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(5/2)*e^4
*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.112312, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {453, 290, 329, 237, 335, 275, 231}

$$-\frac{2\sqrt{ex}(2bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```


```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - (2*(2*b*c - a*d)*Sqrt[e*x])/
(3*a^2*e^3*(a + b*x^2)^(3/4)) + (4*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(5/2)*e^4
*(a + b*x^2)^(3/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{(2bc-ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{(2(2bc-ad))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}\,dx}{3a^2e^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{(4(2bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{e\cdots}\right.}{3a^2e^3}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{\left(4(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int\cdots\right.}{3a^2e^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{\left(4(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int\cdots\right.}{3a^2e^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{\left(2(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int\cdots\right.}{3a^2e^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{4\sqrt{b}(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot\cdots\right.}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0494491, size = 91, normalized size = 0.63

$$\frac{x\left(4x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-2ac+2adx^2-4bcx^2\right)}{3a^2(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(-2*a*c - 4*b*c*x^2 + 2*a*d*x^2 + 4*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(
3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*(a
 + b*x^2)^(3/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{5}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^2e^3x^7+2\,abe^3x^5+a^2e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^3*x^7 + 2*a*b*e^3*x
^5 + a^2*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

3.1125 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=181

$$-\frac{8b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-7ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{7/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt[4]{a+bx^2}(10bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - (2*(10*b*c - 7*a*d))/(21*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (4*(10*b*c - 7*a*d)*(a + b*x^2)^(1/4)
)/(21*a^3*e^3*(e*x)^(3/2)) - (8*b^(3/2)*(10*b*c - 7*a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(7/2)*e^
6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.136564, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {453, 290, 325, 329, 237, 335, 275, 231}

$$-\frac{8b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{7/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt[4]{a+bx^2}(10bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{\cdots}{7ae(e\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - (2*(10*b*c - 7*a*d))/(21*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (4*(10*b*c - 7*a*d)*(a + b*x^2)^(1/4)
)/(21*a^3*e^3*(e*x)^(3/2)) - (8*b^(3/2)*(10*b*c - 7*a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(7/2)*e^
6*(a + b*x^2)^(3/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{(10bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx}{7ae^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{(2(10bc-7ad))\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7a^2e^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} + \frac{(4b(10bc-}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} + \frac{(8b(10bc-}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} + \frac{\left(8b(10bc-\right.}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} - \frac{\left(8b(10bc-\right.}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} - \frac{\left(4b(10bc-\right.}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} - \frac{8b^{3/2}(10bc-}{}$$

Mathematica [C] time = 0.0677751, size = 82, normalized size = 0.45

$$-\frac{2\sqrt{ex}\left(x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(7ad-10bc)\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{1}{4};-\frac{bx^2}{a}\right)+3ac\right)}{21a^2e^5x^4\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(3*a*c + (-10*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hypergeo
metric2F1[-3/4, 7/4, 1/4, -((b*x^2)/a)]))/(21*a^2*e^5*x^4*(a + b*x^2)^(3/4)
)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{9}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)^{\frac{1}{4}} \left(dx^2 + c\right)\sqrt{ex}}{b^2e^5x^9 + 2\,abe^5x^7 + a^2e^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^5*x^9 + 2*a*b*e^5*x
^7 + a^2*e^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

3.1126 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}} \, dx$

Optimal. Leaf size=221

$$-\frac{e^3\sqrt{ex}(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}+\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+$$

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c - 9*a*d)*
e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*a*d)*e*(e*x)^(5/2))/
(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTan[(b^(1/4)*Sqr
t[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/
2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))
```

Rubi [A] time = 0.13475, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {457, 285, 288, 329, 240, 212, 208, 205}

$$-\frac{e^3\sqrt{ex}(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}+\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c - 9*a*d)*
e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*a*d)*e*(e*x)^(5/2))/
(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTan[(b^(1/4)*Sqr
t[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/
2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}dx = \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-2bc+\frac{9ad}{2}\right)\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{5/4}}dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}dx}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^4\right)\int \frac{1}{\sqrt{ex}\sqrt[4]{a+bx^2}}d}{4b^3}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^3\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{a}}\right.}{2b^3}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{b}{e}}\right.}{2b^3}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^4\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{}}\right.}{4b^3}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{(4bc-9ad)e^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+b}}\right.}{4b^{13/4}}$$

Mathematica [C] time = 0.115709, size = 91, normalized size = 0.41

$$\frac{e^3x^4\sqrt{ex}\left(9a^2d+\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(4bc-9ad)\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\frac{bx^2}{a}\right)\right)}{18a^2b\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*x^4*Sqrt[e*x]*(9*a^2*d + (4*b*c - 9*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(
1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -((b*x^2)/a)]))/(18*a^2*b*(a + b*x^2
)^(5/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{7}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [B] time = 2.46799, size = 2306, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/40*(4*(5*b^2*d*e^3*x^4 - 6*(4*b^2*c - 9*a*b*d)*e^3*x^2 - 5*(4*a*b*c - 9*a
^2*d)*e^3)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^
3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*
d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*arctan(((4*b^11*c - 9*a*b^10*d)*(b*x^2
 + a)^(3/4)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664
*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(3/4)*sqrt(e*x)*e^3 + (b^11*x^2 + a
*b^10)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*
b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(3/4)*sqrt(((16*b^2*c^2 - 72*a*b*c*d + 8
1*a^2*d^2)*sqrt(b*x^2 + a)*e^7*x + (b^7*x^2 + a*b^6)*sqrt((256*b^4*c^4 - 23
04*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e
^14/b^13))/(b*x^2 + a)))/((256*b^5*c^4 - 2304*a*b^4*c^3*d + 7776*a^2*b^3*c^
2*d^2 - 11664*a^3*b^2*c*d^3 + 6561*a^4*b*d^4)*e^14*x^2 + (256*a*b^4*c^4 - 2
304*a^2*b^3*c^3*d + 7776*a^3*b^2*c^2*d^2 - 11664*a^4*b*c*d^3 + 6561*a^5*d^4
)*e^14)) + 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c
^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^
(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 + (b^4*x^2 + a*
b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*
c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)) - 5*(b^5*x^4 + 2*a*b^4
*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 1
1664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4)*(
4*b*c - 9*a*d)*sqrt(e*x)*e^3 - (b^4*x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^3
*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13
)^(1/4))/(b*x^2 + a)))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

3.1127 $\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=149

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*Sqrt[e*x])
/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(
a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[
e]*(a + b*x^2)^(1/4))])/b^(9/4)
```

Rubi [A] time = 0.0872398, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {452, 288, 329, 240, 212, 208, 205}

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```


```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*Sqrt[e*x])
/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(
a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[
e]*(a + b*x^2)^(1/4))])/b^(9/4)
```


Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{d\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{b}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{\left(de^2\right)\int \frac{1}{\sqrt{ex}\sqrt[4]{a+bx^2}}\,dx}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{\left(de^2\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2} + \frac{\left(de^2\right)\text{Subst}\left(\int \frac{}{e+ \cdots}\right.}{b \cdots}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}} + \frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}$$

Mathematica [C] time = 0.0809759, size = 77, normalized size = 0.52

$$\frac{2x(ex)^{3/2}\left(5dx^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\frac{bx^2}{a}\right)+9ac\right)}{45a^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*x*(e*x)^(3/2)*(9*a*c + 5*d*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[9/4, 9/4, 13/4, -((b*x^2)/a)]))/(45*a^2*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [B] time = 2.20728, size = 949, normalized size = 6.37

$$-\frac{4\left(5a^2de-\left(b^2c-6abd\right)ex^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}+20\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}b^7de\left(\frac{d^4e^6}{b^9}\right)^{\frac{3}{4}}-\left(b^8x^2\right.}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/10*(4*(5*a^2*d*e - (b^2*c - 6*a*b*d)*e*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)
+ 20*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*arctan(-((b*
x^2 + a)^(3/4)*sqrt(e*x)*b^7*d*e*(d^4*e^6/b^9)^(3/4) - (b^8*x^2 + a*b^7)*(d
^4*e^6/b^9)^(3/4)*sqrt((sqrt(b*x^2 + a)*d^2*e^3*x + (b^5*x^2 + a*b^4)*sqrt(
d^4*e^6/b^9))/(b*x^2 + a)))/(b*d^4*e^6*x^2 + a*d^4*e^6)) - 5*(a*b^4*x^4 + 2
*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x
)*d*e + (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)) + 5*(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(
e*x)*d*e - (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)))/(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

3.1128 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{ex}(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c + a*d)*Sq
rt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0328255, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 264}

$$\frac{2\sqrt{ex}(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c + a*d)*Sq
rt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx &= \frac{2(bc-ad)\sqrt{ex}}{5abe\left(a+bx^2\right)^{5/4}}+\frac{\left(2\left(2bc+\frac{ad}{2}\right)\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{5ab}\\ &= \frac{2(bc-ad)\sqrt{ex}}{5abe\left(a+bx^2\right)^{5/4}}+\frac{2(4bc+ad)\sqrt{ex}}{5a^2be\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0460489, size = 44, normalized size = 0.56

$$\frac{2x\left(5ac+adx^2+4bcx^2\right)}{5a^2\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(5*a*c + 4*b*c*x^2 + a*d*x^2))/(5*a^2*Sqrt[e*x]*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.005, size = 39, normalized size = 0.5

$$\frac{2x\left(adx^2+4bcx^2+5ac\right)}{5a^2}\left(bx^2+a\right)^{-\frac{5}{4}}\frac{1}{\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] 2/5*x*(a*d*x^2+4*b*c*x^2+5*a*c)/(b*x^2+a)^(5/4)/a^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

Fricas [A] time = 2.00099, size = 136, normalized size = 1.72

$$\frac{2\left((4bc+ad)x^2+5ac\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{5\left(a^2b^2ex^4+2a^3bex^2+a^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c + a*d)*x^2 + 5*a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b^2*e*x^4
+ 2*a^3*b*e*x^2 + a^4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{9}{4}} \sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

3.1129 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\sqrt{ex}(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)*Sqrt[e*x]
)/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*Sqrt[e*x])/(15*a^3*e^
3*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0481076, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{8\sqrt{ex}(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)*Sqrt[e*x]
)/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*Sqrt[e*x])/(15*a^3*e^
3*(a + b*x^2)^(1/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{(8bc-3ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx}{3ae^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{(4(8bc-3ad))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{15a^2e^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{8(8bc-3ad)\sqrt{ex}}{15a^3e^3\sqrt[4]{a+bx^2}}$$

Mathematica [A] time = 0.0347551, size = 65, normalized size = 0.62

$$\frac{x\left(-10a^2\left(c-3dx^2\right)+ab\left(24dx^4-80cx^2\right)-64b^2cx^4\right)}{15a^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (x*(-64*b^2*c*x^4 - 10*a^2*(c - 3*d*x^2) + a*b*(-80*c*x^2 + 24*d*x^4)))/(15
*a^3*(e*x)^(5/2)*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(-12\,abdx^4+32\,b^2cx^4-15\,a^2dx^2+40\,abcx^2+5\,a^2c\right)}{15\,a^3}\left(bx^2+a\right)^{-\frac{5}{4}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/15*x*(-12*a*b*d*x^4+32*b^2*c*x^4-15*a^2*d*x^2+40*a*b*c*x^2+5*a^2*c)/(b*x
^2+a)^(5/4)/a^3/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

Fricas [A] time = 1.82257, size = 204, normalized size = 1.96

$$-\frac{2\left(4\left(8b^{2}c-3abd\right)x^{4}+5a^{2}c+5\left(8abc-3a^{2}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{15\left(a^{3}b^{2}e^{3}x^{6}+2a^{4}be^{3}x^{4}+a^{5}e^{3}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/15*(4*(8*b^2*c - 3*a*b*d)*x^4 + 5*a^2*c + 5*(8*a*b*c - 3*a^2*d)*x^2)*(b*
x^2 + a)^(3/4)*sqrt(e*x)/(a^3*b^2*e^3*x^6 + 2*a^4*b*e^3*x^4 + a^5*e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

3.1130 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d))/(35*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a*d))/(35*a^3*e^3*(e*
x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(105*
a^4*e^3*(e*x)^(3/2))
```

Rubi [A] time = 0.06919, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d))/(35*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a*d))/(35*a^3*e^3*(e*
x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(105*
a^4*e^3*(e*x)^(3/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}dx = -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{(12bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}dx}{7ae^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{(8(12bc-7ad))\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}dx}{35a^2e^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{(32(12bc}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} + \frac{64(12bc}{10}$$

Mathematica [A] time = 0.0483768, size = 94, normalized size = 0.67

$$\frac{\sqrt{ex}\left(40a^2bx^2\left(3c-14dx^2\right)-10a^3\left(3c+7dx^2\right)+64ab^2x^4\left(15c-7dx^2\right)+768b^3cx^6\right)}{105a^4e^5x^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (Sqrt[e*x]*(768*b^3*c*x^6 + 40*a^2*b*x^2*(3*c - 14*d*x^2) + 64*a*b^2*x^4*(1
5*c - 7*d*x^2) - 10*a^3*(3*c + 7*d*x^2)))/(105*a^4*e^5*x^4*(a + b*x^2)^(5/4
))
```

Maple [A] time = 0.007, size = 86, normalized size = 0.6

$$-\frac{2x\left(224ab^2dx^6-384b^3cx^6+280a^2bdx^4-480ab^2cx^4+35a^3dx^2-60a^2bcx^2+15ca^3\right)}{105a^4}\left(bx^2+a\right)^{-\frac{5}{4}}(ex)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/105*x*(224*a*b^2*d*x^6-384*b^3*c*x^6+280*a^2*b*d*x^4-480*a*b^2*c*x^4+35*
a^3*d*x^2-60*a^2*b*c*x^2+15*a^3*c)/(b*x^2+a)^(5/4)/a^4/(e*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

Fricas [A] time = 1.58664, size = 261, normalized size = 1.85

$$\frac{2\left(32\left(12\,b^3c-7\,ab^2d\right)x^6+40\left(12\,ab^2c-7\,a^2bd\right)x^4-15\,a^3c+5\left(12\,a^2bc-7\,a^3d\right)x^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{105\left(a^4b^2e^5x^8+2\,a^5be^5x^6+a^6e^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/105*(32*(12*b^3*c - 7*a*b^2*d)*x^6 + 40*(12*a*b^2*c - 7*a^2*b*d)*x^4 - 15
*a^3*c + 5*(12*a^2*b*c - 7*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^4*b^2
*e^5*x^8 + 2*a^5*b*e^5*x^6 + a^6*e^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

3.1131 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}} dx$

Optimal. Leaf size=178

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*a*d))/(55*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c - 11*a*d))/(55*a^3*e^3
*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c - 11*a*d)*(a + b*x^2)^(3/4))/
(55*a^4*e^3*(e*x)^(7/2)) - (256*(16*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a
^5*e^3*(e*x)^(7/2))
```

Rubi [A] time = 0.0903243, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*a*d))/(55*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c - 11*a*d))/(55*a^3*e^3
*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c - 11*a*d)*(a + b*x^2)^(3/4))/
(55*a^4*e^3*(e*x)^(7/2)) - (256*(16*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a
^5*e^3*(e*x)^(7/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{(16bc-11ad)\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{(12(16bc-11ad))\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,d}{55a^2e^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{(96(16bc}{}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{64(16bc-}{55}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{64(16bc-}{55}$$

Mathematica [A] time = 0.054169, size = 79, normalized size = 0.44

$$\frac{2x\left(ax^2\left(20a^2bx^2-5a^3+160ab^2x^4+128b^3x^6\right)(11ad-16bc)-35a^5c\right)}{385a^6(ex)^{13/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-35*a^5*c + a*(-16*b*c + 11*a*d)*x^2*(-5*a^3 + 20*a^2*b*x^2 + 160*a*b
^2*x^4 + 128*b^3*x^6)))/(385*a^6*(e*x)^(13/2)*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.006, size = 110, normalized size = 0.6

$$-\frac{2x\left(-1408\,ab^3dx^8+2048\,b^4cx^8-1760\,a^2b^2dx^6+2560\,ab^3cx^6-220\,a^3bdx^4+320\,a^2b^2cx^4+55\,a^4dx^2-80\,a^3bcx^2+35\right.}{385\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/385*x*(-1408*a*b^3*d*x^8+2048*b^4*c*x^8-1760*a^2*b^2*d*x^6+2560*a*b^3*c*
x^6-220*a^3*b*d*x^4+320*a^2*b^2*c*x^4+55*a^4*d*x^2-80*a^3*b*c*x^2+35*a^4*c)
/(b*x^2+a)^(5/4)/a^5/(e*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

Fricas [A] time = 1.68508, size = 323, normalized size = 1.81

$$-\frac{2\left(128\left(16\,b^{4}c-11\,ab^{3}d\right)x^{8}+160\left(16\,ab^{3}c-11\,a^{2}b^{2}d\right)x^{6}+35\,a^{4}c+20\left(16\,a^{2}b^{2}c-11\,a^{3}bd\right)x^{4}-5\left(16\,a^{3}bc-11\,a^{4}d\right)x}{385\left(a^{5}b^{2}e^{7}x^{10}+2\,a^{6}be^{7}x^{8}+a^{7}e^{7}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/385*(128*(16*b^4*c - 11*a*b^3*d)*x^8 + 160*(16*a*b^3*c - 11*a^2*b^2*d)*x
^6 + 35*a^4*c + 20*(16*a^2*b^2*c - 11*a^3*b*d)*x^4 - 5*(16*a^3*b*c - 11*a^4
*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^5*b^2*e^7*x^10 + 2*a^6*b*e^7*x^8 +
a^7*e^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{9}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

3.1132 $\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=230

$$-\frac{77a^{3/2}e^6\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}+\frac{11e^3(ex)^{7/2}(2bc-3ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{77ae^5(ex)^{3/2}(2bc-3ad)}{60b^4\sqrt[4]{a+bx^2}}-\frac{e(ex)^{11/2}(2b}{5ab^2\sqrt[4]{a}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (77*a*(2*b*c - 3
*a*d)*e^5*(e*x)^(3/2))/(60*b^4*(a + b*x^2)^(1/4)) + (11*(2*b*c - 3*a*d)*e^3
*(e*x)^(7/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*e*(e*x)^(11/2))
/(5*a*b^2*(a + b*x^2)^(1/4)) - (77*a^(3/2)*(2*b*c - 3*a*d)*e^6*(1 + a/(b*x^
2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(9/2
)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.127523, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 285, 284, 335, 196}

$$-\frac{77a^{3/2}e^6\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}+\frac{11e^3(ex)^{7/2}(2bc-3ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{77ae^5(ex)^{3/2}(2bc-3ad)}{60b^4\sqrt[4]{a+bx^2}}-\frac{e(ex)^{11/2}(2b}{5ab^2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```


```
[Out] (2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (77*a*(2*b*c - 3
*a*d)*e^5*(e*x)^(3/2))/(60*b^4*(a + b*x^2)^(1/4)) + (11*(2*b*c - 3*a*d)*e^3
*(e*x)^(7/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*e*(e*x)^(11/2))
/(5*a*b^2*(a + b*x^2)^(1/4)) - (77*a^(3/2)*(2*b*c - 3*a*d)*e^6*(1 + a/(b*x^
2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(9/2
)*(a + b*x^2)^(1/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
```

```
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-5bc+\frac{15ad}{2}\right)\right)\int \frac{(ex)^{13/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}} + \frac{\left(11(2bc-3ad)e^2\right)\int \frac{(ex)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{10b^2}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{\left(77a(2bc-3ad)e^4\right)\int -}{60b^3}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.174434, size = 140, normalized size = 0.61

$$\frac{e^5(ex)^{3/2}\left(-110a^2b\left(7c-3dx^2\right)+1155a^3d-20ab^2x^2\left(11c+3dx^2\right)-385a\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(3ad-2bc)\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\right.\right.}{120b^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^5*(e*x)^(3/2)*(1155*a^3*d - 110*a^2*b*(7*c - 3*d*x^2) + 8*b^3*x^4*(5*c +
 3*d*x^2) - 20*a*b^2*x^2*(11*c + 3*d*x^2) - 385*a*(-2*b*c + 3*a*d)*(a + b*x
^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/
(120*b^4*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c \right) \left(e x \right)^{\frac{13}{2}} \left(b \,x^{2}+a \right)^{-\frac{9}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right)\left(e x\right)^{\frac{13}{2}}}{\left(b x^{2}+a\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\left(d e^{6} x^{8}+c e^{6} x^{6}\right)\left(b x^{2}+a\right)^{\frac{3}{4}} \sqrt{e x}}{b^{3} x^{6}+3 \, a b^{2} x^{4}+3 \, a^{2} b x^{2}+a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^6*x^8 + c*e^6*x^6)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*a
*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(13/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{13}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

3.1133 $\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=192

$$\frac{7e^3(ex)^{3/2}(6bc-11ad)}{30b^3\sqrt[4]{a+bx^2}}+\frac{7\sqrt{a}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{7/2}(6bc-11ad)}{15ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{11/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (7*(6*b*c - 11*a
*d)*e^3*(e*x)^(3/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((6*b*c - 11*a*d)*e*(e*x)
^(7/2))/(15*a*b^2*(a + b*x^2)^(1/4)) + (7*Sqrt[a]*(6*b*c - 11*a*d)*e^4*(1 +
 a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(1
0*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.106291, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 285, 284, 335, 196}

$$\frac{7e^3(ex)^{3/2}(6bc-11ad)}{30b^3\sqrt[4]{a+bx^2}}+\frac{7\sqrt{a}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{7/2}(6bc-11ad)}{15ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{11/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (7*(6*b*c - 11*a
*d)*e^3*(e*x)^(3/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((6*b*c - 11*a*d)*e*(e*x)
^(7/2))/(15*a*b^2*(a + b*x^2)^(1/4)) + (7*Sqrt[a]*(6*b*c - 11*a*d)*e^4*(1 +
 a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(1
0*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
```

```
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-3bc+\frac{11ad}{2}\right)\right)\int \frac{(ex)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{\left(7(6bc-11ad)e^2\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{30b^2}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} - \frac{\left(7a(6bc-11ad)e^4\right)\int \frac{}{(a}}{20b^3}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} - \frac{\left(7a(6bc-11ad)e^4\sqrt[4]{1+}\right.}{20b^4\sqrt[4]{}}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{\left(7a(6bc-11ad)e^4\sqrt[4]{1+}\right.}{}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{7\sqrt{a}(6bc-11ad)e^4\sqrt[4]{1+}}{10b^{7/}}$$

Mathematica [C] time = 0.148515, size = 116, normalized size = 0.6

$$\frac{e^3(ex)^{3/2}\left(-77a^2d+7\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(11ad-6bc)\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+ab\left(42c-22dx^2\right)+4b^2x^2\left(3c+dx^2\right)\right)}{12b^3\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(-77*a^2*d + a*b*(42*c - 22*d*x^2) + 4*b^2*x^2*(3*c + d*x^
2) + 7*(-6*b*c + 11*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F
1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(12*b^3*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \left(e x\right)^{\frac{9}{2}} \left(b \,x^{2}+a\right)^{-\frac{9}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{9}{2}}}{\left(b x^{2}+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d e^{4} x^{6} + c e^{4} x^{4}\right) \left(b x^{2} + a\right)^{\frac{3}{4}} \sqrt{e x}}{b^{3} x^{6} + 3 \, a b^{2} x^{4} + 3 \, a^{2} b x^{2} + a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^4*x^6 + c*e^4*x^4)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*a
*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{9}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

3.1134 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=155

$$-\frac{3e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{3/2}(2bc-7ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c - 7*a*d)*
e*(e*x)^(3/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (3*(2*b*c - 7*a*d)*e^2*(1 + a/
(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sq
rt[a]*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0765485, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 285, 284, 335, 196}

$$-\frac{3e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{3/2}(2bc-7ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```


```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c - 7*a*d)*
e*(e*x)^(3/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (3*(2*b*c - 7*a*d)*e^2*(1 + a/
(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sq
rt[a]*b^(5/2)*(a + b*x^2)^(1/4))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-bc+\frac{7ad}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}} + \frac{\left(3(2bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{10b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}} + \frac{\left(3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{10b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{\left(3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\right.}{10b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.128221, size = 98, normalized size = 0.63

$$\frac{e(ex)^{3/2}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(2bc-7ad)\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a\left(7ad-2bc+2bdx^2\right)\right)}{2ab^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(a*(-2*b*c + 7*a*d + 2*b*d*x^2) + (2*b*c - 7*a*d)*(a + b*x^2
)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(2
*a*b^2*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(de^2x^4+ce^2x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^2*x^4 + c*e^2*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*a
*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

3.1135 $\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}} dx$

Optimal. Leaf size=114

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b*c + 3*a*d
)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.0515974, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 284, 335, 196}

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b*c + 3*a*d
)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{3/2}}{5abe\left(a+bx^2\right)^{5/4}}+\frac{\left(2\left(bc+\frac{3ad}{2}\right)\right)\int\frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$=\frac{2(bc-ad)(ex)^{3/2}}{5abe\left(a+bx^2\right)^{5/4}}+\frac{\left(2\left(bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int\frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5ab^2\sqrt[4]{a+bx^2}}$$

$$=\frac{2(bc-ad)(ex)^{3/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{\left(2\left(bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{5ab^2\sqrt[4]{a+bx^2}}$$

$$=\frac{2(bc-ad)(ex)^{3/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{2(2bc+3ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.10117, size = 86, normalized size = 0.75

$$\frac{x\sqrt{ex}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(3ad+2bc)\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-3a^2d\right)}{3a^2b\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (x*Sqrt[e*x]*(-3*a^2*d + (2*b*c + 3*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*
Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(5/4)
)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*x^6 + 3*a*b^2*x^4 + 3
*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

3.1136 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=142

$$-\frac{2(ex)^{3/2}(6bc-ad)}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - (2*(6*b*c - a*d)*(e*x)^(3/2))/(5
*a^2*e^3*(a + b*x^2)^(5/4)) + (4*(6*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e
*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*Sqrt[b]*e^2*(a
+ b*x^2)^(1/4))
```

Rubi [A] time = 0.0730283, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 290, 284, 335, 196}

$$-\frac{2(ex)^{3/2}(6bc-ad)}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```


```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - (2*(6*b*c - a*d)*(e*x)^(3/2))/(5
*a^2*e^3*(a + b*x^2)^(5/4)) + (4*(6*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e
*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*Sqrt[b]*e^2*(a
+ b*x^2)^(1/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx &= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{(6bc-ad)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{9/4}}\,dx}{ae^2}\\
&= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{(2(6bc-ad))\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{5a^2e^2}\\
&= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{\left(2(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5a^2be^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{\left(2(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/}}\right.}{5a^2be^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{4(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.0416491, size = 85, normalized size = 0.6

$$\frac{2x\left(x^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(ad-6bc)\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-3a^2c\right)}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-3*a^2*c + (-6*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyper
geometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2)^
(5/4))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{3}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^3e^2x^8+3\,ab^2e^2x^6+3\,a^2be^2x^4+a^3e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^2*x^8 + 3*a*b^2*e^2
*x^6 + 3*a^2*b*e^2*x^4 + a^3*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

3.1137 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=181

$$\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{24\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right.}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - (2*(2*b*c - a*d))/(5*a^2*e^3
*Sqrt[e*x]*(a + b*x^2)^(5/4)) + (12*(2*b*c - a*d))/(5*a^3*e^3*Sqrt[e*x]*(a
+ b*x^2)^(1/4)) - (24*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]
*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(7/2)*e^4*(a + b*x^2)^(1
/4))
```

Rubi [A] time = 0.0949316, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 286, 284, 335, 196}

$$\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{24\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```


```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - (2*(2*b*c - a*d))/(5*a^2*e^3
*Sqrt[e*x]*(a + b*x^2)^(5/4)) + (12*(2*b*c - a*d))/(5*a^3*e^3*Sqrt[e*x]*(a
+ b*x^2)^(1/4)) - (24*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]
*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(7/2)*e^4*(a + b*x^2)^(1
/4))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
```

```
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{(2bc-ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} - \frac{(6(2bc-ad))\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{5a^2e^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{(12b(2bc-ad))\int \ldots}{5a^3e^4}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{\left(12(2bc-ad)\sqrt[4]{1+\ldots}\right.}{5a^3e\ldots}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left(12(2bc-ad)\sqrt[4]{1+\ldots}\right.}{\ldots}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{24\sqrt{b}(2bc-ad)\sqrt[4]{1\ldots}}{5a\ldots}$$

Mathematica [C] time = 0.0434594, size = 86, normalized size = 0.48

$$\frac{2x\left(a^2(-c)-5x^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(ad-2bc)\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-(a^2*c) - 5*(-2*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyp
ergeometric2F1[-1/4, 9/4, 3/4, -((b*x^2)/a)]))/(5*a^3*(e*x)^(7/2)*(a + b*x^
2)^(5/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{7}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)\sqrt{ex}}{b^3e^4x^{10}+3\,ab^2e^4x^8+3\,a^2be^4x^6+a^3e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^4*x^10 + 3*a*b^2*e^
4*x^8 + 3*a^2*b*e^4*x^6 + a^3*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{9}{4}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

3.1138 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=219

$$\frac{16b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(14bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2(14bc}{45a^2e^3(ex)^{5/2}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - (2*(14*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (4*(14*b*c - 9*a*d))/(45*a^3*e^3*(e*x
)^(5/2)*(a + b*x^2)^(1/4)) - (8*b*(14*b*c - 9*a*d))/(15*a^4*e^5*Sqrt[e*x]*(
a + b*x^2)^(1/4)) + (16*b^(3/2)*(14*b*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt
[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(9/2)*e^6*(a + b*x
^2)^(1/4))
```

Rubi [A] time = 0.12334, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 286, 284, 335, 196}

$$\frac{16b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(14bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2(14bc}{45a^2e^3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - (2*(14*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (4*(14*b*c - 9*a*d))/(45*a^3*e^3*(e*x
)^(5/2)*(a + b*x^2)^(1/4)) - (8*b*(14*b*c - 9*a*d))/(15*a^4*e^5*Sqrt[e*x]*(
a + b*x^2)^(1/4)) + (16*b^(3/2)*(14*b*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt
[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(9/2)*e^6*(a + b*x
^2)^(1/4))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
```

```
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{(14bc-9ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx}{9ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{(2(14bc-9ad))\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx}{9a^2e^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{(4b(14bc-}{}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b(14bc}{15a^4e^5\sqrt{ex}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b(14bc}{15a^4e^5\sqrt{ex}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b(14bc}{15a^4e^5\sqrt{ex}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b(14bc}{15a^4e^5\sqrt{ex}}$$

Mathematica [C] time = 0.0430659, size = 87, normalized size = 0.4

$$\frac{2x\left(-5a^2c-x^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(9ad-14bc)\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)\right)}{45a^3(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-5*a^2*c - (-14*b*c + 9*a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hy
pergeometric2F1[-5/4, 9/4, -1/4, -((b*x^2)/a)]))/(45*a^3*(e*x)^(11/2)*(a +
b*x^2)^(5/4))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \left(e x\right)^{-\frac{11}{2}} \left(b x^{2}+a\right)^{-\frac{9}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2} + c}{{\left(b x^{2} + a\right)}^{\frac{9}{4}} \left(e x\right)^{\frac{11}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + a\right)}^{\frac{3}{4}} {\left(d x^{2} + c\right)} \sqrt{e x}}{b^{3} e^{6} x^{12} + 3 \, a b^{2} e^{6} x^{10} + 3 \, a^{2} b e^{6} x^{8} + a^{3} e^{6} x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^6*x^12 + 3*a*b^2*e^
6*x^10 + 3*a^2*b*e^6*x^8 + a^3*e^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{9}{4}} (ex)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

3.1139 $\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=101

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 +
 m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q)
```

Rubi [A] time = 0.0790094, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 +
 m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int (ex)^m\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int (ex)^m\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx\\ &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int (ex)^m\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,d\\ &= \frac{(ex)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1+m}{2};-p,-q;\frac{3+m}{2};-\frac{bx^2}{a},-\right.}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.113158, size = 97, normalized size = 0.96

$$\frac{x(ex)^m \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 + m)/
2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^
q)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

3.1140 $\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=84

$$\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -
((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0693624, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -
((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx\\
&= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx\\
&= \frac{1}{5}x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)
\end{aligned}$$

Mathematica [A] time = 0.0582581, size = 86, normalized size = 1.02

$$\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -
((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^4 \left(b x^2+a\right)^p \left(d x^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2+a\right)^p \left(d x^2+c\right)^q x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^2+a\right)^p \left(d x^2+c\right)^q x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

3.1141 $\int x^2 (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0700929, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{1}{3}x^3 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 (a+bx^2)^p (c+dx^2)^q \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^2 \left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q \, dx \\
&= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int x^2 \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q \, dx \\
&= \frac{1}{3}x^3 (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)
\end{aligned}$$

Mathematica [A] time = 0.0482629, size = 86, normalized size = 1.02

$$\frac{1}{3}x^3 (a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

3.1142 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0421726, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^p\left(c+dx^2\right)^q \, dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q \, dx\\
&= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q \, dx\\
&= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)
\end{aligned}$$

Mathematica [B] time = 0.0698361, size = 172, normalized size = 2.18

$$\frac{3acx\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a
), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)
/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p \left(dx^2 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1143 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} dx$

Optimal. Leaf size=82

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

Rubi [A] time = 0.0686437, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2} dx \\
&= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^2} dx \\
&= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.054382, size = 84, normalized size = 1.02

$$-\frac{\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(x*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

3.1144 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

```
[Out] -((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(3*x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0696001, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

```
[Out] -((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(3*x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p(c+dx^2)^q}{x^4}\,dx\\
&= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{x^4}\,dx\\
&= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0867701, size = 86, normalized size = 1.02

$$-\frac{\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

```
[Out] -((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(3*x^3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

3.1145 $\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=242

$$\frac{\left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^q \left(a^2d^2\left(q^2+3q+2\right)+2abcd(p+1)(q+1)+b^2c^2\left(p^2+3p+2\right)\right) \left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+\right.}{2b^3d^2(p+1)(p+q+2)(p+q+3)}$$

```
[Out] -((b*c*(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b
^2*d^2*(2 + p + q)*(3 + p + q)) + (x^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 +
 q))/(2*b*d*(3 + p + q)) + ((b^2*c^2*(2 + 3*p + p^2) + 2*a*b*c*d*(1 + p)*(1
 + q) + a^2*d^2*(2 + 3*q + q^2))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeo
metric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^3*d^2*(1
+ p)*(2 + p + q)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.310226, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 90, 80, 70, 69}

$$\frac{\left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^q \left(a^2d^2\left(q^2+3q+2\right)+2abcd(p+1)(q+1)+b^2c^2\left(p^2+3p+2\right)\right) \left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+\right.}{2b^3d^2(p+1)(p+q+2)(p+q+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] -((b*c*(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b
^2*d^2*(2 + p + q)*(3 + p + q)) + (x^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 +
 q))/(2*b*d*(3 + p + q)) + ((b^2*c^2*(2 + 3*p + p^2) + 2*a*b*c*d*(1 + p)*(1
 + q) + a^2*d^2*(2 + 3*q + q^2))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeo
metric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^3*d^2*(1
+ p)*(2 + p + q)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \frac{1}{2} \text{Subst}\left(\int x^2(a+bx)^p(c+dx)^q \, dx, x, x^2\right)$$
$$= \frac{x^2 \left(a+bx^2\right)^{1+p} \left(c+dx^2\right)^{1+q}}{2bd(3+p+q)} + \frac{\text{Subst}\left(\int (a+bx)^p(c+dx)^q(-ac-(bc(2+p)+ad(2+q)\right.}{2bd(3+p+q)}$$
$$= -\frac{(bc(2+p)+ad(2+q))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(3+p+q)} + \frac{(b^2}{}$$
$$= -\frac{(bc(2+p)+ad(2+q))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(3+p+q)} + \frac{((b}{}$$
$$= -\frac{(bc(2+p)+ad(2+q))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(3+p+q)} + \frac{(b^2}{}$$

Mathematica [A] time = 0.270206, size = 195, normalized size = 0.81

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{\left(a^2d^2\left(q^2+3q+2\right)+2abcd(p+1)(q+1)+b^2c^2\left(p^2+3p+2\right)\right)\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{b^2d(p+1)(p+q+2)} - \frac{\left(c+dx^2\right)(ad(q+2)+bc(p+2))}{bd(p+q+2)} + \right.}{2bd(p+q+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(-(((b*c*(2 + p) + a*d*(2 + q))*(c + d*x
^2))/(b*d*(2 + p + q))) + x^2*(c + d*x^2) + ((b^2*c^2*(2 + 3*p + p^2) + 2*a
*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*Hypergeometric2F1[1 + p,
-q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/(b^2*d*(1 + p)*(2 + p + q)*((b*
(c + d*x^2))/(b*c - a*d))^q)))/(2*b*d*(3 + p + q))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int x^5 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p (dx^2 + c)^q x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p (dx^2 + c)^q x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p (dx^2 + c)^q x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

3.1146 $\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=146

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+2)}-\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q(ad(q+1)+bc(p+1))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^2d(p+1)(p+q+2)}$$

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b*d*(2 + p + q)) - ((b*c*(1 +
p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p
, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.123154, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 70, 69}

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+2)}-\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q(ad(q+1)+bc(p+1))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^2d(p+1)(p+q+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b*d*(2 + p + q)) - ((b*c*(1 +
p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p
, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p(c+dx)^q\,dx,x,x^2\right)$$
$$=\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(2+p+q)}-\frac{(bc(1+p)+ad(1+q))\,\text{Subst}\left(\int(a+bx)^p(c+dx)^q\,dx,x,\right.}{2bd(2+p+q)}$$
$$=\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(2+p+q)}-\frac{\left((bc(1+p)+ad(1+q))\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\right.}{2bd(2+p+q)}$$
$$=\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2bd(2+p+q)}-\frac{(bc(1+p)+ad(1+q))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)}{2b^2d(1+p)(2+p+q)}$$

Mathematica [A] time = 0.0915058, size = 118, normalized size = 0.81

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(b\left(c+dx^2\right)-\frac{(ad(q+1)+bc(p+1))\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{p+1}\right)}{2b^2d(p+q+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(b*(c + d*x^2) - ((b*c*(1 + p) + a*d*(1
+ q))*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/
((1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)))/(2*b^2*d*(2 + p + q))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int x^3\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p\left(dx^2+c\right)^qx^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^qx^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

3.1147 $\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx$

Optimal. Leaf size=85

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d\left(bx^2+a\right)}{bc-ad}\right)}{2b(p+1)}$$

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d
*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.062654, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {444, 70, 69}

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d\left(bx^2+a\right)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d
*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^p(c+dx)^q\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\right)\operatorname{Subst}\left(\int(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q\,dx,x,x^2\right)$$
$$=\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+bx^2\right)}{bc-ad}\right)}{2b(1+p)}$$

Mathematica [A] time = 0.0216457, size = 84, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(
a + b*x^2))/(-(b*c) + a*d)])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(bx^2+a\right)^p\left(dx^2+c\right)^q x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p \left(dx^2 + c\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

3.1148 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x}\,dx$

Optimal. Leaf size=97

$$-\frac{(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

```
[Out] -((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
 b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a*(1 + p)*((b*(c + d*x^2))/(b*c -
 a*d))^q)
```

Rubi [A] time = 0.0934395, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$-\frac{(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] -((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
 b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a*(1 + p)*((b*(c + d*x^2))/(b*c -
 a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^p (c+dx)^q}{x} dx, x, x^2\right)$$
$$= \frac{1}{2}\left((c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\right) \text{Subst}\left(\int \frac{(a+bx)^p \left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x} dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,1;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a(1+p)}$$

Mathematica [A] time = 0.0712278, size = 95, normalized size = 0.98

$$\frac{\left(\frac{a}{bx^2}+1\right)^{-p} (a+bx^2)^p \left(\frac{c}{dx^2}+1\right)^{-q} (c+dx^2)^q F_1\left(-p-q;-p,-q;-p-q+1;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(p+q)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^2
)), -(c/(d*x^2))])/(2*(p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p (dx^2 + c)^q}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

3.1149 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^3}\,dx$

Optimal. Leaf size=98

$$\frac{b\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

```
[Out] (b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
+ b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*((b*(c + d*x^2))/(b*
c - a*d))^q)
```

Rubi [A] time = 0.0884946, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$\frac{b\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
+ b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*((b*(c + d*x^2))/(b*
c - a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p\left(c+dx^2\right)^q}{x^3}\,dx=\frac{1}{2}\,\mathrm{Subst}\left(\int\frac{(a+bx)^p(c+dx)^q}{x^2}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\left(\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\right)\mathrm{Subst}\left(\int\frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx,x,x^2\right)$$

$$=\frac{b\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+bx^2\right)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a^2(1+p)}$$

Mathematica [A] time = 0.0702007, size = 100, normalized size = 1.02

$$\frac{\left(\frac{a}{bx^2}+1\right)^{-p}\left(a+bx^2\right)^p\left(\frac{c}{dx^2}+1\right)^{-q}\left(c+dx^2\right)^qF_1\left(-p-q+1;-p,-q;-p-q+2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2x^2(p+q-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/(b*
x^2)), -(c/(d*x^2))])/(2*(-1 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q*x
^2)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

3.1150 $\int \frac{\left(a+bx^2\right)^p\left(c+dx^2\right)^q}{x^5}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(bx^2+a\right)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

```
[Out] -(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*
(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^3*(1 + p)*((b*(c + d*x^2))/
(b*c - a*d))^q)
```

Rubi [A] time = 0.0891192, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$-\frac{b^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(bx^2+a\right)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

```
[Out] -(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*
(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^3*(1 + p)*((b*(c + d*x^2))/
(b*c - a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} dx = \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^p (c+dx)^q}{x^3} dx, x, x^2\right)$$
$$= \frac{1}{2}\left((c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\right) \text{Subst}\left(\int \frac{(a+bx)^p \left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3} dx, x, x^2\right)$$
$$= -\frac{b^2 (a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(1+p; -q, 3; 2+p; -\frac{d(a+bx^2)}{bc-ad}, \frac{a+bx^2}{a}\right)}{2a^3(1+p)}$$

Mathematica [A] time = 0.109734, size = 100, normalized size = 1.

$$\frac{\left(\frac{a}{bx^2}+1\right)^{-p} (a+bx^2)^p \left(\frac{c}{dx^2}+1\right)^{-q} (c+dx^2)^q F_1\left(-p-q+2; -p, -q; -p-q+3; -\frac{a}{bx^2}, -\frac{c}{dx^2}\right)}{2x^4(p+q-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[2 - p - q, -p, -q, 3 - p - q, -(a/(b*
x^2)), -(c/(d*x^2))])/(2*(-2 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q*x
^4)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p(dx^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

3.1151 $\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

```
[Out] (2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -((b
*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0781048, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -((b
*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int (ex)^{5/2} \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx \\
&= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int (ex)^{5/2} \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q \\
&= \frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}
\end{aligned}$$

Mathematica [A] time = 0.0707725, size = 91, normalized size = 1.

$$\frac{2}{7}x(ex)^{5/2} \left(a+bx^2\right)^p \left(\frac{a+bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*x*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -(
(b*x^2)/a), -((d*x^2)/c)])/(7*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex}\left(bx^2+a\right)^p \left(dx^2+c\right)^q e^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1152 $\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{5/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

```
[Out] (2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((b*
x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0747216, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{2(ex)^{5/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((b*
x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int (ex)^{3/2} \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx\\
&= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int (ex)^{3/2} \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q \, dx\\
&= \frac{2(ex)^{5/2} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}
\end{aligned}$$

Mathematica [A] time = 0.0550889, size = 91, normalized size = 1.

$$\frac{2}{5}x(ex)^{3/2} \left(a+bx^2\right)^p \left(\frac{a+bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*x*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((
b*x^2)/a), -((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2 + a\right)^p \left(dx^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2 + a\right)^p \left(dx^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}\left(bx^2 + a\right)^p \left(dx^2 + c\right)^q ex, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1153 $\int \sqrt{ex} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{3/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

```
[Out] (2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0716873, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{2(ex)^{3/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{ex} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \sqrt{ex} \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx\\
&= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \sqrt{ex} \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q \, dx\\
&= \frac{2(ex)^{3/2} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.0516127, size = 91, normalized size = 1.

$$\frac{2}{3} x\sqrt{ex} \left(a+bx^2\right)^p \left(\frac{a+bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*x*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1154 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{ex}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

```
[Out] (2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^
2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0720401, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{2\sqrt{ex}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^
2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p(c+dx^2)^q}{\sqrt{ex}}\,dx\\
&= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{\sqrt{ex}}\,dx\\
&= \frac{2\sqrt{ex}\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.0495254, size = 89, normalized size = 1.

$$\frac{2x\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^2)/a), -
((d*x^2)/c)])/(Sqrt[e*x]*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q\frac{1}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

3.1155 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a), -
((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0762054, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a), -
((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^p\left(c+dx^2\right)^q}{(ex)^{3/2}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q}{(ex)^{3/2}}\,dx\\
&= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{(ex)^{3/2}}\,dx\\
&= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}
\end{aligned}$$

Mathematica [A] time = 0.0572155, size = 89, normalized size = 1.

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a),
 -((d*x^2)/c)])/((e*x)^(3/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^q(ex)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p \left(dx^2 + c\right)^q}{\left(ex\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

3.1156 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a), -
((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.076363, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a), -
((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^p\left(c+dx^2\right)^q}{(ex)^{5/2}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q}{(ex)^{5/2}}\,dx\\
&= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{(ex)^{5/2}}\,dx\\
&= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0579623, size = 91, normalized size = 1.

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a),
 -((d*x^2)/c)])/(3*(e*x)^(5/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q(ex)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p\left(dx^2+c\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```